Pontifícia Universidade Católica
do Rio de Janeiro

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Monetary Policy and Core Inflation: Assessing the Impact of Alternative Measures within a Multisectoral New Keynesian Model

Dissertação de Mestrado

Thesis presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Carlos Viana de Carvalho

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Abstract

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This paper evaluates alternative measures of core inflation in a multi-sectoral New Keynesian model. Stabilizing sticky-price inflation is considered optimal; however, there are different measures of rigid price inflation in a multi-sector economy. Central banks commonly monitor limited influence estimators that exclude volatile sectors' price changes. This would provide a measure of core that mostly correlates to the rigid sectors of the economy, and consequently, could provide welfare gains. Based on the multi-sector model of Carvalho, Lee e Park (2021), a proposed modification is considered: an interest rate rule that responds to a measure of core inflation instead of the usual headline. This modification introduces a non-linearity we tackle using frontier machine learning methods. We calibrate sector-specific supply shocks through indirect inference to match the moments observed in CPI sectoral inflation. Our results show that responding to alternative core measures can reduce consumer welfare losses.

Keywords

Monetary Policy; Core Inflation; Multi Sector Models.

Resumo

von Ungern-Sternberg,Vitor Henrique; Viana de Carvalho, Carlos. **Política Monetária e Núcleos de Inflação: Avaliando o Impacto de Medidas Alternativas dentro de um Modelo Multissetorial Novo Keynesiano**. Rio de Janeiro, 2023. 45p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Este artigo avalia medidas alternativas de núcleo de inflação em um modelo multissetorial Novo Keynesiano. A estabilização da inflação de preços rígidos é considerada ótima; no entanto, existem diferentes medidas de núcleo de inflação em uma economia multissetorial. Os bancos centrais geralmente monitoram estimadores de influência limitada que excluem mudanças de preços em setores voláteis. Isso forneceria uma medida de núcleo que se correlaciona principalmente com os setores rígidos da economia e, conseqüentemente, poderia fornecer ganhos de bem-estar. Com base no modelo multissetorial de Carvalho, Lee e Park (2021), considera-se uma modificação: uma regra de taxa de juros que responda a uma medida de núcleo da inflação em vez da inflação cheia. Essa modificação introduz uma não linearidade que lidamos usando métodos de aprendizado de maquina. Calibramos choques de oferta específicos do setor por meio de inferência indireta para corresponder aos momentos observados na inflação setorial do CPI. Nossos resultados mostram que responder a medidas de núcleo alternativas podem reduzir as perdas de bem-estar do consumidor.

Palavras-chave

Política Monetaria; Núcleo de Inflação; Modelos Multi Setoriais.

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1
Introduction

In the realm of monetary policy, interest rates play a pivotal role in shaping economic conditions and influencing inflation dynamics. Traditionally, interest rates have been set based on various economic indicators, with core inflation being a central used measure of price dynamics.[1] Central banks commonly rely on core inflation measures as they offer a more accurate depiction of underlying inflation dynamics by filtering out transitory or idiosyncratic price movements. By excluding volatile components, core inflation provides a more reliable representation of the measures to which central banks respond. It captures the persistent inflation trends that are deemed more relevant for policy decision-making.

The classical core measure excludes the most volatile components of headline inflation (i.e. food and energy). However, several central banks have developed other measures of core to capture persistent inflation trends and guide their monetary policy decisions. Inside the Federal Reserve System (USA), the Cleveland Fed produces the Median CPI, which measures the weighted median price change across a broad range of consumer goods and services, excluding extreme price movements. The Federal Reserve Bank of Dallas constructs the "Trimmed Mean PCE Inflation Rate" which excludes a certain percentage of the most extreme price changes from the personal consumption expenditures (PCE) index. These measures are also persistently observed by central banks outside the US, such as the European Central Bank, Bank of Canada, Reserve Bank of Australia, Bank of England, who also compute and consider various other measures of core inflation, trimmed means, weighted median as alternative measures that guide their monetary policy decisions.[2]

In this paper, we delve into the implications of using alternative measures of core inflation as an alternative indicator for interest rate response in a multi-sectoral New Keynesian model. By exploring this alternative approach, we aim to shed light on the potential advantages associated with adopting these measures of core as a policy-relevant measure of inflation dynamics. Ultimately, this research seeks to contribute to the ongoing discussion on enhancing the accuracy and effectiveness of monetary policy decision-making in a dynamic economic environment.

[1]See Mishkin (2007) for a perspective of a policy-maker.
[2]Available at Dallas Fed, Cleveland Fed, ECB, RBA, and Bank of Canada

In the New Keynesian framework, the Taylor Rule links inflation dynamics to interest rate adjustments. The relevant measure conventionally used is the headline inflation, reflecting the average price level across all sectors of the economy. However, it is crucial to recognize that not all sectors contribute equally to overall price stability. In an economy characterized by varying degrees of price rigidity among sectors, it becomes essential for monetary policy to prioritize stabilizing inflation in sectors with stickier prices. This is because it is in these sectors that real distortions are more significant.

We consider a multi-sector model developed by Carvalho, Lee e Park (2021) with sectoral disaggregation as a laboratory to understand the effect of the adoption of interest rate rules that respond to different measures of core inflation. That is, we propose the general modification:

$$i_t = \phi_\pi M(\pi_{1t}, ..., \pi_{kt}) \tag{1-1}$$

where $M(\pi_{1t}, ..., \pi_{Kt})$ can be any measure of core inflation, based on the realized inflation of all sectors (k) in the economy. This paper seeks to introduce non-linear measures, specifically limited influence estimators of core, as an alternative approach to capturing the underlying price dynamics. This is motivated by the fact these alternative measures would frequently exclude volatile sectors of the model. This would provide a measure of core that mostly correlates to the rigid sectors of the economy; consequently, the interest rate responding to it could offer welfare gains.

In our study, we employ the indirect inference approach to estimate the non-usual parameters of the model. Our estimation focuses on matching the moments observed in sectoral inflation series of the Consumer Price Index (CPI) with the moments generated by the model. By achieving a close alignment between the model's predictions and the observed characteristics of the data, we ensure that the proposed modifications provide informative insights into the potential reactions of the economy. Through this process, we find that the model is capable of fairly replicating the data and other comovements we observe between headline and core inflation.

To tackle the technical difficulty of introducing the proposed modification, we turn to the literature of frontier numerical methods that apply machine learning techniques for solving macro models. We make use of projection methods to approximate agents' decision rules with neural networks. These methods have become more prevalent in solving dynamic models and can be traced to, among others, Fernández-Villaverde, Hurtado e Nuño (2019), Maliar, Maliar e Winant (2021) and Kahou et al. (2021).

Our findings indicate that responding to the core measures we have examined can result in significant improvements in overall welfare. In the specific context under consideration, exclusion measures appear to result in a lower loss of welfare compared to other measures. When considering non-linear measures, the weighted median emerges as a preferable choice over trimmed means. Little impact is seen on the headline volatility, most of the impact of the considered policy is reflected on the output volatility.

The paper is divided as follows. After this introduction, section 1.1 reviews the rationale for using alternative measures of core inflation. Section 2 presents the sectoral model developed by Carvalho, Lee e Park (2021) used in this paper, while section 2.5 examines the solution method and the numerical approximation. Section 3 presents the calibration and estimation of the model, and 4 the main comovements observed between headline and core the model is able to replicate. A welfare evaluation of the considered measures are presented in section 5. Section 6 concludes.

1.1
Alternative measures of core inflation

Relative price changes caused by sectoral shocks directly affect inflation fluctuation. The heterogeneity in the price movement of goods from different sectors may carry different information about the current state of the economy. Headline inflation measures the nominal cost of living; however, it might be too unstable due to its vulnerability to relative price changes since it doesn't exclude volatile sectors or transitory shocks. The persistence and extension of such shocks in the rest of the economy raise the question of which variable is relevant for monetary policy to react and stabilize prices.

To address these challenges, central banks closely monitor core inflation, a measure designed to capture the central tendency of inflation by isolating temporary relative price changes. As pointed by Eusepi, Hobijn e Tambalotti (2011), attempts to stabilize an inflation measure that includes such volatile items can lead to unnecessary policy tightening and increased volatility in inflation, which may not be optimal.

The theoretical justification for responding to core is supported by Aoki (2001), who shows that in an economy with two sectors, a flexible-price sector and a sticky-price sector, stabilizing sticky-price inflation is optimal. Building upon this, Benigno (2004) extends the analysis to a two-region setting with monetary union, showcasing that an inflation targeting policy that assigns higher weights to inflation in regions with greater degrees of nominal rigidity is nearly optimal. Eusepi, Hobijn e Tambalotti (2011) delve deeper into this concept by constructing a price index whose weights minimize the welfare cost of nominal price stickiness: a cost-of-nominal-distortions index (CONDI). They compute these weights in a 15-sector New Keynesian model for the US economy and show that their weights depend mainly on the degree of price stickiness. Therefore, monetary policy should focus on goods with stickier prices, where nominal rigidity is causing the most distortions.

The practical application and usefulness of constructing core measures also depend on other factors. In the view of Cleveland's Fed, Carroll e Verbrugge (2019) evaluate the usefulness of a measure of core by the transparency of construction, facility to replicate, lower volatility, and the historical ability to track the underlying inflation trend.

Traditionally, the most widely used measure of core inflation has been the exclusion of food and energy prices. Nevertheless, as pointed out by Ball e Mazumder (2019), this variable still considers many other industries, which also experience significant price changes, that materially influence headline inflation. The literature traces back to Bryan e Cecchetti (1994), who defends limited influence estimators, such as weighted median or trimmed means. This is motivated by the empirical evidence presented by Ball e Mankiw (1995), who observed substantial variation in the third moment of the cross-sectional distribution of price changes.

Further exploring this topic, Alves (2014) examine the effects of a monetary rule that responds to changes in the volatility-weighted measure as opposed to headline inflation. This alternative measure involves revising the weights assigned to different sectors by downplaying the influence of more volatile items, which may provide less informative signals about underlying inflation. The revised weights are determined by pondering the sector sizes by the inverse of the inflation variance exhibited in each sector. This approach aims to mitigate the potential distortion caused by highly volatile components and is also considered in our analysis.

By utilizing alternative measures of core inflation that exclude more volatile sectors more frequently, it becomes possible to better capture the behavior of rigid sectors in the economy. This approach holds the potential to yield welfare gains by providing a clearer understanding of underlying inflation trends. The objective of this paper is to assess the merits of such measures and understand the implications of their adoption. Through a evaluation of the effectiveness of these measures, we aim to gain insights into their potential contributions.

2
Multisectoral model

Carvalho, Lee e Park (2021) develop a multi-sector sticky-price DSGE model with the heterogeneity in the price movement necessary for addressing the question proposed by this paper. There are infinite firms which aggregate into a finite number of sectors that differ with respect to the heterogeneity in price stickiness, and sector-specific supply shocks. Firms use intermediate inputs in production, and sector-specific labor as inputs for production. A description of the main elements of the model is presented in this section, together with the solution method we use for the resolving the non-linear modification. We refer the reader to the original paper for a complete derivation of the benchmark model.

2.1
Representative Household

The representative household maximizes expected utility function, discounted by β:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \sum_{k=1}^{K} \omega_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi} \right],$$

where C_t denotes the household's consumption of the composite consumption good. It supplies different types of labor to firms in different sectors, and the relative disutilities of supplying hours to sector k is given by the parameter $\{\omega_k\}_{k=1}^{K}$. $H_{k,t}$ denotes the hours of labor services supplied to sector k. Labor is fully mobile within each sector, but immobile across sectors. The parameter φ, is the inverse of the (Frisch) elasticity of labor supply.

The agent has access to a complete set of Arrow-Debreu state-contingent claims. Following Woodford (2003) in working with the cashless limit of a monetary economy. The budget constraint of the household may therefore be written as:

$$P_t C_t + E_t\left[Q_{t,t+1} B_{t+1}\right] = B_t + \sum_{k=1}^{K} W_{k,t} H_{k,t} + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t}(i) di$$

where P_t denotes the aggregate price level, which represents the price of the composite consumption good. $W_{k,t}$ is the wage rate in sector k, and $\Pi_{k,t}(i)$ denotes profits of firm ik. Households can trade nominal securities with arbitrary patterns of state-contingent payoffs. B_{t+1} denotes household's payoffs of the portfolio of one-period state-contingent nominal securities. $Q_{t,t+1}$ denotes the nominal stochastic discount factor, that prices these payoffs in period t.

The aggregate consumption composite C_t is a Dixit-Stigliz index of these differentiated goods:

$$C_t = \left[\sum_{k=1}^{K} (n_k)^{1/\eta} \, C_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$$

where η denotes the elasticity of substitution between the sectoral consumption composites, and $n_{k,t} > 0$ are the sector sizes, satisfying $\sum_{k=1}^{K} n_k = 1$.

The households face a intratemporal problem that minimizes total expenditure $P_t C_t$, for which it can purchase the given consumption aggregate C_t. This implies that the demand for the sectoral good is

$$C_{k,t} = n_k \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t.$$

where $P_{k,t}$ denotes the sectoral price index, taken as given by the consumer:

$$P_t = \left[\sum_{k=1}^{K} (n_k) \, P_{k,t}^{1-\eta} \right]^{1/(1-\eta)}$$

$C_{k,t}$ is the sectoral consumption in sector k, which is a composite of differentiated goods produced in the sector. With associated sectoral price index $P_{k,t}$:

$$C_{k,t} = \left[\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{2-1}$$

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}$$

Consumers also minimize expenditure on differentiated goods $C_{k,t}(i)$, taking as given the desired sectoral consumption $C_{k,t}$ and prices $P_{k,t}$ and $P_{k,t}(i)$. This yields the optimal demand for type-i good in sector k:

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} C_{k,t}. \tag{2-2}$$

The first order conditions of the household's maximization problem are:

$$Q_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}}$$

$$\frac{W_{k,t}}{P_t} = \omega_k H_{k,t}^{\varphi} C_t.$$

and a transversality condition. The first equation describes the relationship between asset prices and the time path of consumption, whereas the second is the labor supplied by the agent.

2.2 Firms

There is a continuum of firms indexed by $i \in [0,1]$ and each firm belongs to one of K sectors indexed by $k \in \{1,\ldots,K\}$. Each sector produces a differentiated good that is used for consumption and as an intermediate input for other firms.

Firm i that belongs to sector k has a production function that uses sector-specific labor and intermediate goods in production according to the following technology:

$$Y_{k,t}(i) = A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}(i)^{\delta},$$

where $Y_{k,t}(i)$ is the production of firm ik, $A_{k,t}$ is sector-specific productivity, $H_{k,t}(i)$ denotes hours of labor that firm ik employs, $Z_{k,t}(i)$ is firm ik 's usage of other goods as intermediate inputs, and δ is the elasticity of output with respect to intermediate inputs.

The index of intermediate inputs is defined as follows,

$$Z_{k,t}(i) = \left[\sum_{k'=1}^{K} (n_{k'})^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$$

with the same across-sector elasticity of substitution as the one between consumption varieties (η), where the sectoral intermediate input, $Z_{k,k',t}(i)$, denotes the amount of firm ik 's usage of sector-k' goods as intermediate inputs. Firms combine the varieties of goods to form composites of sectoral intermediate inputs. The amount of sector k' goods used as intermediate inputs by firm ik is given by

$$Z_{k,k',t}(i) = \left[\left(\frac{1}{n_{k'}}\right)^{1/\theta} \int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta}\, di'\right]^{\theta/(\theta-1)}$$

with the within-sector elasticity of substitution between different varieties given by θ, and $Z_{k,k',t}(i,i')$ is the quantity of firm $i'k'$ output purchased by firm ik.

To build the composite $Z_{k,k',t}(i)$ firms minimize total expenditure on good $Z_{k,k',t}(i,i')$, taking as given its price $P_{k',t}(i')$. This gives the optimal demand for sectoral intermediate input:

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}}\left(\frac{P_{k',t}(i')}{P_{k',t}}\right)^{-\theta} Z_{k,k',t}(i) \tag{2-3}$$

These are further assembled to build $Z_{k,t}(i)$ composite intermediate input, that can be used for production. For that, firms minimize expenditure on $Z_{k,k',t}(i)$, paying the given price $P_{k',t}$ for it. This cost-minimization problem yields the optimal demand for intermediate inputs:

$$Z_{k,k',t}(i) = n_{k'} \left(\frac{P_{k',t}}{P_t}\right)^{-\eta} Z_{k,t}(i), \tag{2-4}$$

Firm ik's nominal profit is given by

$$\Pi_{k,t}(i) = P_{k,t}(i)Y_{k,t}(i) - W_{k,t}H_{k,t}(i) - P_t Z_{k,t}(i)$$

the first-order condition of the nominal profit maximization problem implies the following relationship:

$$Z_{k,t}(i) = \frac{\delta}{1-\delta}\frac{W_{k,t}}{P_t}H_{k,t}(i)$$

which is the labor demand of firm ik. All labor demanded in each sector must, therefore, be equal to the labor supplied by the households in each sector. This yields the labor market clearing condition:

$$H_{k,t} = \int_{\mathcal{I}_k} H_{k,t}(i)di \quad \forall k$$

Note that firm ik 's total output has to satisfy the sum of household consumption and demand by all other firms, since all firms' product are used as both final output and inputs into the production of other products. Which yields the labor market clearing condition for each sector:

$$Y_{k,t}(i) = C_{k,t}(i) + \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',k,t}\left(i', i\right) di' \tag{2-5}$$

2.3 Price Setting

The prices adjust following a time dependent rule as in Calvo (1983). Let $\mathcal{I}_k$ to denote the set that contains the indices of firms that belong to sector k (so that $\bigcup_{k=1}^{K} \mathcal{I}_k = [0,1]$). Its measure, denoted n_k, gives the mass of firms belonging to each sector. Each firm in sector k faces a per-period probability of adjusting prices given by $(1-\alpha_k)$. A fraction α_k of firms do not change their prices. The sectoral price level $P_{k,t}$ evolves as

$$\begin{aligned} P_{k,t} &= \left[\frac{1}{n_k}\int_{\mathcal{I}^*_{k,t}} P^{*\,1-\theta}_{k,t} di + \frac{1}{n_k}\int_{\mathcal{I}_k - \mathcal{I}^*_{k,t}} P_{k,t-1}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \\ &= \left[(1-\alpha_k)\, P^{*\,1-\theta}_{k,t} + \alpha_k P^{1-\theta}_{k,t-1}\right]^{\frac{1}{1-\theta}}, \end{aligned} \tag{2-6}$$

where $P_{k,t}^{*}$ is the common optimal price chosen by sector k firms that adjust in period t. These firms are a random subset of all firms in the sector, and their indexes are collected in the set $\mathcal{I}_{k,t}^{*}$, whose measure is $n_k\left(1-\alpha_k\right)$.

Firms that are called to adjust their prices at time t maximize expected discounted profits:

$$\max_{P_{k,t}^{*}(i)} E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} \Pi_{k,t+s}(i)$$

where $Q_{t,t+s}$ and $\Pi_{k,t+s}(i)$ are respectively the stochastic discount factor between time t and $t+s$ and firm ik 's nominal profit at time $t+s$ given that the price chosen at time t is still being charged.

Defining aggregate output as $Y_t = C_t + Z_t$, together with market clearing condition (2-5) and optimality conditions (2-1), (2-2), (2-3), and (2-4), we derive the first-order condition for the firm's profit maximization problem:

$$E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} D_{k,t+s} \left(\frac{P_{k,t}^{*}}{P_{k,t+s}}\right)^{-\theta} \left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s} \left[P_{k,t}^{*} - \left(\frac{\theta}{\theta-1}\right) MC_{k,t+s}\right] = 0$$

where the nominal marginal cost in period $t+s$ is given by

$$MC_{k,t+s} = P_{t+s} \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t+s}}{P_{t+s}}\right)^{1-\delta} \frac{1}{A_{k,t+s}}$$

The first-order condition above and the sectoral price level in (2-6) together determine equilibrium dynamics of sectoral prices. Aggregate price dynamics are then determined by aggregation of such sectoral prices.

An allocation of quantities and prices that satisfy the households' optimality conditions and budget constraint, the firms' optimality conditions, the monetary policy rule, and market-clearing conditions characterize the equilibrium of this economy.

2.4
Log-linearized Dynamics

We derive the first order conditions around the deterministic zero inflation steady state. Then, on top of the linearized model, we introduce the proposed modification. Initially we assume that monetary policy rule is characterized by a Taylor-type interest rate rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t$$

where the nominal interest rate i_t responds to headline inflation and to last quarter interest rate. This facilitates the calibration process and identifying the model elements necessary for replicating the data, since we can easily find the model solution.

The equilibrium is given by a sequence of aggregate variables:

$$\{c_t, \pi_t, i_t, z_t, h_t, (w_t - p_t)\}$$

where the variables denote, respectively, aggregate consumption, aggregate inflation, nominal interest rate, usage of other goods as intermediate inputs, hours worked and real wage. And also the following sectoral variables, sectoral consumption and sectoral inflation:

$$\{c_{k,t}, \pi_{k,t}\}_{k=1}^{K}$$

The following $6 + 2K$ equations determine the equilibrium dynamics of those variables:

$$c_t = E_t \left[c_{t+1}\right] - \left(i_t - E_t \pi_{t+1}\right), \tag{2-7}$$

$$w_t - p_t = \varphi h_t + c_t, \tag{2-8}$$

$$(1-\psi)c_t + \psi z_t = \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)h_t + \delta z_t, \tag{2-9}$$

$$w_t - p_t = z_t - h_t, \tag{2-10}$$

$$\begin{aligned}\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \Bigg\{ & \left[\frac{(1-\delta)\varphi}{1+\delta\varphi} + \frac{1}{\eta}\right] c_{k,t} \\ & + \left[\frac{(1-\delta)(1-\psi\varphi)}{1+\delta\varphi} - \frac{1}{\eta}\right] c_t \\ & + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi} z_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} \Bigg\},\end{aligned} \tag{2-11}$$

$$\pi_t = \sum_{k=1}^{K} n_k \pi_{k,t}, \tag{2-12}$$

$$\Delta\left(c_{k,t} - c_t\right) = -\eta\left(\pi_{k,t} - \pi_t\right), \tag{2-13}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t. \tag{2-14}$$

Where $\psi \equiv \delta(\theta - 1)/\theta$. In the model equation (2-7) is the household's consumption Euler equation or dynamic IS; (2-8) is the aggregate labor supply, obtained by aggregating the household's intra-temporal optimality conditions over sectors; (2-9) is obtained by integrating the production function over all firms; (2-10) is the aggregation of cost minimisation conditions, or labour demand; (2-11) is the sectoral Phillips curve; (2-12) delivers aggregate inflation; (2-13) the demand function for sectoral consumption goods; finally (2-14) the interest rate rule.

2.5 Solution method

This section presents the solution method chosen to solve the model with the proposed modification. An interest rate rule responding to a non-linear measure such as median or trimmed mean inflation requires, in the rational expectations equilibrium, agents' decision rules to be consistent with the non-linear policy adopted by the monetary authority. That is, we need to find the zero for the functional that describes this economy.

The approach taken is similar to classical projection methods that solve Euler-residual minimization problems found in the literature of numerical methods[1]. In our case, however, decision functions need to be flexible enough to accommodate the median to which interest rates respond. We make use of neural networks to approximate agents' decision rules. This is justified because of their property as universal approximators; that is, provided sufficiently many hidden units available, a feed-forward neural network can approximate any Borel-measurable function (famous result shown by Cybenko (1989), Hornik, Stinchcombe e White (1989)).

As in Maliar, Maliar e Winant (2021) we cast our problem in the general framework of solving the functional given by the first order conditions (Euler equations) that describe the optimization problem, given by equations (2-7) through (2-14), for J optimality conditions:

$$\mathbb{E}\left[f_j(m, s, x, m', s', x')\right] = 0, \qquad j = 1, ..., J \tag{2-15}$$

where $m' = M(m, \epsilon')$ is the exogenous state and $s' = S(m, s, x, m')$ the endogenous state, which is controlled by a choice $x \in X(m, s)$. The expectation operator $\mathbb{E}\left[\cdot\right]$ is taken with respect to the next-period shock ϵ'. In our case, we have the exogenous state defined by the sectoral shocks, $(\{a_{kt}\}_{k=1}^{K})$, and our control is given by the decision functions of $(c_t, \{\pi_{kt}\}_{k=1}^{K})$, which respond to the exogenous state and all other predetermined variables $(\{c_{k,t-1}\}_{k=1}^{K})$ of the model.

We need to find θ for a parametric decision rule $x = \varphi(m, s; \theta) \in X(m, s)$ that provides an accurate approximation of the optimal φ in the relevant domain. For a given decision rule $\varphi(\cdot; \theta)$ and the domain for the state variables (m, s), define the Euler-residuals:

$$\mathcal{E}_j(m, s; \theta) = \mathbb{E}\left[f_j(m, s, \varphi(m, s; \theta), m', s', \varphi(m', s'; \theta))\right], \qquad j = 1, ..., J \tag{2-16}$$

[1]For excellent surveys on the literature see Maliar e Maliar (2014) and Fernández-Villaverde, Rubio-Ramírez e Schorfheide (2016)

We evaluate $\mathcal{E}_j(m, s; \theta)$ on a set of points of N points (m_n, s_n) drawn from a Sobol quasirandom sequence. There are K exogenous shocks, to evaluate the integrals we employ a degree three Monomial rule proposed by Stroud (1971). This is the fastest method by approximating integrals and usually can be applied when the exogenous shocks are Gaussian. The architecture of the chosen neural network to approximate $\varphi(\cdot; \theta)$ has one hidden layer, with a ReLu activation function.

Define a loss function, as the expected squared sum of Euler-equations residuals:

$$\Xi(\theta) = \frac{1}{N} \sum_{n=1}^{N} \sum_{j=1}^{J} v_j \left(\mathcal{E}_j(m_n, s_n; \theta)\right)^2 \tag{2-17}$$

where $(v_1, ..., v_j)$ is a vector of weights on J optimality conditions. Our goal is to find a decision rule $\varphi(\cdot; \theta)$ that solves $min_{\theta \in \Theta}\ \Xi(\theta)$. Since we restrict the dimensionality of the problem to only nine sectors, we are able to use Levenberg–Marquardt algorithm to minimize the loss function. However, other optimization algorithms are also suitable, commonly known for training neural networks, the limited-memory BFGS also performed relatively well.

We first apply the method in solving the benchmark model, where the interest rate is responding to headline inflation, in which we know the model solution. After achieving a good numerical approximation for this problem, we introduce the proposed modification of an interest rate that responds to the alternative measures of price inflation and use the previous solution as an initial guess to our minimization problem.

Following the literature, we present the Euler-residuals MSE of our approximation. It serves as a precision measurement of how the first-order conditions are being solved in equality. Table 2.1 shows the approximation for the three models that we consider: an interest rate responding to the weighted median, to the trimmed mean, first trimming 60% of the sectors and second only trimming 20%. For the linear models we are able to achieve a precise solution of around 3e-30, that is almost the exact, and it matches exactly the solution given by gensys. When we introduce the modification, the precision of the approximation deteriorates. This is expected since the sort function that defines these measures are not straightforward to approximate, however, these values seem close to values found in the literature that apply similar methods, as in Kahou et al. (2021), and seems sufficient for some intuition of the dynamics caused by the modification.

Table 2.1: Euler Residuals MSE

Median	1.5e-7
Trimmean 60	4.2e-7
Trimmean 20	1.4e-7

3
Calibration and estimation

Structural parameters are set to conventional values found in the literature, shown in Table 3.1. The discount factor, β, equals 0.99, to achieve a 4% annual steady state interest rate. The inverse of the (Frisch) elasticity of labor supply, φ, is set to 0.5, between the linear specification typical of the RBC literature Hansen (1985) and the low elasticities of labor supply usually estimated by the empirical literature, which might suggest values for φ around 2, as in one of the specifications in Carvalho, Lee e Park (2021). The within-sector elasticity of substitution between different varieties, θ, to 6, which implies a 20 percent steady-state markup for the firms. The across-sector elasticity of substitution, η, is set equal to 2. The elasticity of output with respect to intermediate inputs, δ, is set to 0.7. We adopt standard values for the policy parameters, setting ϕ_π and ρ_i equal to 1.5 and 0.5, respectively.

Table 3.1: Benchmark parameters

Parameter	Description	Value
β	Discount factor	0.99
φ	Inverse of Frisch elasticity	0.5
θ	Elasticity of substitution between varieties	6
η	Across sector elasticity of substitution	2
δ	Elasticity of output with respect to intermediate inputs	0.7
ϕ_π	Monetary Policy response to inflation	1.5
ρ_i	Interest rate smoothing	0.5

The model consists of a parameterized economy with twelve sectors. Each sector differs in mainly two aspects. The shocks it receives, productivity shock, and the nominal rigidity of each sector. Responsible for generating the differences in the observed price responses to shocks, the second source of heterogeneity is the probability of readjusting prices, given by the parameter $(1-\alpha_k)$. We construct in twelve sectors to represent the studied economy, where groups of similar price changes characteristics are aggregated, shown in Table 3.2.

Each sector price stickiness is constructed aggregating the frequency of price changes of the entry level items (ELI) calculated by Nakamura e Steinsson (2008). The distribution of the price stickiness is responsible for generating the difference of how sectors react to sectoral productivity shocks. Figure 3.1

Table 3.2: Twelve-sector model calibration

Sector	n_k (%)	$(1-\alpha_k)$ (%)
Energy	10.42	68.00
Transportation(1)	12.33	42.12
Unprocessed food	8.01	22.42
Education and Communication	7.31	18.62
Housing	11.64	17.19
Transportation(2)	7.19	10.32
Processed food	6.16	9.50
Other goods and services	6.48	9.26
Recreation	7.90	7.75
Medical Care	7.70	6.42
Food away from home	8.41	4.97
Apparel	6.48	3.62

represents this feature, and shows that flexible price sectors dissipate shocks faster and present a more volatile response than rigid price sectors.

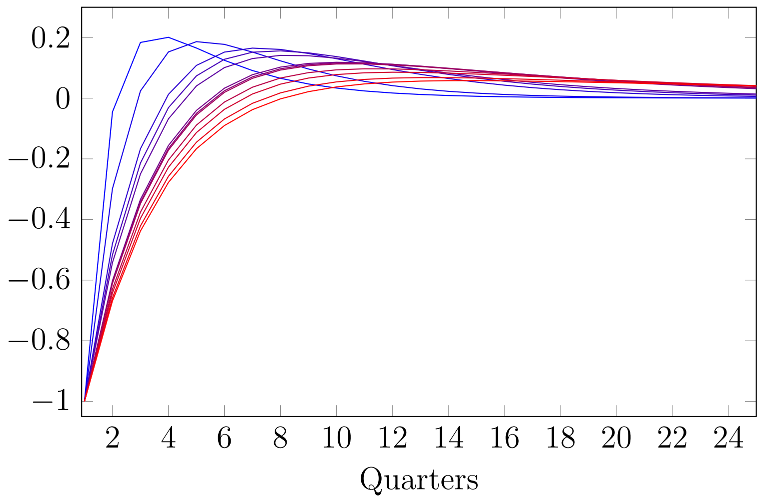


Figure 3.1: Response of sectoral prices to sectoral productivity shocks
Note: More rigid (red) to more flexible (blue) sectors
$\rho_{ak} = 0.7 \ \forall k$

3.0.1 Estimated parameters

The economy is hit by K productivity shocks (a_{kt}). Assumed to follow the stationary process described in equation (3-1):

$$a_{k,t} = \rho_{ak} a_{k,t-1} + \sigma_{ak}\varepsilon_t^{ak}, \quad \varepsilon^{ak} \sim \mathcal{N}(0,1) \tag{3-1}$$

The persistence (ρ_{ak}) and volatility (σ_{ak}) parameters are estimated using the simulated method of moments (SMM). SMM is a simulation-based technique that aims to align the statistical characteristics of the model-generated data with those observed in the real-world data. We utilize aggregated CPI price series data (quarterly, Jan 2011 - Dec 2022) for the twelve mentioned sectors. From these price series, we compute the standard deviation and autocorrelation of price changes within each sector.

SMM estimates the parameters solving the following minimization process.

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}}[\Psi(\boldsymbol{\theta})]'\mathbf{W}[\Psi(\boldsymbol{\theta})]$$

where:

$$\Psi(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{\sigma}\left(\pi_k^d\right) - \boldsymbol{\sigma}\left(\pi_k^m(\boldsymbol{\theta})\right) \\ \boldsymbol{\rho}\left(\pi_k^d\right) - \boldsymbol{\rho}\left(\pi_k^m(\boldsymbol{\theta})\right) \end{bmatrix}$$

in the equation, the estimated parameters characterizing the stochastic processes of sectoral productivity shocks are represented by the parameter vector $\boldsymbol{\theta} \in \mathbb{R}^{2k}$, which is defined as $\boldsymbol{\theta} = [\boldsymbol{\sigma}'_{ak}, \boldsymbol{\rho}'_{ak}]'$. The standard deviation of price changes for each sector k, computed from the data, is denoted as $\boldsymbol{\sigma}\left(\pi_k^d\right)$, while the model counterpart is represented as $\boldsymbol{\sigma}\left(\pi_k^m(\boldsymbol{\theta})\right)$. Similarly, the autocorrelation of the price changes of each sector k, calculated from the data and the model counterpart, are denoted as $\boldsymbol{\rho}\left(\pi_k^d\right)$ and $\boldsymbol{\rho}\left(\pi_k^m(\boldsymbol{\theta})\right)$, respectively . Estimates are consistent for any full rank matrix $\mathbf{W}$, we use the identity matrix, implying that all moments have equal weight.

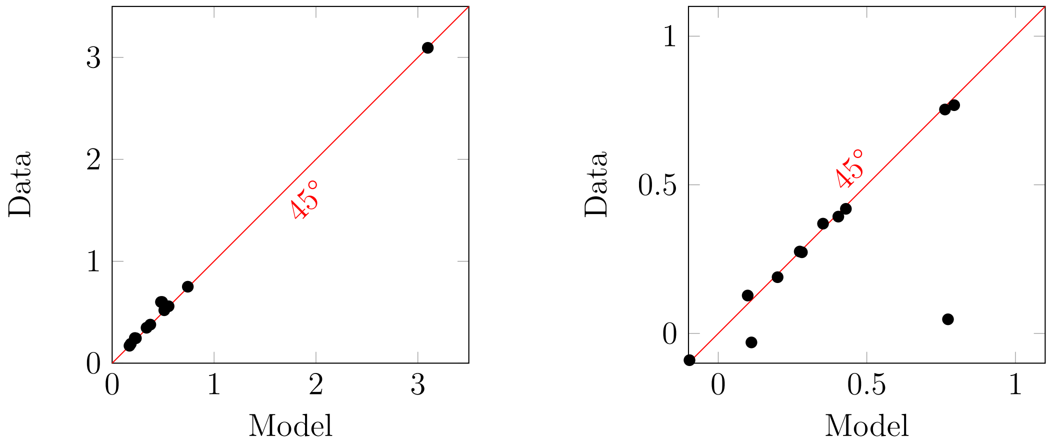
For the matching moment exercise, we assume that monetary policy is conducted through an interest rate rule that responds to headline inflation, as described in equation 2-14. Since the interest rate is responding to a linear measure of inflation, it is possible to solve the model easily using gensys from Sims (2002). Table 3.3 presents the estimated parameters.

Table 3.3: Estimated productivity shocks

Sector	ρ_{ak}	SE	σ_{ak} (%)	SE(%)
Energy	0.68	0.021	4.59	0.048
Transportation(1)	0.00	0.019	2.87	0.046
Unprocessed food	0.37	0.021	5.07	0.113
Education and Communication	0.00	0.062	3.35	0.095
Housing	0.54	0.018	6.72	0.201
Transportation(2)	0.13	0.018	25.0	0.398
Processed food	0.85	0.013	4.35	0.242
Other goods and services	0.42	0.018	13.9	0.397
Recreation	0.17	0.026	16.4	0.480
Medical Care	0.00	0.017	21.2	0.808
Food away from home	0.81	0.025	5.18	0.457
Apparel	0.81	0.033	25.0	0.363

4
Comparing the model and observed data

Table 4.1 presents a scatter plot comparing the moments derived from the observed data with those generated by the model. The calibration process demonstrates a remarkable ability to replicate the observed moments accurately. Notably, several sectors exhibit high alignment between the model-generated and observed moments.



(a) $\sigma(\pi_k)$ (b) $\rho(\pi_k)$

Figure 4.1: Sectoral autocorrelation and volatility in model versus in the data

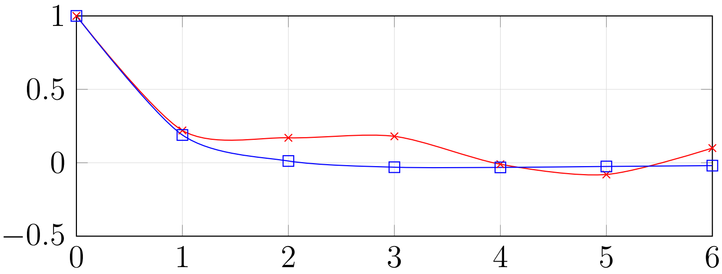
Initially, we examine the relationship between the estimated parameters and the targeted moments. The standard deviation and autocorrelation of price changes are directly linked to the stochastic processes governing sectoral productivity shocks. However, due to interdependencies between sectors arising from intermediate inputs in their production functions and the aggregate price level affecting marginal costs, this relationship is not entirely straightforward. Consequently, the calibration of productivity shocks does not have a one-to-one impact on the observed moments of sectoral inflation. Despite this, the calibration is still able to achieve a reasonable level of fit between the targeted moments.

The model's performance is evaluated through a comparison of the co-movement between median and headline inflation. Achieving a close alignment between the model's predictions and the observed characteristics of the data is crucial to ensure that the proposed modification provides informative insights into the potential reactions of the economy. To accomplish this, we analyze the correlations between headline and median CPI. Additionally, we examine the relationship between the observed moments in the data and the frequency of price changes, which determines the nominal distortions inherent in each sector. By investigating these interconnected aspects, we gain a comprehensive understanding of the model's effectiveness in capturing key economic dynamics and the associations between these variables.

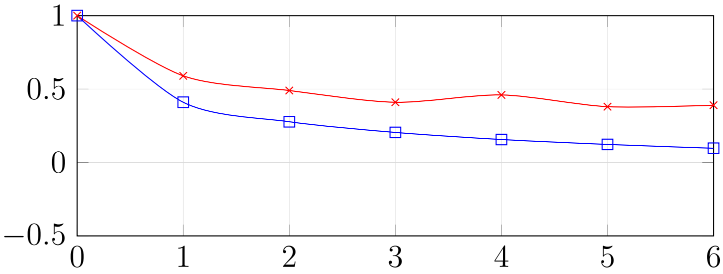
Motivated by the notion that central banks prioritize monitoring core inflation due to its superior predictive ability for future headline inflation, we investigate the relationship between these inflation measures. Although these moments were not explicitly targeted in the calibration process, they exhibit a noteworthy level of resemblance between the model and the data, aligning with the desired characteristics. Figure 4.2 provides a visual comparison of the comovement between weighted median and headline inflation. Panel (a) portrays the autocorrelation of headline inflation, while panel (b) depicts the autocorrelation of the weighted median. The data demonstrates that median inflation exhibits higher autocorrelation, indicating a stronger dependence on its past values compared to headline inflation. Furthermore, panel (c) highlights a robust cross-correlation between median inflation and future headline inflation, demonstrating the utility of this relationship for assessing both the current and future state of the economy.

The presence of nominal rigidity in the model introduces certain constraints that affect the characteristics of observed volatility and autocorrelation in sectoral inflation. Within the model framework, rigid sectors generally exhibit lower volatility compared to their flexible counterparts, which have the capacity to adjust prices more frequently. Furthermore, flexible sectors demonstrate lower autocorrelation due to their ability to respond not only to shocks within their own sector but also to shocks originating from other sectors.

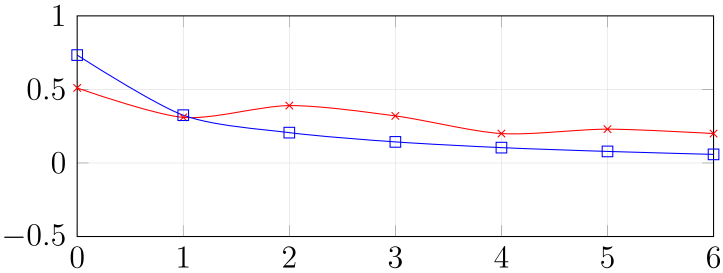
This empirical pattern is observed in the data, albeit with slightly more noise compared to the model's output. Figure 4.3, presented in the left panel, depicts the probability of price readjustment against the standard deviation of sectoral inflation, while the right panel displays the probability of price readjustment against the autocorrelation of sectoral inflation, accompanied by their respective trend lines. The calibration of productivity shocks aims to align the model more closely with the empirical data. Notably, the aforementioned trend persists in both the model and the observed data. This relationship could be explored to give reason to the construction of the double weighted inflation measure. Assigning sectoral weights based on the inverse of inflation variance within each sector would allocate greater importance to less volatile and more rigid sectors. This approach aligns with the notion that monetary policy should respond to the behavior of rigid sectors. Implementing such a measure has the potential to generate welfare gains.



(a) Headline autocorrelation



(b) Median autocorrelation



(c) Headline and median cross-correlation

Figure 4.2: Correlogram: median and headline inflation
Note: blue squares: model, red stars: data

The use of limited influence estimators is motivated by the proposition that the median, excluding large variation of price changes, would be a measure more correlated with sticky prices. Therefore, something central banks should respond to in the pursuit of price stability. Figure 4.4 and 4.5 explicitly shows this difference. Both figures are IRF of the model in which the interest rate responds to headline inflation. Figure 4.4 presents the response to a productivity shock in the most flexible sector; on the other hand, Figure 4.5 presents the response to a productivity shock in the most rigid sector of the economy. The increase in productivity puts downward pressure on prices, primarily affecting the sector that receives the shock and indirectly affecting the economy's other prices. The headline inflation follows this response, and as a consequence, the interest rate responds to it.

Quantitatively, there is a big difference in both responses. Flexible prices respond much more intensively to the shock, this movement is much more

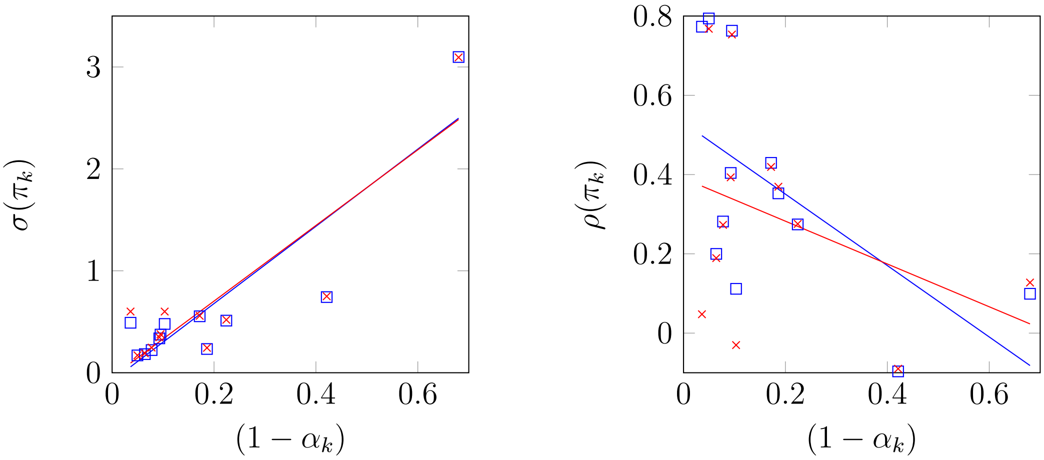
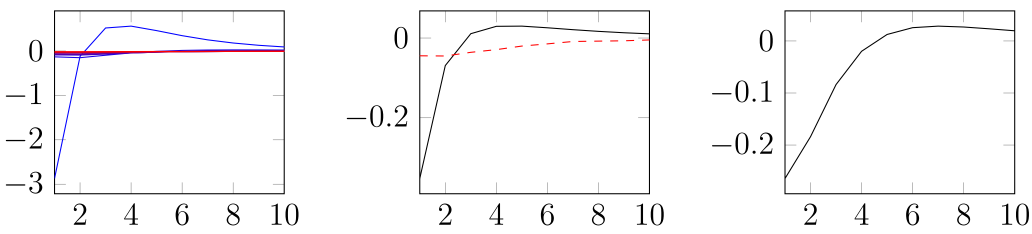
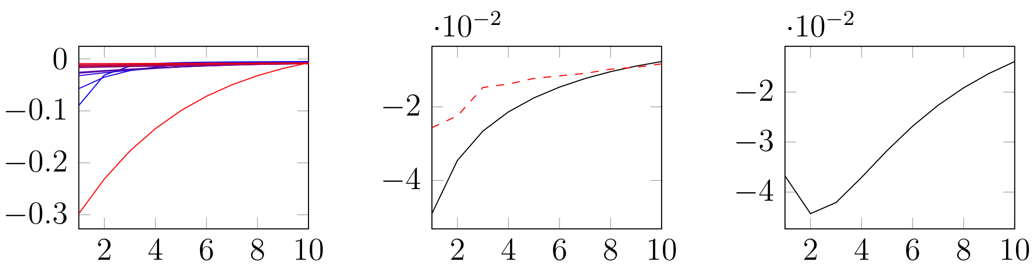


Figure 4.3: Frequency of price changes and sectoral inflation
Note: blue squares: model, red stars: data. Inflation volatility (on the left) and autocorrelation (on the right), with data trend line



(a) Sectoral inflation (b) Headline and median (c) Interest rate

Figure 4.4: Sectoral response to productivity shock: flexible sector



(a) Sectoral inflation (b) Headline and median (c) Interest rate

Figure 4.5: Sectoral response to productivity shock: rigid sector

constrained in the rigid sector. In the red dashed line, the median price is computed in contrast to the headline inflation. The median mainly responds to rigid sector inflation, the persistent inflation component. Observing the rigid sector shock on Figure 4.5 headline inflation is responding more intensively to the flexible sector, that responds to the shock with greater volatility than all the other sectors. Setting the interest rate to respond to such volatile measure as the headline would further pressure the other sectors to adjust prices. However, they won't be able to respond due to the nominal rigidity constraining this adjustment. Therefore, a measure that seeks to stabilize rigid price inflation could generate welfare improvements.

5
Welfare evaluation

To comprehensively evaluate the performance of each policy rule, we employ two main metrics: welfare loss and the introduced change in volatility experienced by the agent. These metrics provide valuable insights into the effectiveness and implications of the different policy rules.

To begin the evaluation, we consider the agent's utility function, which is represented by the equation:

$$U_t = \log(C_t) - \sum_{k=1}^{K} \omega_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi}$$

As in Galí (2015), we evaluate the performance of the policy rules as the utility losses experienced by the representative consumer as a consequence of deviations of the efficient allocation. By analyzing the utility losses, we gain insights into how each policy rule impacts the well-being of the representative consumer. To quantify the welfare losses, we opt for a numerical evaluation approach, which allows for a comparison of the utility losses associated with each policy rule. The welfare losses are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady state consumption:

$$\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{U_t - U}{U_c C}\right] \approx \sum_{i=1}^{N} \sum_{t=0}^{T} \beta^t \left[\frac{U_t - U}{U_c C}\right]$$

Simulating for a period long enough such that future utility is almost irrelevant to the agent (T = 750) and for a high number of samples (N = 10.000) to ensure some robustness in our analysis.[1] We calculate the utility loss for each policy, using the same shocks on every model for consistency.

The results are displayed on Table 5.1. This table provides a comparison of different policy rules based on various metrics, including welfare loss ($\mathscr{L}$), standard deviation of consumption ($\sigma(c_t)$), standard deviation of headline inflation ($\sigma(\pi_t)$), and standard deviation of the relevant measure of core that monetary policy is responding to ($\sigma(M(\pi_{kt}))$).

The first model is the benchmark model, where the interest rate is responding to headline inflation. Based on it we evaluate if deviations bring welfare gains to the agent. Second, the "Exclude 1:4" is a exclusion measure

[1]See Table A.8 in the Appendix A.2 where we show why we believe that the simulations were sufficiently long enough.

Table 5.1: Evaluation of monetary policy rules

Model	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.269	1.015	0.428	0.428
Exclude food and energy	-3.212	1.103	0.418	0.254
Exclude 1:4	-3.218	1.106	0.417	0.194
Median	-3.224	1.096	0.432	0.222
Trimmean 60	-3.244	1.000	0.420	0.215
Double Weighted	-3.243	1.162	0.413	0.129
Trimmean 20	-3.256	1.078	0.427	0.469

where the 4 most flexible sectors are excluded from the index. As expected from the results of Aoki (2001), the exclusion index indicates that responding to fixed price inflation brings clear welfare gains.

Following, the three limited influence measures considered: weighted median, and two trimmed mean measures: the first trims 60% and the second trims 20% of the weighted sectors. These measures seem to bring welfare gains to the consumer with respect to headline inflation. With the best being the weighted median. All three measures seem to bring a slight reduction into consumption and a small increase in headline volatility.

The adjustment of interest rates in response to the double weighted inflation measure. As anticipated, this measure exhibits the lowest volatility among all the measures examined, as it assigns greater weight to the least volatile sectors. In comparison to headline inflation, it appears to result in welfare gains, reducing headline volatility while increasing consumption volatility. Figure 4.3 illustrates a positive correlation between the probability of price readjustment and sectoral volatility, although this correlation is not straightforward or linear. Table 5.2 provides a breakdown of the probability of price changes, sector sizes, sectoral inflation volatility, and the assigned weights for each sector in the double weighted measure (n_k^*). Notably, sectors such as apparel, processed foods and other goods and services demonstrate high volatility despite their relatively small probabilities of price change. This may help explain why the double weighted measure does not achieve the same level of success as the classical exclusion measure.

The last policy under consideration is the classical core that excludes food and energy, items that have historically presented high volatility. The welfare loss is reduced when interest rates responds to it, in fact, our results show that it performs really well. Even better than excluding the four most flexible sectors, which is surprising.

We assess the robustness of our findings by conducting model estimations

Table 5.2: Volatility weighted assigned weights

Sector	$(1-\alpha_k)$ (%)	n_k (%)	$\sigma(\pi_k)$	n_k^* (%)
Energy	68.00	10.42	3.09	0.09
Transportation(1)	42.12	12.33	0.73	1.99
Unprocessed food	22.42	8.01	0.50	2.77
Education and Communication	18.62	7.31	0.21	14.48
Housing	17.19	11.64	0.54	3.38
Transportation(2)	10.32	7.19	0.47	2.75
Processed food	9.50	6.16	0.37	3.89
Other goods and services	9.26	6.48	0.33	5.07
Recreation	7.75	7.90	0.21	14.77
Medical Care	6.42	7.70	0.17	21.71
Food away from home	4.97	8.41	0.16	26.78
Apparel	3.62	6.48	0.49	2.32

under seven different calibration scenarios. Specifically, we vary the parameters related to monetary policy strength (ϕ_π) and persistence (ρ_i). Additionally, we explore the impact of reducing the usage of intermediate inputs, represented by the parameter δ (referred to as calibration I). In each calibration scenario, we re-estimate the parameters of the productivity shocks to ensure consistency with the new settings. This comprehensive analysis allows us to examine the sustainability of the usage of core measures across different policy configurations.

Table 5.3: Considered calibrations

Calibration	ρ_i	ϕ_π
A	0.5	1.50
D	0.5	3
E	0	1.50
F	0	3
G	0.5	1.05
H	0	1.05

We compare the results obtained from different calibration scenarios with the benchmark calibration (referred to as A on Figure 5.1). Across all cases, we consistently observe a decrease in welfare loss when the interest rate responds to inflation measures excluding the four most rigid sectors, the median measure, and the measure excluding food and energy. However, the results for the double weighted measure and the trimming of 20% of price changes vary depending on the calibration. Moreover, the preferred measure does not exhibit consistent ranking across all calibrations. One possible explanation for this inconsistency is the changing calibration of sector-specific productivity shocks, as the models are being re-estimated.

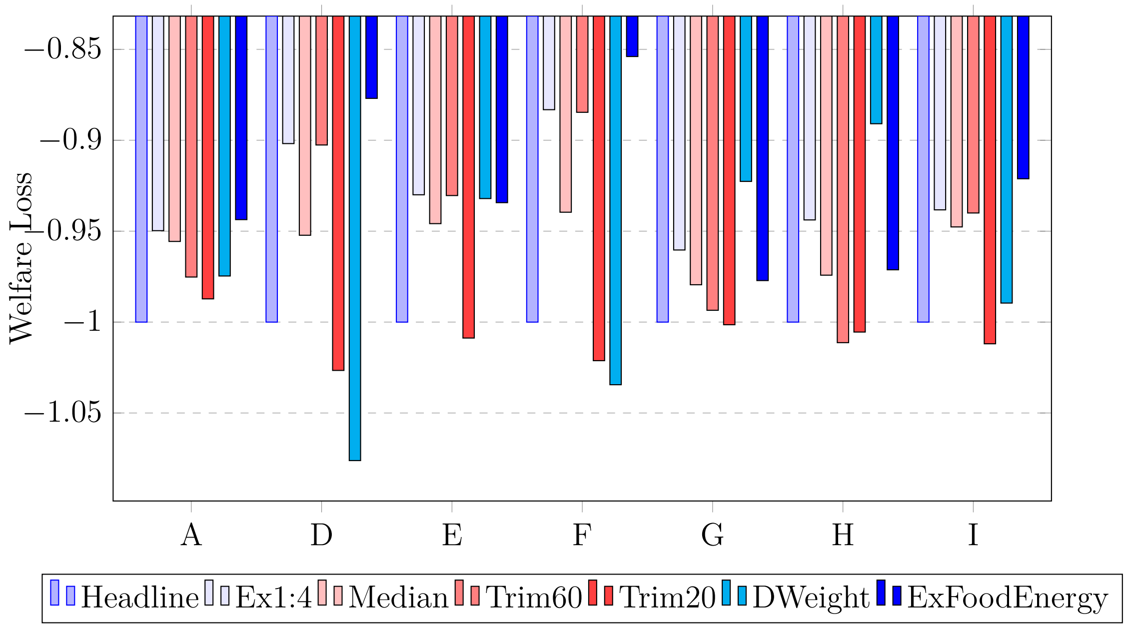


Figure 5.1: Sensibility analysis: changing benchmark parameters
Note: For every calibration, the welfare loss for the model that responds to headline is normalized to -1

The sensitivity of the measures' performance to parameter configurations becomes evident from the observed variations in results. Specifically, the presence of two sources of heterogeneity within sectors, namely nominal rigidity and productivity shocks, significantly influences the choice of preferred core measures for policy response. To provide a visual representation of this phenomenon, we refer to Figure 5.2, which illustrates the differences between these two features.

In Calibration B, we isolate the impact of nominal rigidity by keeping the calibration of productivity shocks consistent across sectors. On the other hand, Calibration C maintains uniform nominal rigidity while introducing distinct parametrizations for the stochastic process of productivity shocks, carefully matching the observed volatility and persistence of sectoral inflation in the data.

By comparing the welfare loss from these calibrations with the benchmark (Calibration A), we gain valuable insights into the performance of the alternative measures. Calibration B highlights the double weighted measure as the most effective, as it assigns greater weight to the least volatile sectors, which are characterized by higher degrees of rigidity, aligning with previous literature. However, Calibration C reveals that the potential improvements achieved by responding to core measures are not present. This indicates that, within our model, the optimal measure for policy response is also contingent on the heterogeneity of productivity shocks.

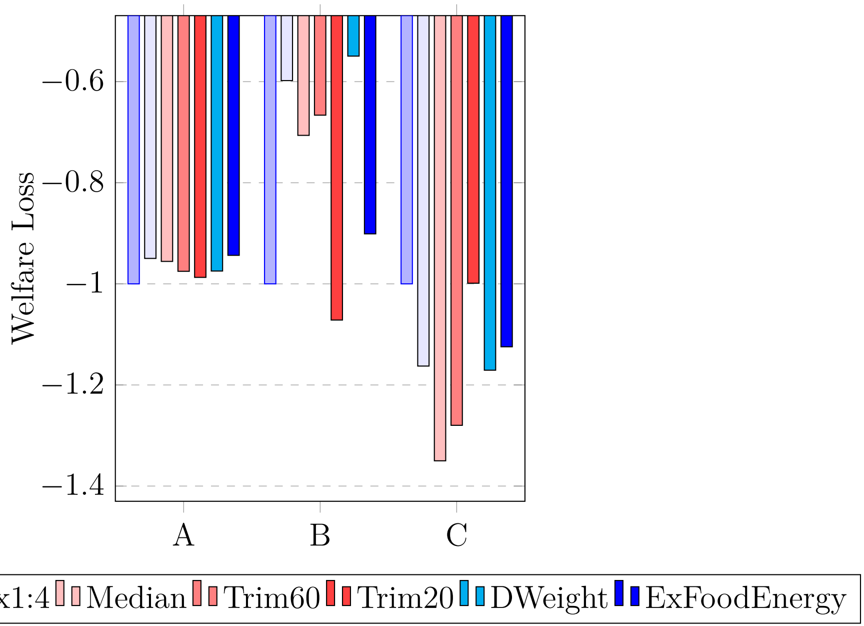


Figure 5.2: Welfare loss under considered calibration
Note: For every calibration, the welfare loss for the model that responds to headline is normalized to -1

When evaluating the performance of core measures, it is crucial to consider the practical aspects of their construction. One aspect that may pose challenges is the determination of whether we can directly observe the probability of price change or the nominal distortion experienced by each sector. This difficulty can complicate our assessment of which sectors are pertinent targets for monetary policy. However, the utilization of double weighted and limited influence estimators appears to yield measures that are more strongly correlated with rigid price inflation. Moreover, these estimators are practical and straightforward to construct. Within the model, these measures demonstrate desirable welfare gains for consumers and might be a good instrument for policy.

6
Conclusion

In conclusion, the heterogeneity of the nominal rigidity present in the economy requires monetary policy to look at a measure of inflation different from the one relevant to consumers. Previous studies have shown that monetary policy should focus on sticky price inflation, where real distortions are more significant. In a multi-sector new Keynesian model with a time-dependent price setting, developed by Carvalho, Lee e Park (2021), we study the performance of limited influence estimators, such as median and trimmed means, to access sticky price inflation. Motivated by the high correlation with rigid sector inflation and its extensive use by central banks, limited influence estimators seem a helpful tool for the monetary authority to respond to.

We first calibrate the model shocks to approximate the model's ability to replicate CPI price inflation. Compared to model simulations, comovements observed between headline and median inflation show that, despite the model being highly stylized, it can still replicate the data reasonably well. We used a projection algorithm to solve for the modification of an interest rate that responds to alternative measures of core inflation. In particular, we used neural networks to approximate agents' decision rules; this was done to accommodate the non-linearity of the introduced modification.

Our results have shown that an interest rate responding to core can bring some reduction into the welfare losses, being positive for the consumer. Inside the model, trimmed means and median price inflation bring desirable gains for the agent, with the weighted median performing slightly better. However these measures still under-perform when compared to classical exclusion measures, indicating that directly responding to shocks happening in rigid sectors might be a better approach to reduce nominal distortions.

7
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A
Appendix

A.1
Expanded results

Table A.1: Calibration A: welfare evaluation

$\rho_i = 0.5$ $\phi_\pi = 1.5$ $\delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.26901	1.015	0.428	0.428
Exclude 1:4	-3.21873	1.106	0.417	0.194
Median	-3.22464	1.096	0.432	0.222
Trimmean 60	-3.24421	1.000	0.420	0.215
Trimmean 20	-3.25622	1.078	0.427	0.469
Double Weighted	-3.24365	1.162	0.413	0.129
Exclude food and energy	-3.21263	1.103	0.418	0.254

Table A.2: Calibration D: welfare evaluation

$\rho_i = 0.5$ $\phi_\pi = 3$ $\delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-2.9279	2.089	0.420	0.420
Exclude 1:4	-2.8297	1.982	0.411	0.173
Median	-2.8801	2.013	0.416	0.186
Trimmean 60	-2.8305	2.005	0.414	0.195
Trimmean 20	-2.9545	2.166	0.425	0.468
Double Weighted	-3.0041	1.769	0.410	0.112
Exclude food and energy	-2.8048	2.006	0.407	0.229

Table A.3: Calibration E: welfare evaluation

$\rho_i = 0$ $\phi_\pi = 1.5$ $\delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.3197	0.888	0.433	0.433
Exclude 1:4	-3.2497	0.956	0.416	0.196
Median	-3.2655	0.959	0.423	0.213
Trimmean 60	-3.2501	0.934	0.420	0.221
Trimmean 20	-3.3285	0.934	0.434	0.477
Double Weighted	-3.2518	1.037	0.409	0.131
Exclude food and energy	-3.2540	0.950	0.418	0.259

Table A.4: Calibration F: welfare evaluation

$\rho_i = 0$ $\phi_\pi = 3$ $\delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-2.6494	2.031	0.427	0.427
Exclude 1:4	-2.5326	1.775	0.403	0.179
Median	-2.5890	1.806	0.408	0.190
Trimmean 60	-2.5341	1.856	0.408	0.201
Trimmean 20	-2.6707	2.151	0.433	0.477
Double Weighted	-2.6839	1.573	0.400	0.113
Exclude food and energy	-2.5034	1.823	0.402	0.237

Table A.5: Calibration G: welfare evaluation

$\rho_i = 0.5$ $\phi_\pi = 1.05$ $\delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.5640	0.586	0.432	0.432
Exclude 1:4	-3.5243	0.751	0.423	0.200
Median	-3.5435	0.686	0.425	0.208
Trimmean 60	-3.5575	0.654	0.420	0.214
Trimmean 20	-3.5654	0.596	0.431	0.473
Double Weighted	-3.4866	0.882	0.417	0.135
Exclude food and energy	-3.5412	0.729	0.426	0.265

Table A.6: Calibration H: welfare evaluation

$\rho_i = 0 \quad \phi_\pi = 1.05 \quad \delta = 0.7$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.5452	0.469	0.435	0.435
Exclude 1:4	-3.4890	0.654	0.422	0.201
Median	-3.5194	0.613	0.422	0.202
Trimmean 60	-3.5565	0.551	0.426	0.221
Trimmean 20	-3.5507	0.487	0.436	0.478
Double Weighted	-3.4361	0.791	0.415	0.136
Exclude food and energy	-3.5164	0.635	0.426	0.267

Table A.7: Calibration I: welfare evaluation

$\rho_i = 0.5 \quad \phi_\pi = 1.5 \quad \delta = 0.5$	$\mathscr{L}$	$\sigma(c_t)$	$\sigma(\pi_t)$	$\sigma(M(\pi_{kt}))$
Headline	-3.8868	0.924	0.422	0.422
Exclude 1:4	-3.8251	1.009	0.406	0.184
Median	-3.8345	0.998	0.414	0.197
Trimmean 60	-3.8268	0.968	0.411	0.205
Trimmean 20	-3.8988	0.956	0.424	0.466
Double Weighted	-3.8764	1.014	0.402	0.117
Exclude food and energy	-3.8080	1.019	0.406	0.239

A.2 Precision of simulations

To ensure the accuracy of our simulations in calculating the statistics for each policy, we compare them to the corresponding "populational" values on Table A.8. In the case of linear models, where the solution assumes a VAR form we use the following approach explainded by Hamilton (1994) to calculate the theoretical variances:

The VAR solution is given by:

$$\boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t,$$

where

$$E\left(\mathbf{v}_t\mathbf{v}_\tau'\right) = \begin{cases} \mathbf{Q} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases}$$

Let $\boldsymbol{\Sigma} = E\left[\boldsymbol{\xi}_t\boldsymbol{\xi}_t'\right]$

$$E\left[\boldsymbol{\xi}_t\boldsymbol{\xi}_t'\right] = E\left[\left(\mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t\right)\left(\mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t\right)'\right] = \mathbf{F}E\left(\boldsymbol{\xi}_{t-1}\boldsymbol{\xi}_{t-1}'\right)\mathbf{F}' + E\left(\mathbf{v}_t\mathbf{v}_t'\right),$$

or

$$\boldsymbol{\Sigma} = \mathbf{F}\boldsymbol{\Sigma}\mathbf{F}' + \mathbf{Q}$$

Applying the properties of the vec() operator and let $\otimes$ be the Kronecker product, we have:

$$\text{vec}(\boldsymbol{\Sigma}) = (\mathbf{F} \otimes \mathbf{F}) \cdot \text{vec}(\boldsymbol{\Sigma}) + \text{vec}(\mathbf{Q}) = \mathbf{A}\,\text{vec}(\boldsymbol{\Sigma}) + \text{vec}(\mathbf{Q})$$

where

$$\mathbf{A} = (\mathbf{F} \otimes \mathbf{F})$$

Let $r = np$, so that $\mathbf{F}$ is an $(r \times r)$ matrix and A is an $(r^2 \times r^2)$ matrix. Equation above has the solution:

$$\text{vec}(\boldsymbol{\Sigma}) = \left[\mathbf{I}_{r^2} - \mathbf{A}\right]^{-1} \text{vec}(\mathbf{Q})$$

Table A.8: Inflation Std (%)

Simulation	Theoric
0.4281549	0.4284856
0.4174684	0.4178545
0.4126146	0.4127642
0.4184244	0.4188630
0.4197990	0.4199465
0.4107180	0.4108763
0.4104441	0.4104528
0.4071283	0.4072868
0.4326514	0.4330080
0.4156942	0.4160936