Pontifícia Universidade Católica
do Rio de Janeiro

Bernardo Fernandes Faria

Domestic and External Shocks in the Brazilian Business Cycle

Dissertação de Mestrado

Masters dissertation presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor : Prof. Yvan Becard
Co-advisor: Prof. Lucas Lima

Rio de Janeiro
April 2025

Pontifícia Universidade Católica
do Rio de Janeiro

Bernardo Fernandes Faria

Domestic and External Shocks in the Brazilian Business Cycle

Masters dissertation presented to the Programa de Pós–graduação em Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia. Approved by the Examination Committee:

Prof. Yvan Becard
Advisor
Departamento de Economia – PUC-Rio

Prof. Lucas Lima
Co-advisor
Departamento de Economia – PUC-Rio

Prof. Márcio Garcia
Departamento de Economia – PUC-Rio

Dr. Danilo Cascaldi-Garcia
Board of Governors – FED

Rio de Janeiro, April 28th, 2025

Bernardo Fernandes Faria

Bachelor in Economics, Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio), 2020

Bibliographic data

Faria, Bernardo Fernandes

Domestic and External Shocks in the Brazilian Business Cycle / Bernardo Fernandes Faria; advisor: Yvan Becard; co-advisor: Lucas Lima. – 2025.

49 f: il. color. ; 30 cm

Dissertação (mestrado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia, 2025.

Inclui bibliografia

1. Economia – Teses. 2. Ciclo Econômico. 3. Choques Externos-Domésticos. 4. VAR Estrutural. 5. Estimação Bayesiana. 6. Dados Atípicos. I. Becard, Yvan. II. Lima, Lucas. III. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. IV. Título.

CDD: 004

Acknowledgments

I would first like to thank my advisors, Yvan Becard and Lucas Lima, for their guidance and all stimulating conversations. I am also grateful to Marcio Garcia, Carlos Viana and Danilo Cascaldi-Garcia for their thoughtful suggestions.

A special mention goes to my fellow classmates for their collaborative spirit throughout this master's program, as well as to all the professors and staff of the Department of Economics.

My deep gratitude to Prof. Augusto Sampaio, who opened the doors of PUC-Rio to me and countless other young students throughout his life.

I dedicate this dissertation to my family — Fatima, Robson, and Mariana — for encouraging my education and their constant support.

And finally, to my love, Hanna, for embarking on this journey by my side and believing in me, even when I did not.

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

Abstract

Faria, Bernardo Fernandes; Becard, Yvan (Advisor); Lima, Lucas (Co-Advisor). **Domestic and External Shocks in the Brazilian Business Cycle**. Rio de Janeiro, 2025. 49p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Political leaders in office usually overstate their influence on the domestic business cycle during growth periods but deflect responsibility to foreign factors during recessions. This thesis seeks to disentangle and quantify the contribution of domestic and external shocks on Brazil's economic activity. We build a structural vector autoregressive model and estimate it using Bayesian techniques over the 1999-2024 sample. We identify the shocks using a block-recursive structure based on the small open economy assumption and decompose Brazil's GDP growth accordingly. We conclude that domestic shocks are the main driver of Brazil's business cycle, though external shocks also play a significant role. Not adjusting for extreme observations registered during the COVID-19 pandemic inflates the perceived influence of foreign factors. In addition, recessions in Brazil are not all alike in terms of the "nationality" of their primary causes. For instance, 2008-09 was more external, while 2014-16 was more domestic.

Keywords

Business Cycle; Global-Domestic Shocks; Structural VAR; Bayesian Estimation; Outlier Data.

Resumo

Faria, Bernardo Fernandes; Becard, Yvan; Lima, Lucas. **Choques Domésticos e Externos no Ciclo Econômico Brasileiro**. Rio de Janeiro, 2025. 49p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Líderes políticos incumbentes frequentemente exageram sua influência sobre o ciclo econômico doméstico durante períodos de crescimento, mas terceirizam a responsabilidade para fatores externos durante recessões. Esta dissertação busca separar e quantificar a contribuição de choques domésticos e externos sobre a atividade econômica do Brasil. Nós empregamos um modelo de vetor autoregressivo estrutural estimado com técnicas Bayesianas entre 1999-2024. Identificamos os choques usando uma estrutura recursiva em blocos baseada na hipótese de pequena economia aberta e decompomos o crescimento do PIB brasileiro entre eles. Concluímos que choques domésticos são o principal determinante do ciclo econômico brasileiro, embora choques externos também desempenhem um papel significativo. Não corrigir pelas observações extremas registradas durante a pandemia de COVID-19 infla a contribuição percebida dos fatores globais. Ademais, as recessões no Brasil não são todas iguais com respeito à "nacionalidade" de suas causas primárias. Por exemplo, a de 2008-09 foi mais externa, enquanto que a de 2014-16 mais doméstica.

Palavras-chave

Ciclo Econômico; Choques Externos-Domésticos; VAR Estrutural; Estimação Bayesiana; Dados Atípicos.

Table of contents

List of figures

List of tables

List of Abbreviations

VAR – Vector Autoregressive
SVAR – Structural Vector Autoregressive
BVAR – Bayesian Vector Autoregressive
FEVD – Forecast Error Variance Decomposition
IRF – Impulse Response Function
SA – Seasonally Adjusted
GDP – Gross Domestic Product
SOE – Small Open Economy
EME – Emerging Market Economy
DEV – Developed Country
GFC – Great Financial Crisis
MCMC – Monte Carlo Markov Chain
EPU – Economic and Policy Uncertainty

1
Introduction

The study of business cycles seeks to uncover the deep and primary forces that drive economic agents to unexpectedly change their behavior, resulting in alternating periods of economic growth and contraction. A central question in empirical macroeconomics is how to identify and quantify the contribution of these unanticipated structural disturbances - commonly called shocks - in shaping economic fluctuations.

In the context of developed and relatively closed economies, such as the United States (US), the literature frequently points, as source of shocks, to: government spending and taxation (Blanchard and Perotti, 2002; Mertens and Ravn, 2012); monetary policy (Romer and Romer, 2004; Christiano, Eichenbaum and Evans, 2005); financial frictions and risk (Gilchrist and Zakrajšek, 2012; Christiano, Motto and Rostagno, 2014); news and confidence (Fujiwara, Hirose and Shintani, 2011; Angeletos, Collard and Dellas, 2018).

For emerging economies like Brazil - characterized by higher volatility and greater exposure to global commodity markets -, the literature typically recognizes the influence of external shocks in addition to conventional domestic ones, such as: terms of trade (Fernández, Schmitt-Grohé and Uribe, 2017; Fernández, González and Rodríguez, 2018); global interest rates (Canova, 2005; Fernández-Villaverde et al., 2011); global financial risk and country spreads (Akinci, 2013); and uncertainty (Carrière-Swallow and Céspedes, 2013).

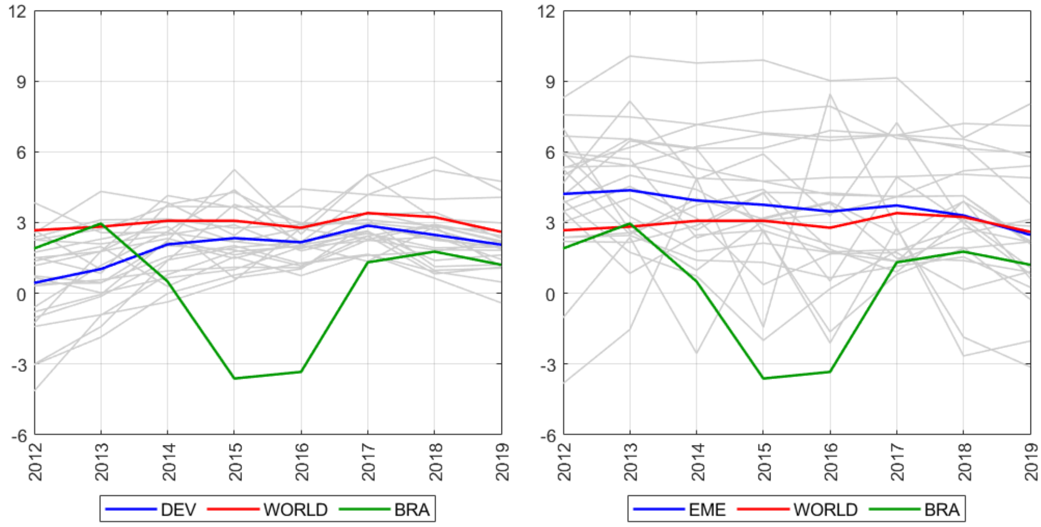
This sensitivity to financial and external conditions is conveniently exploited in political discourse. National leaders in office usually overstate their influence on the domestic business cycle during growth periods, but deflect responsibility to financial markets or, more often, foreign factors during recessions. For instance, amid the 2014-16 recession[1], President Dilma Rousseff delivered a live TV speech attributing the crisis to an adverse external scenario:

> "... we are in the second phase of fighting the most serious international crisis since the Great Depression of 1929..."
>
> "The crisis has severely affected major economies such as the US, the EU and Japan. Even China... has seen its growth slow."
>
> "... there was no way to predict that the international crisis would last this long."
>
> - March 8, 2015

[1]One of the longest and deepest economic downturns in Brazil since the 1980s, according to the Brazilian Business Cycle Dating Committee (CODACE).

In Figure 1.1, we plot the annual GDP growth of a large group of developed nations and emerging economies around those years. We note that during 2014-16 most countries were growing, or at least not contracting as deeply as Brazil. This superficial analysis lead us to believe that the main causes of this recession were negative domestic shocks rather than external ones. Figure 1.2 offers a broader view of Brazil's recent economic history and motivates us to revisit this same question to other notable expansion and recession episodes.

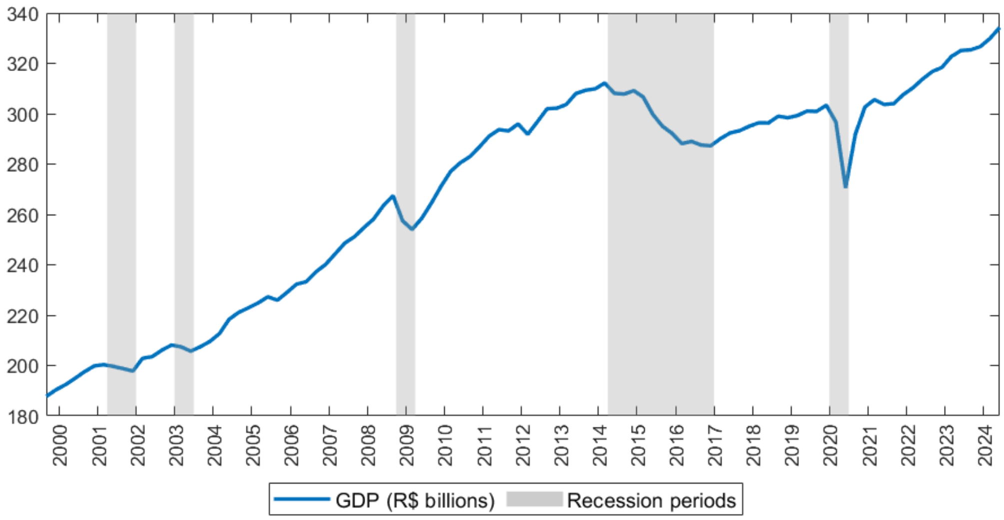
Figure 1.1: Cross-country GDP growth (annual, %)



(a) Developed nations (b) Emerging economies

Note: Green line = Brazil; red line = world aggregation from WB; blue lines = average of developed and emerging economies; and gray lines = other selected countries. Source: World Bank database.

Figure 1.2: Brazil's business cycle chronology



Note: Data at quarterly frequency, in real values and seasonally adjusted. Source: IBGE and CODACE.

This thesis seeks to disentangle and quantify the contribution of domestic and external shocks to Brazil's economic activity imposing a minimum number of hypotheses. We build a structural vector autoregressive (SVAR) model that includes domestic and international variables. We use a block recursive structure based on the small open economy (SOE) assumption to identify two subsets of shocks, one driven only by domestic shocks and the other exclusively by external shocks. Then, we decompose Brazil's GDP growth series into these two components using two structural exercises: the forecast error variance decomposition (FEVD) and the historical decomposition.

We estimate the model over the 1999Q3-2024Q4 period and adopt a Bayesian technique that optimally selects the informativeness of the prior distributions in the spirit of hierarchical modeling. With this we avoid the imposition of ad hoc values to the hyperparameters. We also correct for estimation distortions caused by outlier observations registered during the COVID-19 pandemic. We base ourselves on two approaches recently proposed in the literature that interpret these extreme values as shifts in one of the moments of the forecast errors distribution: either its mean or its volatility[2].

Our results suggest that domestic shocks are the main driver of Brazil's business cycle, explaining 60% of the variance in output, although external shocks also play a significant role, being responsible for the remaining 40%. We find that not adjusting for pandemic-related outliers inflates the perceived influence of foreign shocks to approximately 70%. We also show that recessions in Brazil are not all alike, at least in terms of the geographical origin of their underlying causes. The same conclusion is also valid for expansion periods. Regarding specifically the three most recent recession episodes in Brazil, they had different primary causes: 2008-09 was driven by global shocks, 2014-16 was related mainly to domestic causes, while 2020 had a more mixed influence.

The literature on the effects of external shocks in emerging market economies (EMEs) has early contributions from Mendoza (1995) and Kose (2002), both of which analyze terms of trade shocks via a SOE business cycle model. Since then, the papers gradually increase the scope of what is defined as the external sector. They also vary in terms of the empirical modeling adopted, the countries they analyze, and the time window used. Akinci (2013) and Shousha (2016), for example, use panel data from 6 EMEs, including Brazil, so their conclusions are based on average results for these groups. In contrast, works such as Schmitt-Grohé and Uribe (2018), Fernández, Schmitt-Grohé and Uribe (2017), Fernández, González and Rodríguez (2018) and Ferreira

[2]We thank Domenico Giannone, Michele Lenza, Giorgio Primiceri and Danilo Cascaldi-Garcia for sharing their replication codes.

and Valério (2023) offer individual results for Brazil. In general, they suggest that external shocks account for 24-48% of output fluctuations in Brazil. In addition, virtually all of them point to domestic shocks as the main driver.

We contribute on four fronts. First, by including a larger set of variables in the international block, we provide a broader coverage of external shocks compared to earlier studies that focus only on one or two international shocks, mitigating concerns about omitted variable bias. Second, we complement the more usual FEVD analysis with a historical decomposition, moving beyond average effects to provide a more detailed and episode-specific assessment of how external shocks impacted Brazil. Third, our Bayesian approach departs from the standard use of a Minnesota prior combined with a Normal-Inverse Wishart distribution by jointly estimating both parameters and hyperparameters within a hierarchical modeling framework. With this, we significantly reduce the number and relevance of subjective choices when specifying priors. Lastly, as far as we are concerned, we are the first to show that pandemic-related outlier observations can lead to distorted results in a Brazilian macroeconomic context. We prove that if we correct for these extreme values - by downplaying their informativeness - we can reconcile our results with those of the existing literature and the model's own estimates prior to the COVID-19 pandemic.

The remainder of this thesis is structured as follows. In Chapter 2 we explain our model, identification hypothesis, and estimation method. Chapter 3 presents the data and two approaches to correct for the pandemic-related outlier observations. Chapter 4 contains the results, in the form of variance and historical decompositions. In Chapter 5 we discuss the impacts of not adjusting for outliers. Chapter 6 concludes.

2
Model

2.1
Structural VAR

The use of SVAR models to investigate the sources of business cycle fluctuations traces back to the seminal paper of Blanchard and Quah (1989). They revolutionize the macroeconomic field by linking the innovations in VAR models to theoretically motivated structural shocks[1]. This approach allows us to interpret a VAR(p) model as the reduced-form representation of data generated by a structural VAR(p) process, which can be written as follows[2]:

$$\mathrm{A}_0\mathrm{y}_t = \nu + \sum_{k=1}^{p} \mathrm{A}_k\mathrm{y}_{t-k} + \epsilon_t \tag{2-1}$$

where y_t is a n-size vector of stationary endogenous variables, p is the lag order and ν is a vector of n constants. A_0 is the matrix that governs the contemporaneous interaction between the endogenous variables while $\{\mathrm{A}_1,...,\mathrm{A}_p\}$ are matrices that dictate the historical relationship with its lags. ϵ_t is a random vector of n (structural) shocks with:

$$\begin{cases} \mathbb{E}[\epsilon_t] = 0, \quad \forall t \\ \mathbb{E}[\epsilon_t\epsilon'_{t-j}] = 0, \quad \forall j < t \\ \Sigma_\epsilon = \mathbb{E}[\epsilon_t\epsilon'_t] = \mathrm{D}, \quad \forall t \quad \text{(a diagonal matrix)} \end{cases} \tag{2-2}$$

Unfortunately, the shocks themselves are not directly observable, leading to the so-called identification problem. But, under certain conditions they can be recovered from the VAR residuals. Assuming that A_0 is not singular and denoting $\mathrm{A} = \mathrm{A}_0^{-1}$, we can obtain the model's reduced-form representation:

$$\begin{aligned} \mathrm{y}_t &= \mathrm{A}_0^{-1}\nu + \sum_{k=1}^{p} \mathrm{A}_0^{-1}\mathrm{A}_k\mathrm{y}_{t-k} + \mathrm{A}_0^{-1}\epsilon_t \\ &= \mathrm{c} + \sum_{k=1}^{p} \mathrm{B}_k\mathrm{y}_{t-k} + \mathrm{A}\epsilon_t \\ &= \mathrm{c} + \mathrm{B(L)}\mathrm{y}_{t-1} + \mathrm{u}_t \end{aligned} \tag{2-3}$$

[1]Bernanke (1986) defines structural shocks as primitive, exogenous forces that are economically meaningful and mutually uncorrelated.

[2]We follow the derivations in Kilian and Lutkepohl (2017) textbook.

where B(L) is the lag polynomial of the reduced-form coefficient matrices $\{B_1, ..., B_p\}$. Equation (2-4) highlights that the reduced-form errors u_t are linear combinations of structural shocks ϵ_t, with the elements of A working as weights. One way to recover the structural shocks from the reduced-form errors is to impose restrictions[3] on the elements of A.

$$\begin{aligned} u_t &= A\epsilon_t \\ \Sigma_u &= A\Sigma_\epsilon A' = ADA' \end{aligned} \tag{2-4}$$

2.2 Identification

The international economics literature usually considers that emerging economies are small players in global markets, in the sense that they cannot significantly impact global variables. Known as the small open economy (SOE) assumption, its use in the SVAR framework has roots on Cushman and Zha (1997) and Zha (1999). In this study, we assume that Brazil is one of these SOEs[4] and use this theoretical premise as identification hypothesis. We incorporate the SOE assumption by dividing the n variables from Equation (2-3) into two groups: a foreign y_t^f and a domestic y_t^d. Then, we set $A_{f,d} = 0$ to create a recursive block structure:

$$\begin{bmatrix} y_t^f \\ y_t^d \end{bmatrix} = \begin{bmatrix} c^f \\ c^d \end{bmatrix} + \begin{bmatrix} B_{f,f}(L) & B_{f,d}(L) \\ B_{d,f}(L) & B_{d,d}(L) \end{bmatrix} \begin{bmatrix} y_{t-1}^f \\ y_{t-1}^d \end{bmatrix} + \begin{bmatrix} A_{f,f} & 0 \\ A_{d,f} & A_{d,d} \end{bmatrix} \begin{bmatrix} \epsilon_t^f \\ \epsilon_t^d \end{bmatrix} \tag{2-5}$$

We identify the matrix A through a Cholesky decomposition on the estimated covariance matrix. However, this strategy imposes a structural impact ordering between variables that is more restrictive than the ordering between blocks required to obtain the foreign vs. domestic dichotomy. If we sum all Cholesky-identified shocks from the foreign equations and separately summing those from the domestic equations, we obtain two synthetic shocks — each representing a linear combination of all foreign and all domestic shocks, respectively[5]. This scheme imposes the minimum number of hypotheses required to credibly reach our objective.

[3] Most types of identification used in the SVAR literature are theoretical hypotheses that are translated into some kind of restriction on A itself or on a matrix derived from it, such as zero restrictions (short and long-run) and sign restrictions.

[4] In Appendix A we discuss a common critic that considers Brazil as a price maker on the international commodity market

[5] Changing the within-block order of the variables do not alter our results as long as we respect the between-blocks order, keeping all foreign variables before any domestic.

Then, we decompose the Brazilian output into these two identified shock series by implementing two common exercises of SVAR models: the forecast error variance decomposition (FEVD) and the historical decomposition[6]. The former illustrates how much of the variance of one variable can be explained by exogenous shocks to the other variables of the system, while the latter is useful to assess the cumulative effect of each shock in explaining particular ups and downs of a time series.

2.3 Bayesian Estimation

One issue with VAR models is that their dense parameterization usually leads to overfitting problems for models with many variables and few observations. This concern is particularly relevant when working with Brazilian time series data due to the relatively short sample window available[7]. The Bayesian approach helps to overcome this curse of dimensionality by combining the likelihood function with some informative prior distributions. Bayesian VARs (BVARs) incorporate this external information into the estimation to shrink the richly parameterized model toward a more parsimonious one.

Most of the literature on BVARs uses some version of the Minnesota Prior, introduced by the seminal work of Litterman (1986). Its basic principle is that each variable is centered around a random walk, which is imposed by setting the following moments for the slope coefficients:

$$\mathbb{E}\left[(\mathrm{B}_k)_{ij}\right] = \begin{cases} \delta_i, & j = i, k = 1 \\ 0, & \text{otherwise} \end{cases} \qquad \mathbb{V}\left[(\mathrm{B}_k)_{ij}\right] = \begin{cases} \frac{\lambda^2}{k^2}, & j = i \\ \vartheta \frac{\lambda^2}{k^2} \frac{\sigma_i^2}{\sigma_j^2}, & \text{otherwise} \end{cases} \tag{2-6}$$

Originally, Litterman set $\{\delta_i = 1, \forall i\}$ as he assumes high persistence in all variables. However, we estimate the BVAR with stationary variables, so we impose the prior belief of a white noise, setting $\{\delta_i = 0, \forall i\}$. The prior variance is designed to decrease with the lag order according to $\frac{1}{k^2}$, while $\frac{\sigma_i^2}{\sigma_j^2}$ accounts for scale differences[8]. The hyperparameter $\vartheta \in (0, 1)$ controls the importance of the lags of other variables relative to their own lags. λ is the hyperparameter that determines the overall tightness of the prior distribution around its mean. So, it governs the relative importance between prior beliefs and the information from the data. When $\lambda = 0$, the posterior equals the prior, and the data do not influence the estimates. Conversely, as $\lambda \to \infty$, posterior expectations converge to maximum likelihood estimates (MLE).

[6] Appendix B contains details and derivations of these exercises.
[7] In this thesis we have around 1400 observations and at least 900 parameters.
[8] Where σ_i^2 is the variance of a AR(1) residual estimated with data from variable i.

For structural analysis, the standard approach in the literature is to impose that the parameters belong to the Normal-Inverse Wishart prior family[9], as in Kadiyala and Karlsson (1997). It preserves the core principles of the Minnesota prior while allowing for a more flexible covariance structure than Litterman's fixed diagonal covariance matrix[10], which do not account for potential correlations between residuals. Denoting $\mathcal{B} = vec(\mathrm{B})$:

$$\begin{aligned} \mathcal{B} \mid \Sigma_{\mathrm{u}} &\sim \mathcal{N}\left(\beta_0, \Sigma_{\mathrm{u}} \otimes \Omega_0\right) \\ \Sigma_{\mathrm{u}} &\sim \mathcal{IW}\left(S_0, \nu_0\right) \end{aligned} \tag{2-7}$$

where β_0, Ω_0, S_0 (the prior scale matrix) and ν_0 (its degree of freedom) are functions of a lower dimensional vector of hyperparameters, set to be consistent with Equation (2-6) and $\mathbb{E}\left[\Sigma_u\right] = diag(\sigma_1^2, \cdots, \sigma_n^2)$.

From this setup, literature has proposed various approaches to select the informativeness of the prior distribution for the VAR coefficients - the value of shrinkage parameter λ. Doan and Sims (1984) set the prior tightness by maximizing the model's out-of-sample forecasting performance. In contrast, Banbura, Giannone and Reichlin (2010) suggest controlling for overfitting. We follow the procedure of Giannone, Lenza and Primiceri (2015), who account for uncertainty in the hyperparameters by treating them as additional parameters and performing a full posterior simulation using a Markov chain Monte Carlo (MCMC) algorithm. We adopt this estimation method because it automatically selects the appropriate amount of shrinkage (λ), taking into account the trade-off between in-sample fit and model complexity, obtaining the maximum informativeness of the priors in the spirit of hierarchical modeling[11]. The significant reduction in the number and relevance of subjective choices when specifying priors will be useful in Chapter 3 when we include more hyperparameters in the model.

[9] This is possible under the assumption of $\vartheta = 1$.

[10] In the original Minnesota prior structure, slope coefficients $\mathrm{B} = (\mathrm{B}_1, \ldots, \mathrm{B}_p)'$ are assumed to be independent and normally distributed, the prior on the intercept c is set to be diffuse and the covariance matrix of the residuals is assumed to be diagonal, fixed and known: $\Sigma_u = diag(\sigma_1^2, \cdots, \sigma_n^2)$.

[11] Appendix C contains a longer discussion on this topic.

3
Data

3.1
Baseline Dataset

This work uses quarterly data, as GDP — the key business cycle indicator — is typically released every quarter. Four lags were used in the model specification and the time window spans 1999Q3-2024Q4. We deliberately discarded the observation period before 1999 because we understand that the transition of the exchange rate regime from fixed to free-floating in January 1999 and the implementation of the inflation-targeting monetary policy regime in June 1999 represent structural changes in the economy's parameters.

Table 3.1: Data description (baseline variables)

Variables	**Source**
Domestic	
Consumer price index (IPCA)	IPEA Data, IBGE
Brazil real GDP	IPEA Data, IBGE
Primary fiscal result	BCB-SGS
BNDES disbursement	BNDES
Selic target interest rate	BCB-SGS
Ibovespa index	Refinitiv Workspace
Brazil EPU index	Baker, Bloom and Davis (2016)
BRL/USD exchange rate	BCB-SGS
International	
Export commodity price index	SECEX
US real GDP	Fed St. Luis, BEA
China real GDP	Refinitiv Workspace, NBS
Fed Funds effective rate	Fed St. Luis
Fed Funds shadow rate	Wu and Xia (2016)
S&P 500 index	Refinitiv Workspace
CBOE volatility index (VIX)	Fed St. Luis
US dollar index (DXY)	Refinitiv Workspace

Note: Primary fiscal result includes the balance from federal government, Central Bank and sub-national governments (states and municipalities); BNDES disbursement is added to the fiscal result; Economic policy uncertainty (EPU) indexes count the number of specific words, like policy and uncertainty, on newspaper articles; Commodity price index is calculated using the FOB value and the net weight of Brazil's exported commodities; We use the Fed Funds rate but substitute it by the Wu-Xia shadow rate whenever the US monetary authority hits the zero-lower bound.

Table 3.1 describes the variables of our baseline specification. We include traditional variables, following previous papers, but also unconventional ones that are relevant for the Brazilian context such as BNDES disbursements and China's GDP[1]. We include a considerable large number of variables in the baseline model to cover the main shocks that affect the Brazilian economy, minimizing concerns about omitted variables. We test including other variables[2], but observe only minor changes in results. After carrying out unit root tests, we log-differentiate the non-stationary variables with the exception of interest rates, which we only take the first difference. We employ the X13 Arima-Seats to seasonally adjust the data series.

3.2 Outlier Observations

The COVID-19 pandemic was a global health crisis that also severely disrupted the economy. It caused unprecedented shifts in several macroeconomic indicators during the first months of lockdown. These abnormal values pose a challenge for macroeconomists, as they can lead to distorted and even unstable coefficients. So, how should we treat the observations associated with the pandemic?

Schorfheide and Song (2020) recognize that economists have two options: increase the model complexity or simply exclude these extreme observations. Noting that VAR forecasts performed well from July 2020 onward, on par with pre-pandemic sample, they propose excluding the observations most affected by the pandemic (March/2020 - June/2020).

We opt to follow two approaches that do not fully discard these observations, but rather downplay their informativeness in some degree. Both works interpret the abnormal large innovations of that period as shifts in one of the moments of the forecast-errors distribution: either its volatility - Lenza and Primiceri (2022) - or its mean - Cascaldi-Garcia (2022).

3.2.1 Volatility Shift

Lenza and Primiceri (2022) conjecture that the innovations from the first months of the pandemic presented a volatility that was substantially higher than their historical patterns, which justified the abnormal values observed. The authors adapt the VAR model from Equation (2-3) to obtain a heteroskedastic model with the help of s_t:

$$\mathrm{y}_t = \mathrm{c} + \sum_{k=1}^{p} \mathrm{B}_k \mathrm{y}_{t-k} + \mathrm{s}_t \mathrm{u}_t \tag{3-1}$$

[1]Appendix D provide a deeper discussion on this.

[2]A full description of these alternative variables is available in Appendix D.

This framework allows the residual covariance matrix to scale up its regular value of Σ_u by a factor of s_t^2 during the pandemic period. Let t^* denote the last period before the outbreak (2019Q4 in our case) and a the number of periods with extreme values (set to 3, covering 2020Q1–2020Q3). Then:

$$\mathrm{s}_t = \begin{cases} 1, & t \leq t^* \\ \theta_1, & t = t^* + 1 \\ \vdots & \\ \theta_a, & t = t^* + a \\ 1, & t > t^* + a \end{cases} \tag{3-2}$$

This parametrization allows the scaling factor s_t to take different values in each of the a periods[3]. If $\{\mathrm{s}_t = 1, \quad \forall t\}$, including the pandemic periods, the model collapses back to our standard BVAR. One drawback is that, in each of these a periods, all shocks are assumed to have their volatility scaled up equally, the so-called commonality assumption.

Once we observe $\theta \equiv [\theta_1, ..., \theta_a]$, we can reweight our dataset, as shown in Equation (3-3) and proceed with a standard Bayesian estimation. However, θ is not known in advance. Instead of postulating ad hoc values, Lenza and Primiceri (2022) adapt Giannone, Lenza and Primiceri (2015) Bayesian approach to treat the model's hyperparameters - including θ - as additional parameters, optimizing them at each draw.

$$\frac{\mathrm{y}_t}{s_t} = \frac{\mathrm{c}}{s_t} + \sum_{k=1}^{p} \mathrm{B}_k \frac{\mathrm{y}_{t-k}}{s_t} + \mathrm{u}_t \tag{3-3}$$

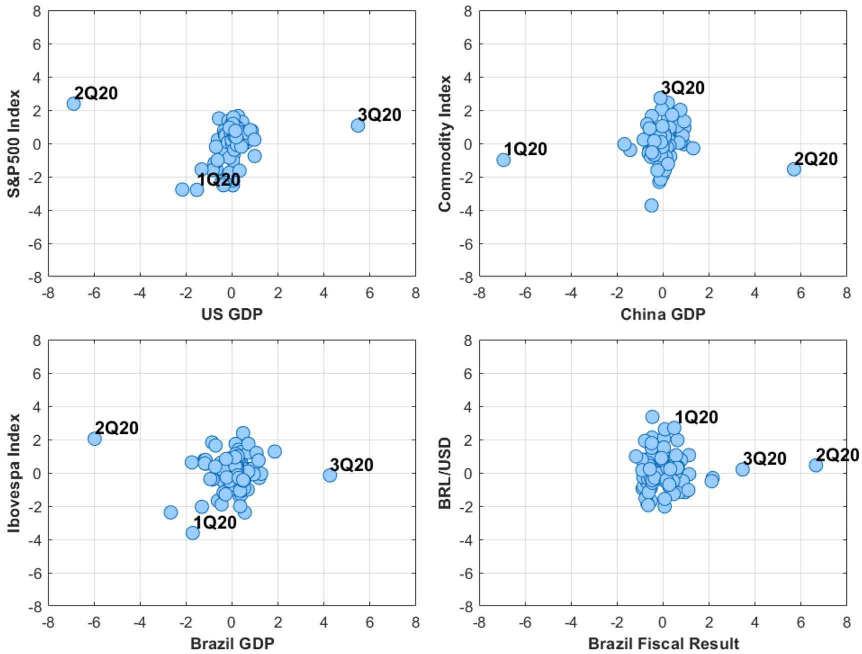
3.2.2 Mean Shift

The commonality assumption is a good approximation when all variables exhibit significant variations simultaneously. However, this was not the case during the COVID-19 pandemic. As Figure 3.1 shows, several macroeconomic variables did not display historically abnormal values.

Cascaldi-Garcia (2022) proposes a more parsimonious homoskedastic approach, called Pandemic Priors, which removes the need for the commonality assumption. Basically, he allows for direct intercept (mean) shifts during the pandemic period using individual time dummies. Hence, each variable can potentially present different shifts, even in the same period.

[3]The original paper also imposes an exponential decay on the scaling factor after these a periods at a rate of ρ. However, an ex-post analysis reveals that the data from 2020Q4 onward did not exhibit abnormal values. Therefore, we remove this feature.

Figure 3.1: Comparative dispersion of selected pairs of variables



Note: Z-score is a standardized statistical measure to compare the deviation of an observation to its sample average. Mathematically, the z-score of observation i from variable j is: $z\text{-}score_i^j = (y_i^j - \bar{y}^j)/\sigma^j$.

Adapting Equation (2-3), the VAR(p) stays:

$$\mathrm{y}_t = \sum_{\tau=t^*+1}^{t^*+a} \mathrm{d}_\tau \mathbf{1}_\tau + \mathrm{c} + \sum_{k=1}^{p} \mathrm{B}_k \mathrm{y}_{t-k} + \mathrm{u}_t \tag{3-4}$$

where $\mathbf{1}_\tau$ is an indicator function that assumes 1 if $t = \tau$ and 0 otherwise and d_τ is a vector of n time dummies for period $t = \tau$.

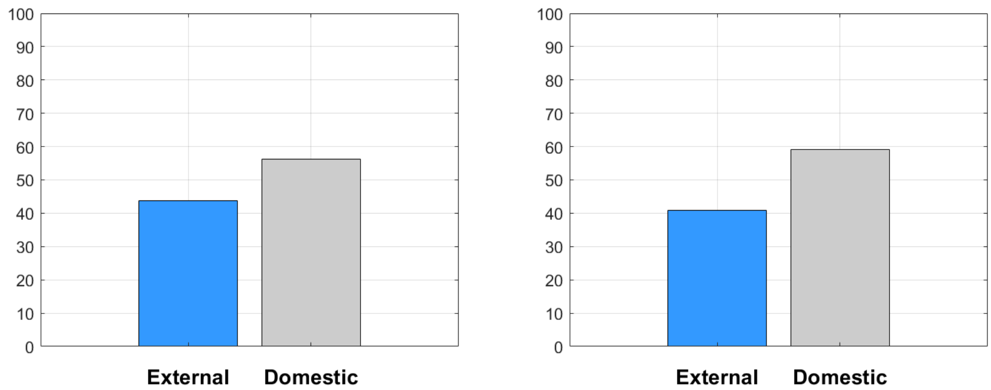
In addition, Cascaldi-Garcia (2022) introduces the parameter ϕ that governs the tightness of the prior associated with the time dummies. When $\phi = 0$, the time dummies absorb all the variance of the pandemic period, meaning no signal is taken from it. Conversely, when $\phi \to \infty$, the time dummies shrink toward zero, allowing full signal to be taken from those observations[4]. Although the selection of ϕ can be arbitrary, Giannone, Lenza and Primiceri (2015) procedure can be adapted to incorporate the Pandemic Prior framework. Thus, we can estimate the optimal shrinkage level of the pandemic observations (ϕ) alongside other hyperparameters.

[4]As a result, the Pandemic Prior nests the boundary cases of Schorfheide and Song (2020), where pandemic observations are ignored, and of a conventional Minnesota Prior, where pandemic information is treated as any other observation.

4
Results

Figure 4.1 plots the variance decomposition exercise of Brazil's GDP series into the two aggregated shocks that we identify. First, we note that results are quite similar under both methods - Lenza and Primiceri (2022) and Cascaldi-Garcia (2022). Second, it indicates that domestic shocks are the main driver of Brazil's business cycle, explaining approximately 60% of the variance in domestic GDP, while the contribution of external shocks is smaller but still significant, closer to 40%[1] .

Figure 4.1: Variance decomposition of Brazil's GDP growth (%)



(a) Lenza and Primiceri (2022)　　(b) Cascaldi-Garcia (2022)

Note: Forecasting horizon of 16 periods (4 years).

The results we present are consistent with the findings of previous studies cited in Chapter 1. In general, they estimate that external shocks account for 24-48% of output fluctuations in Brazil, so our paper falls at the upper end of this range. In addition, virtually all of them point to domestic shocks as the main driver. The importance of domestic shocks in the Brazilian business cycle can be understood in light of the significant role of domestic absorption — particularly private consumption — in the output composition (Table 4.1).

Table 4.1: Brazil's GDP components: Expenditure side (% of GDP)

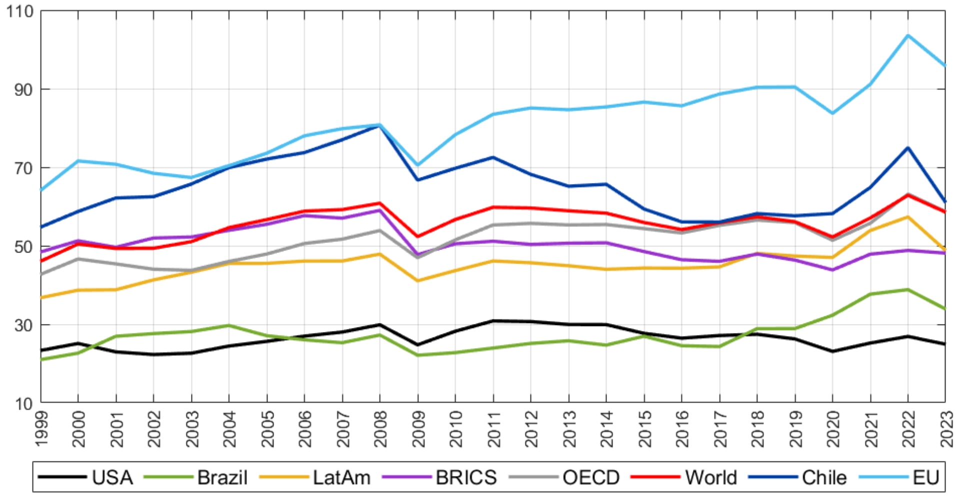
Personal consumption	**Government spending**	**Investments**	**Exports**	**Imports**
65	19	16	12	-12

Note: Average over 1999-2024. Investments = gross fixed capital formation (GFKF) + inventories change. Source: IBGE National Accounts.

[1]Conditional FEVD results for different forecasting horizons at the business cycle frequency are very stable and quickly converge to the unconditional variance levels.

The fact that Brazil remains a relatively closed economy (Figure 4.2), even compared to its emerging peers, further supports the relevance of domestic shocks. In line with this, Fernández, González and Rodríguez (2018) finds that external shocks account for 48% of the GDP variance in Brazil - similar to us — but 77% in Chile, a Latin American emerging economy that is markedly more open to trade than Brazil.

Figure 4.2: Trade (% of GDP)

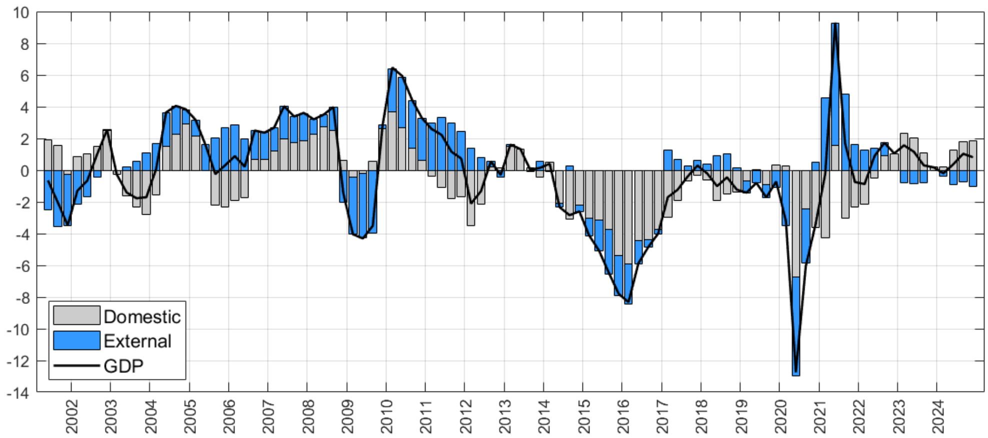


Note: BRICS countries' average excludes Brazil. Source: World Bank database.

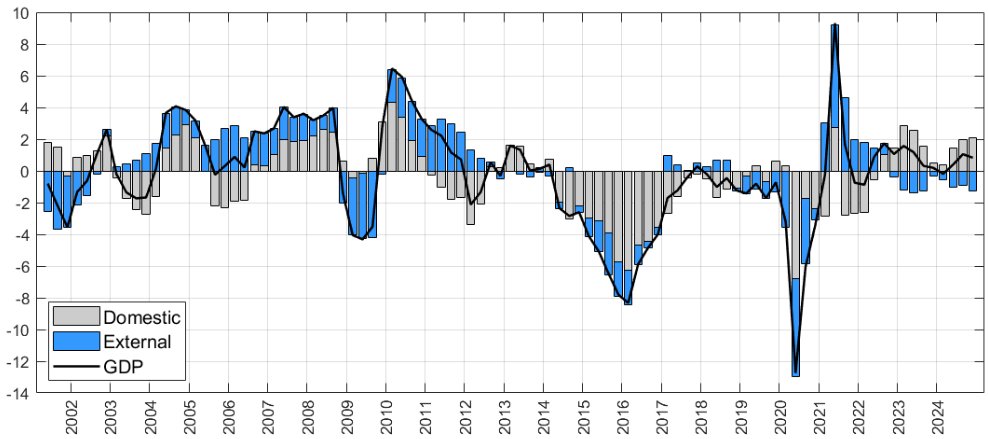
Figure 4.3 plots Brazil's GDP times series decomposed according to the contribution of external and domestic shocks over the selected sample. This historical decomposition exercise is useful to assess the cumulative effect of each shock in explaining particular ups and downs of a time series. As in the FEVD exercise, the historical decomposition results estimated using Lenza and Primiceri (2022) and Cascaldi-Garcia (2022) methods are very similar. Several key insights that arise when we analyze the cumulative contribution of external and domestic shocks during famous economic episodes in Brazil align with the conventional wisdom of economists:

1. Examining the 2000-11 years, for instance, we observe that external shocks systematically contributed in a positive way to economic activity, except during the Great Financial Crisis. This finding reinforces the argument that Brazil's economic expansion during those years was partially fueled by an increase in Chinese demand and a global commodity boom. Carasco, Mello and Duarte (2014) highlight the importance of this favorable external environment in shaping the growth of emerging markets, although they suggest that Brazil would have profit less than other peers.

Figure 4.3: Historical decomposition of Brazil's GDP growth (%)



(a) Lenza and Primiceri (2022)



(b) Cascaldi-Garcia (2022)

Note: GDP growth series is demeaned and accumulated in 4 quarters.

2. During the 2008–09 global financial crisis, our results confirm that the recession observed in Brazil was largely driven by external shocks, with domestic factors playing only a marginal role. This finding reinforces the view that the internal downturn was primarily a spillover from the international financial system crash, following a sharp drop in capital flows, investor confidence, and global trade that reverberated across emerging markets. The negligible contribution of domestic shocks supports the narrative of some policymakers who claimed that Brazil's economic fundamentals remained relatively sound despite an adverse international scenario, limiting the scope and duration of the internal contraction.

3. In contrast, the 2014–2016 recession presents a markedly different pattern. Our results indicate that domestic shocks played the dominant role in the downturn, overshadowing negative external influences such

as falling commodity and oil prices, which had only a limited contribution. This finding is consistent with the preliminary evidence discussed in Chapter 1 and suggests that the roots of the recession were largely home-grown, ranging from political uncertainty and increased risk premia to worse inflationary conditions and declining business confidence. This challenges some political narratives, which attributed the crisis to foreign factors, as illustrated in Chapter 1.

4. The COVID-19 pandemic stands out as a case in which we observe a more mixed contribution to that recession, illustrating the complex nature of this episode. On the external front, a sharp drop in international trade severely affected supply chains, as many of Brazil's key trading partners imposed lockdowns and suspended production. At the same time, domestic disruptions, triggered by temporary closure of non-essential businesses, led to a contraction in household consumption and business investment. Domestic and international financial instability further exacerbated macroeconomic imbalances with capital flight, exchange rate depreciation, and declining investor confidence.

5. We also identify several noteworthy episodes in which domestic and external shocks moved in opposite directions — for instance, during 2005–06 and 2023–24. In the former, the *Mensalão* corruption scandal stands out as a possible trigger of negative domestic shocks while the terms of trade bonanza supported positive external shocks. In the latter case, we observe a combination of strong positive domestic shocks — partially driven by expansive fiscal measures[2] — with an adverse external scenario - possibly stemming from a highly contractionary US monetary and a worsening of the terms of trade.

This historical analysis we provide shows that, despite domestic shocks being the main driver of GDP fluctuations, this composition is not constant throughout the years. In particular, we can infer that the recessions in Brazil are not all alike and have different primary causes, at least in terms of the geographical origin of their underlying shocks. The same conclusion also applies to periods of economic expansion. This variability underscores the need for careful analysis when attributing the observed economic dynamics to either domestic or external sources.

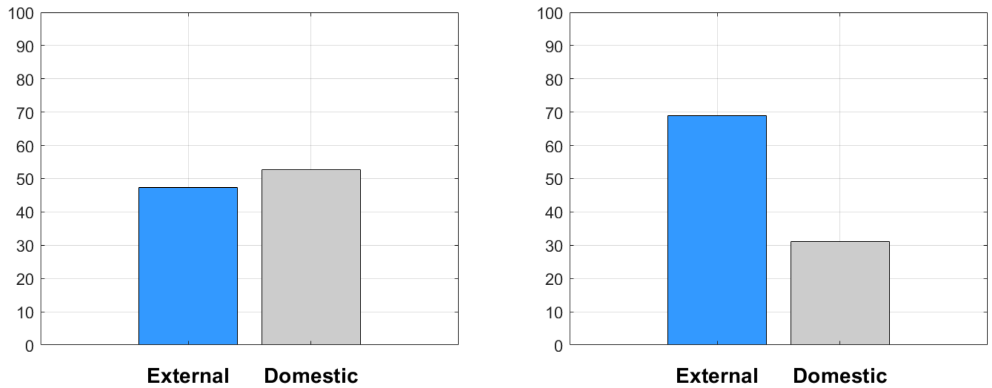
[2]Examples include increased spending on social security, civil servant salaries, and minimum wage-related expenses, as well as on programs such as *Minha Casa Minha Vida*, *Pé de Meia*, and others.

To complement the analysis, Appendix E presents some additional results of this baseline estimate. In special, we plot the correspondent historical decomposition charts without accumulating the series, providing a more real-time, though volatile, perspective of each quarter. We also include a placebo exercise to reinforce the robustness of our model. Additionally, in Appendix F we re-estimate the model at a monthly frequency making the necessary adaptations.

5
Discussion

One might question whether adjusting for pandemic-related outliers is necessary, especially given the additional complexity it introduces to the model. Figure 5.1 plots the result of the variance decomposition exercise based solely on the estimation procedure of Giannone, Lenza and Primiceri (2015), without any correction for outliers. Figure 5.1(a) restricts the end of the sample to 2019Q4, the last observation before the outbreak of the COVID-19, to provide a view of what the model was indicating prior to the pandemic. Using this pre-COVID sample, external shocks explain 47% of Brazil's GDP variance, slightly above our baseline results from Chapter 4 but still conveying the same general message.

Figure 5.1: Variance decomposition of Brazil's GDP growth (%)



(a) Sample: 1999Q3 - 2019Q4 (b) Sample: 1999Q3 - 2024Q4

Note: Forecast window of 16 periods (4 years).

However, when we incorporate the entire available sample, represented in Figure 5.1(b), our model suggests that external shocks are in fact the primary driver of domestic economic fluctuations. More specifically, foreign shocks would account for nearly 70% of the variance in Brazil's GDP, leaving domestic shocks as a relatively minor source of variation. This conclusion contrasts sharply with our baseline results, our pre-COVID sample estimation, and the existing literature cited in Chapter 1, effectively switching the importance order between the shocks.

Table 5.1 provides a direct comparison between the available results for Brazil. It highlights that once we adjust for the abnormal values recorded during the COVID-19 period, regardless of the chosen methodology, the results are again consistent with what the existing literature and our pre-COVID estimation suggest. Our findings prove that failing to downweight pandemic-related data leads to an overestimation of the role of the external component in shaping Brazil's business cycle while understating the importance of domestic factors. As far as we are concerned, we are the first study to demonstrate that these extreme values can lead to distorted results in a Brazilian macroeconomic context.

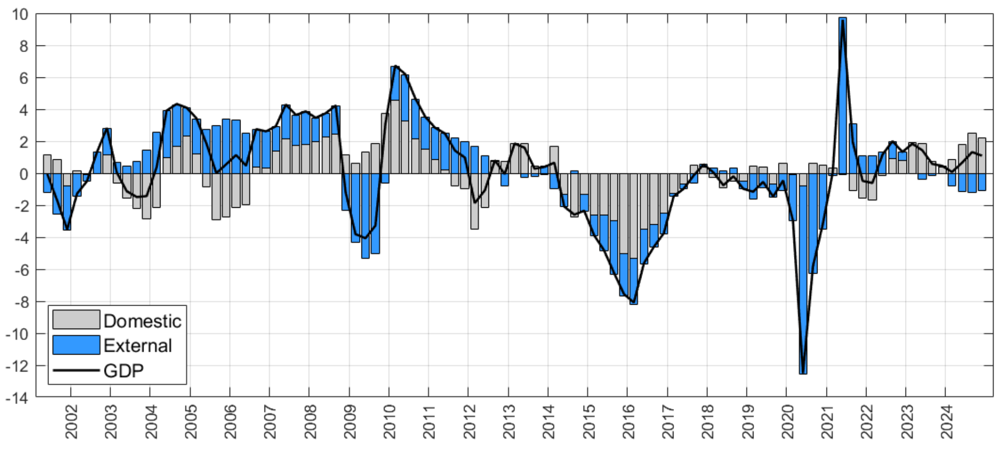
Table 5.1: FEVD comparison: Share of external shocks (%)

Literature:	
Schmitt-Grohé and Uribe (2018)	16
Fernández, Schmitt-Grohé and Uribe (2017)	40
Shousha (2016)	28*
Akinci (2013)	24*
Fernández, González and Rodríguez (2018)	48
Ferreira and Valério (2023)	27-63**
Our Paper:	
Giannone, Lenza and Primiceri (2015): pre-COVID	47
Giannone, Lenza and Primiceri (2015)	69
Lenza and Primiceri (2022)	44
Cascaldi-Garcia (2022)	41

Note: Share of external shocks in explaining the variance in domestic output. *Average result for a group of 6 EMEs (Brazil included). **Their results exhibit a large variability depending on the forecast horizon of the FEVD.

In Figure 5.2, we plot the historical decomposition of the model estimated using just Giannone, Lenza and Primiceri (2015) procedure over the entire sample. If we compare it with Figure 4.3, we note that not correcting for the COVID-19 extreme observations inflates the foreign contribution to domestic GDP mainly during the pandemic, but also in other episodes such as the Great Financial Crisis, albeit less markedly.

Figure 5.2: Historical decomposition of Brazil's GDP growth (%)



Note: GDP growth series is demeaned and accumulated in 4 quarters.

Focusing on the COVID-19 period, it suggests that external shocks were the sole drivers of the 2020 recession and its subsequent recovery, barely leaving room for domestic influence in that period. This is puzzling, given that regional lockdowns likely triggered significant domestic shocks rather than being merely an endogenous response to foreign developments. That is, internal and external lockdowns constituted different exogenous shocks. In our view, the above result would be more consistent with an imaginary counterfactual scenario in which other countries implemented lockdowns while Brazilian authorities did not. One explanation for this might be due to the model's own structure - specifically, the assumption that external variables affect domestic variables, but not the other way around. As a direct consequence, the only way the model can reconcile the fact that the COVID-19 downturn occurred simultaneously in Brazil and abroad is by attributing the shocks triggered by domestic lockdowns to external sources.

6
Conclusion

The aim of this thesis is to disentangle and quantify the contribution of domestic and external shocks to Brazil's economic activity imposing a minimum number of hypotheses. We build a SVAR model that includes domestic and international variables with a block-recursive structure. We base ourselves on the small open economy assumption, arguing that Brazil is a relatively minor player in global markets, to identify two aggregated shocks, one driven only by domestic shocks and the other exclusively by external shocks. Then we decompose Brazil's GDP growth series into these two components using the forecast error variance decomposition and the historical decomposition.

We estimate the model over the period of 1999Q3-2024Q4 with a Bayesian technique that optimally selects the informativeness of the prior distributions in the spirit of hierarchical modeling. With this, we significantly reduce the number and relevance of subjective choices of the hyperparameter values. We show that outlier observations registered during the COVID-19 pandemic can lead to distorted results and correct for it using two new methodologies that downplay the informativeness of these extreme observations to some extent.

Our results suggest that, on average, domestic shocks are the main driver of Brazil's business cycle in our sample, explaining nearly 60% of GDP variance, although external shocks also play a significant role, being responsible for the remaining 40%. Not adjusting for pandemic-related outliers inflates the perceived influence of foreign shocks to approximately 70%, in stark contrast to both the previous literature and our estimates based on a pre-COVID sample.

We also find that the recessions in Brazil are not all alike, at least in terms of the geographical origin of their underlying shocks. The same conclusion is also valid for expansion periods. In particular, the three most recent recession episodes in Brazil had different primary causes: 2008-09 was basically driven by global shocks, 2014-16 was related mainly to domestic causes, while 2020 had a more mixed influence. These and other insights from our historical analysis align with the conventional wisdom of economists.

7
Bibliography

AHIR, H.; BLOOM, N.; FURCERI, D. **The World Uncertainty Index**. [S.l.], 2022.

AKINCI, Global financial conditions, country spreads and macroeconomic fluctuations in emerging countries. **Journal of International Economics**, 2013.

ANGELETOS, G.-M.; COLLARD, F.; DELLAS, H. Quantifying confidence. **Econometrica**, v. 86, n. 5, p. 1689–1726, 2018.

BAKER, S. R.; BLOOM, N.; DAVIS, S. J. Measuring economic policy uncertainty. **The Quarterly Journal of Economics**, v. 131, n. 4, p. 1593–1636, 2016.

BANBURA, M.; GIANNONE, D.; REICHLIN, L. Large bayesian vector auto regressions. **Journal of Applied Econometrics**, 2010.

BARCELONA, W. et al. **What Happens in China Does Not Stay in China**. [S.l.], 2022.

BERNANKE, B. S. Alternative explanations of the money-income correlation. **Carnegie-Rochester Conference Series on Public Policy**, v. 25, p. 49–99, 1986.

BLANCHARD, O.; PEROTTI, R. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. **The Quarterly Journal of Economics**, Oxford University Press, v. 117, n. 4, p. 1329–1368, 2002.

BLANCHARD, O. J.; QUAH, D. The dynamic effects of aggregate demand and supply disturbances. **American Economic Review**, 1989.

CANOVA, F. The transmission of us shocks to latin america. **Journal of Applied Econometrics**, v. 20, n. 2, p. 229–251, 2005.

CARASCO, V. N.; MELLO, J. M. P.; DUARTE, I. F. **A Década Perdida: 2003 – 2012**. [S.l.], 2014.

CARRIèRE-SWALLOW, Y.; CéSPEDES, L. F. The impact of uncertainty shocks in emerging economies. **Journal of International Economics**, v. 90, n. 2, p. 316–325, 2013.

CASCALDI-GARCIA, D. **Pandemic Priors**. [S.l.], 2022.

CHRISTIANO, L. J.; EICHENBAUM, M.; EVANS, C. L. Nominal rigidities and the dynamic effects of a shock to monetary policy. **Journal of Political Economy**, 2005.

CHRISTIANO, L. J.; MOTTO, R.; ROSTAGNO, M. Risk shocks. **American Economic Review**, v. 104, n. 1, p. 27–65, 2014.

CUSHMAN, D. O.; ZHA, T. Identifying monetary policy in a small open economy under flexible exchange rates. **Journal of Monetary Economics**, v. 39, n. 3, p. 433–448, 1997.

DOAN, R. L. T.; SIMS, C. Forecasting and conditional projection using realistic prior distributions. **Econometric Reviews**, Taylor & Francis, v. 3, n. 1, p. 1–100, 1984.

FERNáNDEZ, A.; GONZáLEZ, A.; RODRíGUEZ, D. Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies. **Journal of International Economics**, 2018.

FERNáNDEZ, A.; SCHMITT-GROHé, S.; URIBE, M. World shocks, world prices, and business cycles: An empirical investigation. **Journal of International Economics**, 2017.

FERNáNDEZ-VILLAVERDE, J. et al. Risk matters: The real effects of volatility shocks. **American Economic Review**, 2011.

FERREIRA, M. S.; VALéRIO, A. C. Global shocks and emerging economies: Disentangling the commodity roller coaster. **Revista Brasileira de Economia - RBE**, 2023.

FUJIWARA, I.; HIROSE, Y.; SHINTANI, M. Can news be a major source of aggregate fluctuations? a bayesian dsge approach. **Journal of Money, Credit and Banking**, v. 43, n. 1, p. 1–29, 2011.

GIANNONE, D.; LENZA, M.; PRIMICERI, G. E. Prior selection for vector autoregressions. **The Review of Economics and Statistics**, 2015.

GILCHRIST, S.; ZAKRAJšEK, E. Credit spreads and business cycle fluctuations. **American Economic Review**, v. 102, n. 4, p. 1692–1720, 2012.

GRUSS, B.; KEBHAJ, S. **Commodity Terms of Trade: A New Database**. [S.l.], 2019.

KADIYALA, K. R.; KARLSSON, S. Numerical methods for estimation and inference in bayesian var-models. **Journal of Applied Econometrics**, Wiley, v. 12, n. 2, p. 99–132, 1997.

KILIAN, L.; LUTKEPOHL, H. **Structural Vector Autoregressive Analysis**. [S.l.]: Cambridge University Press, 2017.

KOSE, M. Explaining business cycles in small open economies: 'how much do world prices matter?'. **Journal of International Economics**, v. 56, n. 2, p. 299–327, 2002.

LENZA, M.; PRIMICERI, G. E. How to estimate a vector autoregression after march 2020. **Journal of Applied Econometrics**, 2022.

LITTERMAN, R. B. Forecasting with bayesian vector autoregressions: Five years of experience. **Journal of Business and Economic Statistics**, 1986.

MENDOZA, E. G. The terms of trade, the real exchange rate, and economic fluctuations. **International Economic Review**, v. 36, p. 101–137, 1995.

MERTENS, K.; RAVN, M. O. Empirical evidence on the aggregate effects of anticipated and unanticipated us tax policy shocks. **American Economic Journal: Economic Policy**, v. 4, n. 2, p. 145–81, 2012.

MIRANDA-AGRIPPINO, S.; REY, H. Chapter 1 - the global financial cycle. In: GOPINATH, G.; HELPMAN, E.; ROGOFF, K. (Ed.). **Handbook of International Economics: International Macroeconomics, Volume 6**. [S.l.]: Elsevier, 2022, (Handbook of International Economics, v. 6). p. 1–43.

ROBBINS, H. An empirical bayes approach to statistics. In: ______. **Volume 1 Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, Volume I**. Berkeley: University of California Press, 1956. p. 157–164.

ROMER, C. D.; ROMER, D. H. A new measure of monetary shocks: Derivation and implications. **American Economic Review**, v. 94, n. 4, p. 1055–1084, 2004.

SCHMITT-GROHé, S.; URIBE, M. How important are terms-of-trade shocks? **International Economic Review**, 2018.

SCHORFHEIDE, F.; SONG, D. **Real-Time Forecasting with a (Standard) Mixed-Frequency VAR During a Pandemic**. [S.l.], 2020.

SHOUSHA, S. Macroeconomic effects of commodity booms and busts: The role of financial frictions. **Unpublished manuscript**, 2016.

STEIN, C. Inadimissibility of the usual estimator for the mean of a multivariate normal distribution. In: ______. **Volume 1 Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, Volume I**. Berkeley: University of California Press, 1956. p. 197–206.

STOCK, J.; WATSON, M. Chapter 8 - dynamic factor models, factor-augmented vector autoregressions, and structural vector autoregressions in macroeconomics. In: TAYLOR, J. B.; UHLIG, H. (Ed.). **Handbook of Macroeconomics**. [S.l.]: Elsevier, 2016. v. 2.

WU, J. C.; XIA, F. D. Measuring the macroeconomic impact of monetary policy at the zero lower bound. **Journal of Money, Credit and Banking**, 2016.

ZHA, T. Block recursion and structural vector autoregressions. **Journal of Econometrics**, 1999.

A
Small Open Economy

A common critique of small open economy models concerns the treatment of commodity prices as exogenous to the economic developments of major commodity-exporting countries, like Brazil. In other words, it is sometimes argued that Brazil has monopolistic power in the international commodity market. In fact, commodities account for a large fraction of Brazil's exports but are considerably diffuse in several different products, as per Table A.1. In addition, total world exports of commodities are typically not entirely concentrated in just one or two countries, as shown by the HH index. Even if Brazil accounts for a large share of global exports to a specific commodity, such as poultry meat, it does not necessarily mean that it has monopolistic power in that market.

Table A.1: Brazil exports data

Commodity	Share in total Brazil's exports (%)	Share in total world exports (%)	Index of market concentration (HHI)
Crude Oil	13	4	low (666)
Soybean	13	57	high (4143)
Iron Ore	9	21	high (3287)
Raw Sugar	6	40	medium (1831)
Refined Petroleum	3	1	low (443)
Coffee	3	19	low (722)
Frozen Bovine Meat	3	26	low (1265)
Sulfate Chemical	3	23	low (1088)
Soybean Meal	3	31	medium (1946)
Poultry Meat	3	26	low (1140)
Corn	2	25	medium (1571)
Raw Cotton	2	19	medium (1925)
Others (with <2%)	37	-	-
Year of data	2024	2023	2023

Note: The Herfindahl-Hirschman Index (HHI) is used to determine market competitiveness. A market with an HHI < 1,500 is considered competitive, an 1,500 < HHI < 2,500 is moderately concentrated and an HHI > 2,500 is highly concentrated. Source: Observatory of Economic Complexity (OEC).

The few exceptions are soybean and iron ore, in which Brazil is a major player trading in a highly concentrated market. In theory, domestic supply shocks could affect international prices of these products and thus their own commodity terms of trade. However, both items represent a fraction not higher than 22% of Brazil's exports matrix. So, even if domestic supply shocks do affect these international prices, the effect on its overall commodity terms-of-trade may be somewhat disregarded.

B
Decomposition Exercises

There are some interesting structural applications for SVAR models. In this thesis, we resort to two of these empirical exercises: the forecast error variance decomposition (FEVD) and the historical decomposition. Below, we follow Kilian and Lutkepohl (2017) textbook derivations. To begin with, consider the VAR(1) representation of the VAR(p) process from Equation (2-3):

$$\mathrm{Y}_t = \mathrm{C} + \mathrm{B}\mathrm{Y}_{t-1} + \mathrm{U}_t, \quad \text{with}$$
$$\mathrm{C} = \mathrm{Jc}, \quad \text{and} \quad \mathrm{U}_t = \mathrm{Ju}_t \quad \text{where}$$

$$\mathrm{Y}_t \equiv \begin{bmatrix} \mathrm{y}_t \\ \mathrm{y}_{t-1} \\ \vdots \\ \mathrm{y}_{t-p+1} \end{bmatrix}, \quad \mathrm{Y}_{t-1} \equiv \begin{bmatrix} \mathrm{y}_{t-1} \\ \mathrm{y}_{t-2} \\ \vdots \\ \mathrm{y}_{t-p} \end{bmatrix}, \quad \mathrm{B} \equiv \begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \cdots & \mathrm{B}_{p-1} & \mathrm{B}_p \\ \mathrm{I}_n & 0_n & \cdots & 0_n & 0_n \\ 0_n & \mathrm{I}_n & \cdots & 0_n & 0_n \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_n & 0_n & \cdots & \mathrm{I}_n & 0_n \end{bmatrix}, \quad \mathrm{J} \equiv \begin{bmatrix} \mathrm{I}_n \\ 0_n \\ \vdots \\ 0_n \end{bmatrix} \tag{B-1}$$

B.1
Variance Decomposition

The FEVD illustrates how much of the variance of one variable can be explained by exogenous shocks to the other variables of the system. If we advance Equation (B-1) h periods and recursively substitute Y_{t+h-1} until Y_{t-1}:

$$\begin{aligned} \mathrm{Y}_{t+h} &= \sum_{k=0}^{h-1} \mathrm{B}^k\mathrm{C} + \mathrm{B}^h\mathrm{Y}_{t-1} + \sum_{k=0}^{h-1} \mathrm{B}^k\mathrm{U}_{t+h-k} \\ \mathrm{J}'\mathrm{Y}_{t+h} &= \sum_{k=0}^{h-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^h\mathrm{Y}_{t-1} + \sum_{k=0}^{h-1} \mathrm{J}'\mathrm{B}^k\mathrm{U}_{t+h-k} \\ \mathrm{y}_{t+h} &= \sum_{k=0}^{h-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^h\mathrm{Y}_{t-1} + \sum_{k=0}^{h-1} \mathrm{J}'\mathrm{B}^k\mathrm{Ju}_{t+h-k} \end{aligned} \tag{B-2}$$

Then, the h-step ahead forecast error at period T (the last observation of the sample) is:

$$\begin{aligned} \mathrm{y}_{T+h} - \mathrm{y}_{T+h|T} &= \mathrm{y}_{T+h} - \mathbb{E}[\mathrm{y}_{T+h}|\mathrm{y}_T, \mathrm{y}_{T-1}, \ldots] \\ &= \sum_{k=0}^{h-1} \mathrm{J}'\mathrm{B}^k\mathrm{Ju}_{T+h-k} = \sum_{k=0}^{h-1} \Phi_k\mathrm{u}_{T+h-k} \\ &= \sum_{k=0}^{h-1} \Phi_k\mathrm{A}\epsilon_{T+h-k} = \sum_{k=0}^{h-1} \Theta_k\epsilon_{T+h-k} \end{aligned} \tag{B-3}$$

where $\Phi_k = [\phi_{ij,k}] = \mathrm{J}'\mathrm{B}^k\mathrm{J}$ is the k-th reduced-form impulse response and $\Theta_k = [\theta_{ij,k}] = \Phi_k\mathrm{A}$ is the k-th structural impulse response, in the sense that $\theta_{ij,k}$ represents the effect of a shock j on the variable i after k periods.

Since the errors have mean zero (predictions are unbiased), the forecast error covariance - or mean-squared prediction error (MSPE) - matrix at horizon h is:

$$\begin{aligned} MSPE(h) &= \mathbb{E}[(\mathrm{y}_{T+h} - \mathrm{y}_{T+h|T})(\mathrm{y}_{T+h} - \mathrm{y}_{T+h|T})'] \\ &= \mathbb{V}[(\mathrm{y}_{T+h} - \mathrm{y}_{T+h|T})] \\ &= \sum_{k=0}^{h-1} \Theta_k \Sigma_\epsilon \Theta_k' \\ &= \sum_{k=0}^{h-1} \Theta_k \Theta_k' \end{aligned} \tag{B-4}$$

where we assume $\Sigma_\epsilon = D = I$, a scale normalization[1] necessary due to the unobserved nature of the shocks. So, the fraction of the contribution of shock j to the forecast error variance (MSPE) of variable i at the prediction horizon h is:

$$FEVD_j^i(h) = \frac{MSPE_j^i(h)}{MSPE^i(h)} = \frac{\sum_{k=0}^{h-1} \theta_{ij,k}}{\sum_{j=1}^{n} \sum_{k=0}^{h-1} \theta_{ij,k}} \tag{B-5}$$

Ignoring rounding error, the sum of the $FEVD_j^i(h)$ for all shocks $j = \{1, ..., n\}$ gives us 1 by construction, and if we multiply the fractions $FEVD_j^i(h)$ by 100 we obtain percentages. In a stationary model, the limit of the $FEVD_j^i(h)$, as $h \to \infty$, is the variance decomposition of y_t because the forecast error covariance matrix (or MSPE) converges to the unconditional covariance matrix of y_t.

B.2
Historical Decomposition

While structural FEVDs and IRFs capture average movements in the data, they do not address how the shocks contributed to the historical fluctuations over time. Often, the goal is to assess the cumulative impact of a specific structural shock on a variable at a particular point in time. Returning to the VAR(1) representation from Equation (B-1) and recursively substituting Y_{t-1}:

[1] We use the unit-standard deviation normalization, where each structural shock has unit variance and the main diagonal elements of matrix A are left unrestricted. In contrast, Stock and Watson (2016) advocates for the unit-effect normalization, which keeps the structural shocks' covariance matrix unrestricted $\Sigma_\epsilon = D$ but constrains the diagonal of matrix A to be a unit vector. While the choice between these normalizations has no impact on FEVDs or historical decompositions, it is crucial for IRFs. Under the former, a one-unit shock corresponds to a contemporaneous impulse of one-standard-deviation in its associated observed variable; under the latter, it corresponds to a contemporaneous impulse of one-unit in the variable, which tends to offer a clearer economic interpretation and is particularly valuable for policy analysis.

$$
\begin{aligned}
\mathrm{Y}_t &= \sum_{k=0}^{t-p-1} \mathrm{B}^k\mathrm{C} + \mathrm{B}^{t-p}\mathrm{Y}_p + \sum_{k=0}^{t-p-1} \mathrm{B}^k\mathrm{U}_{t-k} \\
\mathrm{J}'\mathrm{Y}_t &= \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^{t-p}\mathrm{Y}_p + \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{U}_{t-k} \\
\mathrm{y}_t &= \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^{t-p}\mathrm{Y}_p + \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{J}\mathrm{u}_{t-k} \\
\mathrm{y}_t &= \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^{t-p}\mathrm{Y}_p + \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{J}\mathrm{A}\epsilon_{t-k} \\
\mathrm{y}_t &= \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{C} + \mathrm{J}'\mathrm{B}^{t-p}\mathrm{Y}_p + \sum_{k=0}^{t-p-1} \Theta_k\epsilon_{t-k}
\end{aligned} \tag{B-6}
$$

In other words, the value of y_t depends on a deterministic component, on the sequence of shocks $\{\epsilon_p, \ldots, \epsilon_t\}$ - which can be estimated - and on the initial conditions of the sample Y_p. These initial conditions, in turn, reflect the influence of shocks that occurred before the sample begins and, therefore, cannot be estimated. Because the moving average (MA) coefficients decay over time, the impact of pre-sample shocks on y_t diminishes as t increases. Denoting $\widehat{\mathrm{y}}_t = \mathrm{y}_t - \sum_{k=0}^{t-p-1} \mathrm{J}'\mathrm{B}^k\mathrm{C}$ (after subtracting the deterministic term) and dropping the pre-sample term, we obtain the approximation:

$$
\widehat{\mathrm{y}}_t \approx \sum_{k=0}^{t-p-1} \Theta_k\epsilon_{t-k} \tag{B-7}
$$

It is important to note that this truncation of the MA representation introduces an approximation error, especially at the beginning of the sample when much of the shock history is unknown. As time progresses and the influence of earlier unobserved shocks fades, the approximation becomes more accurate. The speed of this convergence depends on the persistence of the underlying process. Consequently, when the sample is short and the data exhibit high persistence, historical decomposition should be applied with caution.

C
Hierarchical Modeling

The hierarchical modeling (HM) literature has roots in the frequentist Empirical Bayes (EB) framework, designed for contexts involving multiple parallel estimation problems. Pioneered by Robbins (1956) and Stein (1956), the EB was frequently used to estimate hyperparameter values from data instead of specifying ad hoc values. With the advance of computation techniques, the EB has been mostly supplanted by a fully Bayesian treatment in hierarchical analyses.

In a simple Bayesian estimation, we use the Bayes' law to describe the posterior of a parameter vector of interest α:

$$\mathrm{p}(\alpha|y) = \frac{\mathrm{p}(y,\alpha)}{\mathrm{p}(y)} \propto \mathrm{p}(y|\alpha)\mathrm{p}_\gamma(\alpha) \tag{C-1}$$

where $\mathrm{p}_\gamma(\alpha)$ is the parameter's prior distribution, γ collects its hyperparameters - those coefficients that parameterize the prior distribution but are not considered random variables -, and $\mathrm{p}(y|\alpha)$ is the model's likelihood function.

In order to account for hyperparameter uncertainty, we need to interpret the model in a hierarchical way, replacing $\mathrm{p}_\gamma(\alpha)$ by $\mathrm{p}(\alpha|\gamma)$ and adopting a prior density on the hyperparameters - $\mathrm{p}(\gamma)$ -, also known as the hyperprior. Then, we can evaluate its posterior using the same Bayes law:

$$\begin{gathered} \mathrm{p}(\gamma|y) \propto \mathrm{p}(y|\gamma)\mathrm{p}(\gamma) \\ \mathrm{p}(y|\gamma) = \int \mathrm{p}(y|\alpha,\gamma)\mathrm{p}(\alpha|\gamma)d\alpha \end{gathered} \tag{C-2}$$

where $\mathrm{p}(y|\gamma)$ is the so-called marginal likelihood (ML), which has an analytical expression for VAR models with conjugate priors.

Giannone, Lenza and Primiceri (2015) argue that their hierarchical procedure selects the optimal value of any hyperparameter in terms of the trade-off between model fit and complexity. To prove that, they break the ML into two terms: one that depends on the in-sample fit and the other that penalizes imprecise out-of-sample forecasts:

$$\mathrm{p}(y|\gamma) \propto \{\text{model fit}\} \cdot \{\text{penalty for complexity}\} \tag{C-3}$$

In our set up, $\alpha = (\mathcal{B}, \Sigma_{\mathrm{u}})$ and $\gamma = \{\lambda, (\lambda,\theta), (\lambda,\phi)\}$, depending on if and which method to correct for outliers we use. After the authors have selected the respective hyperpriors - Gamma distribution for λ and ϕ and Pareto distribution for θ -, they employ a Metropolis-step algorithm to sample a hyperparameter vector from the simulated posterior. Conditional on them, the VAR coefficients $(\mathcal{B}, \Sigma_{\mathrm{u}})$ are drawn from their posterior distribution, which follows a Normal-Inverse Wishart form. The authors show that their method outperforms other popular approaches in terms of out-of-sample forecasting and accuracy of structural impulse response functions.

D
Variables

The Brazilian Development Bank (BNDES) is a public institution that provides long-term credit in local currency for investments in sectors considered strategic to boost productivity, create jobs, and reduce inequalities. Its disbursements refer to the amounts effectively paid by the bank to finance projects, acting as a direct injection of resources into the economy. Given their impact on public debt and economic activity, yet exclusion from the traditional public budget, BNDES disbursements are regarded as one of Brazil's main instruments of para-fiscal policy. For example, between 2008-14, the volume of subsidized credit (at interest rates below market levels) offered by the bank was so high that BNDES disbursements reached the equivalent of 10% of the General Government's budgetary expenditures (or 4% of GDP). Omitting this variable from our model would therefore risk excluding a significant source of economic shocks to Brazil.

The Chinese economy holds first-order importance for Brazil, having become its main trading partner in 2009 — surpassing the United States — and now accounting for nearly 30% of Brazilian exports. More than 75% of these exports to China consist of soybeans, iron ore, and crude oil. While Miranda-Agrippino and Rey (2022) initially argue that China's influence on the world's economy is limited to the global trade and commodity cycle, Barcelona et al. (2022) provide evidence that China's credit policies are also key drivers of the global financial cycle, alongside US monetary policy. Therefore, any comprehensive assessment of external shocks affecting the Brazilian economy must include a measure of China's business cycle.

We also include in the international block a variable that captures the strength of the US dollar — the DXY index — in order to explicitly control for its influence and isolate the portion of BRL/USD exchange rate fluctuations driven exclusively by domestic factors. Since exchange rate movements reflect the relative dynamics between two currencies, failing to account for the dollar's behavior can cloud the identification of shocks specific to the Brazilian real. By including the DXY and assuming that domestic variables do not affect international ones, we are able to recover these domestic shocks.

Table D.1: Data description (alternative variables)

Variable	Source
Domestic	
EMBI+ Brazil	IPEA Data, JP Morgan
Economic uncertainty index (IIE-Br)	FGV-IBRE
Brazil unemployment rate	IPEA Data, IBGE
Brazil real effective exchange rate	BCB-SGS
Brazil gross debt (DBGG)	BCB-SGS
International	
World real GDP	World Bank
Trade-weighted world real GDP	World Bank
European Union real GDP	Fed St. Louis
Japan real GDP	Fed St. Louis
US consumer price index (CPI)	Fed St. Louis
Trade-weighted commodity price index	IMF, Gruss and Kebhaj (2019)
CRB commodity index	Refinitiv Workspace
World uncertainty index	Ahir, Bloom and Furceri (2022)
Global EPU index	Baker, Bloom and Davis (2016)

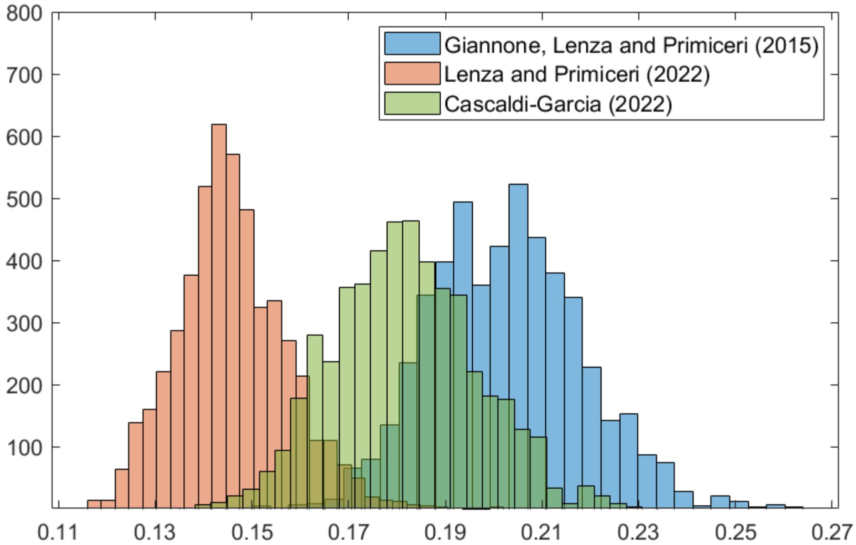
Note: EMBI+ index measures the sovereign spread between the yields of US and Brazilian public bonds, a proxy for country risk; The unemployment rate measure is an retropolation between PME and PNAD indicators, the former discontinued in 2015 and the latter starting in 2012; WB measure of world GDP disregards Brazil's specific trading partners, having a low correlation with domestic GDP; We built our own trade-weighted world GDP series, but it captures only 70% of Brazil's trading partners due to data availability; Trade-weighted commodity index uses rolling weights according to Brazil's exports profile changes; Economic policy uncertainty (EPU) indexes count the number of specific words, like policy and uncertainty, on newspaper articles.

E
Additional Results

E.1
Posterior Draws

Below, we present the histogram of all the posterior draws of the hyperparameters. Focusing on the Minnesota Prior parameter λ, which controls the tightness of the VAR slope coefficients around the postulated priors, this visualization allows us to compare the optimal level of shrinkage selected in each approach. The results displayed in Figure E.1 indicate that the estimation proposed by Giannone, Lenza and Primiceri (2015) shrinks the slope coefficients slightly less than the two methods that address the COVID-19 outlier data. Litterman (1986) originally finds that $\lambda = 0.2$ works well to forecast US macroeconomic variables with a BVAR model, considerably close to our own result.

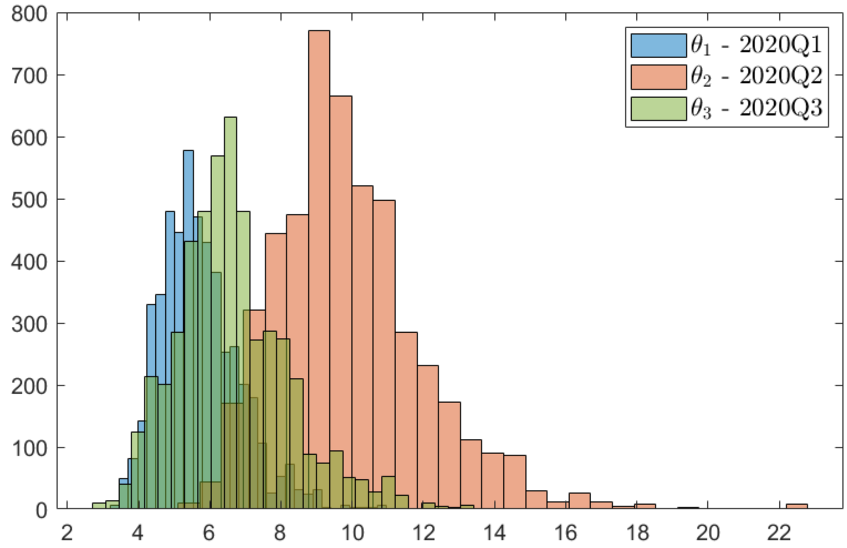
Figure E.1: Posterior draws for λ



Note: Histograms of the Minnesota Prior tightness parameter in each estimation. Constructed after 10,000 draws from the posterior distribution with burn-in period of 5,000.

Examining the volatility shift parameters $\theta_1, \theta_2, \theta_3$ from Lenza and Primiceri (2022) in Figure E.2, we observe that the information contained in the data from 2020Q2 is downplayed more significantly than those from 2020Q1 and 2020Q3. This reflects the fact that the first COVID-19 wave and lockdowns occurred mainly between March and June 2020. Later waves saw milder macroeconomic and financial fluctuations as restrictions eased and governments and markets adapted.

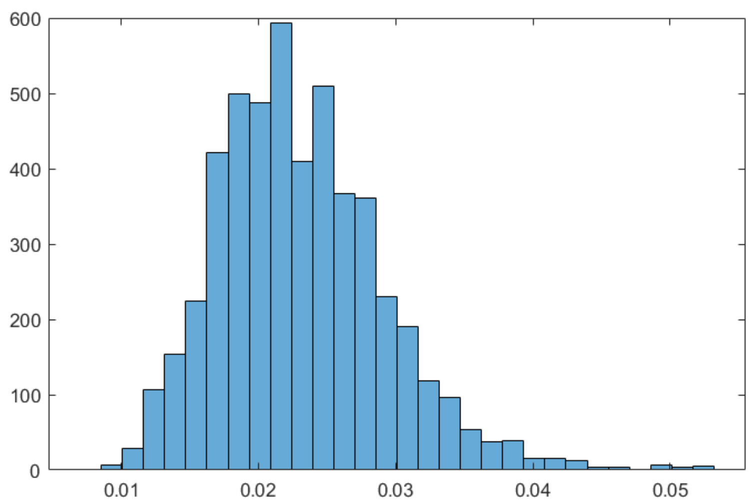
Figure E.2: Posterior draws for θ



Note: Histograms of the volatility shift parameters from Lenza and Primiceri (2022). Constructed after 10,000 draws from the posterior distribution with burn-in period of 5,000.

The selection of ϕ, the parameter that governs the prior associated with the time dummies of Cascaldi-Garcia (2022), in Figure E.3 indicates a considerable degree of shrinkage, though some information from the pandemic observations is still useful. For comparison, the author shows that $\phi = 0.001$ mimics the no-signal case, while $\phi = 5$ is close to the full-signal case. In its original paper, he works with US data and finds a mean value for ϕ of 0.002.

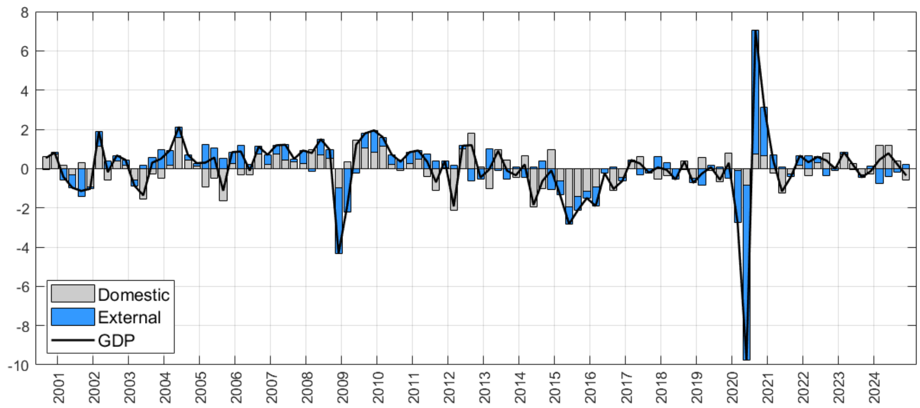
Figure E.3: Posterior draws for ϕ



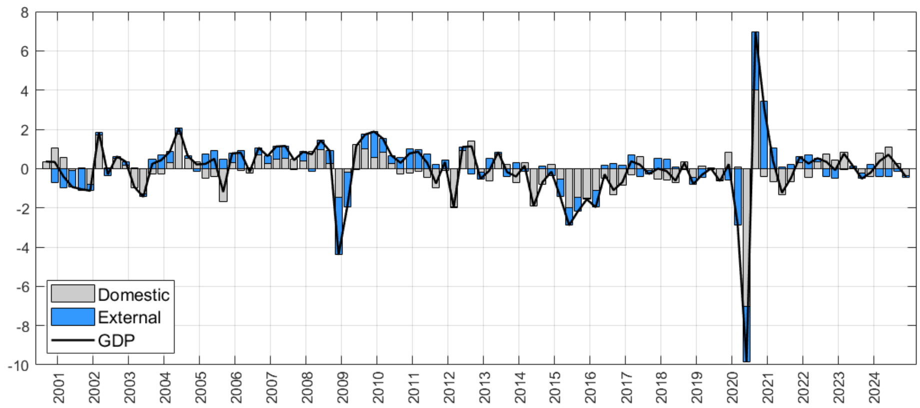
Note: Histogram of the mean shift parameter from Cascaldi-Garcia (2022). Constructed after 10,000 draws from the posterior distribution with burn-in period of 5,000.

E.2
Quarterly (Not Accumulated)

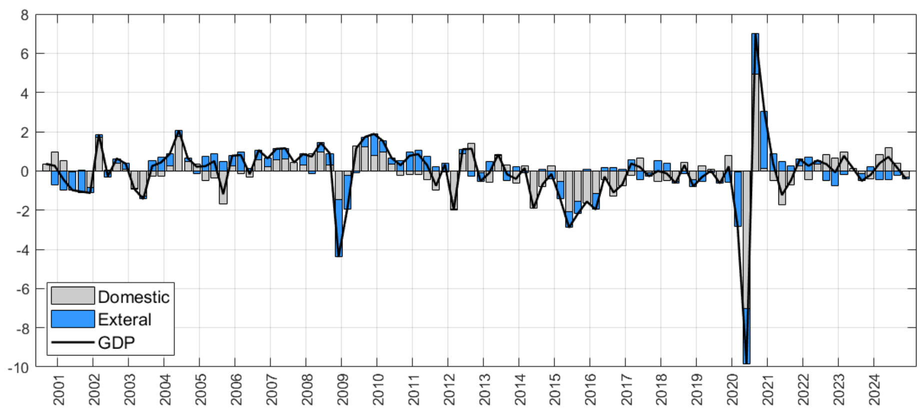
Figure E.4: Historical decomposition of Brazil's quarterly GDP growth



(a) Giannone, Lenza and Primiceri (2015)



(b) Lenza and Primiceri (2022)



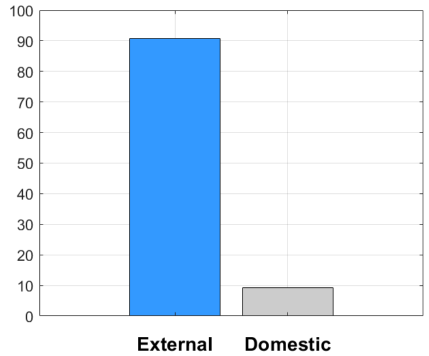
(c) Cascaldi-Garcia (2022)

Note: GDP growth series is demeaned but not accumulated.

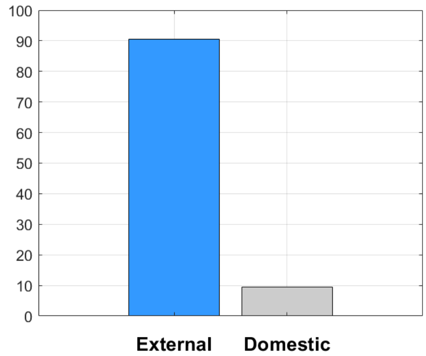
E.3
Robustness

One way to ensure that our model is not capturing spurious relationships — and to reinforce the validity of our conclusions — is through a type of placebo exercise. A simple and informative test using the same model and estimates involves plotting the variance decompositions of US and China's GDP instead of Brazil's. Under our identification assumption, external shocks influence domestic variables, but domestic shocks do not affect external variables. Therefore, if the model is correctly specified, we should expect Brazilian domestic shocks to have little to no explanatory power over the variance of these two foreign countries' GDP. Instead, their fluctuations should be exclusively driven by the external shocks of our model.

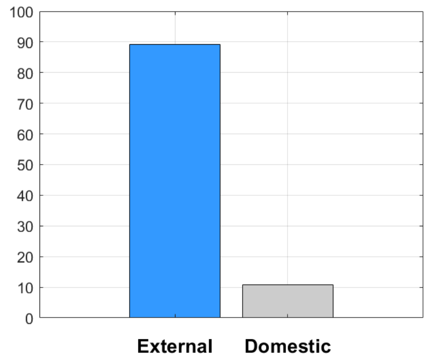
Figure E.5: Variance decomposition of other countries GDP growth (%)



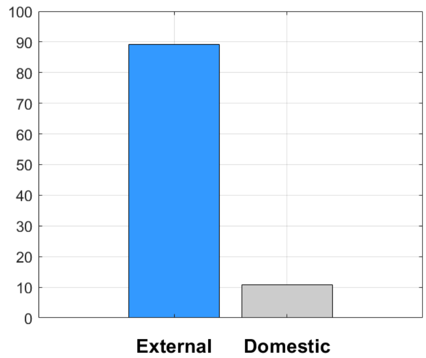
(a) USA - Lenza and Primiceri (2022)



(b) USA - Cascaldi-Garcia (2022)



(c) China - Lenza and Primiceri (2022)



(d) China - Cascaldi-Garcia (2022)

Note: Forecast window of 16 periods (4 years).

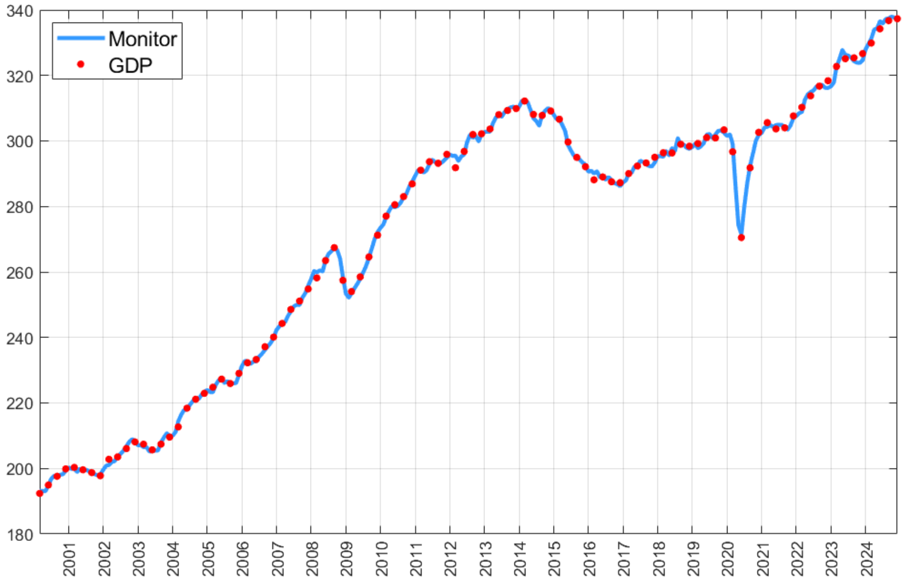
As shown in Figure E.5, external shocks account for approximately 90% of the business cycle fluctuations in the US and China, while the contribution of Brazilian domestic shocks is minimal, though not exactly zero. This small residual influence arises because the SOE assumption was implemented by imposing zero restrictions only on the contemporaneous impact matrix A. In contrast, the reduced-form lagged coefficient matrices $\{\mathrm{B}_1, \ldots, \mathrm{B}_p\}$ were left unrestricted. This modeling choice allows for the possibility of lagged effects from domestic variables on external ones. However, as the figures demonstrate, the impact of these lagged relations is small and can be somewhat disregarded, indicating that our baseline results are satisfactorily robust and credible.

We chose not to restrict the matrices $\{\mathrm{B}_1, \ldots, \mathrm{B}_p\}$ because doing so would imply that the equations in our system have different dependent variables. This would break the natural conjugacy of the prior distributions, meaning the posteriors would no longer belong to the same distributional family of the priors and we would lose closed-form expressions for posterior moments. Given this substantial increase in complexity, combined with the empirically small size of these lagged coefficients effect, we opt for the more parsimonious and tractable specification.

F
Monthly Frequency

We repeat the exercise using data at a monthly frequency. For that we substitute the Quarterly National Accounts (CNT) GDP indicator from IBGE for the GDP monitor calculated by FGV-IBRE. This monthly publication has a considerable fit with the official quarterly measure as seen in Figure F.1.

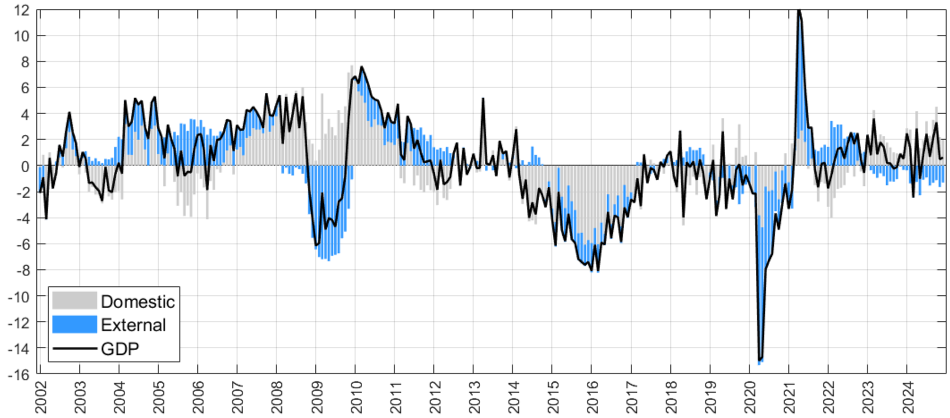
Figure F.1: Quarterly and monthly GDP measures (R$ billions)



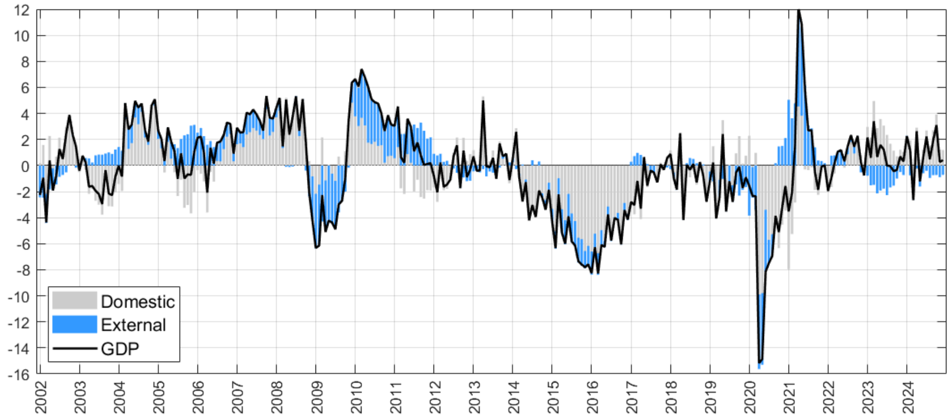
Note: Monthly monitor values represent a 3-month accumulation. Data in real terms and seasonally adjusted. Source: IBGE and FGV-IBRE

However, finding monthly counterparts for the US and China GDP indicators is not an easy task due to low availability, methodological differences and, specifically in the case of China, data credibility. That said, we use the Chicago Fed national activity index and the Chinese industrial production index, which are common options when dealing with similar situations. The results using monthly frequency preserve the general conclusion when we analyze the main economic episodes of the domestic business cycle in Figure F.2.

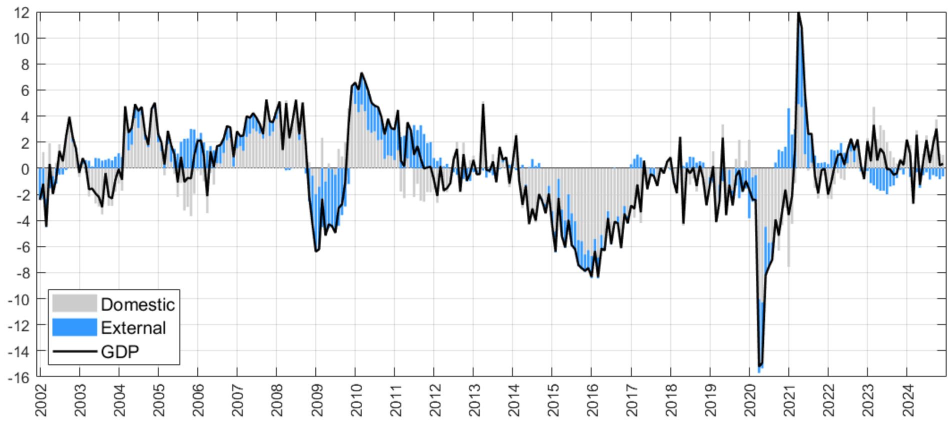
Figure F.2: Historical decomposition of Brazil's GDP growth



(a) Giannone, Lenza and Primiceri (2015)



(b) Lenza and Primiceri (2022)



(c) Cascaldi-Garcia (2022)

Note: GDP growth series is demeaned and accumulated in 12 months.