Pontifícia Universidade Católica do Rio de Janeiro

Jonas Caldara Pelajo

Innovative Decision Models for Energy Commercialization

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Administração de Empresas of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Administração de Empresas.

Advisor: Prof. Leonardo Lima Gomes

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Prof. Leonardo Lima Gomes
Advisor
PUC-Rio

Prof[a]. Cristina Pimenta de Mello Spineti Luz
UFRJ

Prof. Javier Gutierres Castro
UFSC

Prof[a]. Katia Maria Carlos Rocha
IPEA

Prof. Luiz Eduardo Teixeira Brandão
PUC-Rio

Rio de Janeiro, April 17th, 2024

Jonas Caldara Pelajo

I hold a Master of Science (M.Sc.) degree in Business Administration, awarded by the Pontifical Catholic University of Rio de Janeiro in 2019. Additionally, I completed my Bachelor of Science (B.S.) in Industrial Engineering at the Federal University of Rio de Janeiro in 2007.

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Abstract

Pelajo, Jonas Caldara; Gomes, Leonardo Lima. **Innovative Decision Models for Energy Commercialization** (Advisor). Rio de Janeiro, 2024. 155p. Tese de Doutorado - Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

In the last decade, the Brazilian electricity sector has faced regulatory and operational challenges due to the need to adapt to changes in the energy matrix, which shows a growing share of intermittent renewable energies, such as solar photovoltaic and wind energy. Additionally, social and environmental restrictions on the construction of new hydroelectric reservoirs require the development of new models for hydrological risk management. This thesis comprises four studies and aims to develop decision support models that contribute to the management of the national interconnected system and the optimization of relevant processes in the sector, considering the current scenario. The first study, by defining a methodology for accessing the parameters of the ECP_G functional, contributes to the innovation and improvement of theoretical models, with practical results for the sector. The second study contributes to the process of seasonalizing the physical guarantee and reveals an optimal strategy that simultaneously maximizes the results of generating agents, preventing reductions in payoffs resulting from individual movements of competitors. The third study proposes a commercialization portfolio optimization model, which allows agents to adequately expose themselves to risk, contributing to more efficient commercial management. Finally, the fourth study presents an operational model for an energy futures clearing house, offering valuable insights for stakeholders interested in establishing such a project in Brazil, where no energy futures clearing house currently exists.

Keywords

Energy commercialization; utility function; physical guarantee; hedge; Monte Carlo simulation.

Resumo

Pelajo, Jonas Caldara; Gomes, Leonardo Lima. **Modelos de Decisão Inovadores para Comercialização de Energia**. Rio de Janeiro, 2024. 155p. Tese de Doutorado - Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

Na última década, o setor elétrico brasileiro tem enfrentado desafios regulatórios e operacionais devido à necessidade de adaptação às mudanças na matriz elétrica, que apresenta uma participação crescente de energias renováveis intermitentes, como a solar fotovoltaica e a eólica. Além disso, restrições sociais e ambientais para a construção de novos reservatórios hidrelétricos exigem o desenvolvimento de novos modelos para a gestão do risco hidrológico. Esta tese é composta por quatro estudos e tem como objetivo desenvolver modelos de apoio à decisão que contribuam para a gestão do sistema interligado nacional e para a otimização de processos relevantes do setor, considerando o cenário atual. O primeiro estudo, ao definir uma metodologia de acesso aos parâmetros do funcional ECP_G, contribui para a inovação e o aprimoramento de modelos teóricos, com resultados práticos para o setor. O segundo estudo contribui para o processo de sazonalização da garantia física e revela uma estratégia ótima que maximiza simultaneamente os resultados dos agentes geradores, prevenindo reduções nos *payoffs* resultantes de movimentos individuais de concorrentes. O terceiro estudo propõe um modelo de otimização de portfólio de comercialização, que permite aos agentes uma exposição adequada ao risco, contribuindo para uma gestão comercial mais eficiente. Finalmente, o quarto estudo apresenta um modelo de operação de uma bolsa de futuros de energia, fornecendo informações relevantes para agentes interessados em implementar um empreendimento desse tipo no Brasil, que ainda não possui uma bolsa de futuros de energia.

Palavras-chave

Comercialização de energia; função utilidade; garantia física; Hedge; Simulação de Monte Carlo.

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1
Introduction

The Brazilian electricity grid is predominantly fueled by renewable sources, which accounted for 87.9% of total electricity production in 2022. Notably, solar photovoltaic and wind energy have played significant roles in the recent surge of renewable energy in Brazil's electricity grid, although hydroelectric power remains the predominant source. Photovoltaic solar generation achieved 30.1 terawatt-hours (TWh), representing a staggering 79.8% increase compared to the previous year. Wind generation, on the other hand, reached 81.6 TWh, demonstrating a growth of 12.9% compared to the previous year. Hydroelectric generation contributed 64.3 TWh, marking a substantial 17.7% growth compared to 2021 (EPE, 2023). The expansion in hydroelectric energy production was primarily driven by hydrological conditions rather than an increase in installed capacity.

Although hydraulic generation is currently the most representative in the Brazilian electricity matrix, this source of energy has been experiencing, in recent years, several challenges regarding regulation, mainly concerning the hydrological risk. The reduction in reservoir levels, which became more evident in 2014 and 2015, added to other issues on planning and execution of infrastructure expansion, generated a crisis that resulted in losses for agents, mainly generators.

As a result, many generating agents sued the system operator, which, among other results, suspended a large part of the apportionment of the energy reallocation mechanism (MRE), generating a billionaire debt to the electricity sector, which is still pending in court.

While much of these issues stemmed from prolonged droughts and declining reservoir levels, it is essential to recognize that the electricity sector crisis is not solely tied to reduced reservoir levels. The prevailing model of the Brazilian electrical system's development and expansion is no longer suitable for the current landscape, where constraints on new reservoir construction and the increasing intermittency of load profiles have made system management considerably more complex. In 2022, there was a significant uptick in the proportion of wind and solar energy within the energy matrix, as highlighted in Table 1, maintaining the trajectory of an expanding presence of renewable and intermittent energy sources within the matrix.

Energy Source	2021 (GWh)	2022 (GWh)	Δ 2022/2021
Hydroelectric	362,818	427,114	17.7%
LNG (Liquefied Natural Gas)	86,957	42,110	-51.6%
Wind	72,286	81,632	12.9%
Biomass	52,416	52,223	-0.4%
Nuclear	14,705	14,559	-1.0%
Coal	17,585	7,988	-54.6%
Oil derivates	17,327	7,056	-59.3%
Photovoltaic solar	16,752	30,126	79.8%
Other	15,263	14,364	-5.9%
Total Generation	**656,109**	**677,173**	**3.2%**

Table 1 – Brazilian energy matrix: energy sources comparison between 2022 and 2021

Source: EPE (2023)

Renewable energies, especially wind and solar, present additional complexity to the system, as they depend heavily on meteorological variables and do not have reservoirs, as occurs with most of hydraulic energy generation, what leads to high volatility to the load, the energy produced is automatically inserted into the system, with priority in the dispatch in relation to other types of energy. The national system operator then needs to create several mechanisms to compensate for this high volatility and meet demand for energy. In some cases, the operator does not dispatch the energy from hydroelectric energy plants, even with hydrological conditions for this to have occurred, to compensate for the load conditions of the wind and solar plants.

The crisis showed, therefore, that the hydrological risk management mechanisms needed to be reassessed and the agents involved have been looking for different alternatives to solve this problem. In 2015, the Government allowed the renegotiation of the hydrological risk to the generators, through the Law 13.203/2015 (Brasil, 2015) and ANEEL Normative Resolution No. 684/2015 (ANEEL, 2015), with the aim of reducing the risk to which the energy generators were exposed, sharing it with the consumers. However, it is still necessary to modernize the Brazilian electricity sector to ensure proper operation and the ability to expand the system supporting the country development while maintaining the attractiveness of the investment.

As part of efforts to modernize the electricity sector, in 2019, the government published Ordinance 187 of April 4 (MME, 2019), which establishes important guidelines towards the modernization of the Brazilian electricity sector, promoting perspectives of changes in the market environment and mechanisms to enable the expansion of the Electricity System, price formation mechanisms, energy reallocation mechanism (MRE) and risk cost allocation.

In this context, the various decision support models that were developed need to be rebuilt and new ones will need to be developed for the current regulatory and operational scenario of the electricity sector. The objective of this work is, in the context of the Electricity Sector Modernization Program, to develop new decision support models for energy commercialization, involving the following themes: Parameterization of the new preference function ECP_G for the electricity sector proposed by Luz (2016), which is a generalization of the Extended CVaR Preference functional (ECP) proposed by Street (2010); physical guarantee seasonalization; portfolio optimization for commercialization; operating model for an energy futures clearing house in Brazil.

1.1
Research objectives

The objective of this work is to develop, in the light of the new regulatory scenario of the Brazilian electricity sector:

a) A methodology to define the parameters of ECP_G functional and its underlying utility function, proposed by Street (2016).
b) A new decision model for the seasonalization of the physical guarantee of hydroelectric plants within the scope of the energy reallocation mechanism (MRE). The model will also be applied to a scenario of big player of Brazilian marketing.
c) A decision model to hedge the energy generation portfolio and contracts. The model will also be applied to a scenario of a big player of Brazilian marketing.

d) An operation model for a clearing house to Brazilian futures energy market.

1.2 Scope delimitation

This work was carried out jointly with a large company that operates in the electricity generation and commercialization sector. The models were developed and tested for this company, but they were developed in the most generic way as possible so that they can be used by other agents in the electricity sector.

1.3 Research outline

The research is quantitative and involves decision making under uncertainty. The population of interest are hydroelectric power plants and energy traders. The research was carried out with a company that has its own generation power plant portfolio and commercialization contracts in sufficient volume for the application of the models developed. The models were made as generically as possible. The power plants selected for the study have size, location and technology characteristics that make them a good representative of the hydroelectric power plants population.

1.4 Main contributions

This work was developed with the objective of promoting academic advances that may be applicable to practical cases related to participants in the electricity sector. The main contributions and novelty of this work to the state of the art are:

a) The ECP functional has been widely used since 2013 by the System Operator for hydrothermal dispatch optimization and, also, in several other problems regarding the electricity sector. One of its main advantages is the ease of implementation in dual stochastic programming problems (Luz, 2016). However, the parameters that reflect the preferences of each agent are defined based on the experience of the operators and there is currently no method to determine these parameters described in the literature. By defining a methodology to access the functional parameters, this work contributes to the innovation and improvement of theoretical models, with practical results for the sector.

b) The development of a seasonalization model, which will help hydroelectric energy generators to optimize the seasonalization process, which is performed once a year for each power plant. Currently, this process is performed relying on the experience of specialists of power plants, but there is no consolidated optimization model to support the decision process. The seasonalization process is important because it can lead to large financial loses or gains depending on how volatile the price of energy through the year is.

c) Energy traders often own power generation plants. An optimization model for simultaneously hedging the generation portfolio and contracts will help these participants to have a more profitable operation, adequate to the risk levels they are willing to face, according to their risk profile. Currently, a large part of the portfolio optimization process is done with intuitive assessment to the risk profile, which can lead to inadequate risk exposure. The proposed optimization model allows agents to be adequately exposed to their risk profile and optimize their returns, considering the portfolio of generation contracts.

2
Parameter Estimation for Generalized Extended CVaR Preference Function

2.1
Introduction

The Extended CVaR Preference functional (ECP), proposed by Street (2010), is an optimization measure that weighs the expected revenue value and the conditional Value-at-Risk ($CVaR_\alpha$) distribution at a given significance level α (equation (1)). The ECP functional became a very important measure for Brazilian electricity sector as it started to be applied, in 2013, for the optimization of a variety of problems by the national system operator, including the dispatch of the hydrothermal energy. Dual stochastic problems are particularly suitable to be solved using ECP functional due the optimization implementation simplicity.

$$ECP_{\alpha,\lambda} = (1-\lambda)E[\tilde{x}] + \lambda CVaR_\alpha;\ \lambda \in [0,1] \quad (1)$$

where λ is the aversion risk parameter, $E[\tilde{x}]$ is the expected value of $\tilde{x}$, $CVaR_\alpha$ is the Conditional Value-at-Risk for level α and $\tilde{x}$ is the set of possible outcomes assigned to the random variable x.

Luz (2016) proposed a generalization of ECP functional, ECP_G, shown in equation (2), and its underlying utility function, a generalization of Street's proposed functional, for one period, which results directly in the generalization of the ECP to N levels or reference points of risk aversion.

$$ECP_{G_{\alpha,\lambda}} = E[u(\tilde{x})] = \lambda_0 E(\tilde{x}) + \sum_{n=1}^{N} \lambda_n CVaR_{\alpha_n}; \quad (2)$$

$$\lambda_n \geq 0\ e\ \sum_{n=1}^{N} \lambda_n = 1$$

The concept behind the ECP_G, along with its underlying utility function, aims to facilitate the representation of distinct risk aversion behaviors exhibited by each agent across various risk levels. Typically, the utility function doesn't exhibit a linear behavior across the entire range of potential outcomes. The introduction of a piecewise linear function serves the purpose of capturing these nonlinear patterns comprehensively. This approach proves to be highly advantageous, offering a valuable combination of usefulness, flexibility, and precision. Importantly, it maintains the simplicity associated with the use of linear functions, ensuring that neither the mathematical complexity nor the processing time of optimization programs is increased. Risk aversion and its different components are extensively detailed in Raiffa and Keeney (1975) and the idea of using piecewise linear functions for this type of problem is found in Rabin (2000).

Despite the wide acceptance of the piecewise linear function as a form of representation for the agent's utility function, and its extensively within the electricity sector, notably by the national system operator in their optimization programs, there is still a theoretical gap, which concerns to the method to determine the risk aversion parameters and also the quantity and location of the N risk levels to be considered. There is not a detailed description of how to determine the parameters of underlying utility functions ECP and ECP_G in the literature. The parameters are essential elements for the correct capture of the agent's risk preferences and, consequently, for the assessment of the utility function. In this way, an inadequate parameter determination procedure, regarding the risk aversion parameters, λ's, can lead to an optimization process where there is excess or lack of agent's exposure to risk, considering the actual risk profile of the agents, what can generate inconsistent results and outcomes that are not optimized.

The main contribution of this work is to provide a methodology to determine the underlying utility function parameters, which will allow market agents to determine with greater clarity, uniformity, and precision the risk preferences of the system participants, which are used to support decisions in various scenarios, especially in dual stochastic optimization problems, and also in various applications in the management of the national interconnected system. The proposed approach is based on interviews with risk managers and hypothetical lotteries through which these managers express their preference between two types of investment with varying probabilities of occurrence. It will also be possible to assess the suitability of the number of levels chosen, through 50-50 lotteries. Given the great acceptance of the models proposed by Street (2010) and Luz (2016) by the Brazilian electricity market, especially by the national system operator, there is great potential for improving the results of the optimization models that are current being applied for the Brazilian system operation, as a result of a more adequate and uniform procedure to determine the aversion parameters risk.

2.2
Utility theory

Von Neumann and Morgenstern (1944), Friedman and Savage (1948), and more recently Fishburn (2013) have been profoundly influential in shaping utility theory and the contemporary understanding of decision-making under risk. These eminent scholars have played pivotal roles alongside other distinguished authors in advancing these fields (Bassett Jr et al., 2004). While it has faced criticism from certain authors, including Rabin (2000), who contends that expected utility theory provides an implausible explanation for significant risk aversion when dealing with relatively small stakes, it's worth noting that expected utility theory remains one of the most robust approaches for risk management and modeling risk aversion.

One of the main concepts in utility theory is Von Neumann-Morgenstern (vN-M) utility function, which is defined as a function of the investor's wealth level. For each level of investor wealth (W), there is an associated value, defined by its utility function $u(W)$, which is a specific function that reflects the value that the investor personally attributes to a given level of wealth. Utility is a psychic measure and, therefore, the utility function is extremely personal and reflects your satisfaction or happiness associated with a certain wealth level.

From the development of the concept of utility function, it was possible to establish preferences between random variables, or lotteries, through an analytical comparison between them. One random variable is preferable to another if its associated utility is greater than the utility associated with the other random variable.

The computation of expected utility is usually made through the Lebesgue integral or through the Choquet integral. The latter permit the probability weights associated with the least favorable outcomes to be accentuated, yielding a more pessimistic decision criterion (Bassett Jr et al., 2004).

2.2.1 Risk aversion

From the utility function it is also possible to evaluate the risk aversion, neutrality, and proneness. Mathematically, we use the second derivative of the utility function to characterize the risk profile. When the utility function is concave, the investor is risk averse, when it is convex, risk prone, and when it is linear, risk neutral. The investor may also present different preferences profiles according to the level of wealth, being, for example, prone to risk in a certain segment and averse to risk in another segment, within the possible values of observed wealth. We can define, according to the inequalities of equations 3, 5 and 7, respectively, risk-averse, neutral, and risk-prone investors.

The risk-averse investor is psychologically more sensitive to losses than gains (equation (3)), starting from an initial reference of wealth w_o, which means that, a decision maker that is risk averse will prefer the expected consequence of a nondegenerate lottery to that lottery (equation 4). Considering a 50-50 lottery, the investor is risk averse if he prefers the expected consequence to the lottery. The decision maker is risk averse if, and only if, his utility function is concave.

$$|u(w_o + d) - u(w_o)| \leq |u(w_o) - u(w_o - d)| \quad (3)$$

$$u[E(\tilde{x})] > E[u(\tilde{x})] \quad (4)$$

The risk-neutral investor (equation (5)) is psychologically indifferent to losses and gains, starting from an initial reference of wealth W_o. which means that, a decision maker that is risk neutral will be indifferent between the expected consequence of a nondegenerate lottery to that lottery (equation (6)). Considering a 50-50 lottery, the investor is risk neutral if he is indifferent between the expected consequence to the lottery.

$$|u(w_o + d) - u(w_o)| = |u(w_o) - u(w_o - d)| \quad (5)$$

$$E[u(\tilde{x})] = u[E(\tilde{x})] \tag{6}$$

The risk-prone investor (equation (7)) is psychologically more sensitive to gains than losses, starting from an initial reference of wealth W_o, which means that, a decision maker that is risk prone will prefer the lottery to the expected consequence of a nondegenerate lottery (equation (8)). Considering a 50-50 lottery, the investor is risk prone if he prefers the lottery to expected consequence. The decision maker is risk averse if, and only if, his utility function is convex.

$$|u(w_o + d) - u(w_o)| \geq |u(w_o) - u(w_o - d)| \tag{7}$$

$$E[u(\tilde{x})] > u[E(\tilde{x})] \tag{8}$$

In Figure 1 we present illustrative utility functions for investors with varying risk preferences. These functions represent the relationship between wealth and utility for risk-averse, risk-prone, and risk-neutral investors:

1. Risk-Averse Investor: This investor exhibits a greater aversion to losses than an inclination towards gains. Consequently, their utility function takes on a concave shape, indicating diminishing marginal utility as wealth increases.
2. Risk-Prone Investor: In contrast, the risk-prone investor is more inclined toward gains and less deterred by losses. Their utility function exhibits a convex shape, illustrating increasing marginal utility as wealth grows.
3. Risk-Neutral Investor: For the risk-neutral investor, the sensitivity to losses and gains is balanced. As a result, their utility function is linear or straight, signifying a consistent marginal utility for both gains and losses.

These graphical representations help us visualize how different investor types evaluate and respond to risk in their decision-making processes.

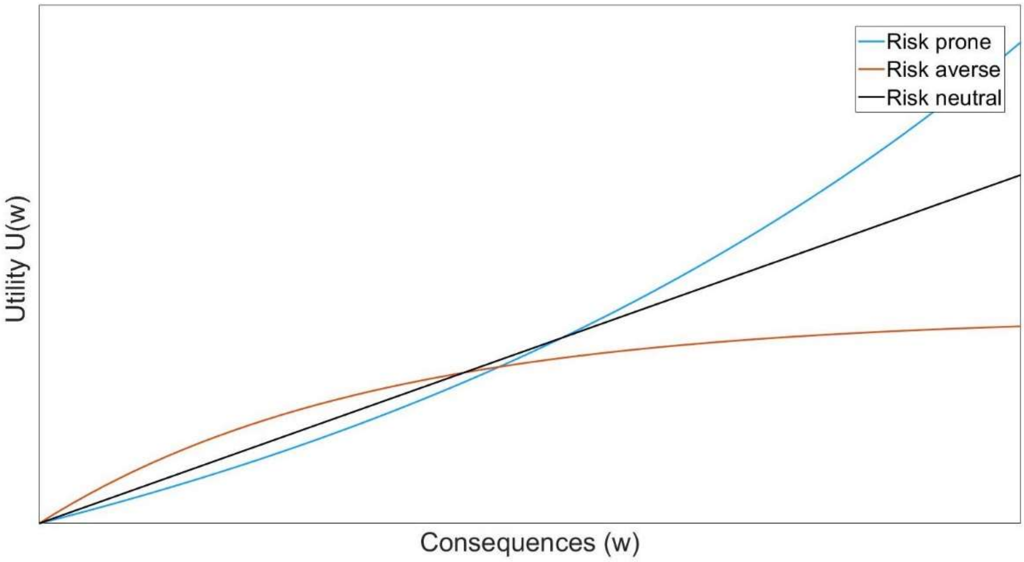

Figure 1 – Example of utility functions of risk averse, risk prone and risk neutral investors

2.2.2 Certainty equivalent

The certainty equivalent ($\hat{x}$) is a concept widely used in utility theory. Raiffa and Keeney (1975) define the certainty equivalent of a lottery L as an amount $\hat{x}$ such the decision maker is indifferent between L and the amount $\hat{x}$ for certain. Mathematically we can define the certainty equivalent ($\hat{x}$) by equating the utility of the certainty equivalent to the expected value of the utility of the lottery (equation (9)):

$$u(\hat{x}) = E[u(\tilde{x})] \tag{9}$$

where $\tilde{x}$ denote the uncertain consequences of the lottery (random variable).

The expected consequence of the lottery ($\bar{X}$) is determined by the equation (10) and the expected utility of the lottery ($E(u(\tilde{x}))$) is determined by the equation (11):

$$\bar{X} = E(\tilde{x}) = \sum_{i=1}^{n} p_i x_i \tag{10}$$

$$E(u(\tilde{x})) = \sum_{i=1}^{n} p_i u(x_i) \tag{11}$$

where p_i is the probability of the consequence x_i.

The certainty equivalent of a lottery is unique for monotonic utility functions. The development of utility theory has been concerned with the development of utility functions with monetary consequences, so the terms selling price and cash equivalent are found in the literature as the same as certainly equivalent of a lottery representing monetary amounts (Raiffa & Keeney, 1975).

The equation (10) and the equation (11) represent a lottery with discrete number of consequences. For lotteries with consequences described by a probability density function, $\bar{X}$ and $E(u(\tilde{x}))$ are determined by the equation (12) and the equation (13), respectively.

$$\bar{X} = E(\tilde{x}) = \int xf(x)dx \tag{12}$$

$$E(u(\tilde{x})) = \int u(x)f(x)dx \tag{13}$$

where f is the probability density function.

The certainty equivalent can also be obtained using the equation (14). This approach to calculate the value of the certainty equivalent can be very useful in solving many problems involving utility functions and lotteries:

$$\hat{x} = u^{-1}(E[u(\tilde{x})]) \tag{14}$$

where u^{-1} is the inverse function of u.

2.2.3 Risk premium

The risk premium of a lottery is defined as the difference between the expected consequence of the lottery and its certainty equivalent (equation (15)).

$$Risk\ Premium = \bar{x} - \hat{x} = E(\tilde{x}) - u^{-1}(E[u(\tilde{x})]) \qquad (15)$$

The risk premium can be interpreted as the amount that the investor is willing to give up in relation to the expected value of the lottery (average of possible consequences) in order to not have to take the risk of the lottery, receiving a fixed value, consequently.

2.2.4 Insurance premium

The insurance premium concept is remarkably similar to the certainty equivalent concept but refers to a situation where the investor is faced with an unfavorable lottery, and he wants to avoid the financial responsibilities of this lottery by passing it on to someone else. The insurance premium is therefore the negative of the certainty equivalent of the lottery (equation (16))

$$Insurance\ Premium = -\hat{x} = -u^{-1}(E[u(\tilde{x})]) \qquad (16)$$

2.2.5 Strategically equivalent

The strategically equivalent is another important concept of utility theory. Two utility functions, $u_1$and u_2, are strategically equivalent ($u_1 \sim u_2$) if we can find constants h and $k > 0$, such that (equation (17)):

$$u_1(x) = h + ku_2(x),\ for\ all\ x. \qquad (17)$$

It is possible to prove that if two utility functions are strategically equivalent, then the certainly equivalents for any lottery implied by u_1 and u_2 are the same. The consequence is that, if we two or more strategically equivalent utility functions, their preferences regarding two lotteries, $\tilde{x}_1$and $\tilde{x}_2$will be the same. So, if lottery $\tilde{x}_1$is preferred to lottery $\tilde{x}_2$ considering utility function u_1, the preference will be the same considering utility function u_2 or any other since $u_1\sim u_2$. The practical implication is that strategically equivalent utility functions will lead to the same action strategy for the decision maker.

2.2.6
Proportional risk aversion

Another important concept in utility theory concerning risk, described in Raiffa and Keeney (1975), is proportional risk aversion. As an example, we have a class of investments in which the investor places an m proportion of his assets (x_0) in a double or nothing bet where the probability of success is p and of losing is $1-p$. The result of your investment is described in equation (18):

$$\tilde{z}_m = \begin{cases} (1-m)x_0 + 2mx_0 = (1+m)x_0\,, with\ probability\ p \\ (1-m)x_0\,, \qquad\qquad\qquad with\ probability\ 1-p \end{cases} \quad (18)$$

where $\tilde{z}$ is the result of the lottery where z represents a nonnegative random variable.

For this type of investment, the authors prove that the optimal investment strategy does not depend on the initial amount (x_0) to be invested. They explore four classes of utility functions for which the investment plans don't depend on x_0 and show that the following statements are equivalent:

i. $xr(x)$ is constant.
ii. $u(x)\sim \log x$, or x^{1-c} for $0 \neq c < 1$, or $-x^{-(c-1)}$ for $c > 1$, or $u(x) = x$.
iii. The optimum investment plan is independent of the assets.

The expression $xr(x)$ is called the proportional local risk aversion, determined by the equation (19):

$$xr(x) \cong -x\left(\frac{u''(x)}{u'\,(x)}\right) \tag{19}$$

2.2.7
Parameters of proportional risk aversion utility functions

When utility functions exhibit constant proportional risk aversion, it is possible to determine the appropriate parameter c using questions to be answered by the decision marker involving certainty equivalent and 50-50 lottery. Assuming that the initial wealth of the decision maker is x_0, we ask him to choose between two options:

a) x_0 for certain,
b) A 50-50 lottery with will
   a. Double the initial value to $2x_0$ or,
   b. Reduce the wealth to ρx_0

If the investor is indifferent between a and b when $\rho = 0.5$, then $c = 1$ or $u(x) = \log x$. If the investor prefers option a keeping $\rho = 0.5$ then, $c > 1$. If the investor prefers b, then $c < 1$.

Supposing $\rho > 0.5$ and $c > 1$, it is possible to calculate c by solving equation (20). This equation equates utility of certainty equivalent to the expected value of the utility of the lottery.

$$-{x_0}^{-(c-1)} = \frac{1}{2}\left[-\rho {x_0}^{-(c-1)} - 2{x_0}^{-(c-1)}\right] \tag{20}$$
$$2 = \rho^{-(c-1)} + 2^{-(c-1)}$$

Using the same procedure, for $\rho < 0.5$ and $c < 1$, we can determine de parameter c solving equation (21).

$${x_0}^{1-c} = \frac{1}{2}\left[\rho {x_0}^{1-c} + 2{x_0}^{1-c}\right] \tag{21}$$
$$2 = \rho^{1-c} + 2^{1-c}$$

2.3 Risk measures

A key component when formulating strategies for action in the face of lotteries is the risk management. An investor needs to maximize his result, which is usually done in terms of utility, but at the same time he needs to manage the risk to avoid losses that are not adequate to his profile. Due to its application in finance and many other areas, risk management is a subject with a very well-developed literature and the idea of this section is to briefly present the main concepts of Value-at-Risk and Conditional Value-at-Risk measures that are useful for the understand of performance measures, ECP and ECP_G functionals, that will be presented in the following section.

2.3.1 Value-at-Risk

An extremely popular measure in risk management, mainly due to its simplicity and ease of application, is the Value-at-Risk (VaR). The VaR value represents the maximum loss, in percentage terms of the possible scenarios, that the investor is willing to face, given an investment. It is expressed in terms of α, which represents the $1-\alpha$ worst case scenarios that the investor is unwilling to accept, so the unacceptable losses will not exceed $1-\alpha$ scenarios.

For a given lottery and a given period t, if the investor defines α value, for example 95%, that corresponds to a v value for VaR, then there is a probability of $1-\alpha$ (5%) that the lottery return is below v within a period t. The calculation of the VaR is done in a simple way, through the percentile of the distribution of the possible returns once the investor knows the distribution returns.

A convenient definition of VaR and CVaR measures – CVaR will be discussed in the next topic – for the use as a component of preference functional is proposed by Street (2010). The measure is calculated for the revenue context and assumes a probability space (Ω, Γ, P). The revenues are stochastic and defined as Γ-measurable functions $R\colon \Omega \to \mathrm{Q}$ that map elements from Ω (all possible states) to Q (set of possible revenue outcomes), where $\mathrm{Q} \subset \Re$. The cumulative probability function of a random revenue R is shown in equation (22):

$$F_R(r) = P\{\omega \in \Omega \mid R(\omega) \leq r\} \quad \forall r \in Q \tag{22}$$

where r is the revenue of the asset or portfolio.

The VaR, that denotes the quantile function, for the revenue context can be mathematically defined as (equation (23)):

$$VaR_\alpha(R) = F_R^{-1}(1 - \alpha) \tag{23}$$
$$= inf\{r \in Q \mid F_R(r) \geq (1 - \alpha)\}$$

Despite being a very simple measure to calculate, when the distribution of possible outcomes is known, VaR has some problems, such as not being a coherent risk measure because it does not fulfill the axiom of subadditivity (Acerbi & Tasche, 2002) and it does not provide information about the loss when it occurs. Jorion (1996) points out that VaR is a necessary but not a sufficient measure, which means that it is recommended to use VaR along with other risk measures and risk management procedures to achieve ideal results when managing risky investments or portfolios.

2.3.2 Conditional Value-at-Risk

One of the main problems of VaR as a measure of risk is the lack of desirable properties, such as differentiability and convexity, making it difficult to implement in mathematical programming models, and not present the subadditivity property, so the VaR of a combination of random variables may be greater than the sum of the VaR of each of them, which makes the VaR a non-coherent risk measure (ROCHA, 2013). For coherent risk measure that presents subadditivity, portfolio diversification should lead to risk reduction while, for measures which violate subadditivity, diversification can produce an increase in the measure of risk even when partial risks are triggered by mutually exclusive events (Acerbi et al., 2001).

Another problem of VaR measure is that it does not provide information about the magnitude of loss, such as its expected value once it occurs beyond the cut off value. Thus, distributions with quite different shapes may have the same VaR value. One investment could have high depth events while the other not. The former could lead the investor or company to bankruptcy, while the latter would not, however, would have the same measure of associated risk, which could lead to an erroneous assessment of risks, when comparing the two investments.

The Conditional Value-at-Risk (CVaR), also known as expected shortfall (ES) (Acerbi & Tasche, 2002) and tail conditional expectation (Artzner et al., 1999), is a measure that indicates the amount of tail risk and provides the information of the weighted average of the losses in the tail of the revenue distribution, beyond the Value-at-Risk (VaR) critical value, for a specific period. CVaR can also be written as a particular case of Choquet Expected Utility defined as the negative Choquet ϱ_{v_α} (equation (24)) expected return (Bassett Jr et al., 2004).

$$\varrho_{v_\alpha}(X) = -\int_0^1 F^{-1}(t)dv(t) = -\alpha^{-1}\int_0^\alpha F^{-1}(t)dt \tag{24}$$

where $v(t) = min\left\{\frac{t}{\alpha}, 1\right\}$.

The risk measure CVaR has become an well-known measure in the last two decades, after the development of an approach for portfolio optimization for financial instruments proposed by Rockafellar and Uryasev (2000), whose objective is the minimization of CVaR, through a technique that calculates VaR and optimizes the CVaR simultaneously.

The CVaR is a coherent risk measure (Artzner et al., 1999) and tends to be more pessimist than VaR as CVaR incorporates even the most catastrophic, high depth, events into the calculation. CVaR can be determined, for the revenue context, as shown in equation (25) (Street, 2010) and represents the conditional expectation of portfolio losses greater than VaR measure.

$$CVaR_\alpha(R) = E[R|R \leq VaR_\alpha] \tag{25}$$

where $E[.|.]$ is the expectation operator conditioned to a given set. The operator is determined as shown in equation (26), for a set $A \subseteq Q$.

$$E[R|R \in A] = \int r \frac{dF_R(r)}{P\{\omega \in \Omega\,|R(\omega) \in A\}} \quad (26)$$

The Figure 2 shows an example of VaR and CVaR configuration for a smooth cumulative function and a given α quantile.

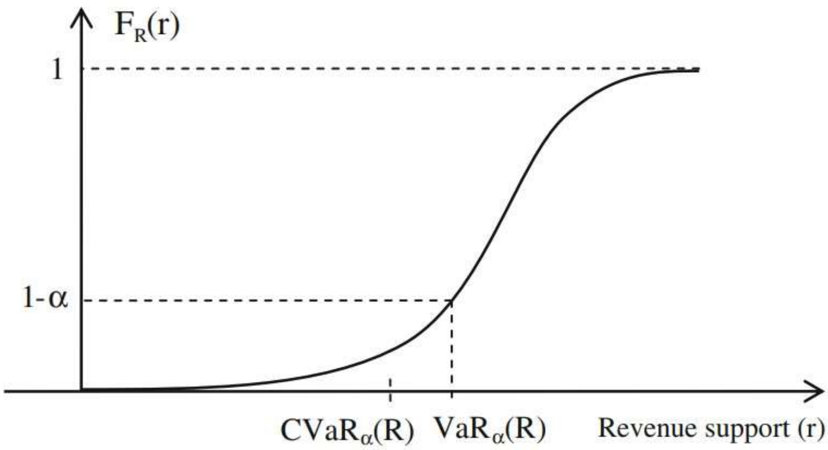

Figure 2 – Example of VaR and CVaR for a smooth revenue cumulative probability function

Source: Street (2010)

The Figure 3 shows two distributions A and B with quite different shapes, and which have the same VaR risk measure. Using the VaR measure alone, we could conclude that these are two investments with the same risk. However, the B distribution has a longer left tail, with a greater presence of high-depth, catastrophic events. The CVaR manages to capture this difference, indicating a greater risk for distribution B, compared to A.

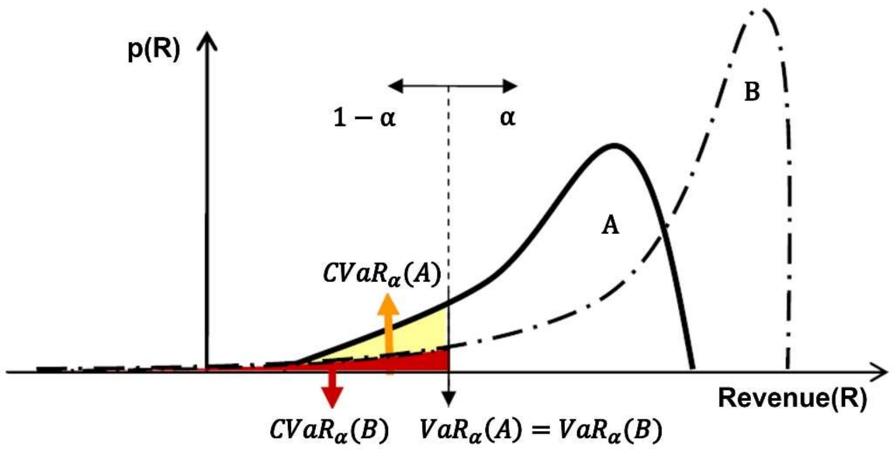

Figure 3 – Example of two distributions with the same VaR metric and different CVaR metrics

Source: Street (2008)

2.3.3
Coherent measures of risk

The coherent measures of risk have desirable characteristics for risk measurement, represented by axioms. Artzner et al. (1999) describe the axioms that represents a coherent risk measure. A risk measure satisfying the axioms of translation invariance, subadditivity, positive homogeneity, and monotonicity is coherent.

Let's consider Ω the set of outcomes of an experiment and compute the final net worth of a position for each of its elements, a random variable denoted by $X(w)$. The indicator function of state is w. Consider δ be the set of all risks, that is the set of all real-valued functions on Ω. A measure of risk $\rho(X)$ is a mapping from δ into ℝ.

The axion T is the translation invariance, shown in equation (27), which means that adding the sure initial amount α to the initial position and investing it in the reference instrument simply decreases the risk measure by α.

$$For\ all\ X\ \in\ \delta,\ \rho(X+\alpha) \leq \rho(X) - \alpha \qquad (27)$$

The axiom S is the subadditivity, shown in equation (28). Subadditivity is a property that indicates that the total risk measure of a set of assets is less than or equal to the risk measure of the individual sum of assets in the portfolio, so that no additional risk is created when mixing investments.

$$For\ all\ X_1\ and\ X_2\ \in\ \delta, \qquad \rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \tag{28}$$

The axiom PH is the positive homogeneity, shown in equation (29), implies that one increases the size of each portfolio position the portfolio risk increases in equal proportion.

$$For\ \lambda \geq 0\ and\ all\ X \in\ \delta, \qquad \rho(\lambda X) \leq \lambda\rho(X) \tag{29}$$

The axiom M is the monotonicity, shown in equation (30). The interpretation of this axiom is that if the gains in portfolio X are smaller than those in portfolio Y for all possible scenarios, then the risk in portfolio X is greater than in portfolio Y.

$$For\ all\ X\ and\ Y \in\ \delta\ with\ X \leq Y\ we\ have\ \rho(X) \geq \rho(Y) \tag{30}$$

2.4
Decisions under uncertainty using CVaR and expected utility

Most works involving expected utility theory and the CVaR coherent risk measure at a certain point were divided into two strands that aimed to build decision models under uncertainty. While a group of finance researchers developed towards the evolution of controlling the risk of the positions taken, the other group focused on the development of economics works through the theory of utility. None of the purposes of the research groups is more or less relevant and they are connected by a common objective, which is to characterize the agent risk profile of so that he is able to make decisions under uncertainty (Street, 2008).

Street (2008) proposes a methodology to integrate the two approaches, through the creation of a piecewise linear function that derives from a set of risk constraints, which aims to control the CVaR for different significance levels. The strategy is to formulate a problem of expected net revenue maximization, subject to M CVaR constraints (equation (31)).

$$CVaR_{\alpha_i}(R) \geq R_i^{min}, \qquad i = 1, \dots, M \tag{31}$$

where $R(x, \xi)$ is the stochastic net revenue for the contracted amount (x) and other uncertainty factors (ξ).

The maximization problem can be formulated as shown in the equation (32), equation (33), equation (34), equation (35) and equation (36).

$$Max_{(x,b,z)} \sum_s p_s \cdot R(x, \xi_s) \tag{32}$$

Subject to:

$$\beta_{i,s} \leq 0 \;\; for\ all\ i = 1, \dots, M\ and\ s = 1, \dots, S \tag{33}$$

$$\beta_{i,s} \leq (1 - \alpha_i)^{-1} \cdot [R(x, \xi_s) - z_i] \;\; for\ all\ i = 1, \dots, M\ and\ s = 1, \dots, S \tag{34}$$

$$z_i + \sum_s p_s \cdot \beta_{i,s} \; \geq \; R_i^{min}\ for\ all\ i = 1, \dots, M \tag{35}$$

$$G \cdot x \leq d \tag{36}$$

where $\alpha_i \geq \alpha_{i+1}, R_i^{min} \leq R_{i+1}^{min}$ for all $i = 1, \dots, M$.

Inserting in the objective function a penalty λ , the new objective function is (equation (37)):

$$F_{obj} = \sum_s p_s \cdot R(x, \xi_s) + \sum_i \lambda_i \cdot \left(z_i + \sum_s p_s \cdot \beta_{i,s} \; - \; R_i^{min} \right) \tag{37}$$

$$= \sum_s p_s \cdot \left[R(x,\xi_s) + \sum_i \lambda_i \cdot \beta_{i,s} \right] + \sum_i \lambda_i \cdot \left(z_i - R_i^{min} \right)$$

The objective function is a piecewise function of net revenue function $R(x, \xi_s)$ and it has $M + 1$ segments. The coefficients (a_k, b_k), of piecewise utility linear function implicit to the net revenue maximization problem is expressed in the equation (38) and equation (39). The utility function for a given net revenue value r is shown in equation (40):

$$a_k = 1 + \sum_{i:k \le i \le M} \lambda_i \cdot (1 - \alpha_i)^{-1}, for\ all\ k = 1, \dots, M + 1 \tag{38}$$

$$b_k = -\sum_{i:k \le i \le M} \lambda_i \cdot (1 - \alpha_i)^{-1} \cdot z_i^* \ , for\ all\ k = 1, \dots, M + 1 \tag{39}$$

$$U(r) = r \cdot \left(1 + \sum_{i:k \le i \le M} \lambda_i \cdot (1 - \alpha_i)^{-1} \right) - \sum_{i:k \le i \le M} \lambda_i \cdot (1 - \alpha_i)^{-1} \cdot z_i^* \tag{40}$$

where $z_i^* = VaR_{\alpha_i}[R(x, \xi_s)],\ i = 1, \dots, M.$

The piecewise linear utility function, implicit to the maximization problem is shown in Figure 4. This function is a classical Neumann-Morgenstern function (vN-M) and does not depend on the evaluated stochastic variable (Street, 2008).

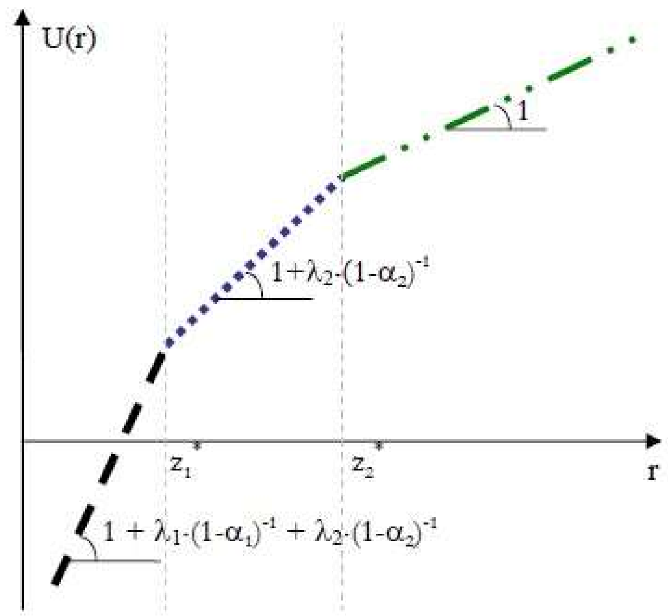

Figure 4 – Piecewise linear utility function implicit to the maximization problem
Source: Street (2008)

2.4.1 Extended CVaR Preference (ECP) Functional

The Extended CVaR Preference (ECP) Functional was propose by Street (2010) and has become largely adopted in Brazilian electricity market for a variety of problems, mainly dual stochastic optimization problems and currently is the objective function that is used by the national system operator in the optimization of the hydrothermal dispatch. The inclusion of this functional as an objective function in the national system occurred in August 2013.

The intuition behind the functional is to weigh the expected gains with the average losses above the critical point, arriving at an optimal solution so that the functional can simultaneously optimize the revenue, represented by the expected value of the net revenue, and manage the risk involved in investment, represented by CVaR metric. For two lotteries with the same CVaR metric for α and different expected revenue values (unconditioned expectation), the agent would choose the one which provides the greatest expected value, as the risk metric is the same for both (Street, 2010). On the other hand, if two lotteries have the same expected values, the investor would choose the one that has the lower risk of losses beyond the critical value.

Extended CVaR Preference (ECP) is defined as (equation (41)):

$$\Phi_{\alpha,\lambda}(R) = \lambda \cdot CVaR_{\alpha} + (1-\lambda) \cdot E[R];\ \lambda \in [0,1] \quad (41)$$

The ECP probability-dependent expected utility form is shown in equation (42):

$$\Phi_{\alpha,\lambda}(R) = \lambda \cdot E\{z^*(\alpha,R) - [R - z^*(\alpha,R)]|^{-}/(1-\alpha)\} + (1-\lambda) \cdot E[R] \quad (42)$$

The underlying utility function for the functional and probability-dependent expected utility form of the ECP functional are shown in equation (43) and equation (44), respectively:

$$\mathrm{U}_{\alpha,\lambda}(r, F_R) = \lambda \cdot E\left\{{F_R}^{-1}(1-\alpha) - \left[r - {F_R}^{-1}(1-\alpha)\right]\right|^{-}/(1-\alpha)\right\} + (1-\lambda) \cdot r \quad (43)$$

$$\Phi_{\alpha,\lambda}(R) = E\left\{\mathrm{U}_{\alpha,\lambda}\,[R, F_R(.)]\right\} \quad (44)$$

The Figure 5 provides a graphical visualization of the utility function (equation 43) for a fixed revenue distribution $F_R(.)$. There are two separated segments $r \leq {F_R}^{-1}(1-\alpha)$ and $r > {F_R}^{-1}(1-\alpha)$. Note that r is continuous on $r = {F_R}^{-1}(1-\alpha)$.

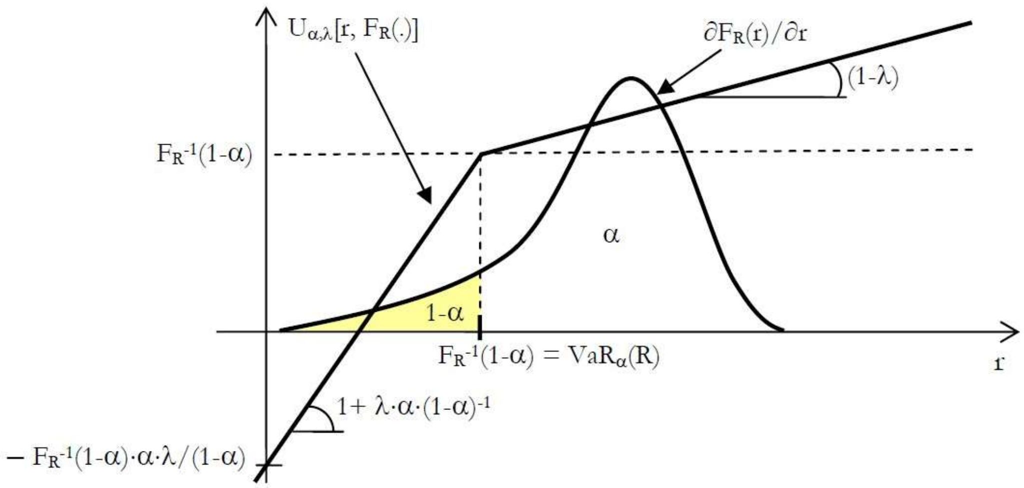

Figure 5 – Example of local piecewise linear utility function

Source: Street (2010)

2.4.2 Generalized Extended CVaR Preference (ECP_G) Functional

The generalization of the ECP functional (equation (45)), described in the previous topic, was proposed by Luz (2016), for a period and N reference points and its denomination is ECP_G. The motivation behind this generalization is rooted in the idea that investors may exhibit varying degrees of risk aversion across distinct risk ranges, each characterized by the CVaR metric.

$$ECP_{G_{\alpha,\lambda}} = E[u(\tilde{x})] = \lambda_0 E(\tilde{x}) + \sum_{n=1}^{N} \lambda_n CVaR_{\alpha_n}; \quad (45)$$

$$\lambda_n \geq 0 \; and \; \sum_{n=1}^{N} \lambda_n = 1$$

The ECP_G functional is computed from the expected value of the utility function shown in equation 46, and is capable to capture N risk levels through $N + 1$ linear utility functions that cover all values in $\mathbb{R}$.

$$\boldsymbol{Utility\ function\ U}: \mathbb{R} \rightarrow \mathbb{R}$$

$$U(x)\begin{cases} \lambda_0 x + \sum_{i=1}^{N} \lambda_i VaR_{\alpha_i} + \sum_{i=1}^{n} \frac{\lambda_i}{(1-\alpha_i)}\left(x - VaR_{\alpha_i}\right);\ (a) \\ \lambda_0 x + \sum_{i=1}^{N} \lambda_i VaR_{\alpha_i};\ (b) \end{cases} \tag{46}$$

$$(a): x\ \in\ \left]VaR_{\alpha_{n+1}}; VaR_{\alpha_n}\right],\ x\ \in [1; N] \subset \mathbb{N}, VaR_{\alpha_{N+1}} \rightarrow\ -\infty$$
$$(b): x\ \in\ ]VaR_1; VaR_0], VaR_{\alpha_0} \rightarrow\ +\infty$$

Where $\sum_i \lambda_i = 1\ , \lambda_i \geq 0, x\ \in\ [0, N], \alpha_i \subset\ ]0,1[\ , \alpha_{N+1} = 1, \alpha_0 = 0.$

The Figure 6 shows an example of the utility function from equation 46 and $N = 2$. The intersection of the lines occurs at the points where $x_i = VaR_{\alpha_i}$. Each linear function of the piecewise function is defined by λ_0 for $x\ \in\ ]VaR_1; VaR_0]$ and $\lambda_0 + \sum_{i=1}^{n} \frac{\lambda_i}{(1-\alpha_i)}$ for $x\ \in \left]VaR_{\alpha_{n+1}}; VaR_{\alpha_n}\right]$:

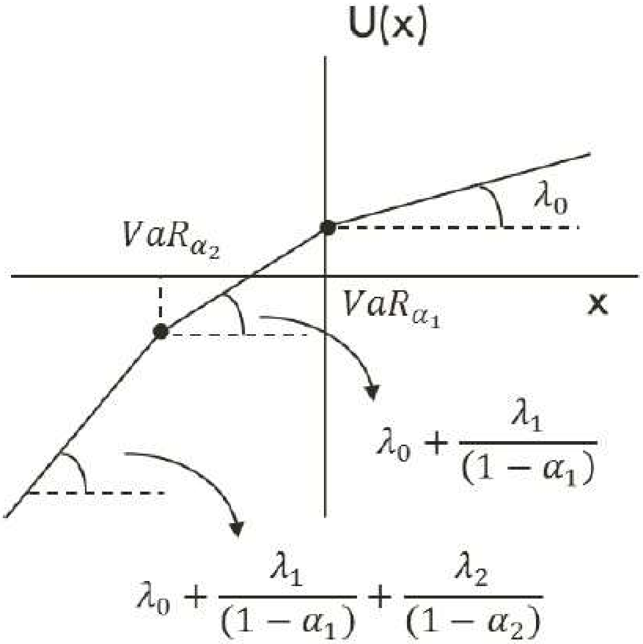

Figure 6 – Example of underlying utility function of ECP_G functional for N=2

Source: Luz (2016)

The underlying utility function of ECP_G (equation (46)) is not a classical Von Neumann-Morgenstern utility function since it depends on the revenue distribution (Luz, 2016). It is continuous in $\mathbb{R}$ and as consequence it is possible to integrate the cumulative probability function $F(x)$ through the probability space.

The investor's risk profile is reflected in the utility function through the lambdas λ_i risk aversion parameters that change the slope of each of the $N+1$ segments that represent the function.

The functional ECP_G is obtained from the computation of expected utility and is a coherent risk measure as it is a convex combination of $CVaR_\alpha$, where $\sum_{.} \lambda_i = 1$. Oliveira (2009) demonstrates that for a family of coherent risk measures $\{\rho_\tau\}$ $\tau \in$ T, any convex combination ρ is also a coherent measure of risk (equation (47)).

$$\rho = \sum_{\tau \in \mathrm{T}} \lambda_\tau \rho_\tau \qquad (47)$$

$$\sum_{\tau \in \mathrm{T}} \lambda_\tau = 1 \; and \; \lambda_\tau \geq 0$$

The certainly equivalent and the risk premium of lotteries can be obtained from the utility function from equation 46, and are shown in equations (48) and (49), respectively.

$$Eq = U^{-1}(E[U(x)]) = U^{-1}\left(\lambda_0 E[\tilde{x}] + \sum_{n=1}^{N} \lambda_n CVaR_{\alpha_n}\right) \qquad (48)$$

$$Rp = E[\tilde{x}] - Eq \qquad (49)$$

The risk premium demonstrates that the function presents global risk aversion although locally it is composed of N linear functions and each linear function segment is risk neutral. So, it is not possible to calculate the Arrow-Pratt classical risk aversion (Arrow, 1965; Pratt, 1978) measure ($\gamma = U''(x)/U'(x)$) as the utility function is not differentiable for the full set of possible outcomes, and it is not differentiable at the points where the function's slope changes. Thus, the aversion occurs from one segment to another and can be computed as shown in equation (50) (Luz, 2016)

$$\gamma_n = -\frac{(s_n - s_{n+1})}{s_{n+1}} \tag{50}$$

$$s_0 = \lambda_0 \text{ and}$$

$$s_n = \lambda_0 + \sum_{i=1}^{n} \frac{\lambda_i}{(1-\alpha_i)}$$

$$\gamma_n = \frac{\lambda_{n+1}}{1-\alpha_{n+1}} \times \left(\lambda_0 + \sum_{i=1}^{n+1} \frac{\lambda_i}{(1-\alpha_i)}\right)^{-1}$$

where s_n is the slope each of N segments of function and λ_0 is the slope of $s_0$segment.

2.4.3 Parameter estimation: objective and subjective approaches

The determination of the utility function weight parameters is among the most important steps in risk management since the allocation decisions are compromised when the risk parameters do not correctly reflect the investor preferences. Techniques for estimating the function parameters have been widely discussed in the literature and are divided manly in two groups: subjective and objective approaches. Subjective approaches use opinion from experts and decision makers on the criteria weight to determine the parameters while objective approaches uses actual data from the decision matrix and disregard the opinion of expert and decision makers (Žižović et al., 2020).

Keeney and Raiffa (Keeney & Raiffa, 1976) describe a subjective approach to estimate the parameter of the utility function using 50-50 lotteries that is very intuitive and practical. The decision maker just needs to choose between two equally likely consequences each time. The process is recursive and through it the certainty equivalent and the parameters of the functions are estimated. The method that uses the certainty equivalent compared to lotteries has many variants, some of them use probabilities that differ from 50% for each outcome, keeping the sum of probabilities equal to 100%.

The Simple Multi-Attribute Rating Technique (SMART) (Edwards, 1977) and other improved variants as SMARTS and SMARTER (Edwards & Barron, 1994) are another group of techniques widely explored in the literature that use the subjective approach to estimate the function parameters. SMART is a linear additive model and a multi-criteria decision method. It consists of a set of criteria and weights that permit comparison, allowing the decision maker to assess the alternatives and take the optimum decisions (Siregar et al., 2017). The equation (51) shows the multilinear model function model of SMART:

$$Maximize \sum_{j=1}^{k} w_j \cdot u_{ij} \tag{51}$$

where w_j is the weighted value j of k criterion and u_{ij} is the alternative utility value i on criterion j.

The pairwise comparison methods are another type of subjective approach based on pairwise comparison, exclusively. One of the first developments of this method that was observed in the literature occurred in 1927, by Thurstone (Thurstone, 1994). The idea behind the method is to let the decision maker, who can be a person or a team, establish relative importance between two alternatives that are presented to them at a time, so that, in the end, it is possible to establish the relative significance of all alternatives.

One of the most popular pairwise comparison methods used is the Analytical Hierarchy Processes (AHP), proposed by Saaty (T. L. Saaty, 1980) widely applied in multicriteria decision making. The AHP is a nonlinear framework that take several factors into consideration, which carries both deductive and inductive thinking, and can make numerical tradeoffs to support the conclusions. It is method that allows the establishment of measures for social or physical domains (R. W. Saaty, 1987). The AHP model has special concern with consistency and measurements were developed and tested to estimate the level of consistency obtained.

The AHP method can also be applied to discrete and continuous cases and dominances matrices are applied to the former and kernels of Fredholm operators to the latter (T. L. Saaty & Vargas, 1987). The main idea of the method is to use pairwise comparison to determine the relative weights of each criterion using a nine-level scale. Them, for each alternative, it is calculated a value that reflects the weight of each criterion and the specific values assigned to the alternative (Table 2).

Intensity of importance	Definition
1	Equal importance
2	Weak or slight
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very strong or demonstrated Importance
8	Very, very strong
9	Extreme importance
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i
Rationals	Ratios arising from the scale

Table 2 – AHP fundamental scale of absolute numbers

Source: Saaty (2004)

The decision to utilize subjective methods often stems from the prevalent lack of readily available observable data in numerous scenarios, or the inherent challenges associated with acquiring and processing such data. In these scenarios, the utilization of subjective models, which rely on the expertise of decision-makers, can prove to be highly appropriate. Conversely, objective approaches rely solely on data to estimate parameters, obviating the need for input from experts and decision-makers. In such cases, criteria weights are derived exclusively from the criteria values associated with alternatives. Objective approaches prioritize the analysis of the decision matrix, which portrays the values of the considered alternatives in relation to a defined set of criteria. (Žižović et al., 2020).

One of the main methods that uses objective approach to support decision making is the Criteria Importance Through Intercriteria Correlation (CRITIC) method. CRITIC is a method for the determination of objective weights of relative importance in Multi Criteria Decision Making (MCDM) problems and is based on analytical investigation of the evaluation matrix to extract the information of the evaluation criteria (Diakoulaki et al., 1995). The multicriteria problem can be written as shown in equation (52):

$$Max\{f_1(a), f_2(a), \dots, f_m(a) | a \in A\} \quad (52)$$

where A is a set of finite alternatives and m is a system of evaluation criteria f_j.

The membership function x_j maps the values of f_j to the interval $[0,1]$. The value x_{aj} (equation (53)) represents the degree to which the alternative a is close to the ideal value f_{jb} (best performance in criterion) and far from f_{jw} (worst performance in criterion). The initial matrix is, therefore, converted into a matrix of relative scores.

$$x_{aj} = \frac{f_j(a) - f_{jw}}{f_{jb} - f_{jw}} \quad (53)$$

For each criterion j , a vector x_j (equation (54)) is generated denoting the scores of all n alternatives. The standard deviation σ_j is also computed and represents the contrast intensity for the criterion and is a measure of the criterion value to the decision-making process.

$$x_j = (x_j(1), x_j(2), \dots, x_j(n)) \quad (54)$$

Then a $m \times m$ matrix is generated using the linear correlation between the vectors x_j and x_k for the value of r_{jk} element. The measure of conflict created by criterion j with respect to the rest of criteria is measure according to equation (55):

$$\sum_{k=1}^{m} (1 - r_{jk}) \quad (55)$$

The information of MCDM problem C_j (equation (56)) is related to contrast (σ_j) and conflict (equation (55)).

$$C_j = \sigma_j \cdot \sum_{k=1}^{m} (1 - r_{jk}) \quad (56)$$

The objective weighs w_j are obtained according to equation (57). The higher is C_j more information is transmitted by criterion j and higher is the importance of w_j for the decision-making process.

$$w_j = \frac{C_j}{\sum_{k=1}^{m} C_k} \quad (57)$$

There are other well-known objectives methods, as the entropy method (Shannon, 1948), FANMA (Ma et al., 1999) and CRITIC variations as CRITIC-M (Žižović et al., 2020) and all of them rely only on data to define the weight of the functions and do not need the opinion of decision makers or experts.

2.5
Parameter estimation: ECP and ECP_G functionals

The parameters estimation for the ECP (ECP_G for $N = 1$) functional is extremely important for the Brazilian electricity sector since the national integrated system uses the ECP functional to optimize several system processes. The settlement price of the differences is calculated from the optimization of the hydrothermal dispatch for the Brazilian electrical system, considering a horizon of up to 5 (five) years and the optimization objective function (equation (58)) is defined, for each stage, since 2013, as the sum of 75% of expected price and 25% of $CVaR_{50\%}$ (Luz, 2016).

$$ECP_{50\%,25\%} = (0.75)E[\tilde{x}] + (0.25)CVaR_{50\%} \quad (58)$$

The risk aversion parameters were defined through a public audience (ANEEL, 2013) in 2013, the process of defining the risk aversion parameters α and λ was subject to several criticisms, as non-active participation of sector agents since the beginning of this process. The process was carried out through the generation of scenarios for which the α and λ values were obtained by trial and error, since there was no established procedure in the literature for estimating the parameters of ECP functional underlying utility function.

The definition of parameters for the Brazilian electricity sector has an additional difficulty, as their implementation in the NEWAVE optimization software is made through a multistage problem, so for each stage there is a set of scenarios, an expected value $E[\tilde{x}]$ and a calculated $CVaR$.

Luz (2016) suggested the use of AHP method and its 9-point scale and showed an example of 2 levels of risk, one representing a moderate level (20%, acceptable) and the other an extreme level (5%, not acceptable). The moderate level could be represented by the expected result of the worst 20% scenarios and the extreme risk of the worst 5%.

The AHP method, despite having great potential decision-making problems under uncertainty, can be difficult to implement, especially considering the dynamics of the electricity sector, where the participation of several agents in the definition of risk parameters is required. There is also the possibility of generating outcomes with low consistency, which would require the revision of the assigned value, according to the 9-point scale that could make the process even more time consuming. There is, therefore, a challenge to develop a methodology to determine the parameters that is simple and easy for the agents to understand, as they need to participate in the process and validate the risk aversion parameters obtained.

First, we need to understand whether it would be adequate to use an objective or subjective method to determine the parameters. An objective method would be quite convenient as it would avoid any kind of bias and the need for consensus among the different decision makers who participate in the process, since the data themselves would indicate the ideal weights for the parameters.

The use of an objective method requires, however, the data to be large and varied so that it makes possible to investigate and to establish adequate correlations and extract the information of the evaluation criteria based on analytical investigation of the evaluation matrix. unfortunately, the existing data set today does not allow this approach. The data present until today represent the application of the same parameters for α and λ defined in since 2013, 50% and 25%, respectively, and there is not a variety of different decisions and results that allow the creation of an investigation matrix for objective determination of the criteria. The subjective alternative therefore becomes the recommended alternative.

Among the subjective approaches, a well-established technique for estimating the parameters of the utility function and, consequently, the ECP_G functional parameters, is the use of hypothetical lotteries through which the decision maker indicates the value to which he becomes indifferent between the lottery and an exact amount (certainly equivalent). This approach will be detailed further on, but before that, it is necessary to discuss the 3 types of estimation that are involved in the parameterization of the ECP_G, which require different strategies.

The parameters estimation is divided into three decisions (Luz, 2016):

- Determination of the number of levels N
- Determination of α_i's
- Estimation of λ_i's

2.5.1 Determination of the number of levels N and estimation of α's

The ECP_G functional and its piecewise underlying utility function can be interpreted as an approximation of the decision maker preferences. In studies involving the estimation of the utility function, usually, the assessed function is smooth (Keeney & Raiffa, 1976) and the change in slope along the curve is gradual. There are no non-differentiable points where, abruptly, there is a change in the slope of the linear function, that should be empirically observed, as occurs in a piecewise linear function between each segment. However, from a practical point of view, choosing a piecewise linear function to represent the investor's utility function is quite adequate. This approach gives a lot of flexibility in the shape of the utility function and, at the same time, from an optimization point of view, it also allows the adoption of simple and efficient maximization strategies, which is desirable from a computational perspective.

The greater the number of linear functions ($N + 1$), the better is the adherence to the investor's actual preferences, as the overall shape becomes closer to a smooth curve. On the other hand, an exceptionally large number of levels N implies a greater effort to capture the different degrees of risk aversion within each segment and it may become difficult for the decision maker and the analyst or researcher to capture very subtle changes from one segment to another.

Thus, there is a balance between the level of approximation of the decision maker utility function and the practicality and viability of assessing the segments of the utility function that needs to be achieved. Street (2008) suggests using at least 5 levels to define the agent's preferences.

The definition of α's, in the same way, also impacts on the assessment accuracy the underlying utility function. The choice of α's should be done to capture the segments in which there is little or no change in the decision maker's risk preferences i.e., segments where empirical data points converge towards a linear trend.

Many companies have formalized loss limits, such as unacceptable losses, undesirable losses, and acceptable losses, which may be, for example, referring to $VaR_{80\%}$, $VaR_{90\%}$ and $VaR_{85\%}$, respectively and these limits can be used as initial reference values for the α's.

Keeney and Raiffa (1976) point out that choosing an utility function is, somewhat, of a heuristic search process, and there is no clear cut procedures for solving the problem. The goal is to find a utility function which satisfies almost all the constrains and is not grossly incompatible with the others. This process can, therefore, lead to different outcomes for the utility function, but all equally appropriate for the decision maker to operate. This occurs mainly due to the subjectivity involved in the process. A suggested approach for defining the number of levels N and α's, for a single agent, is shown as follows:

- Use the 5 levels suggested by Street (2008) as a starting point.
- If there are formalized milestones for losses use these milestones even if there are more than 5.
- If there are less than 5 loss milestones, use those milestones plus the amount to achieve 5 levels, filling in the largest remaining segments evenly spaced.

- If there are no loss milestones, use 5 levels as starting points. As extreme losses are particularly important to be controlled, it is suggested to use more α levels close to extreme losses such as α levels 80% 90% and 95%. For 5 levels, possible α values to be used are: 40%, 65%, 80%, 90% and 95%. The value of 99% is not suggested, as well as no value with a scale below 5%, as some studies, such as Spetzler (1968), found that decision makers cannot differentiate values lower than 5%.
- To check if the number of levels N and α's was chosen correctly, a 50-50 lottery can be used, using α's correspondent revenue as reference points and the certainty equivalent. For each segment, if the decision makers indicate the certainty equivalent as being the average of segment revenue value for each segment, then there is no need to change the number of levels or α's.
- If the decision makers indicate the certainty equivalent to be a different value from segment average revenue value, then it is necessary to change the number of levels and α's. The change can be made by dividing the analyzed segment into two segments of the same size, which would increase the number of levels by one and add one more α value to the model.
- The procedure is repeated until all certain equivalents chosen by decision makers are the mean of the revenue of the evaluated segment.

As, for each segment, the utility function is a linear function ($u(x) \sim x$), so within each segment, there is proportional (constant) risk aversion as shown in equation (19), the 50-50 lottery and the certainty equivalent comparison strategy can be used to support the definition of the number of levels and α's as described above.

The certainty equivalent obtained from the assessment in some cases will diverge just a bit from the computed value, which is the average of the segment. A significance or approximation margin can be established for which the empirical value obtained is considered as the computed value, even if they are not exactly the same. For example, a 5% margin can be considered, relative to the distance from the extremes of the segment to the midpoint or $\pm 2.5\%$ considering $VaR_{\%}$ scale and, if the empirical value of the certainty equivalent falls within this margin, the risk neutrality condition is considered to be fulfilled, within the segment.

The approach described above applies only to the assessment of utility function for a single agent. For the definition of the number of levels N and α's for the national system, the previously suggested approach would be not recommended and the definition of the number of levels and α's does not necessarily need to be directly obtained through the interviews with the agents. The determination of the parameters of number of levels N and α's using interviews would require the agents to respond interviews in a two-stage process. First stage for the definition of the number of levels N and α's and second stage for the definition of λ's. The stages should occur separately as it would be necessary to consolidate the data and determine the parameters of number of levels and α's before starting the interviews to assess the risk aversion parameters λ's because the number of levels and α's must be the same for all agents.

Implementing this two-step process may incur significant costs and consume valuable time, with relatively little benefit in terms of obtaining the utility function. This is because the crucial risk aversion parameters, which hold the utmost importance, are exclusively defined by the λ's. The number of levels N and the α's parameters are related to the level of approximation of a smooth utility curve and to the risk milestones (e.g., 95%) that are specific to each agent and do not make sense for a large number of agents, as occurs on national system.

Thus, an adequate strategy for a system with many agents is to previously establish the number of levels and α values and, later, access the λ's values, through the interviews, in a single stage process. For example, it can be used $N = 5$ and the following α's: 30%, 60%, 80%, 90% and 95% or 40%, 65%, 80%, 90% and 95%.

2.5.2
Estimation of utility function risk aversion parameters λ's

Several works were developed to assess the utility function, among them the pioneering and seminal works of Meyer and Pratt (1968), Swalm (1966) and Spetzler (1968). Significant research related to the assessment of utility functions continues to this day, not only within the financial and economic sectors but also across various other fields. For instance, Feeny et al. (2002) describe the estimation of the Health Utilities Index Mark 3 (HUI3) scoring system. This system comprises eight single-attribute utility functions and one multiplicative multi-attribute utility function. HUI3 is grounded in the von Neumann-Morgenstern (vN-M) expected utility theory, with extensions to accommodate multi-attribute functions.

Another study in the health sector involving the assessment of utility functions was conducted by van Dam et al. (2020) . This study evaluates utility functions to provide clinicians with information for decision-making regarding the use of opioid analgesics for pain relief. Furthermore, utility functions find application in the development of new opioids and the selection of opioids for treatment, as opioids with positive utility functions are favored over those with negative utility functions.

Numerous studies also exist within the field of management. Harrison et al. (2010), for instance, explores factors that facilitate acquisition of knowledge about stakeholder utility functions and analyze how firms that prioritize stakeholders management can competitively benefit from this guidance over long term, sustainably.

The strategies for assessing utility functions are quite diverse, and there is no one-size-fits-all approach due to significant variations in evaluated conditions and data structures from one case to another. One of the most intuitive and straightforward methods to assess utility functions involves lotteries and certainty equivalents. The decision-maker expresses their preference for the outcome of a given lottery, typically with a 50-50 percent probability, or its certainty equivalent, allowing to map one utility function empirical point. Repeating the process for other lotteries, it is possible to map a set of empirical points that will be later used to fit the utility function.

The methodology usually employed for the empirical assessment of points along the utility curve requires the utilization of a specific questioning approach and is described detailed in Keeney and Raiffa (1976). This approach involved the selection of choices between 50-50 lotteries featuring two potential outcomes, and a certain outcome. Subsequently, the value associated with the certain outcome is systematically adjusted through successive inquiries until the decision maker reach a state of indifference between the certain outcome and the lottery (i.e., the determination of the lottery's certainty equivalent). The utility of the consequences of the lottery is set arbitrarily.

The utility of certainty equivalent is found easily as the utility assigned to the certainty equivalent must be equal to expected utility of the lottery. This method leads to an empirical point of the utility function. Then, the certainty equivalent can be used to evaluate the utilities of other consequences. Through this iterative process, multiple data points along the utility function can be estimated. Finally, a coherent and continuous curve is fitted to the empirical data to provide a smooth representation of the utility function. Following there is an example of a questioning quoted from Swalm (Swalm, 1966):

> Suppose your company is being sued for patent infringement. Your lawyer's best judgment is that your chances of winning the suit are 50-50; if you win, you will lose nothing, but if you lose, it will cost the company $1,000,000. Your opponent has offered to settle out of court for $200,000. Would you fight or settle?

The Figure 7 shows a didactic example of utility function fit for a risk neutral investor using 7 empirical points. Equation (59) shows the utility function fitted using the least squares method:

$$u(x) = 2.36x + 10 \tag{59}$$

$$R^2 = 0.96$$

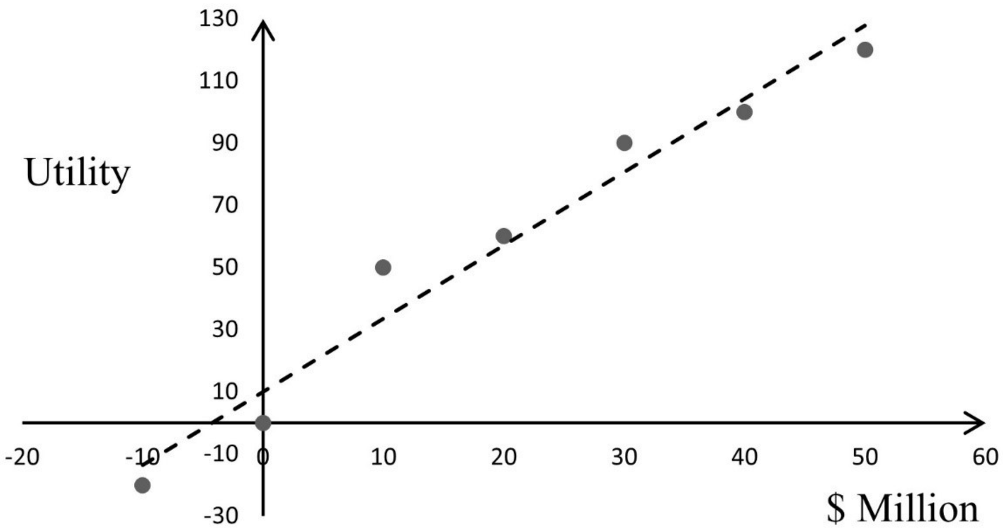

Figure 7 – Example of utility function fitted to empirical data

A variant of this approach involves the utilization of lotteries in which the probabilities of success and failure are subject to variation. The decision maker is presented with a choice between an investment with a success probability of p and a failure probability of $1 - p$, accompanied by consequences x_s in the event of success, and x_f in the event of failure. The decision maker is then required to determine whether to proceed with the investment or not. The probability p is successively adjusted until the decision maker reaches a probability of indifference p_0 regarding the decision to engage in the investment or abstain from it. The utility for p_0 is given by equation (60) (Keeney & Raiffa, 1976):

$$u(0) = p_0 u(x_s) + (1 - p_0) u(x_f) \tag{60}$$

Repeating the procedure for a range of lotteries or investment opportunities, it is possible to empirically record points of utility function and then fit a smooth curve to represent the decision maker's utility function.

Respondents may encounter challenges, particularly when dealing with probabilities deviating from the 50% mark. Understanding and quantitatively interpreting probabilities, especially those at values like 20% or 30%, can pose difficulties for decision-makers. Clarifying what these probabilities truly signify may present a hurdle in the decision-making process. Thus, if the questioning strategy involves probabilities that differ from 50%, the use of graphic tools, such as the pie chart, may be appropriate to help the respondent understand these probabilities through graphic visualization.

An example of how the certainty equivalent strategy can be applied is elaborated by Spetzler (1968) in which and individual stated that he is indifferent between the following alternatives A and B (Table 3).

	Alternative A		**Alternative B**
	Probability	**Outcome**	**Outcome**
1	0.5	$ 30 million	A sure $ 4 million
	0.5	0	
2	0.8	$ 30 million	A sure $ 10 million
	0.2	0	
2	0.7	$ 30 million	A sure $ 4 million
	0.3	$ -2 million	

Table 3 – Certainty equivalent strategy to assess the utility function
Source: Spetzler (1968)

As the decision maker is indifferent between A and B alternatives, the utility of certainty equivalent (alternative B) is equal to the expected value of the utility of the lottery (alternative A), as shown in equation (61), equation (62) and equation (63):

$$U(\$\ 4\ million) = 0.5U(\$\ 30\ million) + 0.5U(\$\ 0) \tag{61}$$

$$U(\$\ 10\ million) = 0.8U(\$\ 30\ million) + 0.2U(\$\ 0) \tag{62}$$

$$U(\$\ 4\ million) = 0.7U(\$\ 30\ million) + 0.2U(\$ - 2\ million) \tag{63}$$

Two values on the scale can be arbitrary chosen, for example $U(\$\ 30\ million) = 100$ and $U(\$\ 0) = 0$. Then it is possible to calculate the utility values for $\$\ 4\ million$, $\$\ 10\ million$ and $\$ - 2\ million$: $50,\ 70$ and $-66\ 2/3$, respectively. These points are empirical points and will be used to build a plot and proper fit a utility function that represents the decision maker's preferences.

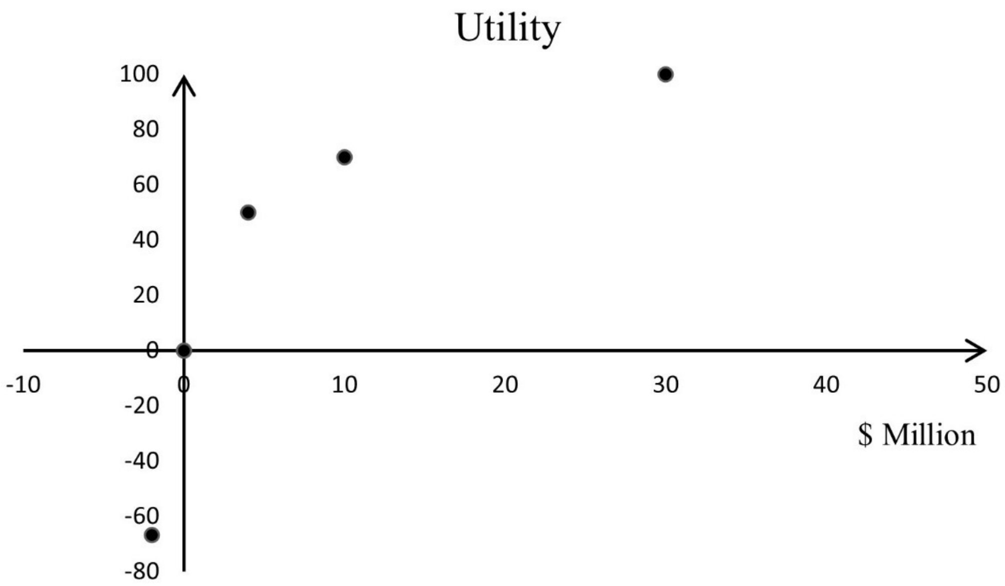

Figure 8 – Example of utility function assessment
Source: (Spetzler, 1968)

The plot (Figure 8) can be used to build a utility model, fit the utility function and to check if the points obtained are consistent with expected results from the expected utility model. The pattern of points describes the aversion risk profile from the decision maker. For example, a straight line would represent a risk neutral decision maker. The more concave is the pattern, the more the decision maker is risk averse. Sometimes the results from the plot will show inconsistencies with the utility model and one of the assumptions of the model is that the plot will predict future behavior, so, if it is identified any inconsistent behavior by analyzing the plot, it is expected that the inconsistent behavior will persist.

Another variant of this approach uses a mathematical form of the utility function prior to plotting the graph which will necessarily be used to fit the parameters based on the empirical data resulting from the interviews. The main disadvantage of this approach is that it assumes a previous pattern of behavior by the decision maker reflected in the form of the utility function, which may not be true. The chosen function needs to have a high level of shape flexibility, in order to accommodate a range of different types of response and behavior patterns that can be obtained (Spetzler, 1968).

One of the biggest challenges when assessing a utility function using questions about the certainty equivalent and alternatives with associated probabilities is the construction of adequate questions. The questions must be created so that the responses would be the same to an actual decision. Another challenge is to ensure that the decision maker's responses reflect the preferences of the organization and not the individual decision maker, so the decision maker must respond according to what he believes he should decide, considering the organizational objectives and not his own objectives.

There are also some alternative approaches as hybrid assessment methods and paired-gamble methods that are robust against different forms of bias (Farquhar, 1984). These methods can be an appropriate alternative when biases are identified. Bias emerges when individuals exhibit a tendency to disproportionately favor certain outcomes over those linked with probabilities. Additionally, biases are apparent in the assessment of ventures with risk. Notably, outcomes with lower associated probabilities, e.g., 20%, are often overestimated, whereas higher probabilities, like 80%, tend to be underestimated. (Tversky, 1967).

Unlike number of levels N and the α values, which are more directly related to the level of approximation or granularity of the utility function, λ's are directly related to the level of risk aversion in each segment and, consequently, to the shape of the utility function. It is therefore the most important parameter to be estimated between the 3 types (number of levels N, α's and λ's).

For the number of levels N and α values, even without knowing the decision maker's preferences, it would be possible, simply by increasing the number of levels and choosing α's equally spaced, to improve the adherence of the utility function. It is possible to start with generic values, such as the 5 proposed levels as mentioned on previous topic, and it will be a good starting point for modeling the utility function.

On the other hand, it is not possible to do the same for λ's as they are related to levels of risk aversion, it would not be appropriate to establish arbitrary initial values and these values need to be determined directly, asking a set of questions to the decision makers. Although there is no prior information about the λ's, it is important that the function obtained is globally risk averse and that the λ's obtained are all positive. These assumptions are part of the ECP and ECP_G formulation and need to be satisfied.

For the estimation of λ's for the ECP_G functional and its underlying utility function, it is necessary to consider that there is already a predetermined mathematical form of the utility function (equation (46)) and that it does not depend on any type of prior evaluation of risk preferences of the decision makers. The mathematical form was constructed in a way that allows a large flexibility of shape of the utility function.

There are some significant differences between the present work and those that are usually carried out to assess the utility function of decision makers, which need to be considered before we suggest the approach to estimate the risk preferences. Usually, research tries to develop a prescriptive utility function that has the following characteristics (Spetzler, 1968):

1. Continuous and twice differentiable over the range of x.
2. Lead to a risk aversion measure function $\gamma(x) = U''(x)/U'(x)$ that is positive or equal to zero over the range of x.
3. $\gamma(x)$ should be constant or, at least, monotonically decreasing ($\gamma'(x) \leq 0$) over the range of x.

The ECP_G underlying utility function, as a piecewise function, is not twice differentiable over the points where the slope of the function changes (the endpoints of each segment). Each segment of the function is risk neutral, and the Arrow-Pratt classical risk aversion measure (Arrow, 1965; Pratt, 1978) ($\gamma = U''(x)/U'(x)$) is not appropriate or useful as $\gamma = 0$ for all segments and it does not capture the global risk aversion of the function. The measure of function risk aversion pattern for the piecewise function is possible only when two or more segments are compared, as the function is globally risk averse but not locally risk averse. The risk aversion of each segment is calculated comparing it to the previous segment of the utility function. The measure obtained should be monotonically decreasing over the range of possible segments. An appropriate measure is described by Luz (2016) and needs to evaluate the risk aversion considering two segments of the function and the risk measure compares one segment to another to evaluate the changes in risk aversion. The measure for piecewise function that can capture risk aversion from one segment to another is shown in equation (50). In this way, any utility function will be proper fitted to the empirical data will only if, indeed, the decision makers exhibit risk aversion behavior and if the pattern of risk aversion decreases from one function segment to the next.

Despite these important differences, with the necessary adaptations and constraints imposed, such as the alternative way to measure the aversion (equation (50)), that the overall behavior of the piecewise linear utility function is quite similar to the behavior of a usual prescriptive utility function and we can use similar strategies to assess the utility function.

2.5.2.1 Experimental approach

The strategy for mapping empirical points using certain equivalents and lotteries or investments with associated probabilities is quite adequate and promising, given its simplicity and ease of understanding by decision makers. The proposed experimental approach for the assessment of the utility function is:

1. Map a set of experimental utility points using indifference probabilities responses for hypothetical investments. The financial level of investments should be as close as possible to the level the agent usually decides. The hypothetical investments should vary in value so that the plot cover all segments of the utility function.
2. Analyze the plot to check any inconsistencies. The points should present an overall pattern of risk aversion. Reassess with the decision maker any points that are very distant from the pattern found (outliers).
3. Fit the linear function for each segment using the mathematical form of equation as reference. The utility function should change slope exactly on the points determined by α's.
4. After fitting the utility function, it is possible to determine the λ's values using the slopes of each segment. All the λ's values must be positive, and any negative values indicate the need to redo the procedures to assess the agent's preferences.

As the utility function only presents risk aversion when considering two or more segments so the approach to determine the utility function parameters involves framing questions in which the outcomes of investments or lotteries are, at least, in adjacent pairs of the $N + 1$ segments. The outcomes of the hypothetical investments should not be within the same function segment, as the responses, in this case, would not allow the calculation of risk aversion parameters. The questions, using outcomes of two separate segments will allow to assess the empirical utility function values for any function outcome.

The points in which the slopes of the utility function changes are expressed in terms of VaR, which is related to α's, for example $VaR_{95\%}$ is the VaR related to the α = 95%. Therefore, it is necessary to find the VaR values for the revenue of each agent. The challenge lies in the availability of values for all potential revenue outcomes from the agent's investments. These values are crucial for enabling a direct calculation of VaR's and their associated revenue values corresponding to each α previously determined along with the number of levels N.

Developing a method to estimate the output distribution becomes imperative due to their limited accessibility, as they hardly would be available completely. As the estimates of the utility function parameters will be inputted to the programs of the national system operator, a good reference for the distribution of revenue is the PLD (*Preço de Liquidação das Diferenças)* simulation, which is generated through the NEWAVE program, developed by CEPEL (*Centro de Pesquisas de Energia Elétrica*) for the national system operator.

The Nwlistop is a NEWAVE complementary program, and it generates 2,000 price simulation for each submarket. As the agent whose risk preferences are being assessed is an electricity market agent directly influenced by the settlement price of differences (PLD) variation, it is reasonable to assume that its distribution of revenue will be similar to the distribution of the simulated PLD for the submarket in which the agent operates. If the agent operates in more than one submarket it will be necessary to assess the parameter for each submarket as the risk aversion can be different from one market to another. The NEWAVE PLD simulation should be generated for period closest to the investment payback period.

To convert the PLD simulated values to revenue values, it is necessary to calculate a conversion factor, which will be applied to all PLD's simulated by NEWAVE. The conversion factor (CF) can be calculated as follows (equation 64):

$$CF = \frac{\bar{R}}{\left(\frac{\sum_{i=1}^{2000} PLD_i}{2000}\right)} \qquad 64$$

where $\bar{R}$ is the average revenue of the agent investments or projects.

The average revenue of the agent is a value that needs to be estimated based on the revenue of the agent's investments in the electricity sector. The period considered can be, for example, the last 2 or 5 years.

By applying the conversion factor to all PLD's it is possible to define the revenue values of the agent related to each VaR_{α_i}. The conversion is necessary as the values of α values are originally expressed in percentages rather than in terms of revenue. The utilization of the conversion factor enables the evaluation of preferences among agents with diverse scales and financial capacities.

To use the conversion factor and find the VaR value for the agent scale, we first define the α value that will to be converted. Then we find the value for the PLD distribution and finally we multiply the value by the conversion factor (equation (65)).

$$VaR_{\alpha j} = \left[\frac{\bar{R}_j}{\left(\frac{\sum_{i=1}^{2000} PLD_i}{2000}\right)}\right] \cdot VaR_{\alpha P} \qquad (65)$$

where $VaR_{\alpha j}$is the VaR_{α} converted for agent j, $VaR_{\alpha P}$is the VaR_{α} obtained from PLD distribution and $\bar{R}_j$is the average of revenue from agent j.

Using the certainty equivalent method, described previously, and making the correct adjustments to the distribution, using the conversion factor, for the assessment of utility function, it is possible to assess the utility function factors λ's for a variety of agents of each submarket. It is important to assess the utility function of agents that varies on size and types of projects. After the assessment of the utility function and its parameters for a variety of agents it will be possible to obtain a distribution of empirical utility points. The compilation of final λ values that will be used for the national system operator can be obtained through some methods that will be discussed later.

2.5.2.2 Interview approach

As mentioned earlier, the proposed interview approach employs hypothetical high-risk investments linked to two potential outcomes (x_i), each with its corresponding probabilities ($p(x_i)$). The goal for each hypothetical investment is to determine the indifference point for the decision maker – whether to pursue the investment or not. The certainty equivalent (CE) is used to determine the experimental points as the utility of the certainty equivalent must be equal to the expected utility of the lottery (equation (66)):

$$U(CE) = \sum_{i=1}^{2} p(x_i)U(x_i) \tag{66}$$

To map the experimental points using the certainty equivalent approach there are two main techniques that are applied:

1. Employ 50-50 lotteries to determine the certainty equivalent through successive inquiries until the decision maker reach a state of indifference between the certain outcome and the lottery. Subsequently, compute the utility of the certainty equivalent using equation 66.
2. Utilize a fixed certainty equivalent (typically the value of the investment or project) and systematically adjust the probabilities of success and failure until the investor reaches a state of indifference regarding whether to invest in the project or not.

The approach 1 has the advantage of providing probabilities that are easy to understand and interpret by the decision maker. This is, therefore, a simpler and more intuitive approach for the interviewee. On the other hand, approach 2 is closer to the actual decisions made by managers in which, usually, investments have a determined value and the outcomes have varied probabilities of success. An advantage of approach 2 is that there is control over the location on the position of the empirical certainty equivalent over the x-axis. It is possible to choose, for example, equally spaced values on this axis, which may be desirable for later function fitting.

There is no right or wrong approach between 1 and 2, and each application require the evaluation of the more suitable approach. For the assessment of ECP_G underlying utility function, we chose approach 2 as it is convenient to control the number of empirical points within each segment to proper fit the function to each segment. While approach 1 permits the certainty equivalent (empirical point) to be positioned anywhere amidst the potential outcomes, approach 2 offers the advantage of precisely defining both the quantity and placement of empirical points. This, in turn, greatly facilitates the fitting of linear functions to each respective segment and avoid segments without measured utility.

Another choice, which is quite important, is the number of questions to pose during each interview. A very small number of questions can affect the function's fit for the segment. Therefore, a greater number of data points acquired tends to yield more robust results. However, a very large number of questions can be tiring for the interviewee, who may answer, after a while, automatically without correctly interpreting the questions and data. In this way, a large number of questions and, consequently, empirical points obtained are desired, but not to large that becomes exhaustive for the interviewee. An alternative to increase the number of empirical points obtained, without making each interview too tiring, is to increase the number of interviewees from the same agent.

There is also no exact number of people or agents who need to be interviewed in each utility function assessment. It depends on factors such as the availability of respondents and the number of decision makers within each company. Swalm (1966) interviewed about 100 executives while Spetzler (1968) interviewed 36, and Green (1963), 16 executives.

For the assessment of the ECP_G underlying utility function is suggested, for each interview, 10 questions for each adjacent pair of linear functions. For 5 levels (6 segments), it would be necessary to determine 60 empirical utility points, but, as will be elucidated next section, for the first segment ($x \in \ ]VaR_1; VaR_0], VaR_{\alpha_0} \rightarrow +\infty$) we are going to choose two arbitrary utility points. So, for the first segment, we don't need to determine any empirical point and, for 5 levels we will determine 50 utility points and not 60. The number of interviews per agent suggested is one, as the focus is to determine the parameters for the system and not for a single agent, it is better to include more agents then increase the number of empirical points from the same agent.

There is also a concern with standardization, that why the same number of utility point is required. If, within the same agent, the decisions regarding the low-end and the high-end level of investments are taken by different management levels, it is possible to divide the interview in more than one, but it is important to gather the same number of utility points per agent after the assessment. It is also important to gather the same number of experimental points per agent to allow the proper treatment when consolidating the values of the parameters. The number of agents to have the utility function assessed should be as large as possible and it is important that the agents have different profiles and sizes.

2.5.2.3
Defining arbitrary values for the utility function

To assess the utility function, we need to begin by establishing two initial arbitrary utility points. Subsequent utility function points are then derived based on these initial values. It's important to note that all segments and values within the underlying utility function of the functional ECP_G are contingent upon the definition of λ's values, which are unknown at the outset of the assessment. There are no pre-established fixed utility values, by the definition of ECP_G underlying utility function, from which we can derive other utility points. On the other side, by selecting two arbitrary values we cannot guarantee, in the first moment, that the assessment will yield the desired mathematical structure outlined in equation (46), which includes constraints such as the requirement of $\sum \lambda_i = 1$. However, to perform the interviews, it is imperative to determine two initial utility values, otherwise we cannot determine any empirical point. So, by the end of this process, we will have obtained a piecewise linear function but not necessarily with the desired mathematical form. Instead, we will have a utility function that is strategically equivalent and possesses the same risk preferences. Therefore, it will be necessary to transform the obtained utility function into another one that possesses the desired mathematical form, as defined in equation (46).

Since each linear function exhibits local risk neutrality while the utility function demonstrates global risk aversion, we must compare the slopes between segments to reflect risk aversion behavior accurately. This conversion is executed using an alternative risk aversion coefficient, as defined in equation 50, which compares the slope of one segment to the next. The goal of this conversion is to ensure that the calculated risk aversion coefficient remains consistent across all segments in both functions (the function obtained, and the function converted to the desired format). Throughout this conversion process, we must also maintain the constraint $\sum \lambda_i = 1$.

We suggest establishing the two arbitrary values for the first segment, which contains the λ_0 slope. By stablishing two arbitrary values, an arbitrary λ_0 will, consequently, be chosen. Through the interviews, the other empirical points will be determined, and the function fit will provide the values for the other λ_i's stablishing the slope for the other function segments.

One possible and recommended approach to determine the two arbitrary utility points is to use a 45-degree slope ($\lambda_0 = 1$) and a make $U(VaR_{\alpha_1}) = VaR_{\alpha_1}$. This approach allows us to select two points within the first segment and determine the experimental utility points of the other function segments. The Figure 9 shown an illustrative example of assessment of utility function using two arbitrary values where $\alpha_1 = 50\%$ and $VaR_{\alpha_1} = 20$; $\alpha_2 = 95\%$ and $VaR_{\alpha_2} = 5$. The lambdas are $\lambda_0 = 1$, $\lambda_1 = 30\%$ and $\lambda_2 = 15\%$. The slopes are $s_0 = 1$, $s_1 = 1{,}6$ and $s_2 = 4{,}6$.

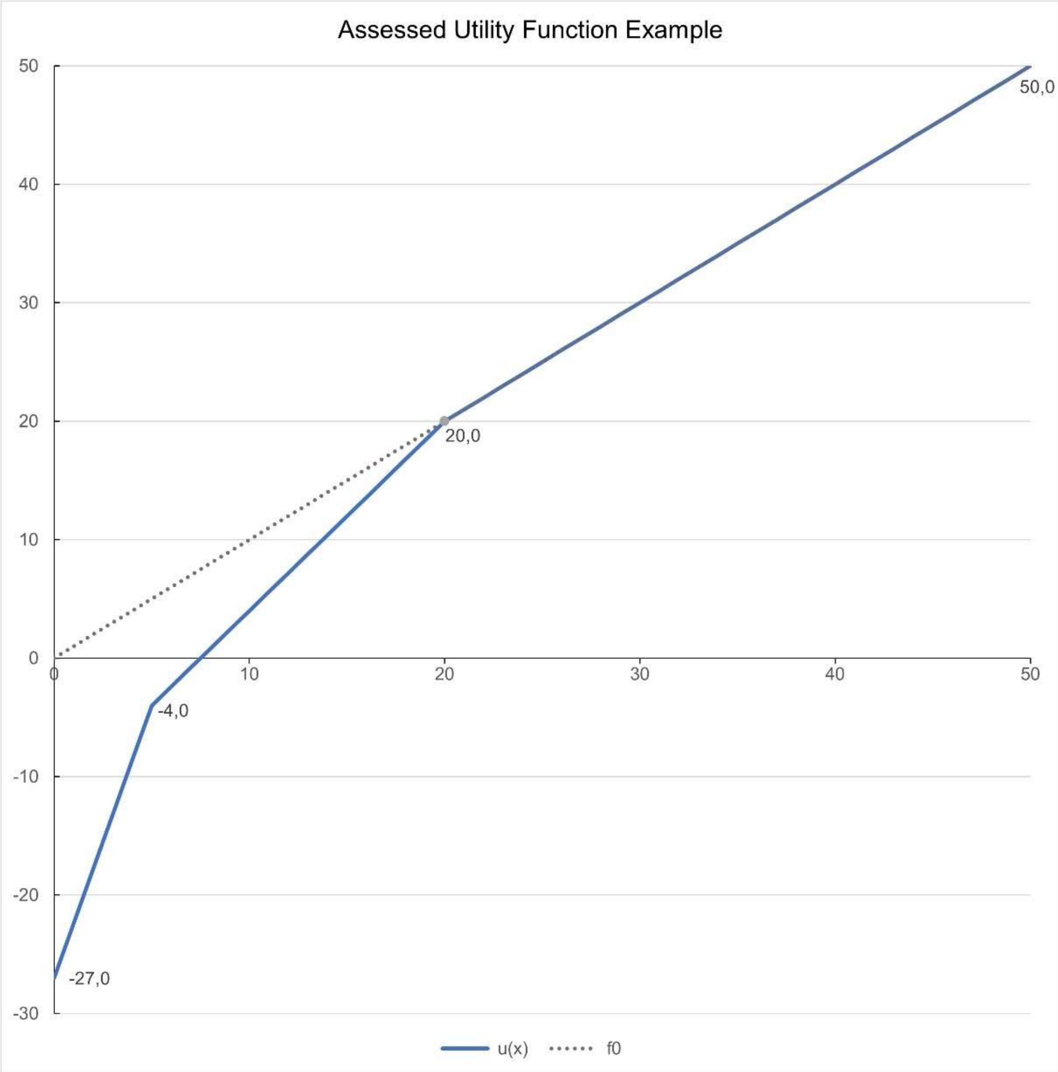

Figure 9 – Example of assessed utility function using two arbitrary utility points

The slopes of all the utility function segments are higher than 1 so, clearly, the constraint $\sum \lambda_i = 1$ is not met. So, we need to compute another utility function from the assessed function that has the required mathematical form and the same risk preferences from the original one. The procedure will be shown next, after we present the approach to consolidate the utility parameters after the interviews.

2.5.2.4
Consolidating the utility function parameters

After the interview process is completed, it is necessary to calculate the value of the parameters. The objective, at this stage, is to calculate the consolidated λ's. As previously discussed, the number of levels and α's should be defined before the interviews. There are two main approaches that can be used to calculate the consolidated λ's:

1. Calculate the λ's for each agent, using an optimization program to fit the best linear function for each segment, and then determine the λ's for the agent using the slopes of each segment obtained. Finally weight the values for the system using a set of criteria.
2. Convert all the utility function empirical points for the same base, using the conversion factor (equation (64)) and then, directly fit the linear functions of each segment using an optimization program. and then determine the λ's for the agent using the slopes of each segment obtained.

The advantage of the approach 2 over the approach 1 is that the function fit is optimized for all data and the optimizer will consider all empirical points at once. The approach 1 optimizes the results only for the agent and not necessarily for the system. Despite that, approach 1 makes possible to weight the λ's, using one or more criterion, for example, revenue of the agent. If we need to weight the parameters using approach 2 it will be necessary to increase or decrease the number of empirical points of each agent to reflect the weigh desired for each agent.

We recommend Approach 2 for the consolidation of λ's as it allows to achieve a better function fit to the data and will lead to a more adherent λ's to the decision makers preferences. To put the empirical data on the same scale, all revenue values related to the determined empirical points from the agent is divided by the average revenue from that agent as shown in equation (67):

$$\bar{\bar{x}}_{ij} = \frac{x_{ij}}{\bar{R}_j} \tag{67}$$

where x_{ij} is the standardized x-axis of utility of the empirical point i of agent j, x_{ij} is the x-axis of the empirical determined utility of point i of agent j and $\bar{R}_j$is the average of revenue from agent j.

To fit the utility function using the approach 2 is necessary to fit each segment at a time. The fit is very similar to any other function fit to empirical data and we should choose an adequate optimizer method for linear regression. One of the most common methods to perform a linear regression is the mean square root but there are many other methods to perform the linear regression available for being used with python, R, MATLAB etc.

The only difference from a traditional linear regression of data fitting is that there is an additional constraint, normally not present in this type of problem. To ensure the continuity of the piecewise linear function as a whole, the utility of the points where there is a change in slope of the piecewise linear function, the utility value must match for the two linear pieces of the function at these points of slope change.

To fit the piecewise utility function and cope with this constraint, we propose the following:

1. Convert all data and aggregate it in one single set of data.
2. Estimate de utility function initially for the first segment, the one with the less desired outcomes, $\left]VaR_{\alpha_{N+1}}; VaR_{\alpha_N}\right]; VaR_{\alpha_{N+1}} \rightarrow -\infty$. This first segment is the most important as it can lead, in some cases the company to bankrupt. The first segment will have the utility function fitted without any additional constraint mentioned above.
3. The next adjacent segment will have the lower bound point that can be obtained using the utility function from the first segment. This point is the same for the two adjacent segments as the function is continuous. The optimizer must fit the function for de second segment making the function to pass through the utility point calculated for the lower bound.
4. Repeat 3 for the next segments until segment 0. The segment 0 does not need to have the function fitted as the slope is determined by the two arbitrary utility points.
5. Use the slope of each segment to calculate the λ values (equation (68)), that will represent the values that will be used ECP_G functional, after the adjustments shown in the next topic.

$$s_0 = \lambda_0 \qquad (68)$$
$$s_1 = \lambda_0 + \frac{\lambda_1}{(1-\alpha_1)}$$
$$s_n = \lambda_0 + \frac{\lambda_1}{(1-\alpha_1)} + \cdots + \frac{\lambda_n}{(1-\alpha_n)}$$

where s_i is the slope of each of $N+1$ linear functions fitted to the data.

As all linear functions slopes s_i will be determined after the function fit to data, the λ can be easily determined by solving the system of equations as shown in equation (68).

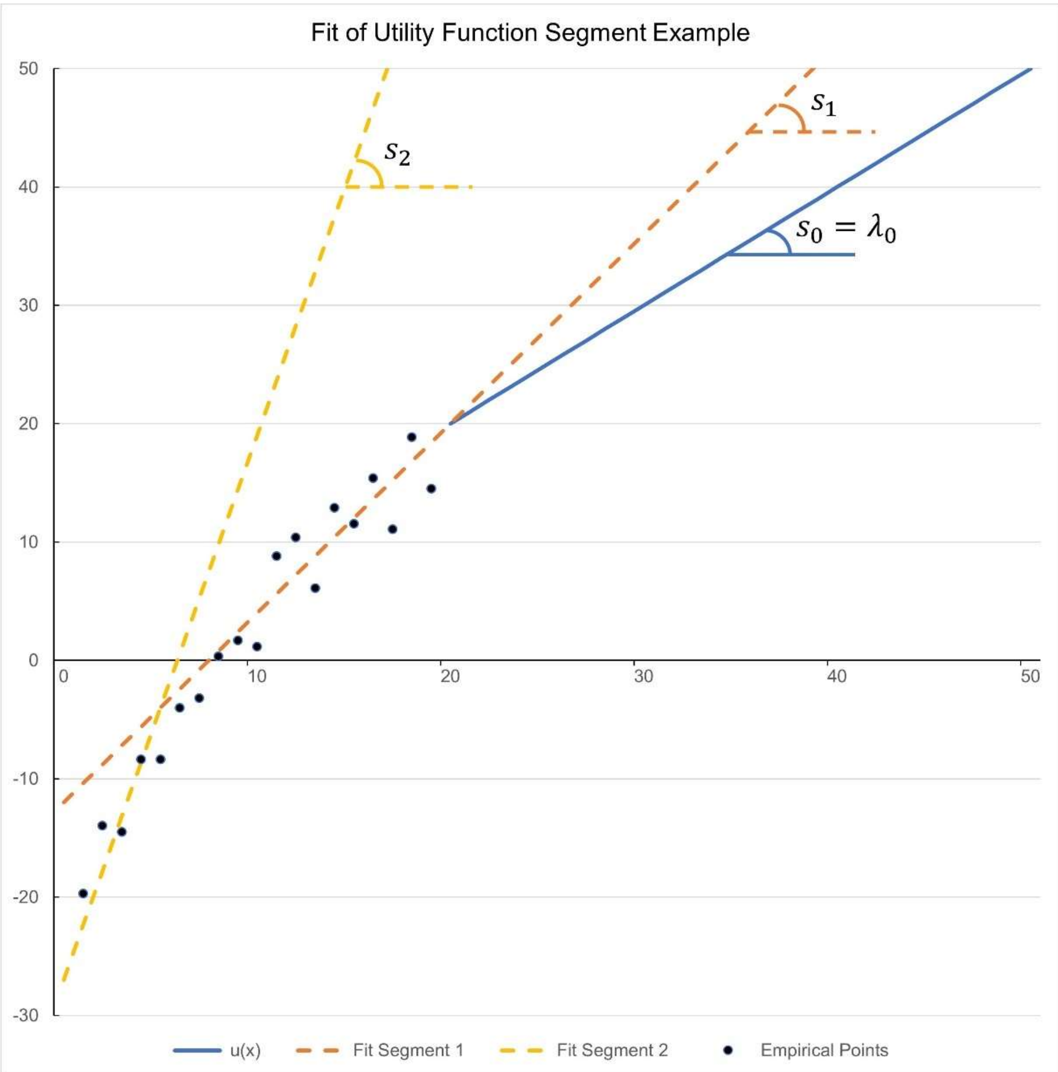

Figure 10 – Example of function fit for a piecewise linear utility function with two levels and 3 segments

The Figure 10 shows an illustrative example of function fit starting by the segment with the less desirable outcomes (segment 2,]0, 5]). Then the fit is performed for the next segment (segment 1,]5,20]). Note that we don't have empirical points within segment 0 as the λ_0 is arbitrary and, in this case, equal to 1. Fewer empirical points are purposely presented than those suggested for better visualization.

2.5.2.5 Crafting the ultimate utility function

As mentioned previously, the utility function assessed, despite being a piecewise linear function, lacks the required complete mathematical form shown in equation (46) and does not meet the $\sum \lambda_i = 1$ constraint. So, we must construct another utility function that has the same risk preferences from the assessed one but meet the constraints and mathematical form required.

The first procedure is to calculate the risk parameters for the utility function, using equation (50). From the example shown in Figure 9, where $\lambda_0 = 1, \lambda_1 = 30\%$ and $\lambda_2 = 15\%$, we have the following slopes and risk aversion measures (Table 4):

Function Segment	Slope (s_n)	Risk Aversion (γ_n)
0	$s_0 = \lambda_0 = 1$	–
1	$s_1 = 1.6$	$\gamma_1 = -\frac{1 - 1.6}{1.6} = 0.375$
2	$s_2 = 4.6$	$\gamma_2 = -\frac{1.6 - 4.6}{4.6} = 0.652$

Table 4 – Example of slope and risk aversion measures for assessed utility function

Then, the parameters of the utility function that we will build from the assessed utility function, can be determined solving the system of linear equations (equation (69)), as follows, where the risk parameters are fixed values and the new slopes are calculated using the equation (50) and the s_i's are written in terms of λ_i's and α_i's . The same procedure can be used for any number of utility function levels. To make possible to solve the system of equations we add the condition $\sum \lambda_i = 1$

and the number of variables are equal to the number of equations to be solved, in our case, 3 equations and 3 variables.

$$-\frac{s_0 - s_1}{s_1} = 0.375 \therefore s_0 = 0{,}625\ s_1 \tag{69}$$
$$-\frac{s_1 - s_2}{s_2} = 0.652 \therefore s_1 = 0{,}348\ s_2$$
$$\lambda_0 + \lambda_1 + \lambda_2 = 1$$

where $s_0 = \lambda_0$ and $s_1 = \lambda_0 + \frac{\lambda_1}{(1-\alpha_1)} = \lambda_0 + \frac{\lambda_1}{0.5}$ and $s_2 = a_1 + \frac{\lambda_2}{(1-\alpha_2)} = \lambda_0 + \frac{\lambda_1}{0.05}$.

Solving the systems of equations, we find the final parameters of the new utility function, which have the proper mathematical form and the same risk preferences as the assessed utility function. The parameters are:

- $\lambda_0 = 0.690$
- $\lambda_1 = 0.207$
- $\lambda_2 = 0.103$

Note that $\lambda_0 + \lambda_1 + \lambda_2 = 1$ and that the risk measure is the same: $\gamma_1 = 0.375$ and $\gamma_2 = 0.652$. The slopes of each segment are:

- $s_0 = 0.690$
- $s_1 = 0.103$
- $s_2 = 3.172$

The graphical representation of the new utility function is shown in Figure 11:

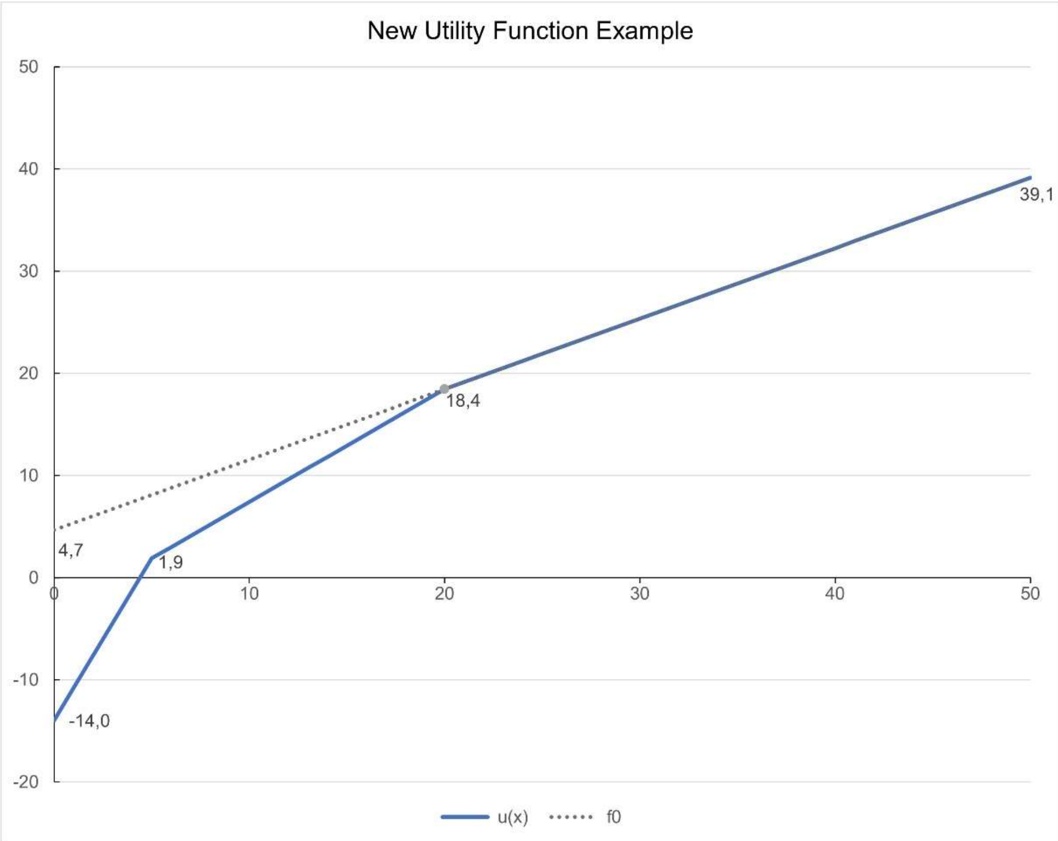

Figure 11 – Example of adjusted utility function that meets the mathematical form required and constraints, from assessed utility function

The mathematical description of utility function from the example is shown in equation (70):

$$U(x)\begin{cases} 0.690x + 4.655 + 0.414(x-20) + 2.069(x-5)\ (a) \\ 0.690x + 4.655 + 0.414(x-20)\ \ (b) \\ 0.690x + 4.655\ (c) \end{cases} \tag{70}$$

$(a): x \in \,]-\infty; 5]$

$(b): x \in \,]5; 20]$

$(c): x \in [20; +\infty[$

2.6 Conclusion

The primary contribution of this research was to present a methodology aimed at determining the parameters from the ECP_G functional underlying utility function. This novel approach empowers market agents to discern, with heightened clarity, uniformity, and precision, the risk preferences of the system participants. These preferences play a pivotal role in supporting decisions across various scenarios, notably in dual stochastic optimization problems, and find applications in the management of the national interconnected system. The proposed methodology was rooted in interviews with risk managers and involves hypothetical lotteries, wherein these managers express their preferences regarding two types of investment with varying probabilities of occurrence. This approach not only enhances decision-making processes but also facilitates a nuanced understanding of risk preferences in diverse applications within the national interconnected system.

Furthermore, the methodology allows for the evaluation of the appropriateness of the selected number of levels, employing 50-50 lotteries. This additional layer of assessment ensures a comprehensive understanding of the chosen risk levels and their alignment with the decision-making context. The substantial acceptance of models introduced by Street (2010) and Luz (2016) within the Brazilian electricity market, particularly by the national system operator, underscores the considerable potential for enhancing the outcomes of ongoing optimization models applied in the operational framework of the Brazilian system. This potential improvement is a direct consequence of adopting a more fitting, comprehensive, and uniform procedure for determining risk aversion parameters, thus contributing to the refinement of the decision-making landscape in the Brazilian electricity sector.

The main challenge for applying the proposal's approach to the national system is the adherence of a large and varied number of generating agents, both in size and location, which is fundamental for the adequate capture of risk parameters. However, as generating agents directly benefit from an optimized risk management system, it is possible that adherence to a utility function assessment will be high.

Resources such as the use of video conferencing and online questionnaires can be used to make the interview process more practical, faster and economically efficient, without losing information relevant to the process. The use of these tools is recommended due to the large number of generating agents and geographic dispersion.

As suggestions for future work, we suggest implementing the proposed methodology, initially, for a small group of agents and, subsequently, implementing the proposed methodology for the national interconnection system. The application of the methodology to the national interconnected system represents a great opportunity in terms of system risk management, as it guarantees better adherence of the system programming to the risk profiles of the generating agents, which ultimately represents greater security for the operation of the system in the short and long term, preserving the financial health of the system participants.

3
A Game Theoretical Approach for Seasonalization of Hydropower Plants Physical Guarantee

3.1
Introduction

In 2022, Brazil harnessed an impressive 440.0 terawatt-hours (TWh) of hydraulic energy, constituting a substantial 61.9% share within the total national electricity matrix, which stood at 690.1 TWh (EPE, 2023). Hydropower, being a renewable energy source, holds a crucial position in the country's energy landscape. Nonetheless, this dominance also brings forth notable complexities in the field of system management, mainly due to the dynamic nature of river flows and reservoir levels.

Within this intricate context, the effective management of physical guarantees and their seasonalization emerges as a critical facet in the orchestration of the Brazilian energy system. The concept of physical guarantee represents a cornerstone in the management of hydroelectric power generation within the Brazilian Interconnected System (SIN - *Sistema Interligado Nacional*). It embodies the maximum energy commitment a power plant can make in its contracts over a one-year period.

Although the total physical guarantee cannot change for the year, there is a flexibility for the individual plants to allocate and distribute their annual physical guarantees on a monthly basis, with the total allocation remaining constant. This practice allows power plants to better align with their energy supply contracts and risk profiles, a process referred to as the seasonalization of the physical guarantee.

This work presents a novel approach employing game theory principles and time series forecasting models to optimize the allocation decisions concerning the seasonalization of the physical guarantee. Our approach incorporates insights from regulatory agencies, market forecasts, and industry expertise.

It's important to note that the seasonalization of the physical guarantee is an annual process with significant financial implications for the market participants, especially for the hydroelectric power plants. This process directly influences the exposure of each stakeholder to the weekly spot price, PLD, that is contingent on factors like anticipated rainfall patterns and reservoir levels. For instance, the most risk-averse strategy involves strictly aligning the yearly physical guarantee with the monthly requirements of energy supply contracts, effectively mitigating individual exposure to the PLD. Conversely, agents with a higher risk appetite may opt for a more dynamic approach, allocating more physical guarantee to months where spot prices are expected to peak and accepting increased exposure to price fluctuations.

The proposed approach empowers decision-makers to calculate the monthly physical guarantee for their power plants, which consider the other agent's decisions. Notably, we show that regardless of other agent's risk preferences, the optimal decision consistently converges to the same strategy, known as the Nash Equilibrium, optimizing overall results.

3.2
Brazilian context

3.2.1
Price formation and difference settlement price

The operational management of energy generation and transmission in Brazil, apart from a portion of the northern region, is carried out through the SIN and comprises four subsystems: South, Southeast/Center-West, Northeast, and Northern regions. The energy dispatch decisions within the SIN are made according to the merit order, but also considers the present and future energy supply security.

Accordingly, sources with lower marginal cost have priority in dispatching. Thus, hydro power plants, which have a marginal cost close to zero, are usually the first to be dispatched. However, when reservoir levels are low or there is a forecasted drop in these levels, the operator may choose to preserve this energy source and use other one with higher marginal cost to avoid future energy shortage. When renewable sources that rely on climatic conditions are used, such as wind and photovoltaic energy, the generated energy cannot be stored. Therefore, the energy produced by these sources is automatically fed into the system as it becomes available and takes priority over other energy sources.

The price formation follows the dispatch order, where the current price is the price of the last batch supplied to the market and represents the lowest possible price for a certain energy load level, generating a balance between supply and demand. The last marginal price that meets the demand at a given time is the spot price, PLD, and it is used for short-term operations. The PLD has maximum and minimum limits, defined for each year in ANEEL Resolution 858/19 (ANEEL, 2019).

The PLD holds a central role in the modeling of this work, as it significantly impacts the decision-making of power-generating agents when assessing their exposure to PLD during the process of seasonalization of their physical guarantees. Additionally, we will delve into the Energy Reallocation Mechanism (*MRE – Mecanismo de Realocação de Energia*) shortly, which likewise subjects power generators to fluctuations in PLD.

3.1.2
Energy reallocation mechanism, physical guarantee and Generation Scaling Factor

The predominance of the hydraulic source in the Brazilian electricity matrix brings challenges for the management of the SIN. Due to the large extension of the national territory, some regions will have more favorable hydrological conditions for hydraulic energy production than others. Thus, in the interconnected system, the dispatch prioritizes power plants that are under the more favorable hydrological conditions and a centralized operation allows for a better treatment of hydrological risk, a lower marginal cost, and a greater system safety. It also allows better management of resources when there are plants in sequence in the same river basin. In these cases, there is an optimal dispatching order that globally optimizes the use of resources. The optimized outcome can only be obtained when dispatch management is centralized (CCEE, 2018).

Under this arrangement, the generating agents have no control over the level of power generation of their own power plants since the dispatch decision is centrally made by the National Electricity System Operator (*ONS – Operador Nacional do Sistema Elétrico*). Thus, depending on the ONS dispatch decision, the financial result of the plant's operation may suffer large oscillations. To minimize the financial risk associated with the centralized management of the system, the

MRE was created so that the hydrological risk could be shared among all the generating agents.

One of the main components of the MRE is the system physical guarantee, which is the sum of the physical guarantees of each hydroelectric power plant within the SIN. It corresponds to the maximum energy that the system can safely supply, considering the supply guarantee criteria in force (MME, 2016). For each new plant included in the SIN, it is necessary to calculate its respective physical guarantee, which will be part of the concession contract and is the maximum amount of energy that the power plant can commit to deliver in contracts. The determination of the physical guarantee, as well as its periodical revisions, is carried out by the Energy Research Company (*EPE – Empresa de Pesquisa Energética*) and follows specific and regulated methodologies (EPE, 2020b).

Within the MRE, participating plants receive revenue that is directly proportional to the total generation of the entire system. The allocation of this revenue is determined by assessing each plant's physical guarantee as a percentage of the SIN's overall physical guarantee. MRE functions as an insurance for the participating power plants. Through this mechanism, the plants that generate less energy than their physical guarantee in a specific period receive compensation from those that surpass their guaranteed generation levels. Any gains or losses associated with surplus or deficit of generation, either above or below the system's physical guarantee, are also distributed proportionally based on each plant's individual physical guarantee. Any discrepancies are reconciled at the spot price prevailing in the market.

The adjustment factor of the generation, above or below the physical guarantee, is known as Generation Scaling Factor (GSF) and is obtained from the division of the aggregated hydroelectric generation by the physical guarantee. The system is organized so that the risk of the hydroelectric generation staying bellow the physical guarantee, in the period of 1 year, is considerably low. Nonetheless, it is conceivable that hydroelectric generation for the year could fall below the physical guarantee, particularly during extended periods of drought. In such circumstances, the average GSF may drop below 1, which will have an adverse effect on the revenue of the power plants. When the average GSF falls below 1, it indicates that the sum of energy produced by all the hydroelectric plants in the SIN is below the system physical guarantee and is insufficient to cover the contracts, so, that other sources were needed to supply the energy contracted. The shortfall between the energy generated and the physical guarantee is adjusted in the short-term market, for the price of PLD, and the cost is passed on to MRE participants, proportionally to their physical guarantees.

3.1.2
Seasonalization of the Physical Guarantee

The physical guarantee is determined for the period of one year. However, the adjustments of the MRE occur on a monthly basis and the contracts are also invoiced monthly. Generating agents are granted the flexibility to perform monthly seasonalization of their physical guarantee. Seasonalization occurs once annually, with the results delivered in December and applicable for the entire following year. Once implemented, it remains static throughout the year without any allowance for mid-year adjustments.

There are specific limitations imposed on the seasonalization process. The sum of the 12-month physical guarantee must not surpass the plant's overall physical guarantee. Furthermore, for any given month, the seasonalized physical guarantee cannot exceed the Effective Power, unless there's no established Effective Power defined in the regulatory act. Negative values physical guarantee are also not permitted.

Seasonalization allows each agent to make the decision they consider most appropriate to their risk and contract profiles. By seasonalizing, agents can vary the exposure to the risk of GSF and PLD variations. A typically conservative agent will seek to match, each month, the physical guarantee to the plant's contracts. In doing so, the agent tries only to be exposed to the GSF risk as the physical guarantee allocated is aligned with contracts profile and the physical guarantee will be enough to cover the contracts when GSF is higher or equal to 1.

There are two main risks to be considered when deciding on seasonalization. The first risk is the individual risk, represented by exposure to the PLD resulting from a mismatch between the seasonalized physical guarantee and the plant's contracts for a given month. The differences, positive or negative, are settled at the current PLD. Agents less averse to risk will seek to allocate greater physical guarantee to the months with the highest PLD forecasts, expecting generating surpluses that can be settled at the prevailing PLD. But the players must also consider the systemic impact of the seasonalization decision, that is the risk associated with the GSF (second type of risk).

The GSF risk also exposes the power plant to the PLD, but this risk is distributed across all plants within the system. It is important to emphasize that the GSF is not a fixed value resulted from the physical guarantee seasonalization as it is determined by the total energy generated, that is uncertain. Decisions related to seasonalization directly impact the accounting of GSF, which has financial repercussions for all participating members. For instance, if in a given month, hydroelectric energy generation falls below the sum of all seasonalized physical guarantees, GSF will dip below 1. In such cases, the difference between the seasonalized physical guarantee and the actual energy generated is reconciled at the price of PLD and allocated in the pool, based on each plant's seasonalized physical guarantee. Therefore, seasonalization decisions encompass both an individual component (physical guarantee versus power plant contracts) and a systemic component (combined system physical guarantee versus actual system energy generation).

When considering the seasonalization decisions, the agents are faced with the challenge of anticipating the seasonalization decisions of the other power plants to make the most advantageous selection of physical guarantee per month. This scenario can be aptly represented as a strategic game, wherein agents are tasked with devising an optimal seasonalization strategy. Furthermore, the resulting outcomes hinge on the seasonalization strategies adopted by all generation agents.

3.2 Game-theoretic approach for the seasonalization of the physical guarantee

Game Theory delves into the intricate realm of decision-making among interacting agents, whether their intentions lean towards cooperation or conflict. Within this framework, agents engage in rational choices, carefully evaluating the outcomes of their decisions alongside those made by their counterparts. Perfect competition materializes when decisions are made within an environment where firms act as price takers, manufacturing homogeneous commodities. Within this context, prices are determined by the balance of market demand and industry production, with individual firms holding no sway over the equilibrium price. Entrants into this arena face minimal barriers, and collusion remains absent from the landscape. However, in the realm of imperfect competition, a multitude of scenarios may unfurl, including the emergence of collusion and collaborative ventures among the agents. Some of these scenarios are analyzed for commodity oil by Guimarães Dias and Teixeira (2010).

Game theory has found a multitude of applications within the electricity sector, serving as a crucial tool for tasks such as determining energy prices within specific markets and establishing the values in various auction scenarios. An innovative approach in this domain has been presented by Abapour et al. (2020), who leveraged game theory to devise a strategy for Demand Response (DR) aggregators. This strategy aims to maximize profit by harnessing the flexibility of customers' appliances to curtail energy costs, while also employing game theory to model competition between aggregators.

Furthermore, game theory has extended its influence into the realms of blockchain technology and energy grids, where consumers can dynamically engage in energy consumption, production, sharing, and even storage. This transformative category, often referred to as prosumers, was scrutinized by Jiang et al. (2020). Their work introduced a game theory-based pricing model for peer-to-peer (P2P) electricity trading, considering interactions between sellers and buyers, as well as among different sellers.

The prosumer landscape and community-based market mechanisms have also been subjects of analysis by Mitridati et al. (2021). Employing game-theoretic tools, they developed a consumer-centric market mechanism that optimizes community energy management, offering valuable insights for efficient energy utilization within such communities.

Leonel et al. (2019) introduced an innovative approach employing game theory to address the annual seasonalization process in Brazil. Within their study, they harnessed a game theory model to scrutinize three distinct seasonalization strategies: allocation in accordance with contracts, heightened allocation during the wet season, and increased allocation in the dry season. Their analysis encompassed two distinct risk profiles, identifying the strategies that constitute the Nash equilibrium.

However, it's crucial to note that their research operated under the assumption that these strategies remained independent of price forecasts and MRE apportionment results, variables inherently tied to financial outcomes. In contrast, our study takes a different trajectory by delving into the repercussions of seasonalization on MRE apportionment. This, in turn, directly impacts the Generation Scaling Factor (GSF), which undergoes alterations based on agents' seasonalization decisions. Consequently, GSF forms an integral component of our model, providing a more comprehensive understanding of the complex interplay between seasonalization and its implications on the energy landscape.

3.2.1 Seasonalization game overview

The process of monthly seasonalization, conducted once a year, can be formulated as a strategic game. Every participant's decision during the seasonalization process have a direct impact on other participants results, through the GSF. For instance, if one agent maximizes their power plant output for a particular month based on a bullish forecast for the PLD, and all other agents do the same movement, a mismatch between generated energy and the allocated physical guarantee may occur. This imbalance could lead to the GSF dropping below 1, as the physical guarantee probably will surpasses the energy generated for that month. When the GSF falls below 1, it results in financial penalties for all participants.

Therefore, successful seasonalization strategies must consider the price forecast and the decisions of other agents, which can significantly influence the GSF and impacts the power plant revenue. This complex situation can be aptly modeled as a strategic game. For the analysis of the dynamics involving the physical guarantee seasonalization, we will consider the perfect competition environment, where the participants are free to enter and leave the market and there is no possibility of collaboration or collusion between the agents (non-cooperative game). This assumption is reasonable given that the market for hydro power generation is well established and has many participants.

New participants can freely enter the market through the Regulated Contracting Environment (*ACR – Ambiente de Contratação Regulada*) or the Free Contracting Environment (*ACL – Ambiente de Contratação Livre*). For non-cooperative games, the most important concept developed is Nash Equilibrium, which represents the set of strategies in which, each player cannot individually get their individual result (payoff) better by changing the strategy, unilaterally. This equilibrium can be deterministic or probabilistic.

When it comes to the nature of information within the seasonalization game, we classify it as a game of imperfect information. This categorization stems from the fact that each agent's seasonalization decisions are revealed simultaneously, with no opportunity for subsequent revisions. Information is deemed perfect when all participants are fully aware of the actions taken by their counterparts. In the context of seasonalization, however, agents only become privy to the decisions made by other participants after they themselves have committed to their choices, given that these decisions occur concurrently.

While it's true that there are several pieces of information held by individual agents that remain inaccessible to others, the critical information required for informed decision-making during the seasonalization process is openly available. This includes the set of potential strategies, making this game fall under the category of complete information games.

3.2.2
Designing the framework for seasonalization game strategies

Before delving into the intricacies of the seasonalization game, it's essential to establish two fundamental concepts: the GSF and the payoff for each agent. The GSF serves as a crucial index, gauging the extent of coverage the system offers for the contracted energy. It can be precisely defined using the equation (71).

$$GSF_j = \frac{\sum_{i=1}^{n} \tau_{ij}}{\sum_{i=1}^{n} g_{ij}} \tag{71}$$

where:

$\sum_{j=1}^{12} g_{ij} = G_i$, G_i is the annal physical guarantee of power plant i, GSF_j is the GSF for the month j, $\tau_{ij}$is the total generation of power plant i for the month j, $g_{ij}$is the physical guarantee allocated by power plant i for the month j (this value is obtained from the seasonalization of the physical guarantee) and n is the number of power plants in the energy relocation mechanism.

The annual payoff for each agent (π), as a result from the seasonalization decision, is determined by summing the power plant's results over the course of the twelve months in a year (equation (72)).

$$\pi_i = \sum_{j=1}^{12} P_j \cdot \left[\left(g_{ij} \times GSF_j\right) - C_{ij}\right] \tag{72}$$

where P_j is the spot price (PLD) for the month j, C_{ij} is the contracted amount from power plant i for the month j.

To illustrate the seasonalization process, we will employ a simplified example featuring two time periods (1 and 2) and involving two participants, denoted as 'a' and 'b'. The power plants payoffs for each month are presented in Table 5:

	Period 1 payoff	**Period 2 payoff**	**Total payoff (π)**
a	$P_1(g_{a1} \times GSF_1)$ $-P_1 \times C_{a1}$	$P_2(g_{a2} \times GSF_2)$ $-P_2 \times C_{a2}$	$P_1[(g_{a1} \times GSF_1) - C_{a1}] +$ $P_2[(g_{a2} \times GSF_2) - C_{a2}]$
b	$P_1(g_{b1} \times GSF_1)$ $-P_1 \times C_{b1}$	$P_2(g_{b2} \times GSF_2)$ $-P_{2\times}C_{b2}$	$P_1[(g_{b1} \times GSF_1) - C_{b1}] +$ $P_2[(g_{b2} \times GSF_2) - C_{b2}]$

Table 5 – Example of seasonalization outcomes with 2 powerplants for 2 periods

There are only 2 periods to be considered, so if the decision maker chooses the amount of physical guarantee to be allocated to month 1 then, automatically, the amount of physical guarantee allocated to month 2 is determined, as shown in equation (73) and equation (74).

$$G_a = g_{a1} + g_{a2} \therefore \quad g_{a2} = G_a - g_{a1} \tag{73}$$

$$G_b = g_{b1} + g_{b2} \therefore \quad g_{b2} = G_b - g_{b1} \tag{74}$$

The total payoffs can be expressed as shown in equation (75) and equation (76):

$$\pi_a(g_{a1}) = P_1[(g_{a1} \times GSF_1) - C_{a1}] \quad (75)$$
$$+P_2[(G_a - g_{a1}) \times GSF_2 - C_{a2}] \therefore$$
$$\pi_a(g_{a1}) = P_1\left[\left(g_{a1} \times \frac{\tau_{a1+}\tau_{b1}}{g_{a1} + g_{b1}}\right) - C_{a1}\right]$$
$$+P_2\left[(G_a - g_{a1}) \times \frac{\tau_{a2+}\tau_{b2}}{(G_a - g_{a1}) + (G_b - g_{b1})} - C_{a2}\right]$$

$$\pi_b(g_{b1}) = P_1[(g_{b1} \times GSF_1) - C_{b1}] \quad (76)$$
$$+P_2[(G_b - g_{b1}) \times GSF_2 - C_{b2}] \therefore$$
$$\pi_b(g_{b1}) = P_1\left[\left(g_{b1} \times \frac{\tau_{a1+}\tau_{b1}}{g_{a1} + g_{b1}}\right) - C_{b1}\right]$$
$$+P_2\left[(G_b - g_{b1}) \times \frac{\tau_{a2+}\tau_{b2}}{(G_a - g_{a1}) + (G_b - g_{b1})} - C_{b2}\right]$$

The total generation of each month 1 is $\tau_{a1+}\tau_{b1}$. This value can be replaced by k_1, that is the generation forecast for month 1. The same can be done for month 2 and we can substitute $\tau_{a2+}\tau_{b2}$ by k_2 as shown in equation (77) and equation (78):

$$\pi_a(g_{a1}) = P_1\left[\left(g_{a1} \times \frac{k_1}{g_{a1} + g_{b1}}\right) - C_{a1}\right] \quad (77)$$
$$+P_2\left[(G_a - g_{a1}) \times \frac{k_2}{(G_a - g_{a1}) + (G_b - g_{b1})} - C_{a2}\right] \therefore$$

$$\pi_a(g_{a1}) = \frac{g_{a1}}{g_{a1} + g_{b1}} \times P_1 k_1$$
$$+\frac{G_a - g_{a1}}{(G_a - g_{a1}) + (G_b - g_{b1})} \times P_2 k_2$$
$$-(P_1 \times C_{a1} + P_2 \times C_{a2})$$

$$\pi_b(g_{b1}) = P_1\left[\left(g_{b1} \times \frac{k_1}{g_{a1} + g_{b1}}\right) - C_{b1}\right] \quad (78)$$

$$+P_2\left[(\,G_b - g_{b1})\times\frac{k_2}{(\,G_a - g_{a1}) + (\,G_b - g_{b1})} - C_{b2}\right] \therefore$$

$$\pi_b(g_{b1}) = \frac{g_{b1}}{g_{a1} + g_{b1}} \times P_1 k_1$$
$$+\frac{G_b - g_{b1}}{(\,G_a - g_{a1}) + (\,G_b - g_{b1})} \times P_2 k_2$$
$$-(P_1 \times C_{b1} + P_2 \times C_{b2})$$

The spot prices (PLD) P_j and the generation of each power plant i for the month j τ_{ij} are random variables. For simplicity, we considered the PLD and the generation ($\sum_{i=1}^{n} \tau_{ij}$) forecasts as fixed values, not as variables, and that the forecasted values (k_1 and k_2) will be the same for all agents. We understand that, even if the decision makers use different forecast models, these forecasts should lead to very similar results. So, the forecasted spot prices and system generation should be very close, even if the players use different forecast methods. We also assume that the forecasted values should be close to the actual values and that, a simpler and efficient approach, would be to consider the forecasted values as the actual values. Future works may evaluate if there is any relevant improvement if we consider the prices and generation as random variables, which will lead to a random payoff.

The Nash Equilibrium, when it exists, represents the set of strategies in which, each player cannot individually get their individual result (payoff) better by changing the strategy, unilaterally. The analytical solution the problem must be one of the solutions in which the first derivative of the payoff function equals zero as shown in equation (79). The same applies both to player 'a' and player 'b'.

$$\frac{d\pi_a}{dg_{a1}} = 0 \qquad (79)$$
$$\frac{d\pi_b}{dg_{b1}} = 0$$

Our approach was to find the function points where the derivative of the function is zero and that are the maximum of the payoff function, what means that the second derivative of the function must be negative. Thus, we calculated the point where the derivative is equal to zero for the payoff function of one player and then we substituted the results obtained on the other player payoff function. The Nash Equilibrium is found if we can make sure that the optimal solution cannot be improved, for both players, by individually changing their position.

There are some constraints that need to be considered: the prices (P_i), the power plant generation (k_i), the physical guarantee (g_{ij}), the contracted amount (c_{ij}) and the physical guarantee (G_i) must be positive values. We also must consider that the sum of physical guarantee allocate to each month by each power plant is equal to physical guarantee (equation (80)).

$$G_i = \sum_{j=1}^{12} g_{ij} \tag{80}$$

For the two players and two periods problem, the first derivative of the payoff function, for player 'a' and player 'b' are expressed by the equation (81) and equation (82), respectively:

$$\frac{d\pi_a}{dg_{a1}} = -\frac{g_{a1}P_1k_1}{(g_{a1}+g_{b1})^2} + \frac{P_1k_1}{(g_{a1}+g_{b1})} + \frac{P_2k_2(G_a - g_{a1})}{(G_a + G_b - g_{a1} - g_{b1})^2} - \frac{P_2k_2}{(G_a + G_b - g_{a1} - g_{b1})} \tag{81}$$

$$\frac{d\pi_b}{dg_{b1}} = -\frac{g_{b1}P_1k_1}{(g_{a1}+g_{b1})^2} + \frac{P_1k_1}{(g_{a1}+g_{b1})} + \frac{P_2k_2(G_b - g_{b1})}{(G_a + G_b - g_{a1} - g_{b1})^2} - \frac{P_2k_2}{(G_a + G_b - g_{a1} - g_{b1})} \tag{82}$$

Equaling the payoff derivative function to zero ($\frac{d\pi_b}{dg_{b1}} = 0$) we can find the expression for the g_{b1}. There are two solutions, $g_{b1}*$ and $g_{b1}**$, that make $\frac{d\pi_b}{dg_{b1}} = 0$, as shown in equation (83) and equation (84), respectively:

$$g_{b1} *= \frac{\left(\begin{array}{c} g_{a1}P_1k_1G_a + g_{a1}P_2k_2G_a + G_a\sqrt{(g_{a1}G_aP_1k_1P_2k_2 - P_1k_1P_2k_2g_{a1}{}^2)} \\ +g_{a1}P_1k_1G_b + G_b\sqrt{(g_{a1}G_aP_1k_1P_2k_2 - P_1k_1P_2k_2g_{a1}{}^2)} \\ -P_1k_1g_{a1}{}^2 - P_1k_1g_{a1}{}^2 \end{array}\right)}{(-P_2k_2G_a + g_{a1}P_1k_1 + g_{a1}P_2k_2)} \tag{83}$$

$$g_{b1} **= \frac{\left(\begin{array}{c} g_{a1}P_1k_1G_a + g_{a1}P_2k_2G_a - G_a\sqrt{(g_{a1}G_aP_1k_1P_2k_2 - P_1k_1P_2k_2g_{a1}{}^2)} \\ +g_{a1}P_1k_1G_b - G_b\sqrt{(g_{a1}G_aP_1k_1P_2k_2 - P_1k_1P_2k_2g_{a1}{}^2)} \\ -P_1k_1g_{a1}{}^2 - P_1k_1g_{a1}{}^2 \end{array}\right)}{(-P_2k_2G_a + g_{a1}P_1k_1 + g_{a1}P_2k_2)} \tag{84}$$

Then we can substitute the two solutions, $g_{b1}*$ and $g_{b1}**$, in the equation that represents the derivative of the payoff function of player 'a', equation (81) and solve it equaling the expression to zero. We arrive at the same solution for the physical guarantee allocated by player 'a' to period 1, denoted by $g_{a1}{}^*$ and $g_{a1}{}^{**}$, expressed by equation (85) below:

$$g_{a1}{}^* = g_{a1}{}^{**} = G_a \times \frac{P_1k_1}{P_1k_1 + P_2k_2} \tag{85}$$

The physical guarantee allocated by the player 'a' to period 2, $g_{a2}*$, is obtained by subtracting the physical guarantee allocated to the period 1, $g_{a1}{}^*$, from the total physical guarantee G_a (equation 86):

$$g_{a2}{}^{*} = g_{a2}{}^{**} = G_a - G_a \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2} \tag{86}$$
$$= G_a \left(1 - \frac{P_1 k_1}{P_1 k_1 + P_2 k_2}\right) = G_a \left(\frac{P_2 k_2}{P_1 k_1 + P_2 k_2}\right)$$

To find the physical guarantee allocated by player 'b' to period 1 we follow the same approach that was used for player 'a'. Equaling the payoff derivative function to zero ($\frac{d\pi_a}{dg_{a1}} = 0$) we found the expression for the g_{a1} values. There are also two solutions, $g_{a1}*$ and $g_{a1}**$, that make $\frac{d\pi_a}{dg_{a1}} = 0$, as shown in equation (87) and equation (88), respectively:

$$g_{a1} *= \tag{87}$$
$$\frac{\begin{pmatrix} g_{b1} P_1 k_1 G_a + G_a \sqrt{(g_{b1} G_b P_1 k_1 P_2 k_2 - P_1 k_1 P_2 k_2 g_{b1}{}^{2})} \\ + g_{b1} P_1 k_1 G_b + g_{b1} P_2 k_2 G_b + G_b \sqrt{(g_{b1} G_b P_1 k_1 P_2 k_2 - P_1 k_1 P_2 k_2 g_{b1}{}^{2})} \\ - P_1 k_1 g_{a1}{}^{2} - P_1 k_1 g_{a1}{}^{2} \end{pmatrix}}{(-P_2 k_2 G_b + g_{b1} P_1 k_1 + g_{b1} P_2 k_2)}$$

$$g_{a1} **= \tag{88}$$
$$\frac{\begin{pmatrix} g_{b1} P_1 k_1 G_a - G_a \sqrt{(g_{b1} G_b P_1 k_1 P_2 k_2 - P_1 k_1 P_2 k_2 g_{b1}{}^{2})} \\ + g_{b1} P_1 k_1 G_b + g_{b1} P_2 k_2 G_b - G_b \sqrt{(g_{b1} G_b P_1 k_1 P_2 k_2 - P_1 k_1 P_2 k_2 g_{b1}{}^{2})} \\ - P_1 k_1 g_{a1}{}^{2} - P_1 k_1 g_{a1}{}^{2} \end{pmatrix}}{(-P_2 k_2 G_b + g_{b1} P_1 k_1 + g_{b1} P_2 k_2)}$$

Substituting the two possible solutions in the equation that represents the derivative of the payoff function of player 'b' (equation 82), we obtain the same result for the expression for the physical guarantee allocated by player "b" to period 1, denoted by $g_{b1}{}^{*}$ and $g_{b1}{}^{**}$, expressed by equation (89) below:

$$g_{b1}{}^{*} = g_{b1}{}^{**} = \tag{89}$$
$$G_b \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2}$$

The physical guarantee allocated by the player 'b' to period 2, $g_{b2}*$, is obtained by subtracting the physical guarantee allocated to the period 1, g_{b1}^{*}, from the total physical guarantee G_b (equation (90)):

$$g_{b2}^{*} = g_{b2}^{**} = G_b - G_b \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2} = G_b \left(1 - \frac{P_1 k_1}{P_1 k_1 + P_2 k_2}\right) = G_b \left(\frac{P_2 k_2}{P_1 k_1 + P_2 k_2}\right) \quad (90)$$

To ascertain whether the obtained values represent a Nash Equilibrium, it's crucial to assess whether a move made by an individual player can enhance their outcome at the expense of others. If the players cannot improve their result by individually changing their decisions, then we have the Nash Equilibrium for the game. This assessment involves analyzing the second derivative of the payoff function to determine its implications. A positive second derivative of the function signifies that a point where the first derivative is zero represents a local minimum. Conversely, a negative second derivative of the function indicates a local maximum, while a second derivative equal to zero renders the test inconclusive.

For a Nash equilibrium to exist, the identified point must be a local maximum that cannot be negatively affected by the movements of other players and no other point (decision) within the function, meeting valid parameters, should yield a better financial outcome for the player. Hence, it's imperative to establish that this point represents a global maximum within the possible values of physical guarantee and the constraints of the problem.

The second derivative of the payoff function for player 'a' and player 'b' is shown in equation (91) and equation (92), respectively:

$$\frac{d^2\pi_a}{dg_{a1}^{\,2}} = \frac{P_1 k_1 g_{a1}}{(g_{a1} + g_{b1})^3} - \frac{2P_1 k_1}{(g_{a1} + g_{b1})^2} + \frac{2P_2 k_2 (G_a - g_{a1})}{(G_a + G_b - g_{a1} - g_{b1})^3} - \frac{2P_2 k_2}{(G_a + G_b - g_{a1} - g_{b1})^2} \quad (91)$$

$$\frac{d^2\pi_b}{dg_{b1}{}^2} = \frac{2P_1k_1g_{b1}}{(g_{a1}+g_{b1})^3} - \frac{2P_1k_1}{(g_{a1}+g_{b1})^2} + \frac{2P_2k_2(G_b-g_{b1})}{(G_a+G_b-g_{a1}-g_{b1})^3} - \frac{2P_2k_2}{(G_a+G_b-g_{a1}-g_{b1})^2} \tag{92}$$

The second derivative of the payoff function is always negative for the range of possible values of g_{a1} and g_{b1}, for both players, considering that all variables are positive, $G_a \geq g_{a1}$ and $G_a \geq g_{a1}$. We are going to prove for $\frac{d^2\pi_a}{dg_{a1}{}^2}$ but the same procedure can be done for $\frac{d^2\pi_b}{dg_{b1}{}^2}$.

Let's start analyzing the first 2 terms of $\frac{d^2\pi_a}{dg_{a1}{}^2}$ and rewrite them appropriately, as shown in equation (93):

$$\frac{2P_1k_1g_{a1}}{(g_{a1}+g_{b1})^3} - \frac{2P_1k_1}{(g_{a1}+g_{b1})^2} = \frac{2P_1k_1}{(g_{a1}+g_{b1})^2} \times \left(\frac{g_{a1}}{g_{a1}+g_{b1}} - 1\right) \tag{93}$$

As all the variables are positive, the expression $\frac{2P_1k_1}{(g_{a1}+g_{b1})^2}$ must be also positive. The expression $\frac{g_{a1}}{g_{a1}+g_{b1}} \leq 1$, as g_{a1} and g_{b1} are positive values. So the expression $\frac{g_{a1}}{g_{a1}+g_{b1}} - 1$ must be negative and also the expression $\frac{2P_1k_1}{(g_{a1}+g_{b1})^2} \times \left(\frac{g_{a1}}{g_{a1}+g_{b1}} - 1\right)$. Then we know that the first 2 terms of $\frac{d^2\pi_a}{dg_{a1}{}^2}$ represent a negative value.

Then we analyze the last 2 terms of $\frac{d^2\pi_b}{dg_{b1}{}^2}$ and rewrite them appropriately, as shown in equation (94):

$$\frac{2P_2k_2(G_a-g_{a1})}{(G_a+G_b-g_{a1}-g_{b1})^3} - \frac{2P_2k_2}{(G_a+G_b-g_{a1}-g_{b1})^2} = \frac{2P_2k_2}{(G_a+G_b-g_{a1}-g_{b1})^2} \times \left(\frac{G_a-g_{a1}}{G_a-g_{a1}+G_b-g_{b1}} - 1\right) \tag{94}$$

As all the variables are positive, $G_a - g_{a1} \geq 0$ and $G_b - g_{b1}$, the expression $\frac{2P_2k_2}{(G_a+G_b-g_{a1}-g_{b1})^2}$ must be also positive. The expression $\frac{G_a-g_{a1}}{G_a+G_b-g_{a1}-g_{b1}} \leq 1$, as $G_a - g_{a1}$ and $G_b - g_{b1}$ are positive values. So the expression $\frac{G_a-g_{a1}}{G_a-g_{a1}+G_b-g_{b1}} - 1$ must be negative and also the expression $\frac{2P_2k_2}{(G_a+G_b-g_{a1}-g_{b1})^2} \times \left(\frac{G_a-g_{a1}}{G_a-g_{a1}+G_b-g_{b1}} - 1\right)$. Then we know that the first 2 and the last 2 terms of $\frac{d^2\pi_a}{dg_{a1}{}^2}$ represent a negative value. So $\frac{d^2\pi_a}{dg_{a1}{}^2}$ is negative for all possible values of g_{a1} and g_{b1}. This indicates that the point where the first derivative of the function is equal to zero is a global maximum as there is no point where the curve changes the concavity form concave to convex, as the second derivative is always negative within the range of possible values of physical guarantee.

Using the same approach, we can prove that $\frac{d^2\pi_b}{dg_{b1}{}^2}$ is negative for all the possible values of values of g_{a1} and g_{b1} considering the constraints for the optimization problem. Therefore, the solution for the seasonalization problem expressed by equation (85), equation (86) (optimal solution for player 'a'), equation (89) and equation (90) (optimal solution for player 'b') is also the Nash Equilibrium as it represents the set of strategies in which, each player cannot individually get their individual result (payoff) better by changing the strategy, unilaterally.

The Nash Equilibrium for the physical guarantee seasonalization game with 2 players and 2 periods is shown in Table 6:

	Period 1 Physical Guarantee	Period 2 Physical Guarantee
Player a	$G_a \times \frac{P_1k_1}{P_1k_1 + P_2k_2}$	$G_a \times \frac{P_2k_2}{P_1k_1 + P_2k_2}$
Player b	$G_b \times \frac{P_1k_1}{P_1k_1 + P_2k_2}$	$G_b \times \frac{P_2k_2}{P_1k_1 + P_2k_2}$

Table 6 – Nash Equilibrium for the physical guarantee seasonalization game

It is possible to calculate the GSF for the Nash Equilibrium for period 1 and period 2 using the physical guarantee from the Nash Equilibrium and the equation (71) for $n = 2$. The equation (95) and equation (96) show the GSF for period 1 and period 2 respectively:

$$GSF_1 = \frac{k_1}{g_{a1}{}^* + g_{b1}{}^*} = \tag{95}$$

$$= \frac{k_1}{G_a \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2} + G_b \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2}}$$

$$= \frac{k_1}{(G_a + G_b) \times \frac{P_1 k_1}{P_1 k_1 + P_2 k_2}} = \frac{P_1 k_1 + P_2 k_2}{(G_a + G_b) \times P_1}$$

$$GSF_2 = \frac{k_2}{g_{a2}{}^* + g_{b2}{}^*} = \tag{96}$$

$$= \frac{k_2}{G_a \times \frac{P_2 k_2}{P_1 k_1 + P_2 k_2} + G_b \times \frac{P_2 k_2}{P_1 k_1 + P_2 k_2}}$$

$$= \frac{k_2}{(G_a + G_b) \times \frac{P_2 k_2}{P_1 k_1 + P_2 k_2}} = \frac{P_1 k_1 + P_2 k_2}{(G_a + G_b) \times P_2}$$

3.2.3
The energy equivalent prices

When the GSF is below one for a certain period, it means that the system was not capable of generating the total energy to supply the physical guarantee allocated during that period. This will affect the payoff, as the physical guarantee allocated for the period is multiplied by the GSF value when the payoff is calculated, as shown in equation (72). So, even if the player allocated the exact amount of the contracted energy, the GSF below 1 will cause a shortfall, as explained previously. Therefore, the actual revenue does not depend only on the price and the allocated physical guarantee but the price in conjunction with the GSF. The energy price multiplied by the GSF and multiplied by the physical guarantee is the actual revenue that the player expects to receive when he allocates the physical guarantee to a month. We can interpret that the GSF multiplied by the energy price for the month is the actual price that the player will deal for the allocated physical guarantee, that we will denote Equivalent Price (EP). The payoff function in terms of EP is shown in equation (97):

$$\pi_i = \sum_{j=1}^{12} P_j \cdot \left[\left(g_{ij} \times GSF_j \right) - C_{ij} \right] \tag{97}$$
$$= \sum_{j=1}^{12} P_j \times g_{ij} \times GSF_j - P_j \times C_{ij}$$
$$= \sum_{j=1}^{12} EP_j \times g_{ij} - P_j \times C_{ij}$$

The Equivalent Price of Nash Equilibrium has an interesting feature, which will help in obtaining the optimal solution to the problem with different periods and different participants: for all periods, the equivalent price is the same. The interpretation is that the equivalent prices need to be equal because, otherwise, there would be an opportunity for arbitration on the part of one or more players, and with this, an opportunity to individually improve the result, which cannot occur in the Nash Equilibrium. Therefore, the equivalent prices need to be equal to guarantee the Nash Equilibrium and avoid any opportunity for a unilateral movement by any player, which would improve their result. For the simplified example featuring two time periods (1 and 2) and involving two participants, 'a' and 'b', the equivalent prices for period 1 and period 2 are shown in equation (98) and equation (99), respectively:

$$EP_1 = P_1 \times GSF_1 \tag{98}$$
$$= P_1 \times \frac{P_1 k_1 + P_2 k_2}{(G_a + G_b) \times P_1} = \frac{P_1 k_1 + P_2 k_2}{G_a + G_b}$$

$$EP_2 = P_2 \times GSF_2 \tag{99}$$
$$= P_2 \times \frac{P_1 k_1 + P_2 k_2}{(G_a + G_b) \times P_2} = \frac{P_1 k_1 + P_2 k_2}{G_a + G_b}$$

The same approach used to determine the equivalent prices for n players and m periods. The equivalent price for the general case is determined by equation (100):

$$EP_j = P_j \times GSF_j \tag{100}$$
$$= P_j \times \frac{\sum_{j=1}^{m} P_j k_j}{(\sum_{i=1}^{n} G_i) \times P_j} = \frac{\sum_{j=1}^{m} P_j k_j}{\sum_{i=1}^{n} G_i}$$

3.2.4 Physical guarantee seasonalization for the SIN

The equivalent prices are the key to the optimization of physical guarantee seasonalization for the Brazilian System. As the equivalent prices are the same for all the periods when the players choose the Nash Equilibrium strategies, we can calculate the equivalent price of Nash Equilibrium (EP^*) using equation (100) and them determinate the GSF for each month (equation (101)):

$$GSF_j = \frac{P_j}{EP^*} \tag{101}$$

Then we can use equation (71) to determine the total physical guarantee (Γ_j) allocated to each month (equation (102)):

$$GSF_j = \frac{P_j}{EP^*} = \frac{\sum_{i=1}^{n} \tau_{ij}}{\sum_{i=1}^{n} g_{ij}} \therefore \tag{102}$$
$$\frac{P_j}{EP^*} = \frac{k_j}{\sum_{i=1}^{n} g_{ij}} \therefore$$
$$\sum_{i=1}^{n} g_{ij} = \frac{k_j \times EP^*}{P_j}$$
$$\Gamma_j = \frac{k_j \times EP^*}{P_j}$$

The physical guarantee allocated by each player can be easily determined, as we know the contribution of each player to the total physical guarantee. The procedure is to determine, for the player j, the percentage of the total physical guarantee ($\%TFG_i$) using equation (103) and then apply the same percentage to each month to find the seasonalized physical guarantee for the player.

$$\%TFG_i = \frac{G_i}{\sum_{i=1}^{n} G_i} \tag{103}$$

Then monthly physical guarantee that the player will allocate is determined by equation (104) below:

$$g_{ij} = \%TFG_i \times \frac{k_j \times EP^*}{P_j} \tag{104}$$

3.3 Seasonalization game for the Brazilian interconnected system

To apply the suggested game approach to the Brazilian Interconnected System (*SIN-Sistema Interligado Nacional*) we need to estimate the settlement price of the differences (PLD) for each month and the generation of the hydroelectric power plants in the system. The generation of hydroelectric plants in the system can be obtained by difference. There are other sources of energy that contribute to the system generation, so the hydroelectric plants generation is obtained subtracting the generation of other sources of energy from the system energy load. The agents' monthly income will depend on the generation of the hydroelectric plants (η_j), and also from the result of the GSF and the decision to seasonalize each agent. The generation for each month j can be determined using equation (105).

$$\eta_j = \varepsilon_j - \left(\zeta_j + \omega_j + \beta_j + \phi_j + s_j\right) \tag{105}$$

where ε_j is the energy load, ζ_j is the thermal generation, ω_j is the wind generation, β_j is the biomass generation, ϕ_j is the solar generation, s_j represents the generation of small hydroelectric plants.

The GSF for month j is obtained from the division of the generation (η_j) by the seasonalized physical guarantee (Γ_j) for the whole system, as shown in equation (106). The seasonalized physical guarantee for the system is determined by the sum of seasonalized physical guarantees of each agent as shown in equation (107).

$$GSF_j = \frac{\eta_j}{\Gamma_j} \tag{106}$$

$$\Gamma_j = \sum_{i=1}^{n} (g_{ij}) \tag{107}$$

The generation needs to be estimated for each period and to estimate it, is necessary to estimate the energy load, the thermal generation, the wind generation, the biomass generation, the solar generation and the generation of small hydroelectric plants (*PCH – Pequenas Centrais Hidrelétricas*). There is an energy load forecast that is public and done by ONS. However, for this work, a model of Self-regressive Integrated Mobile Average (ARIMA) forecast with an exogenous variable was adjusted to identify the occurrence of regime change. The periods of crisis generate a demand shock and cause generation to decrease substantially, lowering energy prices sharply.

The other parameters to estimate the gross generation (thermal generation, wind generation, biomass generation, solar generation, and generation of small hydroelectric plants) are obtained from the reports published by the Electrical Energy Clearing Chamber and EPE. These reports are public and periodically updated by the entities responsible for managing the system. After the load forecast and the consolidation of data regarding other energy sources, the next step is to determine, for each month, the generation of the hydroelectric plants using the equation (105).

Then we apply the equation 100 to determine the equivalent prices, that will be the same for each month. After that we calculate the GSF for each month, using equation 101. The seasonalized physical guarantee for the system is determined using equation 102. Equation 103 is used to determine the percentage of the physical guarantee that is added by each system participant. Finally, the equation 104 is used to determine the seasonalization of the physical guarantee for each month for each participant.

We can also consider the opportunity cost by adding a monthly intertemporal discount rate (r) (equation (108)). For the numerical application we decided to not use de discount rate as the difference of equivalent prices would small between months.

$$GSF_j \times PLD_j = \frac{GSF_{j+1} \times PLD_{j+1}}{1 + r}, \qquad j = 1, \dots, 11 \tag{108}$$

1.1.3.4
Numerical application

For the numerical application we selected an important player in Brazil. The application of our optimization model was carried out in collaboration with the company's management team, ensuring close oversight throughout the process. To tailor the model to our specific needs, we initiated the process by utilizing load forecasts for the Brazilian Interconnected System (SIN). These forecasts were divided into two distinct time series: one for training the forecasting model and the other for testing its performance. The training series covers the period from January 1999 to December 2017, while the test series spans from January 2018 to April 2020.

We conducted the model fitting using Python 3.8 (64-bit) software, employing the pmdarima 1.6.0 module and the auto_arima function. The selection of the optimal model was based on the Akaike Information Criterion (AIC), where the model with the lowest AIC was chosen. In this case, the selected model was SARIMA with the following parameters: order = (0, 1, 0) and seasonal_order = (1, 0, 1, 12).

The remaining parameters necessary for estimating gross energy generation were sourced from reports published by the CCEE (*Câmara de Compensação de Energia Elétrica*) and EPE (*Empresa de Pesquisa Energética*).

The energy load forecast, generated through our SARIMA model, served as input for calculating the generation of hydroelectric plants (η_j) using equation 105. Table 7 presents the load (ε_j) forecasts produced by our SARIMA model, as well as other forecasts obtained from CCEE and EPE, including thermal generation (ζ_j), wind generation (ω_j), biomass generation (β_j), solar generation (ϕ_j), and small hydroelectric plants generation (s_j) (CCEE, 2020; EPE, 2020c; ONS, 2020). These forecasts were generated on December 5th, 2020.

	Jan	Feb	Mar	Apr	Mai	Jun
ε	71,139	72,152	72,017	70,687	69,406	68,879
ζ	10,901	8,139	8,152	6,190	6,273	6,982
ω	6,423	4,544	4,480	5,093	6,584	8,388
β	824	760	798	771	783	816
ϕ	1,790	1,760	2,150	3,929	4,902	5,286
s	1,511	1,527	1,548	1,397	1,306	1,193
$\boldsymbol{\eta}$	**49,690**	**55,422**	**54,889**	**53,307**	**49,558**	**46,214**

	Jul	Aug	Sep	Oct	Nov	Dec
ε	68,855	69,665	70,439	71,066	71,119	71,039
ζ	7,206	7,974	8,418	8,070	7,619	6,802
ω	8,851	10,136	10,269	9,650	9,003	7,711
β	843	922	1,027	970	950	968
ϕ	5,520	5,628	5,548	5,215	4,586	2,959
s	1,011	883	884	1,060	1,303	1,447
$\boldsymbol{\eta}$	**45,424**	**44,122**	**44,293**	**46,101**	**47,658**	**51,152**

Table 7 – The energy load forecast and estimation for system generation (Avg MW)

The PLD generation forecast was conducted using the NEWAVE software, a government-developed tool accessible to all agents. The forecast generated is the same for all agents, given the same inputs. We used the 2000 series forecasted for PLD by NEWAVE and calculated the expected spot price for each month. We used expected PLD values (E(P)) obtained from NEWAVE generated on December 5th, 2020 (Table 8).

	Jan	Feb	Mar	Apr	Mai	Jun
E(P)	255.48	206.30	179.50	165.74	142.68	161.54
Hours (H)	744	672	744	720	744	720

	Jul	Aug	Sep	Oct	Nov	Dec
E(P)	202.48	207.25	221.74	233.68	219.20	151.66
Hours (H)	744	744	720	744	720	744

Table 8 – NEWAVE PLD expected values (E(P)) (BRL/MWh) for 2021

Note: Forecast generated on December 5th, 2020.

In our optimization process for the year 2021, we employed an average total physical guarantee of 58,202 megawatts. This value was sourced from MRE (EPE, 2020a). It's essential to note that the estimated gross generation, which stands at an average of 48,945 megawatts, falls short of the total system's physical guarantee. This shortfall indicates an overall deficit within the system, which is poised to have adverse implications on the results and payoffs for system participants.

We applied the procedure described in last section and set upper and lower boundaries of physical guarantee using MRE data to not exceed maximum and minimum seasonalization restrictions.

The seasonalized physical guarantee Γ_j, the GSF for each month and the equivalent prices (EP) are shown in Table 9:

	Jan	Feb	Mar	Apr	Mai	Jun
Γ	77,530	69,827	60,172	53,958	43,183	45,593
GSF	0.64	0.79	0.91	0.99	1.15	1.01
EP	163.74	163.74	163.74	163.74	163.74	163.74

	Jul	Aug	Sep	Oct	Nov	Dec
Γ	56,171	55,846	59,982	65,792	63,800	51,152
GSF	0.81	0.79	0.74	0.70	0.75	1.08
EP	163.74	163.74	163.74	163.74	163.74	163.74

Table 9 – Seasonalization of physical guarantee outcomes

Note: seasonalized physical guarantee (Γ) (Avg MW) GSF for each month and equivalent prices (EP) (BRL/MWh) obtained from optimization process.

The seasonalized physical guarantee (Γ_j) for the system, as well as the Generation Scaling Factor (GSF) and equivalent prices (EP) derived from the optimization process, serve as crucial inputs for conducting the seasonalization process by the agents (players). The chosen player for the numerical application possesses an installed capacity of 700 average Megawatts, with a corresponding physical guarantee of 500 average Megawatts. By considering the proportion of the total physical guarantee contributed by this player to the entire system, we can determine the optimal allocation of physical guarantee for the player, along with the associated monthly payoffs in BRL MM. These results are outlined in Table 10, considering the contracted amount (C) for the player.

	Jan	Feb	Mar	Apr	Mai	Jun
Γ	666	600	517	464	371	392
C	475	450	450	390	390	390
π	-9.15	3.62	2.88	8.11	3.79	0.82

	Jul	Aug	Sept	Oct	Nov	Dec
Γ	483	480	515	565	548	407
C	400	430	430	430	430	425
π	-1.47	-7.86	-7,9	-5.9	-3.25	1.63

Table 10 – Physical guarantee allocation (Avg MW), amount contracted (C) (Avg MW) and the monthly payoffs (BRL MM)

The player total payoff for the seasonalization game is BRL -14.68 MM. This value is consistent with prediction for the system condition, where the gross generation will be substantially lower than the physical guarantee.

The Figure 12 shows the seasonalized physical guarantee, the contracted amount, and the GSF-corrected seasonalization. The GSF-corrected seasonalization is obtained multiplying the physical guarantee allocated to the month by the computed GSF for that month. It represents the revenue of the player, in terms of average MW, already considering the GSF effects. The results consider the optimization of the equivalent price for the whole system.

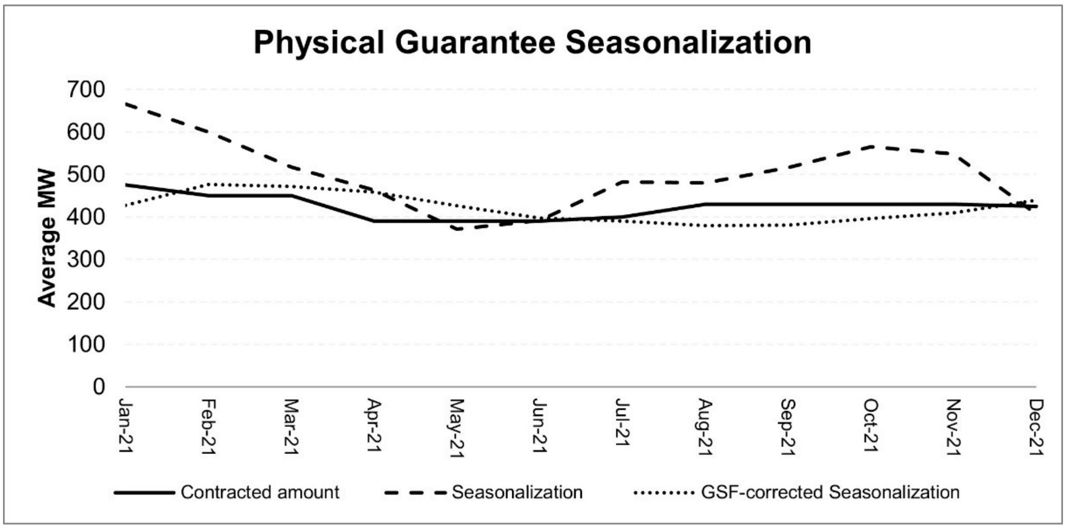

Figure 12 – Seasonalization of physical guarantee decision for an actual player of the Brazilian market

3.2
Conclusion

The seasonalization of physical guarantees is a critical annual process for hydroelectric plants. It grants agents the flexibility needed to align their physical guarantees with their contractual obligations. This process resembles a strategic game, where individual decisions impact other Market Risk Exposure (MRE) participants by influencing the Gross Settlement Figure (GSF), a factor that directly affects participants' revenue.

Typically, risk-averse participants tend to align their seasonalization process closely with their contract profiles. This cautious approach aims to minimize the risk associated with variations in the PLD due to discrepancies between the physical guarantee and the contract terms. However, this strategy doesn't optimize outcomes nor fully shield participants from GSF-related risks.

Our research reveals an optimal strategy that simultaneously maximizes a player's results while safeguarding them from potential reductions in payoffs resulting from other participants' actions. This strategy revolves around achieving equal equivalent prices, where the equivalent price for a given month is determined by multiplying the GSF by the PLD. Remarkably, this strategy, which optimizes payoffs, is also the most conservative and is advisable even for risk-averse participants.

In our study, we applied this optimization model to a key market player. We designed a proprietary model to predict energy demand and estimate gross energy generation, employing a Seasonal Autoregressive Integrated Moving Average (SARIMA) model with dummies. The player authorized and monitored our model's application, and the results significantly informed their seasonalization process for the year 2021.

Regulatory considerations remain a pivotal concern, given the sector's evolving landscape. As the industry undergoes substantial changes, the mechanisms for risk control and sharing may also evolve. Consequently, adjustments to our proposed model may become necessary soon.

For future research, we propose evaluating the seasonalization game formulation using utility as a payoff metric, which would be determined based on a participant's risk profile, rather than focusing solely on financial returns. Additionally, exploring the existence of a Nash equilibrium and identifying dominant strategies within this context could further enhance our understanding of this complex process. In future research, it also may be valuable to examine the equilibrium by considering energy load and PLD forecasts as stochastic variables. This approach could potentially yield novel insights distinct from those presented in this current study.

4
Optimizing Electrical Energy Commercialization: A Decision-Making Model for Hedge Transactions

4.1
Introduction

In the 1990s, a transformative wave of restructuring swept through the global electricity sector, ushering in the era of competitive electricity markets (Joskow, 2006). In Brazil, this transformation took shape with the launch of the Restructuring Project of the Brazilian Electricity Sector (*Reestruturação do Setor Elétrico Brasileiro* – RE-SEB Project), spearheaded by the Ministry of Mines and Energy (MME). This groundbreaking initiative led to the creation of a free energy marketplace, uniting generators, marketers, and energy-conscious consumers. Subsequently, the Free Contracting Environment emerged, allowing these stakeholders to freely negotiate bilateral contracts for energy procurement and sales, all within the bounds of established regulations and guidelines (CCEE, 2021).

Given the inherent volatility of electricity spot prices, market participants naturally seek ways to mitigate their exposure to price fluctuations (Luz et al., 2012; Matsumoto & Yamada, 2021) by negotiating forward contracts. Conversely, in certain countries, market restructuring has given rise to broader free markets and energy exchanges, where a wide array of energy derivatives is traded for hedging purposes.

Within the electricity sector, including players such as generators, traders, and consumers, it's not uncommon to find individuals or entities in a short position due to various factors. This situation can arise for a multitude of reasons, including power generation shortfalls due to low levels of Natural Energy Inflow (*Energia Natural Afluente – ENA*) or reservoir level reductions during droughts (in the case of hydroelectric generators), uncertainty in wind speed (for wind generators), speculative motives (for traders), and construction delays in new power plants. Consequently, these participants are compelled to engage in spot market energy purchases to meet their contractual commitments, thereby exposing themselves to price risk.

One viable strategy for these participants is to hedge against risk by entering forward contracts, allowing them to secure part or all of their position at a predetermined price. If the hedge is only partial, the remaining balance must be settled at the spot price upon maturity, referred to as the Price Differences Settlement (*Preço de Liquidação das Diferenças – PLD*), which exposes the participant to energy price variations. Conversely, a full hedge eliminates this risk but also precludes the possibility of capitalizing on favorable spot market price movements.

This article endeavors to scrutinize the decision-making process concerning hedge operations, with the goal of maximizing the participant's profits while maintaining a certain level of risk protection. To this end, we assume that the participant's risk aversion can be quantified through α percentiles of Value at Risk (VaR). As such, the primary contribution of this study is the development of a decision support tool tailored to market participants in a short position in the electricity market, considering their willingness to pay for risk protection and the associated transaction costs.

The structure of this article is as follows: after this introduction, we present an extensive review of the relevant literature in the field. Section 3 delves into the development of a decision-making model for hedge transactions in electrical energy commerce. In Section 4, we present a numerical application and engage in a comprehensive discussion of the results. Finally, we conclude our analysis.

4.2
Utility functions and risk hedging in electricity markets: a literature review

According to Oliveira, Arfux, and Teive (2006), in a competitive electricity market environment, risk analysis is an essential tool to guide investors in the decision-making process, considering both contract uncertainties and spot market energy prices. In their study, they propose three risk measures: mean variance, maximum loss, and maximum average loss, applied to the issue of energy commercialization for investment analysis purposes. The results indicate that these measures complement the techniques presented by Markowitz (1952) and the theories of Value at Risk (VaR) and Conditional Value at Risk (CVaR), thereby improving the quality of decision-making in the field of energy commercialization.

Luz (2016) asserts that in the electricity market, it is necessary to determine performance and risk measures that can assist in decision-making and to extract functions from these measures for the calculation of risk premiums, which can serve as a guide for pricing hedge operations. Following this rationale, Benth, Cartea, and Kiesel (2008) provide a framework that enables us to explain how the risk preferences of market players explain the sign and magnitude of the market risk premium across different forward contract maturities. When considering the German electricity market, the authors observe that the producer's market power and the market risk premium exhibit a term structure that decreases as the time to maturity of the forward contract increases.

Deng and Oren (2006) argue that exposure to energy price risks can generate devastating consequences for agents in the electricity sector. The authors analyze different types of financial instruments that allow the sharing and control of these risks through appropriately structured hedge strategies and conclude that the main challenge in the electricity market is to increase its liquidity, using derivatives for economic efficiency. On the other hand, Pineda, Conejo, and Carrión (2010) analyze a specific financial instrument that is widely used as risk mitigation tool: the insurance contracts. They evaluate the convenience of signing an insurance contract against unexpected failures in electricity production units and their impact on contracting decisions through a stochastic programming model. Their findings indicate that insurance reduces financial risk and that the greater the risk aversion, the greater the value (premium) that the producer is willing to pay for a particular insurance contract.

Other studies propose their own risk mitigation strategies. For example, Cotter and Hanly (2010) developed a GARCH-in-Mean model to estimate a time-varying coefficient of relative risk aversion based on the observed risk preferences of both short and long energy hedgers. Their empirical results show that, when explicit risk aversion is taken into consideration, there are significant differences in expected utility and risk-minimizing hedge strategies. Additionally, they find that, in general, long hedgers are more risk-averse than short hedgers. Woo, Horowitz, and Hoang (2001) also developed an innovative tool to address the challenge of a risk-averse energy trader who offers a fixed-price forward contract to provide electricity purchased from a potentially volatile and unpredictable fledgling spot energy market. This tool is based on a cross-hedging strategy that reduces the contract's profit variance and determines the forward-contract price as a risk-adjusted price.

Oum and Oren (2010) proposed a method to mitigate volumetric risk faced by regulated load-serving entities (LSEs) and traders of default service contracts when providing load-following services to their customers at fixed or regulated prices while purchasing electricity or facing an opportunity cost at volatile wholesale prices. They extended the work developed by Oum and Oren (2009) and presented a static hedging strategy for the LSE's retail positions using electricity standard derivatives such as forwards, calls, and puts. The authors determined the optimal hedging strategy based on expected utility maximization, and through several numerical examples, they demonstrated the impact of each price-based financial energy instrument on portfolio optimization.

Zhang and Wang (2009) assert that effective risk management fosters well-functioning electricity markets. They demonstrate, through a risk-constrained electricity procurement model based on the CVaR methodology, that hedge contracts, serving as price protection products, offer customers financial or physical insurance against their exposure to energy price risk. This comes in exchange for a risk premium they are willing to pay. Since a utility function is a suitable tool to consider customer preferences regarding decisions under uncertainty, in a recent study, Niromandfam, Yazdankhah, and Kazemzadeh (2020) apply principles of the utility function to identify the preferences and behavior of agents in the electricity sector concerning various risk hedging contracts. Their results suggest that the proposed risk hedging mechanism reduces the average market price of electricity and its fluctuations, enabling customers to manage electricity costs effectively.

In a study closely related to ours, Cotter and Hanly (2012) also emphasize that a key issue in the estimation of energy hedges is the hedgers' attitude towards risk, which is encapsulated in the form of the hedgers' utility function. They address this issue by estimating and applying energy market-based risk aversion to three different utility functions: quadratic, exponential, and logarithmic. Their results show that significant differences exist between hedge strategies depending on the risk attitudes of energy hedgers as represented by different utility functions. Like these authors, we extend the literature in this field and address the issue of how an agent in the electricity sector, exposed to variations in energy prices, can optimally hedge their commercialization decision. However, we propose a tool to support the hedge decision for these agents, considering a preference function that allows modeling the variation of the risk aversion level of an agent across different preference bands.

4.2 Quantifying risk aversion: a preference function approach

We present a novel model founded upon a preference function, facilitating the exploration of variations in an agent's risk aversion level across different preference bands. Central to this model is the imperative task of risk assessment, achieved through the utilization of measures such as VaR (Value at Risk), CVaR (Conditional Value at Risk), and ES (Expected Shortfall). In accordance with Jorion's seminal work (1996), $CVaR_\alpha$ represents the expected loss beyond the VaR_α threshold, which is established as the maximum acceptable risk level at a given confidence level α.

Despite its widespread adoption, VaR has encountered criticism for its limitations, particularly in instances where it fails to adequately account for sub-aggregation issues arising from specific conditions within the financial position distribution.

The equation (109) and equation (110) provide definitions for VaR_α and $CVaR_\alpha$, respectively, emphasizing their absolute values. It is noteworthy that in many cases, these definitions are conventionally presented with values in their absolute form, as they inherently denote negative values, signifying potential losses from a risk perspective.

$$VaR_\alpha(X) = inf\{m|P[X - m > 0] \leq \alpha\} \qquad (109)$$
$$= inf\{m|P[X - m < 0] \leq 1 - \alpha\}$$

$$CVaR_\alpha(X) = E[X|X \leq VaR_\alpha] \qquad (110)$$
$$= \frac{1}{1-\alpha}\int_{-\infty}^{VaR_\alpha} xf(x)dx = \frac{1}{1-\alpha}\int_{0}^{1-\alpha} VaR_u(X)du$$

where $\alpha \in [0,1[$ and $m \in \mathbb{R}$.

We used the preference functional introduced by Luz (2016), enabling us to model the fluctuations in an agent's risk aversion level while accommodating various preference bands. The preference functional is formally defined within equation (111):

$$ECP\_G_{\alpha,\lambda} = E[u(\tilde{x})] = \lambda_0 E(\tilde{x}) + \sum_{n=1}^{N} \lambda_n CVaR_{\alpha_n}; \quad (111)$$
$$\lambda_n \geq 0 \; and \; \sum_{n=1}^{N} \lambda_n = 1$$

Considering that the agents are in a short position in the electricity market, the financial position X can be expressed by equation (112):

$$X = \sum_{t=1}^{\tau} [(-(1-\delta)\pi_t - \delta\phi_t + \chi)\upsilon_t \eta_t] \quad (112)$$

where δ represents the percentage of the purchase decision of the hedge transaction; π_t is the spot energy price (BRL/MWh); ϕ_t is the future price estimated by the forward energy price curve (BRL/MWh); χ is the opportunity cost (BRL/MWh); υ_t is the uncontracted amount (MW); and η_t is the number of hours in month t.

From the utility function developed by Luz (2016), we can define the Certainty Equivalent (φ) and the Risk Premium (γ), which are presented, respectively, in equation (113) and equation (114). The Certainty Equivalent reflects the situation of the agent's indifference between hedging and being exposed to the energy price risk. On the other hand, the Risk Premium is the difference between the average of the financial position X and the Certainty Equivalent, which represents the premium required by the hedge transaction.

$$\varphi = U^{-1}(E[U(X)]) = U^{-1}(\lambda_0 E[X] + \sum_{n=1}^{N} \lambda_n CVaR_{\alpha_n}(X)) \tag{113}$$

$$\gamma = E[X] - \varphi \tag{114}$$

To parametrize the ECP_G function, we use the AHP method, since it is based on the decomposition and synthesis of the peer-to-peer relationships among criteria. This method seeks to prioritize alternatives by distilling them into a single measure of performance, as elucidated by Saaty (1991). The AHP method offers several notable advantages, including its flexibility, simplicity, intuitive appeal to decision-makers, and the hierarchical organization of criteria based on their assigned attributes, as corroborated by Ishizaka and Labib (2011), Macharis et al. (2004), and Ramanathan (2001).

The AHP methodology recommends a systematic approach, beginning with the identification of the problem, the delineation of its objectives, the enumeration of available alternatives, the specification of decision-making criteria, and the designation of decision-makers. Subsequently, it involves a critical phase in which decision-makers assess the relative importance of each criterion in relation to the others, culminating in the construction of a judgment matrix. Finally, normalization of these judgments is carried out, yielding weight matrices that assign values to pairwise comparisons (preference matrices) and to each of the criteria initially defined in the process.

In assessing the validity of the weight matrices, Saaty (1991) introduced two crucial metrics: the average consistency index (CI) and the consistency ratio (CR). According to the author's stipulation, if the resulting consistency ratio falls within or below the 10% threshold, it indicates acceptable consistency within the pairwise comparison matrix, affirming the validity of the derived weights for practical use. The equation (115) and the equation (116) provide the formal expressions for these pivotal measures:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{115}$$

$$CR = \frac{CI}{RI(n)} \quad (116)$$

where λ_{max} is the major eigenvalue of the judgment matrix, n is the order of the judgment matrix and $RI(n)$ is the random consistency index for matrices of order n, which approximate the results found by Saaty (1991), described in Table 11:

n	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$RI(n)$	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Table 11 – Random consistency indices

Source: Saaty (1991)

To illustrate the application of the proposed model, we employ a numerical example focusing on a prominent player in the Brazilian electricity sector. This player faces a critical hedge decision for the latter half of the year. We explore two distinct decision scenarios: (a) making the hedge choice in July to cover exposure until September and subsequently making another hedge decision in October to safeguard against year-end risks, and (b) opting to hedge in July to mitigate exposure throughout the entire latter half of the year. Our goal is to determine the optimal hedge level that simultaneously maximizes the agent's profits while ensuring a predefined level of risk protection for each of these temporal blocks: July to September, October to December, and July to December.

Initially, we consider a fixed opportunity cost of 201.00 (BRL/MWh) alongside the values detailed in Table 12:

Month	υ	ϕ	η
July	-35.44	325.00	744
August	-61.30	325.00	744
September	-66.51	325.00	720
October	-92.54	310.00	744
November	-75.90	310.00	720
December	-59.32	310.00	744

Table 12 – Input values for the numerical application

Note: υ is the uncontracted amount (MW); ϕ is the forward price (R$/MWh); and η is the number of hours

Furthermore, as a proxy for energy prices in the spot market, we consider the Price for Load Dispatch (PLD) determined by the CCEE. In this context, we rely on the monthly simulation conducted by the National System Operator for the year 2021, specifically focusing on the energy market within the Southeast/Center-West region of Brazil. This submarket represents the highest demand and liquidity within the National Interconnected System. The pertinent data used in this analysis is summarized in Table 13:

Blocks	Start	End	$\bar{\pi}$	$\bar{\upsilon}$
1	July	September	350.05	-54.29
2	October	December	230.72	-75.92
3	July	December	290.39	-65.10

Table 13 – Summary data

Note: $\bar{\pi}$ represents the average spot energy price in each block (BRL/MWh) and $\bar{\upsilon}$ is the weighted average of the uncontracted amount in each block (MW average).

Next, we establish two target risk levels: one representing intermediate risk and the other signifying extreme risk. Our approach defines intermediate risk as the expected outcome of the worst 30% of scenarios, while extreme risk encapsulates the worst 5% of outcomes. In essence, we treat intermediate risk as the expected value of negative results, and extreme risk as the expected value of results that fall below the threshold of maximum bearable loss. Consequently, we determine the following values: $N = 3$, $\alpha_1 = 70\%$, and $\alpha_2 = 90\%$.

These intermediate and extreme risk levels, along with the expected value of outcomes, serve as the criteria for comparison and prioritization in selecting the optimal strategy to evaluate operational performance. To facilitate this decision-making process, we apply the Analytic Hierarchy Process (AHP) method employing a 9-point Saaty scale. The Table 14 outlines the judgment matrix utilized in this analysis:

	Extreme Risk	Intermediate Risk	Expected Value
Extreme Risk	1.00	1.50	3.00
Intermediate Risk	0.67	1.00	1.50
Expected Value	0.33	0.67	1.00
Total	**2.00**	**3.17**	**5.50**

Table 14 – Matrix of Judgments

The values adopted in this matrix were defined by the team responsible for the hedge decisions of this major agent that we are analyzing based on their experiences in the energy sector. For more details on AHP applications, we recommend Ishizaka and Labib (2011) and Rafaeli and Müller (2007).

After this, we determine the weight matrix (Table 15), which is calculated from the division of each value defined in the matrix of judgments by the sum found in each column.

	Extreme Risk	Intermediate Risk	Expected Value
Extreme Risk	0.50	0.47	0.55
Intermediate Risk	0.33	0.32	0.27
Expected Value	0.17	0.21	0.18

Table 15 – Matrix of weights

Subsequently, we proceed to compute the criterion weight and consistency values, as outlined in Table 16. Criterion weight values are determined as the mean of each row in the weight matrix. Meanwhile, the consistency values are calculated by multiplying the judgment matrix by the criterion weight values and then dividing by the criterion weight found in each row.

	Criterion Weight	Consistency
Extreme Risk	0.51	3.01
Intermediate Risk	0.31	3.01
Expected Value	0.19	3.01

Table 16 – Criterion weight and consistency

As a result of these calculations, we derive an average consistency index (CI = 0.0046) and subsequently, the consistency ratio (CR = 0.79%). It's worth noting that the consistency ratio falls well below the 10.00% threshold, indicating that the pairwise comparison matrix exhibits acceptable consistency, thus affirming the validity and usability of the derived weights. Furthermore, Table 17 provides the values assigned to probability, along with the corresponding probabilistic weights, calculated as the ratio of each probability to their collective sum.

	Criterion Weight	Probabilistic Weight
Extreme Risk	0.05	0.04
Intermediate Risk	0.30	0.22
Expected Value	1.00	0.74

Table 17 – Probability and probabilistic weight

From these calculations, we emphasize the significance of the criteria by weighing each criterion and multiplying it by its respective probabilistic weight. It's essential to note that by normalizing these values, we derive the following weights: $\lambda_0 = 61.33\%$, $\lambda_1 = 30.34\%$, and $\lambda_2 = 8.33\%$. With this information, we proceed to optimize equation (112) to estimate the influence of the hedge transaction on each of the analyzed blocks in the purchase decision.

Subsequently, we utilize equations (113) and equation (114) to determine the Certainty Equivalent, the Risk Premium, and the value that the agent is willing to allocate for the hedge in each block. The primary results are summarized in Table 18.

	$\boldsymbol{\delta}$ %	**Mean BRL x1000**	$\boldsymbol{CVaR_{70\%}}$ **BRL x1000**	$\boldsymbol{CVaR_{95\%}}$ **BRL x1000**	$\boldsymbol{\varphi}$ **BRL x1000**	$\boldsymbol{\gamma}$ **BRL x1000**	$\boldsymbol{\omega}$ **BRL /MWh**
1	100.0	-14,864	-14,863	-14,863	-14,863	0.00	350
2	0.0	-6,435	-37,309	-58,621	-19,022	75.09	306
3	45.3	-27,845	-55,185	-71,286	-37,789	34.59	325

Table 18 – Results

Note: In this context, δ represents the optimal percentage for the purchase decision in the hedge transaction, while "Mean" denotes the arithmetic mean of the financial position X, as defined in equation (4). CVaR70% signifies the average loss in the worst 30% of case scenarios for the financial position X, and CVaR95% represents the average loss in the most adverse 5% of case scenarios for this financial position. Furthermore, ϕ (phi) symbolizes the Certainty Equivalent, γ (gamma) represents the Risk Premium, and ω (omega) signifies the value the agent is willing to pay for the hedge. This value, ω, is determined as the sum of the average spot energy price and the Risk Premium.

4.3
Conclusions

Electricity market participants often grapple with short positions, exposing them to the volatilities of energy prices, which could lead to financial losses. One avenue to mitigate this risk involves hedging these positions through forward contracts, allowing agents to secure a fixed price for all or part of their short exposure.

With this in mind, we introduce a model designed to assist in the hedge decision-making process, with the overarching goal of optimizing an agent's profits while maintaining a desirable level of risk protection. To achieve this, we factor in the agent's risk aversion, quantifiable through α percentiles of Value at Risk (VaR). Our model is underpinned by a preference function, enabling us to adapt to variations in the risk aversion levels across diverse preference ranges.

To illustrate the model application, we apply it to a real-world scenario involving a major player in the Brazilian electricity sector, tasked with making a critical hedge decision for the latter half of the year. Our findings indicate that if the agent chooses to implement a two-stage hedge strategy (one in July, covering exposure through September, and another in October, safeguarding year-end exposure), a recommended approach would be to contract 100.00% of the uncontracted amount for the 3rd quarter, averaging 54.29 MW, with no hedging required in the 4th quarter.

However, should the agent opt for a single hedge decision in July, covering exposure throughout the entire latter half of the year, our model suggests purchasing 45.30% of their position. In this scenario, the optimal decision would entail buying an average of 29.49 MW (equivalent to 45.30% of the uncontracted amount in the 2nd semester) at a rate of up to 325.00 BRL/MWh.

This decision support tool empowers agents with short positions in the electricity market to make well-informed and optimal hedge transaction decisions. It is essential to note that while the model is robust, its primary limitation lies in the parameterization of the preference function. In our study, we employ the widely discussed AHP method.

5
A Framework for Operations and Premium Estimation in a Brazilian Energy Futures Clearing House

5.1
Introduction

A major overhaul of the Brazilian electricity sector began in 1997, when the first privatizations were initiated. The changes included the process of de-verticalizing electricity production, transmission, distribution and commercialization, part of the Brazilian electricity sector restructuring project. An environment for accounting and liquidation of electricity was also created, the Wholesale Energy Market, which is currently the CCEE.

In 2004, the Free Contracting Environment (*Ambiente de Contratação Livre - ACL*) and the Regulated Contracting Environment (*Ambiente de Contratação Regulada - ACR*) were created. Before that, the free market had very low liquidity, and with the creation of the ACL, it started to see growth, but, even so, the futures market continued to feature bilateral trades and the over-the-counter (OTC) format. In 2005, the first trading session for energy contracts in the country was created by the Brazilian Mercantile and Futures Exchange (BM&F), which, due to low liquidity, was discontinued. In 2012, BRIX and BBCE (*Balcão Brasileiro de Comercialização de Energia*), two important electronic trading platforms, started operations, which brought more agility to the business and standardization of contracts. However, these are platforms that still operate in an OTC format.

During the overhaul of the Brazilian electricity sector, attempts were made to develop an energy exchange in which investors could trade futures contracts, but these failed, possibly due to the lack of an adequate methodology and technological tools that are available today, and due to characteristics of the market, such as the high volatility of electricity prices and the difficulty of establishing a mark-to-market consensus. The high volatility of energy prices in the Brazilian market is a characteristic that is difficult to change, as the Brazilian electricity matrix features hydraulic energy as its main source, which is extremely dependent on climatic factors, which are difficult to predict. Periods of drought or prolonged rain, for example, can cause a sharp rise or fall in short-term energy prices and the futures market.

The high volatility of energy prices undermines the process of securing multilateral guarantees and, today, some of the most sophisticated methods to deal with this issue are used in European and American energy exchanges. The main method is cascading, where futures contracts with longer periods expire on previously agreed dates and are replaced by shorter, equivalent contracts, promoting greater security and liquidity in the market. This methodology can be adapted to Brazilian specificities to provide less volatility and increase market liquidity.

Another alternative to increase the security and liquidity of the market, with the objective of making the operation of a Brazilian energy exchange viable, is to structure an operation in which the clearing house assumes the positions of clients that do not respond to the margin call, after the mark-to-market. As the purpose of the transaction is not speculative and, in this model, the clearing house closes the positions it has taken as soon as possible. This strategy makes the existence of a symmetric order in the order book unnecessary to close the position of a client who does not respond to the margin call. In a highly liquid market, the manager would not need to take any position and would simply close the position of these clients using other clients' open orders, from the order book. However, in the Brazilian energy market, due to low liquidity, it would not always be possible to close the position immediately and it would be necessary, to protect the clearing house and the clients, to set up an operation in which the clearing house would, provisionally, assume the position of that client, removing them from the operation.

Recently, Souza et al. (2021) analyzed the economic preconditions for the Brazilian electricity market, perceptions, and expectations of agents about a specific future electricity market, through an exploratory study. In this study, questionnaires were applied to market agents to estimate the perception and expectation of the conditions for the implementation of an electricity futures exchange in Brazil. The study identified mostly positive perceptions about the economic preconditions for the creation of a future electricity market, which reinforces the importance of developing works that seek to build frameworks that help in the feasibility of an electricity futures exchange.

The objective of this work is to develop a framework for an electricity futures exchange and simulate its operation, adhering to the Brazilian reality of low liquidity and high volatility, where the clearing house assumes the positions of clients that do not respond to the margin call, after marking to the market, and generate relevant information for agents who are willing to create an enterprise of this type in Brazil.

For the work, we produced simulations of a set of possible scenarios, including bullish and bearish shocks and different levels of liquidity. We also analyzed the sensitivity of returns in relation to the expected level of liquidity, given the characteristics of the Brazilian market, and we evaluated the option of taking out insurance to protect the clearing house. The insurance premium was calculated using a Monte Carlo simulation, a well-established and widely used method in studies of derivatives and futures markets, used in studies such as those of Irwin et al. (1996), Cortazar and Schwartz (1998), Abadie and Chamorro (2009) and Pelajo et al. (2019). The analysis of the simulation results and the insurance calculation are novel and important information for market agents interested in the creation of a Brazilian electricity futures clearing house.

The article is divided as follows: the first chapter presents the theoretical framework of the work; chapter 5.2 presents the methodology used; chapter 5.3 discusses the simulation results; and chapter 5.4 presents the conclusions of the work.

5.2
Theoretical foundation

5.2.1
Central Counterparty Clearinghouses

Central counterparty clearing houses (CCPs) are formed by companies that intermediate in operations in a market and operate in a structure in which they assume the credit risk of all parties, but as their net position is always zero, they do not assume the market risk (Bliss & Steigerwald, 2006). The CCP becomes the intermediary for all trades (Figure 13) between the members of a given market, converting all trades into symmetrical contracts between the parties involved and the CCP. Settlement of positions is made much easier with this model, as the position taker only needs to find a symmetrical position in the CCP's order book to close it. This eliminates the need for bilateral contract termination.

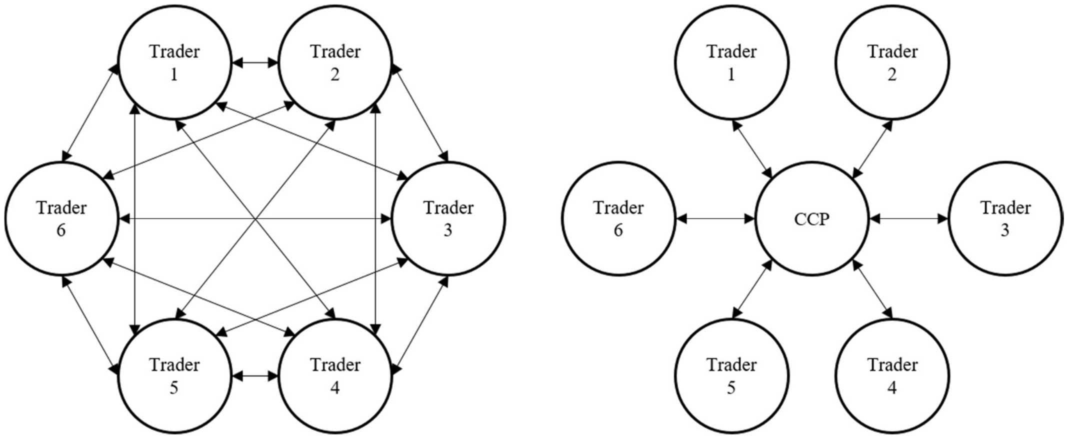

Figure 13 – Over-the-counter OTC (L) and central counterparty clearing CCP (R)
Note: Adapted from Cont and Kokholm (2012).

When contracts are centrally managed, there is a reduction in the need for collateral, as only the net positions of agents are considered, reducing risk when compared to a non-centralized structure (Cont & Kokholm, 2012). Another advantage of a CCP is the reduction of operating costs (when compared to bilateral contracts), transparency and the mitigation of default risk, as the CCP protects members against this type of risk (Nosal, 2010). The CCP is also responsible for marking the position of customers to the market and ensuring the transfer of amounts according to the result of the operation, associated with their contracts (Pirrong, 2009).

5.2.2
Energy futures: risk premiums and pricing

A classic approach to explaining commodity futures prices, where future price expectations are uncertain, is to assign an inventory carrying cost (such as inventory cost and depreciation) and a premium risk charged by the speculators, so that the futures price includes these components. By selling futures, asset holders get rid of the risk of future variations and pass them on to speculators, who are remunerated with a premium for the risk they assume. When we consider the cost of carrying inventory, it is possible to highlight an important component that is considered separately in the models, which is the convenience fee. A convenience fee exists when the possession of a certain stock can be useful to its holder. For example, it may be useful for a producer to have inventory to meet unexpected demand (Fama & French, 1987; Kaldor, 1939). Algebraically, we can define the relationship between future price (PF), expected price (EP) and current price (CP) as a function of the interest rate i, the cost of carrying inventories c' , the risk premium r and the rate of convenience q, as presented in equation (117) (all terms are marginal):

$$EP - CP = i + c' - q + r \quad (117)$$

In markets where hedgers are future sellers, we have the following relationship, presented in equation (118) and equation (119):

$$FP - CP = i + c' - q \quad (118)$$

$$FP = EP - r \quad (119)$$

When the hedgers are future buyers, we can use equation (120) and equation (121) to describe the relationship between future price (PF), expected price (EP) and current price (CP) as a function of the interest rate i, the cost of carrying inventories c', the risk premium r and the rate of convenience q:

$$FP - CP = i + c' \quad (120)$$

$$FP = EP - r + q \quad (121)$$

The market behavior, where the futures price falls below the expected future price and the futures price converges to the spot price, at maturity, from underneath, is known as normal backwardation. In general, the behavior of normal backwardation is associated with circumstances where there is a low level of supply and/or a low level of inventory. When the opposite occurs, that is, when hedgers are on the long end and speculators are on the short end, the situation is reversed, and futures prices converge to the spot price from above. This market behavior is known as contango. Contango behavior is usually associated with circumstances where there is an immediate abundant commodity supply (Benth et al., 2008).

In another extremely popular approach to explaining the behavior of commodity futures prices, the price of the futures contract is divided into two components, one component related to the forecast of spot prices of the commodity for the future date and another related to the risk premium. The risk premium in this case would be related to the ability of a restricted subset of speculators to better predict the futures price than other market participants and futures contract prices would be unbiased predictors of the futures price. This theory is known as forecasting theory and its advocates argue that there would be no clear price movement trend in futures markets and that the proportion of profits relative to contango or normal backwardation would be zero (Lee & Zhang, 2009).

Lee and Zhang (2009) examine the characteristics of the price movements of 29 markets and present evidence of the validity of the mechanisms explaining the prices of futures contracts proposed by the two theories, simultaneously showing that, depending on market conditions, one theory is dominant to explain the behavior of prices. The common view that two theories are mutually exclusive is replaced by an interpretation that they complement each other and, in a way, compete. It is observed that the presence of normal backwardation, contango or forecasting is related to the specific characteristics of each market and, according to its specificities, one type of mechanism can become dominant.

The electricity markets are part of the commodity markets, and are characterized by the limitation in storing electricity, which directly influences the behavior of spot prices for electricity and derivatives, which futures and forward markets are part of. The price behavior differs from other commodities, where it is possible to wait, increasing storage, for the most opportune moment to offer them to the market. Without this option, arbitrage arising from storage capacity is extremely limited and prices are expected to be highly dependent on demand and specific local conditions, such as weather conditions and the level of local economic activity (Lucia & Schwartz, 2002). Models based on storage capacity thus have limited power to explain prices in electricity markets.

In electricity markets where hydroelectric generation with reservoirs is predominant, there is evidence that the theory of inventory cost and convenience rate is relevant, as the reservoirs of hydroelectric plants work to store electricity.

Empirical results from Nord Pool, one of the most important in Europe and with contracts traded in more than 14 countries, show that, for weekly futures contracts, hydrological conditions have a great influence on market behavior. The convenience rate is positive (normal backwardation) when reservoir levels are low, in the first half of the year, and negative (contango) when levels are high, in the second half of the year. On average, the observed convenience rate is negative and spot prices tend to be below the prices of futures contracts, in contango, with risk premiums having a negative sign over the analysis period, from 1996 to 2006 (Botterud et al., 2010).

5.2.3 Diffusion processes with jumps and asset pricing

Distributions of returns on some assets traded in an exchange environment, such as stocks, have thick tails (a leptokurtic format) when compared to the normal distribution. This phenomenon can be caused by the presence of price jumps, which are infrequent and large-scale movements (Yan, 2011). The energy market has well-known characteristics of asset price behavior, such as mean reversion, high volatility and the presence of jumps, and the models developed need to consider these characteristics.

A classic modeling of this type of process, where there is a discontinuity in the pattern of movements, with the presence of jumps, was developed by Merton (1976). This approach allows the calculation of the option value (insurance) in the presence of a jump, which was not possible until then, with the diffusion model developed by Black and Scholes (1973).

This approach proposes two components that explain the variation in the price of the asset, one linked to a normal variation, with marginal effects, such as adjustments between supply and demand or changes in interest rates; and the other based on important and completely new information to the market, which causes more than marginal effects on prices. The first component, with marginal effects, is governed by the Wiener process and the second is modeled as a Poisson process, where the event is the arrival of new information on the market about a given stock.

New information arrives independently and is evenly distributed. The differential stochastic equation representing stock returns is presented in equation (122):

$$dS/S = (\alpha - \lambda\kappa)dt + \sigma dz + dq \qquad (122)$$

Where S represents the asset price, dS is the price change, α is the expected instantaneous return, σ is the instantaneous volatility of the returns, dz is the Gauss-Wiener process, and dq is the Poisson process. Processes dz and dq are independent. The average amount of new information (number of hops) per unit of time is represented by λ. κ is the expected value of (Y-1), a random variable that represents the percentage change if the Poisson event occurs.

If λ, κ and σ are constant, the returns on stock prices over time t can be described according to equation (123):

$$(S(t))/S = exp[(\alpha - 1/2\sigma\hat{}2 - \lambda\kappa)t + \sigma z(t)]\, Y(n) \qquad (123)$$

When Y(n) is a lognormal distribution, $\frac{S(t)}{S}$ also assumes a lognormal distribution. Merton (1976) demonstrates that the European option value can be defined according to equation (124):

$$F(s,t) = \sum_{n=0}^{\infty} e^{-\lambda t}(\lambda t)^n \left[\varepsilon_n\{w(sx_n e^{\lambda\kappa t}, t; E, \sigma^2, r)\}\right] \qquad (124)$$

Where $\varepsilon_n\{w(sx_n e^{\lambda\kappa t}, t; E, \sigma^2, r)\}$ represents the value of the option according to the Black-Scholes formula for the exercise price E and risk-free rate r.

5.3
Methodology

5.3.1
Modeling the clearing house process: a simulation approach

For the simulation of the electricity futures clearing house operation, we used the contract and financial volume data from the Brazilian Electricity Trading Agency (BBCE) referring to January 2020 (Appendix I. Supplementary Data 1 – Monthly Volumes), obtained from the company's website. The volume of contracts traded by BBCE in January 2020 was 6,245 contracts, through which a financial volume of BRL 4.2 billion and 17,446,000 MWh of energy were traded, representing a weekly average of 1,419 contracts, BRL 954.5 million in financial volume and 3,965,000 MWh. The settlement of differences price (*Preço de Liquidação das Diferenças – PLD*) is determined on a weekly basis, considering the load levels of each submarket of the Brazilian electrical system. Monthly traded contracts and financial volume data are not normally made available to the market. This is private and difficult to access information. We obtained the data from a news item on the company's website and therefore used them as an estimate. However, there is no data available for other months and the use of the January 2020 volume for the other months is the best estimate that was possible for this work.

We obtained the weekly volumes through the daily average of the total volume transacted through BBCE in January and the rate of new contracts considered in the model is constant over the weeks. Seasonality is not considered in the rate of new contracts. The choice of a constant rate of creation of new contracts was due to the absence of information on monthly volumes for BBCE. Future works may include this feature if necessary and if the data to support the choice of seasonality in the rate of new contracts are available.

The choice of BBCE data as a basis for volumes is due to it being the most important OTC for electricity futures contracts in Brazil. Thus, it is an excellent reference of volume for simulating the clearing house. We inserted the weekly volumes in the model considering the selling and buying ends and therefore the number of positions is twice the number of contracts.

To perform the calculations and generate the graphs, we developed a Python code (Appendix I. Supplementary Data 2 – Python Code) in version 3.8 64-Bit and the following packages: numpy version 1.18.1; pandas version 1.0.1; matplotlib version 3.1.3; IPython version 7.12.0; and ipywidgets version 7.5.1.

5.3.2
Simulation of contract price trajectories and weekly returns

As the electricity futures market in Brazil continues to trade bilaterally and in the OTC format, information on futures market trading prices is not reliably available, as it does not come from a liquid and transparent market, with public information. This information is available to a very limited extent.

Today there is a private company (DCIDE) that consolidates some information and expectations of market participants to sell periodical bulletins containing, among other information, the forward curve projected by it. However, the data used are neither public nor complete.

On the other hand, energy spot price paths are public and reliable, as are spot price forecasts, which rely on a robust methodology and forecast models fully available to market agents. It is to be expected, therefore, that in a future operation of an energy exchange the forward already includes all market information regarding prices and expectations and that future variations result only from the inclusion of new information.

The presence of jumps can occur, as these are also the result of the arrival of new information to the market. Thus, we assume that the jump diffusion process is the best representation for the movement of prices in the futures market. The use of another process to represent the price variation of futures contracts, such as an autoregressive model, could go against the non-arbitrage argument, which is valid in a highly liquid and transparent environment.

The price trajectory modeling was based on the model proposed by Merton (1976); however, we forced the jumps associated with the Poisson process, in the proposed modeling, to occur in two of the three scenarios of the simulated operation for 1 year. This is because it is important to assess the financial impact when the jump in prices occurs throughout the year, compared to the scenario where the jumps do not occur. Thus, we modeled three price trajectories, using the Brownian geometric movement to represent the returns. In the base scenario, we evaluated the operation in a year where there are no jumps. The other two scenarios evaluate the operation where there is a jump of 50% positive and 50% negative, respectively, in one of the randomly chosen weeks in each price trajectory.

The PLD has a maximum and minimum value, which for 2020 was set at BRL 559.75 and BRL 39.68, respectively (CCEE, 2020). However, for futures contracts, we do not impose this price restriction, as risk premiums may be applied, depending on the behavior of the market, causing prices to have to be adjusted and, eventually, they may exceed these legal limits of the spot market.

We started from the premise that there is no type of convenience fee for holding a futures contract. We made this choice since the behavior of the risk premium can change due to several factors, including hydrological ones, as seen in the Nord Pool market (Botterud et al., 2010) Market behavior can also change according to market hedging needs and speculators' positioning. Although some studies such as those of Luz et al. (2012) and Costa (2018) identified the presence of contango in the Brazilian forward market, the data used for the studies are quite limited and scarce, because the forward market is in the OTC format, without information transparency and full standardization. Thus, inferring a premium in the behavior of price paths would be premature. Therefore, we consider the drift to be zero, so that only the component of the Wiener process has an influence on the price variation.

Below, we show equation (125) of returns (R_t), used in the base scenario, where there are no jumps and which follows a stochastic Wiener process (S_t), where α is the drift, t is the time and σ is the deviation pattern. Eliminating the drift and using $\Delta t = 1/52$, the return at t is according to equation (126), in which are independent Gaussian variables with a mean of 0 and variance of 1. Prices at $t + 1$ are determined according to equation (127).

$$R_t = \alpha dt + \sigma dz \tag{125}$$

$$R_t = \sigma \epsilon \sqrt{\Delta t} \tag{126}$$

$$P_{t+1} = P_t * e^{R_t * 1} \tag{127}$$

5.3.3
Calculation of positions taken by the clearing house

In the proposed model, the clearing house assumes the position of clients that do not respond to the margin call. To calculate the positions provisionally assumed by the clearing house, it is necessary to calculate the probability of customer default. The assumption adopted is that the clearing house works with a maximum probability of default of 1%, referring to a weekly variation of 100% (positive or negative). To represent this behavior, we chose the default percentage function $d_{\%}$ defined as $d_{\%} = \sqrt{|\, r_t \,|}$, in equation (128), where $d_{\%}$ is the percentage of default for the absolute percentage return $|\, r_t \,|$ that week. Note that for a 100% variation from one week to another, the default percentage is 1% ($d_{\%} = \sqrt{|\, 1 \,|} = 1$). For a weekly return of 50%, the default percentage is 0.71% ($d_{\%} = \sqrt{|\, 0{,}5 \,|} = 0{,}71$). This function generates a default percentage that varies according to the weekly return. When the weekly return is small, the number of customers in default is small for that week. When the weekly return is large, the number of customers in default increases.

$$d_{\%} = \sqrt{|\, r_t \,|} \tag{128}$$

The number of positions (x_t) taken by the clearing house, given the weekly volume of 3,965,000 MWh (7,930,000 MWh considering both ends, but only one end that is called margin), is given by equation (129), where the variable side is assigned as -1 for negative returns and 1 for positive returns.

$$x_t = \sqrt{|\, r_t \,|} * 3.965.00 * side \tag{129}$$

$$side = \begin{cases} -1, para\ r_t < 0 \\ 1, para\ r_t >= 0 \end{cases}$$

The positions opened weekly, based on BBCE's January 2020 data, are as follows:

- 1,419 new contracts per week.
- 1,419*2 = 2,838 open contracts (considering both ends).
- 3,965,000 MWh traded weekly (7,930,000 considering both ends).

5.3.4
Clearing house

The total net position depends not only on volatility, but also on the time it takes to close positions. We define the liquidity ratio L as the average time it takes the clearing house to dispose of contracts. The scenario of greater liquidity is associated with the index L=1 period and the scenario of less liquidity is associated with the index L=4 periods. The position is given by equation (130), which represents the sum of positions assumed and not yet closed, in period   for the liquidity ratio L.

$$X_t = \sum_{k=1}^{L} x_{t-k} \tag{130}$$

5.3.5
Scenarios

Twelve scenarios were analyzed, obtained based on three price levels (neutral, bullish shock and bearish shock) and four liquidity levels, represented by the number of periods for the clearing house to dispose of the position (1, 2, 3 and 4 weeks). We simulated the bullish and bearish shocks as follows: a week is randomly chosen in the year for a bullish or bearish shock of 50%. The distribution of the binary variable is uniform, so that for each trajectory, each week of the year has an equal probability of being selected for the occurrence of the shock. The bullish or bearish shock is a regime change that is represented in the model according to equation (131):

$$r_t = \sigma\epsilon\sqrt{\Delta t} * (1 \pm 0{,}5 * I_{\{0,1\}}^{t}) \tag{131}$$

where σ is the standard deviation, ϵ are independent Gaussian variables with a mean of 0 and variance of 1, Δt is the time interval and $I_{\{0,1\}}^{t}$ is the binary variable that assumes the value of 1 when regime change occurs and 0 when it does not. The objective is to simulate the occurrence of stress, which can be caused, for example, by a sudden drop in domestic consumption, as occurred recently due to the pandemic caused by Covid-19.

We considered volatility of 17.03%, obtained from Argento (2020), who analyzed weekly forward contract returns data from 2012 to 2019, obtained from DCIDE. Volatility was obtained by applying the Markov-switching variance model. Even though the data are from the OTC market, these data are the best reference available for the simulation. The value of 17.03% is conservative for the purpose of this work and refers to the volatility of contracts maturing in one month (M1). The volatility of contracts with longer maturities is lower. As the volume segregated by maturity is not known, the choice of the highest volatility, in this case, is the most appropriate and conservative, to represent the possible variation for the set of contracts. For each scenario analyzed, we simulated 10,000 return and exercise paths for the insurance option. We simulated a period of 52 weeks, representing 1 year of operation. The Table 19 summarizes the 12 scenarios analyzed.

	Liquidity			
Returns	**Period 1**	**Period 2**	**Period 3**	**Period 4**
Bearish Shock	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Neutral	Scenario 5	Scenario 6	Scenario 7	Scenario 8
Bullish shock	Scenario 9	Scenario 10	Scenario 11	Scenario 12

Table 19 – Analyzed scenarios

5.3.6
Defining operation's financial outcome

The financial result π_t for the proposed operation, between a period t and another $t-1$ is given by equation (132) and the accumulated result Π_t in the period is given by equation (133):

$$\pi_t = (P_t - P_{t-1}) * X_t \quad (132)$$

$$\Pi_T = \sum_{t=0}^{T} \pi_t \quad (133)$$

where X_t is the total net position, defined in Equation 130, and P_t and P_{t-1} are, respectively, the prices of the contracts at times t and $t-1$.

5.3.7 Insurance

We performed the insurance premium calculation using the Monte Carlo simulation, where, for each trajectory of returns, the insurance cover helps the clearing house to recover financially from any loss at the end of the 52-week period. The amount received from the insurance contract, when insurance cover is necessary, is equal to the absolute value of the accumulated loss. In the case of positive returns, at the end of the period, the clearing house does not trigger the insurance and the amount received from the insurance company is zero.

Insurance for the clearing house can then be modeled as a put with strike = 0. The payoff α_s of the option, at the end of the period T, is given by equation (134):

$$\alpha_s = max(-\Pi_T^s, 0) \quad (134)$$

The payoff for the average of the s scenarios is given by equation (135):

$$\alpha = \frac{1}{S} * \sum_{s=1}^{S} \alpha_s \quad (135)$$

At time $T = 0$, the present value of the option brought at the risk-free rate is given by equation (136):

$$p = \alpha * e^{-T * r_f} \quad (136)$$

5.4 Results

We ran the simulation model for the 12 scenarios. Below we present the graphs representing the results of the four liquidity levels, for the bearish shock (Figure 14), neutral (Figure 15) and bullish shock (Figure 16) price regimes. As expected, we observed that the lower the liquidity, which in practice translates into a longer time for the clearing house to dispose of positions, the greater the average size of the clearing house's positions. This behavior is observed for the three regimes (bearish shock, neutral and bullish shock).

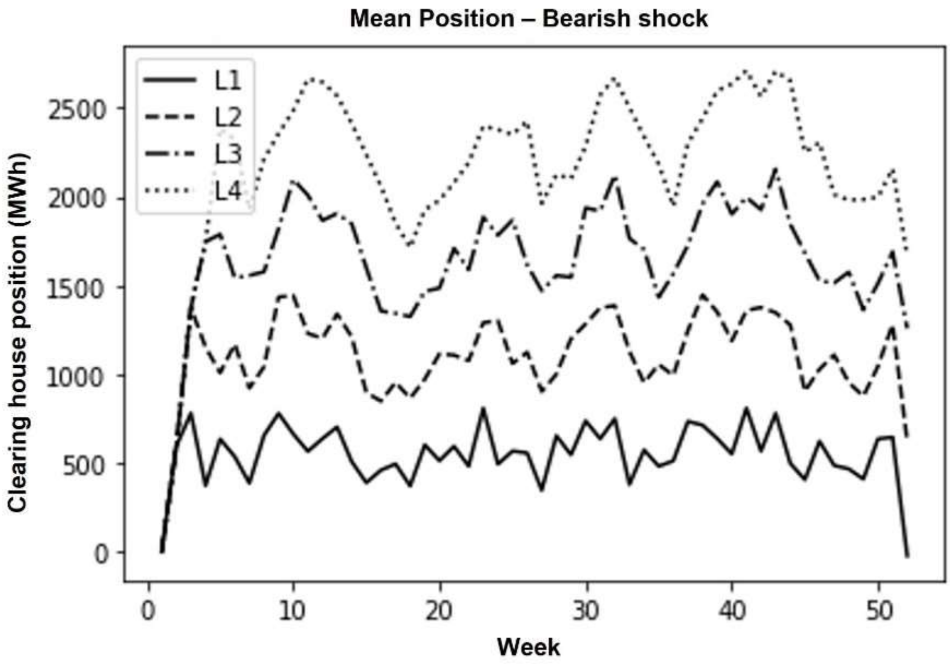

Figure 14 – Mean position - bearish shock (MWh)

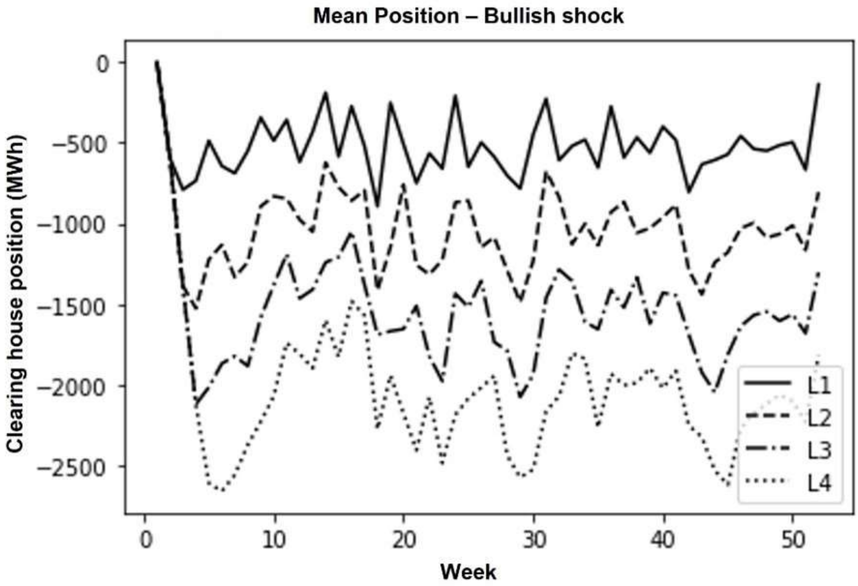

Figure 15 – Mean position - bullish shock (MWh)

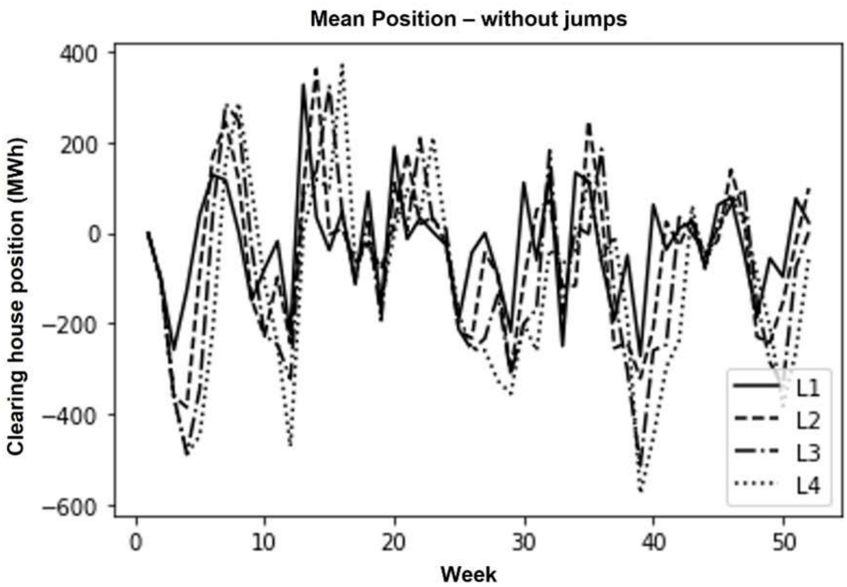

Figure 16 – Mean position - neutral market (MWh)

The Figure 17 below presents the three graphs on the same scale for comparison purposes. It is possible to observe that, in the neutral regime, the positions vary less compared to the other two regimes, which was an expected result, since there is a sudden variation of 50% in the price in all the bullish and bearish shock simulations.

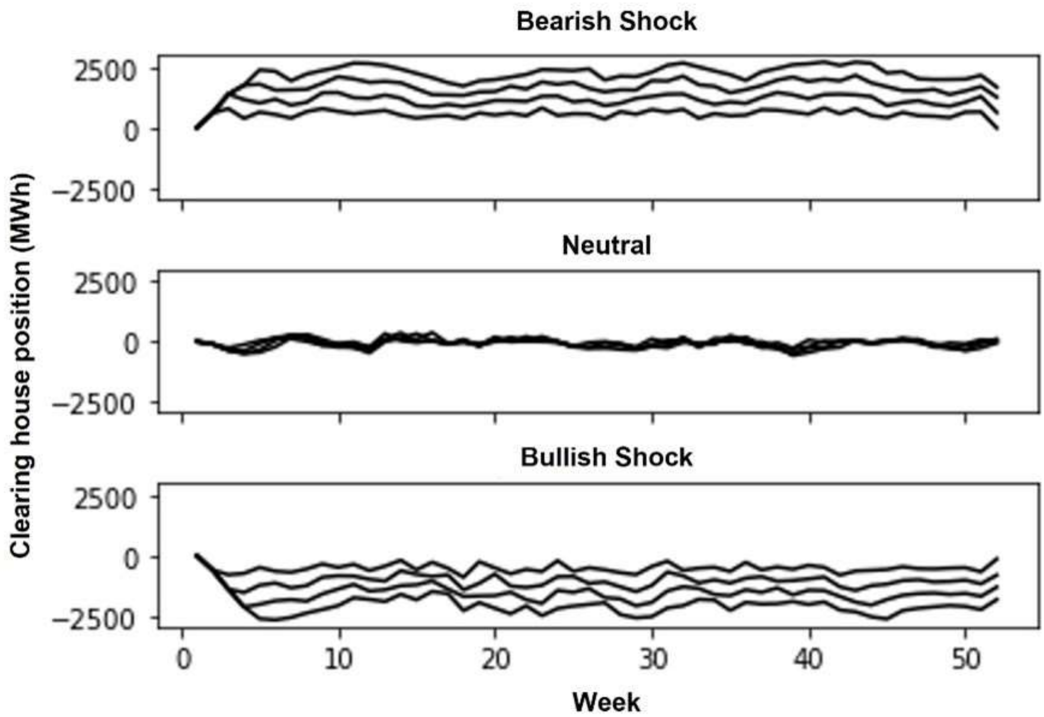

Figure 17 – Results for the 3 price scenarios (MWh)

5.4.1 Calculation of cumulative returns

The clearing house operation generates, at the end of the period, an accumulated result that will be positive or negative, but it is expected to be close to zero or not significant in relation to the total volume transacted. The purpose of the operation is not to make a profit, but to enable a market that has low liquidity. For this work, we calculated the accumulated returns of each of the trajectories for each scenario (Appendix I. Supplementary Data 3 – Accumulated Returns). The average of the returns, for each scenario, is presented in Table 20, where we can observe that the higher the liquidity level, the higher the modulus of average return is. The neutral scenario is the one with the lowest accumulated values, in absolute terms.

The values, however, are slightly negative, but very small in relation to the total amount transacted in the 52 weeks, of BRL 49.63 billion (BRL 954.5 million weekly). The fact that there is a floor for the PLD, which cannot be negative, generates slight asymmetry in the profile of returns, since the PLD, when it reaches zero, can only assume positive values. The same does not occur for very high SDP values, when there is always the possibility that they will continue to rise.

For scenarios where there is a 50% bearish shock, there is a greater volume of long positions that need to be taken over by the clearing house and, for the same reason as above, generate a positive return. Similarly, when there is a bullish shock, the clearing house must take on a large volume of short positions and the return tends to be negative. When there is a bearish or bullish shock, the values, whether positive or negative, are higher in absolute terms than in the neutral scenario, but they are still small when compared to the financial volume traded in a year.

For the same type of returns (bullish shock, neutral, or bearish shock), the lower level of liquidity is related to greater risk, with an increase in the positive and negative values of the accumulated returns.

Prices/Liquidity	1 Period	2 Periods	3 periods	4 Periods
Bearish Shock	231,369.96	400,853.01	615,791.92	837,066.17
Neutral	-113,536.15	-142,096.15	-154,850.71	-175,348.16
Bullish Shock	-276,514.78	-539,550.52	-733,045.02	-887,398.66

Table 20 – Accumulated returns for 52 weeks (BRL)

Considering that the clearing house's revenue would be 0.5% of the total contracts and both ends, it would have a revenue of BRL 496.36 million and the average loss, in the worst scenario, would represent only 0.19% of the revenue, which appears be quite reasonable in terms of cost, given the importance of the operation to the viability of the electricity futures clearing house.

5.4.2
Calculation of the risk-free rate

We calculated the risk-free rate following the methodology adopted until 2020 by ANEEL (ANEEL, 2020) to calculate the weighted average cost of capital (WACC) of generation, transmission, and distribution projects, where the fixed income bond used is the US Treasury Bond type “USTB10,” to estimate the risk-free rate. We obtained the series of annual data on the price of this security from the period from January 1995 to December 2012, and we calculated the arithmetic average, obtaining an average annual interest rate of 4.59%.

Country risk, of 3.52%, is calculated using the median referring to the EMBI+BR from January 2000 to December 2012. We estimated American inflation using the average of the period from 1995 to 2012, obtaining the value of 2.47% per year. Considering this inflation, the real interest rate is 2.07%, which, added to the country risk, results in a risk-free rate of 5.59% per year for Brazil.

5.4.3
Insurance premium calculation

We used the real options methodology to calculate the insurance premium, where the insurance premium is calculated as a put option. We calculated it for the 12 scenarios, with the put value following the methodology presented in sections 2.6 and 2.7. The put represents the option to sell the insurance at the value of Π_T^S (the result of the operation) when it is less than zero at the end of the 52-week period. The value is obtained by bringing the average payoff at the risk-free rate to the present value. Table 21 below shows the calculation of the average payoff, in BRL, for each of the 12 scenarios:

Prices/Liquidity	1 Period	2 Periods	3 periods	4 Periods
Bearish Shock	1,155,485.50	1,570,661.12	1,800,629.69	1,952,360.92
Neutral	1,594,821.36	2,202,501.27	2,676,045.19	3,073,308.72
Bullish Shock	2,157,544.80	3,110,220.91	3,822,222.31	4,422,000.43

Table 21 – Average payoffs in BRL for the 12 scenarios

The Table 22 shows the percentage of simulations in which the insurance was activated. Note that the value is high (close to 50%), which is understandable given that the generation of prices follows a geometric Brownian motion (GBM) and that the only restriction is that prices cannot go below zero. Values slightly below 50%, similarly to what happens with returns, are justified by the restriction of negative prices, which impose a barrier to falling prices, a restriction that does not exist for positive values.

Prices/Liquidity	1 Period	2 Periods	3 periods	4 Periods
Bearish Shock	45.48%	42.59%	40.40%	39.35%
Neutral	47.63%	44.43%	42.62%	40.65%
Bullish Shock	47.78%	44.13%	42.42%	40.21%

Table 22 – Percentages of simulations that triggered the insurance

Considering the payoffs and the risk-free rate of 5.59% per year, we calculated the insurance value for each of the scenarios. The insurance calculation results are shown in Table 23:

Prices/Liquidity	1 Period	2 Periods	3 periods	4 Periods
Bearish Shock	1,092,666.02	1,485,270.08	1,702,736.11	1,846,218.27
Neutral	1,508,116.81	2,082,759.41	2,530,558.50	2,906,224.28
Bullish Shock	2,040,247.06	2,941,129.69	3,614,422.20	4,181,592.60

Table 23 – Insurance calculation in BRL for the 12 scenarios

As, in the model, the insurance is annual, the values of the average payoffs are close to the value of the insurance itself. The amounts for the insurance of the operation are also small when compared to the total volume transacted, of BRL 49.63 billion. Considering billing of BRL 496.3 million, according to the rationale already presented, the maximum insurance value of BRL 4.2 million represents only 0.84% of the projected revenue, a value that could possibly be incorporated without major problems to the operation.

5.5
Conclusions

In this work, we proposed an operating model for a clearing house for trading electricity futures that would adhere to the Brazilian reality, marked by high volatility in energy prices and low liquidity of futures contracts. In the proposed model, the clearing house provisionally assumes the positions of clients that do not respond to the margin call, after marking to market, thus avoiding contract default. This strategy allows the clearing house to manage default risk even in a market with low liquidity, removing customers who do not respond to the margin call from the operation. The model can be used in conjunction with cascading, which can provide the market with increased liquidity, as higher value contracts, as they approach the expiration date, transform into several smaller, more accessible contracts that adhere to other customers.

We performed the simulations to reflect normal operating conditions and bullish and bearish shock scenarios in spot prices. These shocks need to be considered in the model due to the Brazilian electricity matrix, where there is a predominance of hydraulic sources, and the price formation mechanism based on marginal costs. We modeled the financial returns of the positions as a GBM, associated with the constraint of positive prices. We demonstrated that the average returns, for different levels of liquidity, generate a total return in 52 weeks that, in absolute terms, is small compared to the revenue and volume transacted. The worst average return occurred for the bullish shock scenario and liquidity associated with 4 days to dispose of the position, where the average return was BRL -0.89 million, compared to BRL 49.64 billion traded in the period of 52 weeks.

We also calculated the insurance value for each scenario, for the case in which the clearing house does not want to assume the risk of unexpected returns. The insurance needs to cover the most unlikely events and therefore its calculated value, for all scenarios, is above the average value of the clearing house's returns, however, it protects it against all negative results. Even so, the value of the maximum insurance, in the bullish shock scenario and with 4 days to close the positions, is BRL 4.18 million (0.84% of estimated revenue), an apparently reasonable value in terms of cost, considering the importance of the operation.

This work therefore showed that this operating model, where the clearing house assumes customer positions, generates relatively small additional costs and they may be easily absorbed by the clearing house. Even the insurance option presented small values for each scenario and compared to the total volume of the operation and projected billing. These data are of great use to agents interested in opening a venture of this type in Brazil, which still lacks an electricity futures exchange.

Among the limitations of the study, we highlight the non-availability of detailed data on volumes and values of contracts traded. As the market is private and in the over-the-counter format, the volume of operations of the main OTC markets is not available and the contracts are bilateral. Therefore, we used constant contract volumes, without considering any type of seasonality in the volume of futures contracts.

For future work, we suggest a search for more accurate data on contract volumes from the main companies that provide the over-the-counter market in a way that makes it possible to build a model with seasonalized volume data. Another suggestion is to adapt the model to a market where contango or normal backwardation predominates.

6
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7
Appendix

Appendix I. Supplementary Data 1 – Monthly Volumes

Contratos BBCE
Mês: Jan. 2020

Tipo	Qte Contratos	Qte MWh	Volume Financeiro	Média Contrato MWh	Média Contrato R$
Tela	3.637	4.727.000	1.100.000.000	1.300	302.447
Boleta	2.608	12.719.000	3.100.000.000	4.877	1.188.650
Total	**6.245**	**17.446.000**	**4.200.000.000**	**2.794**	**672.538**
média diária	284	793.000	190.909.091		
média semanal	1419	3.965.000	954.545.455		
Cenário Base (1% default)	14,19	39.650	9.545.455	2.794	672.538

Appendix I. Supplementary Data 2 – Python Code

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from IPython.display import display
import ipywidgets as widgets
np.random.seed(seed=43)
get_ipython().run_line_magic('matplotlib', 'inline')

def gbm(n_periods = 52, n_scenarios=10000, mu=0.00, sigma=0.1703, steps_per_period=1, s_0=191.89):
    dt = 1/steps_per_period
    n_steps = int(n_periods*steps_per_period) + 1

    rets_plus_1 = np.random.normal(loc=(1+mu)**dt, scale=(sigma*np.sqrt(dt)), size=(n_steps, n_scenarios))
    rets_plus_1[0] = 1
    return s_0*pd.DataFrame(rets_plus_1).cumprod(), pd.DataFrame(rets_plus_1-1)
```

```
def regimen_change(n_periods = 52, n_scenarios=10000, mu=0.00, sigma=0.1703,
steps_per_period=1, s_0=191.89, delta_mu=0.5):
    t_star = np.random.randint(1, n_periods-1, (1, n_scenarios))

    dt = 1/steps_per_period
    n_steps = int(n_periods*steps_per_period) + 1

    rets_plus_1 = np.random.normal(loc=(1+mu)**dt, scale=(sigma*np.sqrt(dt)),
size=(n_steps, n_scenarios))
    rets_plus_1[t_star, np.array(range(n_scenarios))] = (1 + delta_mu)**dt

    rets_plus_1[0] = 1
    return s_0*pd.DataFrame(rets_plus_1).cumprod(), pd.DataFrame(rets_plus_1-
1)

def default_position (ret, open_positions = 3965000):
    return ret.applymap(lambda x: -np.sqrt(abs(x))*open_positions/100 if x>0 else
np.sqrt(abs(x))*open_positions/100)

def cumulative_clearing_position (default, L):
    return default.shift(1).rolling(L,min_periods=1).sum()

def wealth_change (prices, X):
    return (prices - prices.shift(1))*X

def cumulative_wealth (wealth):
    return wealth.cumsum()

def average_payoff(cum_pl):
    return cum_pl.iloc[-1].apply(lambda x: max(-x, 0 )).mean()

def present_value(value, rate, years = 1):
    return value*np.exp(-1*rate*years)
```

```
def plot_wealth(wealth, title, L):
    y_max=wealth.values.max()
    terminal_wealth = wealth.iloc[-1]

    tw_mean = terminal_wealth.mean()
    tw_median = terminal_wealth.median()
    failure_mask = np.less(terminal_wealth, 0)
    n_failures = failure_mask.sum()
    p_fail = n_failures/len(terminal_wealth)

    e_shortfall = np.dot(terminal_wealth, failure_mask)/n_failures if n_failures > 0
else 0.0
    fig, (wealth_ax, hist_ax) = plt.subplots(nrows=1, ncols=2, sharey=True,
gridspec_kw={'width_ratios':[3,2]}, figsize=(24, 9))
    fig.suptitle("Resultado Financeiro Clearing ("+ title +" - L = "+str(L)+")",
fontsize=16)
    plt.subplots_adjust(wspace=0.0)

    wealth.plot(ax=wealth_ax, legend=False, alpha=0.3, color="black")
    wealth_ax.axhline(y=0, ls="--", color="black")

    wealth_ax.set_ylim(top=y_max)

    terminal_wealth.plot.hist(ax=hist_ax, bins=50, ec='w', fc='black',
orientation='horizontal')
    hist_ax.axhline(y=0, ls="--", color="black")
    hist_ax.axhline(y=tw_mean, ls=":", color="black")
    hist_ax.axhline(y=tw_median, ls=":", color="black")
    #hist_ax.annotate(f"Mean: ${int(tw_mean)}", xy=(.7, .9),xycoords='axes
fraction', fontsize=24)
    #hist_ax.annotate(f"Median: ${int(tw_median)}", xy=(.7, .85),xycoords='axes
fraction', fontsize=24)
```

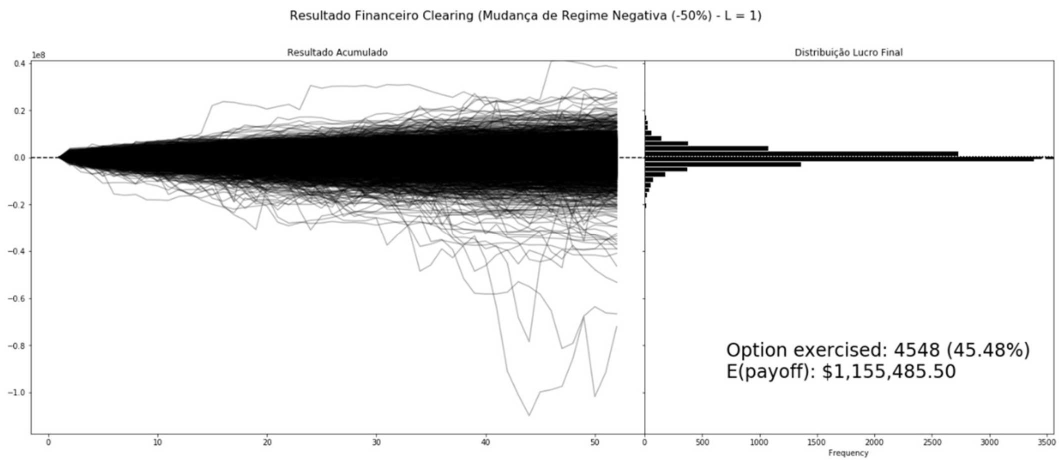
```
    wealth_ax.set_title('Resultado Acumulado')
    hist_ax.set_title('Distribuição Lucro Final')
    hist_ax.axhline(y=0, ls="--", color="black", linewidth=3)
    hist_ax.annotate(f"Option exercised: {n_failures}
({p_fail*100:2.2f}%)\nE(payoff): ${average_payoff(wealth):,.2f}", xy=(.2, .15),
xycoords='axes fraction', fontsize=24)

price_up, returns_up = regimen_change(delta_mu=0.5)
price_base, returns_base = gbm(mu=0)
price_down, returns_down = regimen_change(delta_mu=-0.5)

def average_position(returns, L):
    x = default_position(returns)
    X = cumulative_clearing_position(x, L)
    return X.mean(axis=1)

avg_neg = {'L1': average_position(returns_down, L=1),
        'L2': average_position(returns_down, L=2),
        'L3': average_position(returns_down, L=3),
        'L4': average_position(returns_down, L=4)}

ax_down = pd.DataFrame(avg_neg).plot.line(color="black",style=["-","--","-
.",":"])
ax_down.set(xlabel='semana', ylabel='posição (MWh)',
title='Posição Média da Clearing (Choque de Baixa(-50%))')

avg_base = {'L1': average_position(returns_base, L=1),
        'L2': average_position(returns_base, L=2),
        'L3': average_position(returns_base, L=3),
        'L4': average_position(returns_base, L=4)}
```

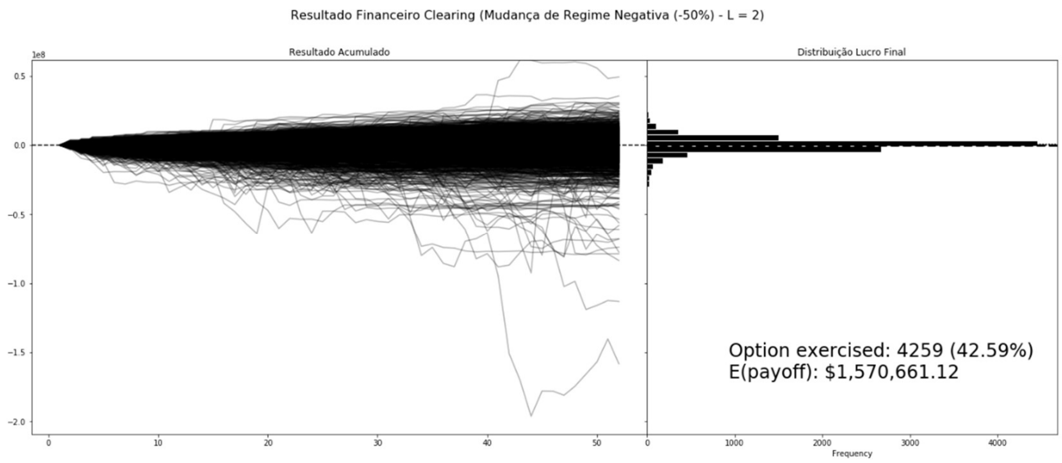
```
ax_down = pd.DataFrame(avg_base).plot.line(color="black",style=["-","--","-
.",":"])
ax_down.set(xlabel='semana', ylabel='posição (MWh)',
title='Posição Média da Clearing (Sem Jumps)')

avg_pos = {'L1': average_position(returns_up, L=1),
        'L2': average_position(returns_up, L=2),
        'L3': average_position(returns_up, L=3),
        'L4': average_position(returns_up, L=4)}

ax_down = pd.DataFrame(avg_pos).plot.line(color="black",style=["-","--","-
.",":"])
ax_down.set(xlabel='semana', ylabel='posição (MWh)',
title='Posição Média da Clearing (Choque de Alta (50%))')

type(avg_base)

f, (ax1, ax2, ax3) = plt.subplots(3, sharex=True, sharey=True)
ax1.plot(pd.DataFrame(avg_neg),'-',color='black')
ax2.plot(pd.DataFrame(avg_base),'-',color='black')
ax3.plot(pd.DataFrame(avg_pos),'-',color='black')

ax1.set_title('Posição Média da Clearing \n Choque de Baixa (-50%)')
ax2.set_title('Sem Jumps')
ax3.set_title('Choque de Alta (50%)')
ax3.set(xlabel='semana')
ax2.set(ylabel='posição (MWh)')
plt.subplots_adjust(hspace = 0.5)

def option_value (prices, returns, L, title):
    x = default_position(returns)
    X = cumulative_clearing_position(x, L)
    pi = wealth_change(prices, X).dropna()
    PI = cumulative_wealth(pi).dropna()
```

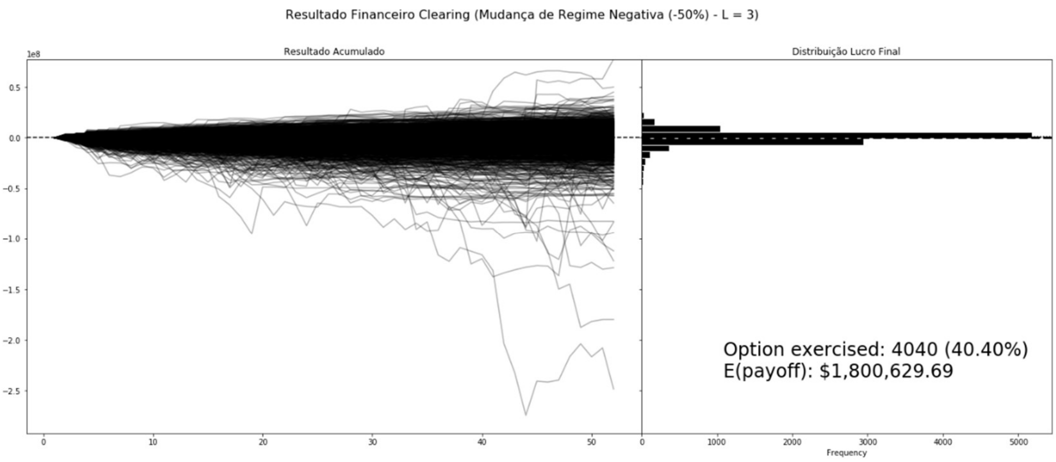
```
    plot_wealth(PI, title, L)
    return(PI.iloc[-1].mean())

DownL1 = option_value(price_down,returns_down, 1, 'Mudança de Regime Negativa (-50%)')
DownL2 = option_value(price_down,returns_down, 2, 'Mudança de Regime Negativa (-50%)')
DownL3 = option_value(price_down,returns_down, 3, 'Mudança de Regime Negativa (-50%)')
DownL4 = option_value(price_down,returns_down, 4, 'Mudança de Regime Negativa (-50%)')
BaseL1 = option_value(price_base,returns_base, 1, 'Caso Base')
BaseL2 = option_value(price_base,returns_base, 2, 'Caso Base')
BaseL3 = option_value(price_base,returns_base, 3, 'Caso Base')
BaseL4 = option_value(price_base,returns_base, 4, 'Caso Base')
UPL1 = option_value(price_up,returns_up, 1, 'Mudança de Regime Positiva (50%)')
UPL2 = option_value(price_up,returns_up, 2, 'Mudança de Regime Positiva (50%)'
)
UPL3 = option_value(price_up,returns_up, 3, 'Mudança de Regime Positiva (50%)')
UPL4 = option_value(price_up,returns_up, 4, 'Mudança de Regime Positiva (50%)')

print('Down L1 = ' + str(DownL1))
print('Down L2 = ' + str(DownL2))
print('Down L3 = ' + str(DownL3))
print('Down L4 = ' + str(DownL4))
print('Base L1 = ' + str(BaseL1))
print('Base L2 = ' + str(BaseL2))
print('Base L3 = ' + str(BaseL3))
print('Base L4 = ' + str(BaseL4))
print('UP L1 = ' + str(UPL1))
print('UP L2 = ' + str(UPL2))
```

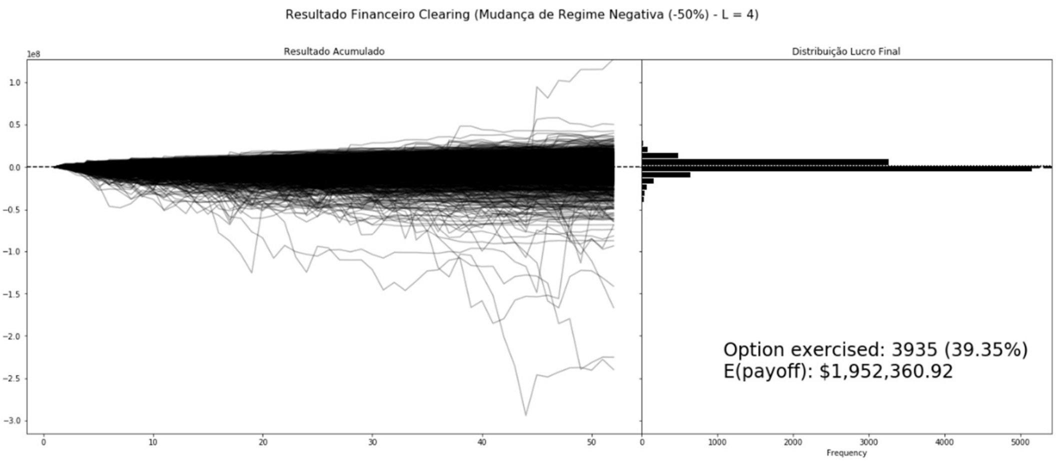
```
print('UP L3 = ' + str(UPL3))
print('UP L4 = ' + str(UPL4))
```

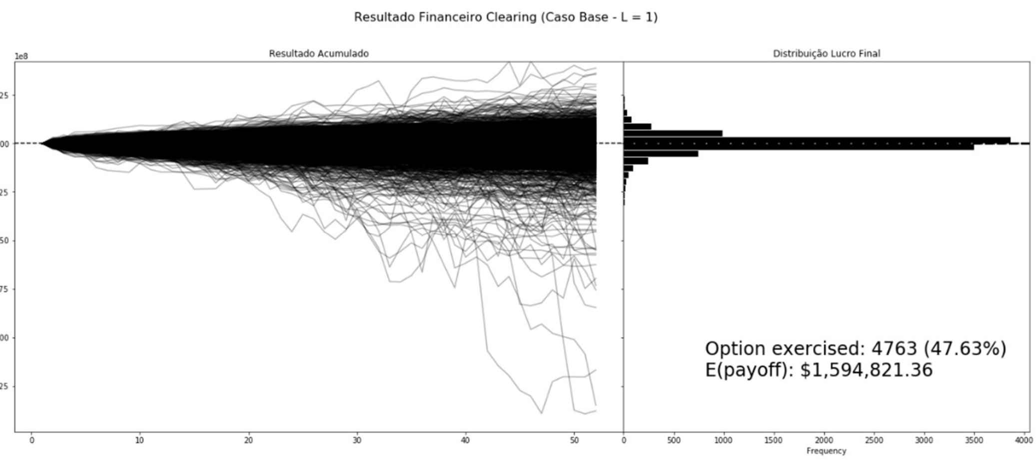
Appendix I. Supplementary Data 3 – Accumulated Returns

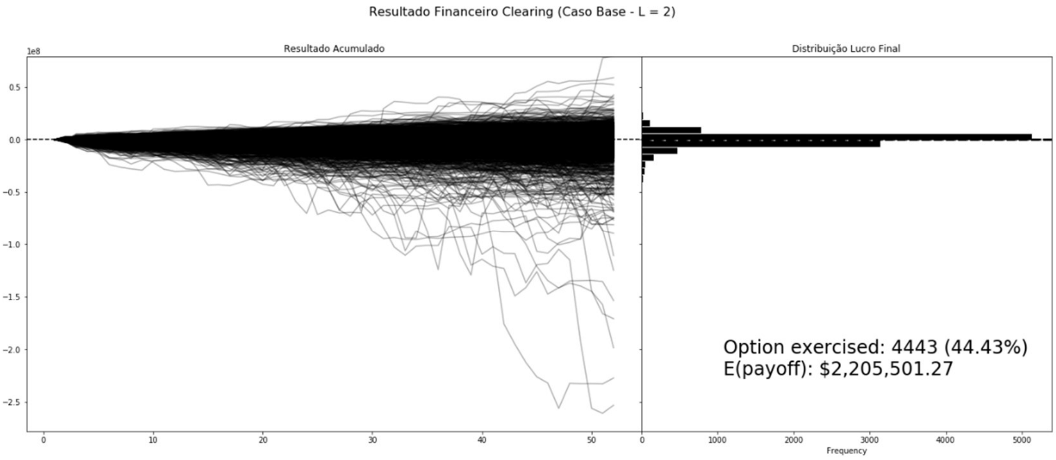


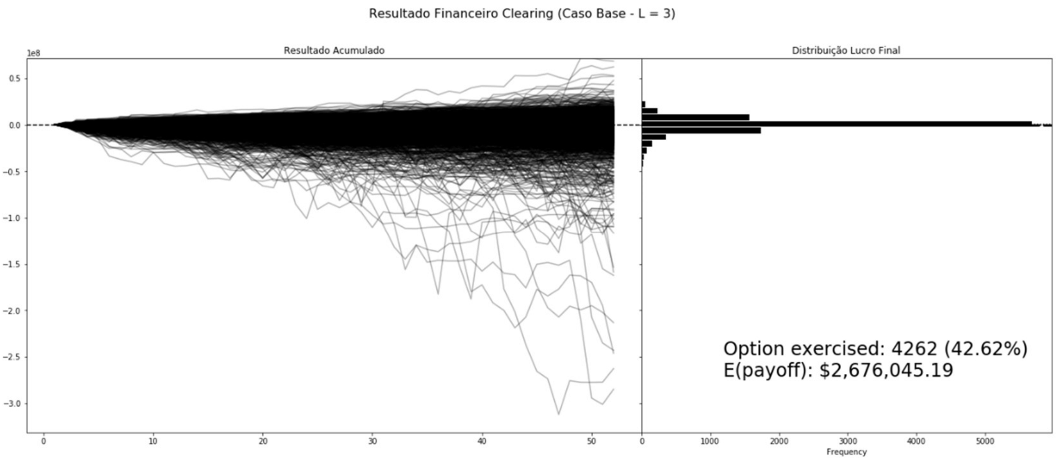


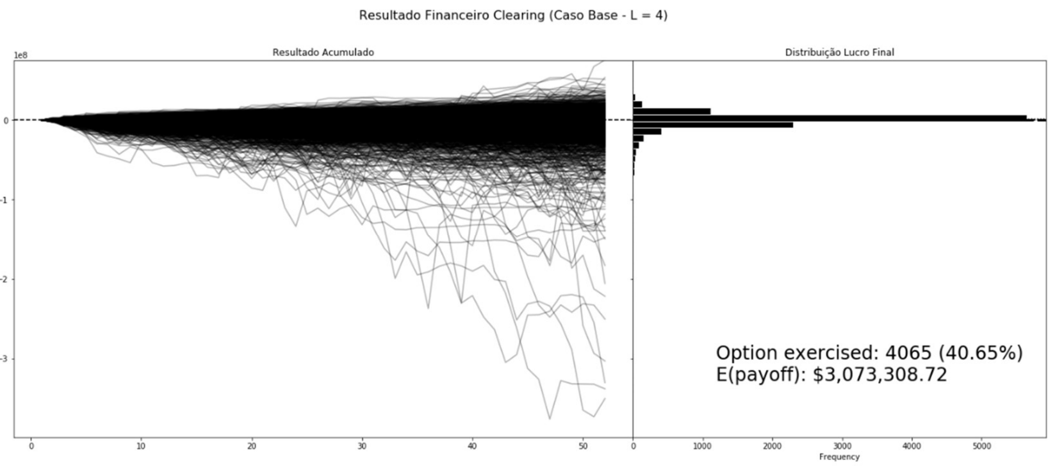

Resultado Financeiro Clearing (Mudança de Regime Negativa (-50%) - L = 3)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 4040 (40.40%)
E(payoff): $1,800,629.69

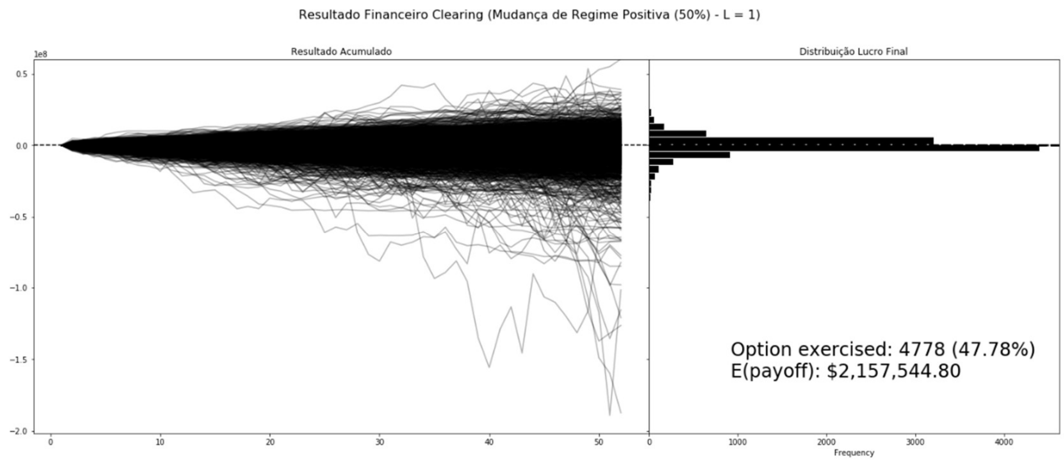

Resultado Financeiro Clearing (Mudança de Regime Negativa (-50%) - L = 4)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 3935 (39.35%)
E(payoff): $1,952,360.92

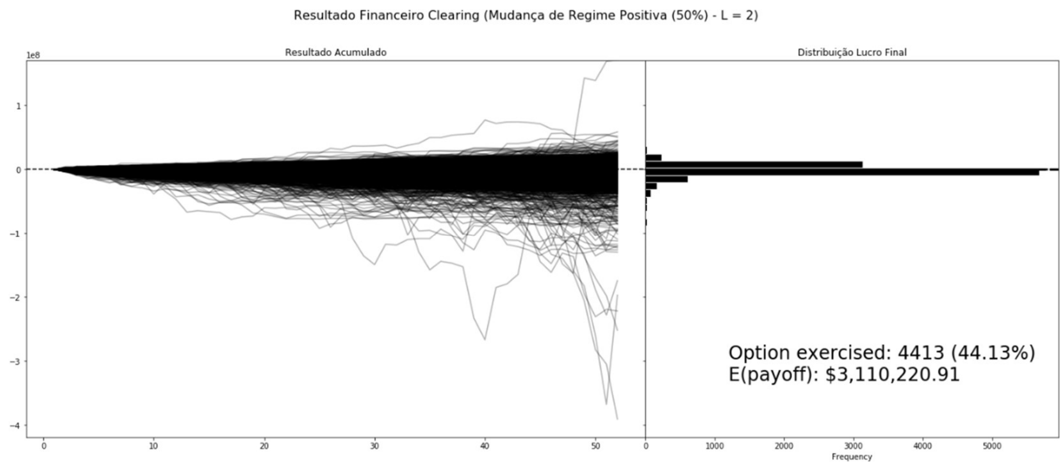

Resultado Financeiro Clearing (Caso Base - L = 1)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 4763 (47.63%)
E(payoff): $1,594,821.36

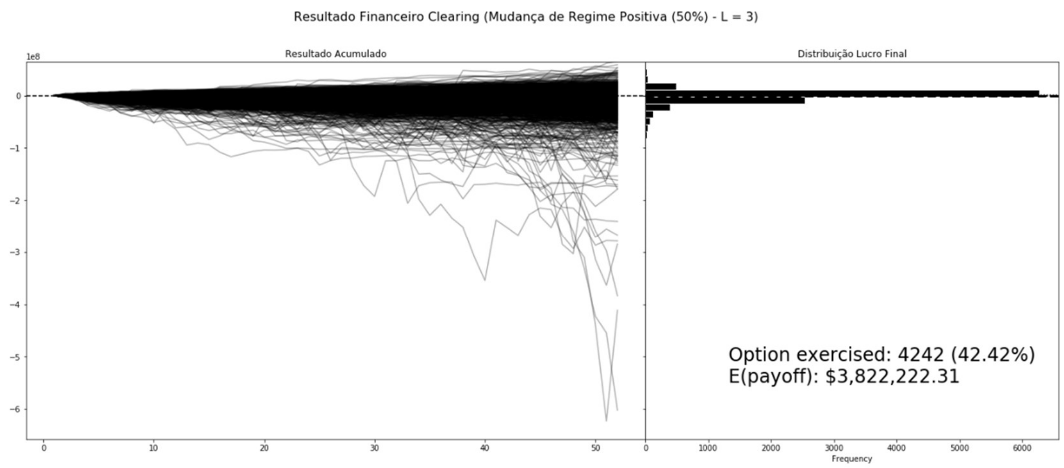

Resultado Financeiro Clearing (Caso Base - L = 2)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 4443 (44.43%)
E(payoff): $2,205,501.27

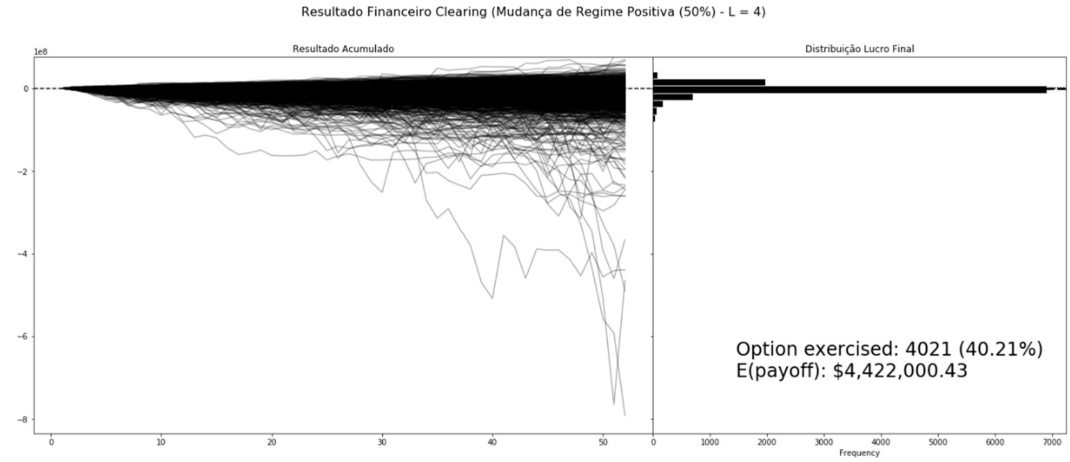

Resultado Financeiro Clearing (Caso Base - L = 3)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 4262 (42.62%)
E(payoff): $2,676,045.19


Resultado Financeiro Clearing (Caso Base - L = 4)
Resultado Acumulado
Distribuição Lucro Final
1e8
Frequency
Option exercised: 4065 (40.65%)
E(payoff): $3,073,308.72


Resultado Financeiro Clearing (Mudança de Regime Positiva (50%) - L = 1)
Resultado Acumulado
Distribuição Lucro Final
1e8
Option exercised: 4778 (47.78%)
E(payoff): $2,157,544.80
Frequency


Resultado Financeiro Clearing (Mudança de Regime Positiva (50%) - L = 2)
Resultado Acumulado
Distribuição Lucro Final
1e8
Option exercised: 4413 (44.13%)
E(payoff): $3,110,220.91
Frequency


Resultado Financeiro Clearing (Mudança de Regime Positiva (50%) - L = 3)
Resultado Acumulado
Distribuição Lucro Final
1e8
Option exercised: 4242 (42.42%)
E(payoff): $3,822,222.31
Frequency


Resultado Financeiro Clearing (Mudança de Regime Positiva (50%) - L = 4)
Resultado Acumulado
Distribuição Lucro Final
1e8
Option exercised: 4021 (40.21%)
E(payoff): $4,422,000.43
Frequency