7
Examples

In this section we will present an illustration of the modeling techniques discussed in this paper. The first example considers only the in-sample fitting and the second one considers one-step ahead forecasts.

7.1
Example 1: Canadian Lynx.

The first set analyzed is the 10-based logarithm of the annual record of the numbers of Canadian lynx trapped in the Mackenzie River district of north-west Canada for the period 1821-1934 inclusively (114 observations). For further details and background history, see (48), (58), and Wong and Li (1999,2000).We report only the results for in-sample fitting because the number of observations is rather small and most of the previous studies in literature have only considered the in-sample analysis.

Many models have been proposed for this dataset. It is commonly accepted that the data are cyclical, with a period of 9-10 years. Furthermore, the histogram also shows obvious multimodality. A multimodality test was carried out by (12).

The variables were selected by using the same methodology used by (47). The estimated tree using either AIC or BIC is a 1-split tree, whose the transition variable is the y_{t-2}.

$$
\begin{aligned}
g_0(\cdot) &= g(y_{t-2}; 9.9826, 2, 3.2655), \\
y_{1t} &= 0.5465 + 1.319 y_{t-1} - 0.4655 y_{t-2} + \hat{\varepsilon}_{1t} \quad \hat{\varepsilon}_{1t} \sim N(0, 0.0325), \\
y_{2t} &= 0.9892 + 1.5173 y_{t-1} - 0.8832 y_{t-2} + \hat{\varepsilon}_{2t} \quad \hat{\varepsilon}_{2t} \sim N(0, 0.0493).
\end{aligned}
$$

This class of models has the ability to forecast the conditional distribution, so we need to define another measure to compare it with other models. This measure is the empirical coverage $(1-\alpha)100\%$ 1-step ahead.

We have selected some models that were applied previously for comparison with our model. The first model is an AR(2) model used in (48). Then, we compare it with the Self-Exciting Threshold Autoregressive (SETAR) model in (58) and we also compare it with the Mixture Autoregressive (MAR) in (66) and with the Generalized Mixture Autoregressive (GMAR) in (65).

The models MAR and GMAR have a mixture of Gaussian models as the conditional density and the others have a Gaussian conditional density. It is important to note that the Tree-MM model has the same number of regimes and the same transition variable as the models SETAR and GMAR.

Tabela 7.1: EXAMPLE 1: EMPIRICAL COVERAGE.
The table shows the empirical coverage as well as the mean absolute error (MAE) for a set of different models.

	Empirical Coverage $(1-\alpha)100\%$						
Model	50	60	70	80	90	95	MAE
AR(2)	50.00	58.93	68.75	75.89	88.61	92.86	1.99
SETAR	44.86	56.07	69.16	81.31	90.65	95.33	2.27
MAR	52.68	63.39	70.54	82.14	88.39	96.43	2.36
GMAR	47.32	58.93	68.75	82.14	92.86	93.75	2.25
Tree-MM	48.21	57.14	67.86	79.46	89.29	96.43	1.89

It is not simple to select a winner among these models because every one has good coverage, so we need to use a measure such as the mean absolute error (MAE) to choose the model with the best coverage. The model with the best coverage is the Tree-MM, which has the smallest MAE, followed by AR(2), GMAR, SETAR and MAR.

7.2
Example 2: Brazilian Financial Dataset.

In this section we apply the techniques developed in this paper in automatic trading with exchange rates and stocks from the Brazilian market. We also compare the results with an artificial neural network model (NN) built using Bayesian regularization (MacKay 1992a,b), with the ARMA model, and the naïve method (the forecast for any period equals the previous period's actual value). We selected an asset which tracks the BOVESPA Index (IBOVESPA). IBOVESPA is an index of 50 stocks traded at the São Paulo Stock Exchange, Brazil. The choice was made taking into account the liquidity of the asset and its economic relevance. The selected asset is the Petrobras PN (PETR4). The observations cover the period from 01/20/1999 to 12/30/2004 (1476 observations). The sample is divided into two groups. The first one consists of 1227 observations (from 01/20/1999 to 12/30/2003) and is used to estimate the model. The second group consists of 249 observations (from 02/02/2004 to 12/30/2004) and it is used for out-of-sample evaluation.

The set of covariates is composed of the first 10 lags of the log-return of the asset, the first 10 lags of the volatility, the first 10 lags of the traded volume between 2 days, the first difference of the 10-day moving average of the return (MA10) and 20-day moving average of the return (MA20), and the first difference of the following 10 exogenous variables: IBOVESPA, S&P 500 Index (S& P), US Dollar (DOL), Treasury 10 years (T10), C-Bond (C-BOND), spread between C-Bond and T10 (SOT), Oil NY (OIL), Swap 360 (SW360), Set of commodities (CRY) and the Developing Countries Stock Index (BINDEX).

The statistial measures used to evaluate the model are the mean absolute error (MAE), the mean absolute percentage error ($MAPE$), the root mean square error ($RMSE$), Theil's inequality coefficient (U), and the correct direction change (CDC). The financial measures are the mean return ($\bar{R}$), the annual return (R^A), the accumulate return (R^C), the annual volatility (σ^A), the Sharpe index (SR), and the percentage of winning trades (WT). Furthermore, we present the coverage of the model and the statistics of the coverage for the NN and TreeMM models.

We first select the set of regressors using the procedure proposed by (52), then estimate the TREE-MM model using the technique proposed here. We consider that the series has the first lag of log return, daily volatility and trade volume between 2 days as variables, so the table shows the other selected variables, which are MA10 and CRY.

The estimated neural network has two hidden units and uses the whole set of variables. The estimated Tree-MM model has one split and the transition variable is the first lag of the daily volatility. The model is the following:

$$
\begin{aligned}
g_0(\cdot) &= g(v_{t-1}; 5.2572, 2, 0.0318), \\
y_{1t} &= -7.9906 \times 10^{-4} - 0.0542 y_{t-1} + 0.0775 v_{t-1} + 5.0897 \times 10^{-4} q_{t-1} \\
&\quad -3.6653 \times 10^{-6} MA10 + 0.0180 CRY + \varepsilon_{1t} \quad \varepsilon_{1t} \sim N(0, 1.9374 \times 10^{-4}), \\
y_{2t} &= -6.956 \times 10^{-3} + 0.3382 y_{t-1} + 0.1673 v_{t-1} + 1.5762 \times 10^{-3} q_{t-1} \\
&\quad -1.6057 \times 10^{-5} MA10 + 0.0167 CRY + \varepsilon_{2t} \quad \varepsilon_{2t} \sim N(0, 6.5584 \times 10^{-4}).
\end{aligned}
$$

Table 7.2 shows the statistics for the models. The Tree-MM model, the Neural Network model and the linear model have similar performance. The financial measures, shown in Table 7.3, indicate that the Tree-MM model has the best performance among the other models.

Tabela 7.2: STATISTICAL RESULTS

This table shows the statistical results for the different models.

Series		ARMA	Naive	NN	TREE-MM
	MAE	0.012	0.016	0.012	0.012
PETR4	RMSE	0.017	0.022	0.017	0.017
	CDC	60.48	58.065	65.73	62.50

Tabela 7.3: FINANCIAL RESULTS

This table shows the financial results for the different models.

Series		ARMA	Naive	NN	TREE-MM
PETR4	$\bar{R}$	0.68	0.50	1.65	1.45
	R^A	41.64	26.38	60.47	61.69
	R^C	40.98	25.96	59.51	60.71
	σ^A	24.45	22.59	18.47	18.31
	SR	1.70	1.17	3.27	3.37
	$\#T$	60	52	36	42
	WT	55.00	46.67	75.00	76.19

Next we present the coverage results for the neural network model and the Tree-MM model. The results in Table 7.4 clarify precisely the difference between the normal conditional model and the conditional mixture of models. While the Tree-MM has very good coverage, the Neural Network model with Gaussian error has a very poor one. The means of these two models describe the behavior of the asset very well.

Tabela 7.4: COVERAGE FOR EVALUATED NN AND TREE-MM MODELS.

This table shows the empirical coverage and p-value of the Christoffersen statistics for the estimated NN and Tree-MM models.

		Empirical Coverage							
Model	Percentile	50.0	60.0	70.0	80.0	90.0	95.0	97.5	99.0
Tree-MM	Est. Percentile	47.79	57.91	63.93	80.52	90.02	95.04	97.56	98.98
	LR_{uc}	.0953	.0979	.9380	.6306	1.000	1.000	1.000	1.000
	LR_{cc}	.0386	.0236	.9644	.7485	.8872	.9410	.9872	0.000
NN	Est. Percentile	42.23	57.91	65.92	78.14	89.55	94.77	97.35	98.57
	LR_{uc}	0.000	0.000	.0007	.0754	.5607	.6978	.7426	.1257
	LR_{cc}	0.000	0.000	0.000	.0088	.3002	.8547	.8863	0.000

We conclude that the Tree-MM model works better than the other models selected for modeling this asset. Furthermore, it captures better the characteristics of the time series, being a good choice for modeling financial risk.