4
Yield and Failure Criteria

This chapter presents a review of material yield and failure criteria. In order to predict material failure or yielding of the steel casing, cement and salt rock, a *yield* or *fracture* criterion must be established for each one. Frictional materials such as rock and cement may (but are certainly not limited to) be characterized as brittle or ductile, for this is dependent upon the hydrostatic stress level upon which they are subjected to. For further reading, consult Hibbeler (2003) and Dowling (1999).

4.1
Mohr-Coulomb Fracture Criterion

The Mohr-Coulomb criterion developed by the German engineer *Otto Mohr* (1835-1918) is the most simple and widely used fracture criterion for determining the normal and shear stress at failure on a loaded frictional material. In order to take into account the cement's frictional properties, the Mohr-Coulomb criterion was selected for this research. Its simplicity is found in the graphical stress circle known as the *Mohr Circle* (see Figure 4-1).

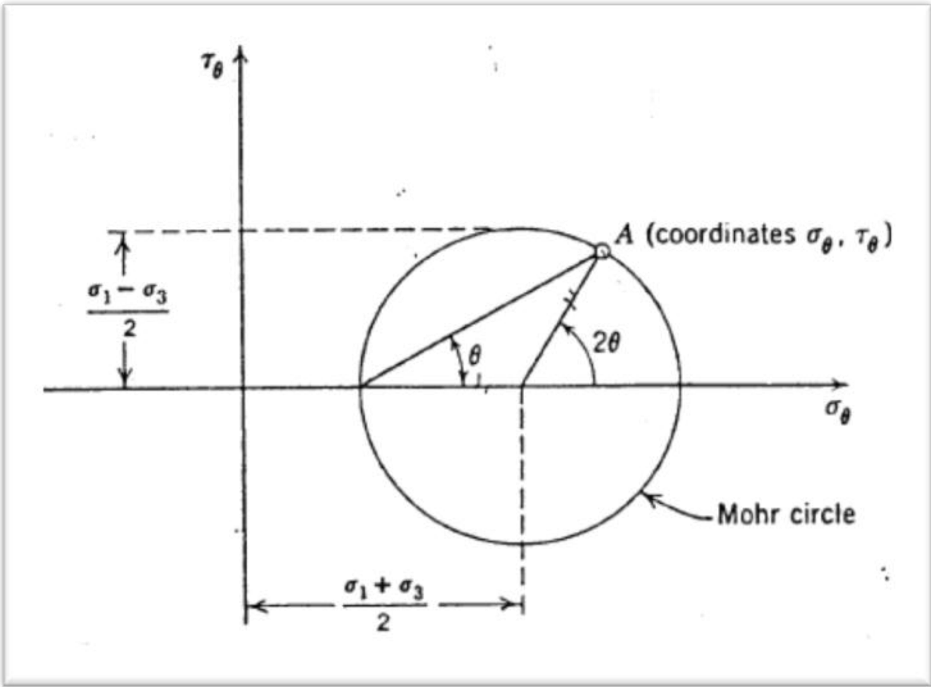

Figure 4-1: Mohr Circle (Lambe et al., 1979).

To further explain the use of the Mohr circle, assume that the in situ stresses on a rock are to be analyzed. The stresses on its four sides (assuming a square cutting) each make an angle with the surface. If the observer is rotated to a certain angle, the observed shear stresses will vanish and the stresses upon each face are the principal stresses, namely the major (σ_1) and the minor (σ_3). By knowing σ_1 and σ_3, the Mohr circle can be drawn. Once constructed, the Mohr circle is used to find the values of σ and τ at any point on the circle. A point represents the orientation 2θ to the principal stresses.

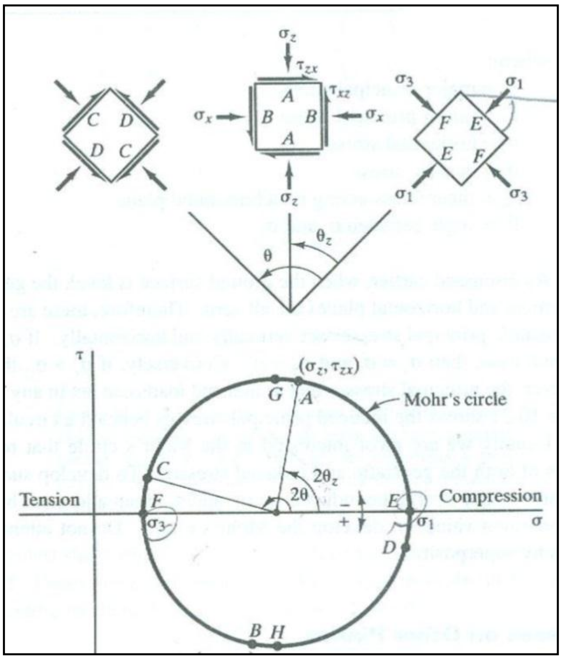

Figure 4-2: A specimen’s stress angle θ corresponds to the circle (Coduto, 1999).

By knowing the principal stresses and their directions, the Mohr Circle facilitates the determination of the state of stress at any plane on a continuous material. In other words, it gives the *stress tensor* since it describes the state of stress at a point in terms of shear stress and normal stress. The principal equation for this criterion is expressed as

$$\boldsymbol{\tau} = \boldsymbol{c} + \boldsymbol{\sigma} \tan \boldsymbol{\varphi} \tag{4.1}$$

Where

$\tau =$ shear stress at failure;

$c =$ cohesive strength of the material;

$\sigma =$ normal stress upon the failure plane; and

$\varphi =$ angle of internal friction.

Once several laboratory triaxial failure test results at different confining pressures are conducted, a linear envelope can be drawn tangent to all Mohr’s circles (Desai et al., 1984). The angle made between the linear envelope and the horizontal axis represents φ, which is the angle of internal friction at which failure occurs. This linear envelope extends to the ordinate axis, which indicates the apparent cohesive strength c of the specimen. The point at which the linear envelope is tangent with a given circle indicates the normal stress σ and shear stress τ at failure for the corresponding specimen.

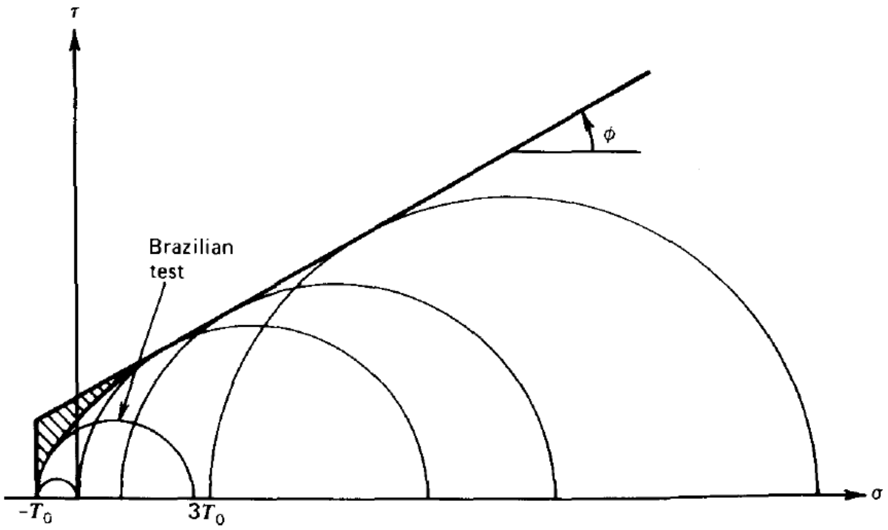

Figure 4-3: Mohr-Coulomb failure criterion with tension cutoff (Goodman, 1989).

It is worth noting that the failure envelope is actually a curve, but a straight line gives an approximation and is widely used in practice (Lambe et al., 1979). Extending the line up to the maximum tensile stress at failure, T_0, is known as the tension cutoff, where σ_3 is never greater than T_0. The following equations are also used for the Mohr circle:

$$\boldsymbol{\tau}_{max} = \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_3}{2} \tag{4.2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4.3}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4.4}$$

4.2
Von Mises Yield Criterion

The Von Mises yield criterion states that the maximum yield stress in a ductile material is reached when the distortion energy per unit volume of the material equals or surpasses the distortion energy per unit volume of the same material when it is subjected to yielding in a uniaxial tension test (Hibbeler, 2003). It is therefore known also as the *maximum distortion energy criterion*.

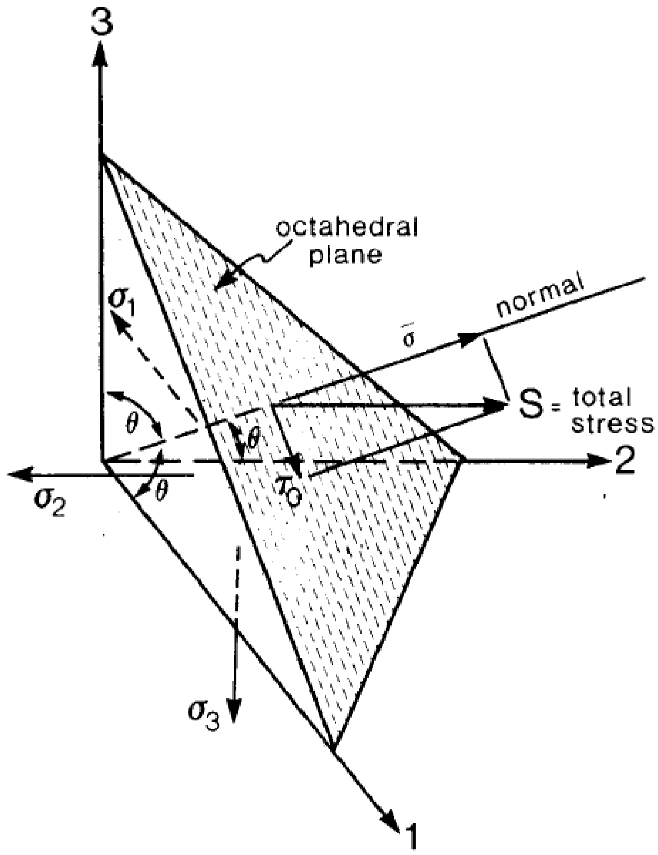

Figure 4-4: Octahedral plane (Infante et al., 1989).

This criterion is very useful for studying and testing mechanical behavior of materials because the yield envelope is composed by a unique expression. The von Mises Criterion states that yielding begins once the *octahedral shear stress* reaches a critical value, τ_o, which varies for each material. For this reason the Von Mises-Hencky criterion is also known as the *maximum octahedral shear stress criterion*. As depicted in Figure 4-4, the octahedral shear stress lies on a unique plane in which its normal stress (or mean stress) makes an angle of 54.7 degrees with all three principal stress directions.. The shear stress on the octahedral plane is expressed as

$$\tau_0 = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \tag{4.5}$$

By substituting the uniaxial stress state with $\sigma_1 = \sigma_{uniaxial}$ into the octahedral shear criterion gives

$$\tau_o = \frac{\sqrt{2}}{3}\sigma_{uniaxial} \tag{4.6}$$

The plane on which the uniaxial stress acts is related to the octahedral plane by a rotation through the angle θ of Figure 4-4, where

$$\theta = cos^{(-1)}\left(\frac{1}{\sqrt{3}}\right) = 54.7° \tag{4.7}$$

The octahedral normal stress, σ_{vm}, is determined with the following expression:

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_1-\sigma_3\right)^2} \tag{4.8}$$

The Mohr circle also provides this result by noting that in uniaxial tension the mean stress on the octahedral plane is

$$\bar{\sigma} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{\sigma_1}{3} \tag{4.9}$$

Knowing this stress value on the Mohr Circle allows for the determination of angle θ and τ_o.

Setting $\sigma_2 = \sigma_3 = 0$, Eq. **Error! Reference source not found.** becomes $\sigma_{vm} = \sigma_1$ and the von Mises stress σ_{vm} is obtained for uniaxial loading. The von Mises stress may be thought of as converting any loading state into the uniaxial loading state in order for comparison. If, for example, the tensile (uniaxial) yield stress for a metal is known but it is not subjected under uniaxial loading—say biaxial or triaxial—the von Mises yield stress σ_{vm} enables the comparison to be made with the uniaxial yield strength. In other words, The von Mises criterion establishes stress combinations at a given point that will cause failure. Another advantage about the von Mises stress equation is that it does not require that the principal stresses be known and Eq. **Error! Reference source not found.** may be rewritten as:

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}{}^2+\tau_{yz}{}^2+\tau_{zx}{}^2\right)} \tag{4.10}$$

(4.11)

4.3
Tresca Yield criterion

An alternative to the von Mises criterion is the Tresca yield criterion. The Tresca yield criterion is also based upon results of metal specimens, but it differs from the von Mises criterion by stating that yielding is provoked by the maximum shear stress that acts on a plane oriented 45 degrees from the normal principal stress (Dowling, 1999):

$$\tau_{\max} = Max\left(\left|\frac{\sigma_1-\sigma_2}{2}\right|,\left|\frac{\sigma_2-\sigma_3}{2}\right|,\left|\frac{\sigma_1-\sigma_3}{2}\right|\right) \tag{4.12}$$

Hence, it is also called the *maximum shear stress yield criteria*. The maximum shear stress is a critical value dependent upon the material. Like the von Mises theory, the Tresca theory is capable of predicting failure stress for a material subjected to any type of loading (Hibbeler, 2003).. This is practical for salt as well as metal, for the gliding mechanism between the crystal planes is controlled by shear stress (Dowling, 1999). In the case of uniaxial loading, the equation for the Tresca yield criteria is reduced to

$$\tau_{max} = \frac{\sigma_0}{2} \tag{4.13}$$

Where σ_0 (or σ_1) is the yield tensile stress.

An argument in favor of the von Mises Criterion is that there exists a greater probability of yielding on the octahedral planes, in which the shear stress at yielding occurs on four planes on the tetrahedron as opposed to only two by the Tresca Criterion (Dowling, 1999). In addition, the von Mises Criterion is used in the API Bulletin 5C3 (1994) regarding formulas and calculations for casing, tubing, drill pipe and line properties. For this reason, the von Mises criterion will be used for the steel casing in this research, while the salt formation will not require a yield criterion since it does not attain failure.

4.4 Chapter Summary

The Mohr-Coulomb criterion is applicable for brittle materials while the von Mises and Tresca criteria are applicable for ductile materials. The first two criteria will be used in this research for simulating failure scenarios for the cement and casing. Tresca is preferably used as a measure for deviatoric stress found in the salt creep double mechanism equation (See Chapter 3.8.4). Because salt does not reach failure, no yield or failure criterion is assigned for it. Although there does exist other available criteria, these three are widely used in engineering applications.