4
Monte Carlo Study

This section is exclusively to report the results of a simulation study to verify the behavior of the proposed tests (LM type test of linearity, no error autocorrelation and no remaining nonlinearity). In addition, a simulation experiment will be carried out in order to check the ability of an information criteria (AIC and BIC) to select the correct number of amplitudes in the Fourier form expressed in model (5). Two models were simulated with different specifications. The first 100 observations were discarded to avoid any initialization effects.

Model 1:

$$y_t = \phi_{1s} + \phi_{2s} y_{t-1} + \varepsilon_t$$

where $\phi_{1s} = 3.0 - 0.30 + 0.2 + 0.15 - 0.6 - 0.8 - 0.25 + 0.80 + 0.05 - 0.35 + 0.10$ and

$$\begin{aligned}\phi_{2s} &= 0.05 - 0.15 * \sin(2\pi(dt/48)) - 0.20 * \cos(2\pi(dt/48)) + \\ &+ 0.10 * \sin(4\pi(dt/48)) + 0.25 * \cos(4\pi(dt/48)) - \\ &- 0.08 * \sin(6\pi(dt/48)) + 0.11 * \cos(6\pi(dt/48)) + \\ &+ 0.15 * \sin(8\pi(dt/48)) - 0.06 * \cos(8\pi(dt/48)) - \\ &- 0.35 * \sin(10\pi(dt/48)) + 0.26 * \cos(10\pi(dt/48)) \quad and \quad \varepsilon_t \sim NID(0, 0.25^2)\end{aligned}$$

Model 2:

$$y_t = \phi_{11s} + \phi_{12s} y_{t-1} + (\phi_{21s} + \phi_{22s} y_{t-1}) \times F(20(y_{t-1} - 0.25)) + \varepsilon_t$$

where $\phi_{11s} = 3.0 - 0.30 + 0.2 + 0.15 - 0.6 - 0.8 - 0.25 + 0.80 + 0.05 - 0.35 + 0.10$,

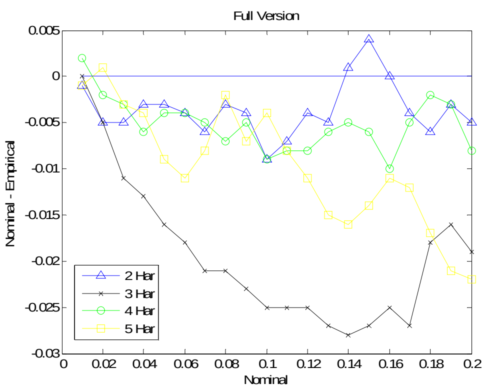
$$\phi_{21s} = 2.3 + 0.50 - 0.1 + 0.35 - 0.4 - 0.2 + 0.45 + 0.60 - 0.15 + 0.35 + 0.20,$$

$$\begin{aligned}\phi_{12s} &= 0.75 - 0.65 * \sin(2\pi(dt/48)) - 0.70 * \cos(2\pi(dt/48)) + \\ &+ 0.80 * \sin(4\pi(dt/48)) + 0.25 * \cos(4\pi(dt/48)) - \\ &- 0.58 * \sin(6\pi(dt/48)) + 0.41 * \cos(6\pi(dt/48)) + \\ &+ 0.05 * \sin(8\pi(dt/48)) - 0.16 * \cos(8\pi(dt/48)) - \\ &- 0.35 * \sin(10\pi(dt/48)) + 0.20 * \cos(10\pi(dt/48))\end{aligned},$$

$$\phi_{22s} = -0.80 + 0.55 * \sin(2\pi(dt/48)) - 0.45 * \cos(2\pi(dt/48)) -$$
$$+ 0.60 * \sin(4\pi(dt/48)) + 0.35 * \cos(4\pi(dt/48)) +$$
$$+ 0.25 * \sin(6\pi(dt/48)) - 0.15 * \cos(6\pi(dt/48)) +$$
$$+ 0.33 * \sin(8\pi(dt/48)) - 0.25 * \cos(8\pi(dt/48)) +$$
$$+ 0.35 * \sin(10\pi(dt/48)) - 0.10 * \cos(10\pi(dt/48)) \quad and \quad \varepsilon_t \sim NID(0, 0.35^2)$$

Model 1 is a stationary linear periodical autoregressive model and is used only to verify the empirical size of the linearity test. Model 2 is one specification of the STPAR model. Several harmonics were used instead of just 5. To test 4 until 2 harmonics, we simply removed the part of the model related to those amplitudes. The size of the sample tested was 500. It was also tested the full and economic version of the test and, finally, the results are based on 1000 replications of each model. It is important to mention that we assumed that the elements of w_t were correctly specified in both size and power situations.

First, figures 1, 2, 3 and 4 show the distortion between the nominal and empirical size of the linearity test. Figures 1 and 2 consider the correct transition variable and figures 3 and 4 consider the transition variable y_{t-2}. In the x axis is the nominal size and it was tested from 80% to 99% of confidence level. By looking at the plots we can conclude that the difference is acceptable in any specification. The difference in the nominal minus the empirical size is no more than 4% in any case. It was also tested the economic version in which regression (10) is computed and (9) for the full version of the test.



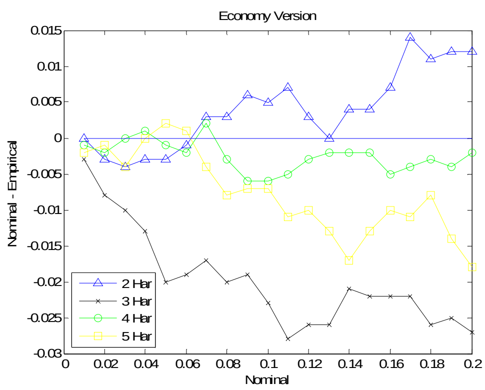


Figure (1 and 2) – (Nominal size - Empirical size) – Transition variable: y_{t-1} - 500 observations – Linearity test

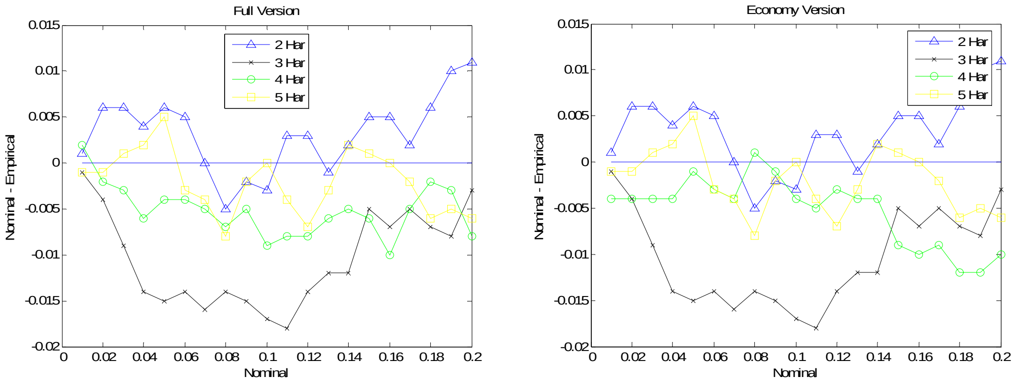


Figure (3 and 4) – (Nominal size - Empirical size) – Transition variable: y_{t-2} - 500 observations – Linearity test

Next, figures 5 to 8, we present the power size curve of the linearity test using data generated from Model 2. Again the x axis represents the nominal size varying from 0.01 to 0.2.

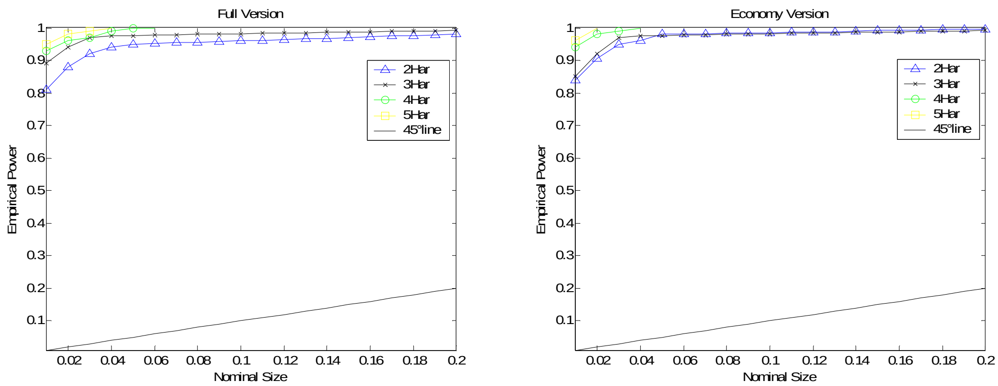


Figure (5 and 6) – Power size curve of the linearity test – Transition variable: y_{t-1} - 500 observations

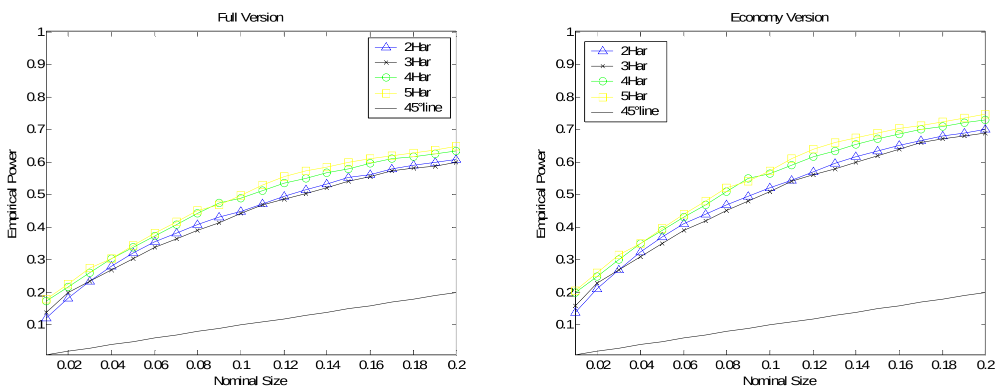


Figure (7 and 8) – Power size curve of the linearity test – Transition variable: y_{t-2} - 500 observations

When the transition variable is correctly chosen the power is almost one for any nominal significance level (Figure 5 e 6). When the transition variable is not correct, the power decreases substantially.

Continuing with the simulations, the next step is to check the empirical size and power of the LM-type test of no error auto correlation. For that, we have generated data from model 2, but assumed that the errors followed an AR(1) process $\varepsilon_t = \rho\varepsilon_{t-1} + \mu_t$, $\mu_t \sim NID(0,1^2)$. Figures 9, 10, 11 and 12 show the results for the empirical size with different values of q, 1, 2, 4 and 12. We conclude that the size of the test is acceptable in any specification. Figures from 13 to 20 show the power results when $\rho = 0.2$ (13, 14, 15 and 16) and $\rho = 0.4$ (17, 18, 19 and 20). As expected, the power increases when the value of ρ increases.

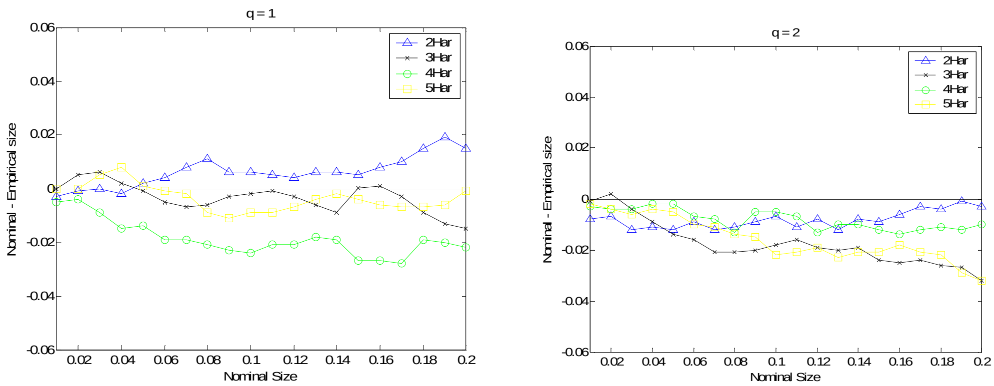


Figure (9 and 10) – Difference between Nominal and Empirical Size, q = 1 and 2, $\rho = 0$.

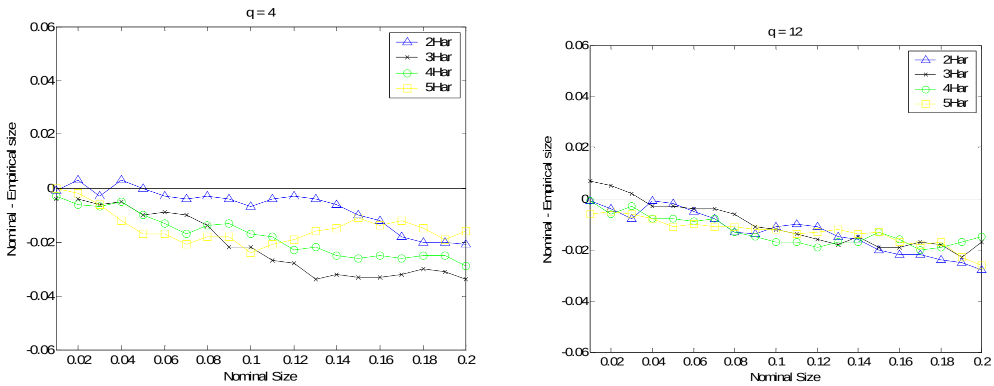


Figure (11 and 12) – Difference between Nominal and Empirical Size, q = 4 and 12, $\rho = 0$.

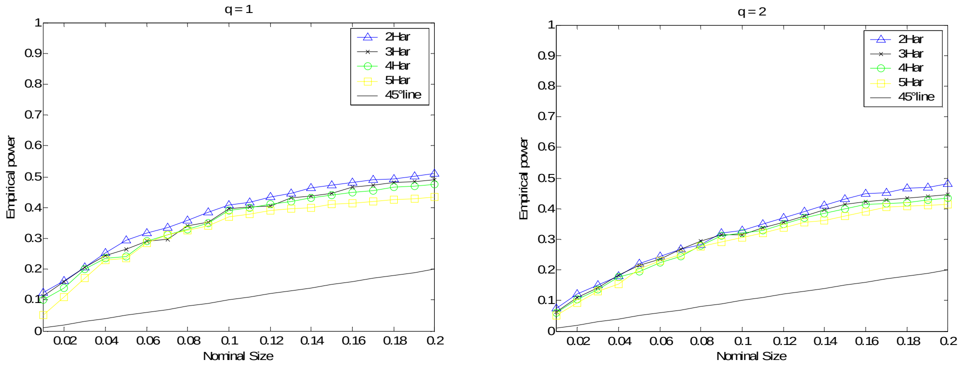


Figure (13 and 14) – Power size curve of no error autocorrelation test with $\rho = 0.2$ when q = 1 and 2.

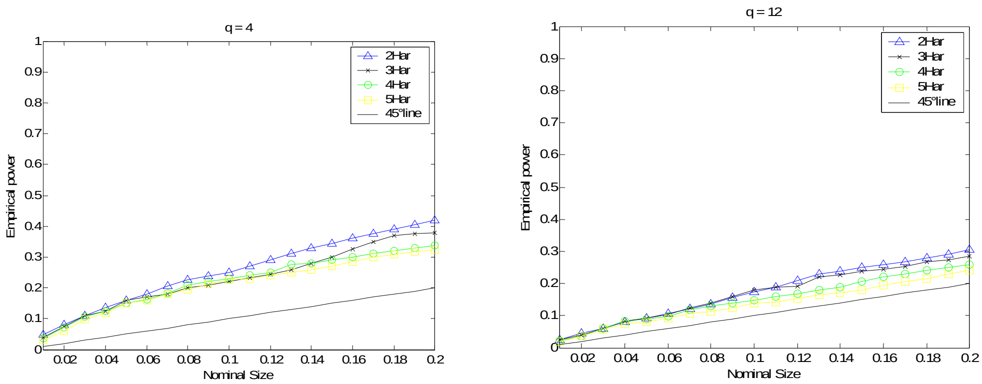


Figure (15 and 16) – Power size curve of no error autocorrelation test with $\rho = 0.2$ when q = 4 and 12.

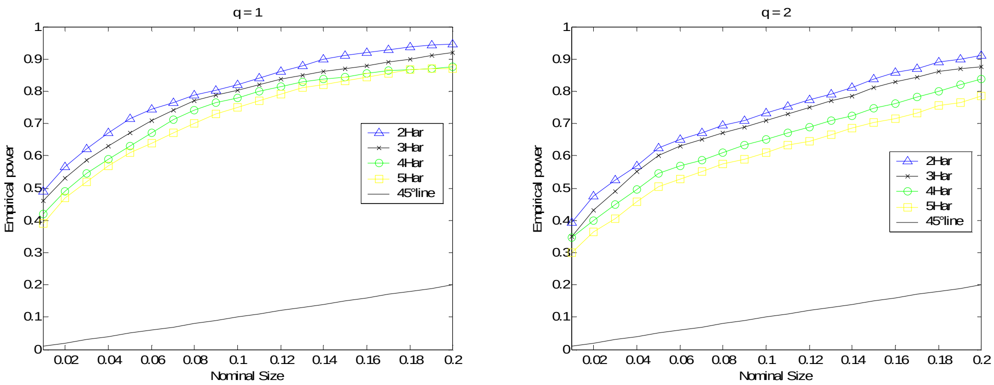


Figure (17 and 18) – Power size curve of no error autocorrelation test with $\rho = 0.4$ when q = 1 and 2.

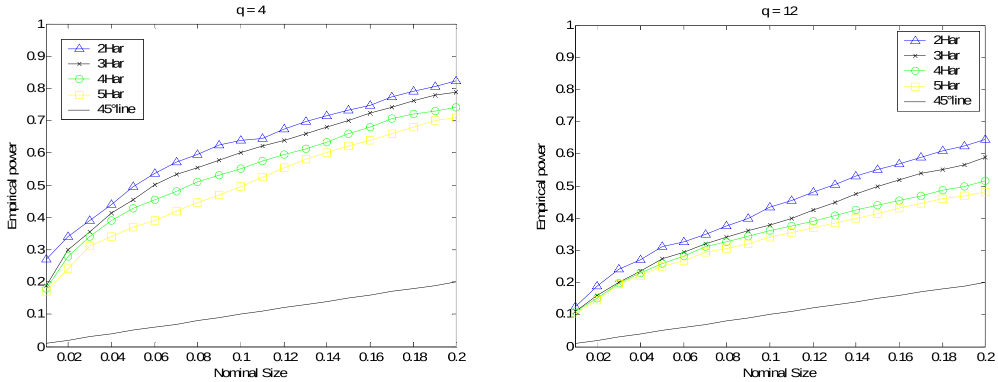


Figure (19 and 20) – Power size curve of no error autocorrelation test with $\rho = 0.4$ when q = 1 and 2.

The last simulation concerning the LM-type tests was carried out for the test of no remaining nonlinearity. Figure 21 show the empirical size curve and 22 presents the power curve. In the power simulation we assume that model 2 will have a third regime.

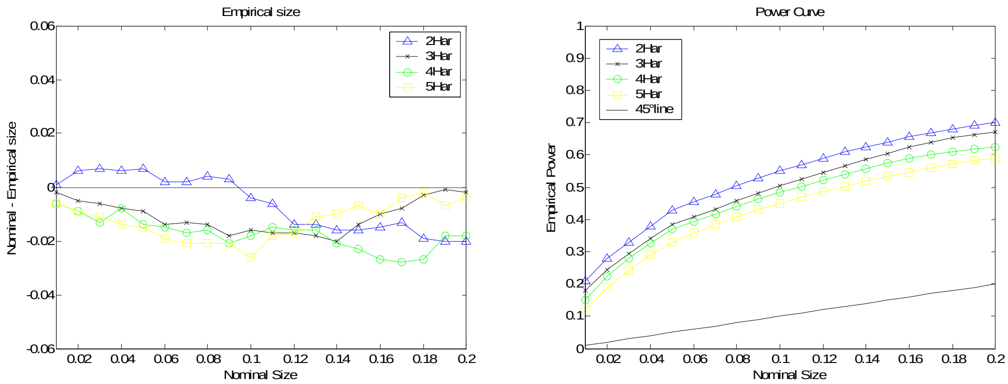


Figure (21 and 22) – Empirical and Power size curve of no remaining nonlinearity test.

Finally, the last simulation will concern the number of harmonics to be chosen in (5) using an information criteria such as the Akaike Information Criterion (AIC) (Akaike, 1974, Akaike, 1981) and the Bayesian Information Criterion (BIC). In the modeling cycle of the model the AIC and BIC were helpful in selecting the p order of the model, however in this procedure we are interested in discovering how accurate these criteria are in the selection of the correct number of harmonics. The two criteria are defined as

$$BIC = -2\sum_{t=1}^{T} \log f(y_t \mid w_t; \hat{\psi}) + K \log T \qquad (28)$$

$$AIC = -2\sum_{t=1}^{T} \log f(y_t \mid w_t; \hat{\psi}) + 2K \qquad (29)$$

where K is the number of estimated parameters.

It was simulated 100 models 1 and 2 with h = 2, 3, 4 and 5. We assume that the order of the model (p) and the periodicity represented in the step function were correctly chosen. Table 1 and 2 illustrate the results for Model 1 and 2, respectively.

Table 1 – Selection of Harmonics using AIC and BIC – Model 1 – 500 observations

	AIC		BIC	
	Correct	**Incorrect**	**Correct**	**Incorrect**
h = 2	98	2	100	0
h = 3	95	5	98	2
h = 4	90	10	95	5
h = 5	88	12	93	7

Table 2 – Selection of Harmonics using AIC and BIC – Model 2 – 500 observations

	AIC		BIC	
	Correct	**Incorrect**	**Correct**	**Incorrect**
h = 2	86	14	91	9
h = 3	82	18	87	13
h = 4	69	31	78	22
h = 5	58	42	74	26

It is clear that BIC has a better performance than AIC in all situations. The information criteria (AIC and BIC) perform much better, as they should, when the model is just a linear periodic autoregressive. And other conclusion that it is worth mentioning is that more amplitudes mean more incorrect selections using both information criteria.