

4

Resultados

Nesse capítulo é feita uma comparação entre os seguintes métodos:

1. *gradient one fitting*.
2. *gradient one fitting* com *ridge regression*.
3. O novo método usando *gradient one fitting*.
4. O novo método usando *gradient one fitting* com *ridge regression*.

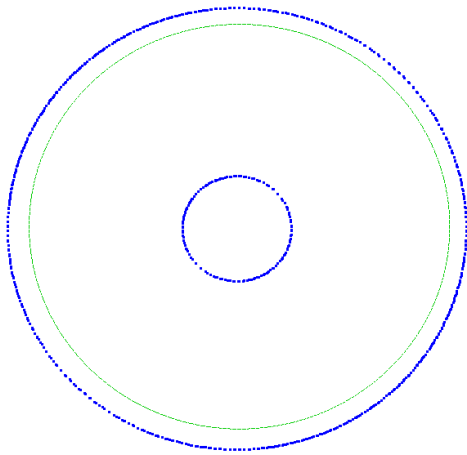
Essa numeração para os métodos será utilizada nas legendas das figuras a seguir.

Primeiramente serão mostrados os resultados para curvas suaves. Depois foram escolhidas curvas com singularidades, para exemplificar o desempenho do método mesmo para esses casos, onde a aproximação não deveria funcionar. Finalmente, são exemplificados resultados do método considerando curvas incompletas (faltando alguns pontos).

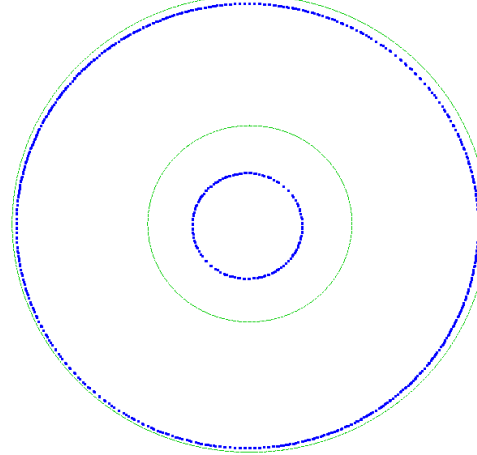
4.1

Curvas Suaves

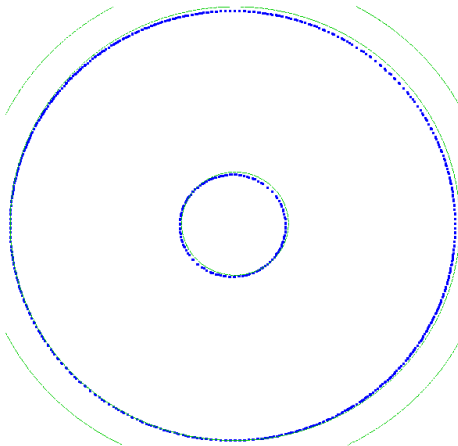
Os dois exemplos a ser mostrados consideram pontos em duas componentes conexas.



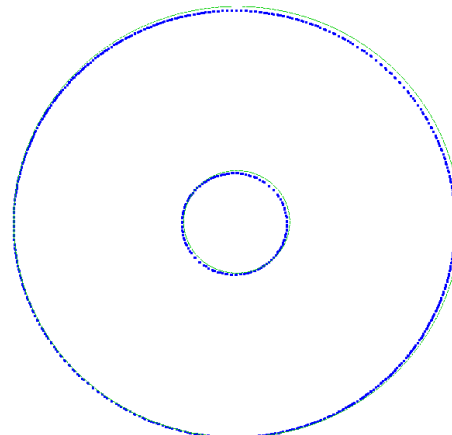
4.1(a): $d = 2$



4.1(b): $d = 4$



4.1(c): $d = 6$



4.1(d): $d = 8$

Figura 4.1: Dois círculos: método 1 usando um polinômio de grau d com $\mu = 0.125$.

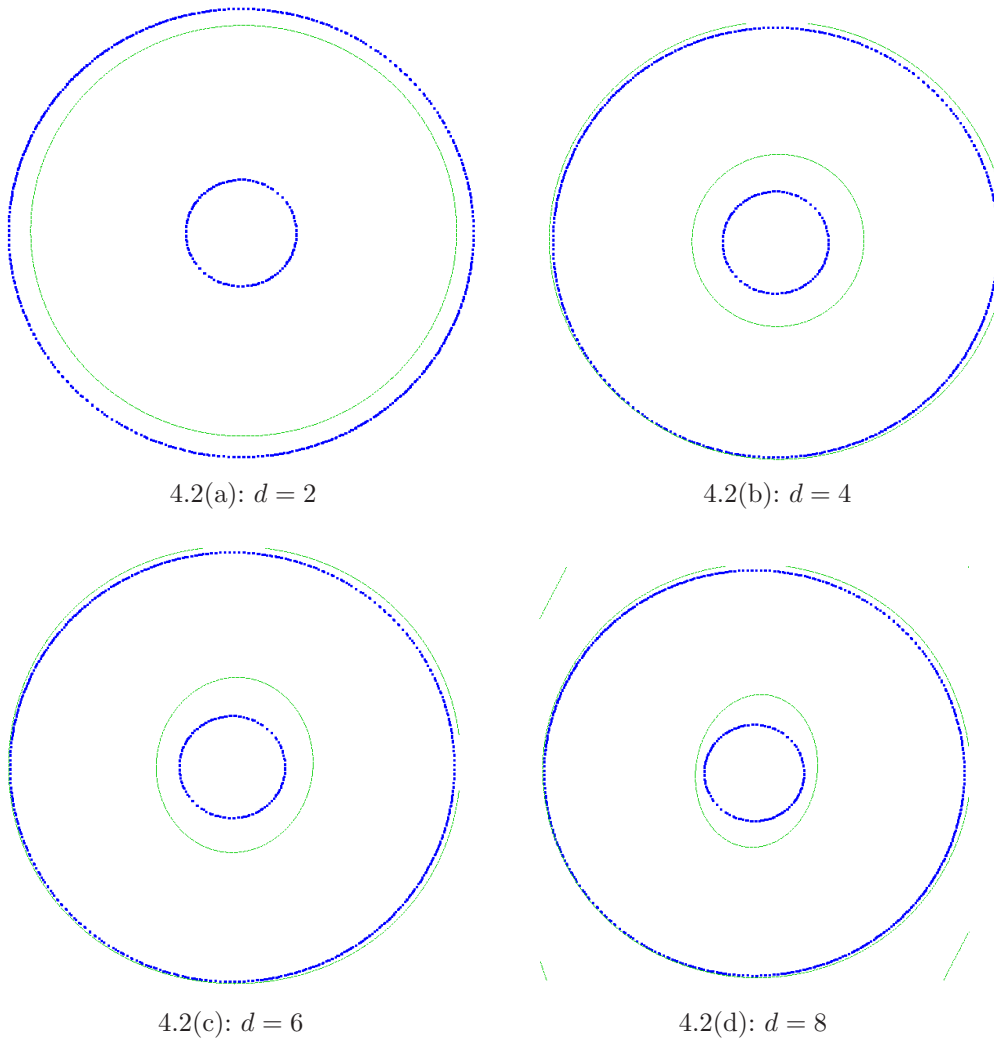


Figura 4.2: Dois círculos: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

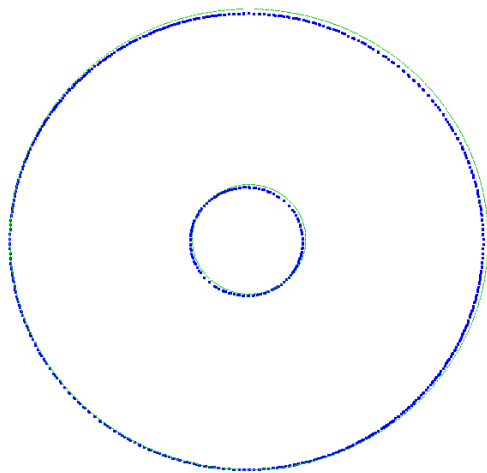
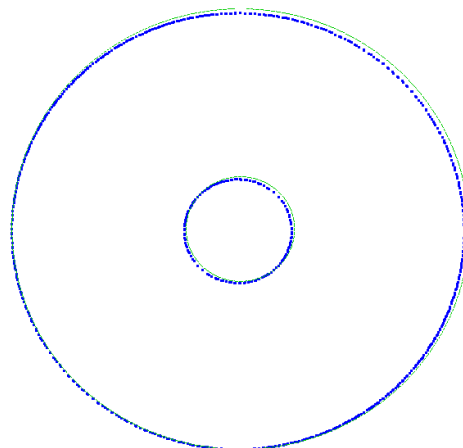
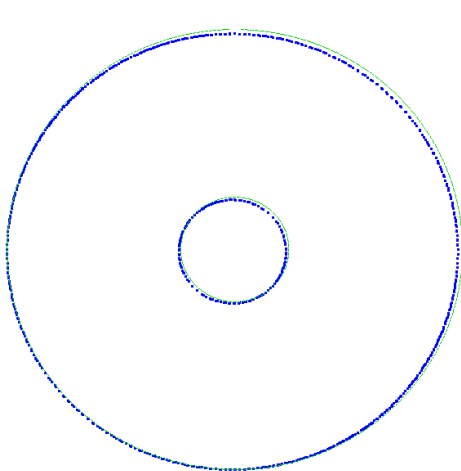
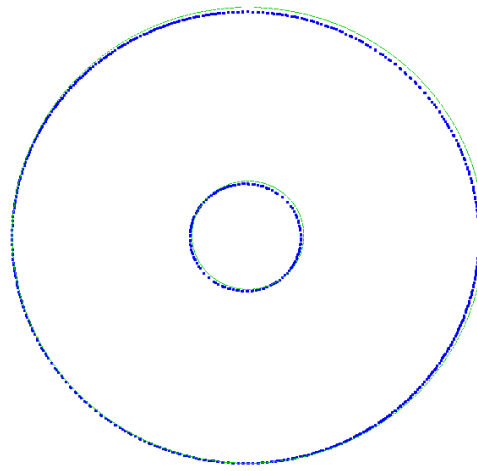
4.3(a): $d = 2$; $l_{max} = 2$ 4.3(b): $d = 2$; $l_{max} = 3$ 4.3(c): $d = 2$; $l_{max} = 4$ 4.3(d): $d = 2$; $l_{max} = 5$

Figura 4.3: Dois círculos: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

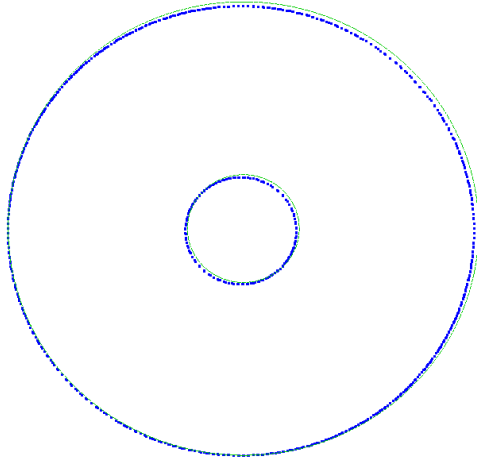
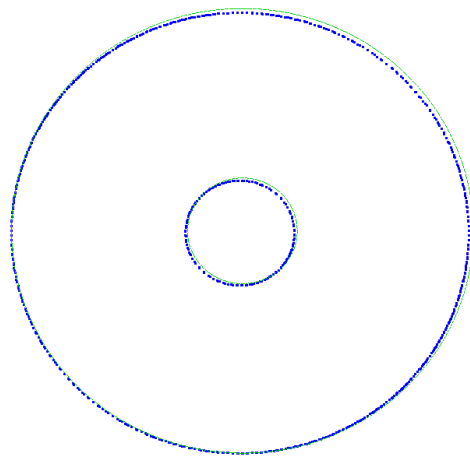
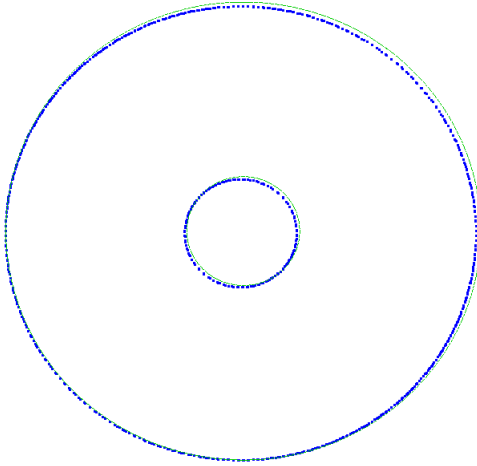
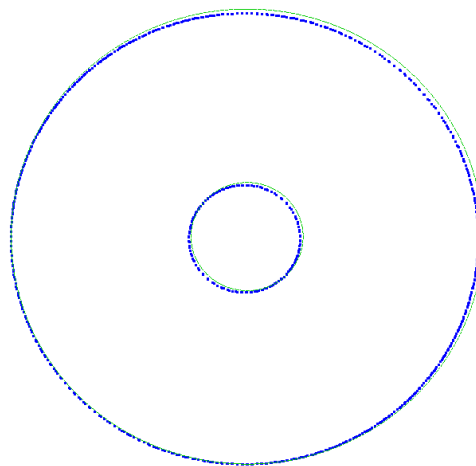
4.4(a): $d = 2$; $l_{max} = 2$ 4.4(b): $d = 2$; $l_{max} = 3$ 4.4(c): $d = 2$; $l_{max} = 4$ 4.4(d): $d = 2$; $l_{max} = 5$

Figura 4.4: Dois círculos: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

Este outro exemplo a ser mostrado ilustra os pontos e suas respectivas normais, além da aproximação feita pelos métodos.

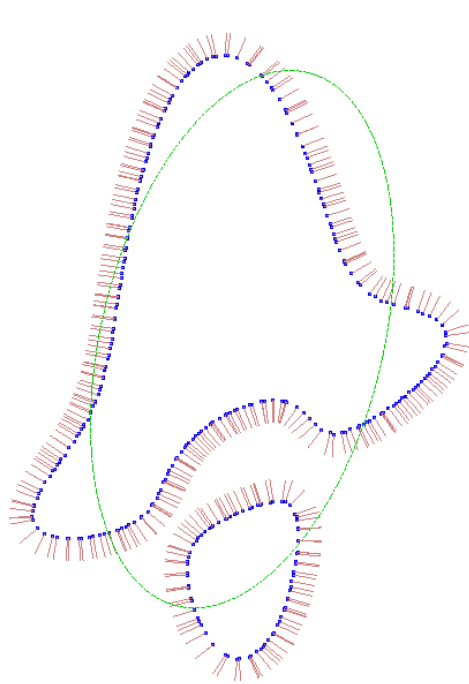
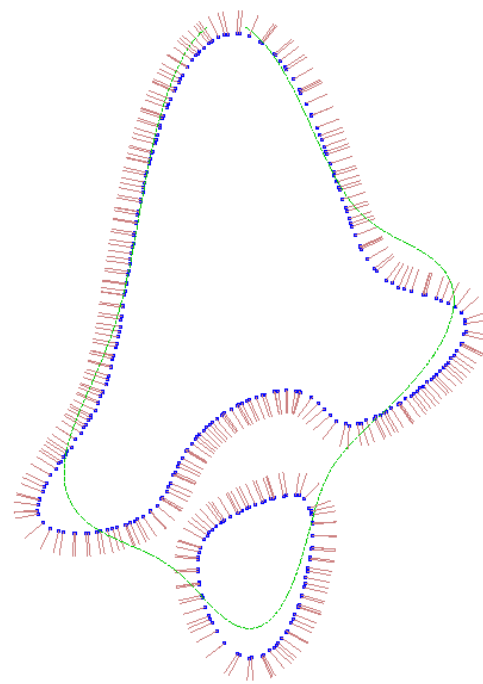
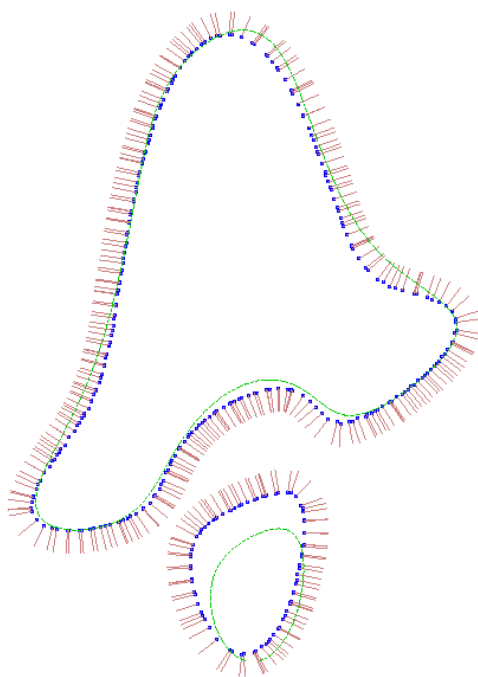
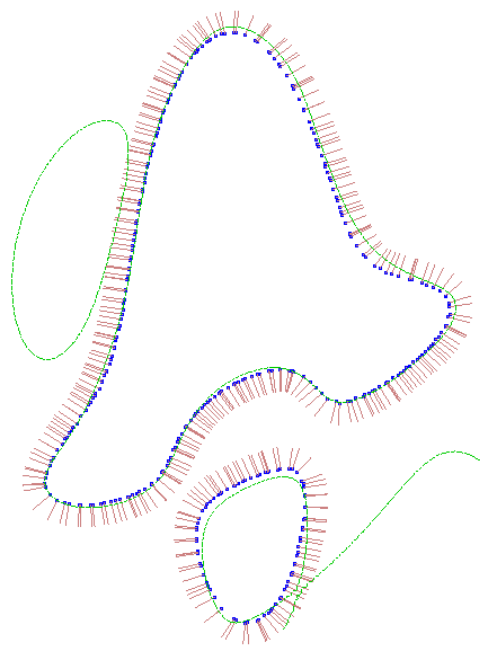
4.5(a): $d = 2$ 4.5(b): $d = 4$ 4.5(c): $d = 6$ 4.5(d): $d = 8$

Figura 4.5: Taubin: método 1 usando um polinômio de grau d com $\mu = 0.125$.

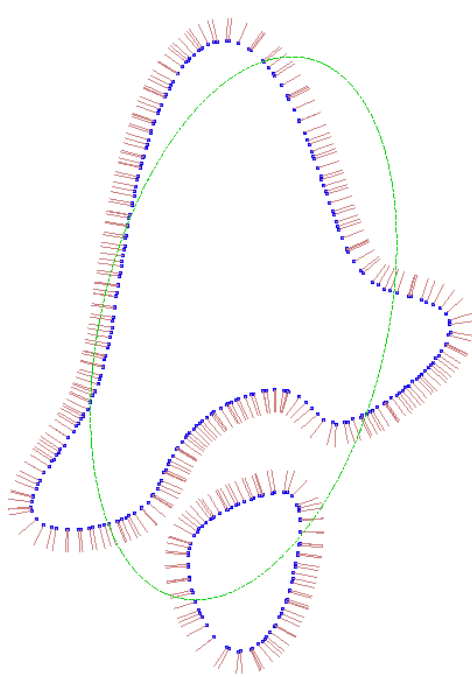
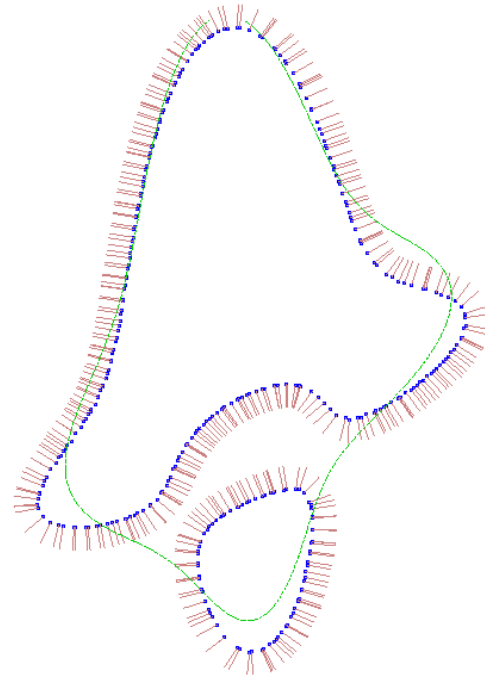
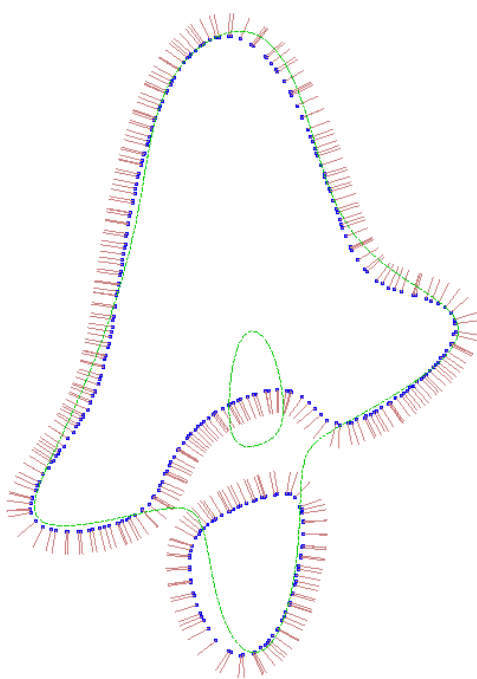
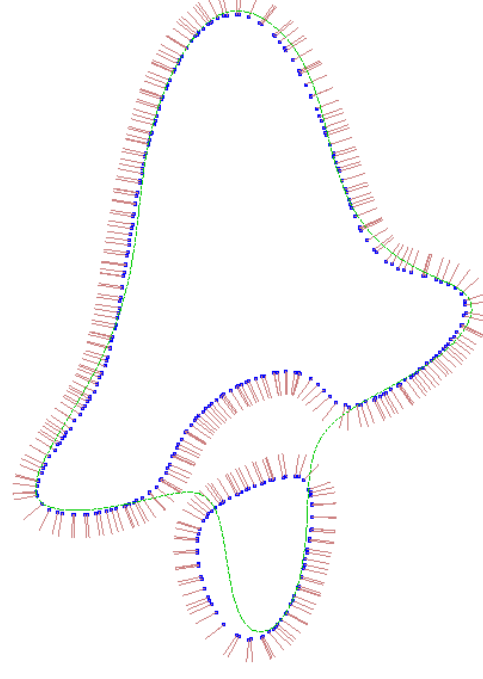
4.6(a): $d = 2$ 4.6(b): $d = 4$ 4.6(c): $d = 6$ 4.6(d): $d = 8$

Figura 4.6: Taubin: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

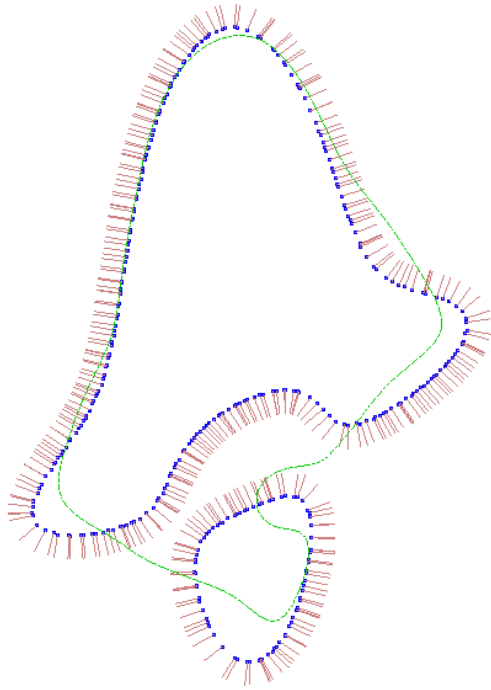
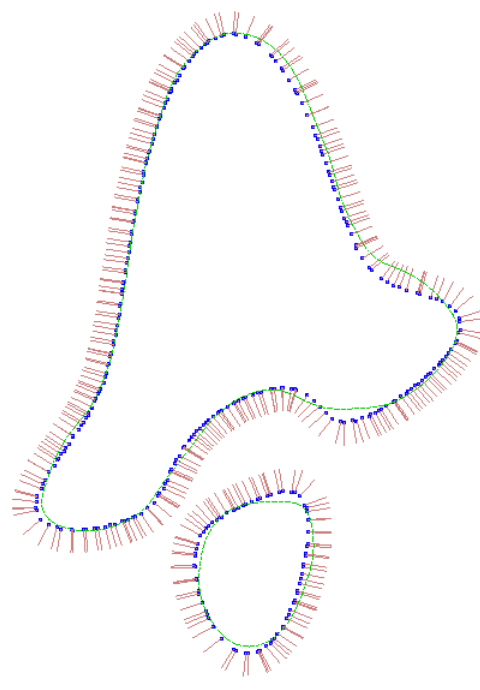
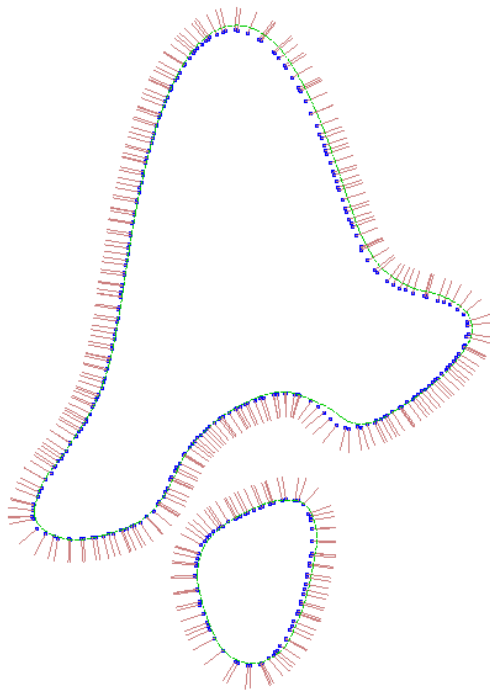
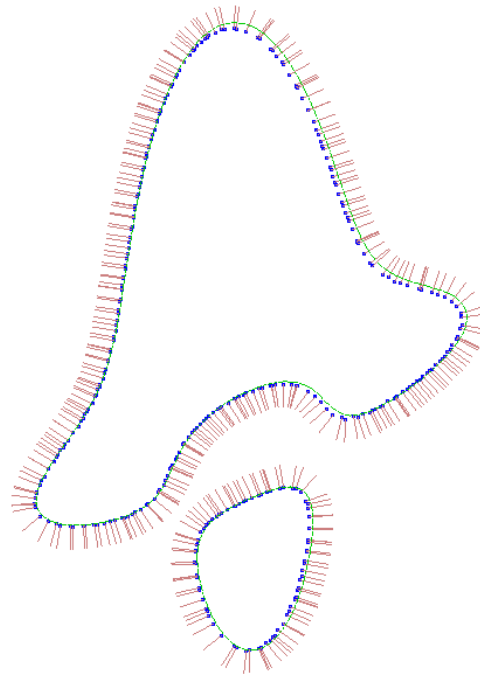
4.7(a): $d = 2$; $l_{max} = 2$ 4.7(b): $d = 2$; $l_{max} = 3$ 4.7(c): $d = 2$; $l_{max} = 4$ 4.7(d): $d = 2$; $l_{max} = 5$

Figura 4.7: Taubin: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

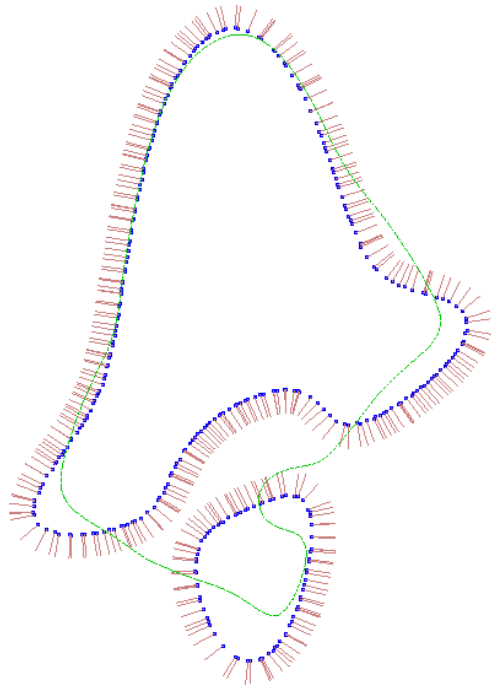
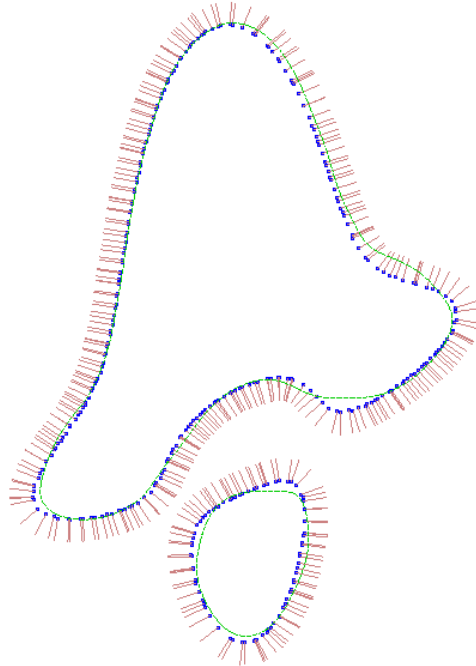
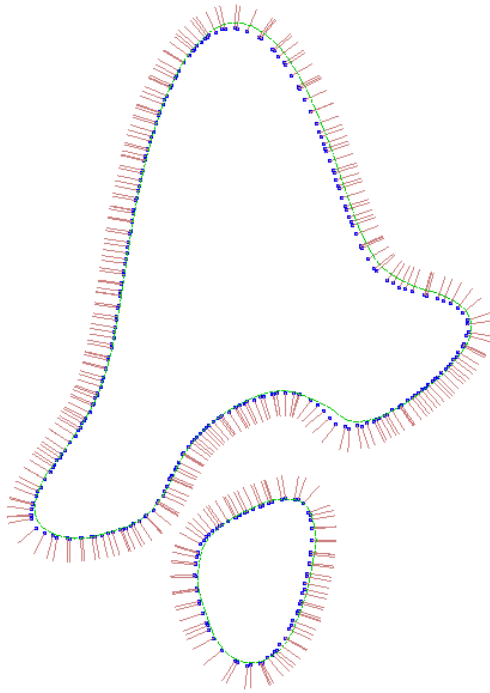
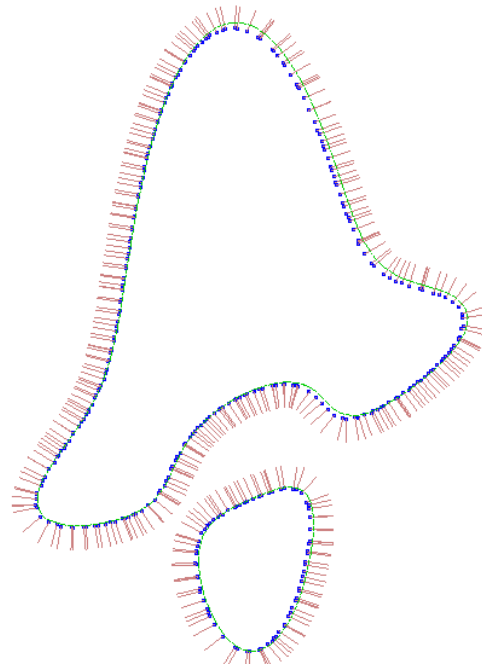
4.8(a): $d = 2$; $l_{max} = 2$ 4.8(b): $d = 2$; $l_{max} = 3$ 4.8(c): $d = 2$; $l_{max} = 4$ 4.8(d): $d = 2$; $l_{max} = 5$

Figura 4.8: Taubin: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

Os dois próximos exemplos são de curvas suaves com apenas uma componente conexa.

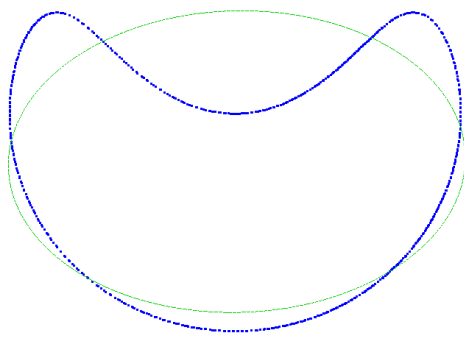
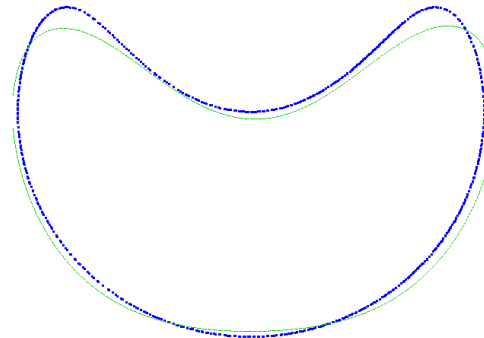
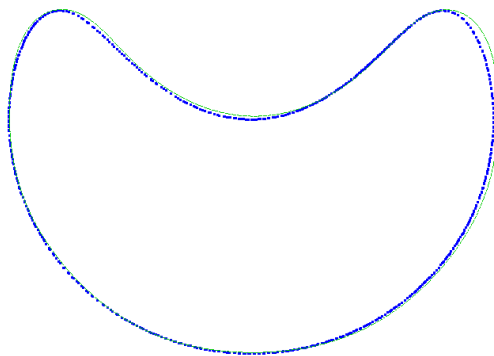
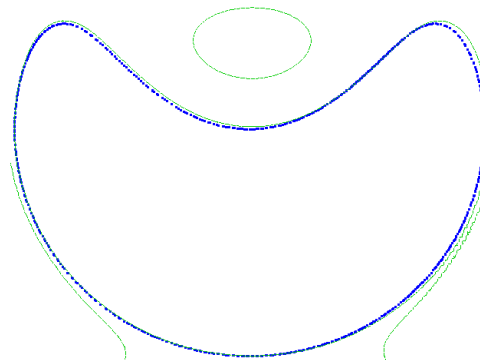
4.9(a): $d = 2$ 4.9(b): $d = 4$ 4.9(c): $d = 6$ 4.9(d): $d = 8$

Figura 4.9: Sorriso: método 1 usando um polinômio de grau d com $\mu = 0.125$.

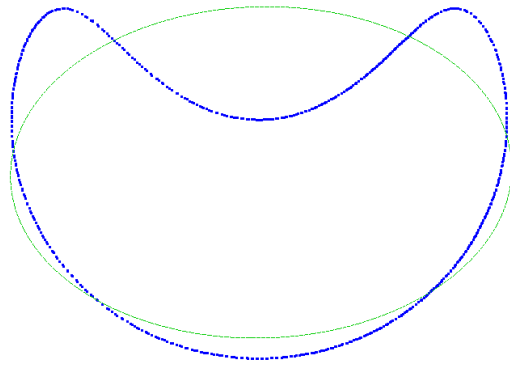
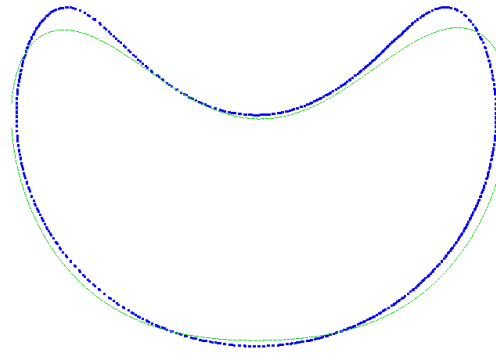
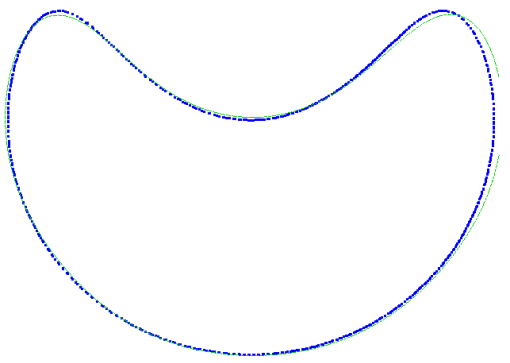
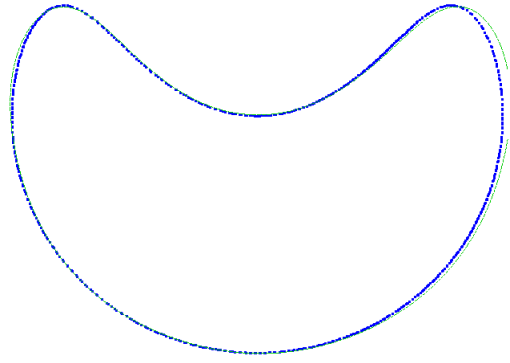
4.10(a): $d = 2$ 4.10(b): $d = 4$ 4.10(c): $d = 6$ 4.10(d): $d = 8$

Figura 4.10: Sorriso: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

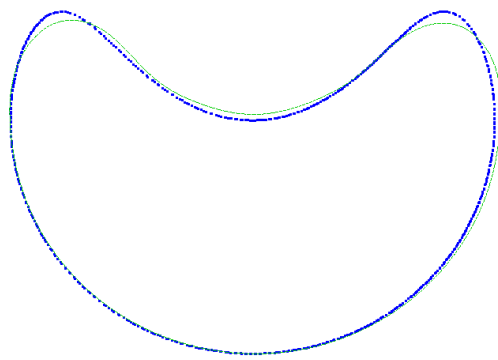
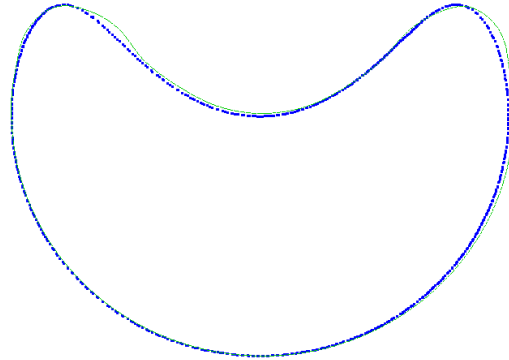
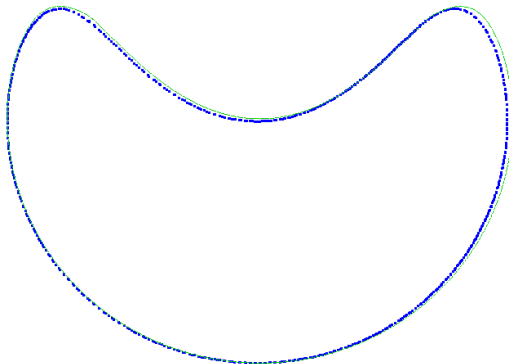
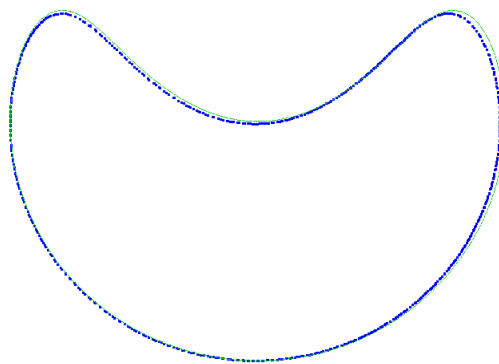
4.11(a): $d = 2$; $l_{max} = 2$ 4.11(b): $d = 2$; $l_{max} = 3$ 4.11(c): $d = 2$; $l_{max} = 4$ 4.11(d): $d = 2$; $l_{max} = 5$

Figura 4.11: Sorriso: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

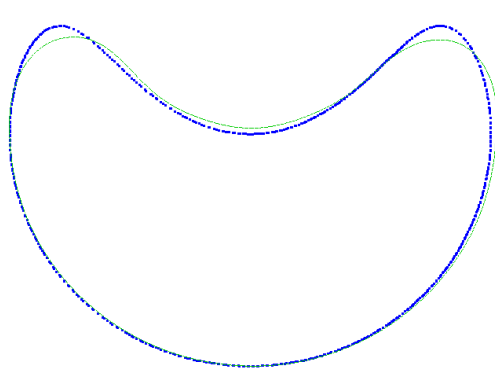
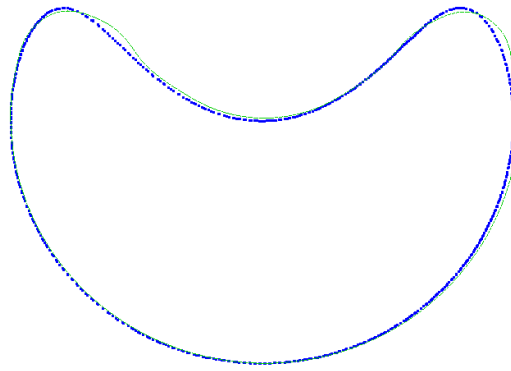
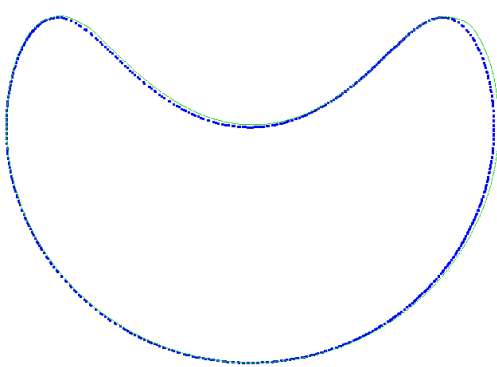
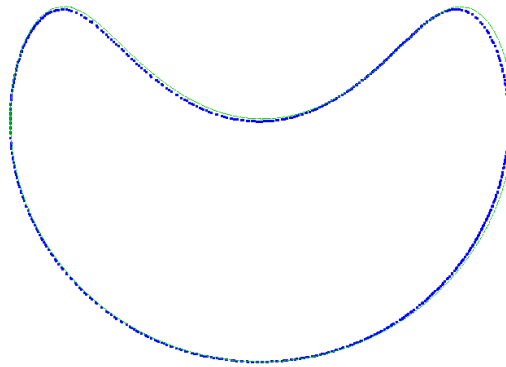
4.12(a): $d = 2$; $l_{max} = 2$ 4.12(b): $d = 2$; $l_{max} = 3$ 4.12(c): $d = 2$; $l_{max} = 4$ 4.12(d): $d = 2$; $l_{max} = 5$

Figura 4.12: Sorriso: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

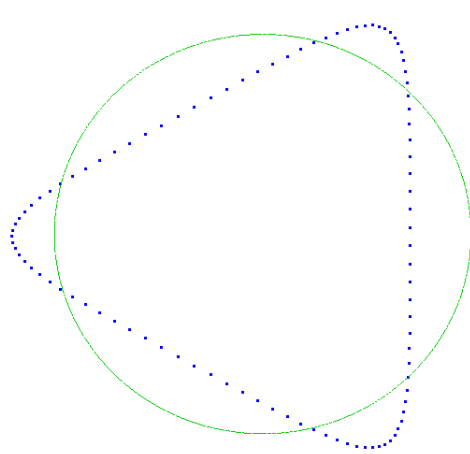
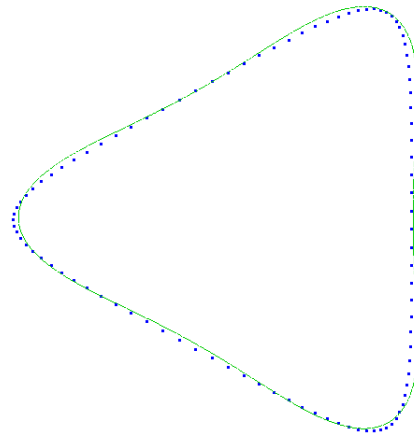
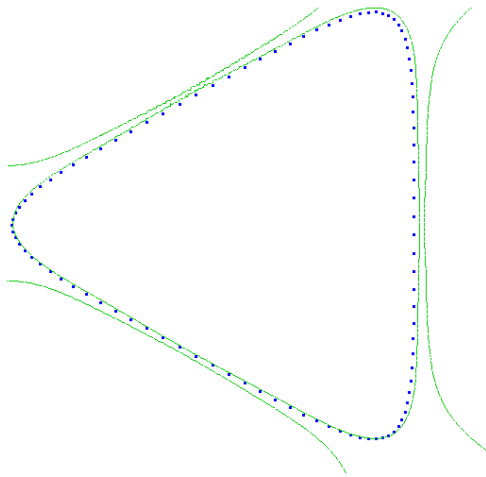
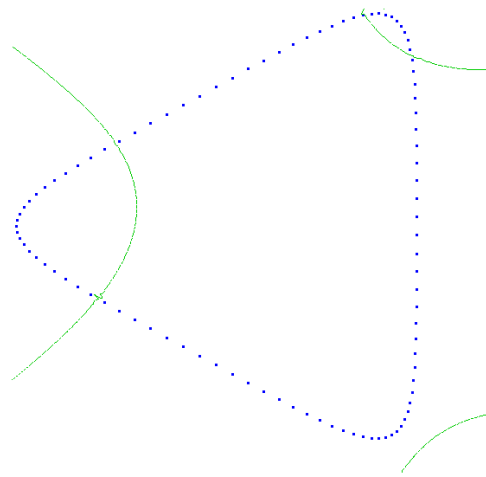
4.13(a): $d = 2$ 4.13(b): $d = 4$ 4.13(c): $d = 6$ 4.13(d): $d = 10$

Figura 4.13: Hipociclóide: método 1 usando um polinômio de grau d com $\mu = 0.125$.

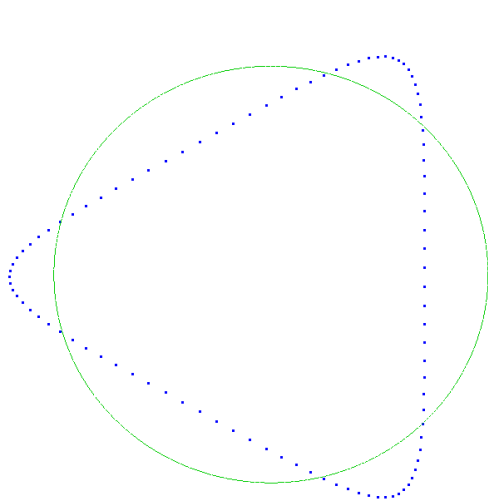
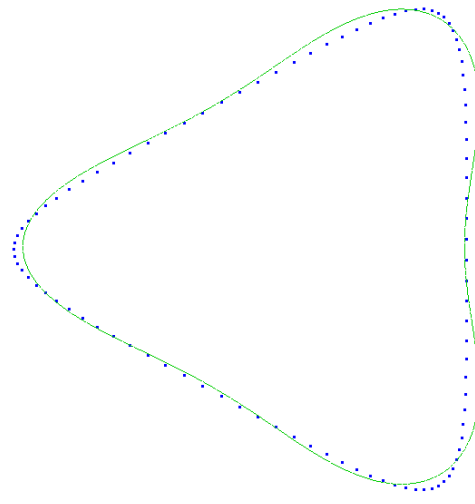
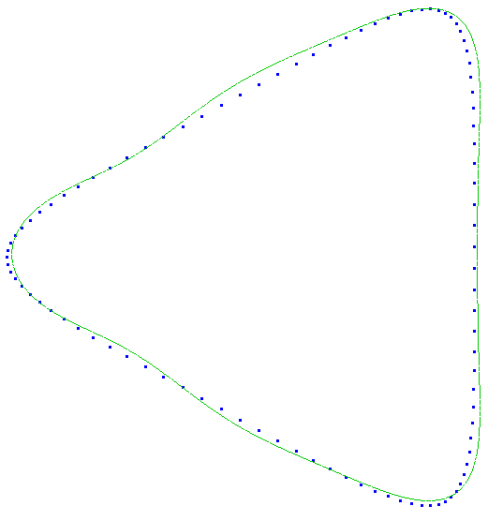
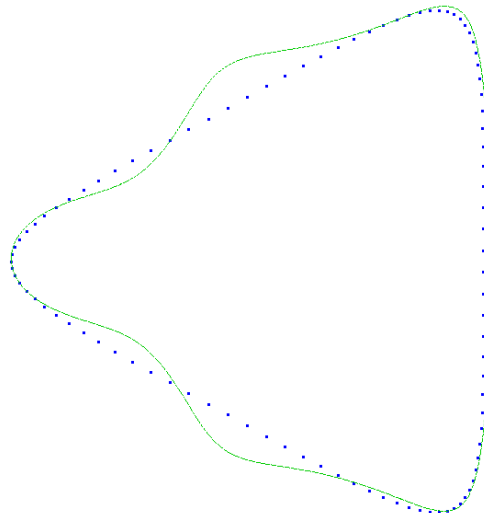
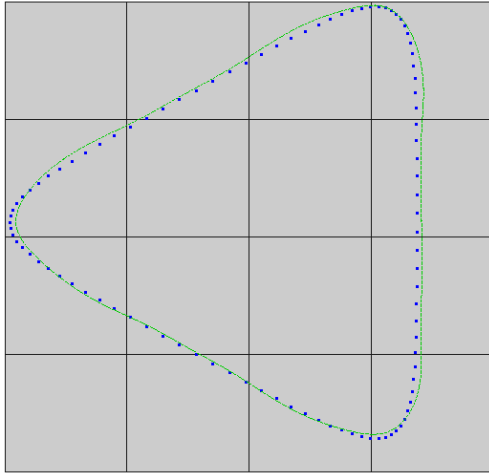
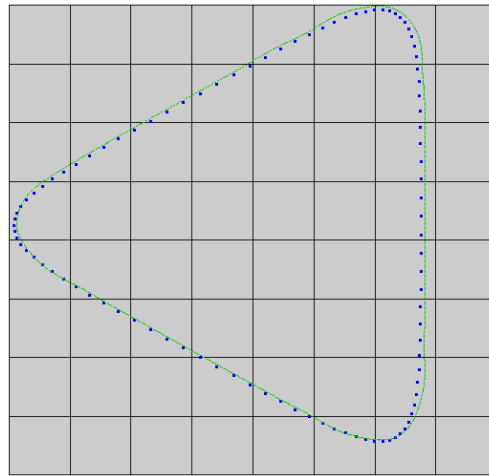
4.14(a): $d = 2$ 4.14(b): $d = 4$ 4.14(c): $d = 6$ 4.14(d): $d = 10$

Figura 4.14: Hipociclóide: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

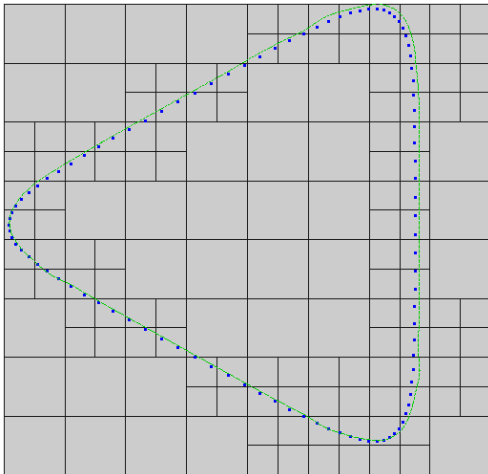
Neste exemplo as aproximações dos métodos estará acompanhada da construção da árvore *Quad-Tree*.



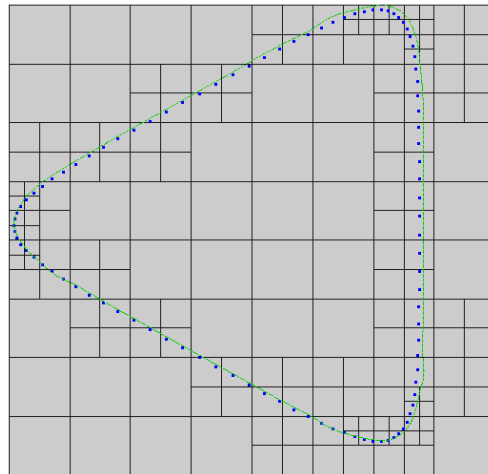
4.15(a): $d = 2$; $l_{max} = 2$



4.15(b): $d = 2$; $l_{max} = 3$



4.15(c): $d = 2$; $l_{max} = 4$



4.15(d): $d = 2$; $l_{max} = 5$

Figura 4.15: Hipociclóide: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

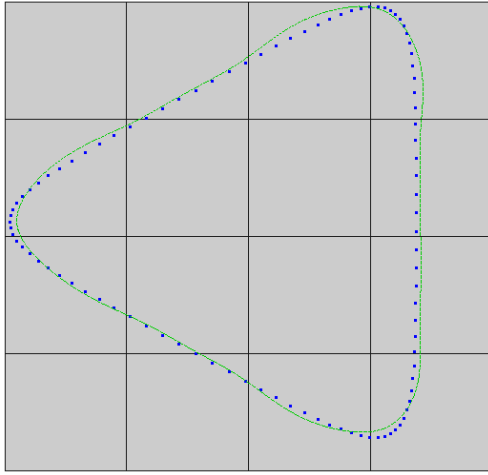
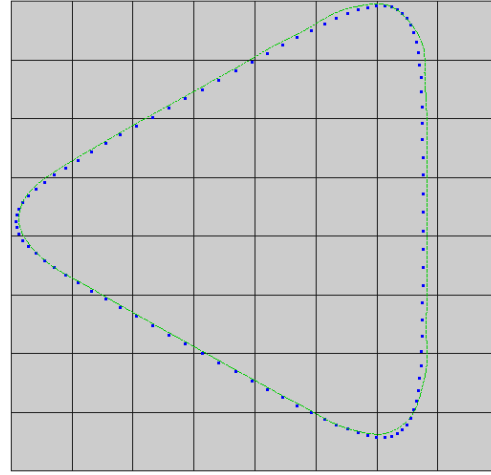
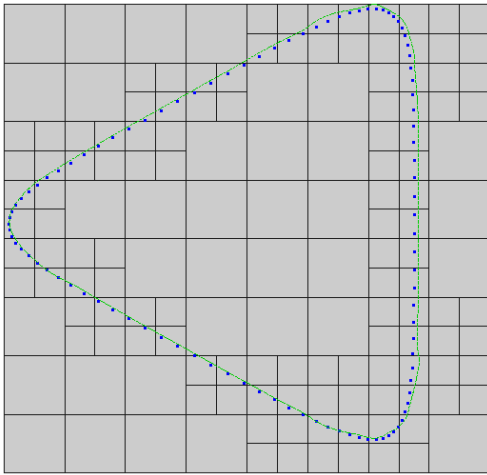
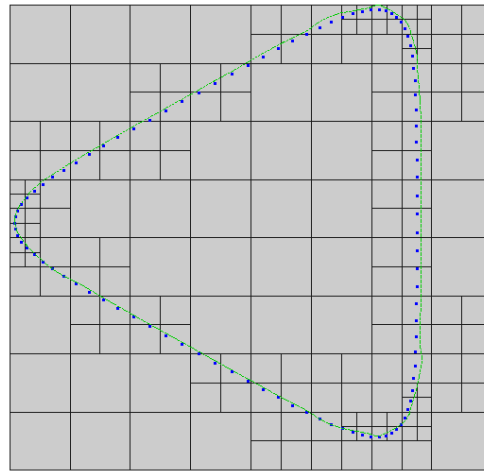
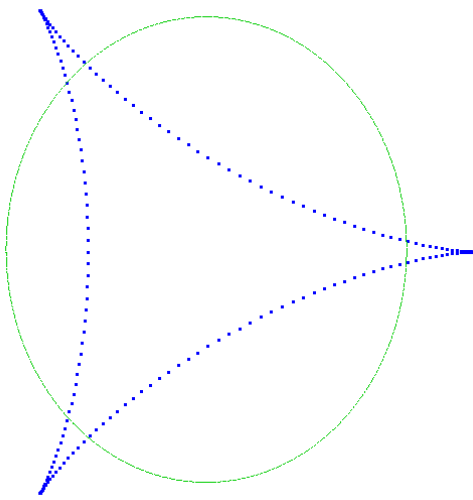
4.16(a): $d = 2$; $l_{max} = 2$ 4.16(b): $d = 2$; $l_{max} = 3$ 4.16(c): $d = 2$; $l_{max} = 4$ 4.16(d): $d = 2$; $l_{max} = 5$

Figura 4.16: Hipociclóide: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

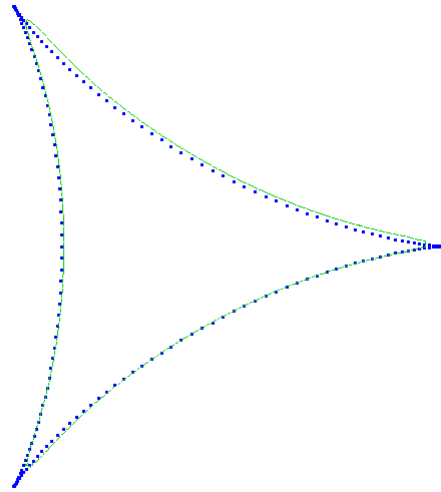
4.2

Curvas com singularidades

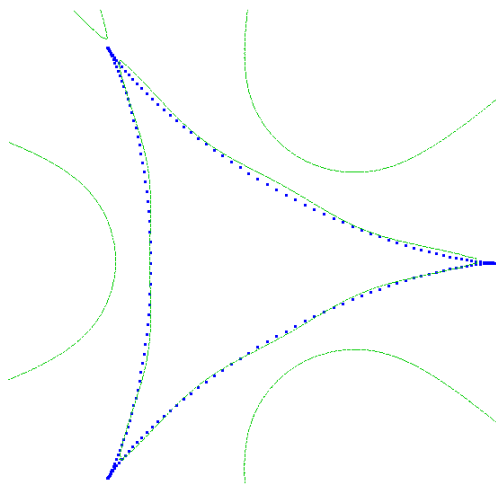
Agora serão ilustradas curvas com singularidades. Sob ponto de vista teórico, todos esses métodos não deveriam funcionar. Mas mesmo assim, o novo método proposto obteve boas aproximações.



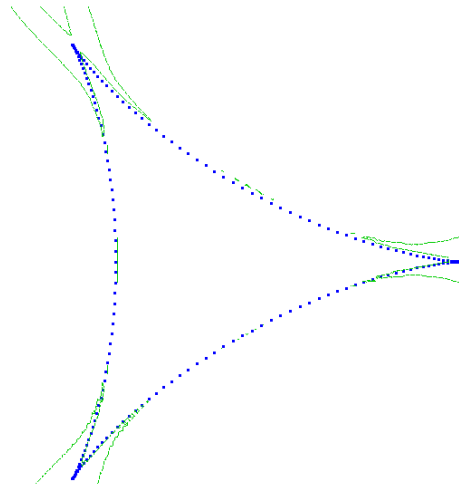
4.17(a): $d = 2$



4.17(b): $d = 4$



4.17(c): $d = 6$



4.17(d): $d = 8$

Figura 4.17: Cúspide: método 1 usando um polinômio de grau d com $\mu = 0.125$.

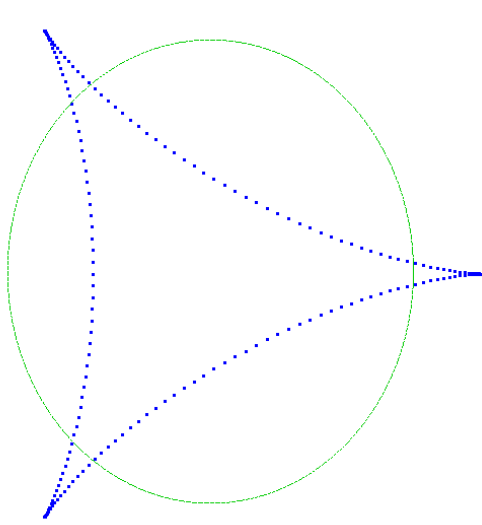
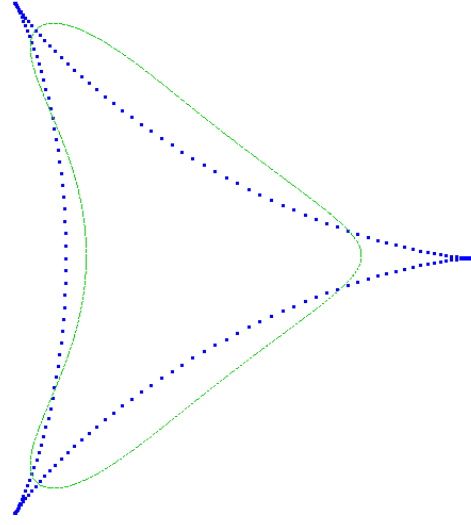
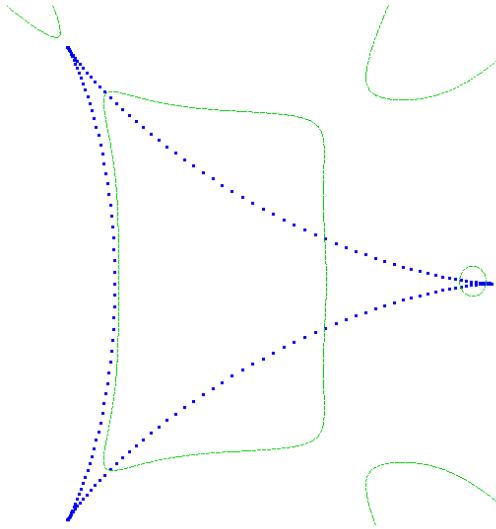
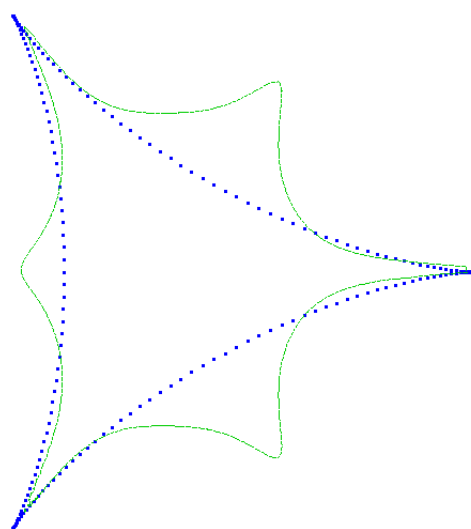
4.18(a): $d = 2$ 4.18(b): $d = 4$ 4.18(c): $d = 6$ 4.18(d): $d = 8$

Figura 4.18: Cúspide: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

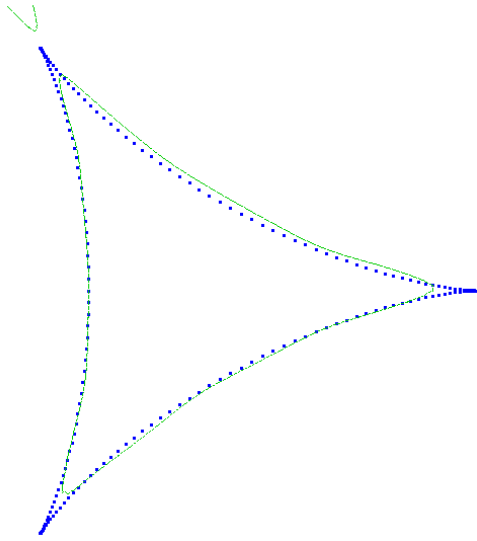
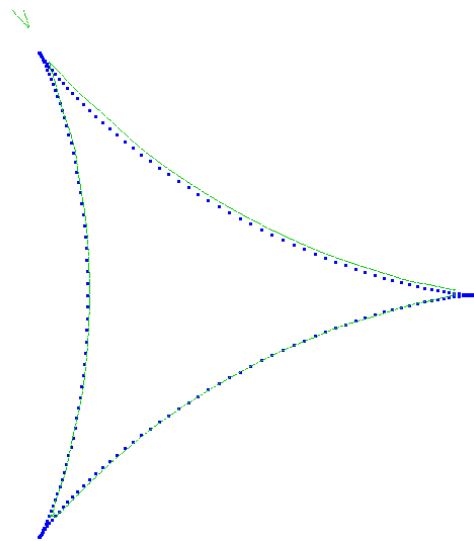
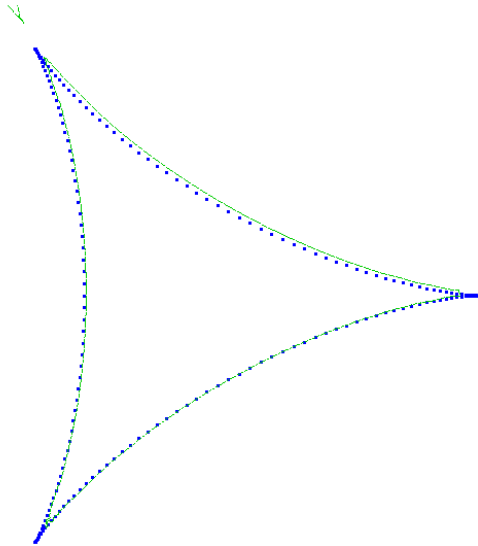
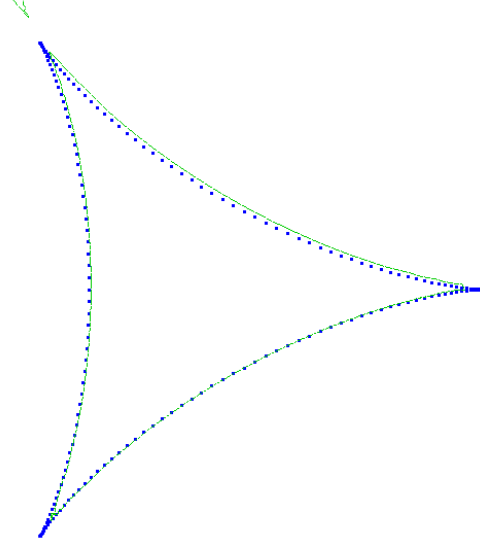
4.19(a): $d = 2$; $l_{max} = 2$ 4.19(b): $d = 2$; $l_{max} = 3$ 4.19(c): $d = 2$; $l_{max} = 4$ 4.19(d): $d = 2$; $l_{max} = 5$

Figura 4.19: Cúspide: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

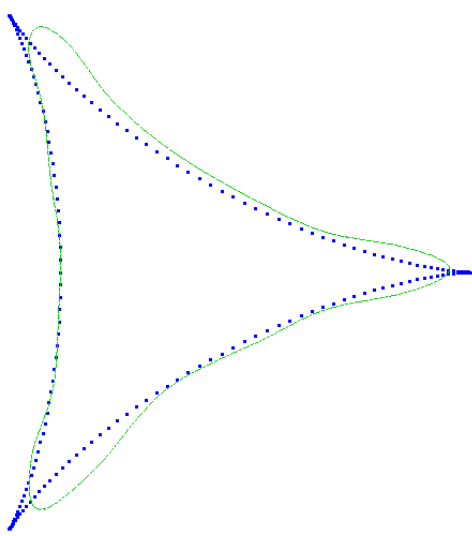
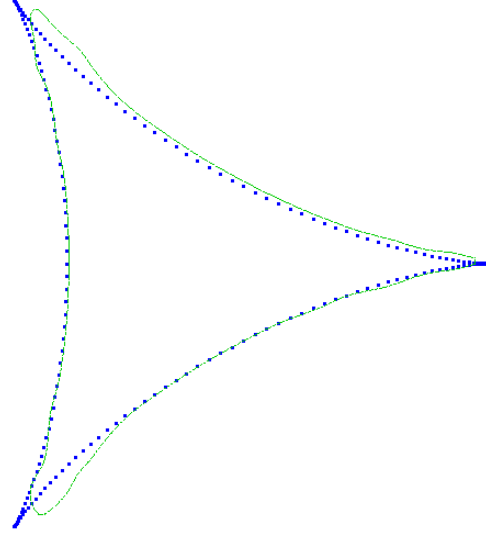
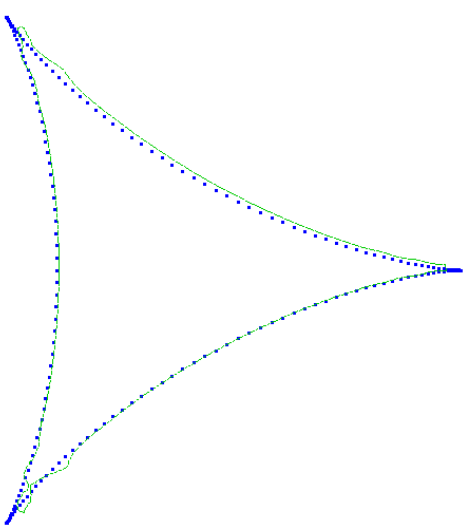
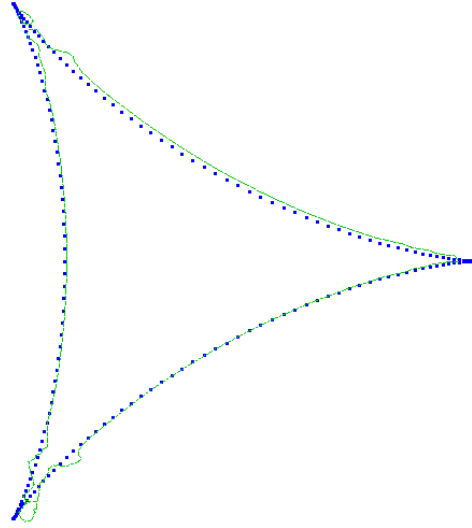
4.20(a): $d = 2$; $l_{max} = 2$ 4.20(b): $d = 2$; $l_{max} = 3$ 4.20(c): $d = 2$; $l_{max} = 4$ 4.20(d): $d = 2$; $l_{max} = 5$

Figura 4.20: Cúspide: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

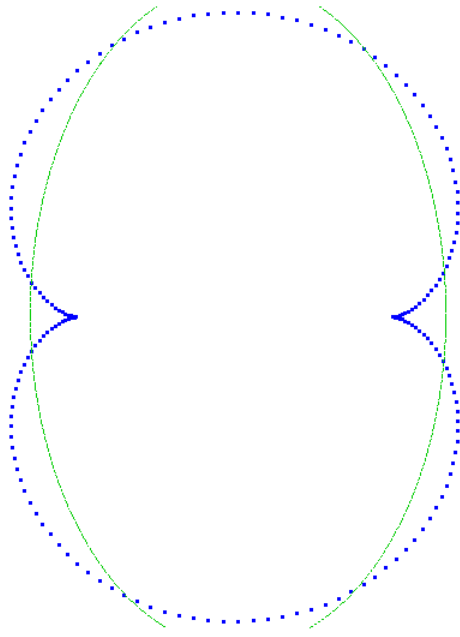
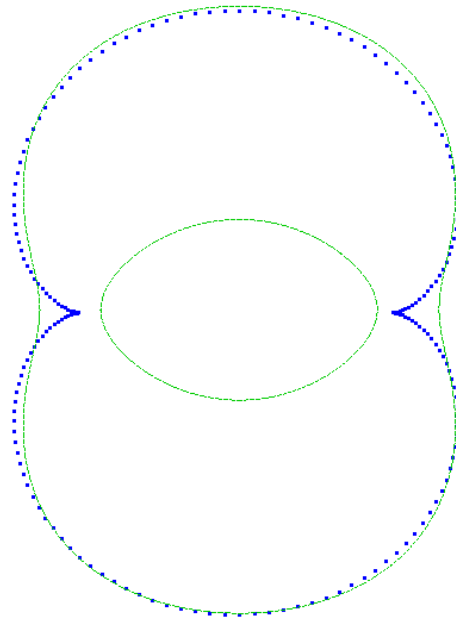
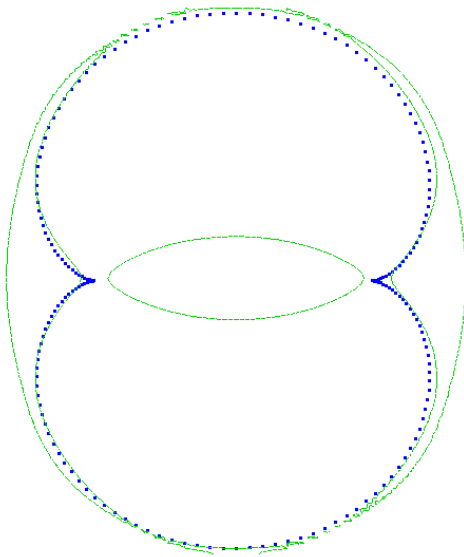
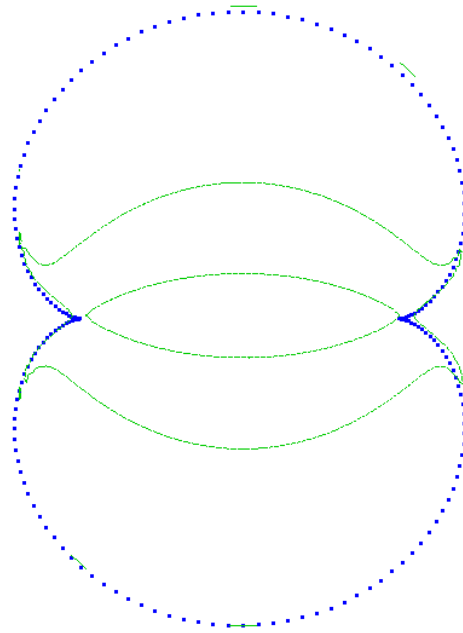
4.21(a): $d = 2$ 4.21(b): $d = 4$ 4.21(c): $d = 6$ 4.21(d): $d = 8$

Figura 4.21: Nephroid: método 1 usando um polinômio de grau d com $\mu = 0.125$.

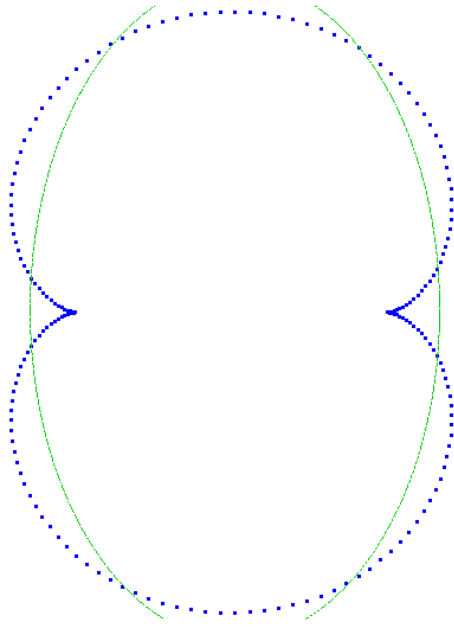
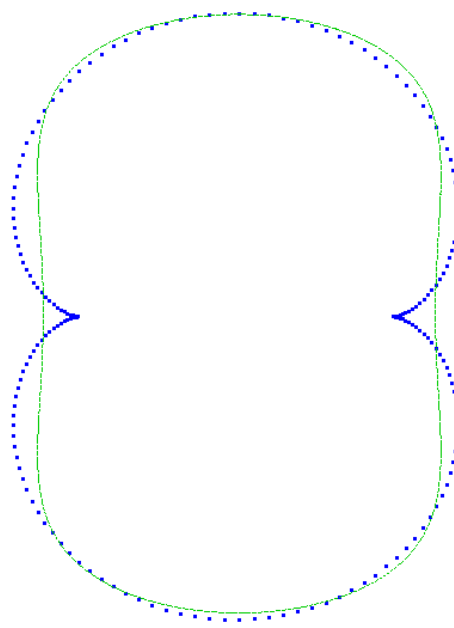
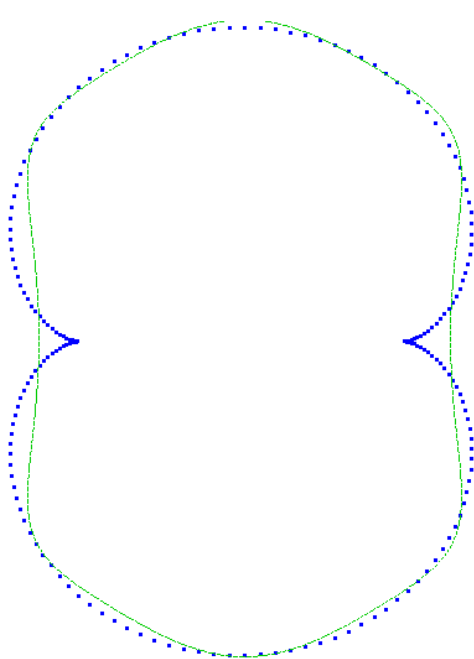
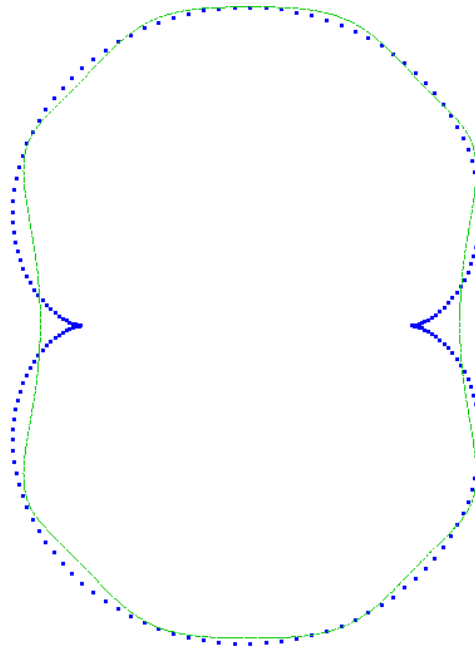
4.22(a): $d = 2$ 4.22(b): $d = 4$ 4.22(c): $d = 6$ 4.22(d): $d = 8$

Figura 4.22: Nephroid: método 2 usando um polinômio de grau d com $\mu = 0.125$ e $\kappa = 0.001$.

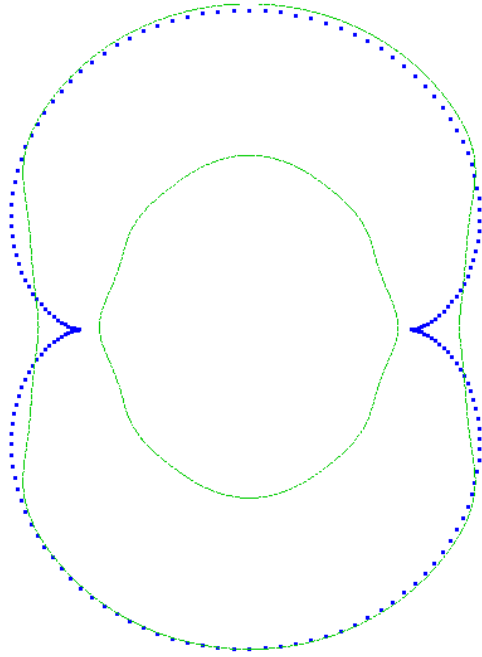
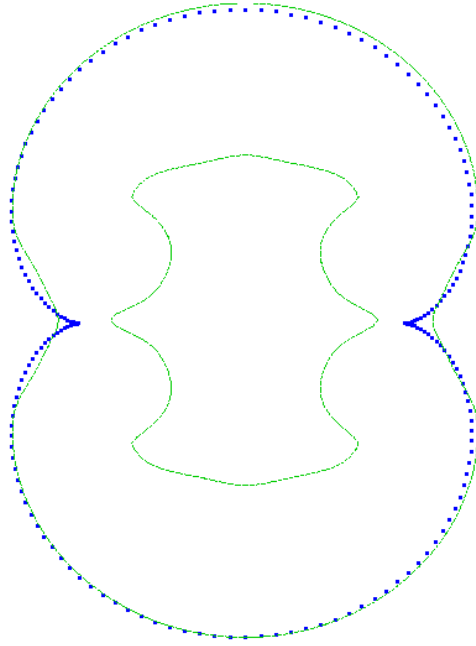
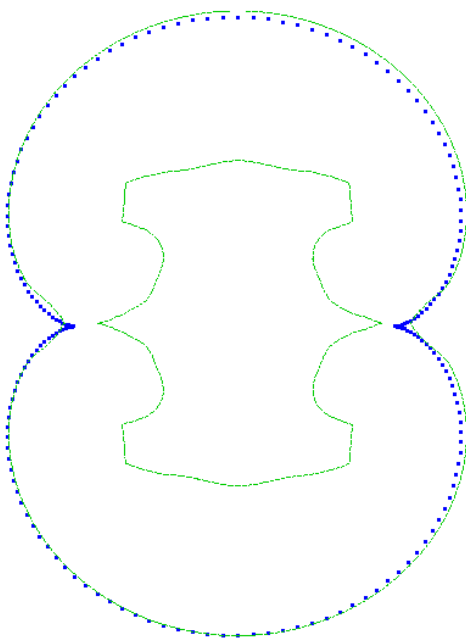
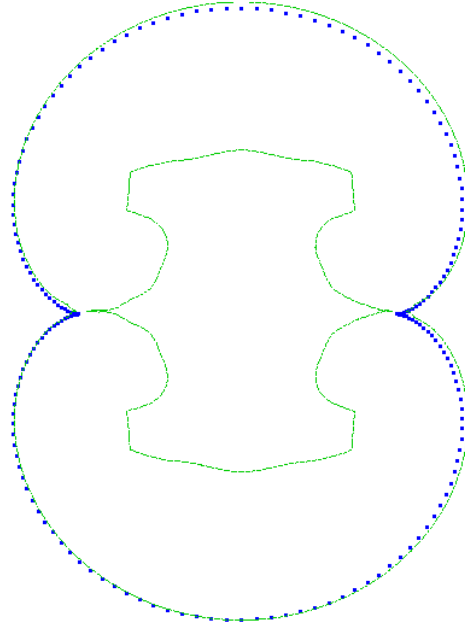
4.23(a): $d = 2$; $l_{max} = 2$ 4.23(b): $d = 2$; $l_{max} = 3$ 4.23(c): $d = 2$; $l_{max} = 4$ 4.23(d): $d = 2$; $l_{max} = 5$

Figura 4.23: Nephroid: método 3 usando localmente um polinômio de grau d com $\mu = 0.125$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

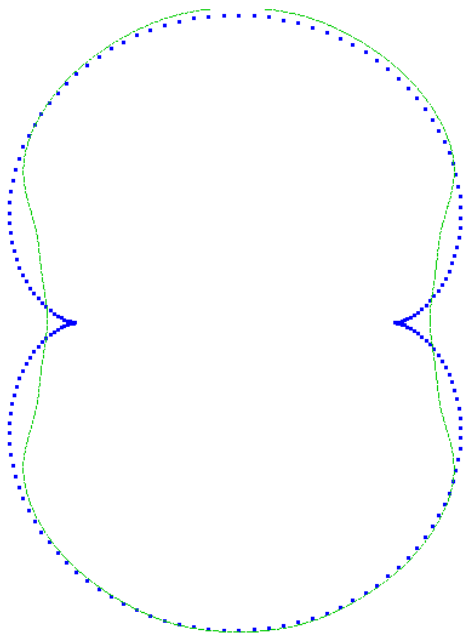
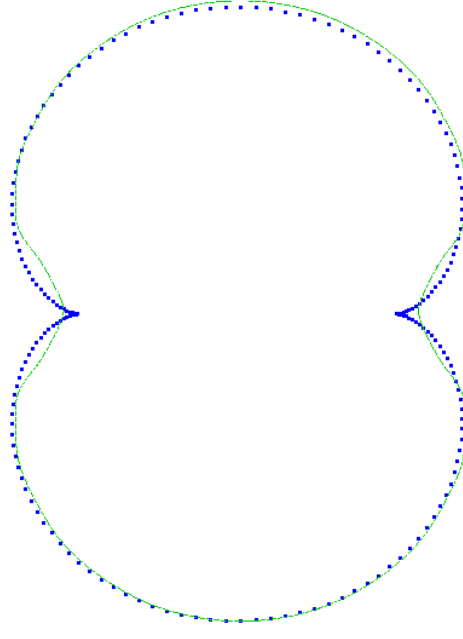
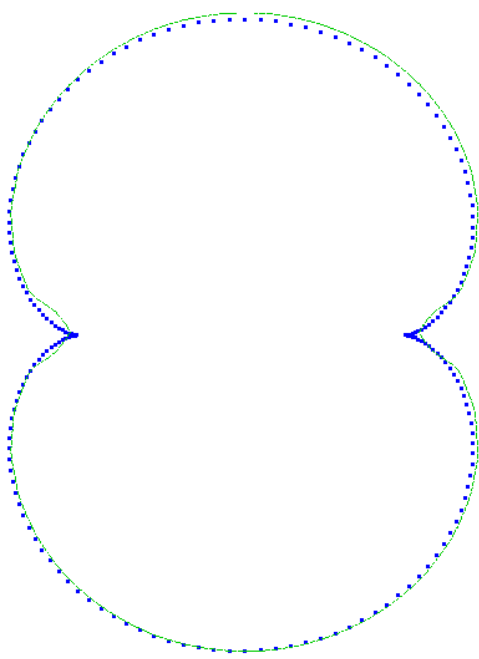
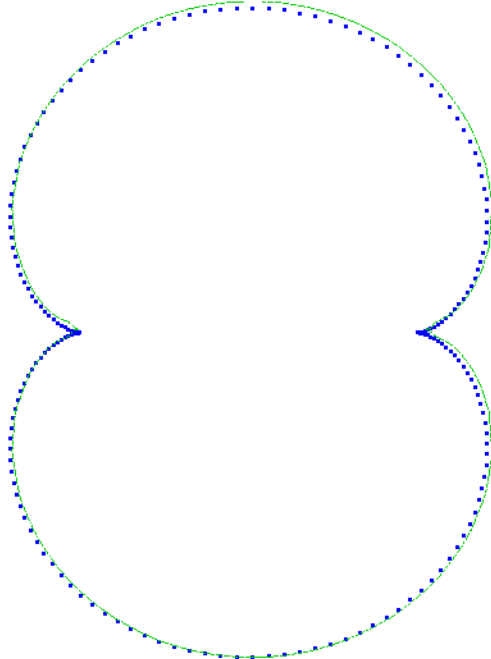
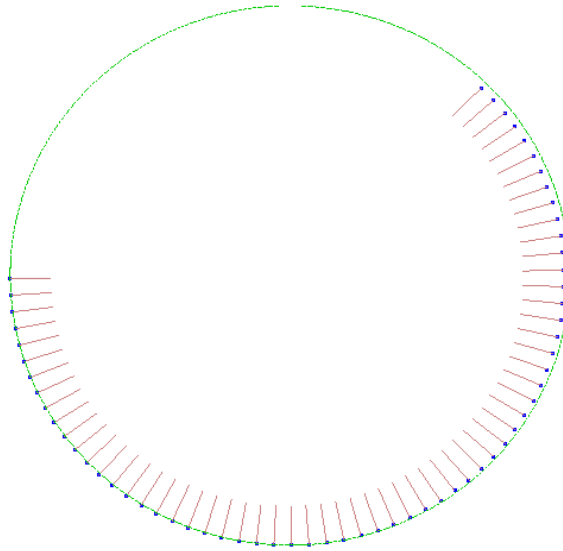
4.24(a): $d = 2$; $l_{max} = 2$ 4.24(b): $d = 2$; $l_{max} = 3$ 4.24(c): $d = 2$; $l_{max} = 4$ 4.24(d): $d = 2$; $l_{max} = 5$

Figura 4.24: Nephroid: método 4 usando localmente um polinômio de grau d com $\mu = 0.125$, $\kappa = 0.001$ e uma *Quad-Tree* com nível máximo igual a l_{max} .

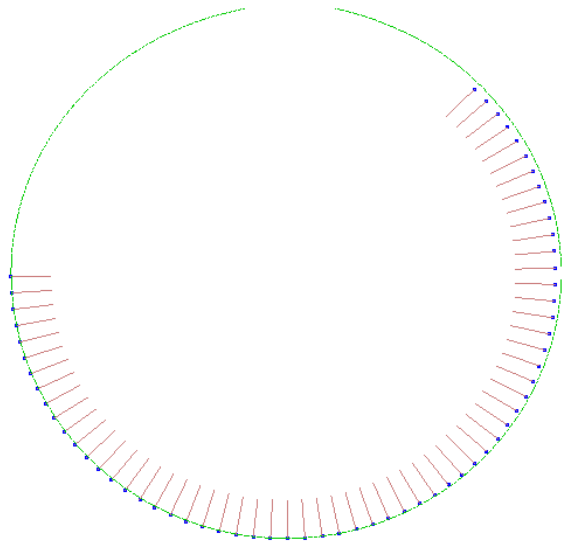
4.3

Completando buracos

Muitas vezes os dados vindos dos dispositivos vêm com ruído ou com buracos. Esses buracos podem ser muitas vezes completados diretamente por esses métodos implícitos. Seguem alguns exemplos para ilustrar a performance do novo método na solução desse problema.

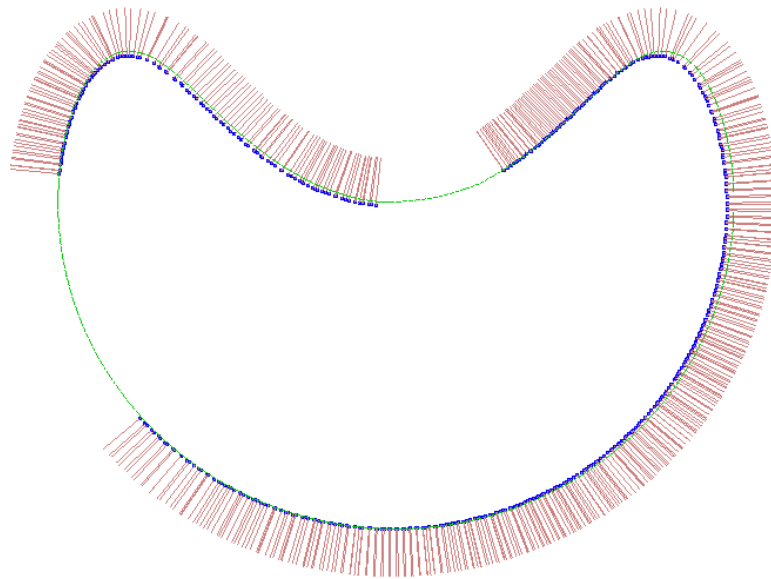


4.25(a): Utilizando o método 3

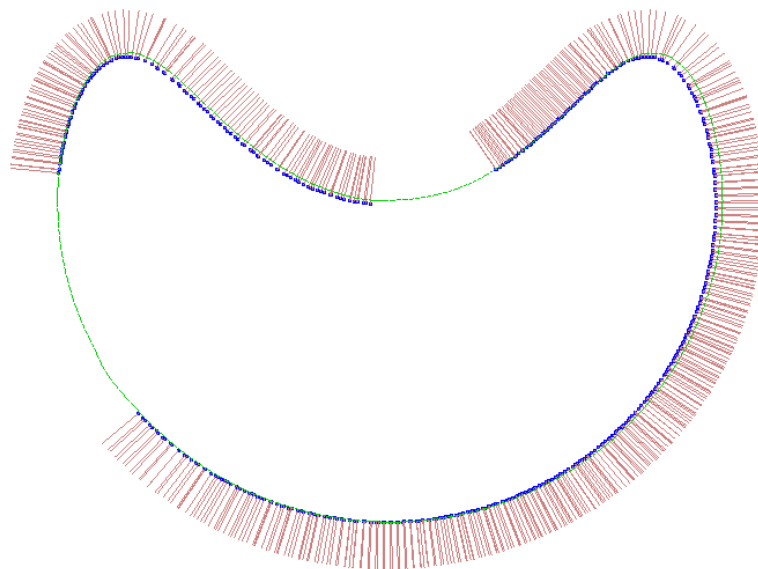


4.25(b): Utilizando o método 4

Figura 4.25: Círculo: (a) método 3 e (b) método 4 usando localmente um polinômio de grau $d = 2$ com $\mu = 0.17$, $\kappa = 0.01$ e uma *Quad-Tree* com nível máximo igual a 5

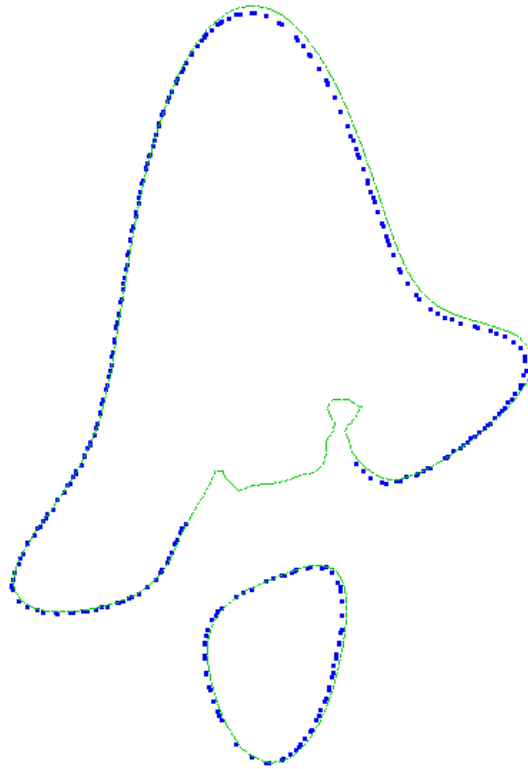


4.26(a): Utilizando o método 3

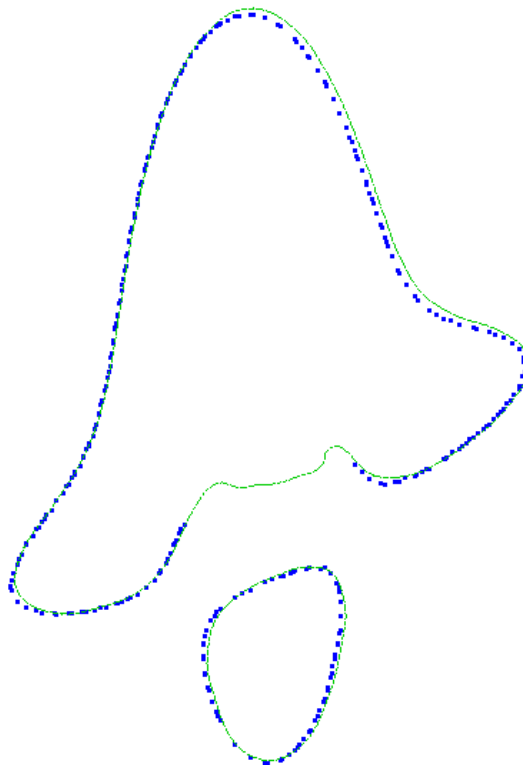


4.26(b): Utilizando o método 4

Figura 4.26: Sorriso: (a) método 3 e (b) método 4 usando localmente um polinômio de grau $d = 2$ com $\mu = 0.17$, $\kappa = 0.01$ e uma *Quad-Tree* com nível máximo igual a 5



4.27(a): Utilizando o método 3



4.27(b): Utilizando o método 4

Figura 4.27: Taubin: (a) método 3 e (b) método 4 usando localmente um polinômio de grau $d = 2$ com $\mu = 0.17$, $\kappa = 0.01$ e uma *Quad-Tree* com nível máximo igual a 4