Pontifícia Universidade Católica
do Rio de Janeiro

João Paulo Lima Gomes

Monetary policy transmission under high bank spread

Dissertação de Mestrado

Masters dissertation presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Yvan Bécard

Rio de Janeiro
April 2024

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Abstract

Gomes, João Paulo Lima; Bécard, Yvan (Advisor). **Monetary policy transmission under high bank spread**. Rio de Janeiro, 2024. 69p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

We analyze the influence of high credit spread on the transmission of monetary policy. We develop a macroeconomic model that generates endogenous credit spread with two main ingredients: banks that operate in Cournot competition and defaulting firms whose assets exhibit low recovery rate. The model implies that imperfect banking competition mitigates the effects of a monetary shock while credit frictions amplify them, indicating the presence of two transmission channels acting in opposite directions. A calibration consistent with Brazil shows that the second channel dominates the first, revealing that high credit spreads amplify the impact of a monetary shock on inflation but at a considerable cost in terms of lost output. To deal with such inefficiency, we propose an alternative monetary rule that takes into account changes in the spread. Our results suggest that this alternative rule reduces welfare losses, offering a promising avenue for central banks to balance inflation control and output concerns.

Keywords

DSGE; Financial friction; Banking market power; Credit spread; Monetary shock; Conditional welfare.

Resumo

Gomes, João Paulo Lima; Bécard, Yvan. **Transmissão de política monetária sob alto spread bancário**. Rio de Janeiro, 2024. 69p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Nós analisamos a influência do alto spread de crédito na transmissão da política monetária. Nós desenvolvemos um modelo macroeconômico que gera um spread de crédito endógeno com dois ingredientes principais: bancos que operam em concorrência de Cournot e empresas inadimplentes cujos ativos apresentam baixa taxa de recuperação. O modelo implica que a concorrência bancária imperfeita atenua os efeitos de um choque monetário, enquanto as fricções de crédito os amplificam, indicando a presença de dois canais de transmissão atuando em direções opostas. Uma calibração consistente com o Brasil mostra que o segundo canal domina o primeiro, revelando que os altos spreads de crédito amplificam o impacto de um choque monetário sobre a inflação, mas a um custo considerável em termos de perda de produto. Para lidar com essa ineficiência, propomos uma regra monetária alternativa que leva em conta as mudanças no spread. Nossos resultados sugerem que essa regra alternativa reduz as perdas de bem-estar, oferecendo um caminho promissor para os bancos centrais equilibrarem o controle da inflação com as preocupações com o produto.

Palavras-chave

DSGE; Fricção financeira; Poder de mercado bancário; Spread de crédito; Choque monetário; Bem-estar condicional.

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1
Introduction

Brazil has one of the world's high credit spread, the difference between deposit and loan rates: the average annual spread in Brazil is 26.8%, second only to Madagascar (World Bank 2020). How does a high spread affect the transmission of monetary policy? Does it amplify or dampen monetary shocks? Is the central bank of Brazil helped or hindered by a banking sector with high spread? We study these questions in this paper.

Brazil's financial sector has two characteristics that set it apart from other economies and which may explain the high spread. First, the banking sector is highly concentrated, with the three largest banks holding around 70% of total assets in 2021 (see figure 1.1). Coelho et al. (2017) estimate elasticities of demand and supply of credit and conclude that there is little banking competition in Brazil. Joaquim et al. (2019) show that a reduction in banking competition results in an increase in the lending rates. However, other countries such as Australia, Canada and Germany have highly concentrated banking sectors, but with low spreads. Therefore, the high spread in Brazil cannot be explained solely by high banking concentration.

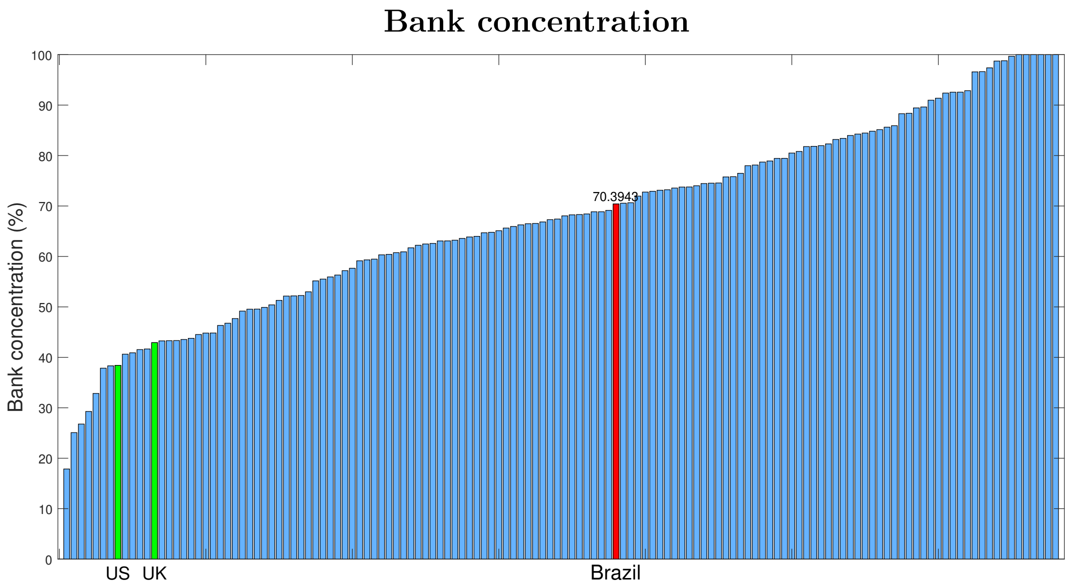

Figure 1.1: Fraction of total banking assets held by the three largest banks in 2021. Source: World Bank.

The second characteristic is Brazil's loan recovery rate, which is much lower than in most other countries. The loan recovery rate measures how many cents on the dollar are recovered by creditors through reorganization, liquidation or debt enforcement proceedings. Figure 1.2 illustrates the position of Brazil and the other countries in ascending order of recovery rate. It is clear that the Brazilian economy occupies an atypical position in relation to other countries. Capeleti, Garcia and Sanches (2018) show that the recovery rate and the banking spread are negatively correlated across countries, so that this metric, being very low for Brazil, may be an important factor in determining the high Brazilian spread.

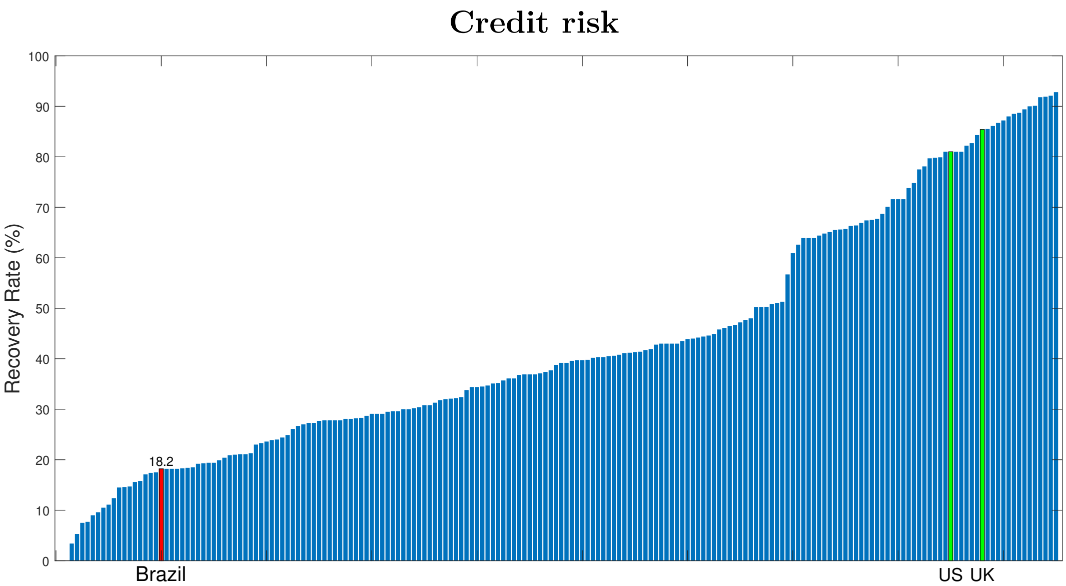

Figure 1.2: Recovery rate. Source: World Bank 2019.

This paper develops a macroeconomic model that generates an endogenous credit spread based on these two characteristics of the Brazilian credit market. The model has two main ingredients: (1) a banking sector that operates in Cournot competition and (2) defaulting entrepreneurs whose assets exhibit low recovery rate. Thus, banks with market power recover only a fraction of assets from defaulting entrepreneurs. We use a calibration suitable for Brazil and simulate the model with a monetary shock.

Our main finding is as follows: the model features two transmission channels, who triggers opposite effects. The first channel, resulting from imperfect banking competition, mitigates the effects of the monetary shock. The second channel, resulting from low recovery rate, amplifies the effects of the shock. In our baseline calibration, the second channel dominates the first. The result is that monetary policy under a high spread has greater power over inflation, but is disproportionately costly in terms of output.

Based on this result, we look for a way to mitigate the cost of the monetary shock on output, while maintaining its amplified effects on inflation. To do this, we conduct a welfare analysis using an alternative monetary rule, which takes into account changes in the spread caused by the monetary shock. Comparing the model under a standard monetary rule and under this alternative rule, we find that the latter reduces welfare losses; the new rule also maintains the power of monetary policy on inflation, but the cost of lost output is considerably reduced. We conclude that a monetary rule that takes into account movements in the banking spread, in addition to deviations in inflation and output, is able to maintain the high power of monetary policy on inflation without a significant reduction in economic activity.

This paper contributes to the literature on monetary policy and credit market frictions. One line of research consists of models with financial constraints (Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Iacoviello (2005), Christiano, Motto and Rostagno (2014), Iacoviello (2015), Elenev, Landvoigt and Nieuwerburgh (2021)). Another line of research develops models with banks that exhibit market power (Gerali et al. (2010), Dib (2010), Andrés and Arce (2012), Li (2021), Wang et al. (2022)). Our contribution is to combine these two lines of research in a single framework, which results in a model that presents two transmission channels for monetary shocks, both with opposite effects.

In addition, there are few papers that focus directly on the relationship between monetary policy, credit spread and welfare (Curdia and Woodford (2010), Fiore and Tristani (2013), Cúrdia and Woodford (2016)). We find that incorporating financial variables into monetary rules can improve welfare.

This paper is organized as follows. Section 2 presents the model. Section 3 details the calibration and offers comparative statics. Section 4 studies the transmission of monetary policy. Section 5 shows that a monetary rule that considers changes in the spread is welfare improving. Section 6 concludes.

2
Model

The model presents a banking sector that operates in Cournot competition as in Li (2021). Entrepreneurs borrow from banks, are subject to borrowing constraints as in Iacoviello (2005) and may default as in Bernanke, Gertler and Gilchrist (1999). To close the model, we used a device employed by Fujisima (2021) which consists of recycling firms which acquire physical capital after a default has occurred.

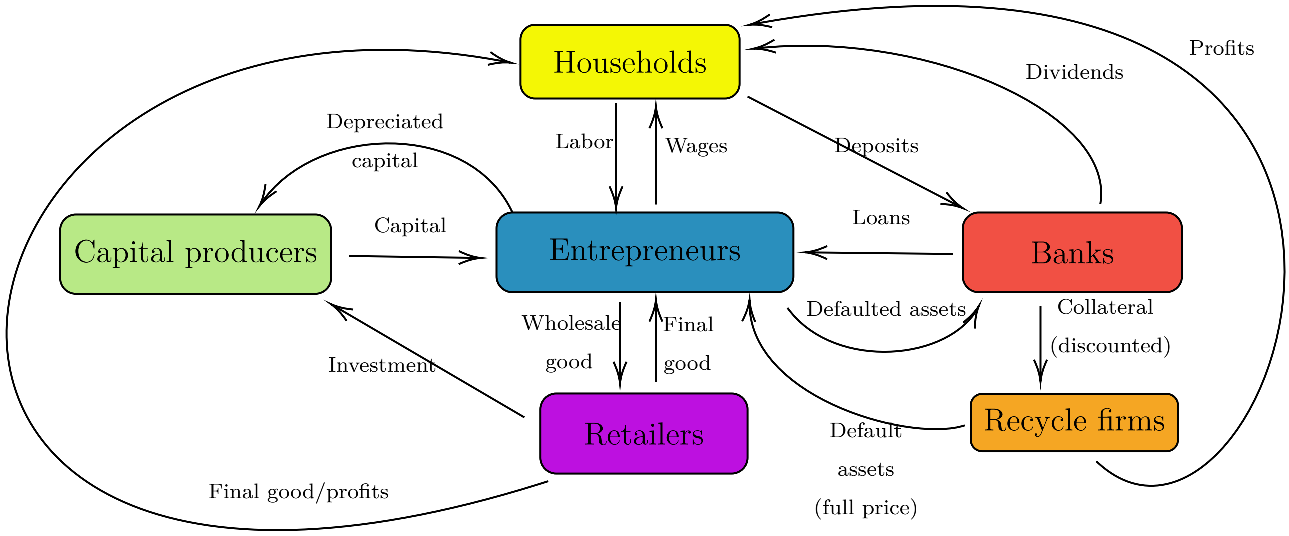

Figure 2.1: Model diagram

2.1
Households

The representative household maximizes its expected utility given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \,\Gamma_{t+s} \left(\log(C_{t+s} - h^H C_{t-1+s}) - \frac{L_{t+s}^{1+\phi}}{1+\phi} \right) \tag{2.1.1}$$

which depends on current consumption C_t, lagged consumption C_{t-1} and labor L_t. The parameter h^H measures the degree of external habit formation in consumption, and ϕ is the inverse of labor supply elasticity. The household discount factor is given by $\beta \in (0,1)$, and Γ_t is an intertemporal preference shock. The logarithm of the Γ_t shock follows an AR(1) process given by:

$$\log(\Gamma_t) = (1-\rho_\Gamma)\,\log(\Gamma) \,+\, \rho_\Gamma\,\log(\Gamma_{t-1}) \,+\, \sigma_\Gamma\,\varepsilon_t^\Gamma\,, \qquad \varepsilon_t^\Gamma \,\sim\, \mathcal{N}(0,1)$$

where $\Gamma = 1$ consists of the steady-state value of the preference shock.

In each period t, the household consumes C_t, saves D_t in real terms of bank deposits, and offers L_t of labor to entrepreneurs. The bank deposit D_{t-1} in nominal terms is yielded by a nominal gross interest rate R_{t-1} at the beginning of period t.

Let P_t be the aggregate price index of final goods, so that the gross inflation rate is $\Pi_t = \dfrac{P_t}{P_{t-1}}$. We assume that households own producers of physical capital, retailers (producers of final goods), banks and recycle firms. Given the real gross return on bank deposits $\dfrac{R_{t-1}\, D_{t-1}}{\Pi_t}$, the real labor income $W_t\, L_t$, the real profit T_t^{CP} of physical capital producers, the real profit T_t^R of retailers, the fraction $1-\omega^B$ of the real profit T_t^B of banks and of the real income $\mu\, G(\bar{\omega}_t^E)\, Q_t\, (1-\delta^E)\, K_{t-1}^E$ of banks generated by the collateral after default, as well as the real profit $(1-\mu)\, G(\bar{\omega}_t^E)\, Q_t\, (1-\delta^E)\, K_{t-1}^E$ of recycle firms, the representative household faces the following budget constraint:

$$
\begin{aligned}
C_t + D_t \leq \frac{R_{t-1}\, D_{t-1}}{\Pi_t} &+ W_t\, L_t + T_t^{CP} + T_t^R + (1-\omega^B)\,[\,T_t^B + \mu\, G(\bar{\omega}_t^E)\, Q_t\, (1-\delta^E)\, K_{t-1}^E\,] \\
&+ (1-\mu)\, G(\bar{\omega}_t^E)\, Q_t\, (1-\delta^E)\, K_{t-1}^E
\end{aligned}
\tag{2.1.2}
$$

Therefore, the representative household's problem is to choose, in period t, consumption, bank deposits and labor supply so as to maximize its objective function (2.1.1) subject to the budget constraint (2.1.2). The first-order conditions for C_t, L_t and D_t are provided in Appendix A.1.

2.2 Banks in Cournot competition

We use a Cournot banking sector to characterize banking competition and capture the market power of the banks. We assume that there are N banks in this economy, each indexed by h. There is no market power in the bank deposit market, so that the nominal interest rate on deposits is equal to the policy rate R_t, controlled directly by the central bank. Bank h receives deposits $D_t(h)$ from households and offers credit $B_t(h)$ to entrepreneurs. In addition, bank h accumulates bank capital $K_t^B(h)$, so that the accounting identity is given by:

$$
B_t(h) = D_t(h) + K_t^B(h) \tag{2.2.1}
$$

In each period t, the total outflow of funds, consisting of real profit $T_t^B(h)$ (which will be used to build up bank capital and to pay dividends to households), credit $B_t(h)$ granted to entrepreneurs and real gross interest payments on deposits to households given by $\frac{R_{t-1}\, D_{t-1}(h)}{\Pi_t}$, equals the total inflow of funds, consisting of deposits $D_t(h)$ made by households and real gross interest payments on loans made to entrepreneurs who have not defaulted, $(1 - F(\bar{\omega}_t^E))\, \frac{R_{t-1}^B\, B_{t-1}(h)}{\Pi_t}$. We also assume that the bank incurs a cost whenever it deviates its equity/assets ratio from a capital requirement level given by v^B in the previous period. This cost is expressed by:

$$\Omega_t^B(h) \;=\; \frac{\kappa_{KB}}{2} \left(\frac{K_t^B(h)}{B_t(h)} \,-\, v^B \right)^2 K_t^B(h)\,, \tag{2.2.2}$$

where κ_{KB} is the adjustment cost parameter of the equity/assets ratio. Therefore, the real profit of bank h is:

$$T_t^B(h) \;=\; (1 - F(\bar{\omega}_t^E))\, \frac{R_{t-1}^B\, B_{t-1}(h)}{\Pi_t} - \frac{R_{t-1}\, D_{t-1}(h)}{\Pi_t} - B_t(h) + D_t(h) - \frac{\Omega_{t-1}^B(h)}{\Pi_t}$$

The N banks operate in Cournot competition. Each individual bank h takes the loan quantities offered by the other banks $m \neq h$ as given. The bank takes into account the effect of its choice to offer $B_t(h)$ on the partial equilibrium in the credit market, through the total quantity of credit B_t and the loan market rate R_t^B; but ignores general equilibrium effects and takes other aggregate prices and quantities as given. Each bank h sets its credit quantity $B_t(h)$ to maximize the sum of the expected discounted present value of future real profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\, T_{t+s}^B(h)$$

subject to (2.2.1) and (2.2.2), where:

$$\begin{aligned} T_t^B(h) \;=\; & (1 \,-\, F(\bar{\omega}_t^E))\, \frac{R_{t-1}^B(B_{t-1}(h) \,+\, \sum_{m\neq h} B_{t-1}(m))\; B_{t-1}(h)}{\Pi_t} \,-\, \frac{R_{t-1}\, D_{t-1}(h)}{\Pi_t} \\ & -\, B_t(h) \,+\, D_t(h) \,-\, \frac{\Omega_{t-1}^B(h)}{\Pi_t} \end{aligned}$$

Profit $T_t^B(h)$ is positive due to imperfect competition, and an exogenous fraction $(1 - \omega^B)$ will be the households' dividends. A key aspect is that $R_t^B(\cdot)$ represents the inverse demand function for loans, which depends on B_t and therefore on $B_t(h)$. This is crucial for introducing imperfect banking competition. The dependence of $R_t^B(h)$ on $B_t(h)$ means that each bank h exercises some control over the equilibrium gross interest rate on loans by changing its own supply of credit given the other banks' quantities of credit, and this is taken into account by bank h under Cournot competition when it chooses $B_t(h)$. As in Gerali et al. (2010), we assume that the dividend policy is exogenous, so that bank capital is not a choice variable for h.

We impose that an ω^B fraction of the real profits and income generated by default be used to build new bank capital from non-depreciated bank capital from the previous period. The dynamics of bank capital are given, in real terms, by:

$$K_t^B(h) = (1 - \delta^B) \frac{K_{t-1}^B(h)}{\Pi_t} + \omega^B \left[T_t^B(h) + \mu \, G(\bar{\omega}_t^E) \, Q_t \, (1 - \delta^E) \, K_{t-1}^E(h) \right]$$

where δ^B is the rate of depreciation of bank capital.

The first-order condition is, after some algebraic manipulation, given by (see appendix A.2):

$$R_t^B = \frac{\left[R_t - \kappa_{KB} \left(\frac{K_t^B}{B_t} - v^B \right) \left(\frac{K_t^B}{B_t} \right)^2 \right] \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} \right\}}{\left[1 - \left(\frac{1}{PED_t \, N} \right) \right] \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} (1 - F(\bar{\omega}_{t+1}^E)) \right\}}$$

where N is the number of banks and PED_t is the interest elasticity of market demand for credit, in absolute terms. Therefore, $PED_t \, N$ is the elasticity of demand for credit, in absolute terms, faced by each bank h.

In the expression of the loan rate, three key parameters determine R_t^B and, therefore, the bank spread. Firstly, although it doesn't appear in the expression, a very small recovery rate μ reduces bank capital formation K^B in the steady-state, reducing the $\frac{K^B}{B}$ ratio, which leads to an increase in the loan rate and, therefore, a higher spread in the steady-state. Secondly, the σ parameter determines the fraction $F(\bar{\omega}^E)$ of loans in default in the steady-state, so that a higher $F(\bar{\omega}^E)$ leads to a higher loan rate R^B and thus a higher spread. Finally, a small number of banks N directly determines a higher spread in the steady-state; on the other hand, in the case of perfect banking competition, each bank faces a perfectly elastic demand for credit, so that $PED \, N \to \infty$. Thus, together with $\mu = 1$ and $\sigma \approx 0$, adopting an arbitrarily high N allows us to reproduce the situation of perfect banking competition, since the banks' optimizing behavior results in $R_t^B = R_t$ and, therefore, zero spread and zero real profit.

The interest elasticity of the market demand for credit, PED_t, is calculated from the demand for credit of the representative entrepreneur (see appendix A.3). Therefore, the elasticity of demand for credit is endogenous in the model, reflecting the degree of financial constraint on the entrepreneur dependent on shocks.

2.3 Entrepreneurs

Entrepreneurs produce a homogeneous intermediate good that is sold under perfect competition to retailers, producers of final goods. In period $t-1$, entrepreneurs acquire physical capital K^E_{t-1} from physical capital producers at the real price Q_{t-1} for production in period t. The capital K^E_{t-1} and the labor L_t hired from the representative household are used to produce the wholesale (intermediate) good $Y_{w,t}$ via a Cobb-Douglas production technology with constant returns to scale:

$$Y_{w,t} = A_t\,(K^E_{t-1})^{\alpha}\,(L_t)^{1-\alpha}\,, \tag{2.3.1}$$

where $\alpha \in (0,1)$ is the output elasticity of physical capital.

At the nominal price $P_{w,t}$, the homogeneous wholesale good is then sold to retailers, who produce differentiated final goods. Productivity A_t, common to all entrepreneurs, follows an AR(1) process in logarithm:

$$\log(A_t) = (1-\rho_a)\,\log(A) \,+\, \rho_a\,\log(A_{t-1}) \,+\, \sigma_a\,\varepsilon^a_t\,, \qquad \varepsilon^a_t \sim \mathcal{N}(0,1)$$

where $A = 1$ represents the steady-state value of A_t, and $\rho_a \in (0,1)$ is the persistence parameter of the process.

Each entrepreneur finances his acquisition of physical capital using loans from banks and its net worth. The collateral of the bank loan B_t taken by the entrepreneur is the physical capital K^E_t held by the entrepreneur, and the value of the collateral is subject to an idiosyncratic shock ω^E_t, which is an independent random variable identically distributed over time and entrepreneurs. This shock follows a cumulative distribution function $F(\omega^E_t)$ on a non-negative support ($\omega^E_t > 0$) whose mean is equal to 1. In particular, $log(\omega^E_t) \sim \mathcal{N}(-\frac{\sigma^2_t}{2}, \sigma^2_t)$. We add this idiosyncratic shock, to which the physical capital served as collateral is subject, in order to generate a credit default. Thus, a default cut-off rule is given by:

$$\frac{R^B_{t-1}\,B_{t-1}}{\Pi_t} = \bar{\omega}^E_t\,\left[Q_t\,(1-\delta^E)\,K^E_{t-1}\right]$$

The above rule defines, for each entrepreneur, a default threshold $\bar{\omega}^E_t$ for the shock. If $\omega^E_t < \bar{\omega}^E_t$, the entrepreneur defaults and loses its collateral to the banks. However, if $\omega_t^E > \bar{\omega}_t^E$, the entrepreneur prefers to pay off his loan debt and thus retains possession of the collateral.

As in Christiano, Motto and Rostagno (2014), we define risk shock, σ_t, as the standard deviation of $\log(\omega^E)$ in period t. Risk shock follows an AR(1) process in logarithm:

$$\log(\sigma_t) = (1-\rho_\sigma)\,\log(\sigma) + \rho_\sigma\,\log(\sigma_{t-1}) + \sigma_\sigma\,\varepsilon_t^\sigma\,, \qquad \varepsilon_t^\sigma \sim \mathcal{N}(0,1)$$

We assume perfect insurance between entrepreneurs, so that default threshold and decisions to consume, purchase physical capital, hire labor and take out credit are the same for all entrepreneurs. This is a way of circumventing heterogeneity immediately after the idiosyncratic shocks have taken place[1]. We can then aggregate the continuum of entrepreneurs into a representative entrepreneur.

In each period t, the representative entrepreneur consumes C_t^E of final goods, has cost $W_t\,L_t$ in real terms with the households' labor and acquires physical capital K_t^E at the real price Q_t from the capital producers. The entrepreneur also has to pay off its total debt (see appendix A.4). On the other hand, the entrepreneur has a realized product, in terms of units of consumption of final goods, given by $\frac{Y_{w,t}}{X_t}$, where $X_t \equiv \frac{P_t}{P_{w,t}}$ is the markup of the retail sector. The entrepreneur also obtains income from the sale of the stock of non-depreciated physical capital to capital producers given by $Q_t\,(1-\delta^E)\,K_{t-1}^E$, where δ^E is the depreciation rate of physical capital. Finally, the entrepreneur obtains credit B_t from the banks. His budget constraint is:

$$C_t^E + W_t\,L_t + Q_t\,K_t^E + (1 - F(\bar{\omega}_t^E))\,\frac{R_{t-1}^B\,B_{t-1}}{\Pi_t} \;\leq\; \frac{Y_{w,t}}{X_t} + (1 - G(\bar{\omega}_t^E))\,Q_t\,(1-\delta^E)\,K_{t-1}^E + B_t \tag{2.3.2}$$

In the above expression, $F(\bar{\omega}_t^E)$ indicates the fraction of credit taken out by entrepreneurs in $t-1$ that is not repaid in t (i.e. it measures the entrepreneur's default in t), and $G(\bar{\omega}_t^E)$ is the fraction of collateral that is confiscated in t due to default. Therefore, $1 - G(\bar{\omega}_{t+1}^E)$ represents the fraction of physical capital that is expected to be available as collateral in period $t+1$. The borrowing constraint is then given by:

[1]We assume that all entrepreneurs insure each other through transfers. These transfers, however, only take place after each entrepreneur has decided to default. The entrepreneur who receives a collateral shock smaller than his default threshold will default, while another entrepreneur who receives a shock larger than his default threshold will honor his loan debt. Transfers then take place, making it possible to aggregate the continuum of entrepreneurs. This allows us to consider that the continuum of entrepreneurs has the same default threshold.

$$B_t \leq M\,\mathbb{E}_t \left\{ (1 - G(\bar{\omega}^E_{t+1}))\, \frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)\,K^E_t}{R^B_t} \right\}, \tag{2.3.3}$$

where M is the loan-to-value (LTV) or pledgeability ratio of banks, and R^B_t is the nominal gross interest rate for bank credit taken by entrepreneurs.

The objective of the representative entrepreneur is to maximize its expected lifetime utility with external habit formation in consumption

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta^E)^s \log(C^E_{t+s} - h^E C^E_{t-1+s}) \tag{2.3.4}$$

subject to the budget constraint (2.3.2), its production function (2.3.1) and its borrowing constraint (2.3.3). The parameter β^E is the representative entrepreneur's discount factor. We assume that $\beta^E < \beta$, as is standard in borrowing constraint models, so that entrepreneurs are net borrowers.

The intertemporal choice of the representative entrepreneur is distorted when the borrowing constraint is binding (see the first-order condition for B_t in appendix A.4). By calibrating the model so that $\dfrac{\beta}{\beta^E} > (1 - F(\bar{\omega}^E))\,\dfrac{R^B}{R}$, we impose that the borrowing constraint is always binding in the neighborhood of the steady-state and therefore guarantee that the entrepreneur needs to borrow to finance his physical capital (see appendix A.5). In addition, we guarantee that the entrepreneur's ability to smooth his consumption depends on the financial tightening resulting from the borrowing constraint.

We define the entrepreneur's net worth N^E_t as the return on physical capital net of the loan repayment. We can rewrite the entrepreneur's budget constraint from his net worth (see appendix A.6):

$$C^E_t + Q_t\,K^E_t = B_t + N^E_t$$

Therefore, the entrepreneur uses both bank credit and his net worth to acquire physical capital and consume differentiated final goods.

2.4 Recycling firms

In the event of default, the banks confiscate the collateral but only manage to take a fraction μ of the value of the collateral. The μ parameter adds the recovery rate to the model, a relevant factor in a modeling proposal for Brazil's banking sector. Recycling firms are a device that allows this factor to be included in the model, without the destruction of collateral after default.

After confiscating the collateral, the banks sell it to the recycling firms, but at a discount rate of μ. Therefore, only a proportion μ of the value of the default collateral is added to the banks' real profit from financial intermediation under spread. In possession of the collateral, the recycling firms then resell it to the entrepreneurs, this time charging the full real price Q_t. Therefore, the real profit of the recycling firms is given by:

$$T_t^{RF} = G(\bar{\omega}_t^E)\, Q_t\, (1 - \delta^E)\, K_{t-1}^E - \mu\, G(\bar{\omega}_t^E)\, Q_t\, (1 - \delta^E)\, K_{t-1}^E$$
$$T_t^{RF} = (1 - \mu)\, G(\bar{\omega}_t^E)\, Q_t\, (1 - \delta^E)\, K_{t-1}^E$$

The real profit T_t^{RF} is then rebated to households in lump-sum form. As the representative entrepreneur pays a debt of $G(\bar{\omega}_t^E)\, Q_t\, (1 - \delta^E)\, K_{t-1}^E$ resulting from the default, the amount paid to the recycling firms is already taken into account, so their budget constraint does not change.

2.5 Central bank

Monetary policy is implemented by a standard Taylor rule. The central bank adopts a policy rate that depends on the lagged policy rate, the steady-state value R of the policy rate, the deviation of the gross inflation rate from its steady-state level, and the change in the current output level relative to the lagged output level:

$$R_t = (R_{t-1})^{\phi_r} \left[R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\phi_r} e^{\sigma_r\, \varepsilon_t^R}, \qquad \varepsilon_t^R \sim \mathcal{N}(0,1)$$

where ϕ_r is the interest smoothing parameter, ϕ_π is the weight assigned by the central bank to inflation stabilization, and ϕ_y is the weight corresponding to output stabilization. In addition, ε_t^R is an exogenous monetary policy shock.

Since there is no market power of banks in the deposit market, they equate their rate of return on deposits to the policy rate employed by the central bank.

The rest of the model is standard and is provided in the appendices. With regard to retailers, prices are optimized a la Calvo (1983) and non-optimized prices are indexed and corrected by a fraction of lagged gross inflation, as in Castro et al. (2015).

3
Sources of the high spread

We follow estimates for the Brazilian economy to calibrate the model; for some remaining parameters, we calibrate in order to generate reasonable steady-state values for Brazil. Next, we check how changes in the values of the three key parameters of the model determine changes in the steady-state values of the banking spread.

3.1
Calibration

We calibrate most of the parameters with the values estimated by Castro et al. (2015); for other parameters, we follow Gerali et al. (2010) and Ferreira (2013). For the recovery rate, we assign a value close to the latest World Bank estimate. The other parameters are calibrated so that the model generates first moments that are compatible with the Brazilian economy in the period 2011Q2 to 2023Q3. Table 3.1 shows the complete calibration of the model.

Table 3.1: Model calibration

Description	Parameter	Value	Source
Inverse of the elasticity of labor supply	ϕ	1	Castro (2015)
Capital share in production function	α	0.44	Castro (2015)
Persistence of household's habit	h^H	0.74	Castro (2015)
Persistence of entrepreneur's habit	h^E	0.74	Castro (2015)
Adjustment cost of investment	χ	3.42	Castro (2015)
Price indexation	γ	0.33	Castro (2015)
Calvo - price stickness	θ	0.75	Castro (2015)
Interest rate smoothing (Taylor rule)	ϕ_r	0.79	Castro (2015)
Inflation coefficient (Taylor rule)	ϕ_π	2.43	Castro (2015)
Output coefficient (Taylor rule)	ϕ_y	0.16	Castro (2015)
Physical capital depreciation	δ^E	0.025	Gerali (2010)
Bank capital depreciation	δ^B	0.090	Gerali (2010)
Elasticity of substitution (final good)	ϵ	6	Gerali (2010)
Recovery rate	μ	0.20	World Bank (2020)
Bank capital to assets ratio	v^B	0.098	Own calibration
Household's discount factor	β	0.978	$R = 2.15\%$ quarterly
Entrepreneur's discount factor	β^E	0.940	Own calibration
Number of banks	N	5	Own calibration
Loan-to-value physical capital	M	0.30	Ferreira (2013)
Steady-state value of risk shock	σ	0.55	Own calibration
Bank's leverage cost	κ_{KB}	25	Own calibration
Bank's dividend policy	ω^B	0.75	Law 6,404/76

The discount factor β of the representative household is calibrated so that the steady-state value of the policy rate coincides with the quarterly average return on the certificado de depósito bancário (CDB), considering zero inflation in the steady-state. We calibrate β^E to ensure that the entrepreneur's borrowing constraint is always binding in the neighborhood of the steady-state[1]. The v^B parameter is calibrated according to the bank capital requirements in Brazil defined by the BCB: between 2013 and 2019, the requirement varied from 0.11 to 0.08; we adopt the average of these values[2]. We calibrate ω^B following the value in Brazilian law that defines the minimum percentage of profit earmarked for payment of dividends[3]. We assume that the five largest banks in Brazil control the entire credit market, so we calibrate $N = 5$ (see appendix D).

With regard to the recovery rate, we calibrate $\mu = 0.20$, which is close to the value of 0.182 estimated by the World Bank[4]. It is important to note that the World Bank's Doing Business 2020 report does not distinguish between different types of credit when calculating the recovery rate; in the absence of this specification, we consider the World Bank's estimate to be valid for credit to firms whose collateral is physical capital. In addition, the recovery rate estimated by the World Bank consists of cents on the dollar that insured creditors are able to recover from insolvent borrowers after reorganization, liquidation or debt enforcement procedures, which is the approximate concept of the recovery rate that we use in the model.

For the shocks, we calibrate the autocorrelation parameters ρ_Γ, ρ_a and ρ_σ at 0.9 and the corresponding standard deviations σ_Γ, σ_a and σ_σ at 0.01.

3.2 Model fit

The κ_{KB} parameter, which defines the cost paid by banks when there is a deviation from the v^B requirement level, is calibrated so as to generate an equity/assets ratio value in the steady-state that is sufficiently close to the corresponding measure in the data, defined as the average of the Índice de Patrimônio de Referência in Brazil between 2013Q4 and 2023Q2. Oliveira and Ferreira (2018) points out that the Brazilian banking system's data is strongly determined by the largest banks; therefore, we consider this approach to be valid for calibrating κ_{KB}.

[1]For larger values of β^E, the borrowing constraint can be non-binding depending on the size of the shock.

[2]We use the minimum capital requirement adopted in Brazil, which constitutes the Patrimônio de Referência of banks according to Oliveira and Ferreira (2018). See <https://www.bcb.gov.br/pec/appron/apres/basileia(v3).pdf>.

[3]Law 6,404/76, articles 201 to 205, defines a minimum percentage of 25% of adjusted profit for payment of dividends. It is therefore a minimum payout that must be included in the company's bylaws. See <https://www.planalto.gov.br/ccivil_03/leis/l6404consol.htm>.

[4]<https://archive.doingbusiness.org/en/data/exploretopics/resolving-insolvency>.

We calibrate σ so that the steady-state default rate is close to the corresponding one in the data, defined as the average credit default for firms using collateral. This series is not explicitly displayed in the BCB's Time Series Management System (SGS-BCB)[5]. Given the importance of working capital credit for firms in Brazil, we assume that total credit for firms is made up of two modalities: working capital credit (which does not use collateral, and the guarantee consists of the company's future revenues) and credit that uses collateral. The amount of the latter is calculated from the total credit and working capital credit series, both available in the SGS-BCB. Next, we assume that the average default of firms is defined by defaults of the two types of credit, with the weights defined by the share of each type in total credit in each quarter from 2011Q2 to 2023Q3. Therefore, with the average credit default and working capital credit default rates, we obtain the average default in the period for firms that took out credit that uses collateral.

Table 3.2: Steady-state values vs data first moments

		Model	**Data**
Spread	$R^B - R$	1.30	1.05
Recovery rate	μ	0.20	0.18
Default rate	$F(\bar{\omega}^E)$	2.70	2.80
Equity/assets ratio	$\frac{K^B}{B}$	13.72	13.71

Notes: Spread and default in percentage points; ratio in percent. Since the default rate, Índice de Patrimônio de Referência and loan rate are monthly in the SGS-BCB, we adjust the steady-state values to monthly for ease of comparison.

Table 3.2 shows the steady-state values generated by the calibration and the average values of the corresponding variables in the data. The loan rate for credit that uses collateral is also not available explicitly in the SGS-BCB, so we compute its average value from the average loan rate for firms and the loan rate for working capital credit. A description of these series is available in Appendix E.

With the model calibrated, it is possible to carry out comparative statics in order to assess how the three key parameters (N, σ and μ) determine the model variables in the steady-state.

3.3 Comparative statics

In subsection 2.2, from the inspection of the loan rate equilibrium condition, we point out that a small number N of banks determines a high

[5] <https://www3.bcb.gov.br/sgspub/localizarseries/localizarSeries.do?method=prepararTelaLocalizarSeries>.

steady-state spread; alternatively, an arbitrarily high N makes the model reproduce the situation of perfect competition between banks. Figure 3.1 confirms that the steady-state spread is decreasing in N, with the greatest variation occurring when N is small; for $N > 20$, the spread hardly changes with an increase in N. Note that the spread does not reach zero due to the default and the low recovery rate. In any case, the variation of the spread with N shows the importance of this parameter for banks' market power. Credit market demand is more elastic as N increases, so that banks with some market power are prevented from increasing the loan rate in a more competitive credit market.

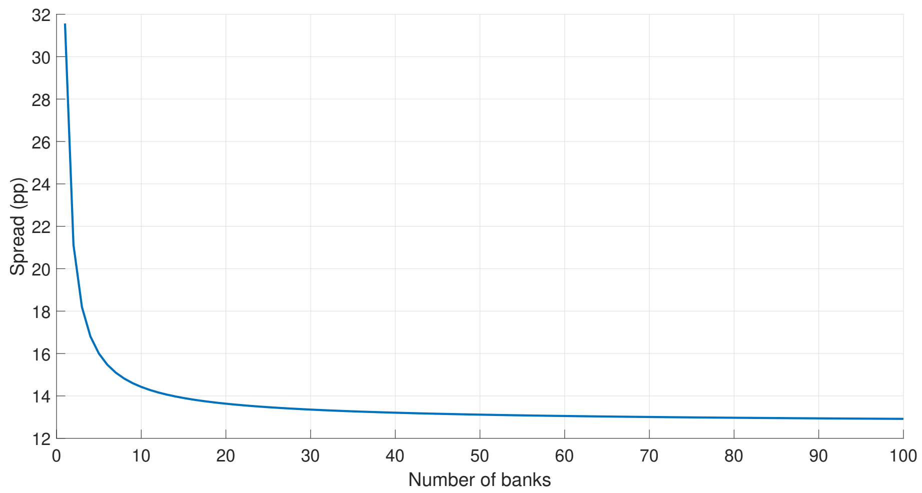

Figure 3.1: Annual bank spread versus number of banks

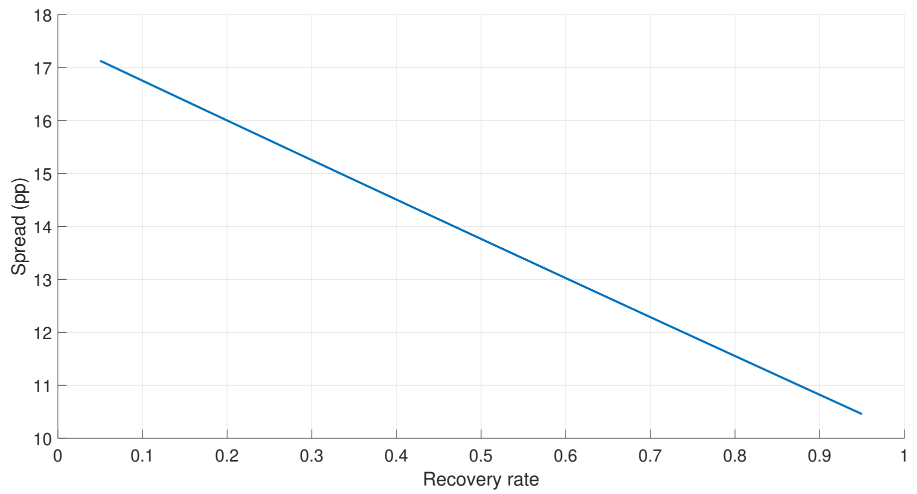

Figure 3.2: Annual bank spread versus recovery rate

Figure 3.2 shows how variations in the recovery rate determine changes in the steady-state spread. Decreasing values of the recovery rate result in increasing values of the spread, indicating that the model exhibits the expected behavior when we vary μ. Note that, all other things being equal, an increase

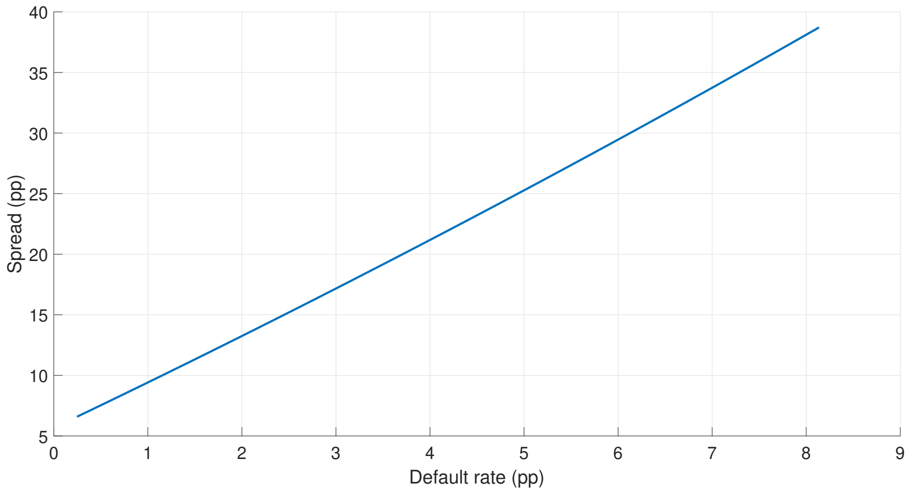

Figure 3.3: Annual bank spread versus default rate

in the recovery rate to the values of the most advanced economies (around 0.8) would result in a drop in the annual spread of more than 4 pp, which shows the relevance of this parameter in determining the spread[6].

With regard to the other key parameter, σ, we expect higher default rates to determine higher spread values. This relationship is shown in figure 3.3. Note that variations in the probability of default result in large changes in the annual steady-state spread. According to the model, reducing the default rate to 1 pp reduces the annual spread from 16 pp to below 10 pp, all else being equal.

[6]As indicated in subsection 2.2, a low recovery rate reduces the capital-asset ratio, which results in a high spread value. In Appendix F, this mechanism is shown in Figure F.1.

4
Monetary policy transmission

Our baseline model nests two other models (see subsection 2.2). We analyze three cases: the baseline model (i), the model whose credit friction is only the Cournot competition of banks (ii), and the model without friction in the loan market (iii). The baseline model (i) consists of the model following the calibration in section 3.1. In order to remove the effects of the credit default, we set $\mu = 1$ and $\sigma = 0.01$, so that we recover the model whose only credit friction is the Cournot competition of banks (ii). Finally, we set $\mu = 1$, $\sigma = 0.01$ and $N = 1000000$ to obtain the model without credit frictions, in which banks operate in perfect competition (iii). We impose a monetary shock of 25 basis points, which is equivalent to a contractionary shock of 1 percent per year in Selic, for the three cases, in order to maintain the consistency of the analysis.

By sequentially excluding the two types of credit frictions, it is possible to identify two transmission channels. The first, called the collateral channel, stems from Cournot competition and can be seen in the IRFs in solid line in figure 4.1. Immediately after the shock, the entrepreneur's borrowing constraint becomes tighter, so that the market demand for loans becomes more sensitive to any change in the loan rate; in other words, the shock increases the

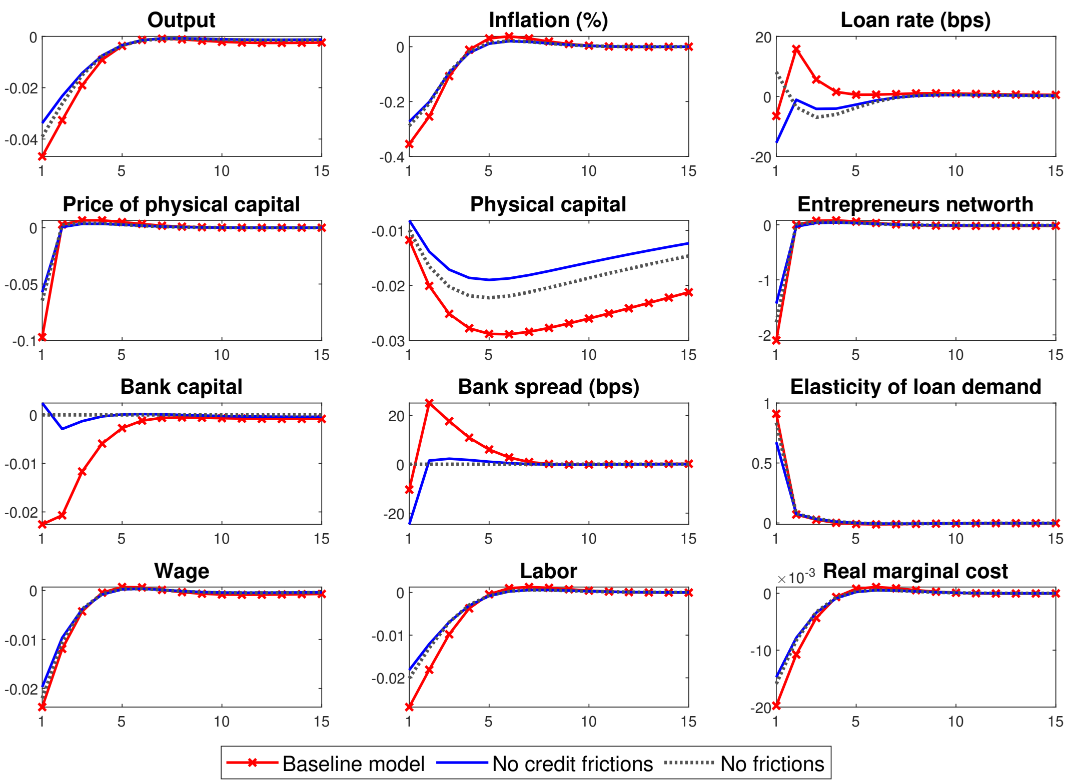

Figure 4.1: Response to monetary policy shock

elasticity of demand for loans. Faced with a more elastic demand for credit, banks lower their loan rate, which reduces the spread. The lower loan rate, in turn, eases the squeeze on the entrepreneur's borrowing constraint. Thus, relative to the case without frictions, the elasticity of market demand for credit increases by a smaller magnitude immediately after the shock. This means that the entrepreneur suffers a smaller drop in his ability to borrow compared to the case without frictions, which mitigates the drop in the acquisition of physical capital, investment and, finally, output. Therefore, imperfect banking competition acts to mitigate the effects of the monetary shock, a result in line with most of the literature[1].

The second transmission channel, called the net worth channel, is defined by the effects of default, which are enhanced by the low recovery rate. The mechanism is identified from the IRFs in line with crosses in figure 4.1. With the shock, the probability of default increases, tightening the entrepreneur's borrowing constraint and thus limiting their ability to take out credit. This leads to a drop in the acquisition of physical capital, which leads to a drop in the real price of capital and, therefore, a reduction in the entrepreneur's net worth. As the entrepreneur uses his net worth to acquire physical capital in addition to loans, there is a further fall in the acquisition of physical capital, which further reduces the real price of capital and, therefore, the net worth. This process repeats itself indefinitely, occurring at the same moment as the shock. There is therefore a feedback effect, called a financial accelerator by Bernanke, Gertler and Gilchrist (1999). The result is an amplification of the effects of the shock on the variables in the model.

Compared to the no frictions case, the two channels have opposite effects. The Cournot competition dampens while the default (and the low recovery rate) amplifies the effects of the shock. Note that the second channel dominates the first, since in the baseline case the variables exhibit greater changes at the time of the shock compared to the no frictions case (a situation in which it is possible to consider that the two channels have exactly the same magnitude). Compared to the case with only Cournot competition of banks, the higher probability of default also mitigates the fall in the loan rate and, consequently, in spread.

The main result is the amplification of the effects of the monetary shock in the baseline model. Monetary policy under high bank spreads has its effects

[1] Our result is in line with Gerali et al. (2010), Dib (2010), Andrés and Arce (2012) and Wang et al. (2022), who conclude that banks with market power mitigate the effects of a monetary shock. One of the bases of our model for Cournot competition in banking, Li (2021) reaches the opposite conclusion. In fact, the paper indicates an inconsistent result, in which low bank competition is accompanied by more elastic loan market demand.

on inflation and output magnified.

In the period following the shock, the probability of default, the real price of capital, the entrepreneur's net worth and the elasticity of loan demand return to their steady-state values. Note, however, that the spread increases abruptly and remains some periods above its steady-state level. This is due to the low levels of bank capital, which increases the loan rate and therefore the spread. This persistence of high spreads contributes to restricting entrepreneurs financially for longer, amplifying the fall in physical capital in the following periods. As a result, the output IRF in the baseline model remains below the IRFs of the other cases, which suggests a greater cumulative loss of output.

Finally, the real marginal cost in the baseline model shows a response to the shock similar to the other models. Although the acquisition of physical capital remains compromised for longer periods, the return of output to its steady-state level re-establishes wages and demand for labor, accelerating the recovery of real marginal cost. As a result, the three models show practically identical inflation dynamics from $t = 2$ onwards.

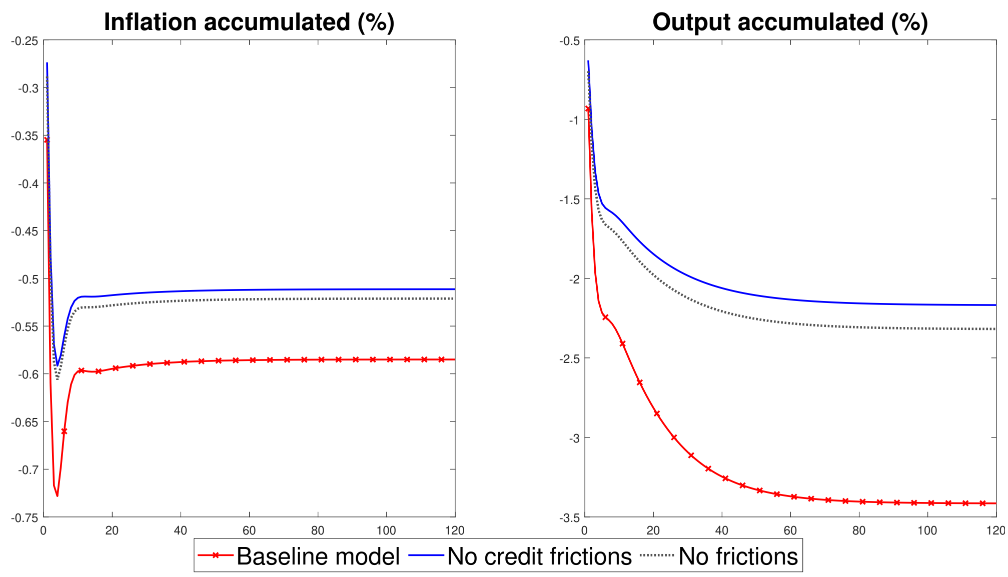

Figure 4.2: Cumulative IRFs for the three model cases

In figure 4.2, we plot the cumulative IRFs of inflation and output, both expressed in percentage deviations from their steady-state levels, to better investigate the effects of the shock for the three cases. Given the effects of the two channels, we expect that the cumulative fall in inflation and the cumulative loss of output are higher in the baseline model and lower in the model whose only credit friction is banking competition in Cournot, with the no frictions model showing intermediate responses. However, the cumulative output loss is in fact disproportionately higher in the baseline model, corroborating our observation in figure 4.1.

5
Welfare analysis

For better evaluation, from figure 4.2 we compute a "shock cost" measure for each case, defined as the ratio between the cumulative output loss and the cumulative fall in inflation.

Table 5.1: The cost of the shock in terms of output

	Inflation acc. (%)	**Output acc. (%)**	y/π
Baseline model	-0.58	-3.41	5.88
No credit frictions	-0.51	-2.17	4.25
No frictions	-0.52	-2.32	4.46

Although it weakens the effects of the monetary shock on accumulated inflation, the case with only banks competing in Cournot shows the lowest cost of the shock. In the baseline model, the effect of the shock is the greatest but the cost of the shock in terms of output is also the highest. We can conclude that, under high spreads, the central bank of Brazil's monetary policy, although powerful in reducing inflation, is quite inefficient in terms of economic activity

Would it be possible to reduce this inefficiency of monetary policy? In other words, would it be possible to keep monetary policy powerful in terms of inflation but less costly in terms of economic activity? To answer these questions, we conduct a welfare analysis to deal with the inefficiency faced by the central bank of Brazil when implementing its monetary policy. Specifically, we evaluate how a small change in the monetary policy rule could generate welfare gains.

Curdia and Woodford (2010) develop a New-Keynesian model with financial intermediation that displays spread, and conclude that a spread-adjusted Taylor rule results in a response to financial disturbances close to that generated by the Ramsey policy. A spread adjustment in a standard Taylor rule results in a closer approximation to the optimal policy. In this paper, we are not interested in the optimal policy, but in whether a spread-adjusted monetary rule comes close to optimality, generating welfare gains; and whether this rule is capable of mitigating the inefficiency faced by the central bank of Brazil.

To proceed with the welfare analysis, we solve the model using a second-order approximation of the equilibrium conditions for a given policy rule and then evaluate the welfare of this solution. The conditional welfare of the representative household and entrepreneur are given, respectively, by[1]:

[1]Since we want to know whether a change in the monetary rule is beneficial in welfare

$$W_t^H \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\log \left(C_{t+s} - h^H C_{t-1+s} \right) - \frac{(L_{t+s})^{1+\phi}}{1+\phi} \right]$$
$$W_t^E \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta^E \right)^s \log \left(C_{t+s}^E - h^E C_{t-1+s}^E \right)$$

Following Rubio and Carrasco-Gallego (2014), we define social welfare as a weighted sum of the welfare of the two representative agents. The weighting of the welfare of each agent depends on the respective discount factor, so that in the steady-state the same level of consumption for both agents would have the same impact on the level of utility[2]:

$$W_t^S = (1 - \beta)\, W_t^H + \left(1 - \beta^E\right) W_t^E$$

The alternative Taylor rule, inspired by Curdia and Woodford (2010), adjusts our standard Taylor rule by incorporating the change in the spread compared to the spread of the previous period:

$$R_t = (R_{t-1})^{\phi_r} \left[R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \left(\frac{S_t}{S_{t-1}} \right)^{-\phi_s} \right]^{1-\phi_r} e^{\sigma_r \, \varepsilon_t^R}, \qquad \varepsilon_t^R \sim \mathcal{N}(0,1)$$

where $\phi_s > 0$ is the weight assigned by the central bank to changes in the banking spread. Note that an increase in the spread determines a reduction in the policy rate.

From the second-order approximation of the non-linear model, we compute the values of the conditional social welfare W_t^S as a function of a grid for the ϕ_s parameter. This grid consists of values for ϕ_s from 0 to 1, with a variation of 0.01 between two consecutive values. The other parameters, including ϕ_r, ϕ_π and ϕ_y, follow the already calibrated values. The idea is that the adjustment between the two monetary rules is only due to the spread, which has zero weight in the standard Taylor rule.

Figure 5.1 shows the gain in conditional social welfare obtained by adopting the alternative Taylor rule compared to the standard Taylor rule. At $\phi_s = 0$, the alternative Taylor rule is identical to the standard one, so there is no gain in social welfare. For $\phi_s > 0$, note that the gain in social welfare is always positive and increasing in ϕ_s. Also note that small values of the ϕ_s grid generate much of the social welfare gain. Although higher values of ϕ_s

terms, we use conditional welfare. We choose the steady-state as the conditioning set. Thus, we start at the steady-state in both Taylor rules and compare the welfare losses resulting from the shocks of the model.

[2]In the model, we express the conditional welfare of each agent recursively. We use these expressions to calculate the respective steady-state values. See appendix H.

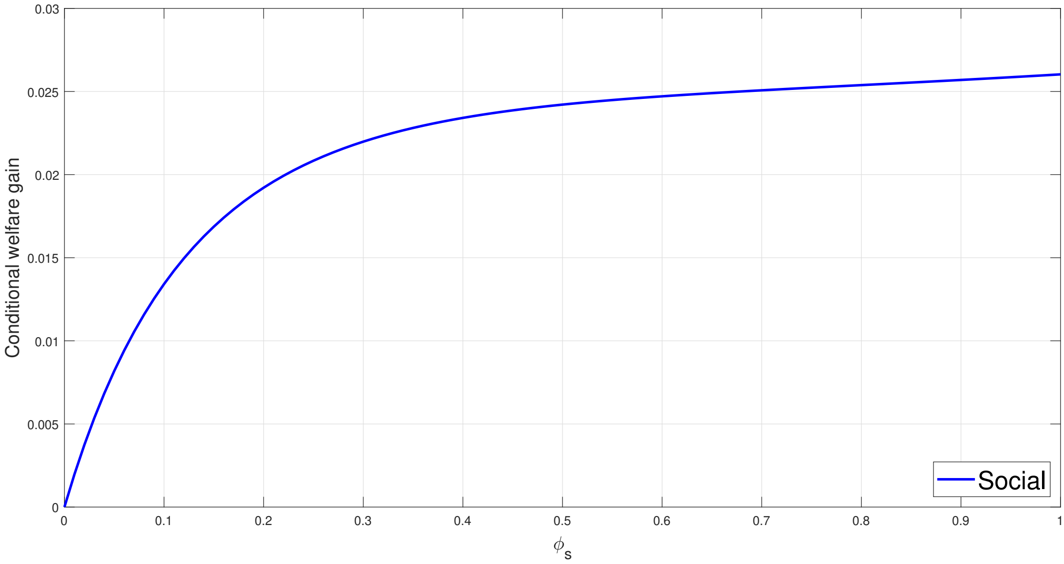

Figure 5.1: Conditional social welfare gain from adopting the alternative Taylor rule

determine a monetary rule that is closer to the optimal rule, we are interested in checking whether an adjustment to the standard monetary rule is able to reduce the inefficiency of monetary policy; we choose $\phi_s = 0.30$. At this value, we obtain most of the social welfare gain; moreover, this calibration gives a weight to the spread that is close to that given to output and much lower than that given to inflation.

For the two monetary rules, table 5.2 shows the loss of social welfare resulting from shocks shown in section 2. The spread-adjusted Taylor rule reduces the loss of social welfare for the monetary shock; for productivity and preference shocks, the reduction in welfare loss is marginal. Therefore, the improvement in welfare provided by the spread-adjusted Taylor rule occurs for shocks that affect credit restrictions or financial conditions more, a result in line with Curdia and Woodford (2010).

Table 5.2: Loss of social welfare resulting from each shock for different monetary rules

	Monetary shock	**Risk shock**	**Productivity shock**	**Preferences shock**
Standard Taylor rule	-2.748	-2.732	-2.737	-2.730
Alternative Taylor rule	-2.735	-2.729	-2.732	-2.729

Notes: Monetary shock has standard deviation of 25 basis points; the other shocks are 100 basis points. All the shocks have a downward effect on output. Thus, productivity and preference shocks are negative (the latter determines an increase in the household's stochastic discount factor); monetary and risk shocks are positive.

5.1 Standard Taylor rule vs alternative Taylor rule

Given that a small spread adjustment in the Taylor rule is capable of reducing the welfare loss caused by the same monetary shock, we compare the dynamics of the baseline model under the adjusted Taylor rule with the already discussed standard Taylor rule. Figure 5.2 shows the IRFs of some variables in the model for the two monetary rules.

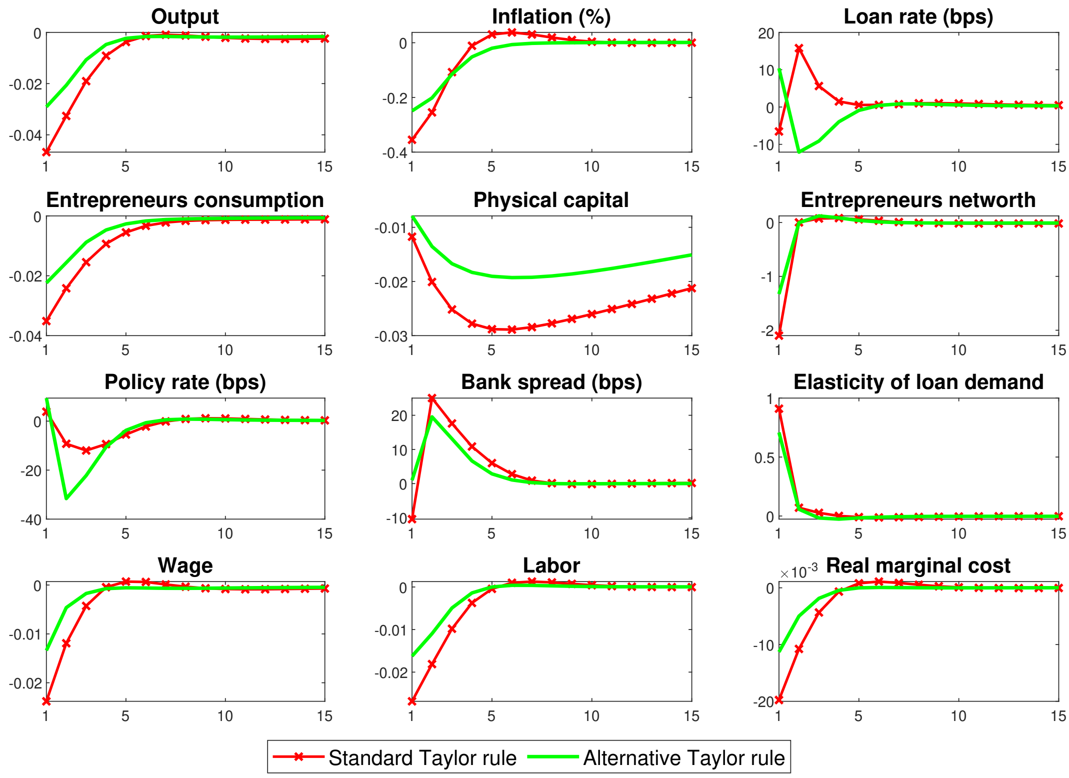

Figure 5.2: Response to monetary policy shock for both Taylor rules

First, we notice a marked difference in the dynamics of the loan rate: between the two Taylor rules, the loan rate follows practically opposite paths. Under our spread-adjusted Taylor rule, at the instant of the shock, the loan rate rises to above the steady-state value. This result stems from the higher policy rate at $t = 1$ compared to the case with the standard Taylor rule: since output falls less and the spread shows a very small increase, the policy rate is mainly driven by the shock itself, which increases the loan rate. We then see a less obstructed transmission between policy rate and loan rate, which suggests a reduction in the effects of credit frictions[3].

Under the alternative Taylor rule, a milder increase in the elasticity of credit demand suggests a greater influence of the banks' market power and therefore of the collateral channel. The representative entrepreneur is less financially restricted by the shock (the borrowing constraint becomes less binding). Even under a higher loan rate compared to the standard Taylor rule, the ability to acquire physical capital is less compromised, which mitigates the fall in the real price of capital and in the entrepreneur's net worth. The effects

[3]Both the elasticity of credit demand and the default rate become less relevant in determining the loan market rate.

of the net worth channel are lessened, meaning that the financial accelerator is less active in this case. This is evident in the smaller drop in physical capital and output at the time of the shock.

Therefore, at the time of the shock, the spread-adjusted Taylor rule mitigates the effects of the net worth channel and highlights those of the collateral channel. The smaller drop in the acquisition of physical capital determines, under the spread-adjusted Taylor rule, a smaller reduction in the demand for labor and, therefore, a smaller drop in wages. As a result, the reduction in the real marginal cost is smaller, which leads to a smaller fall in inflation at the time of the shock.

At $t = 2$, the period following the shock, note that the policy rate shows a significant drop compared to the standard Taylor rule case. This is because, due to the increase in the spread, the central bank reacts aggressively, lowering the policy rate. At this point, the elasticity of loan demand and the default rate return to their steady-state levels; however, the effect of the fall in the policy rate overlaps with that of the low level of bank capital, resulting in a reduction in the loan rate to below steady-state levels. As low bank capital acts to increase the loan rate, the reduction in the loan rate is smaller than that in the policy rate by the central bank, which sustains higher spreads from $t = 2$ onwards. As the spread decreases towards its steady-state level, the central bank reacts by increasing its policy rate. However, this movement is slow, so the policy rate remains below its steady-state level for some periods, defining the low levels of the loan rate. This result ensures that the entrepreneur's borrowing capacity is maintained.

In this way, the alternative Taylor rule weakens the effects of the net worth channel at the time of the shock, as well as sustains loan rate values below the steady-state level in subsequent periods. In terms of welfare, nominal frictions affect creditor agents (households) and credit frictions affect debtor agents (entrepreneurs). Note that, in the equilibrium condition B.7 in appendix B.2, a tighter borrowing constraint (higher $\lambda^E_{2,t}$ value) distorts the entrepreneur's intertemporal consumption decision more, impairing his ability to smooth consumption. This is because the entrepreneur can't smooth his consumption following an Euler Equation like the household, being dependent on the degree of tightness of his borrowing constraint.

Therefore, the monetary rule adjusted to changes in the spread allows the entrepreneur to better smooth its consumption. This is clear in Figure 5.3, in which the welfare of entrepreneurs increases with the adoption of a spread-adjusted Taylor rule. As households are not financially restricted from consuming, the inclusion of this factor in the monetary rule causes a reduction in their level of welfare. However, the gain in welfare for entrepreneurs offsets the fall for households, so that overall the alternative Taylor rule results in less loss of social welfare, as shown in figure 5.1.

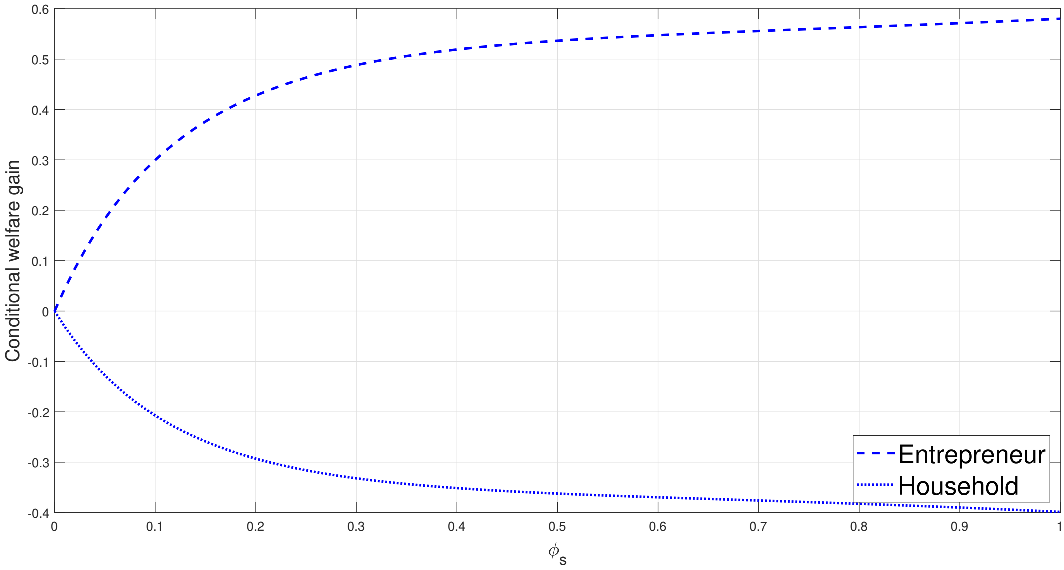

Figure 5.3: Conditional welfare gain for each agent from adopting the alternative Taylor rule

Finally, the entrepreneur's greater ability to smooth its consumption means that output returns more smoothly to its steady-state value. The same happens with labor, wages and, consequently, real marginal cost. This defines inflation returning more smoothly to its steady-state level, with no "rebound effect" as under the standard Taylor rule, prolonging the effect of the monetary shock on inflation.

5.2 Mitigating the inefficiency of monetary policy under high bank spreads

To evaluate the effects of the shock, in figure 5.4 we plot the accumulated IRFs of inflation and output under spread-adjusted Taylor rule together with the IRFs of these variables under the standard Taylor rule. The cumulative inflation is similar to the model under the standard Taylor rule, but the cumulative output loss is considerably lower. The alternative Taylor rule reduces the effects of credit frictions, improving the transmission of monetary policy and easing the financial constraints on entrepreneurs.

Table 5.3: The cost of the shock in terms of output for all cases

	Inflation acc. (%)	**Output acc. (%)**	y/π
Standard Taylor rule	-0.58	-3.41	5.88
Alternative Taylor rule	-0.62	-2.28	3.68

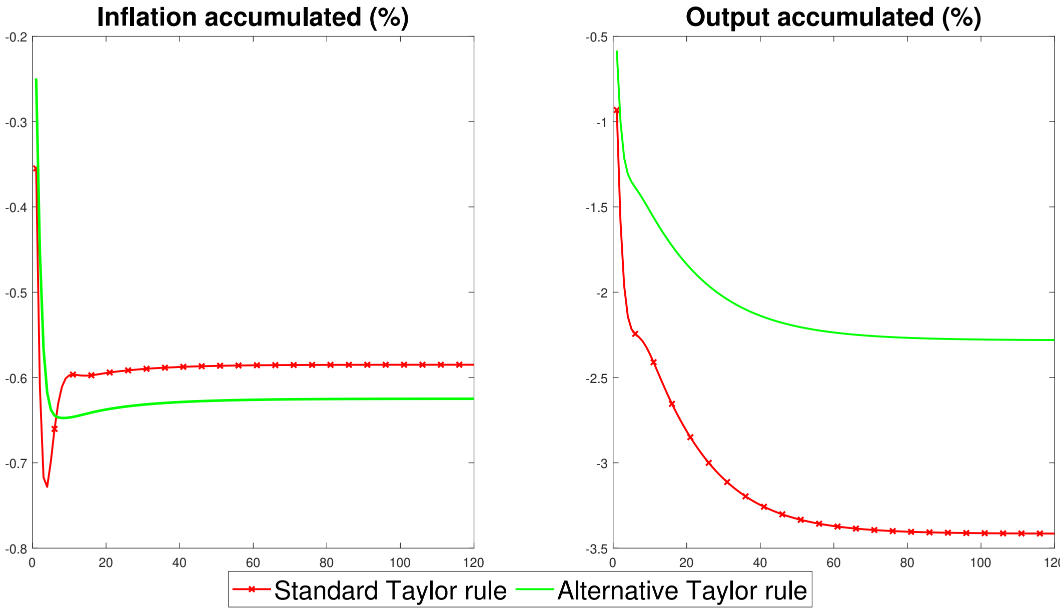

Figure 5.4: Accumulated IRFs of inflation and output for the two monetary policies

Table 5.3 shows the "cost shock" measure for the cases in figure 5.4. In fact, monetary policy is less costly in terms of lost output if changes in the banking spread are taken into account. Furthermore, the shock has greater effects on inflation, so that monetary policy is as powerful under a spread-adjusted rule as under a standard rule. Thus, we conclude that a small change to the monetary rule, consisting of a low-weighted adjustment to the banking spread, improves the efficiency of monetary policy under high bank spreads.

6
Conclusion

This paper proposes to evaluate the transmission of monetary policy under high bank spreads, a striking feature of the Brazilian financial sector. We seek to assess whether high spreads facilitate or hinder the work of the central bank of Brazil, understood here as exercising high control over inflation without incurring large GDP costs. To do this, we develop a New-Keynesian model with a banking sector and agents subject to borrowing constraints. The model generates an endogenous spread based on two ingredients: imperfect banking competition (Cournot) and a low recovery rate after credit default.

Our model allows us to identify two channels of monetary policy transmission, both with opposing effects. The first, due to banking market power, mitigates the effects of the monetary shock. The second, due to credit default, amplifies the effects of the shock. The calibration consistent with Brazil indicates that the second channel outweighs the first, so that the transmission of monetary policy under high bank spreads ends up being powerful. However, although it has a high effect on inflation, contractionary monetary policy is also inefficient, resulting in an excessive loss of output.

To mitigate this inefficiency, we evaluate welfare losses due to monetary shocks under an alternative monetary rule, which takes into account changes in the banking spread. We find that a small adjustment in the monetary rule based on changes in the spread is able to reduce the welfare loss and mitigate inefficiency, maintaining the power of monetary policy and greatly reducing the accumulated output loss.

We hope that this work will contribute to discussions involving the conduct of monetary policy in Brazil. Our model only considers credit for firms, so we leave the development of models that also present credit for households for future research. Earmarked credit, although it has reduced its share of total credit in recent years, is still relevant in Brazil. We believe that models with this type of credit in determining the banking spread and the transmission of monetary policy constitute an important research agenda for policymakers.

7
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A
Model appendix

A.1
Households

The first-order conditions (FOCs) for C_t, L_t and D_t are, respectively:

$$\lambda_t = \frac{1}{C_t \, - \, h^H \, C_{t-1}}$$

$$\lambda_t \, W_t = (L_t)^{\phi}$$

$$\lambda_t = \beta \, \mathbb{E}_t \left\{ \lambda_{t+1} \, \frac{\Gamma_{t+1}}{\Gamma_t} \, \frac{R_t}{\Pi_{t+1}} \right\}$$

where λ_t is the Lagrange multiplier.

Substituting FOC for C_t into FOC for L_t, we obtain the equilibrium condition for labor supply:

$$L_t = \left(\frac{W_t}{C_t \, - \, h^H \, C_{t-1}} \right)^{\frac{1}{\phi}} \tag{A.1.1}$$

Finally, substituting FOC for C_t into FOC for D_t, we obtain the Euler Equation of the representative household:

$$1 = \mathbb{E}_t \left\{ \beta \, \frac{(C_t \, - \, h^H \, C_{t-1})}{(C_{t+1} \, - \, h^H \, C_t)} \, \Psi_{t+1} \, \frac{R_t}{\Pi_{t+1}} \right\}$$

where $\Psi_t \; \equiv \; \dfrac{\Gamma_t}{\Gamma_{t-1}}$.

We denote $\Lambda_{t,t+1}$ as the Stochastic Discount Factor (SDF) of the household in period t for real payoffs in period $t+1$. From the Euler Equation above, the SDF is given by:

$$\Lambda_{t,t+1} \; \equiv \; \beta \, \Psi_{t+1} \, \frac{(C_t \, - \, h^H \, C_{t-1})}{(C_{t+1} \, - \, h^H \, C_t)} \tag{A.1.2}$$

Therefore, the Euler Equation of the representative household is given by:

$$1 = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \, \frac{R_t}{\Pi_{t+1}} \right\} \tag{A.1.3}$$

A.2 First-order condition of bank h

The first-order condition is:

$$\mathbb{E}_t\left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\left[\left(1-F(\bar{\omega}^E_{t+1})\right)\frac{\partial R^B_t}{\partial B_t(h)}B_t(h)+\left(1-F(\bar{\omega}^E_{t+1})\right)R^B_t-R_t-\frac{\partial\,\Omega^B_t(h)}{\partial\,B_t(h)}\right]\right\}=0$$

The optimal total quantity of loans is $B_t = B_t(h) + \sum_{m\neq h} B_t(m)$, and each bank produces a fraction of the total quantity. We assume that the N banks are identical, so that $B_t(h) = \frac{B_t}{N}$ in equilibrium. As $\frac{\partial B_t}{\partial B_t(h)} = 1$, then $\frac{\partial\, R^B_t}{\partial\, B_t(h)} = \frac{\partial\, R^B_t}{\partial\, B_t}\frac{\partial\, B_t}{\partial\, B_t(h)} = \frac{\partial\, R^B_t}{\partial\, B_t}$. We can rewrite the first-order condition as:

$$\left(\frac{\partial\, R^B_t}{\partial\, B_t}\frac{B_t}{N}+R^B_t\right)\mathbb{E}_t\left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\left(1-F(\bar{\omega}^E_{t+1})\right)\right\}=\left(R_t+\frac{\partial\,\Omega^B_t(h)}{\partial\,B_t(h)}\right)\mathbb{E}_t\left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\right\} \tag{A.2.1}$$

On the right-hand side of the above expression, we have:

$$\begin{aligned}\frac{\partial\,\Omega^B_t(h)}{\partial\,B_t(h)} &= -\,\kappa_{KB}\left(\frac{K^B_t(h)}{B_t(h)}-\upsilon^B\right)\left(\frac{K^B_t(h)}{B_t(h)}\right)^2\\ &= -\,\kappa_{KB}\left(\frac{\frac{K^B_t}{N}}{\frac{B_t}{N}}-\upsilon^B\right)\left(\frac{\frac{K^B_t}{N}}{\frac{B_t}{N}}\right)^2\\ &= -\,\kappa_{KB}\left(\frac{K^B_t}{B_t}-\upsilon^B\right)\left(\frac{K^B_t}{B_t}\right)^2\end{aligned} \tag{A.2.2}$$

Also, the elasticity of market demand for credit, in absolute terms, is defined by:

$$PED_t \equiv -\frac{\partial\,B_t}{\partial\,R^B_t}\frac{R^B_t}{B_t} \tag{A.2.3}$$

Substituting expressions (A.2.2) and (A.2.3) into (A.2.1), the first-order condition can be rewritten as:

$$R^B_t=\frac{\left[R_t-\kappa_{KB}\left(\frac{K^B_t}{B_t}-\upsilon^B\right)\left(\frac{K^B_t}{B_t}\right)^2\right]\mathbb{E}_t\left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\right\}}{\left[1-\left(\frac{1}{PED_t\,N}\right)\right]\mathbb{E}_t\left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\left(1-F(\bar{\omega}^E_{t+1})\right)\right\}}$$

A.3
Elasticity of the market demand for credit

The representative entrepreneur's demand for credit is given by:

$$B_t = M\,\mathbb{E}_t\left\{(1 - G(\bar{\omega}^E_{t+1}))\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)\,K^E_t}{R^B_t}\right\} \tag{A.3.1}$$

We see that R^B_t has a direct effect on B_t: an increase in loan rate reduces the entrepreneur's credit-taking capacity, reducing credit demand. In addition, there is also an indirect effect: R^B_t influences the entrepreneur's demand for physical capital (see the first-order condition for K^E_t, (A.4.2)); in turn, the physical capital K^E_t acquired by the entrepreneur, serving as collateral, determines the demand for B_t. Therefore, when bank h chooses $B_t(h)$ - which affects the loan rate R^B_t under Cournot competition - it needs to consider how entrepreneurs will respond by changing their demand for physical capital K^E_t. The real price Q_{t+1} of physical capital is determined by the equilibrium between entrepreneurs and capital producers; therefore, it does not depend on R^B_t.

Taking the derivative of B_t with respect to R^B_t, we obtain:

$$\frac{\partial B_t}{\partial R^B_t} = \underbrace{-\,M\,\mathbb{E}_t\left\{(1 - G(\bar{\omega}^E_{t+1}))\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)\,K^E_t}{(R^B_t)^2}\right\}}_{\text{direct effect}}$$
$$\underbrace{+\,M\,\mathbb{E}_t\left\{(1 - G(\bar{\omega}^E_{t+1}))\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)}{R^B_t}\,\frac{\partial K^E_t}{\partial R^B_t}\right\}}_{\text{indirect effect}}$$

Simplifying the term that represents the direct effect, we have:

$$\frac{\partial B_t}{\partial R^B_t} = -\,\frac{B_t}{R^B_t} + M\,\mathbb{E}_t\left\{\Big(1 - G(\bar{\omega}^E_{t+1})\Big)\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)}{R^B_t}\right\}\frac{\partial K^E_t}{\partial R^B_t} \tag{A.3.2}$$

Now, we need to calculate the partial derivative $\frac{\partial K^E_t}{\partial R^B_t}$, which is how entrepreneurs vary their demand for physical capital based on changes in the loan rate. Substituting (A.4.1) and (A.4.4) into (A.4.2), the first-order condition of the representative entrepreneur with respect to K^E_t is:

$$
\begin{aligned}
& Q_t \,-\, M\,\mathbb{E}_t \left\{ \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) \, \frac{Q_{t+1}\,\Pi_{t+1}\,(1 \,-\, \delta^E)}{R^B_t} \right\} = \\
& = \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \left[\alpha\, \frac{A_{t+1}\,(K^E_t)^{\alpha-1}\,(L_{t+1})^{1-\alpha}}{X_{t+1}} \right] \right\} \\
& + \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) Q_{t+1}\,(1 \,-\, \delta^E) \right\} \\
& - \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \, \frac{\left(1 \,-\, F(\bar{\omega}^E_{t+1})\right)}{\Pi_{t+1}} \right\} M\,\mathbb{E}_t \left\{ \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) Q_{t+1}\,\Pi_{t+1}\,(1 \,-\, \delta^E) \right\}
\end{aligned}
$$

We use the notations below to simplify the above expression:

$$
\begin{aligned}
A_{K,t} &\equiv Q_t \,-\, M\,\mathbb{E}_t \left\{ \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) \, \frac{Q_{t+1}\,\Pi_{t+1}\,(1 \,-\, \delta^E)}{R^B_t} \right\} \\
B_{K,t} &\equiv \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \left[\alpha\, \frac{A_{t+1}\,(L_{t+1})^{1-\alpha}}{X_{t+1}} \right] \right\} \\
C_{K,t} &\equiv \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) Q_{t+1}\,(1 \,-\, \delta^E) \right\} \\
& - \beta^E\,\mathbb{E}_t \left\{ \frac{C^E_t \,-\, h^E\,C^E_{t-1}}{C^E_{t+1} \,-\, h^E\,C^E_t} \, \frac{\left(1 \,-\, F(\bar{\omega}^E_{t+1})\right)}{\Pi_{t+1}} \right\} M\,\mathbb{E}_t \left\{ \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) Q_{t+1}\,\Pi_{t+1}\,(1 \,-\, \delta^E) \right\}
\end{aligned}
$$

The expression for the first-order condition for K^E_t then becomes:

$$
\begin{aligned}
A_{K,t} &= B_{K,t} \left(K^E_t\right)^{\alpha-1} \,+\, C_{K,t} \\
K^E_t &= \left[\frac{A_{K,t} \,-\, C_{K,t}}{B_{K,t}} \right]^{\frac{1}{\alpha-1}}
\end{aligned}
$$

We are interested in $\frac{\partial K^E_t}{\partial R^B_t}$, so let's derive the demand expression for K^E_t above with respect to R^B_t. Note that $A_{K,t}$ is the only term that presents R^B_t. We obtain:

$$
\frac{\partial K^E_t}{\partial R^B_t} = \frac{1}{\alpha \,-\, 1} \left[\frac{A_{K,t} \,-\, C_{K,t}}{B_{K,t}} \right]^{\frac{1}{\alpha-1}} \frac{1}{(A_{K,t} \,-\, C_{K,t})} \, \frac{\partial A_{K,t}}{\partial R^B_t}
$$

We know that:

$$
\frac{\partial A_{K,t}}{\partial R^B_t} = M\,\mathbb{E}_t \left\{ \left(1 \,-\, G(\bar{\omega}^E_{t+1})\right) \, \frac{Q_{t+1}\,\Pi_{t+1}\,(1 \,-\, \delta^E)}{R^B_t} \right\} \frac{1}{R^B_t}
$$

This brings us to:

$$\frac{\partial K_t^E}{\partial R_t^B} = \frac{1}{\alpha - 1} \frac{K_t^E}{(A_{K,t} - C_{K,t})} M \, \mathbb{E}_t \left\{ \left(1 - G(\bar{\omega}_{t+1}^E)\right) \frac{Q_{t+1} \, \Pi_{t+1} \, (1 - \delta^E)}{R_t^B} \right\} \frac{1}{R_t^B}$$

Note that if $A_{K,t} - C_{K,t} > 0$, then $\frac{\partial K_t^E}{\partial R_t^B} < 0$, i.e. the demand curve for physical capital is negatively sloped. This guarantees that $\frac{\partial B_t}{\partial R_t^B} < 0$, i.e. that the demand curve for loans is also negatively sloped. We see that this is always the case. As $K_t^E > 0$, $A_{K,t} - C_{K,t} > 0$ if and only if $B_{K,t} > 0$, meaning that the terms $C_t^E - h^E \, C_{t-1}^E$ and $C_{t+1}^E - h^E \, C_t^E$ must have the same sign. From the entrepreneur's utility function, these two terms must necessarily be positive. This guarantees that $A_{K,t} - C_{K,t} > 0$ for all t.

As $A_{K,t} - C_{K,t} = B_{K,t} \, (K_t^E)^{\alpha - 1}$, we have:

$$A_{K,t} - C_{K,t} = \mathbb{E}_t \left\{ \beta^E \frac{(C_t^E - h^E \, C_{t-1}^E)}{(C_{t+1}^E - h^E \, C_t^E)} \frac{\alpha \, A_{t+1} \, (K_t^E)^{\alpha-1} \, (L_{t+1})^{1-\alpha}}{X_{t+1}} \right\}$$

In the above expression, we can identify the stochastic discount factor of the representative entrepreneur and the marginal productivity of physical capital in real terms, given respectively by:

$$\Phi_{t,t+1} \equiv \beta^E \frac{C_t^E - h^E \, C_{t-1}^E}{C_{t+1}^E - h^E \, C_t^E}$$

$$MPK_t^E = \frac{\alpha \, A_t \, (K_{t-1}^E)^{\alpha-1} \, (L_t)^{1-\alpha}}{X_t}$$

Then, we can write $A_{K,t} - C_{K,t} = \mathbb{E}_t \left\{ \Phi_{t,t+1} \, MPK_{t+1}^E \right\}$, so that:

$$\frac{\partial K_t^E}{\partial R_t^B} = \frac{1}{\alpha - 1} \frac{K_t^E}{R_t^B} \left(\frac{M \, \mathbb{E}_t \left\{ \left(1 - G(\bar{\omega}_{t+1}^E)\right) \frac{Q_{t+1} \, \Pi_{t+1} \, (1 - \delta^E)}{R_t^B} \right\}}{\mathbb{E}_t \left\{ \Phi_{t,t+1} \, MPK_{t+1}^E \right\}} \right) < 0 \tag{A.3.3}$$

Now we can obtain the interest elasticity of the demand for credit. It is expressed in absolute terms by:

$$PED_t \equiv - \frac{\partial B_t}{\partial R_t^B} \frac{R_t^B}{B_t} \tag{A.3.4}$$

Substituting (A.3.2) and then (A.3.1) and (A.3.3) into (A.3.4), we get:

$$PED_t = 1 + \frac{1}{1-\alpha}\left(\frac{M\,\mathbb{E}_t\left\{\left(1 - G(\bar{\omega}^E_{t+1})\right)\frac{Q_{t+1}\,\Pi_{t+1}\,(1-\delta^E)}{R^B_t}\right\}}{\mathbb{E}_t\left\{\Phi_{t,t+1}\,MPK^E_{t+1}\right\}}\right)$$

The expression above consists of the interest elasticity of the market demand for credit, since we used the total credit B_t in its formulation. The demand for credit faced by bank h has a price elasticity given by:

$$PED_t(h) = -\frac{\partial B_t(h)}{\partial R^B_t}\frac{R^B_t}{B_t(h)}$$

Since $\frac{\partial R^B_t}{\partial B_t(h)} = \frac{\partial R^B_t}{\partial B_t}$, we have $\frac{\partial B_t(h)}{\partial R^B_t} = \left[\frac{\partial R^B_t}{\partial B_t(h)}\right]^{-1} = \left[\frac{\partial R^B_t}{\partial B_t}\right]^{-1} = \frac{\partial B_t}{\partial R^B_t}$. In addition, we have $B_t(h) = \frac{B_t}{N}$. Substituting, we get:

$$PED_t(h) = -\frac{\partial B_t}{\partial R^B_t}\frac{R^B_t}{B_t}N = PED_t\,N$$

In the situation of perfect competition in the banking sector, each bank h faces a perfectly elastic demand for credit, so that $PED_t\,N \to \infty$. Therefore, we can impose competitive banks (excluding imperfect banking competition) in the model by calibrating N to an arbitrarily high value. In the first-order condition, we can verify that $N \to \infty$ results in a spread that depends only on the probability of default and the recovery rate, no longer depending on imperfect banking competition.

A.4 Representative entrepreneur

Defining f as the probability density function of ω^E, the total debt payment of the representative entrepreneur is given by:

$$\int_0^{\bar{\omega}^E_t}\left[Q_t\,(1-\delta^E)\,K^E_{t-1}\right]\omega^E f(\omega^E)\,d\omega^E + \int_{\bar{\omega}^E_t}^{\infty}\frac{R^B_{t-1}\,B_{t-1}}{\Pi_t}f(\omega^E)\,d\omega^E =$$

$$= \int_0^{\bar{\omega}^E_t}\left[Q_t\,(1-\delta^E)\,K^E_{t-1}\right]\omega^E\,dF(\omega^E) + \int_{\bar{\omega}^E_t}^{\infty}\frac{R^B_{t-1}\,B_{t-1}}{\Pi_t}\,dF(\omega^E) =$$

$$= \left[Q_t\,(1-\delta^E)\,K^E_{t-1}\right]G(\bar{\omega}^E_t) + \left(1 - F(\bar{\omega}^E_t)\right)\frac{R^B_{t-1}\,B_{t-1}}{\Pi_t},$$

where $G(\bar{\omega}^E_t) = \int_0^{\bar{\omega}^E_t}\omega^E\,dF(\omega^E)$.

The first term consists of the expected value of the debt in the event of default, and the second term represents the expected value of the debt in the event of no default. An alternative way of interpreting this part of the budget constraint is that, under probability $(1 - F(\bar{\omega}_t^E))$, the entrepreneur pays off his credit debt from the previous period; in addition, the entrepreneur acquires fraction $G(\bar{\omega}_t^E)$ of physical capital from recycling firms at the full real price Q_t. As we are dealing with a representative entrepreneur, this last factor coincides with the cost incurred in the event of default.

The first-order conditions with respect to C_t^E, K_t^E, L_t and B_t are, respectively:

$$\lambda_{1,t}^E = \frac{1}{C_t^E - h^E C_{t-1}^E} \tag{A.4.1}$$

$$\begin{aligned} \lambda_{1,t}^E Q_t =& \beta^E \mathbb{E}_t \left\{ \lambda_{1,t+1}^E \left[\frac{A_{t+1} \alpha (K_t^E)^{\alpha-1} (L_{t+1})^{1-\alpha}}{X_{t+1}} + (1 - G(\bar{\omega}_{t+1}^E)) Q_{t+1} (1 - \delta^E) \right] \right\} \\ &+ \lambda_{2,t}^E M \mathbb{E}_t \left\{ (1 - G(\bar{\omega}_{t+1}^E)) \frac{Q_{t+1} \Pi_{t+1} (1 - \delta^E)}{R_t^B} \right\} \end{aligned} \tag{A.4.2}$$

$$W_t = (1 - \alpha) \frac{A_t (K_{t-1}^E)^\alpha (L_t)^{-\alpha}}{X_t} \tag{A.4.3}$$

$$\lambda_{2,t}^E = \lambda_{1,t}^E - \beta^E \mathbb{E}_t \left\{ \lambda_{1,t+1}^E (1 - F(\bar{\omega}_{t+1}^E)) \frac{R_t^B}{\Pi_{t+1}} \right\} \tag{A.4.4}$$

where $\lambda_{1,t}^E$ and $\lambda_{2,t}^E$ are the Lagrange multipliers for the budget constraint and the borrowing constraint, respectively.

Note that the first-order condition for physical capital (A.4.2) equates the marginal cost of a unit of capital, $\lambda_{1,t}^E Q_t$, to its discounted expected marginal benefit. This benefit has three components: (1) the return on the physical capital used in production; (2) the expected future price of the capital available at the end of period $t + 1$, since the physical capital is resold to the capital producers; and (3) the shadow-value of borrowing, since the capital acquired at the beginning of period t can be used as collateral in bank borrowing.

A.5
Borrowing constraint always binds

In particular, the Euler Equations of the household and the entrepreneur evaluated in the steady-state are, respectively:

$$1 = \beta \frac{R}{\Pi} \tag{A.5.1}$$

$$\frac{\lambda_2^E}{\lambda_1^E} = 1 - \beta^E (1 - F(\bar{\omega}^E)) \frac{R^B}{\Pi} \tag{A.5.2}$$

Note that if $\dfrac{\beta}{\beta^E} > (1 - F(\bar{\omega}^E))\,\dfrac{R^B}{R}$, then $\dfrac{\lambda_2^E}{\lambda_1^E} > 0$. Therefore, in the neighborhood of the steady-state, we guarantee that the borrowing constraint is always binding when $\dfrac{\beta}{\beta^E} > (1 - F(\bar{\omega}^E))\,\dfrac{R^B}{R}$.

Given that entrepreneurs are always financially constrained for sufficiently small shocks, we then define that the effective demand for credit is given by the borrowing constraint being binding:

$$B_t = M\,\mathbb{E}_t \left\{ (1 - G(\bar{\omega}_{t+1}^E))\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 - \delta^E)\,K_t^E}{R_t^B} \right\} \tag{A.5.3}$$

In the above expression, an increase in the gross loan rate R_t^B leads to a reduction in the amount of credit demanded by the representative entrepreneur. The demand for credit is also affected by the demand for physical capital K_t^E. Banks competing in Cournot take into account entrepreneurs' demand for physical capital in their choice of credit supply, which in turn determines the gross loan rate.

A.6
Entrepreneur's net worth

We define the entrepreneur's net worth as the return on physical capital (the productivity and revenue obtained from the sale of the physical capital available at the end of period t) net of the loan repayment:

$$N_t^E = \alpha\,\frac{Y_{w,t}}{K_{t-1}^E\,X_t}\,K_{t-1}^E + (1 - G(\bar{\omega}_t^E))\,Q_t\,(1 - \delta^E)\,K_{t-1}^E - (1 - F(\bar{\omega}_t^E))\,\frac{R_{t-1}^B\,B_{t-1}}{\Pi_t}$$

By substituting the first-order condition for L_t, rewritten as $\alpha\,\dfrac{Y_{w,t}}{X_t} = \dfrac{Y_{w,t}}{X_t} - W_t\,L_t$, into the net worth expression, it is possible to rewrite the entrepreneur's budget constraint from the net worth:

$$C_t^E + Q_t\,K_t^E = B_t + N_t^E$$

A.7
Capital Producers

Perfectly competitive capital producers buy non-depreciated capital $(1 - \delta^E)\,K_{t-1}^E$ at real price Q_t from entrepreneurs and I_t units of final consumer good from retailers to produce new physical capital K_t^E at the end of period t:

$$K_t^E = I_t + (1 - \delta^E)\,K_{t-1}^E\,, \tag{A.7.1}$$

where I_t is the gross investment. The new physical capital produced, K_t^E, will be sold back to the entrepreneur at the real price Q_t, being used in the production of the wholesale good in period $t+1$.

As is common in the literature, we assume that capital producers face investment adjustment costs that depend on the gross growth rate of investment $\frac{I_t}{I_{t-1}}$. Assume that old capital can be converted into a one-to-one proportion of new capital, and a quadratic adjustment cost given by

$$g\left(\frac{I_t}{I_{t-1}}\right) = \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

is incurred in the production of new physical capital, where $g(1) = g'(1) = 0$, $g''(1) > 0$ and $\chi > 0$. This specification of the adjustment cost implies that fewer units of new physical capital are produced from one unit of investment whenever the ratio $\frac{I_t}{I_{t-1}}$ deviates from its steady-state value of 1, with the parameter χ reflecting the magnitude of the cost.

The representative capital producer's problem is to choose the level I_t of gross investment to maximize the sum of discounted expected future profits obtained from revenues from physical capital sales, $Q_t\,K_t^E$, net of the input cost $Q_t\,(1 - \delta^E)\,K_{t-1}^E$ and the investment adjustment cost $g\left(\frac{I_t}{I_{t-1}}\right) I_t$:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[Q_t\,K_t^E - Q_t\,(1 - \delta^E)\,K_{t-1}^E - I_t - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t \right] \tag{A.7.2}$$

subject to the production function of the new physical capital (A.7.1).

In the above expression, $\Lambda_{t,t+s} \equiv \beta^s\,\frac{\Gamma_{t+s}}{\Gamma_t}\,\frac{u'(C_{t+s})}{u'(C_t)}$ is the stochastic discount factor (SDF), since households are the owners of the capital producers. The first-order condition for I_t is given by:

$$Q_t = 1 + \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \chi\,\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right) - \chi\,\mathbb{E}_t\left\{\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right\}$$

In the steady-state, the above equilibrium condition results in $Q = 1$, since $I_t = I_{t-1}$. For any period t, the profit of the capital producer is given by

$$T_t^{CP} = (Q_t - 1)\,I_t - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t\,,$$

so that, in the steady-state, the profit of the representative capital producer is zero.

Therefore, the only way for a positive profit to exist is outside the steady-state, with the profit being rebated to households.

A.8 Retailers

Retailers act in monopolistic competition. A continuum of retailers of unit mass indexed by j, $j \in [0,1]$, buy the homogeneous wholesale good from the entrepreneurs at the price $P_{w,t}$ and use this good as an input for the production of differentiated retail goods.

We assume that one unit of wholesale good can produce one unit of differentiated good, so that the marginal cost of production is the real price of the wholesale good, $\frac{P_{w,t}}{P_t}$. Each retailer j produces a differentiated variety $Y_t(j)$ and charges a nominal price $P_t(j)$ for its differentiated good.

The output of the final consumption good Y_t is a CES composition (constant elasticity of substitution) of all the different varieties produced by the retailers, using a framework of Dixit and Stiglitz (1977):

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}} ,$$

where $\epsilon > 1$ is the intratemporal elasticity of substitution between different varieties.

As is standard in the literature, agents maximize the CES composition of the varieties of differentiated goods subject to a given level of expenditure. Assuming that the retailers are identical, and defining P_t as the expenditure required to acquire one unit of Y_t, we obtain:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} \, dj \right]^{\frac{1}{1-\epsilon}}$$

The above expression consists of the aggregate price index. From it, it is possible to obtain the demand curve faced by each retailer j in period t:

$$Y_t(j) = Y_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon}$$

Each retailer j sets its price $P_t(j)$ taking the aggregate price P_t and the demand curve $Y_t(j)$ as given. Under pricing a la Calvo (1983), each retailer j is able to change its price $P_t(j)$ in period t with probability $1-\theta$. The probability of price adjustment is independent of the time since the last adjustment, so that a $1-\theta$ fraction of retailers reset their prices in each period, while a θ fraction of retailers do not optimize and have their prices indexed, being corrected by a fraction of past gross inflation:

$$P_t(j) \;=\; P_{t-1}(j)\,\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}$$

Therefore, $\theta \in (0,1)$ reflects the degree of price rigidity and $\gamma \in (0,1)$ is a price indexation parameter.

A.9
Recursive Phillips curve

Let $P_t^*(j)$ be the optimal price reset by retailer j in period t; then the demand faced by this retailer j that optimizes its price in period t but cannot optimize its prices until period $t+s$ is given, in period $t+s$, by:

$$Y_{t+s}(j) \;=\; \left(\frac{P_t^*(j)}{P_{t+s}}\right)^{-\epsilon}\,\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\gamma\epsilon}\,Y_{t+s} \tag{A.9.1}$$

Retailer j that optimizes its price for the last time in t has its price in $t+s$ given by $P_{t+s}(j) \;=\; P_t^*(j)\,\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma}$. Thus, the expected discount value of future real profits of this retailer (when the last optimization takes place in period t) is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \theta^s\,\Lambda_{t,t+s}\left[\frac{P_t^*(j)}{P_{t+s}}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma} Y_{t+s}(j) \;-\; \frac{1}{X_{t+s}}\,Y_{t+s}(j)\right] \tag{A.9.2}$$

where $\Lambda_{t,t+s}$ is the stochastic discount factor, since households are the owners of retailers; θ^s is the probability of no further price optimization for s periods; and $\frac{1}{X_{t+s}} \;\equiv\; \frac{P_{w,t+s}}{P_{t+s}}$ is the price of the wholesale good in terms of consumption units in $t+s$, or the real marginal cost of production in period $t+s$.

The problem of retailer j is to choose its price $P_t^*(j)$ in period t to maximize (A.9.2) subject to (A.9.1). The first-order condition for retailer j can be written as:

$$P_t^*(j) \;=\; \frac{\epsilon}{\epsilon\,-\,1}\;\frac{\displaystyle\sum_{s=0}^{\infty}\theta^s\,\mathbb{E}_t\left\{\Lambda_{t,t+s}\,Y_{t+s}\,X_{t+s}^{-1}\,P_{t+s}^{\epsilon}\,\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\gamma\epsilon}\right\}}{\displaystyle\sum_{s=0}^{\infty}\theta^s\,\mathbb{E}_t\left\{\Lambda_{t,t+s}\,Y_{t+s}\,P_{t+s}^{\epsilon-1}\,\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma-\gamma\epsilon}\right\}}$$

Our model is non-linear, so the previous first-order condition is written recursively. Assuming that the equilibrium is symmetrical, $P_t^*(j) \;=\; P_t^*$, the first-order condition can be written as:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} \left[\epsilon \frac{1}{X_{t+s}} \left(\frac{P_t^*}{P_{t+s}}\right)^{-\epsilon} \left(\frac{P_{t+s-1}}{P_{t+1}}\right)^{-\gamma\epsilon} + (1-\epsilon) \left(\frac{P_t^*}{P_{t+s}}\right)^{-\epsilon+1} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma-\gamma\epsilon} \right] = 0$$

We define each portion of the infinite sum as:

$$F_{1,t} \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (X_{t+s})^{-1} \left(\frac{P_t^*}{P_{t+s}}\right)^{-\epsilon} \left(\frac{P_{t+s-1}}{P_{t+1}}\right)^{-\gamma\epsilon}$$

$$F_{2,t} \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} \left(\frac{P_t^*}{P_{t+s}}\right)^{-\epsilon+1} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma-\gamma\epsilon}$$

Thus, the first-order condition is:

$$\epsilon F_{1,t} + (1-\epsilon) F_{2,t} = 0$$

We need $\theta^s \Lambda_{t,t+s}$ to go to zero fast enough relative to inflation rates so that $F_{1,t}$ and $F_{2,t}$ are well-defined and stationary. We can write $F_{1,t}$ and $F_{2,t}$ recursively:

$$F_{1,t} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} (X_t)^{-1} Y_t + \mathbb{E}_t \left\{ \theta \Lambda_{t,t+1} \left(\frac{P_t^*}{P_{t+1}^*}\right)^{-\epsilon} \left(\frac{P_t}{P_{t-1}}\right)^{-\gamma\epsilon} F_{1,t+1} \right\}$$

$$F_{2,t} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon+1} Y_t + \mathbb{E}_t \left\{ \theta \Lambda_{t,t+1} \left(\frac{P_t^*}{P_{t+1}^*}\right)^{-\epsilon+1} \left(\frac{P_t}{P_{t-1}}\right)^{\gamma-\gamma\epsilon} F_{2,t+1} \right\}$$

Defining $\Pi_t^* \equiv \dfrac{P_t^*}{P_{t-1}}$ and rearranging, we get the two expressions that correspond to the Phillips Curve written recursively:

$$F_{1,t} = \left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon} (X_t)^{-1} Y_t + \mathbb{E}_t \left\{ \theta \Lambda_{t,t+1} (\Pi_t^*)^{-\epsilon} (\Pi_t)^{\epsilon-\gamma\epsilon} (\Pi_{t+1}^*)^{\epsilon} F_{1,t+1} \right\}$$

$$F_{2,t} = \left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon-1} Y_t + \mathbb{E}_t \left\{ \theta \Lambda_{t,t+1} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right)^{-\epsilon+1} (\Pi_t)^{\epsilon-1} (\Pi_t)^{\gamma-\gamma\epsilon} F_{2,t+1} \right\}$$

A.10
Evolution of the aggregate price index

Rearranging the expression of the aggregate price index, we have:

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} \, dj$$

We follow Sims (2017)[1] to calculate the evolution of the aggregate price. We can break the above integral into two parts by ordering the retailers along the unit interval:

$$P_t^{1-\epsilon} = \int_0^{1-\theta} (P_t^*(j))^{1-\epsilon}\, dj + \int_{1-\theta}^{1} \left[P_{t-1}(j) \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{1-\epsilon} dj$$

As $P_t^*(j) = P_t^*$, we have:

$$P_t^{1-\epsilon} = (1 - \theta)\,(P_t^*)^{1-\epsilon} + \int_{1-\theta}^{1} \left[P_{t-1}(j) \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{1-\epsilon} dj$$

Given the assumptions that the retailers who optimize their prices in each period are chosen at random and that the number of retailers is very large, the integral of individual prices over $[1-\theta,\, 1]$ of the unit interval is equal to the θ proportion of the integral over the entire unit interval, where θ is the length of the interval $[1-\theta,\, 1]$. That is:

$$\begin{aligned}
\int_{1-\theta}^{1} \left[P_{t-1}(j) \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{1-\epsilon} dj &= \theta \int_0^1 \left[P_{t-1}(j) \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{1-\epsilon} dj \\
&= \theta \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma(1-\epsilon)} \int_0^1 P_{t-1}(j)^{1-\epsilon}\, dj \\
&= \theta \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma(1-\epsilon)} P_{t-1}^{1-\epsilon}
\end{aligned}$$

Therefore, the aggregate price index evolves according to the expression below, so there is no need to analyze the price evolution of each retailer:

$$P_t^{1-\epsilon} = (1 - \theta)\,(P_t^*)^{1-\epsilon} + \theta \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma(1-\epsilon)} P_{t-1}^{1-\epsilon}$$

$$P_t = \left[(1 - \theta)\,(P_t^*)^{1-\epsilon} + \theta\, P_{t-1}^{1-\epsilon} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma(1-\epsilon)}\right]^{\frac{1}{1-\epsilon}}$$

A.11
Recursive price dispersion

As the conversion rate from the wholesale good to the differentiated retail goods is one-to-one, in equilibrium the supply of the wholesale good $Y_{w,t}$ is equal to the supply of differentiated final goods over the unit interval of retailers j. Using

[1] <https://sites.nd.edu/esims/files/2023/05/new_keynesian_2017.pdf>

the expression of the demand faced by an individual retailer j, the wholesale good output can be expressed as:

$$
\begin{aligned}
Y_{w,t} &= \int_0^1 Y_t(j)\, dj = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t\, dj \\
Y_{w,t} &= Y_t \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj
\end{aligned}
$$

The previous equation shows that the output of the final consumption good Y_t differs from the output of the wholesale good $Y_{w,t}$ by a price dispersion factor given by:

$$
\Delta_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj
$$

In a zero-inflation steady-state, the price dispersion equals 1 and the final output Y_t equals the wholesale good output $Y_{w,t}$. In our non-linear model, the price dispersion is rewritten recursively. Using Calvo's hypothesis to break the price dispersion expression into two parts, ordering the retailers along the unit interval, we get:

$$
\begin{aligned}
\Delta_t &= \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj \\
\Delta_t &= \int_0^{1-\theta} \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_t}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{-\epsilon} dj \\
\Delta_t &= \int_0^{1-\theta} \left(\frac{P_t^*}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}}{P_t}\frac{P_{t-1}(j)}{P_{t-1}}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right]^{-\epsilon} dj
\end{aligned}
$$

Using the $\Pi_t \equiv \dfrac{P_t}{P_{t-1}}$ and $\Pi_t^* \equiv \dfrac{P_t^*}{P_{t-1}}$ definition, we get:

$$
\Delta_t = \int_0^{1-\theta} \left(\Pi_t^*\,\Pi_t^{-1}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\Pi_t^{-1}\,(\Pi_{t-1})^{\gamma}\,\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\epsilon} dj
$$

Following the method of Sims (2017), we obtain:

$$
\Delta_t = (1-\theta)\,(\Pi_t^*)^{-\epsilon}\,(\Pi_t)^{\epsilon} + \Pi_t^{\epsilon}\,\Pi_{t-1}^{-\gamma\epsilon}\int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\epsilon} dj
$$

The last term of the equation becomes:

$$\int_{1-\theta}^{1} \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\epsilon} dj = \theta \int_{0}^{1} \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\epsilon} dj = \theta\, \Delta_{t-1}$$

Therefore, the price dispersion can be written recursively:

$$\Delta_t = (1 - \theta)\,(\Pi_t^*)^{-\epsilon}\,(\Pi_t)^{\epsilon} + (\Pi_t)^{\epsilon}\,(\Pi_{t-1})^{-\gamma\epsilon}\,\theta\,\Delta_{t-1}$$

The index j is eliminated from the above expression, so there is no need to track individual prices.

A.12 Retailers' real profit

The real profit T_t^R made by the unit mass continuum of retailers is:

$$T_t^R = \int_0^1 \frac{P_t(j)}{P_t}\, Y_t(j)\, dj - \frac{1}{X_t} \int_0^1 Y_t(j)\, dj$$

We know that the demand faced by retailer j is $Y_t(j) = Y_t \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$. We also know that $Y_{w,t} = \int_0^1 Y_t(j)\, dj$. Substituting into the retailer's real profit expression, we get:

$$\begin{aligned}
T_t^R &= \int_0^1 \frac{P_t(j)}{P_t} \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t\, dj - \frac{1}{X_t} Y_{w,t} \\
T_t^R &= \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon} Y_t\, dj - \frac{1}{X_t} Y_{w,t} \\
T_t^R &= P_t^{\epsilon-1}\, Y_t \int_0^1 P_t(j)^{1-\epsilon}\, dj - \frac{1}{X_t} Y_{w,t}
\end{aligned}$$

Substituting the aggregate price index into the above expression, we arrive at:

$$\begin{aligned}
T_t^R &= Y_t - \frac{Y_{w,t}}{X_t} \\
T_t^R &= \left(\frac{1}{\Delta_t} - \frac{1}{X_t}\right) Y_{w,t}
\end{aligned}$$

The retailers' real profit is rebated to households. We see that T_t^R takes on positive values in the steady-state, given that $\Delta = 1$.

A.13
Market clearing

The definition of equilibrium is standard. The aggregate resource constraint is given by:

$$C_t + C_t^E + I_t + \frac{\chi}{2}\left[\left(\frac{I_t}{I_{t-1}}\right) - 1\right]^2 I_t + \frac{\kappa_{KB}}{2}\left(\frac{K_{t-1}^B}{B_{t-1}} - v^B\right)^2 \frac{K_{t-1}^B}{\Pi_t} = Y_t$$

In equilibrium, the supply of labor from households equals the demand for labor from entrepreneurs, and the supply of new physical capital from capital producers equals the demand for physical capital from entrepreneurs. With $B_t(h)$, $D_t(h)$ and $K_t^B(h)$ being the supply of credit, the amount of deposits received and the bank capital of bank h, respectively, under Cournot competition in the banking sector we have $B_t = \sum_{h=1}^N B_t(h)$, $D_t = \sum_{h=1}^N D_t(h)$ and $K_t^B = \sum_{h=1}^N K_t^B(h)$. In equilibrium, the banking sector's supply of credit B_t equals the entrepreneurs' demand for credit, and the banking sector's demand for deposits equals the households' supply of deposits D_t. From the accounting identity, the total credit supply is equal to the total deposit of the banking sector plus the total bank capital, $B_t = D_t + K_t^B$.

B
Equilibrium equations

B.1
Representative household

$$L_t \;=\; \left(\frac{W_t}{C_t \;-\; h^H\, C_{t-1}}\right)^{\frac{1}{\phi}} \tag{B.1}$$

$$\frac{1}{(C_t \;-\; h^H\, C_{t-1})} \;=\; \mathbb{E}_t\left\{\beta\,\frac{1}{(C_{t+1} \;-\; h^H\, C_t)}\;\Psi_{t+1}\,\frac{R_t}{\Pi_{t+1}}\right\} \tag{B.2}$$

$$\begin{aligned} C_t \;+\; D_t \;=\; & \frac{R_{t-1}\,D_{t-1}}{\Pi_t} \;+\; W_t\,L_t \;+\; T_t^{CP} \;+\; T_t^R \;+\; (1 \;-\; \omega^B)\,\left[T_t^B \;+\; \mu\,G(\bar{\omega}_t^E)\,Q_t\,(1 \;-\; \delta^E)\,K_{t-1}^E\right] \\ & +\; (1 \;-\; \mu)\,G(\bar{\omega}_t^E)\,Q_t\,(1 \;-\; \delta^E)\,K_{t-1}^E \end{aligned} \tag{B.3}$$

$$\Psi_t \;=\; \frac{\Gamma_t}{\Gamma_{t-1}} \tag{B.4}$$

$$\log(\Gamma_t) = (1-\rho_\Gamma)\,\log(\Gamma) \;+\; \rho_\Gamma\,\log(\Gamma_{t-1}) \;+\; \sigma_\Gamma\,\varepsilon_t^\Gamma \tag{B.5}$$

$$\Lambda_{t,t+1} \;\equiv\; \beta\,\Psi_{t+1}\,\frac{(C_t \;-\; h^H\, C_{t-1})}{(C_{t+1} \;-\; h^H\, C_t)} \tag{B.6}$$

B.2
Representative entrepreneur

In the following equilibrium conditions, $\lambda_{2,t}^E$ is the Lagrange multiplier associated with the borrowing constraint of the representative entrepreneur and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Lev_t consists of the entrepreneur's leverage.

$$\lambda_{2,t}^E\,\left(C_t^E \;-\; h^E\,C_{t-1}^E\right) \;=\; 1 \;-\; \beta^E\,\mathbb{E}_t\left\{\frac{C_t^E \;-\; h^E\,C_{t-1}^E}{C_{t+1}^E \;-\; h^E\,C_t^E}\,\left(1 \;-\; F(\bar{\omega}_{t+1}^E)\right)\,\frac{R_t^B}{\Pi_{t+1}}\right\} \tag{B.7}$$

$$\begin{aligned} Q_t \;=\; & \beta^E\,\mathbb{E}_t\left\{\frac{C_t^E \;-\; h^E\,C_{t-1}^E}{C_{t+1}^E \;-\; h^E\,C_t^E}\,\left[\alpha\,\frac{Y_{w,t+1}}{X_{t+1}\,K_t^E} \;+\; \left(1 \;-\; G(\bar{\omega}_{t+1}^E)\right)\,Q_{t+1}\,(1 \;-\; \delta^E)\right]\right\} \\ & +\; \lambda_{2,t}^E\,\left(C_t^E \;-\; h^E\,C_{t-1}^E\right)\,M\,\mathbb{E}_t\left\{\left(1 \;-\; G(\bar{\omega}_{t+1}^E)\right)\,\frac{Q_{t+1}\,\Pi_{t+1}\,(1 \;-\; \delta^E)}{R_t^B}\right\} \end{aligned} \tag{B.8}$$

$$W_t \;=\; (1 \;-\; \alpha)\,\frac{Y_{w,t}}{X_t\,L_t} \tag{B.9}$$

$$C_t^E + Q_t K_t^E = B_t + N_t^E \tag{B.10}$$

$$Y_{w,t} = A_t (K_{t-1}^E)^\alpha (L_t)^{1-\alpha} \tag{B.11}$$

$$MPK_t^E = \frac{\alpha A_t (K_{t-1}^E)^{\alpha-1} (L_t)^{1-\alpha}}{X_t} \tag{B.12}$$

$$B_t = M \mathbb{E}_t \left\{ (1 - G(\bar{\omega}_{t+1}^E)) \frac{Q_{t+1} \Pi_{t+1} (1 - \delta^E) K_t^E}{R_t^B} \right\} \tag{B.13}$$

$$\bar{\omega}_t^E = \frac{R_{t-1}^B B_{t-1}}{Q_t \Pi_t (1 - \delta^E) K_{t-1}^E} \tag{B.14}$$

$$G(\bar{\omega}_t^E) = \Phi \left(\frac{\log(\bar{\omega}_t^E) + \frac{\sigma^2}{2}}{\sigma} - \sigma \right) \tag{B.15}$$

$$F(\bar{\omega}_t^E) = \Phi \left(\frac{\log(\bar{\omega}_t^E) + \frac{\sigma^2}{2}}{\sigma} \right) \tag{B.16}$$

$$Lev_t = \frac{B_t}{N_t^E} \tag{B.17}$$

$$\log(A_t) = (1 - \rho_a) \log(A) + \rho_a \log(A_{t-1}) + \sigma_a \varepsilon_t^a \tag{B.18}$$

$$\log(\sigma_t) = (1 - \rho_\sigma) \log(\sigma) + \rho_\sigma \log(\sigma_{t-1}) + \sigma_\sigma \varepsilon_t^\sigma \tag{B.19}$$

B.3 Capital producers

$$K_t^E = I_t + (1 - \delta^E) K_{t-1}^E \tag{B.20}$$

$$Q_t = 1 + \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \chi \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) - \chi \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.21}$$

$$T_t^{CP} = (Q_t - 1) I_t - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \tag{B.22}$$

B.4 Retailers

$$F_{1,t} = \left(\frac{\Pi_t}{\Pi_t^*} \right)^\epsilon \frac{Y_t}{X_t} + \mathbb{E}_t \left\{ \theta \Lambda_{t,t+1} (\Pi_t^*)^{-\epsilon} (\Pi_t)^{\epsilon - \gamma\epsilon} \left(\Pi_{t+1}^* \right)^\epsilon F_{1,t+1} \right\} \tag{B.23}$$

$$F_{2,t} = \left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon-1} Y_t + \mathbb{E}_t \left\{\theta \, \Lambda_{t,t+1} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right)^{-\epsilon+1} (\Pi_t)^{\epsilon-1} (\Pi_t)^{\gamma-\gamma\epsilon} F_{2,t+1}\right\} \tag{B.24}$$

$$F_{1,t} = \frac{\epsilon - 1}{\epsilon} F_{2,t} \tag{B.25}$$

$$1 = (1 - \theta) \left(\frac{\Pi_t^*}{\Pi_t}\right)^{1-\epsilon} + \theta \, (\Pi_t)^{\epsilon-1} (\Pi_{t-1})^{\gamma(1-\epsilon)} \tag{B.26}$$

$$\Delta_t = (1 - \theta) (\Pi_t^*)^{-\epsilon} (\Pi_t)^{\epsilon} + (\Pi_t)^{\epsilon} (\Pi_{t-1})^{-\gamma\epsilon} \theta \, \Delta_{t-1} \tag{B.27}$$

$$Y_t = \frac{Y_{w,t}}{\Delta_t} \tag{B.28}$$

$$T_t^R = \frac{Y_{w,t}}{\Delta_t} - \frac{Y_{w,t}}{X_t} \tag{B.29}$$

B.5
Banks in Cournot competition

$$R_t^B = \frac{\left[R_t - \kappa_{KB} \left(\frac{K_t^B}{B_t} - v^B\right) \left(\frac{K_t^B}{B_t}\right)^2\right] \mathbb{E}_t \left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}\right\}}{\left[1 - \left(\frac{1}{PED_t \, N}\right)\right] \mathbb{E}_t \left\{\frac{\Lambda_{t,t+1}}{\Pi_{t+1}} (1 - F(\bar{\omega}_{t+1}^E))\right\}} \tag{B.30}$$

$$T_t^B = \left(1 - F(\bar{\omega}_t^E)\right) \frac{R_{t-1}^B \, B_{t-1}}{\Pi_t} - \frac{R_{t-1} \, B_{t-1}}{\Pi_t} + \frac{R_{t-1} \, K_{t-1}^B}{\Pi_t} - K_t^B - \frac{\kappa_{KB}}{2 \, \Pi_t} \left[\frac{K_{t-1}^B}{B_{t-1}} - v^B\right]^2 K_{t-1}^B \tag{B.31}$$

$$B_t = D_t + K_t^B \tag{B.32}$$

$$PED_t = 1 + \frac{1}{1 - \alpha} \left(\frac{M \, \mathbb{E}_t \left\{\left(1 - G(\bar{\omega}_{t+1}^E)\right) \frac{Q_{t+1} \, \Pi_{t+1} (1 - \delta^E)}{R_t^B}\right\}}{\mathbb{E}_t \left\{\Phi_{t,t+1} \, MPK_{t+1}^E\right\}} \right) \tag{B.33}$$

$$K_t^B = (1 - \delta^B) \frac{K_{t-1}^B}{\Pi_t} + \omega^B \left[T_t^B + \mu \, G(\bar{\omega}_t^E) \, Q_t \, (1 - \delta^E) \, K_{t-1}^E\right] \tag{B.34}$$

$$\Phi_{t,t+1} = \beta^E \frac{C_t^E - h^E \, C_{t-1}^E}{C_{t+1}^E - h^E \, C_t^E} \tag{B.35}$$

$$S_t = R_t^B - R_t \tag{B.36}$$

B.6
Central bank

$$R_t = (R_{t-1})^{\phi_r} \left[R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\phi_r} e^{\sigma_r \varepsilon_t^R} \tag{B.37}$$

B.7
Market clearing

$$C_t + C_t^E + I_t + \frac{\chi}{2} \left[\left(\frac{I_t}{I_{t-1}} \right) - 1 \right]^2 I_t + \frac{\kappa_{KB}}{2} \left(\frac{K_{t-1}^B}{B_{t-1}} - v^B \right)^2 \frac{K_{t-1}^B}{\Pi_t} = Y_t \tag{B.38}$$

C
Steady-state

Variables without a time index t denote steady-state values. The steady-state values of some variables are given:

$$\Psi = 1 \qquad \Gamma = 1 \qquad \Pi = 1 \qquad \Pi^* = 1 \qquad A = 1$$

$$Q = 1 \qquad T^{CP} = 0 \qquad X = \frac{\epsilon}{\epsilon - 1} \qquad \Lambda = \beta \qquad \Phi = \beta^E$$

Other variables and ratios of variables are, at steady-state, equal to:

$$R = \frac{\Pi}{\beta} \qquad \Delta = \frac{1 - \theta}{1 - \theta\,(\Pi)^{\epsilon - \gamma\epsilon}} \qquad \frac{W\,L}{Y_w} = \frac{(1 - \alpha)}{X}$$

$$\frac{I}{K^E} = \delta^E \qquad \frac{Y_w}{Y} = \Delta \qquad \frac{T^R}{Y} = \left(\frac{1}{\Delta} - \frac{1}{X}\right)\frac{Y_w}{Y}$$

The dynamics of bank capital in steady-state can be written as:

$$\begin{aligned}\frac{K^B}{B} - (1 - \delta^B)\,\frac{K^B}{B}\,\frac{1}{\Pi} - \omega^B\left(1 - F(\bar{\omega}^E)\right)\frac{R^B}{\Pi} + \omega^B\,\frac{R}{\Pi} - \omega^B\,\frac{R}{\Pi}\,\frac{K^B}{B} \\ + \omega^B\,\frac{K^B}{B} + \frac{\omega^B\,\kappa_{KB}}{2\,\Pi}\left(\frac{K^B}{B} - v^B\right)^2\frac{K^B}{B} - \omega^B\,G(\bar{\omega}^E)\,\mu\,Q\,(1 - \delta^E)\,\frac{K^E}{B} = 0\end{aligned} \tag{C.1}$$

The first-order condition for bank h in steady-state is:

$$R^B\left[\left(1 - F(\bar{\omega}^E)\right)\left(1 - \left(\frac{1}{N}\right)(PED)^{-1}\right)\right] - R + \kappa_{KB}\left(\frac{K^B}{B} - v^B\right)\left(\frac{K^B}{B}\right)^2 = 0 \tag{C.2}$$

From the default cut-off rule, we have:

$$\frac{K^E}{B} = \frac{R^B}{\bar{\omega}^E\,\Pi\,Q\,(1 - \delta^E)} \tag{C.3}$$

The elasticity of demand for credit in the steady-state is:

$$PED = 1 + \frac{1}{1 - \alpha}\left(\frac{M\left(1 - G(\bar{\omega}^E)\right)\dfrac{Q\,\Pi\,(1 - \delta^E)}{R^B}}{\beta^E\;MPK^E}\right) \tag{C.4}$$

We know that the marginal productivity of physical capital in the steady-state is given by $MPK^E = \alpha\,\dfrac{Y_w}{X\,K^E}$. Furthermore, the equilibrium condition B.7

in steady-state is $\lambda_2^E \left(C^E - h^E C^E\right) = 1 - \beta^E \left(1 - F(\bar{\omega}^E)\right) \frac{R^B}{\Pi}$. Including these two expressions in the equilibrium condition B.8 evaluated at steady-state, we get:

$$\beta^E MPK^E = Q - \beta^E \left(1 - G(\bar{\omega}^E)\right) Q (1 - \delta^E) - M \left(1 - G(\bar{\omega}^E)\right) \frac{Q\,\Pi\,(1 - \delta^E)}{R^B} + M \left(1 - G(\bar{\omega}^E)\right) Q (1 - \delta^E) \beta^E \left(1 - F(\bar{\omega}^E)\right) \tag{C.5}$$

The demand for credit in the steady-state can be written as:

$$\frac{K^E}{B} = \frac{R^B}{M\,(1 - G(\bar{\omega}^E))\,Q\,\Pi\,(1 - \delta^E)} \tag{C.6}$$

Equating C.3 with C.6, we get:

$$\bar{\omega}^E - M \left(1 - G(\bar{\omega}^E)\right) = 0 \tag{C.7}$$

Now, we include C.5 in C.4 and then the resulting expression for PED is included in C.2. The expression C.3 is included in C.1. We thus obtain two equations which, together with equation C.7, form a non-linear system of three equations whose variables are $\frac{K^B}{B}$, R^B and $\bar{\omega}^E$. The solution to this system is calculated numerically from an initial guess equal to the values of 0.11, 1.0225 and 0.50, respectively.

From the numerical solution, we can calculate the other steady-state values.

$$S = R^B - R$$

$$G(\bar{\omega}^E) = \Phi \left(\frac{log(\bar{\omega}^E) + \frac{\sigma^2}{2}}{\sigma} - \sigma \right)$$

$$F(\bar{\omega}^E) = \Phi \left(\frac{log(\bar{\omega}^E) + \frac{\sigma^2}{2}}{\sigma} \right)$$

$$\lambda_2^E \left(C^E - h^E C^E\right) = 1 - \beta^E \left(1 - F(\bar{\omega}^E)\right) \frac{R^B}{\Pi}$$

$$\frac{K^E}{Y_w} = \frac{\alpha\,\beta^E}{X \left[Q - \beta^E (1 - G(\bar{\omega}^E))\, Q (1 - \delta^E) - \lambda_2^E (C^E - h^E C^E)\, M\, (1 - G(\bar{\omega}^B))\, \frac{Q\,\Pi\,(1 - \delta^E)}{R^B} \right]}$$

$$MPK^E = \frac{\alpha}{X} \left(\frac{K^E}{Y_w} \right)^{-1}$$

$$PED = 1 + \frac{1}{1-\alpha}\left(\frac{M\left(1 - G(\bar{\omega}^E)\right)\frac{Q\,\Pi\,(1 - \delta^E)}{R^B}}{\beta^E\, MPK^E}\right)$$

$$\frac{B}{Y_w} = M\left(1 - G(\bar{\omega}^E)\right)\frac{Q\,\Pi\,(1-\delta^E)}{R^B}\frac{K^E}{Y_w}$$

From the steady-state ratio $\frac{K^B}{B}$, we can obtain the steady-state ratio $\frac{K^B}{Y_w}$ also using K_t^B dynamics:

$$\frac{K^B}{Y_w} = \frac{\frac{\omega^B}{\Pi}\frac{B}{Y_w}\left[\left(1-F(\bar{\omega}^E)\right)R^B - R\right] + \omega^B G(\bar{\omega}^E)\,\mu\,Q\,(1-\delta^E)\frac{K^E}{Y_w}}{1 - (1-\delta^B)\frac{1}{\Pi} - \omega^B\frac{R}{\Pi} + \omega^B + \frac{\omega^B\,\kappa_{KB}}{2\Pi}\left[\frac{K^B}{B} - v^B\right]^2}$$

$$\frac{T^B}{Y_w} = \left(1 - F(\bar{\omega}^E)\right)\frac{R^B}{\Pi}\frac{B}{Y_w} - \frac{R}{\Pi}\frac{B}{Y_w} + \frac{R}{\Pi}\frac{K^B}{Y_w} - \frac{K^B}{Y_w} - \frac{\kappa_{KB}}{2\Pi}\left(\frac{K^B}{B} - v^B\right)^2\frac{K^B}{Y_w}$$

Rewriting some steady-state ratios:

$$\frac{I}{Y} = \frac{I}{K^E}\frac{K^E}{Y_w}\frac{Y_w}{Y} \qquad \frac{B}{Y} = \frac{B}{Y_w}\frac{Y_w}{Y} \qquad \frac{K^E}{Y} = \frac{K^E}{Y_w}\frac{Y_w}{Y}$$

$$\frac{W\,L}{Y} = \frac{W\,L}{Y_w}\frac{Y_w}{Y} \qquad \frac{K^B}{Y} = \frac{K^B}{B}\frac{B}{Y} \qquad \frac{T^B}{Y} = \frac{T^B}{Y_w}\frac{Y_w}{Y}$$

From the equilibrium conditions, we have:

$$\frac{D}{Y} = \frac{B}{Y} - \frac{K^B}{Y}$$

$$\frac{C}{Y} = \left(\frac{R}{\Pi} - 1\right)\frac{D}{Y} + \frac{W\,L}{Y} + \frac{T^{CP}}{Y} + \frac{T^R}{Y} + \left(1 - \omega^B\right)\left[\frac{T^B}{Y} + \mu\,G(\bar{\omega}^E)\,Q\,(1-\delta^E)\frac{K^E}{Y}\right] + (1-\mu)\,G(\bar{\omega}^E)\,Q\,(1-\delta^E)\frac{K^E}{Y}$$

$$\frac{C^E}{Y} = 1 - \frac{C}{Y} - \frac{I}{Y} - \frac{\kappa_{KB}}{2\,\Pi}\left(\frac{K^B}{B} - v^B\right)^2\frac{K^B}{Y}$$

$$\frac{N^E}{Y} = \frac{C^E}{Y} + Q\frac{K^E}{Y} - \frac{B}{Y}$$

As $\dfrac{Y}{C} = \left(\dfrac{C}{Y}\right)^{-1}$, we can calculate L using the equilibrium condition B.1:

$$L = \left[\frac{W\,L}{Y}\,\frac{Y}{C}\,\frac{1}{(1-h^H)}\right]^{\frac{1}{1+\phi}}$$

In the entrepreneur's production function in steady-state, we have:

$$\frac{L}{Y_w} = \left[A\left(\frac{K^E}{Y_w}\right)^{\alpha}\right]^{\frac{1}{\alpha-1}}$$

$$\frac{L}{Y} = \frac{L}{Y_w}\,\frac{Y_w}{Y}$$

We can then calculate Y:

$$Y = \frac{L}{\frac{L}{Y}}$$

With Y, we can compute all variables in steady-state from the ratios:

$$F_1 = \frac{Y}{X\,(1-\theta\,\beta)} \qquad F_2 = \frac{Y}{1-\theta\,\beta} \qquad W = \frac{W\,L}{Y}\,\frac{Y}{L}$$

$$C^E = \frac{C^E}{Y}\,Y \qquad N^E = \frac{N^E}{Y}\,Y \qquad \lambda_2^E = \frac{\lambda_2^E\,(C^E - h^E\,C^E)}{(C^E - h^E\,C^E)}$$

$$K^E = \frac{K^E}{Y}\,Y \qquad K^B = \frac{K^B}{Y}\,Y \qquad Y_w = \frac{Y_w}{Y}\,Y \qquad B = \frac{B}{Y}\,Y$$

$$I = \frac{I}{Y}\,Y \qquad T^R = \frac{T^R}{Y}\,Y \qquad C = \frac{C}{Y}\,Y \qquad D = \frac{D}{Y}\,Y$$

$$T^B = \frac{T^B}{Y}\,Y \qquad Lev = \frac{B}{N^E} \qquad R^r = \frac{R}{\Pi}$$

D
Calibration of N

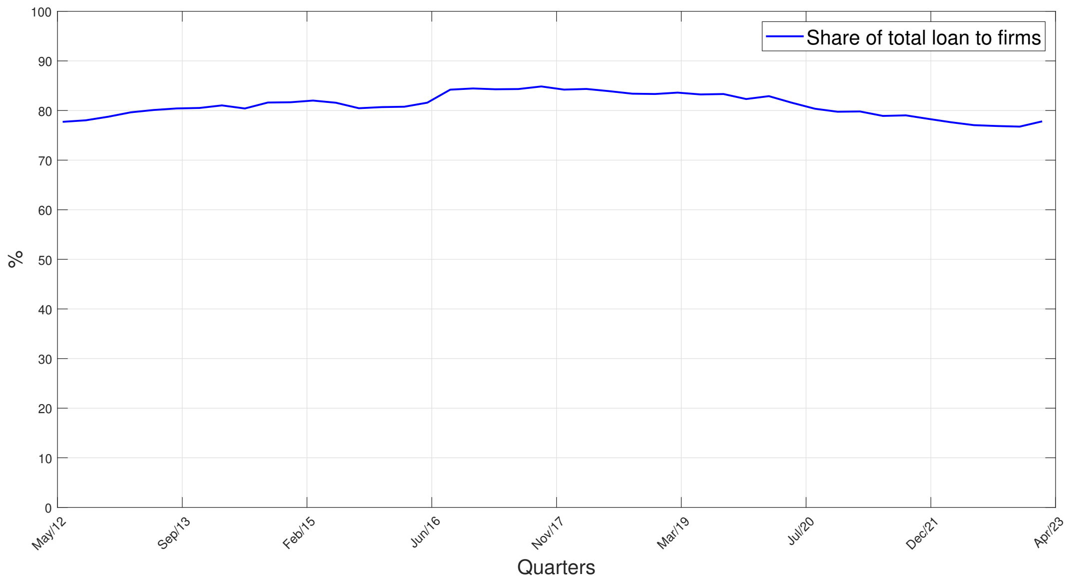

Figure D.1: Concentration of the credit market in Brazil: the five largest banks. Source: COSIF/BCB

Figure D.1 shows that, between 2012Q2 and 2023Q2, the five largest banks were responsible for around 80% of total credit to firms in Brazil. It can be seen that the Brazilian bank credit market is markedly concentrated. Banks controlling most of the credit market have market power, which impacts the loan rate and, therefore, spreads.

E
Series

All the series used are available on the Central Bank of Brazil's Time Series Management System (SGS-BCB)[1].

1. Nonearmarked credit operations outstanding - Non-financial corporations - Total
   - Code: 20543
   - Frequency: Monthly
   - Time Period: mar/2011 - oct/2023
2. Nonearmarked credit operations outstanding - Non-financial corporations - Working capital total
   - Code: 20550
   - Frequency: Monthly
   - Time Period: mar/2011 - oct/2023
3. Month average interest rate of nonearmarked new credit operations - Non-financial corporations - Total
   - Code: 25437
   - Frequency: Monthly
   - Time Period: mar/2011 - oct/2023
4. Month average interest rate of nonearmarked new credit operations - Non-financial corporations - Working capital total
   - Code: 25444
   - Frequency: Monthly
   - Time Period: mar/2011 - oct/2023
5. Percent of 90 days past due loans of nonearmarked credit operations outstanding - Non-financial corporations - Total
   - Code: 21086
   - Frequency: Monthly
   - Time Period: mar/2011 - oct/2023

[1] The Tier1 Ratio series started in Oct/2013 and was deactivated in Jun/2023 by the BCB.

6. Percent of 90 days past due loans of nonearmarked credit operations outstanding - Non-financial corporations - Working capital total
   – Code: 21093
   – Frequency: Monthly
   – Time Period: mar/2011 - oct/2023
7. Accumulated in the year average time deposits rate (CDB/RDB) - Individuals
   – Code: 28585
   – Standard unit: % per month
   – Frequency: Monthly
   – Time Period: mar/2011 - oct/2023
8. I002 - Tier1 Ratio
   – Code: 21801
   – Frequency: Monthly
   – Time Period: mar/2011 - jun/2023

F
Comparative statics

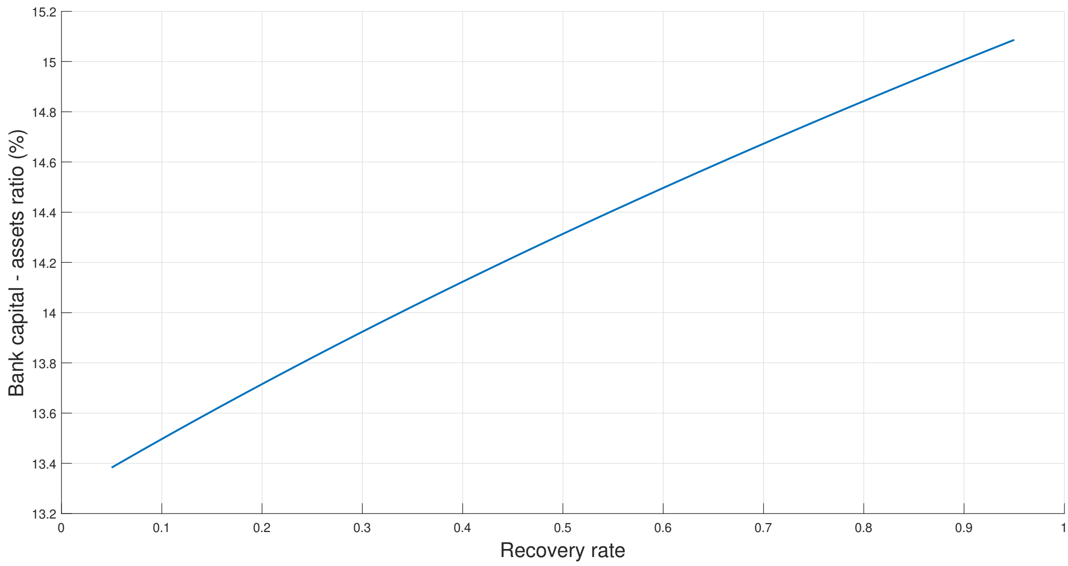

Figure F.1: Bank capital-assets ratio versus recovery rate

The steady-state capital-assets ratio is increasing at the recovery rate, as indicated in figure F.1. A higher recovery rate, all else being equal, increases the construction of bank capital from the default (see equilibrium condition B.34 in Appendix B.5); this results in a higher steady-state capital-assets ratio. The result is a lower steady-state loan rate (see equilibrium condition B.30 in Appendix B.5), determining a lower spread.

G
Steady-state values

Table G.1: Steady-state values of some variables

Description	Baseline Model	No credit frictions	No frictions
Gross inflation rate	1	1	1
Output	5.0189	5.3793	5.6035
Investment	0.5283	0.6075	0.6670
Household's consumption	3.4240	3.5677	3.7206
Entrepreneur's consumption	1.0513	1.1932	1.2159
Entrepreneur's net worth	16.4049	18.6365	20.2649
Entrepreneur's leverage	0.3523	0.3679	0.3767
Loans	5.7791	6.8560	7.6327
Policy rate	1.0225	1.0225	1.0225
Loan interest rate	1.0625	1.0367	1.0225
Bank spread	0.0400	0.0142	0
Bank profit	0.0677	0.1064	0
Bank capital	0.7926	0.8865	0
Default probability	0.0270	0	0
Elasticity of credit demand	6.9657	7.6033	8.0572
Household's welfare	-65.0772	-64.9203	-62.9456
Entrepreneur's welfare	-21.6166	-19.5075	-19.1933
Social welfare	-2.7287	-2.5987	-2.5364

H
Welfare

H.1
Recursive expressions

$$W_t^H = \log\left(C_t - h^H C_{t-1}\right) - \frac{(L_t)^{1+\phi}}{1+\phi} + \beta \, \mathbb{E}_t \left\{W_{t+1}^H\right\}$$

$$W_t^E = \log\left(C_t^E - h^E C_{t-1}^E\right) + \beta^E \, \mathbb{E}_t \left\{W_{t+1}^E\right\}$$

H.2
Steady-state equations

$$W^H = \frac{1}{1-\beta} \left[\log\left(C - h^H C\right) - \frac{(L)^{1+\phi}}{1+\phi}\right]$$

$$W^E = \frac{1}{1-\beta^E} \left[\log\left(C^E - h^E C^E\right)\right]$$

$$W^S = (1 - \beta) \, W^H + \left(1 - \beta^E\right) W^E$$