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Apêndice 1 - Solução das EDOs com distribuição normal

Este apêndice apresenta o detalhamento da solução das EDOs de Ricatti para o caso em que o tamanho dos saltos possui distribuição normal.

Para o modelo contido na seção 3.6.1 e descrito pela eq. (18) resulta em:

$$K_0 = \begin{bmatrix} -\lambda_\chi \\ \mu_\xi^* \end{bmatrix} \qquad K_1 = \begin{bmatrix} -k_\chi & 0 \\ 0 & 0 \end{bmatrix} \qquad \beta'(t) = [\beta_1(t) \quad \beta_2(t)]$$

$$H_0 = \begin{bmatrix} \sigma_\chi^2 & \rho\sigma_\chi\sigma_\xi \\ \rho\sigma_\chi\sigma_\xi & \sigma_\xi^2 \end{bmatrix} \qquad H_1 = \mathbf{0} \qquad \ell_0 = \varpi \qquad \ell_1 = 0$$

Além disso, já havia sido mencionado que a taxa de juros (livre de risco) é constante ($r_1 = 0$), portanto, $R = r_0 = r$.

Como H_1, ℓ_1 e r_1 são nulos, a EDO (5) se reduz a $\dot{\beta}_1(t) = k_\chi \beta_1(t)$ e $\dot{\beta}_2(t) = 0$. A solução é imediata e fornece $\beta_1(t) = \beta_1(\tau)\exp(-k_\chi(\tau - t))$. Como $\beta(\tau) = u = (1 \quad 1) \Rightarrow \beta_1(\tau) = \beta_2(\tau) = 1$. Então resulta em $\beta_1(t) = \exp(-k_\chi(\tau - t))$ e $\beta_2(t) = 1$.

A EDO (6) resulta em

$$\dot{\alpha}(t) = r + \lambda_\chi\beta_1 - \mu_\xi^*\beta_2 - \tfrac{1}{2}\beta_1^2\sigma_\chi^2 - \beta_1\beta_2\sigma_\chi\sigma_\xi\rho - \tfrac{1}{2}\beta_2^2\sigma_\xi^2 - \varpi(\theta(\beta_1) - 1)$$

Usando nesta equação as expressões de β_1 e β_2 encontrados acima:

$$\dot{\alpha}(t) = r + \lambda_\chi e^{-k_\chi(\tau-t)} - \mu_\xi^* - \tfrac{1}{2}\sigma_\chi^2 e^{-2k_\chi(\tau-t)} - \sigma_\chi\sigma_\xi\rho e^{-k_\chi(\tau-t)} - \tfrac{1}{2}\sigma_\xi^2 -$$
$$\varpi(\theta(\beta_1) - 1)$$

Integrando esta equação entre t e τ:

$$\alpha(\tau) - \alpha(t) = r(\tau - t) + \frac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau-t)}) - \mu_\xi^*(\tau - t) - \frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau-t)}) -$$
$$\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1 - e^{-k_\chi(\tau-t)}) - \tfrac{1}{2}\sigma_\xi^2(\tau - t) - \varpi\int_t^\tau (\theta(\beta_1) - 1)ds$$

A condição terminal é $\alpha(\tau) = 0$, logo:

$$\alpha(t) = -r(\tau - t) - \frac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) + \mu_\xi^*(\tau - t) + \frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau - t)}) +$$

$$\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) + \tfrac{1}{2}\sigma_\xi^2(\tau - t) + \varpi\int_t^\tau (\theta(\beta_1) - 1)ds$$

Resta calcular o último termo da equação acima que será denominado $B(\tau - t)$ isto é $B(\tau - t) = \varpi\int_t^\tau (\theta(\beta_1) - 1)ds$. A função característica da distribuição Gaussiana $\nu(\mu_\nu^*, \sigma_\nu^2)$ é $\theta(c) = \exp(\mu_\nu^* c + \tfrac{1}{2}\sigma_\nu^2 c^2)$.

O valor desta função calculado em β_1 é dado por:

$$\theta(\beta_1(t)) = \exp\left(\mu_\nu^* e^{-k_\chi(\tau - t)} + \tfrac{1}{2}\sigma_\nu^2 e^{-2k_\chi(\tau - t)}\right)$$

Logo o resultado será:

$$B(\tau - t) = \varpi\int_t^\tau \left[\exp\left(\mu_\nu^* e^{-k_\chi(\tau - s)} + \tfrac{1}{2}\sigma_\nu^2 e^{-2k_\chi(\tau - s)}\right) - 1\right]ds$$

Agora a equação da transformada em (19) está sob a MME e pode ser escrita como:

$$\psi^Q(\chi_t, \xi_t, t, \tau) = \exp\Big(-r(\tau - t) - \tfrac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) + \mu_\xi^*(\tau - t) +$$

$$\frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau - t)}) + \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) +$$

$$\tfrac{1}{2}\sigma_\xi^2(\tau - t) + e^{-k_\chi(\tau - t)}\chi_t + \xi_t + B(\tau - t)\Big)$$

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Apêndice 2 - Solução das EDOs com distribuição exponencial

Este apêndice apresenta o detalhamento da solução das EDOs de Ricatti para o caso em o tamanho dos saltos possui distribuição exponencial.

A solução das EDOs (5) e (6) para o caso de distribuição exponencial segue as mesmas etapas do Apêndice 1. Os resultados para β_1 e β_2 são os mesmos. Para α o resultado é análogo apenas diferindo nos dois últimos termos que contêm as integrais da função característica da distribuição exponencial:

$$\alpha(t) = -r(\tau - t) - \frac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) + \mu_\xi^*(\tau - t) + \frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau - t)}) +$$

$$\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1 - e^{-k_\chi(\tau - t)}) + \tfrac{1}{2}\sigma_\xi^2(\tau - t) + \varpi_u \int_t^\tau (\theta_u(\beta_1) - 1)ds - \varpi_d \int_t^\tau (\theta_d(\beta_1) - 1)ds$$

Resta calcular a função característica $\theta(\beta_1(t))$ para a distribuição exponencial. Seja Z uma variável aleatória com distribuição exponencial de parâmetro φ:

$$f(z) = \begin{cases} \varphi e^{-\varphi z} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

O valor esperado de Z é $E(Z) = \frac{1}{\varphi} = \eta$. A função característica é dada por:

$$\theta(c) = \int_{\Re} \exp(cz) f(z) dz$$

Substituindo $f(z)$ e integrando de zero a infinito resulta em $\theta(c) = \frac{\varphi}{\varphi - c} = \frac{1}{1 - \eta c}$

Agora serão resolvidas as integrais em $\alpha(t)$. Mas é suficiente resolver apenas uma das integrais. Seja então $\int_t^\tau \varpi(\theta(\beta_1) - 1)ds$ e usando o resultado anterior:

$$\int_t^\tau \varpi(\theta(\beta_1) - 1)ds = \varpi \int_t^\tau \left(\frac{1}{1 - \eta^* \exp(-k_\chi(\tau - s))} - 1 \right) ds = \varpi \int_t^\tau \frac{\eta^* \exp(-k_\chi(\tau - s))}{1 - \eta^* \exp(-k_\chi(\tau - s))} ds$$

ou ainda:

$$\int_t^\tau \varpi(\theta(\beta_1)-1)ds = \frac{\varpi}{k_\chi}\ln\left(\frac{1-\eta^* \exp(-k_\chi(\tau-t))}{1-\eta^*}\right)$$

Logo, o termo $\alpha(t)$ será escrito por:

$$\alpha(t) = -r(\tau-t) - \frac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \mu_\xi^*(\tau-t) + \frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)}) +$$

$$\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \tfrac{1}{2}\sigma_\xi^2(\tau-t) + \frac{\varpi_u}{k_\chi}\ln\left(\frac{1-\eta_u^* \exp(-k_\chi(\tau-t))}{1-\eta_u^*}\right) - \frac{\varpi_d}{k_\chi}\ln\left(\frac{1-\eta_d^* \exp(-k_\chi(\tau-t))}{1-\eta_d^*}\right)$$

Para o caso de mais de um salto para cima e para baixo, escreve-se:

$$\alpha(t) = -r(\tau-t) - \frac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \mu_\xi^*(\tau-t) + \frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)}) +$$

$$\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \tfrac{1}{2}\sigma_\xi^2(\tau-t) + \sum_i \frac{\varpi_{ui}}{k_\chi}\ln\left(\frac{1-\eta_{ui}^* \exp(-k_\chi(\tau-t))}{1-\eta_{ui}^*}\right) + \sum_i \frac{\varpi_{di}}{k_\chi}\ln\left(\frac{1-\eta_{di}^* \exp(-k_\chi(\tau-t))}{1-\eta_{di}^*}\right)$$

A equação da transforma DK, sob a MME, está concluída:

$$\psi^Q(\chi_t,\xi_t,t,\tau) = \exp\Bigg[-r(\tau-t) - \frac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \mu_\xi^*(\tau-t) +$$

$$\frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)}) + \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \frac{1}{2}\sigma_\xi^2(\tau-t) +$$

$$\sum_i \frac{\varpi_{ui}}{k_\chi}\ln\left(\frac{1-\eta_{ui}^* \exp(-k_\chi(\tau-t))}{1-\eta_{ui}^*}\right) + \sum_i \frac{\varpi_{di}}{k_\chi}\ln\left(\frac{1-\eta_{di}^* \exp(-k_\chi(\tau-t))}{1-\eta_{di}^*}\right) + e^{-k_\chi(\tau-t)}\chi_t + \xi_t\Bigg]$$

13 Apêndice 3 - Solução das EDOs com reversão à média para ξ

A solução das EDOs (5) e (6) segue o mesmo procedimento do Apêndice 1. Para a EDO (5), $\beta_1(t)$ e $\beta_2(t)$ são análogos:

$$\dot{\beta}_1(t) = k_\chi \beta_1 \qquad \text{e} \qquad \dot{\beta}_2(t) = k_\xi \beta_2$$

Logo as soluções para $\beta_1(t)$ e $\beta_2(t)$ são:

$$\beta_1 = e^{-k_\chi(\tau-t)} \qquad \text{e} \qquad \beta_2(t) = e^{-k_\xi(\tau-t)}$$

Para a EDO (6) o resultado é análogo ao do Apêndice 1, apenas trocando μ_ξ^* por $k_\xi\hat{\xi}$:

$$\dot{\alpha}(t) = r + \lambda_\chi\beta_1 - k_\xi\hat{\xi}\beta_2 - \tfrac{1}{2}\beta_1^2\sigma_\chi^2 - \beta_1\beta_2\sigma_\chi\sigma_\xi\rho - \tfrac{1}{2}\beta_2^2\sigma_\xi^2 - \varpi(\theta(\beta_1)-1)$$

Usando $\beta_1(t)$ e $\beta_2(t)$ encontrados acima:

$$\dot{\alpha}(t) = r + \lambda_\chi e^{-k_\chi(\tau-t)} - k_\xi\hat{\xi}e^{-k_\xi(\tau-t)} - \tfrac{1}{2}\sigma_\chi^2 e^{-2k_\chi(\tau-t)} - \sigma_\chi\sigma_\xi\rho e^{-(k_\chi+k_\xi)(\tau-t)} -$$
$$\tfrac{1}{2}\sigma_\xi^2 e^{-2k_\xi(\tau-t)} - \varpi(\theta(\beta_1)-1)$$

Integrando esta equação entre t e τ:

$$\alpha(\tau) - \alpha(t) = r(\tau - t) + \frac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau-t)}) - \hat{\xi}(1 - e^{-k_\xi(\tau-t)}) -$$
$$\frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau-t)}) - \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi + k_\xi}(1 - e^{-(k_\chi+k_\xi)(\tau-t)}) - \frac{\sigma_\xi^2}{4k_\xi}(1 - e^{-2k_\xi(\tau-t)}) - B(\tau - t)$$

onde $B(\tau - t)$ foi calculado no Apêndice 1. A condição terminal é $\alpha(\tau) = 0$, logo:

$$\alpha(t) = -r(\tau - t) - \frac{\lambda_\chi}{k_\chi}(1 - e^{-k_\chi(\tau-t)}) + \hat{\xi}(1 - e^{-k_\xi(\tau-t)}) +$$
$$\frac{\sigma_\chi^2}{4k_\chi}(1 - e^{-2k_\chi(\tau-t)}) + \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi + k_\xi}(1 - e^{-(k_\chi+k_\xi)(\tau-t)}) + \frac{\sigma_\xi^2}{4k_\xi}(1 - e^{-2k_\xi(\tau-t)}) + B(\tau - t)$$

A equação da transformada DK sob a MME é dada por:

$$\psi^{Q}(\chi_t,\xi_t,t,\tau)=\exp\Big(-r(\tau-t)-\tfrac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)})+\hat{\xi}(1-e^{-k_\xi(\tau-t)})+$$

$$\frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)})+\frac{\rho\sigma_\chi\sigma_\xi}{k_\chi+k_\xi}(1-e^{-(k_\chi+k_\xi)(\tau-t)})+\frac{\sigma_\xi^2}{4k_\xi}(1-e^{-2k_\xi(\tau-t)})+$$

$$e^{-k_\chi(\tau-t)}\chi_t+e^{-k_\xi(\tau-t)}\xi_t+B(\tau-t)\Big)$$

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Apêndice 4 - Estrutura a termo da volatilidade - Modelo Básico

Este Apêndice apresenta o detalhamento da derivação da estrutura a termo da volatilidade do modelo básico. A estrutura à termo é o comportamento da volatilidade do modelo com a maturidade dos contratos. A eq. (33) da seção 3.7.1 mostra os preços futuros sob a MME. É preciso calcular a variância instantânea dada por $\frac{1}{dt}VAR\left(\frac{dF_{t,\tau}}{F_{t,\tau}}\right)$. Reescrevendo a eq. (33):

$$\ln(F_{\tau,t}) = f(\tau) + e^{-k_\chi(\tau-t)}\chi_t + \xi_t + A(\tau - t)$$

onde:

$$A(\tau-t) = \left(\mu_\xi^*(\tau-t) - \frac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)}) + \right.$$

$$\left. \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \frac{1}{2}\sigma_\xi^2(\tau-t) \right)$$

Pode-se escrever simplificadamente que $\ln(F_{t,\tau}) = g(\chi_t, \xi_t, t)$ onde a função $g(\cdot)$ expressa o segundo membro da equação acima. Usando o lema de Itô e escrevendo as derivadas parciais como subscritos resulta em:

$$d(\ln F_{\tau,t}) = g_t dt + g_\chi d\chi_t + g_\xi d\xi_t + \tfrac{1}{2}g_{\chi\chi}(d\chi_t)^2 + \tfrac{1}{2}g_{\xi\xi}(d\xi_t)^2 + g_{\chi\xi}d\chi_t d\xi_t$$

As derivadas parciais acima são dadas por;

$$g_t = \chi_t(k_\chi e^{-k_\chi(\tau-t)}) - A'(\tau-t) \qquad g_\chi = e^{-k_\chi}(\tau-t) \qquad g_\xi = 1$$

As derivadas de ordem superior são nulas. Introduzindo os valores destas derivadas parciais e as expressões de $d\chi_t$ e $d\xi_t$ na equação acima, resulta em:

$$d(\ln F_{\tau,t}) = \frac{dF_{\tau,t}}{F_{\tau,t}} = \left(k_\chi\chi_t e^{-k_\chi(\tau-t)} - A'(\tau-t)\right)dt + e^{-k_\chi(\tau-t)}\left((-k_\chi\chi_t - \lambda_\chi)dt + \sigma_\chi dW_\chi^*\right)$$

$$+ (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dW_\xi^*$$

Agrupando os termos em dt pode-se escrever:

$$\frac{dF_{\tau,t}}{F_{\tau,t}} = (\cdot)dt + e^{-k_\chi(\tau-t)}\sigma_\chi dW_\chi^* + \sigma_\xi dW_\xi^*$$

Aplicando o operador variância sob a MME na última equação:

$$\tfrac{1}{dt}VAR^Q\left(\tfrac{dF_{\tau,t}}{F_{\tau,t}}\right) = \sigma_\chi^2\, e^{-2k_\chi(\tau-t)} + \sigma_\xi^2 + 2\rho\sigma_\chi\sigma_\xi e^{-k_\chi(\tau-t)}$$

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Apêndice 5 - Estrutura a termo da volatilidade - Modelo Primeira Extensão

A eq. (36) expressa o Modelo Primeira Extensão escrito sob a MME. Esta equação está abaixo:

$$\ln(F_{\tau,t}) = f(\tau) + e^{-k_\chi(\tau-t)}\chi_t + e^{-k_\xi(\tau-t)}\xi_t + A(\tau - t)$$

onde:

$$A(\tau-t) = \left(\hat{\xi}(1-e^{-k_\xi(\tau-t)}) - \frac{\lambda_\chi}{k_\chi}(1-e^{-k_\chi(\tau-t)}) + \frac{\sigma_\chi^2}{4k_\chi}(1-e^{-2k_\chi(\tau-t)}) + \frac{\rho\sigma_\chi\sigma_\xi}{k_\chi+k_\xi}(1-e^{-(k_\chi+k_\xi)(\tau-t)}) + \frac{\sigma_\xi^2}{4k_\xi}(1-e^{-2k_\xi(\tau-t)})\right)$$

Simplificadamente pode-se escrever $\ln(F_{\tau,t}) = g(t,\chi_t,\xi_t)$. Usando o lema de Itô pode-se escrever:

$$d(\ln(F_{\tau,t})) = g_t dt + g_\chi d\chi_t + g_\xi d\xi_t + \tfrac{1}{2}g_{\chi\chi}(d\chi_t)^2 + \tfrac{1}{2}g_{\xi\xi}(d\xi_t)^2 + g_{\chi\xi}(d\chi_t)(d\xi_t)$$

As derivadas parciais acima são:

$$g_t = k_\chi e^{-k_\chi(\tau-t)}\chi_t + k_\xi e^{-k_\xi(\tau-t)}\xi_t - A'(\tau-t)$$

$$g_\chi = e^{-k_\chi(\tau-t)} \qquad g_\xi = e^{-k_\xi(\tau-t)}$$

As derivadas de ordem superior são nulas. Inserindo estas derivadas acima e as expressões de $d\chi_t$ e $d\xi_t$, resulta em

$$\frac{dF_{\tau,t}}{F_{\tau,t}} = \left(-\lambda_\chi e^{-k_\chi(\tau-t)} + k_\xi\hat{\xi}e^{-k_\xi(\tau-t)} - A'(\tau-t)\right)dt + e^{-k_\chi(\tau-t)}\sigma_\chi dW_\chi^* + e^{-k_\xi(\tau-t)}\sigma_\xi dW_\xi^*$$

Aplicando o operador variância sob a MME na última equação:

$$\tfrac{1}{dt}VAR^Q\left(\tfrac{dF_{\tau,t}}{F_{\tau,t}}\right) = \sigma_\chi^2 e^{-2k_\chi(\tau-t)} + \sigma_\xi^2 e^{-2k_\xi(\tau-t)} + 2\rho\sigma_\chi\sigma_\xi e^{-(k_\chi+k_\xi)(\tau-t)}$$