Referências Bibliográficas

[1] ANDERSON, T. W. **Estimating linear restrictions on regression coefficients for multivariate normal distributions**. Ann. Math. Statist. 22 327-351, 1951 [Correction (1980) Ann. Statist. 8 1400].

[2] AHN, S. K.; REINSEL, G. C. **Nested reduced-rank autoregressive models for multiple time series**. J. Amer. Statist. Assoc. 83 849 856. Z, 1988.

[3] BEINE, M.; HECQ, A. **Codependence and Convergence in the EC Economies**. Journal of Policy Modeling, 20 (4), 403-426, 1998.

[4] BOX, G. E. P.; TIAO, G. C. **A canonical analysis of multiple time series**. Biometrika, 64(2):355-365, 1977.

[5] BRANDNER, P.; KUNST, R. M. **Forecasting vector autoregressions - The influence of cointegration: A Monte Carlo Study**. Research Memorandum N0 265, Institute for Advanced Studies, Vienna, 1990.

[6] CAMPBELL, J. Y.; MANKIW, N. G. **Permanent Income, Current Income, and Consumption**, Jounal of Bussines and Economics Statistics, 8, 265-279, 1990.

[7] CHRISTODOULAKIS, N.; DIMELIS, S. P.; KOLLINTZAS, T. **Comparisons of Business Cycles in the EC: Idiosyncrasies and Regularities**. Economica, 62 (245): 1-27, 1995.

[8] CLEMENTS, M. P.; HENDRY, D.F. **On the limitations of comparing mean square forecast errors**, Journal of forecasting, 12, 617-37, 1993.

[9] CUBADDA, G. **Common serial correlation and common business cycles: A cautious note**. Empirical Economics, vol. 24, issue 3, pages 529-535, 1999.

[10] ENGLE; GRANGER **Cointegration and error correction: representation, estimation and testing**. Econometrica, 55:251-76, 1987.

[11] ENGLE; KOZICKI, **Testing for common features**. Journal of Business and Economic Statistics, 11, 369-395, 1993.

[12] ENGLE; ISSLER, J.V. **Estimating Common Sectoral Cycles.** Journal of Monetary Economics, vol. 35, no. 1, pp. 83-113, 1995.

[13] ENGLE; YOO. **Forecasting and testing in cointegrated systems**. Journal of Econometrics, 35, 143-159, 1987.

[14] GUILLÉN; ISSLER; ATHANASOPOULOS. **Forecasting Accuracy and Estimation Uncertainty using VAR Models with Short- and Long-Term Economic Restrictions: A Monte-Carlo Study**. Ensaios Econômicos EPGE no. 589, 2005.

[15] HAMILTON, J. D. **Time Series Analysis**. Princeton University Press, New Jersey, 1994.

[16] HECQ, A.; PALM, F.C.; URBAIN, J.P. **Testing for Common Cyclical Features in VAR Models with Cointegration**. Journal of Econometrics, forthcoming, 2006a.

[17] HECQ, A. **Cointegration and Common Cyclical Features in VAR Models: Comparing Small Sample Performances of the 2-Step and Iterative Approaches**. Mimeo, 2006b.

[18] HECQ, A.. **Common Cyclical Features in Multiple Time Series and Panel Data: Methodological Aspects and Applications**, 2000.

[19] ISSLER, J. V.; VAHID, F. **Common cycles and the importance of transitory shocks to macroeconomic aggregates**. Journal of Monetary Economics, vol. 47, no. 3, pp. 449-475, 2001.

[20] ISSLER, J. V.; GAGLIANONE. **An Econometric Contribution to the Intertemporal Approach of the Current Account, with an Application for the G-7 Countries**. XXVII ENCONTRO BRASILEIRO DE ECONOMETRIA, Natal RN, 2005.

[21] IZENMAN, A. J. **Reduced-rank regression for the multivariate linear model. J. Multivariate**. Anal. 5 248 264. Z .Mathematical Reviews: 51:9381, 1975.

[22] JOHANSEN, S. **Statistical analysis of cointegration vectors**. Journal of Economic Dynamics and Control, 12, 231-54, 1988.

[23] JOHANSEN, S. **Likelihood-based Inference in Cointegrated Vector Autoregressive Models**. Oxford Univ. Press. Z, 1995.

[24] LI, J. X.; WINKER, P. , **Time Series Simulation with Quasi Monte Carlo Methods**, September -27, 2000.

[25] LUTKEPOHL, H. **Introduction to Multiple Time Series Analysis**. Springer, New York. ¨ Z, 1993.

[26] MAKRIDAKIS et al., **The accuracy of extrapolation (time series) methods: Results of a forecasting competition**, Journal of Forecasting, 489-508, 1982.

[27] REINSEL, G. C.; VELU, R. P. . **Multivariate Reduced-rank Regression**. Springer, New York. Z, 1998.

[28] SILIVERSTONVS B. E. T.; HALDRUP, N. (2003), "Lon-run forecasting"

[29] STOCK, J. H.; WATSON, M. W. **Testing for common trends**. Journal of the American Statistical Association, 83(1), 097-107, 1988.

[30] TIAO, G. C.; TSAY, R. S. **A canonical correlation approach to modeling multivariate time series**, *in Proceedings of the Business and Economic Statistics Section, American statistical Association*, 112-120, 1985.

[31] TIAO, G. C.; TSAY, R. S. **Model specification in multivariate time series** (with discussion). J. Roy. Statist. Soc. Ser. B 51, 157-213, 1989.

[32] TOMPSON, P.A. **An MSE statistic for comparing forecast accuracy across series.** International Journal of Forecasting, 6, 219-27, 1990.

[33] TSAY, R. S.; TIAO, G. C. **Use of canonical analysis in time series model identification**. Biometrika 72 299 315. Z, 1985.

[34] TSO, M. K. **Reduced rank regression and Canonical analysis**. Journal of the Royal Statistical Society, Series B, 43, 183-189, 1981.

[35] VAHID, F.; ISSLER, J.V. "**The Importance of Common Cyclical Features in VAR Analysis: A Monte Carlo Study.** Journal of Econometrics, vol. 109, no. 2, pp. 341-363, 2002.

[36] VAHID; ENGLE. **Common trends and common cycles**. Journal of Applied Econometrics, 8, 341-360, 1993.

[37] VELU, R. P., REINSEL, G. C.; WICHERN, D. W. **Reduced rank models for multiple times series**. Biometrika 73, 105 118, 1986.

A. Construção da simulação Monte-Carlo

Considere o modelo VAR(3) :

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + A_3 y_{t-3} + \varepsilon_t \qquad (-1)$$

com parâmetros:

$$A_1 = \begin{bmatrix} a^1_{11} & a^1_{12} & a^1_{12} \\ a^1_{21} & a^1_{22} & a^1_{22} \\ a^1_{31} & a^1_{32} & a^1_{32} \end{bmatrix}, A_2 = \begin{bmatrix} a^2_{11} & a^2_{12} & a^2_{12} \\ a^2_{21} & a^2_{22} & a^2_{22} \\ a^2_{31} & a^2_{32} & a^2_{32} \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} a^3_{11} & a^3_{12} & a^3_{12} \\ a^3_{21} & a^3_{22} & a^3_{22} \\ a^3_{31} & a^3_{32} & a^3_{32} \end{bmatrix}$$

e:

$$\text{Vetor de cointegração} \qquad \beta = \begin{bmatrix} \beta_{11} \\ \beta_{21} \\ \beta_{31} \end{bmatrix}$$

$$\text{Vetor co-característico} \qquad \tilde{\beta} = \begin{bmatrix} \tilde{\beta}_{11} & \tilde{\beta}_{12} \\ \tilde{\beta}_{21} & \tilde{\beta}_{22} \\ \tilde{\beta}_{31} & \tilde{\beta}_{32} \end{bmatrix}$$

$$\text{Matriz de ajustamento} \qquad \alpha = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix}.$$

O coeficiente do termo de longo-prazo é definido por $\alpha\beta' = (A_1 + A_2 + A_3 - I_3)$. A representação VECM é dada por:

$$\Delta y_t = \alpha\beta' y_{t-1} - (A_2 + A_3)\Delta y_{t-1} - A_3 \Delta y_{t-2} + \varepsilon_t \qquad (-2)$$

Ao considerar as restrições de cointegração, a equação (-2) pode ser escrita como VAR(1)

$$\xi_t = F\ \xi_{t-1} + v_t \qquad (-3)$$

$$\text{onde } \xi_t = \begin{bmatrix} \triangle y_t \\ \triangle y_{t-1} \\ \beta' y_t \end{bmatrix}, F = \begin{bmatrix} -(A_2 + A_3) & -A_3 & \alpha \\ I_3 & 0 & 0 \\ -\beta(A_2 + A_3) & -\beta' A_3 & \beta'\alpha + 1 \end{bmatrix} \text{ e}$$

$$v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ \beta'\varepsilon_t \end{bmatrix}$$

1) Restrições de curto-prazo (WF)

Sejam $G = -[R_{21}K + R_{31}]$, $K = [(R_{32} - R_{31})/(R_{21} - R_{22})]$, $R_{j1} = \tilde{\beta}_{j1}/\tilde{\beta}_{11}$, $R_{j2} = \tilde{\beta}_{j2}/\tilde{\beta}_{12}$ $(j = 2, 3)$ e $S = \beta_{11}G + \beta_{21}K + \beta_{31}$

(i) $\tilde{\beta}' A_3 = 0 ==>$

$$A_3 = \begin{bmatrix} -Ga^3_{31} & -Ga^3_{32} & -Ga^3_{33} \\ -Ka^3_{31} & -Ka^3_{32} & -Ka^3_{33} \\ -a^3_{31} & -a^3_{32} & -a^3_{33} \end{bmatrix}$$

(ii) $\tilde{\beta}'(A_2 + A_3) = 0 ==> \tilde{\beta}' A_2 = 0$

$$==> A_2 = \begin{bmatrix} -Ga^2_{31} & -Ga^2_{32} & -Ga^2_{33} \\ -Ka^2_{31} & -Ka^2_{32} & -Ka^2_{33} \\ -a^2_{31} & -a^2_{32} & -a^2_{33} \end{bmatrix}$$

2) Restrições de longo-prazo (Cointegração)

(iv) $\beta'(A_2 + A_3) = [-(a^2_{31} + a^3_{31})S \quad -(a^2_{32} + a^3_{32})S \quad -(a^2_{33} + a^3_{33})S]$
e $\beta' A_3 = [-a^3_{31}S \quad -a^3_{32}S \quad -a^3_{33}S]$

(v) $\beta'\alpha + 1 = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \end{bmatrix} \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix} + 1 = \beta_{11}\alpha_{11} + \beta_{21}\alpha_{21} + \beta_{31}\alpha_{31} + 1$

Portanto, ao considerar as restrições de longo-prazo e curto-prazo, a matriz associada F é representada por:

$$F = \begin{bmatrix} -(A_2 + A_3) & -A_3 & \alpha \\ I_3 & 0 & 0 \\ -\beta(A_2 + A_3) & -\beta' A_3 & \beta'\alpha + 1 \end{bmatrix} =$$

$$\begin{bmatrix} -G(a_{31}^2+a_{31}^3) & -G(a_{32}^2+a_{32}^3) & -G(a_{33}^2+a_{33}^3) & -Ga_{31}^3 & -Ga_{32}^3 & -Ga_{33}^3 & \alpha_{11} \\ -K(a_{31}^2+a_{31}^3) & -G(a_{32}^2+a_{32}^3) & -G(a_{33}^2+a_{33}^3) & -Ka_{31}^3 & -Ka_{32}^3 & -Ka_{33}^3 & \alpha_{21} \\ -(a_{31}^2+a_{31}^3) & -G(a_{32}^2+a_{32}^3) & -(a_{33}^2+a_{33}^3) & -a_{31}^3 & -a_{32}^3 & -a_{33}^3 & \alpha_{31} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -(a_{31}^2+a_{31}^3)S & -(a_{32}^2+a_{32}^3)S & -(a_{33}^2+a_{33}^3)S & -a_{31}^3S & -a_{32}^3S & -a_{33}^3S & b \end{bmatrix}$$

com $b = \beta'\alpha + 1 = \beta_{11}\alpha_{11} + \beta_{21}\alpha_{21} + \beta_{31}\alpha_{31} + 1$

3) Restrições de estacionariedade em covariância

A equação (-3) é estacionária em covariância se todos os autovalores da matriz F encontram-se dentro do círculo unitário. Portanto, os autovalores da matriz F representam um número λ tal que:

$$|F - \lambda I_7| = 0 \qquad \text{(-4)}$$

A solução de (-4) é:

$$\lambda^7 + \Omega\lambda^6 + \Theta\lambda^5 + \Psi\lambda^4 = 0 \qquad \text{(-5)}$$

en que os parâmetros Ω, Θ, e Ψ são: $\Omega = G(a_{31}^2 + a_{31}^3) + K(a_{32}^2 + a_{32}^3) + a_{33}^2 + a_{33}^3 - b$, $\Theta = Ga_{31}^3 + Ka_{32}^3 - (a_{33}^2 + a_{33}^3)b - Gb(a_{31}^2 + a_{31}^3) - Kb(a_{32}^2 + a_{32}^3) + \alpha_{31}S(a_{33}^2 + a_{33}^3) + S\alpha_{21}(a_{32}^2 + a_{32}^3) + S\alpha_{11}(a_{31}^2 + a_{31}^3) + a_{33}^3$ e $\Psi = -a_{33}^3b - Ga_{31}^3b - Ka_{32}^3b + \alpha_{31}a_{33}^3S + a_{32}^3S\alpha_{21} + a_{31}^3S\alpha_{11}$; as primeiras quatro raízes são $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$. Calculam-se as matrizes de coeficientes A_1, A_2 e A_3, bem como as raízes (λ_5, λ_6 e λ_7) e os parâmetros livres. Assim, temos três raízes satisfazendo a equação (-5)

$$\lambda^3 + \Omega\lambda^2 + \Theta\lambda + \Psi = 0$$

para λ_5, a equação: $\lambda_5^3 + \Omega\lambda_5^2 + \Theta\lambda_5 + \Psi = 0$ ....$Eq1$

para λ_6, a equação: $\lambda_6^3 + \Omega\lambda_6^2 + \Theta\lambda_6 + \Psi = 0$ ....$Eq2$

para λ_7, a equação: $\lambda_7^3 + \Omega\lambda_7^2 + \Theta\lambda_7 + \Psi = 0$ ....$Eq3$

Resolvendo $Eq1$, $Eq2$ e $Eq3$ temos: $\Omega = -\lambda_7 - \lambda_6 - \lambda_5$, $\Theta = \lambda_6\lambda_7 + \lambda_6\lambda_5 + \lambda_5\lambda_7$ e $\Psi = -\lambda_5\lambda_6\lambda_7$. Igualando estes parâmetros com as relações acima, temos:

$a^2_{31} = -(-Ka^2_{32} - Ka^2_{32}b + \alpha_{31}Sa^2_{33} - \lambda^6\lambda^7 - \lambda^6 - \lambda^7 - a^2_{33}b - \lambda^5\lambda^6\lambda^7 + b - \lambda^5\lambda^7 - \lambda^5\lambda^6 - a^2_{33} + Sa^2_{32}\alpha_{21} - \lambda^5)/(S\alpha_{11} - G - Gb)$

$a^3_{32} = (-S^2\lambda^7\alpha_{11}\alpha_{31} - b^2\lambda^7G - \lambda^6Gb^2 + b\lambda^7S\alpha_{11} + \lambda^6S\alpha_{11}b - a^3_{31}S\alpha_{11}G + a^3_{31}S^2\alpha^2_{11} - Ga^3_{31}bS\alpha_{11} - \lambda^5Gb^2 + \lambda^5S\alpha_{11}b - \lambda^7\lambda^6\alpha_{31}SG - \lambda^7\lambda^5\alpha_{31}SG - S^2\alpha_{11}\lambda^5\alpha_{31} - S^2\alpha_{11}\lambda^6\alpha_{31} + S\lambda^5Gb\alpha_{31} + S\alpha_{31}\lambda^6Gb - \lambda^5\lambda^7\lambda^6G + \lambda^6\lambda^7Gb + \lambda^5\lambda^7Gb + \lambda^5\lambda^6Gb - SGb^2\alpha_{31} + S^2\alpha_{11}b\alpha_{31} - S^2\alpha_{11}\alpha_{31}a^2_{33} + S^2\alpha^2_{31}a^2_{33}G + SG^2a^3_{31}\alpha_{31} + S\alpha_{11}a^2_{33}b + Gb^3 - S\alpha_{11}b^2 - S^2\alpha_{11}Ka^2_{32}\alpha_{31} - S^2\alpha_{11}\alpha_{31}Ga^3_{31} + S^2a^2_{32}\alpha_{21}G\alpha_{31} - Sa^2_{32}\alpha_{21}Gb + S\alpha_{31}G^2a^3_{31}b - S\alpha_{31}a^2_{33}Gb + S\alpha_{11}Ka^2_{32}b + S\lambda^7Gb\alpha_{31} - \lambda^5\lambda^6\alpha_{31}SG - \lambda^5\lambda^7\lambda^6\alpha_{31}SG + \lambda^5\lambda^7\lambda^6S\alpha_{11})/(S\alpha_{11}K\alpha_{31} - KG\alpha_{31} + bG\alpha_{21} - K\alpha_{31}Gb - S\alpha_{11}\alpha_{21} + G\alpha_{21})/S$

$a^3_{33} = -(Kb^3G - \lambda^5Gb^2K + S\alpha_{11}\lambda^6K\lambda^7\lambda^5 + Kb\lambda^7S\alpha_{11} - Kb^2\lambda^7G - S^2\alpha_{21}\lambda^7\alpha_{11} + \lambda^6GbS\alpha_{21} + S\alpha_{21}\lambda^7Gb - \lambda^6Gb^2K + \lambda^6S\alpha_{11}Kb - \lambda^6S^2\alpha_{11}\alpha_{21} + \lambda^5GbS\alpha_{21} + \lambda^5S\alpha_{11}Kb - \lambda^5S^2\alpha_{11}\alpha_{21} - \lambda^7\lambda^6S\alpha_{21}G + Kb\lambda^7\lambda^6G + Kb\lambda^7\lambda^5G + Kb\lambda^5\lambda^6G - \lambda^7\lambda^6KG\lambda^5 - S^2\alpha_{11}\alpha_{21}Ka^2_{32} + S^2\alpha_{11}\alpha_{21}b - S^2\alpha_{11}\alpha_{21}a^2_{33} + S^2\alpha^2_{21}a^2_{32}G - S\alpha_{11}Kb^2 + S\alpha_{21}G^2a^3_{31} - S\alpha_{21}Gb^2 + S^2a^3_{31}K\alpha^2_{11} - S^2\alpha_{11}\alpha_{21}Ga^3_{31} + S^2\alpha_{21}a^2_{33}G\alpha_{31} + S\alpha_{11}K\hat{\ }2ba^2_{32} + S\alpha_{11}Kba^2_{33} - S\alpha_{11}a^3_{31}KG - S\alpha_{11}KbGa^3_{31} - SKba^2_{33}G\alpha_{31} + S\alpha_{21}G^2a^3_{31}b - S\alpha_{21}\lambda^5\lambda^6G - S\alpha_{21}\lambda^5\lambda^7\lambda^6G - S\alpha_{21}Ka^2_{32}Gb - S\alpha_{21}\lambda^7\lambda^5G)/(S\alpha_{11}K\alpha_{31} - KG\alpha_{31} + bG\alpha_{21} - K\alpha_{31}Gb - S\alpha_{11}\alpha_{21} + G\alpha_{21})/S$

Foram calculados os elementos das matrizes de parâmetros a^2_{31}, a^3_{32}e a^3_{33} fixando o conjunto de autovalores $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ e sorteando de forma independente de uma distribuição uniforme $(-0.9; 0.9)$ os valores a^3_{31}, a^2_{32}, a^2_{33}, λ_5, λ_6 e λ_7. Assim, os elementos das matrizes A_1, A_2 e A_3 são gerados, de forma a construir os processos geradores de dados do modelo VAR(3) sujeito às restrições de cointegração e do tipo WF.