Pontifícia Universidade Católica
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Maria de Fatima Lacerda Barbosa

A Tailored Derivative Instrument to Mitigate the Price-and-Quantity Risk faced by Wind Power Companies

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Engenharia Elétrica, do Departamento de Engenharia Elétrica of PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

Advisor : Prof. Alexandre Street de Aguiar
Co-advisor: Prof. Bruno Fânzeres dos Santos

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Abstract

Barbosa, Maria; Street, Alexandre (Advisor); Fânzeres, Bruno (Co-Advisor). **A Tailored Derivative Instrument to Mitigate the Price-and-Quantity Risk faced by Wind Power Companies**. Rio de Janeiro, 2023. 83p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

The intermittent nature of wind generation combined with the well-known volatility of electricity spot prices expose Wind Power Companies (WPCs) committed to long-term forward contracts to the so-called price-and-quantity risk. Several instruments were designed in the past years to mitigate this risk exposure. However, most of them were mainly constructed to cope with only one of its parts, i.e., price or generation uncertainty. To tackle this issue, in this work, we propose a tailored derivative instrument for WPCs leveraging the principles of options and renewable indexes. The effectiveness and attractiveness of the proposed instrument, referred to as the Wind-Indexed Option (WInd-Op), are evaluated with real data from the Brazilian sector through a general equilibrium setup. We show that Solar Power Companies (SPCs) can be relevant candidates to back these derivatives. Additionally, when compared to the traditional put-and-call options as a benchmark, the results indicate that the equilibrium obtained with the new derivative exhibits a significantly higher total traded volume, lower premium prices, and greater overall welfare.

Keywords

Economic Equilibrium; Energy Trading; Option Contract; Price-and-Quantity Risk; Power System Economics; Renewable Index; Variable Renewable Energy Sources.

Resumo

Barbosa, Maria; Street, Alexandre; Fânzeres, Bruno. **Desenho de um Novo Derivativo para Mitigar o Risco de Preço e Quantidade de Empresas de Produção de Energia Eólica**. Rio de Janeiro, 2023. 83p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

A natureza intermitente da geração eólica combinada com a conhecida volatilidade dos preços de eletricidade expõe as Empresas de Energia Eólica (WPCs) comprometidas com contratos de longo prazo aos chamados riscos de preço e quantidade. Vários instrumentos foram desenvolvidos nos últimos anos para mitigar essa exposição ao risco. No entanto, a maioria deles foi construído para lidar apenas com uma das partes, ou seja, a incerteza de preço ou geração. Para enfrentar essa questão, neste trabalho, propomos um instrumento derivativo customizado para as WPCs, aproveitando os princípios de opções e índices renováveis. A eficácia e atratividade do instrumento proposto, denominado Opção Eólica (WInd-Op), são avaliadas com dados reais do setor brasileiro por meio de um modelo de equilíbrio geral. Mostramos que as Empresas de Energia Solar (SPCs) podem ser candidatas relevantes para respaldar esses derivativos. Além disso, quando comparado com as opções tradicionais de compra e venda, usadas como referência, os resultados indicam que o equilíbrio obtido com o novo derivativo apresenta um volume total de negociação significativamente maior, preços de prêmio mais baixos e maior bem-estar geral.

Palavras-chave

Equilíbrio Econômico; Economia da Energia; Contrato de Opção; Risco de Preço e Quantidade; Índice de Energia Renovável.

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List of Abreviations

ACL – Regulated Contracting Environment

ACR – Free Contracting Environment

ANEEL – Brazilian Electricity Regulatory Agency

ARIMA – Auto-Regressive Integrated Moving Average

B3 – Brazil, Bolsa, Balcão S.A.

BBCE – Balcão Brasileiro de Comercialização de Energia S.A.

CAES – Compressed Air Energy Storage

CCEE – Chamber of Electric Energy Commercialization

CE – Certainty-Equivalent

CMN – National Monetary Council

CVaR – Conditional Value-at-Risk

CVM – Securities and Exchange Commission

DR – Demand Response

ECP – Extended CVaR Preference

EEX – European Energy Exchange

EPE – Energy Research Company

FEC – Firm Energy Certificate

MCP – Mixes Complementary Program

MME – Ministry of Energy and Mines

NDF – non-deliverable forward

NE – Northeast

ONS – National Electric System Operator

OTC – Over-the-counter market

PLD – Price of Differences Settlement

PPA – Power Purchase Agreement

P-Q Risk – Price-and-Quantity Risk

SH – Small-Hydro

SPC – Solar Power Companies

VaR – Value-at-Risk

vRES – Variable Renewable Energy Sources

WInd-Op – Wind-Indexed Option

WPC – Wind Power Companies

WPP-I – Wind Power Performance Index

1
Introduction

1.1
Motivation

The ever-increasing penetration of variable Renewable Energy Sources (vRES) – e.g., solar and wind power plants – in the current electrical generation mix introduce high levels of uncertainty and complexity to the energy portfolio management process of both generation companies and system operators due to their variable nature and limited production predictability (RINGKJØB; HAUGAN; SOLBREKKE, 2018). On top of this supply uncertainty, from an economic perspective, spot prices for electricity in most power markets around the globe are recognized by their high variability and volatility (WERON, 2014). In this context, the decarbonization agenda has driven power systems worldwide towards a massive transition from conventional to renewable generation fleet. This transition towards a zero-marginal cost and variable generation fleet imposes not only operational, but also relevant economic and regulatory challenges (see (PEREIRA; BARROSO; ROSENBLATT, 2004) and (RIBEIRO et al., 2023)).

In Brazil, for instance, due to its hydro-dominated characteristic and a tight-pool-based price formation, the spot price recovers the system's marginal operative cost. Due to the massive participation of zero marginal cost renewable generation (hydro, wind, biomass, and solar generators) in the system (higher than 70% on average), spot prices are frequently at very low levels. Notwithstanding, this pattern is oftentimes broken by unexpected crises with high spot-price spikes due to a number of reasons, such as unexpected droughts, planning bias, etc. As a consequence, renewable generators often enroll in long-term power purchase agreements (PPA), which are financial forward contracts, to mitigate spot market risks and ensure feasible and reliable project finance planning (see (RIBEIRO et al., 2023)). In this context, the Brazilian power sector, albeit singular, offers interesting insights for systems with an increasing share of zero marginal cost generation. Relevant evidence in favor of fixed-price forward contracts to address long-term supply adequacy in these contexts has been recently reported in (WOLAK, 2022) and (RIBEIRO et al., 2023).

However, due to the intermittence of some renewable generation (e.g., wind and solar), they are exposed to mismatch (deficits and surpluses) with respect to their PPA, which may lead to the so-called Price-and-Quantity Risk (PQ-Risk) (see (OUM; OREN; DENG, 2006) and (STREET et al., 2009a)). More specifically, on the one hand, this exposure is materialized whenever a deficit in energy production with respect to the contracted delivery obligation occurs and the renewable agent must clear this deficit in high spot price values. On the other hand, in the case of an energy surplus event, the spot price levels can be significantly lower than the contract price, reducing the income revenue of this surplus. In the past years, several instruments and approaches were introduced to electricity markets aiming at mitigating this risk exposure (STREET et al., 2009a; PINEDA; CONEJO, 2012; FREIRE et al., 2015; BRIGATTO; FANZERES, 2022; ARANHA et al., 2023; XUE et al., 2022; BHATTACHARYA et al., 2016; MATSUMOTO; YAMADA, 2019).

Interestingly, most of the financial instruments were mainly built to cope with price or generation uncertainty, whereas most of the portfolio optimization approaches rely on capital-intensive or centrally coordinated portfolio structures. For instance, while in (STREET et al., 2009a), a portfolio of complementary renewables is centrally coordinated to mitigate the PQ-Risk when selling a long-term forward contract, in (FREIRE et al., 2015), a renewable pool is proposed, and the quotas of future revenue are allocated according to a cooperative game approach. Notwithstanding, these approaches rely on a central party for synergy and risk-mitigation coordination. However, in (PINEDA; CONEJO, 2012), derivatives (call and put options) are studied in a multistage environment, highlighting the benefits of the flexibility of these relevant instruments. This relevant work highlights the benefits of a hedging instrument that is triggered only in exposure situations, thereby being more efficient in addressing the price risk. Following this finding, (BRIGATTO; FANZERES, 2022) proposes to optimally adjust the portfolio levels of renewable sources with call and put options to hedge against the PQ-Risk exposure when selling forward contracts, thereby utilizing derivatives to address the cases where low generation is observed in high spot price scenarios.

Therefore, the objective of this work is to propose a new tailored financial hedging instrument to aid Brazilian WPCs to efficiently mitigate their exposure to the double-sided PQ-Risk when committed to long-term forward contracts. To achieve this goal, we utilize the idea of renewable indices and financial derivatives to develop a new Wind-Indexed Option (Wind-Op). More objectively, we first introduce a novel index called the Wind Power Performance Index (WPP-I), designed to measure production imbalances and generation risk factors. Then, we propose a new derivative that offers a payoff exclusively in instances where the PQ-Risk materializes, meaning in situations where there is either a production deficit alongside a high spot price or a production surplus accompanied by a low spot price. Additionally, the magnitude of the payment corresponds to the financial exposure in both scenarios, thereby resulting in an effective payoff to address the PQ-Risk.

To study the properties of the proposed derivative instrument in a competitive marketplace, we derive a mathematical programming-based problem to identify and study the maximum welfare equilibrium state. Two numerical experiments are conducted to showcase the effectiveness and attractiveness of the proposed WInd-Op using real data from the Brazilian power market. Results indicate that SPCs are relevant candidates to issue this derivative due to their usual hourly complementarity production profile to wind sources. We also perform a numerical comparison with the traditional hedging strategy of acquiring put-and-call options to benchmark the performance of the proposed mechanism. Although tested with Brazilian data, the authors understand that the newly proposed ideas may be of interest to other power systems where long-term contracts are used as a long-term supply adequacy instrument. Therefore, the studies and insights provided in this paper may also contribute to fostering renewables competitiveness, reducing subsidies, and allowing natural complementary resources to address the PQ-Risk in other countries.

1.2
Literature Review

In the past years, several techniques with different solution approaches were proposed in the technical literature to equip renewable agents with efficient managerial tools and decision-support systems to handle the major PQ-Risk factors and exposure. (MO; GJELSVIK; GRUNDT, 2001) explores the synergy in operation and trading to integrate and optimize the scheduling and contract portfolio position of a hydropower generator by means of a risk-constrained optimization model with minimum revenue targets. (PINEDA; CONEJO, 2012) studies the optimal portfolio composition of derivatives to hedge the risk faced by an electricity producer due to spot price uncertainty and availability of its production units. Regarding long-term contracting strategies, the optimal willingness-to-supply curve for a risk-averse renewable agent is proposed in (STREET et al., 2009b). Such concept, which encompasses the idea of finding the optimal contracting level of renewable generators to hedge against the PQ-Risk, has been largely utilized by generation companies in Brazil in iterative long-term contracting auctions carried out in this country since 2004 (BARROSO et al., 2011).

By exploring the usual complementarity in energy production observed between different renewable sources, in (STREET et al., 2011), a novel commercial model for a WPC based on a joint-trading strategy with a run-of-river small hydro is being proposed to the Brazilian energy market. In

(STREET et al., 2009a), the authors propose a joint risk-averse portfolio selection model to combine a biomass generation unit (using sugarcane waste) and a small hydro to support a long-term forward contract. Lately, acknowledging the challenges in appropriately characterizing the uncertainty in energy spot prices, (FANZERES; STREET; BARROSO, 2015; FANZERES; STREET; BARROSO, 2014) extend these ideas and devise a two-stage hybrid robust-stochastic portfolio selection model to combine the generation of a small hydro and a wind generator under an ambiguity-averse decision-making framework, resulting in the first distributional robust paper in energy commercialization. Further on, a portfolio investment strategy was studied in (MAIER; STREET; MCKINNON, 2016) combining investment opportunities in multiple complementary sources and the possibility of selling contracts in the two main long-term contracting environments in Brazil. Similarly, the authors in (NIETA; CONTRERAS; MUÑOZ, 2013) set up a stochastic decision-making process to devise the optimal coordinated operation of a wind farm and a pumped-hydro storage plant in the day-ahead electricity market, with applications to the Iberian system. Aiming to avoid the reliance on capital-intensive portfolio acquisition schemes, (FREIRE et al., 2015) proposes exploring the complementarity of renewable sources through a risk-averse renewable energy hedge pool. More recently, (BRIGATTO; FANZERES, 2022) proposes to optimally adjust the portfolio levels of renewables sources with call/put options to hedge against the price-and-quantity risk exposure when selling forward contracts.

Interestingly, in recent literature, the main instrument types to mitigate the impact of different risk factors in electricity markets and operations are future and option contracts. For instance, (PINEDA; CONEJO, 2013) designed a multi-stage stochastic programming problem to determine the optimal contracting decisions for a risk-averse power producer and demonstrated that options could be more appropriate as a hedging instrument than future contracts. In (HEDMAN; SHEBLE, 2007), a comparison between the following two types of hedging procedures is performed with a focus on the risk a WPC faces in the electricity market: (i) a joint operation with a pumped-storage hydro plant and (ii) option purchasing. The results in (HEDMAN; SHEBLE, 2007) indicate that option contracts are more competitive, considering the high investment costs of storage deployment. Recently, Wind index derivatives, based on wind speed or wind energy, emerged to cope with wind-related volume risks directly. The European Energy Exchange (EEX) introduced in 2016 exchange-traded wind power futures to address this market need. Nasdaq OMX, which is mainly active in the Nordic and UK markets, has also included a German wind power future index in its contract portfolio (KANAMURA; HOMANN; PROKOPCZUK, 2021). In (GERSEMA; WOZABAL, 2017), an equilibrium model for the valuation of wind futures is presented. A standardized contract underlying the capacity factors for the average German wind resource has been devised to help WPCs to cope with weather-related uncertainties. The analysis in (GERSEMA; WOZABAL, 2017) indicated that both wind and conventional power producers benefit from trading the product. The authors in (XUE et al., 2022) designed a standard option contract model based on a different index, a renewable energy price index, and used an Auto-Regressive Integrated Moving Average (ARIMA) framework to forecast the index's future dynamics. Similarly, weather derivatives are also studied as an efficient hedging mechanism to address the quantity risk faced by SPCs. In the particular context of this source, payoffs are in general dependent on the levels of temperature or solar irradiation being higher or lower than an *a-priori*-defined threshold. In (BHATTACHARYA et al., 2016) and (MATSUMOTO; YAMADA, 2019), different weather derivatives are discussed to provide hedging strategies for SPC. More specifically, (BHATTACHARYA et al., 2016) investigates hedging strategies using temperature-based weather derivatives and (MATSUMOTO; YAMADA, 2019) builds a hedging strategy based on a portfolio of derivatives with the crude oil price, solar irradiation, and temperature as the underlying assets.

Despite the relevance of the aforementioned literature, their solution approaches are based on either the physical combination of assets with complementary production profiles or the acquisition of different instrument types that might not be effective in mitigating the exposure to the particular double-sided PQ-Risk. In fact, they are usually designed to act effectively on price or generation uncertainty. Therefore, in the face of a massive transition towards a renewable, intermittent, and zero-marginal cost-based generation fleet, the design of new products tailored to effectively address the PQ-Risk allowing renewables to increase their contracting involvement is key.

1.3 Summary of Contributions

The main contributions of this work are threefold:

1. To craft a financial derivative for WPCs – referred to as a *Wind-Indexed Option* (WInd-Op), to efficiently hedge against both sides of the price-and-quantity risk leveraging the principles of financial options and renewable indexes. For this purpose, a *Wind Power Performance Index* (WPP-I) is proposed based on a regional production profile per unit of Firm Energy Certificates. Then, a double-sided option contract is devised whose payoff dynamics is a function of both underlying variables: (i) WPP-I and (ii) energy spot price.

2. To construct a framework for an appropriate analysis of the proposed hedging instrument's effectiveness based on economic equilibrium concepts. We formalize the process by deriving an equivalent mathematical programming-based formulation to identify the equilibrium state along with a rationale to compute the associated equilibrium prices.

3. To provide insights into the attractiveness of the proposed WInd-Op by means of a numerical experiment with real data from the Brazilian

power system and renewable agents. We show that SPCs are relevant candidates for selling the derivative to WPCs, and both can highly benefit from trading the derivative within a competitive environment. We also compare the performance of the proposed instrument with the traditional strategy of acquiring a put-and-call option as a benchmark. Results indicate that the equilibrium obtained with the new instrument has a significantly higher total traded volume, lower premium prices, and greater overall welfare compared to the put-and-call options benchmark.

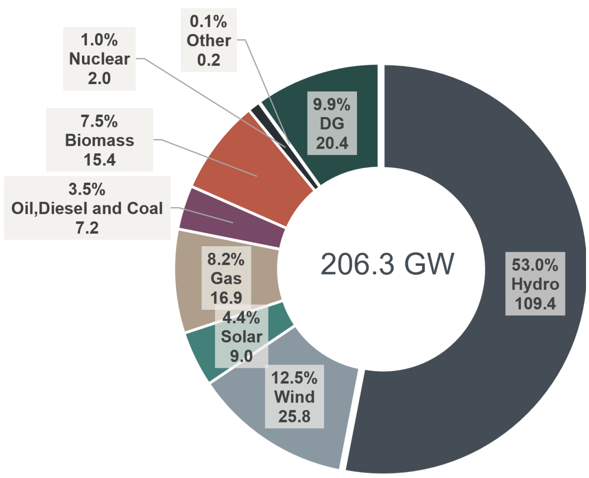
1.4
Work Organization

This dissertation is organized as follows. In chapter 2 we give a brief overview of the Brazilian electricity market and its undergoing changes. Chapter 3 discuss the principles of derivatives in electricity markets. Chapter 4 presents a theoretical background in equilibrium models and uncertainty modeling. Chapter 5 explain the Price-Quantity Risk. Chapter 6 presents the proposed financial hedge mechanism to protect WPP's revenues, in it is detailed the proposed option's payoff formulation and the relationship between it and a conventional put-and-call option. In Chapter 7, the mathematical formulation for the risk-averse profit maximization problem to obtain the optimal contracting strategy for the proposed product by an individual producer is presented. Chapter 8 extends the mathematical formulation of Chapter 7 to a market equilibrium model. Chapter 9 provides numerical results for two case studies using real data from the Brazilian power sector; the first one considers only one WPP and seeks to demonstrate WPP's willingness to demand and SSP's willingness to offer the proposed product, while the second case studies aim to check the viability of the proposed product in an efficient market. Relevant conclusions are drawn in Chapter 11.

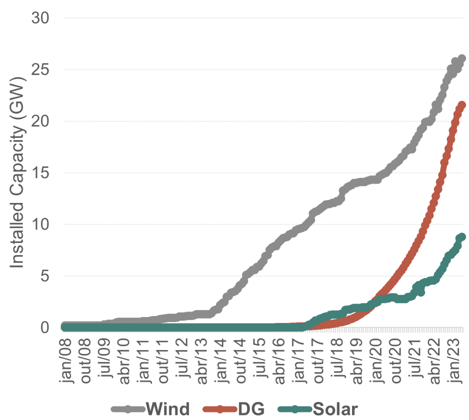
2
Principles of Brazilian Sector

2.1
Brazilian Electricity Market

Abundant hydropower plays a significant role in shaping the power market design in Brazil. Out of the 206.3 GW of installed capacity in 2023, approximately 53% of the energy is generated by hydroelectric plants. The remaining energy sources contribute to the energy mix, with wind accounting for 12.5%, natural gas for 8.2%, biomass for 7.5%, centralized solar for 4.4%, oil for 2.0%, coal for 1.7% and nuclear for 1.0% (ONS, 2023b) – see Figure 2.1a. Additionally, it should be noted that 9.9% of the total installed capacity is attributed to distributed generation, with solar sources being the primary contributor (approximately 98.9%). In total, 87.3% of the installed capacity consists of renewable energy sources. It is worth highlighting the remarkable expansion of wind and solar sources, as well as distributed generation in recent years, which is expected to persist in the upcoming years – see Figure 2.1b.



(a) Total electricity supply by source - June/2023. (ONS, 2023b)



(b) Growth in installed capacity of wind and solar power. (ONS, 2023a) (ANEEL, 2023)

Figure 2.1: Installed Power in Brazil.

The National Electric System Operator (ONS) anticipates an 8.3 GW (93%

growth) increase in installed centralized solar capacity by 2027, alongside a 7.8 GW (30% growth) increase in wind capacity (ONS, 2023b).

An important characteristic of the Brazilian energy market is its concern with supply security. To ensure the supply and enable the expansion of the generation capacity, the current model of the electricity sector, implemented in 2004 by Decree 5,163, is based on two main rules: i) 100% of the energy demand must be covered by contracts, and ii) all contracts must have a physical energy ballast. For the proper functioning of this scheme, the government-owned Energy Research Company (EPE) determines the Firm Energy Certificate (FEC) for each project, indicating the amount of energy each generator is capable of delivering to the system. The FEC value is the maximum amount of energy that the generator can sell in the market.

The energy market in Brazil is divided into two contracting environments: Regulated Contracting Environment (ACR) and Free Contracting Environment (ACL). In the Regulated Contracting Environment, energy negotiations are conducted through centralized auctions, where distributors purchase energy from generators for their concession region. These auctions usually occur 3 or 5 years before the beginning of supply, and long-term contracts (20-30 years) are signed. This model aims to direct the expansion of generation in the country and facilitate the financing of new-generation projects. This market serves small and medium-sized consumers, and the Brazilian Electricity Regulatory Agency (ANEEL) regulates the energy tariffs paid by them.

In the Free Contracting Environment, agreements are established bilaterally between generators, energy traders, and large consumers who have chosen to participate in this environment. Therefore, the terms of the contracts are negotiated between the parties, such as price, duration, and flexibility. This market has grown significantly in recent years by offering more competitive prices to consumers. In 2022, the free market accounted for 36% of total consumption, compared to 25.5% in 2016. The trend is that this market will expand even further in the coming years due to the gradual reduction of requirements for migration from the regulated to the free environment.

Starting from January 2023, a consumer must have a load of at least 500 kW, connected to any voltage level, to migrate to the free market. This requirement was set by MME Ordinance No. 465/2019, which has been gradually easing the criteria in recent years. In another move towards fully opening up the market, MME Ordinance No. 50/2022 allowed all consumers supplied at high voltage to choose any energy supplier from January 1, 2024. In line with this, MME opened Public Consultation No. 137/2022, which proposed greater freedom of choice for low-voltage consumers, including residential, commercial, and industrial ones. The consultation discussed the possibility of residential consumers being able to freely choose their supplier from 2028, and commercial and industrial consumers from 2026. The relaxation of this requirement encourages the growth of the free market and, as a result, the development of the trading market.

When it comes to energy prices in the Brazilian market, it is important to analyze two prices: the market price and the spot price (known as *Preço de Liquidação das Diferenças - PLD*). The former is the price of the bilateral energy contract in the free market and may vary according to energy supply and demand, while the latter is the settlement price for differences in the short-term market. Any surplus or deficit will be valued at the spot price. The price is calculated by the Chamber of Electric Energy Commercialization (CCEE), based on the inputs from the National Electric System Operator (ONS). While generators and sellers can leverage the PLD fluctuation for trading purposes without ignoring risks, this approach is often utilized by generators and trading houses seeking high returns.

Until 2019, the spot price in Brazil was calculated on a weekly basis for three load tiers. However, the increasing use of solar and wind energy sources in the energy mix highlighted the necessity of increasing the granularity of the price. This change enabled the energy market to better reflect real-world operations and provide a more accurate indication of energy value. In July 2019, the Ministry of Energy and Mines (MME) announced through ordinance no. 301, the commencement of hourly operations in 2020 and the implementation of hourly prices from 2021, which are currently in effect.

Currently, in the Brazilian market, the most common form of energy trading is through forward contracts, which, despite not involving physical delivery, need to be registered with the CCEE. In practice, they function as a financial derivative that has evolved within the structure of the Brazilian electricity sector. A new approach gaining momentum involves negotiating future prices using purely financial derivatives. While the market for purely financial derivatives is still in its early stages in Brazil and has limited liquidity, this method offers several advantages over the conventional approach. In more developed markets, trading purely financial derivatives is common, and this market typically coexists with the Balancing market and the Physical Delivery market. Chapter 3 will explore this topic further

2.2 Spot Price

According to the market design, the spot price of energy can be cost-based or bid-based. In the former case, participants are limited to declaring audited costs to the system operator, who models the entire system and optimizes its operation to minimize total operating costs. Energy is then valued based on the marginal cost of operation. In the case of a bid-based market, agents have an active role in price formation, with each agent conducting their own studies to find the opportunity cost and make their bid. Then, the price is determined in a way that maximizes the welfare of the system. The Brazilian system is centrally dispatched by ONS and is cost-based. The main reason for it to follow this market design is the fact that the system is mostly hydrological and with many players in the same river. This characteristic of the system makes it difficult to implement a bid-based system.

The spot prices in Brazil are issued on a day-ahead basis and are derived from the marginal cost of operation (known as CMO) calculated by a unit commitment software (called DESSEM). DESSEM receives as inputs the generation availabilities, the predicted consumption and inflows, and the network data, as well as the cost-to-go function of the mid-term planning model that accounts for the value of the water. The cost-to-go function is obtained by two chained models, a mid- and a long-term hydrothermal dispatch models, namely, DECOMP and NEWAVE, which uses dynamic programming to optimize the reservoirs usage through a five-years horizon. This planning step is centralized based and calculated by the system operator. For more information, we refer to (RIBEIRO et al., 2023).

Additionally, in Brazil, the adopted transmission pricing system follows a regional approach, specifically by submarkets. Currently, the country is divided into four submarkets based on interchange constraints, namely Southeast, South, Northeast, and North. A price difference between the submarkets can arise due to transmission constraints at the interconnection lines. However, if there is no congestion observed in these interconnections, the spot price will be uniform across the submarkets.

Another characteristic of the Brazilian market is that the spot price is subject to an hourly maximum limit, a daily maximum limit, and an hourly minimum limit. The values applicable in 2023 are R$/MWh 1404.77, R$/MWh 684.73, and R$/MWh 69.04, respectively. These limits are annually updated by ANEEL.

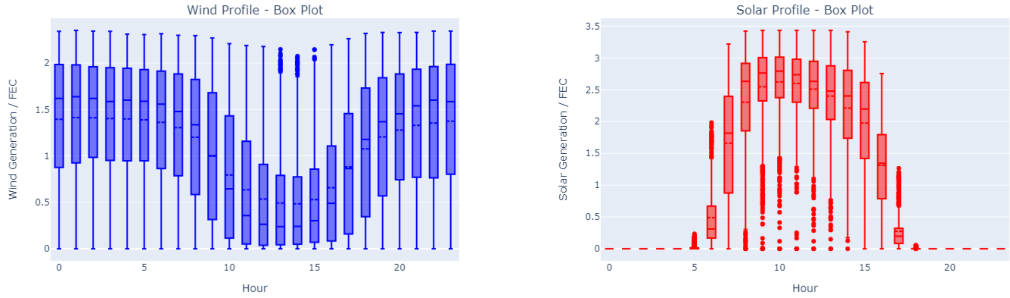
The spot price generally varies according to load, weather conditions, water reservoir levels, power plant availability, energy transmission limits, and system expansion schedule. As a result, the spot price is generally highly volatile. A study by CCEE indicates that hydrology variation and the reservoir's levels account for more than 64% of the volatility of the spot price in Brazil (CCEE, 2022).

As discussed in the previous section, the increasing usage of wind and solar power in the system, sources with a variable hourly profile, has made it necessary to increase the granularity of the settlement period in the Brazilian market. The transition from weekly to hourly pricing has brought benefits and opportunities to the sector. These include enhanced operational flexibility, decreased charges associated with the thermal power plants dispatched out of order of merit, and the opportunity to introduce storage programs and demand response initiatives, among others.

On the other hand, the expansion of renewable sources is expected to contribute to increased price volatility. This is attributed to the intermittency of these sources, which increases the system's reliance on quickly activatable sources, often fulfilled by flexible thermal plants with high variable costs. Large hydroelectric plants also play a significant role in providing system regulation, further emphasizing the importance of effective management of hydraulic resources. However, it is worth noting that the socio-environmental limitations associated with building new large hydro reservoirs hinder the feasibility of expanding hydroelectric power.

2.3 Complementarity of solar and wind generation

The derivative instrument proposed in this work was specifically designed for WPPs situated in northeastern Brazil with a profile similar to the one depicted in Figure 2.2a, which is commonly observed in the region.



(a) Hourly Box Plot profile of $\frac{Wind\ Generation}{FEC}$. (b) Hourly Box Plot profile of $\frac{Solar\ Generation}{FEC}$.

Figure 2.2: Hourly Profile.

Figure 2.2a shows the hourly profile box plot of $\frac{Wind\ Generation}{FEC}$ for WPP Brotas de Macaúbas, situated in the state of Bahia, spanning from July 2019 to July 2021. The median of generation is represented by the solid line within the box, while the dashed line signifies the average. A well-defined pattern can be seen, with a decline in generation during the late morning and afternoon periods.

On the other hand, Figure 2.2b illustrates the hourly profile box plot of $\frac{Solar\ Generation}{FEC}$ for SPP Lapa, located in the state of Bahia, also in the period from July 2019 to July 2021. The graph reveals a characteristic pattern, with an abundance of generation during the day, with the peak around noon.

By observing the hourly generation profiles depicted in Figures 2.2a and 2.2b, it becomes evident that there is a daily complementarity between wind and solar generation, located in this region.While the WPP exhibits a valley in its generation profile between approximately 10 am and 6 pm, with generation levels even falling below the FEC; the SPP generates a significant amount of energy during the day, surpassing the FEC by more than twice its value. Conversely, during the night and early morning, the solar generator ceases its generation, while the wind generator reaches its maximum output during this period, often surpassing the FEC.

Seeking to explore this complementarity, this work proposes a novel hedge instrument for WPPs and assesses the willingness of the SPPs to provide this product to WPPs.

3
Derivatives

Derivatives are financial agreements that derive their value from an underlying asset such as currency, commodities, indices, stocks, or electric energy. They can be linked to the physical delivery of the commodity, or more commonly in the financial market, only to the financial transaction, with their result obtained by the difference between the agreed price and the spot price of the asset (HULL, 2011).

Derivatives are frequently employed for hedging operations and risk management, but can also be utilized for speculations and arbitrage.

These contracts can be traded in a standardized format on a stock exchange, with periodic adjustments to exposure, and with credit risk being assumed by a central counterparty. Alternatively, they can be traded on an organized over-the-counter market, where customized contracts are possible, settled by difference, and involve bilateral credit risk.

In Brazil, the trading of energy has evolved within the framework of the electricity sector, specifically through the CCEE. All contracts for the purchase and sale of energy in the free contracting environment must be registered with the CCEE (registration includes information regarding the quantity of energy and the delivery period, but does not include the price of energy). This institution is responsible for the financial settlement and accounting of all contracts based on the spot price, on a monthly basis. Additionally, the CCEE has the role of monitoring whether all contracts are backed by physical energy, imposing penalties on contracts without backing. These registrations with the CCEE are carried out by generators, consumers, and trading companies. Negotiations take place through bilateral agreements, and subsequently, it is registered with the CCEE. In the case of trading companies, a sales contract can be backed by a purchase contract, as they do not have generation or consumption themselves. Although it is not a purely financial transaction, in practice, these contracts function as financial derivatives (where there is no obligation for physical delivery) of the forward type. The CCEE acts as the entity responsible for contract registration, operating under the supervision of ANEEL. Notably, this form of derivative trading remains the predominant practice in the Brazilian market to this day.

Meanwhile, the Brazilian market for purely financial derivatives indexed to the spot price of energy has been undergoing a gradual development in recent years. In this case, they are considered securities under Law No. 6,358/76, also known as the Capital Market Law. As such, the National Monetary Council (CMN) and the Securities and Exchange Commission (CVM) serve as regulators for the derivatives market, which can only be traded and registered by companies authorized to operate by the CVM, known as financial market infrastructures. Currently, only two such companies are registered with the CVM: B3 (Brazil, Bolsa, Balcão S.A.) and BBCE (Balcão Brasileiro de Comercialização de Energia S.A.).

B3 has been offering registration services for purely financial over-the-counter derivative contracts for electricity in the Forward, Swap, and Option modalities since 2015 (B3, 2023) However, this type of transaction only gained more prominence in early 2021 when driven by the growth of the free contracting environment and the high number of forward contracts linked to CCEE registered (with market turnover reaching up to five times the energy actually consumed), BBCE launched a purely financial derivatives' platform where it began providing registration for this type of contract. The major innovations introduced by BBCE at that time were the screen trading of the product and the possibility of using a standard contract (BBCE, 2023).

The energy derivatives currently available are linked to the spot price (known as *Preço de Liquidação das Diferenças - PLD*) or to the marginal cost of operation (known as *Custo Marginal da Operação - CMO*), calculated, respectively, by CCEE and ONS. The latter began to be made available very recently, in April 2023. The registration systems allow the choice of the term of the contract: weekly, monthly, quarter, semester or annual average, by submarket. Although available, purely financially settled derivatives still have low liquidity in the Brazilian market.

Both platforms, B3 and BBCE, can also be used to negotiate not purely financial forward contracts, which must also be registered with CCEE. BBCE is currently the main platform for screen trading of energy and concentrates a significant portion of the transactions in the free market.

A benefit of purely financial derivatives compared to the common approach currently used in Brazil is the settlement method. The former usually settles based on the difference between the negotiated value and the closing value, rather than the entire value of the operation. This makes agents perceive less risk, attracting new participants and stimulating more trading, ultimately increasing liquidity in the market. Moreover, purely financial derivatives tend to have lower operating costs since they do not need to be registered with CCEE, and reduce the cash requirement of companies, freeing up resources for other transactions. Another advantage is that the structure compatible with the financial market ensures transaction security, drawing in new players such as banks, institutional investors, and investment funds. Additionally, since there is no link to physical settlement, there is no exposure to the ballast penalty (BBCE; ABRACEEL, 2021).

It is also noteworthy that financial derivatives create the opportunity for new products in the energy market. Although energy derivatives are commonly associated with the spot price of energy, it is also possible for them to be derived from another reference related to the electricity sector if agreed upon by the parties and authorized by the regulatory agency.

The main types of derivatives are: forwards, futures, options, and swaps. As this work proposes a new tailored option contract and forwards and futures are the most common derivatives in electricity markets, in this section, we provide a summary of how its derivatives work and how they can serve as effective financial hedging tools.

3.1 Forward

A forward contract is an agreement to purchase or sell a fixed quantity of an asset at a certain future time for a specific price. In the negotiation, one party takes a long position and agrees to buy the underlying asset (a fixed quantity) on a specified future date for a specified price, while the other party assumes a short position and agrees to sell the asset (same fixed quantity of the buyer) on the same date for the same price (HULL, 2011).

The derivative's payoff is linearly correlated to the variation of the spot price, determined by the difference between the negotiated price and the spot price. For this reason, it is classified as a linear derivative. The payoff of a forward purchase contract for a quantity q in MWh, at a pre-defined unit price of S in \$/MWh, on a future date t, is presented in Equation 3-1. The buyer of the contract makes a profit if π_t>S, where π_t is the spot price in \$/MWh at time t. Similarly, the payoff on a forward selling contract is given by the same formula, only by reversing the sign of S and π_t. In this case, the seller will make a profit when π_t<S.

$$Payoff = (\pi_t - S)q \tag{3-1}$$

These contracts are typically negotiated bilaterally in the over-the-counter market (OTC) between generators, consumers, and/or trading companies. The contracts allow customization of prices, terms, quantities, and payment conditions. This flexibility in contracts makes it one of the most classic and widely OTC derivatives. The settlement, most of the time, is by physical delivery, and they are mainly used to hedge against price uncertainty in the short-term market.

Another type of forward contract is the non-deliverable forward (NDF) contract, which does not involve physical delivery and settles at maturity with payment of the difference between the fixed price in the contract and the spot price.

Due to its flexibility, forward contracts are frequently used in the Brazilian electricity market as a tool for risk mitigation. Trading companies are generally risk-taker agents, providing consumers and generators with greater stability and security.

3.2 Future

A future contract is an arrangement between two parties to purchase or sell an asset at a predetermined price and time in the future, akin to a forward contract. However, unlike forward contracts, futures contracts are traded on a market that is operated by an exchange. To enable trading, the exchange defines standard contract features, including quantities traded, deadlines, settlement procedures, and other associated issues. Moreover, futures contracts are usually settled financially, without necessitating physical delivery of the underlying asset (MAYO, 2021).

Since financial contracts are traded on the stock exchange, the counterparties' operations are centralized, which helps reduce credit risk. The parties involved in trading through an exchange can benefit from a clearing house. To reduce the risks of counterparties defaulting, the clearing house mandates that traders deposit funds (called margins) as a guarantee of their ability to meet their obligations.

Additionally, the exchange makes position adjustments that reflect the variation in mark-to-market and these usually are settled daily. In this sense, one of the counterparties has to deposit the difference between the previous day's value and the day's closing value, resulting in a credit to the other counterparty. These daily adjustments are intended to update the financial value of the transactions

By settling profits and losses on a daily basis instead of accumulating them until expiration as in forward contracts, futures contracts reduce credit risk. Additionally, this approach allows the exchange to require lower margin requirements than for the entire contract term and permits participants to enter or exit the market at any time.

3.3 Option

An option is a formal agreement that grants the buyer the right to buy or sell a specific quantity of electricity at a fixed price, known as the strike price, during a predetermined future period known as the delivery period. It is important to note that an option provides the holder with the right to act but not the obligation to do so, distinguishing it from futures and forwards, where the holder is obligated to buy or sell the underlying asset. While there is no cost to enter into a futures or forwards contract, there is a cost associated with acquiring an option, known as the option price, which must be paid regardless of whether the option is exercised (LUENBERGER, 1998).

Calls and puts are the two primary types of options. A call option grants the holder the right to buy a specific quantity of electricity at the strike price, while a put option grants the holder the right to sell a specific quantity of electricity at the strike price. The price of a call option typically decreases as the strike price increases, while the price of a put option typically increases as the strike price increases.

Depending on the delivery period, it is worth noting that the option can be classified as either American or European. American options provide the flexibility to exercise at any point up to the expiration date, while European options can only be exercised on the expiration date itself.

Every option contract has two sides: the agent taking the long position, who purchases either a call or put option, and the agent taking the short position, who sells either a call or put option. The long position grants the right to buy or sell the underlying commodity at the strike price, while the short position undertakes the obligation to buy or sell the underlying commodity at the strike price if the holder of the option chooses to exercise it.

Options are traded on both exchanges and over-the-counter markets. Additionally, the option can be a financial instrument or can have physical delivery.

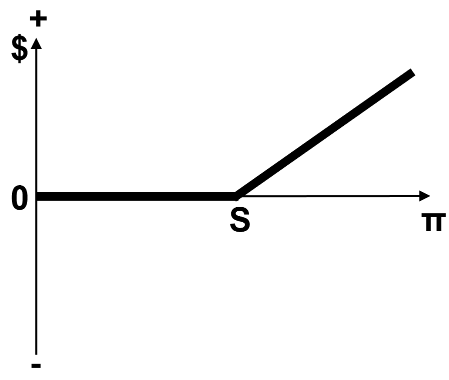
The most relevant feature of options from a hedging perspective is the time delay between the option signing date and its exercise date. This delay allows the option holder to better understand the uncertain parameters during the option delivery period and make an informed decision on whether to exercise the option. Both types of options generally become more valuable as their time to maturity increases.

The typical payoff profile of a call option and put option is shown in Figure 3.1. Suppose you are the owner of a call option on a stock with a strike price of S and the spot price at the delivery period t is π_t. It is apparent that if π_t<S, exercising the option is not beneficial for the buyer since they can purchase the stock at a lower price in the open market. Thus, the option payoff is negative and equal to the option price (c). However, if π_t>S, the option holds value. Exercising the option allows you to buy the stock at a price lower than the market price and protect yourself against excessively high prices. In this case, the value of the option is given by π_t-S-c. Similarly, it is only advantageous for the owner of a put option to exercise the option when S>π_t.

From Figure 3.1, it is evident that the two long positions restrict the potential financial losses to the option price, which is indicative of the conventional conduct of a risk-averse agent. Conversely, the short positions correspond to risk-taker agents who, in return for a specified premium, are willing to bear the risk of the option buyer.

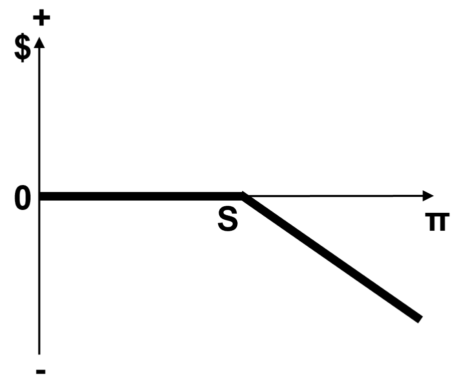
Contracts in the energy market can be utilized for hedging against spot price volatility, thereby enabling predictable profits. For example, a power producer could enter into a forward contract to sell electricity and also purchase a call option for the right to buy electricity during the same delivery period. If the producer's generating unit fails to perform just before the delivery period of both contracts and the pool price is expected to be high, they could exercise the call option to buy electricity. This would enable the producer to fulfill their selling obligation by purchasing electricity through the call option at the strike price, which is likely to be lower than the average pool price during the delivery period. Conversely, if the generating unit performs well or pool prices are expected to decrease below the strike price, the call option would not be exercised.

It is worth noting that while forward contracts and insurance policies are derivatives that reduce either price risk or availability/performance risk, respectively, electricity options are derivatives that can be employed by power producers to hedge against both price and availability risks.



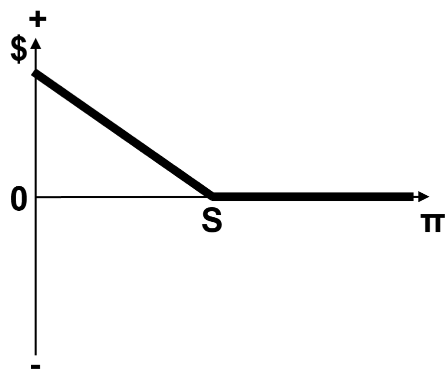
(a) Long call position.

$C_T = max(\pi_t - S, 0)$



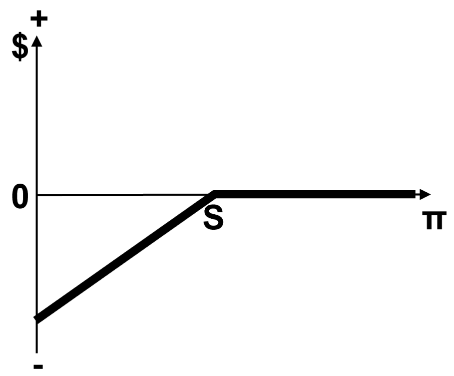
(b) Short call position.

$C_T = -max(\pi_t - S, 0)$



(c) Long put position.

$P_T = max(S - \pi_t, 0)$



(d) Short put position.

$P_T = -max(S - \pi_t, 0)$

Figure 3.1: Payoff diagram of an option depending on the option type and position.

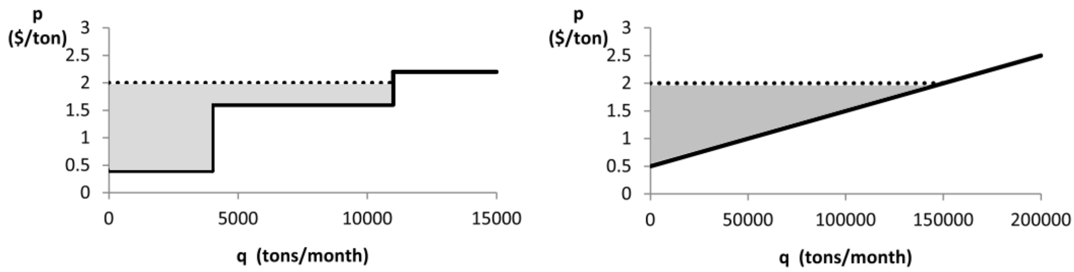
4
Equilibrium Model Principles

This chapter will provide the necessary economic and mathematical foundations for conducting an equilibrium analysis in a competitive market environment. We will also demonstrate how this framework can be formulated as an optimization problem, which can help us assess the effectiveness of the proposed hedging instrument. Additionally, we will introduce important concepts related to risk analysis used in modeling the problem.

4.1
Supply and Demand curves

In a free market, although each company had the legal freedom to set its own prices, charging a higher price than competitors would result in a significant loss of customers, leading to a decrease in revenue. On the other hand, charging less than competitors would attract more customers, but the company would not be able to meet the demand. As a result, companies typically charged the same price as their competitors, with any deviations quickly rectified by managers monitoring the market. This resulted in all firms being regarded as price-takers, and although the price of coal varied from month to month, there was a consensus on the price at any given time. This principle applies to any good or service that is perceived by buyers as being homogeneous, meaning that it is the same regardless of which company supplies it. Examples of such goods or services include wheat, gasoline, standard-sized sheets of plywood, legal services, and more.

The supply curve represents the relationship between the price of a good or service and the quantity that suppliers are willing and able to provide. The solid black line in Figure 4.1a illustrates an example of a supply curve in a market with three suppliers of a specific good. Each of the suppliers has its own production and transportation costs, as well as a specific maximum production capacity. As a result, each supplier is willing to produce a different amount at each price. The lowest-cost supplier among the three are willing to provide up to a maximum of 4000 tons of the good per month, with a variable cost of \$0.40 per ton. The capacity and variable cost of this supplier are depicted by the width and height, respectively, of the first step in Figure 4.1a. The second cheapest supplier can provide up to 7000 tons per month, with a variable cost of \$1.60 per ton, as shown in Figure 4.1a by the second step. Finally, the most expensive supplier can deliver up to 4000 tons per day, with a variable cost of \$2.20 per ton.



(a) Market with three suppliers. (b) Industry-wide.

Figure 4.1: Supply Curve. (GABRIEL et al., 2012)

Additionally, we can conclude from the supply curve that, at a selling price of \$2 per ton, for example, only the cheapest and second cheapest suppliers would be willing to produce, while the most expensive one would not. Thus, in this scenario, the total possible monthly delivery would be 11000 tons. One can still observe that as the price of a good or service increases, the quantity supplied typically increases, and vice versa. This direct relationship between price and quantity supplied is known as the law of supply.

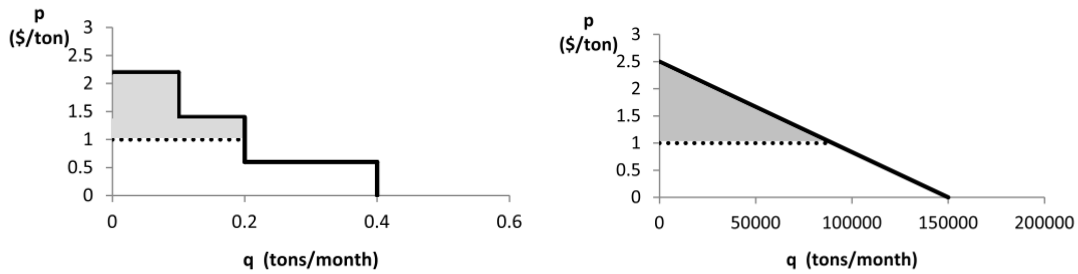
If all companies of this good were taken into consideration, the horizontal axis would have a wider range and the step widths in Figure 4.1a would appear to be relatively small. Furthermore, if there were various costs per ton among all mines, the height of each step would be relatively small too. Therefore, an industry-wide representation could be approximated by a smooth curve, which would have a positive slope, except in cases where a step was particularly wide.

Figure 4.1b illustrates this scenario. The graph depicts a linear correlation; however, it is important to note that other nonlinear correlations are plausible. Another relevant concept is that of company's surplus or producer's surplus. It represents the profit, which is the gross margin minus fixed costs. In Figure 4.1b, the shaded triangular area represents the total producers' surplus of all companies when the price is $2 per ton.

In summary, the fundamental concepts regarding the supply curve are as follows: (a) there typically exists a single price at any particular time; (b) a higher selling price typically leads to a greater quantity of goods being supplied, meaning the supply curve has a non-negative gradient; and (c) the producer surplus can be depicted as the area between the supply curve and the horizontal line at the market price level.

Similarly, the demand curve illustrates the relationship between the price of a good or service and the quantity that consumers are willing and able to buy. Just as suppliers react to prices, consumers also do so, but in this case the relationship is inverse, as the price of a good or service decreases, the quantity demanded typically increases, and vice versa. This inverse relationship between price and quantity demanded is known as the law of demand.

The diagram in Figure 4.2a displays how a particular household customer responded to different prices of a good or service. The behavior of this curve can vary significantly from one consumer to another, depending on the importance of the good to the consumer and whether they can substitute it for a cheaper alternative. When the price exceeded $2.20 per ton, the customer chose not to buy any of the good. When the price was between $1.40 and $2.20 per ton, the household reduced their demand and purchased 0.1 tons per month. At prices higher than $0.60 but no more than $1.40 per ton, the household bought 0.2 tons of the good per month. If the price fell to $0.60 or less per ton, the household was willing to purchase 0.4 tons of the good per month.



(a) Particular household customer. (b) Industry-wide.

Figure 4.2: Equilibrium model implementation. (GABRIEL et al., 2012)

The triangular gray shaded area in Figure 4.2b represents the consumer's surplus, which is the difference between the maximum value that the consumer is willing to pay for a certain quantity of a good or service (intrinsic value) and the actual amount they pay.

4.2 Equilibrium as Intersection of Supply and Demand Curves

The point at which the supply and demand curves intersect is called the equilibrium point, or the market clearing price. At this price, the quantity of the good or service demanded by buyers is equal to the quantity supplied by sellers, and there is no excess supply or demand.

If the price is above the equilibrium point, there will be excess supply, and prices will be pushed down. If the price is below the equilibrium point, there will be excess demand and prices will be pushed up. Hence, the term 'equilibrium' is appropriate for the point $(q^*,\, p^*)$ since any deviation from the price p^* is naturally rectified, and q^* is the quantity that corresponds to p^* on both curves. Figure 4.3 depicts the equilibrium quantity and price, $(q^*,\, p^*)$, which is determined by the intersection of the supply and demand curves.

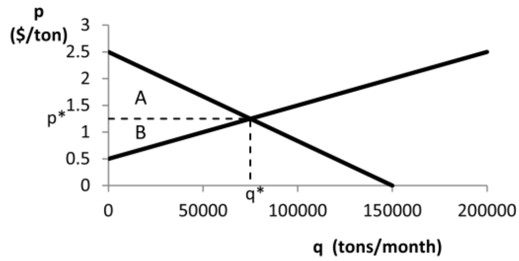


Figure 4.3: Equilibrium point. (GABRIEL et al., 2012)

In most cases, it is reasonable to assume that the prices and quantities observed in a real-world market are equilibrium prices, due to the fact that non-equilibrium situations are unlikely to persist for an extended period.

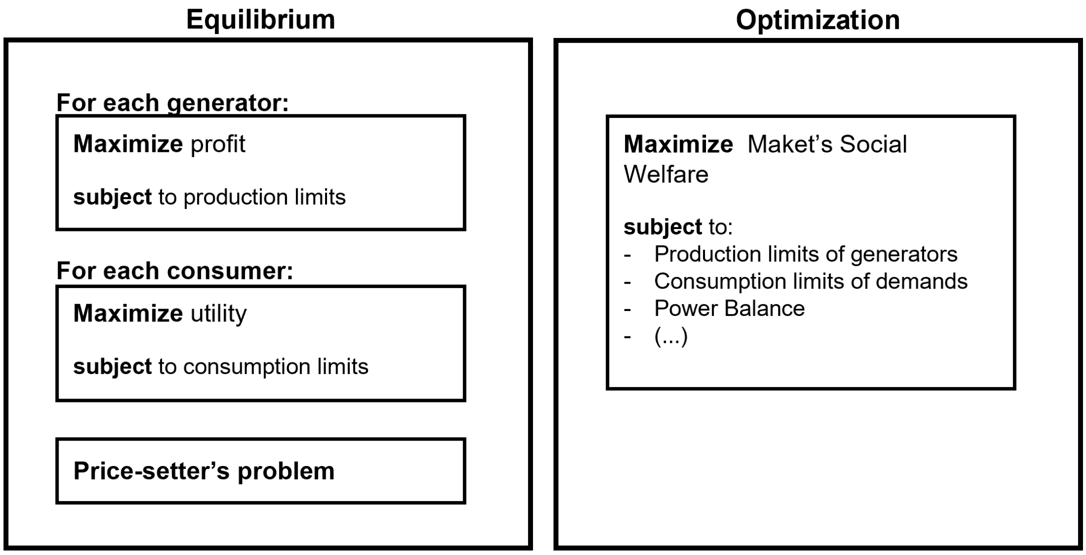
4.3 Equilibrium Solution Using Optimization Problems

As discussed in the previous section, the equilibrium (q^*, p^*) can be calculated by finding the intersection of the demand and supply curves. Another way to conceptualize the competitive equilibrium problem is as a set of optimization problems, where each supplier seeks to optimize its revenue by determining the q^* of production based on the market price and subject to its constraints, while each consumer aims to optimize its utility by determining their q^* of demand based on the market price and its own constraints.

Additionally, it is crucial to add a constraint to the problem to ensure that the quantity of the good or service demanded by consumers is equal to the quantity supplied by producers. Mathematically, this constraint can be transformed into an equivalent unconstrained optimization problem, which we called the price-setter optimization problem. Figure 4.4a shows the overall scheme presented.

These optimization problems cannot be solved separately, since they are linked. We have that for the agents the market price is a parameter and the optimal quantity of the agent is a variable of the problem, but for the price-setter, the equilibrium price is the variable of the problem and the optimal amount of each agent is a parameter of the problem.

Therefore, to solve the problem, the optimization problems must be replaced by KKTs. Then if you put all KKTs together you will have an MCP (Mixes Complementary Program) and will be able to solve all at once. This framework is also known as a game theoretic problem (in this case, a non-cooperative game).



(a) Equilibrium solution.

(b) Equilibrium as Maximization of Social Welfare.

Figure 4.4: Demand Curve. (GABRIEL et al., 2012)

4.4 Equilibrium as Maximization of Social Welfare

Alternatively, if all agents in a single commodity market (consumers and producing companies) act as price-takers (meaning they assume their decisions do not affect the equilibrium price in the market), the equilibrium in the market for that commodity can be modeled as an optimal solution of a mathematical program that maximizes a social welfare function $SW(q)$.

In this context, social welfare is defined as the overall well-being or utility of all agents in the market. The sum of consumer and producer surpluses represents the total social welfare generated by the market. Consumer surplus is the difference between the value that consumers place on a good or service and the price they pay for it, while producer surplus is the difference between the cost of producing a good or service and the price they receive for it. It is possible to evaluate social welfare, at any value of the quantity – there is no need to be restricted to the equilibrium quantity, q^*.

The two methodologies are considered equivalents since the MCP obtained from equilibrium and the KKTs obtained from optimization include identical conditions. Therefore, any solution to the equilibrium problem is also a solution to the optimization problem and vice versa. Figure 4.4a and 4.4b compare the overall scheme of the two methodologies presented. It is important to note that when solving the equilibrium as an optimization model that maximizes social welfare, it is also important to add a commodity balance constraint, ensuring that the sum of demand equals the sum of consumption. Additionally, other necessary constraints to represent the problem can be added, such as production capacity and maximum demand.

The equivalence between the models provides us with very interesting results. We can conclude that at the same time that we maximize social welfare subject to all constraints, we assure that every agent is happy with the solution. We can say that because in equilibrium methodology everyone tries to maximize their own payoff, including the price-setter, we are going to find a Nash Equilibrium Point.

If a solution to our equilibrium problem exists, it will be Nash Equilibrium Point, which means, no market participant can increase its profit by deviating unilaterally from the equilibrium solution. Nash equilibrium is a situation in which no player can improve their payoff by unilaterally changing their strategy, assuming all other players stick to their chosen strategies. In other words, a Nash equilibrium is a state in which no player has an incentive to change their strategy, given that the other players are following a specific strategy. If you optimize it alone maybe you can have a better profit but considering the other participants not. Nash equilibrium is an important concept in game theory and is used to model decision-making in situations of conflict or competition (FACCHINEI; PANG, 2003).

Another relevant point of this equivalence is that, in general, it is computationally much easier to solve the optimization problem than the equilibrium problem. The former can be solved with a simple LP solver, while the latter requires more complex software, such as PATH.

In this study, we developed an equilibrium model to evaluate the attractiveness and effectiveness of the proposed financial instrument. The problem was formulated as an optimization model that seeks to maximize the social welfare of both wind generators (who demand the proposed instrument) and solar generators (who are candidates to offer the proposed instrument).

4.5 Decision under uncertainty

The problem of making decisions in the face of uncertainty involves an agent who needs to establish a policy to guide the future while dealing with uncertainty in certain parameters of the problem. This problem comprises a group of decision variables that must be determined before the uncertainty is realized, known as first-stage variables. Some problems allow for corrective action after the uncertainty is realized. If only one action can be taken, this is referred to as a two-stage problem, and the actions are called second-stage variables. Alternatively, if a series of actions over time is allowed as information is revealed, this is called a multi-stage problem, and the actions are referred to as multi-stage variables (DIXIT; PINDYCK, 1994). For example, in the electricity trading industry, the decision to enter into contracts, which is a first-stage decision, is made prior to the observation of uncertainty (such as spot price, energy generation, etc.), while the second-stage decision involves making adjustments to the short-term market.

A common method to tackle decision-making problems under uncertainty is to replace uncertain parameters with their prediction or expected value, typically derived from the joint distribution. This approach often results in a simple and easily understandable solution. However, when decision variables are particularly susceptible to changes in uncertain parameters that exhibit significant variability, it is imperative that the model used to establish a future guiding policy considers the associated risk (FANZERES, 2014).

Furthermore, every individual (agent) has different preferences or risk profiles, meaning that a distinct value is assigned to the same outcome. Therefore, it is essential to quantitatively handle uncertainty and risk preferences to avoid undesirable solutions.

This section examines one important approach to modeling decision-making problems with uncertainty, called stochastic programming. Additionally, covers the Conditional Value-at-Risk (CVaR), which is a popular risk measure used in both practical and academic contexts. It also explores the concept of Certainty-Equivalent (CE). All these concepts will be used in the equilibrium model implemented in this work to assess the attractiveness and effectiveness of the proposed hedge instrument.

4.5.1 Certainty Equivalent

The Certainty Equivalent (CE) is a concept in decision theory that refers to the lowest deterministic value at which an agent becomes indifferent to a stochastic outcome. (STREET, 2010) demonstrates that the $CVaR_\alpha$ preference index of a random variable is equivalent to its induced certainty equivalent. To prevent weird solutions, with the same $CVaR_\alpha$ value but lower expected returns, a convex combination of the $CVaR_\alpha$ and the agent's unconditioned revenue expectation is introduced, known as the Extended CVaR Preference (ECP).

For a risk-averse agent who adopts the ECP approach, their certainty equivalent $\rho_{\alpha,\beta}(\tilde{R})$ is defined as the ECP of their revenue. Equation 4-1 shows the mathematical formulation.

$$\rho_{\alpha,\beta}(\tilde{R}) = \beta\ CVaR_{\alpha}(\tilde{R}) + (1-\beta)\ \mathbb{E}(\tilde{R}) \tag{4-1}$$

In 4-1, β is the risk aversion parameter and it can range from 0 to 1. If $\beta = 1$, ρ defines a strong risk-averse power producer. On the other hand, if $\beta = 0$, we reach the case of a risk-neutral agent.

4.5.2 Conditional Value-at-Risk

The Conditional Value-at-Risk (CVaR) is calculated as the average of the $(1-\alpha)100\%$ worst scenarios, typically ranging from 1 to 10% (or α from 0.99 to 0.90). This measure is considered a coherent risk metric as it fulfills at least one of the four desirable properties outlined by (ARTZNER et al., 1999): translation invariance, subadditivity, positive homogeneity, and monotonicity. As a result, the CVaR is one of the primary and most promising risk metrics for use in stochastic programming. Moreover, its convex function, which is the result of combining subadditivity and positive homogeneity, can be condensed into a simple linear programming formula proposed by (ROCKAFELLAR; URYASEV, 2002). In this way, for a set Ω of sampled scenarios R_{ω} of the revenue with probability of occurrence p_{ω}, i.e, the pair $(R_{\omega}, p_{\omega})_{\omega\in\Omega}$, the CVaR of a continuous random variable $\tilde{R}$ can be approximated by the following linear programming:

$$\begin{aligned} CVaR_{\alpha}(\tilde{R}) \approx \max_{z,\delta_{\omega}} \quad & z - \sum_{\omega\in\Omega} \frac{\delta_{\omega}\ p_{\omega}}{(1-\alpha)} && \\ \text{s.t.} \quad & \delta_{\omega} \geq z - R_{\omega} && \forall \omega \in \Omega \\ & \delta_{\omega} \geq 0 && \forall \omega \in \Omega \end{aligned} \tag{4-2}$$

Figure 4.5 illustrates two different distributions: distribution A has a shallower lower tail, while distribution B has the potential for very adverse events resulting in significantly negative income. Both distributions share the same Value-at-Risk value of α % (VaR_{α}), which implies that there is an α %

probability of revenue exceeding the VAR value. However, the two curves have different Conditional Value-at-Risk ($CVaR_\alpha$), with $CVaR_\alpha$(B) indicating the presence of high depth events, below the VaR level. This example demonstrates how CVaR addresses the limitations of VaR by providing a more suitable indication of the potential losses that exceed the confidence interval (1-α)100%.

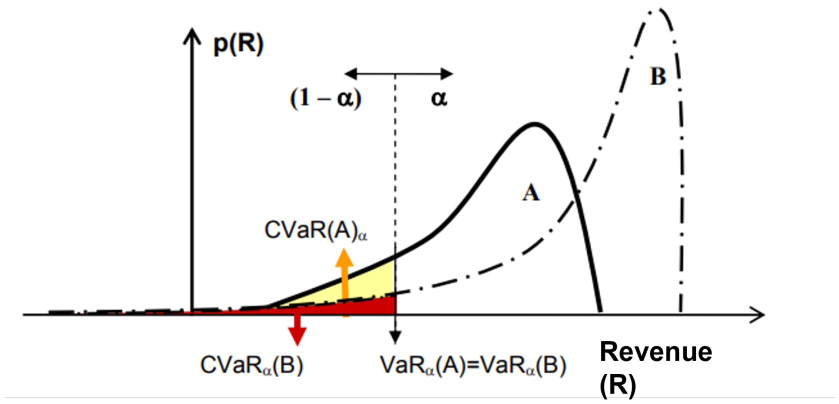


Figure 4.5: Comparison of CVaR for two distributions with the same VaR value. (STREET, 2008)

In general, the CVaR risk metric has been established for loss distributions due to its application in managing financial losses. Consequently, within this framework, it is typically formulated as the conditional expectation of the loss distribution's values that exceed (are greater than) a specified α quantile. Alternatively, within the context of net revenue or financial profit, where preferences are often communicated by agents or decision-makers, the CVaR can be conveniently redefined as the conditional expectation of the most unfavorable distribution scenarios on the left side of revenue, falling below a given (1 - α) quantile—typically ranging from 1% to 10% (or α ranging from 0.99 to 0.90) (STREET, 2010).

In this work, the CVaR measure is utilized in the revenue context and as a utility preference functional. It is important to highlight that in this framework, as presented in (STREET, 2010), the following properties hold for the $CVaR_\alpha$ preference functional:

1. **Translation Invariance:** For $t \in \mathbb{R}$, thus $CVaR_\alpha(R+t) = CVaR_\alpha(R) + t$.

2. **Positive Homogeneity:** For $t \in \mathbb{R}$, $CVaR_\alpha(t \cdot R) = t \cdot CVaR_\alpha(R)$.

3. **Superadditivity:** $CVaR_\alpha(R_1 + R_2) \geq CVaR_\alpha(R_1) + CVaR_\alpha(R_2)$.

4. **Monotonicity**: If $R_1 \leq R_2$, thus, $CVaR_\alpha(R_1) \leq CVaR_\alpha(R_2)$.

5. **Consistency:** For any deterministic random variable t, $CVaR_\alpha t = t$.

For more details about the properties, please refer to (STREET, 2010).

4.5.3
Stochastic Programming

Stochastic programming is the classical approach for modeling decision-making problems in light of the uncertain nature of real problems. The general problem of decision-making, in the context of stochastic programming, can be expressed as a certainty-equivalent ($\rho_{\alpha,\beta}$) maximization (BINGE; LOUVEAUX, 2015). Equation 4-3 shows the mathematical formulation. The uncertain data is represented as a vector of random variables $\tilde{\boldsymbol{\xi}} \in \Omega \rightarrow \Xi$ which maps the set of all possible 'states of nature' into a compact support set $\Xi \subset \mathbb{R}^{\mathbb{K}}$.

$$\max_{x \in X} \ \rho_{\alpha,\beta}\left(R\left(x, \tilde{\boldsymbol{\xi}}\right)\right) \tag{4-3}$$

By considering the CE as defined in section 4.5.1 and the CVaR formulation outlined in section 4.5.2, the optimization of the certain equivalent can be reformulated as:

$$\max_{x,z,\delta_\omega} \quad \beta\left(z - \sum_{\omega \in \Omega} \frac{\delta_\omega \ p_\omega}{(1-\alpha)}\right) + (1-\beta) \sum_{\omega \in \Omega} R\left(x, \tilde{\boldsymbol{\xi}_\omega}\right) p_\omega \tag{4-4}$$

$$\text{s.t.} \quad \delta_\omega \geq z - R\left(x, \tilde{\boldsymbol{\xi}_\omega}\right) \qquad \forall \omega \in \Omega \tag{4-5}$$

$$\delta_\omega \geq 0 \qquad \forall \omega \in \Omega \tag{4-6}$$

5
Supply Contracts backed by vRES and the Price-and-Quantity Risk

In this work, we consider a set $\mathcal{I} = \left\{1, \ldots, n\right\}$ of n vRES committed in long-term supply contracts (hereinafter referred to as a Power Purchase Agreement – PPA) with consumers. The PPA price $\left(P_i\right)$ and volume $\left(V_i\right)$ of each agent $i \in \mathcal{I}$ is considered to be larger than the analysis horizon represented by a set of T hourly, namely, $\mathcal{T} = \{1, \ldots, T\}$. The net revenue, or cash flow function, $f_i\left(\,\cdot\,\right)$, of a contracted renewable agent $i \in \mathcal{I}$ with an uncertain generation profile determined by the random vector $\tilde{\boldsymbol{G}}_i \triangleq \left\{\tilde{G}_{i,t}\right\}_{t \in \mathcal{T}}$, is given by

$$f_i\left(P_i, V_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}\right) = \sum_{t \in \mathcal{T}} \left(P_i\, V_i + \left(\tilde{G}_{i,t} - V_i\right)\tilde{\pi}_t\right). \tag{5-1}$$

Where $\tilde{\boldsymbol{\pi}} \triangleq \left\{\tilde{\pi}_t\right\}_{t \in \mathcal{T}}$ stands for the random vector of energy spot prices for the whole horizon. In (5-1), the first term, $P_i\, V_i$, stands for the PPA fixed cash flow, whereas the second term, $\left(\tilde{G}_{i,t} - V_i\right)\tilde{\pi}_t$, represents the clearing in the spot price of the generation deficit or surplus with respect to the PPA volume.

It should be noted that the cash-flow stream in expression (5-1) explicitly translates the aforementioned double-sided nature of the price-and-quantity risk due to a position in a long-term forward contract. In fact, while a high contracted volume increases the constant payments, it also increases the likelihood of a negative clearing in the short-term market. If, in a given scenario, a negative clearing is accompanied by a high spot price, the total cash flow can be negative. Furthermore, on the other hand, if the renewable agent prefers to avoid a large exposition to the short-term market by contracting a low volume in the PPA, the likelihood of a generation surplus in comparison to the PPA amount is higher. In this context, however, the fixed cash flow is lower. Thus, if the former scenario has an associated low spot price realization, the overall net revenue (fixed plus variable from the short-term market) might not be enough to cover the asset expenses.

Therefore, although long-term supply contracts help in providing generators with more stable cash flows, in the case of renewable generators, with high uncertainty in the generation profile, it also exposes the agents to the PQ-Risk. The PQ-Risk materializes whenever one of these two aforementioned pairwise-linked scenarios occurs, i.e., (generation deficit, high spot price) or (generation surplus, low spot price). In the next section, by targeting this specific double-sided nature of the PQ-Risk, we describe the proposed novel derivative instrument capable of efficiently mitigating the losses in the case of these events.

6
Wind-Indexed Option: Conceptual Design

Aiming to design a derivative instrument to reduce the negative impact of both sides of the previously discussed PQ-Risk, in this section, we describe the conceptual design of the proposed hedging instrument. Firstly, in Subsection 6.1, we establish its foundations, presenting a new *Wind Power Performance Index* (WPP-I). This index is one of the key components to trigger the derivative payoff. Then, in Subsection 6.2, the proposed derivative payoff function is described. Finally, in Subsection 6.3, we devise the overall net revenue stream of a vRES when negotiating the proposed WInd-Op and its associated optimal willingness-to-contract curve.

6.1
Wind Power Performance Index (WPP-I)

Following the quantity risk dynamics discussed in Section 5, if $\tilde{\mathbb{G}}_t$ denotes a representative generation profile, e.g., for a given set of generators in a given region, at an hour $t \in \mathcal{T}$, and $F \in \mathbb{R}_+$ denotes an approximation to the total market amount of traded PPAs in this region, then, the WPP-I associated with this region can be defined as follows:

$$\Delta\Big(\tilde{\mathbb{G}}_t, F\Big) \triangleq \frac{\tilde{\mathbb{G}}_t}{F} - 1 \qquad \forall\, t \in \mathcal{T}. \tag{6-1}$$

Roughly speaking, the WPP-I definition in (6-1) highlights the deficit and surplus condition of a given wind power profile with respect to a reference of involvement in the forward market. Therefore, if at a given hour $t \in \mathcal{T}$, the index is positive – e.g., $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big) > 0$, then it indicates that the generation in that region is in a surplus scenario with respect to the market reference of typical forward involvement. Analogously, if the index is negative at a given hour $t \in \mathcal{T}$ – e.g., $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big) < 0$, then a generation deficit in that region is observed. It is worth highlighting that, in the case of a standardized instrument design, the generation profile $\left\{\tilde{\mathbb{G}}_t\right\}_{t\in\mathcal{T}}$ and the reference F should be of interest to a significant group of generation companies (e.g., the littoral of the Northeast Region of Brazil). Thus, they should be selected according to their representativeness, estimated according to transparent and audited processes, and made available to all market players. However, it can also be the case where specific contracts could be designed for specific companies through private bilateral instruments.

It is beyond the scope of this work to explore all possible formats of estimation processes that could be used to obtain representative generation profiles and the reference to the forward involvement amount. Notwithstanding, we understand the diversity of possible ways that these two elements composing the proposed WPP-I can be estimated as a salient feature of the concept, which allows the market agents interested in creating these products to compete for the attractiveness of their own index. For instance, if one has a now-casting estimation process that better estimates the term $\frac{\tilde{\mathbb{G}}_t}{F}$ for a given set of relevant wind power generators, this agent should generate a more representative WPP-I. In the case study, we test the proposed concept with a practical approach and discuss further possible extensions as future work in the conclusions section. Next, based on the concept of WPP-I defined in this section, we present the proposed hedging instrument.

6.2 WInd-Op Payoff Function

The proposed WInd-Op is built to only trigger a payoff against the two pairwise-linked events (discussed in Section 5) related to the PQ-Risk: (i) a deficit in production with a high spot price, and (ii) a surplus in production with a low spot price. In order to define what is low and high, a reference price S, similar to the strike price of call and put options, is used. Therefore, based on the surplus or deficit amounts, defined by WPP-I, and on the difference between the spot and the reference price, we can define the payoff function of the holder (buyer) of q MW of the proposed derivative, for any period (hour) t within the maturity horizon $\mathcal{T}$, as follows:

$$\Gamma\Big(q_i, \tilde{\mathbb{G}}_t, \tilde{\pi}_t\Big) = \bigg(\max\Big\{\big(S - \tilde{\pi}_t\big)\Delta\big(\tilde{\mathbb{G}}_t, F\big), 0\Big\} - \lambda\bigg)q_i. \qquad \text{(6-2)}$$

The first term of expression (6-2) refers to the payoff of the proposed WInd-Op. The product between the WPP-I and the strike and spot difference highlights the essential dynamics of the WInd-Op to efficiently tackle the double-sided facet of the PQ-Risk by securitizing an amount q_i (in average MW) if both spot price and energy production are against the holder.

On one side, if, at the same time, t, $\Delta\big(\tilde{\mathbb{G}}_t, F\big) < 0$, i.e., there is a deficit in generation with respect to the forward involvement reference, and $\big(S - \tilde{\pi}_t\big) < 0$, i.e., the spot price is higher than the strike price, then the holder has the right to exercise the option. In this case, the holder receives a financial payoff equal to $\Big[\big(S - \tilde{\pi}_t\big)\ \Delta\big(\tilde{\mathbb{G}}_t, F\big)\Big]\ q_i$, which may be closely related to its incurred financial loss if its generation profile and forward involvement are reasonably well approximated by $\mathbb{G}_t$ and F. Note that, in this scenario, the payoff function indicates that the agent is buying the generation-adjusted amount $\Delta\big(\tilde{\mathbb{G}}_t, F\big)q_i$ of energy at the *a priori*-specified strike price S and selling it back in the short-term market by a higher value at the spot price $\tilde{\pi}_t$. This payoff is equivalent to the payoff of a *call option* with a stochastically adjusted delivery amount equal to $\Delta\big(\tilde{\mathbb{G}}_t, F\big)q_i$.

On the other side, if, at the same time, $t \in \mathcal{T}$, $\Delta\big(\tilde{\mathbb{G}}_t, F\big) > 0$ (i.e., a generation surplus with respect to the forward involvement reference) and $\big(S - \tilde{\pi}_t\big) > 0$ (a lower spot price with respect to the strike price reference), then the holder has the right to exercise the option, receiving a financial payoff equal to $\Big[\big(S - \tilde{\pi}_t\big)\ \Delta\big(\tilde{\mathbb{G}}_t, F\big)\Big]\ q_i$, to compensate the lower income for the generation surplus. Interestingly, in this context, the payoff indicates that the agent is buying the generation-adjusted amount $\Delta\big(\tilde{\mathbb{G}}_t, F\big)q_i$ of energy in the short-term market at a spot price $\tilde{\pi}_t$ to sell it at the higher *a priori*-specified strike price S. In this scenario, the derivative has a payoff equivalent to the payoff of a *put option* with a stochastically-adjusted clearing amount equal to $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big)q_i$.

In Figure 6.1, we showcase the payoff of the proposed WInd-Op for a given strike price $S = 90$ \$/MWh as a function of the spot price for different realizations of the WPP-I. For illustrative purposes, we disregard the premium component, i.e., $\lambda = 0$ \$/MWh. It is remarkable the similarity of the payoff function with the standard put-and-call option combination. Interestingly, for the particular events where a 100% surplus or deficit are observed, i.e., $\Big|\Delta\Big(\tilde{\mathbb{G}}_t, F\Big)\Big| = 1$, it recovers exactly the standard combined put-and-call option payoff function, where $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big) = 1$ triggering the call option side and $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big) = -1$ triggering the put option. Nevertheless, according to the proposed derivative payoff function (6-2), different scenario realizations of $\Delta\Big(\tilde{\mathbb{G}}_t, F\Big)$ lead to distinct payoff amounts. For instance, the red lines show the payoff of the call option side of the proposed derivative adjusted by the different levels of the WPP-I ($\Delta\Big(\tilde{\mathbb{G}}_t, F\Big)$). Similarly, the blue lines indicate the adjusted put option payoff side.

It is important to highlight that the payoff ($\Gamma(\cdot)$) of the proposed WInd-Op tends to be lower than the standard put-and-call option combination, which better accommodates the holder's needs to hedge the PQ-Risk. As a consequence, we argue that the proposed instrument is a more fit-for-purpose derivative than the standard call-and-put options, thereby providing a more efficient hedging instrument for the PQ-Risk exposure of WPCs operating in a competitive electricity market. In our case study, we showcase that in the equilibrium, the proposed derivative is cheaper and more effective in increasing the total social welfare than the standard put-and-call option benchmark.

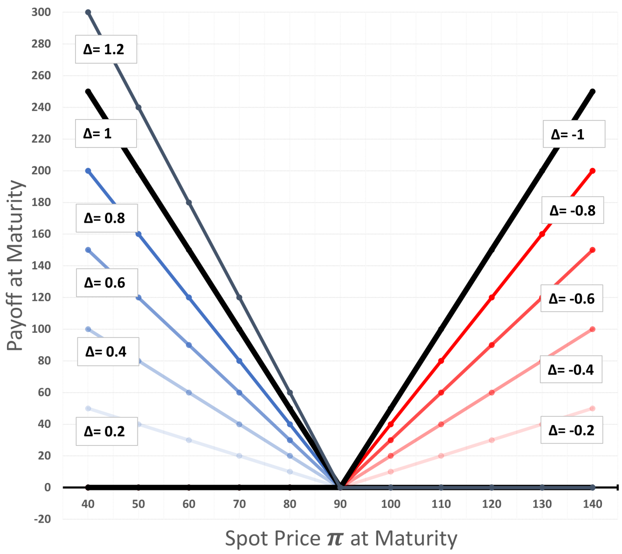


Figure 6.1: Payoff function, $\Gamma(\cdot)$, of the proposed instrument for a strike price $S = 90$ \$/MWh, null premium ($\lambda = 0$ \$/MWh), and different values of the WPP-I.

6.3
Overall Net Revenue

The overall net revenue of a given contracted vRES $i \in \mathcal{I}$ buying the proposed derivative (with $q_i \geq 0$ representing the acquisition of the derivative) can be represented by combining the PPA cash-flow expression (5-1) with the payoff function of the instrument (6-2) as follows:

$$R_i(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}) = \sum_{t \in \mathcal{T}} \Bigg[P_i\, V_i + \Big(\tilde{G}_{i,t} - V_i\Big)\tilde{\pi}_t + \\ \Bigg(\max\Big\{ \Big(S - \tilde{\pi}_t\Big)\, \Delta\Big(\tilde{\mathbb{G}}_t, F\Big),\, 0 \Big\} - \lambda \Bigg) q_i \Bigg]. \tag{6-3}$$

It is worth highlighting that, under the occurrence of the PQ-Risk triggering events (those in which the proposed derivative is exercised), under certain conditions, a specifically designed instrument can fully immunize this agent's net revenue against the spot-price risk factor. This result is formalized next in **Theorem 6.3**.

Assume a WPC $i \in \mathcal{I}$ committed to a long-term PPA with an associated sale price and volume given by P_i and V_i, respectively. Furthermore, consider a

WInd-Op that is designed over the production profile and forward involvement of the WPC, i.e., $\tilde{\mathbb{G}}_t = \tilde{G}_{i,t},\ \forall\, t \in \mathcal{T}$ and $F = V_i$, and the strike price is equal to the PPA price, i.e., $S = P_i$. If the WPC buys the total PPA amount in this WInd-Op, i.e., $q_i = V_i$, then, under the occurrence of the PQ-Risk triggering events (those in which the proposed derivative is exercised), the net revenue (6-3) of the WPC resumes to

$$R_i(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}) = \sum_{t\in\mathcal{T}} \Big(P_i\, \tilde{G}_{i,t} - \lambda\, q_i\Big), \tag{6-4}$$

i.e., the spot-price risk factor vanishes from the cash flow.

Prova. Firstly, for a given hour $t \in \mathcal{T}$ along the maturity of the derivative, under the hypothesis that $\tilde{\mathbb{G}}_t = \tilde{G}_{i,t},\ \forall\, t \in \mathcal{T}$, $q_i = V_i = F$, and $S = P_i$, and the occurrence of a risky event in which the derivative instrument is exercised, the net revenue of the referred WPC is given by

$$\begin{aligned} R_i(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}) = P_i\, F + \Big(\tilde{G}_{i,t} - F\Big)\tilde{\pi}_t\, + \\ \Big(\Big(P_i - \tilde{\pi}_t\Big)\, \Delta\Big(\tilde{G}_{i,t}, F\Big) - \lambda\Big)F. \end{aligned}$$

By using the definition of the WPP-I in (6-1), we have that

$$\begin{aligned} R_i(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}) &= P_i\, F + \Big(\tilde{G}_{i,t} - F\Big)\tilde{\pi}_t\, + \\ &\quad \Big(\Big(P_i - \tilde{\pi}_t\Big)\left(\frac{\tilde{G}_{i,t}}{F} - 1\right) - \lambda\Big)F. \\ &= \tilde{G}_{i,t}\, P_i - \lambda F. \end{aligned}$$

Finally, by summing along the hours of analysis $t \in \mathcal{T}$,

$$R_i(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}) = \sum_{t\in\mathcal{T}} \Big(\tilde{G}_{i,t}\, P_i - \lambda F\Big).$$

■

Theorem 6.3 formalizes two important aspects of the proposed derivative instrument. Firstly, the theorem holds under the hypothesis of $\tilde{\mathbb{G}}_t = \tilde{G}_{i,t}$, $\forall\ t \in \mathcal{T}$, $q_i = V_i = F$, and $S = P_i$. Thus, it states that if the instrument is designed over the production profile of the WPC, the WPC significantly reduces the PQ-Risk of the WPC induced by its uncertain generation when involved in long-term PPAs. As the natural hedge, in the absence of the proposed instrument, is to reduce the forward involvement, the newly proposed derivative should induce higher forward involvements, allowing more long-term contracts to be negotiated in the market by WPCs. Secondly, under the hypothesis that an event in which the instrument is exercised (i.e., a context in which the WPC is exposed to the PQ-Risk), the proposed derivative aims to recover only the losses incurred by the WPC, avoiding extra payments that would be recovered by the premium payment in a market equilibrium situation. So, the proposed derivative is efficient in reducing WPCs losses when selling long-term forward contracts.

7
Optimal Willingness-to-Contract Curve

In order to fully explore the proposed instrument value, each economic agent should pursue a trading strategy that optimizes its risk-adjusted wiliness-to-contract in the WInd-Op given its premium λ. Formally, let ρ_{θ_i} to stand for a θ_i-parameterized coherent risk measure functional that better characterizes the attitude towards risk of a given vRES $i \in \mathcal{I}$ when negotiating the proposed derivative instrument. Then, for a given premium λ, the decision-making problem that defines the optimal amount of the proposed instrument that a renewable agent $i \in \mathcal{I}$ is willing to contract is given by

$$q_i^*\Big(\lambda\Big) \in \underset{\underline{q}_i \leq q_i \leq \overline{q}_i}{\operatorname{argmax}} \left\{ \rho_{\theta_i}\Big(R_i\Big(\lambda, q_i, \tilde{\boldsymbol{G}}_i, \tilde{\boldsymbol{\pi}}\Big)\Big)\right\}. \tag{7-1}$$

In (7-1), $\underline{q}_i$ and $\overline{q}_i$ stand for the minimum and maximum contracting levels, respectively. So, it is important to highlight that if $q_i \geq 0$, it means that agent i is willing to buy the derivative, whereas if $q_i \leq 0$, it means that agent i is willing to sell it. So, by means of a given selection of the bounds, $\underline{q}_i$ and $\overline{q}_i$, we can either define buyers (selecting value such that $\underline{q}_i = 0$ and $\overline{q}_i \geq 0$) and sellers (with $\underline{q}_i \leq 0$ and $\overline{q}_i = 0$), or let agents free to select their role (selecting value such that $\underline{q}_i \leq 0$ and $\overline{q}_i \geq 0$).

More specifically, to characterize each renewable agent's attitude towards risk, we consider in both case studies a convex combination between the *Expected Value* of the net revenue stream (6-3) and the left-tail, α-quantile-based risk functional known as the *Conditional Value-at-Risk* (CVaR_α) (see (STREET, 2010)). More specifically, for each renewable agent $i \in \mathcal{I}$, let $\theta_i \triangleq \{\alpha_i, \beta_i\}$ with $\beta_i \in [0, 1]$ and $\alpha_i \in (0, 1]$. Then, the θ_i-parameterized (coherent) risk functional measure (ρ_{θ_i}) considered in this numerical experiment is defined as follows:

$$\rho_{\theta_i}\Big(\tilde{R}_i\Big) = \beta_i \, \mathrm{CVaR}_{\alpha_i}\Big(\tilde{R}_i\Big) + \Big(1 - \beta_i\Big)\, \mathbb{E}\Big(\tilde{R}_i\Big). \tag{7-2}$$

In (7-2), both α_i and β_i play the role of risk-averse parameters for the renewable agent $i \in \mathcal{I}$. The former (α_i) stands for the confidence level of the CVaR measure, indicating the $(1-\alpha_i)$-quantile up to which the worse net revenues scenarios are averaged. The latter (β_i), on the other hand, balances the weight given to the CVaR measure against the Expected Value. From a risk attitude perspective, according to (STREET, 2010), (7-2) can be interpreted as a Certainty Equivalent functional that assigns a monetary value to a given cash flow. Therefore, an economic agent $i \in \mathcal{I}$ whose risk attitude is well-represented by ρ_{θ_i} aims to select the best amount of WInd-Op by maximizing this functional. Note that the risk measure (7-2) is general enough to map a variety of risk profiles. In fact, if the renewable agent is Risk Neutral, then it can be characterized by setting $\beta_i = 0$, meanwhile increasing the value of β_i induces stronger levels of risk-aversion attitude. For expository purposes, we consider in both case studies $\alpha_i = 0.95, \ \forall \, i \in \mathcal{I}$, and vary only the parameter β_i.

Finally, regarding the scenarios and probabilities used to characterize the uncertainties, we assume that the WInd-Op maturity spans a whole week, $\mathcal{T} = \{1, \ldots, 168\}$, with WPP-I associated with the generation of the wind farm and strike price set at $S = 192$ \$/MWh. To characterize the uncertain factors within the study horizon, we follow the standard stochastic modeling approach and assume a probability space $\Big(\Omega, \mathcal{F}, \mathbb{P}\Big)$ with a finite sample set (plausible scenarios). A pure data-driven (non-parametric) decision-making approach is considered by assigning to the set of scenarios a collection of chronologically coherent historical data with an empirical probability mass equal to $1/|\Omega|$ assigned to each scenario. The scenario data are generated using observed weeks of hourly energy production for all renewable power plants considered in each case study and the energy spot prices for the Northeast (NE) Region of the Brazilian system. The data was extracted from July-2019 up to July-2021, resulting in a total of 104 representative weeks of renewable and spot price scenarios preserving both cross and temporal dependencies.

8
Economic Equilibrium and Market Model

In this section, we outline the setup to study the properties and effectiveness of the proposed WInd-Op when traded in a competitive marketplace. Roughly speaking, we evaluate the performance of the proposed derivative within an *economic equilibrium*, i.e., within a state of the market in which both willingness-to-supply and willingness-to-consume are balanced among all participants. Formally, an economic equilibrium happens whenever a given premium λ is such that

$$\sum_{i \in \mathcal{I}} q_i^*\Big(\lambda\Big) = 0, \tag{8-1}$$

with $q_i^*\Big(\lambda\Big)$ defined as in (7-1). Following the standard economic literature and uniform pricing theory, a price-taker economic equilibrium state of a competitive market can be found by means of solving the following maximum welfare problem:

$$\boldsymbol{q}^* \in \underset{\{q_i\}_{i \in \mathcal{I}}}{\operatorname{argmax}} \sum_{i \in \mathcal{I}} \rho_{\theta_i} \Bigg(\sum_{t \in \mathcal{T}} \Bigg[P_i \, V_i + \Big(\tilde{G}_{i,t} - V_i \Big) \tilde{\pi}_t + \\ \max \Big\{ \Big(S - \tilde{\pi}_t \Big) \, \Delta \Big(\tilde{\mathbb{G}}_t, F \Big), \, 0 \Big\} \, q_i - \lambda q_i \Big) \Bigg] \Bigg) \tag{8-2}$$

subject to:

$$\sum_{i \in \mathcal{I}} q_i = 0. \qquad : \lambda \tag{8-3}$$

$$\underline{q}_i \leq q_i \leq \overline{q}_i, \qquad \forall \, i \in \mathcal{I}. \tag{8-4}$$

Due to the property of translation invariance of coherent risk measures (presented in section 4.5.2), the objective function can be written as:

$$\boldsymbol{q}^* \in \underset{\{q_i\}_{i\in\mathcal{I}}}{\text{argmax}} \sum_{i\in\mathcal{I}} \rho_{\theta_i}\left(\sum_{t\in\mathcal{T}}\left[P_i\,V_i + \left(\tilde{G}_{i,t} - V_i\right)\tilde{\pi}_t + \max\left\{\left(S - \tilde{\pi}_t\right)\Delta\left(\tilde{\mathbb{G}}_t, F\right),\, 0\right\} q_i\right)\right]\right) + \lambda \sum_{i\in\mathcal{I}} q_i \quad \text{(8-5)}$$

Considering the constraint equation 8-3, it is possible to eliminate the term $\lambda \sum_{i\in\mathcal{I}} q_i$ from the equation 8-5. Finally, the proposed maximum welfare problem (8-2)–(8-4) can be written as:

$$\boldsymbol{q}^* \in \underset{\{q_i\}_{i\in\mathcal{I}}}{\text{argmax}} \sum_{i\in\mathcal{I}} \rho_{\theta_i}\left(\sum_{t\in\mathcal{T}}\left[P_i\,V_i + \left(\tilde{G}_{i,t} - V_i\right)\tilde{\pi}_t + \max\left\{\left(S - \tilde{\pi}_t\right)\Delta\left(\tilde{\mathbb{G}}_t, F\right),\, 0\right\} q_i\right)\right]\right) \quad \text{(8-6)}$$

subject to:

$$\sum_{i\in\mathcal{I}} q_i = 0. \qquad : \lambda \quad \text{(8-7)}$$

$$\underline{q}_i \leq q_i \leq \overline{q}_i, \qquad \forall\, i \in \mathcal{I}. \quad \text{(8-8)}$$

The maximum welfare problem (8-6)–(8-8) is obtained by jointly maximizing the risk-adjusted revenue of all players, as per (7-1), considering the equilibrium constraint (8-1). The *equilibrium premium* $\left(\lambda^*\right)$ for the proposed instrument can be computed by solving problem (8-6)–(8-8) and evaluating the dual variable of constraint (8-7). In fact, it recovers the marginal impact in the overall market welfare (among all participants), similar to the standard uniform pricing framework. Furthermore, the associated solution $\boldsymbol{q}^* \triangleq \{q_i^*\}_{i\in\mathcal{I}}$ for (8-6)–(8-8) is a best-response contracting level for each renewable agent $i \in \mathcal{I}$ to the WInd-Op equilibrium premium λ^*, i.e., the optimal $q_i^*\left(\lambda^*\right)$ as in (7-1).

Therefore, the final linear programming model is expressed as:

$$\max_{z_i,\delta_{i,\omega},q_i} \quad \sum_{i\in\mathcal{I}}\left(\beta_i\left(z_i - \sum_{\omega\in\Omega_N}\frac{\delta_{i,\omega}}{N(1-\alpha_i)}\right)+(1-\beta_i)\sum_{\omega\in\Omega_N}\frac{r_{i,\omega}}{N}\right) \tag{8-9}$$

$$\text{s.t.} \quad \delta_{i,\omega} \geq z_i - r_{i,\omega} \qquad \forall i\in\mathcal{I}, \forall\omega\in\Omega_N \tag{8-10}$$

$$\delta_{i,\omega} \geq 0 \qquad \forall i\in\mathcal{I}, \forall\omega\in\Omega_N \tag{8-11}$$

$$r_{i,\omega} = \sum_{t\in\mathcal{T}}\Bigg[P_i\,V_i + \rho_{\theta_i}\Bigg(\Big(\tilde{G}_{i,t,\omega} - V_i\Big)\tilde{\pi}_{t,\omega} \tag{8-12}$$

$$+ max\Big\{\Big(S - \tilde{\pi}_{t,\omega}\Big)\,\Delta\Big(\tilde{\mathbb{G}}_{t,\omega}, F\Big),\ 0\Big\}\,q_i\Bigg)\Bigg] \qquad \forall i\in\mathcal{I}, \forall\omega\in\Omega_N$$

$$\sum_{i\in\mathcal{I}} q_i = 0 \qquad :\lambda \qquad \forall t\in\mathcal{T} \tag{8-13}$$

$$\underline{q}_i \leq q_i \leq \overline{q}_i, \qquad \forall i\in\mathcal{I} \tag{8-14}$$

The presented equilibrium model was implemented using the Julia programming language.

In the next section, we present two numerical studies to illustrate the effectiveness and attractiveness of the proposed instrument using real data from the Brazilian power system.

9
Numerical Experiment

In this section, we illustrate the effectiveness of the proposed WInd-Op by means of two case studies using real data from the Brazilian power sector. In the first one, the equilibrium between a single WPC (buyers) and three SPCs (sellers) is considered. In this case, we assume a "bilateral" trading environment where the derivative is conceived by the single buyer to specifically hedge its PQ-Risk, i.e., the WPP-I is based on the forward involvement and generation profile of the buyer. In the second case study, a wider trading environment is considered, with multiple WPCs (buyers) and SPCs (sellers). In this case, the derivative is based on the average generation profile and forward involvement of all WPCs of the considered region. In both case studies, we analyze the benefits introduced by the proposed instrument and benchmark it with the traditional put-and-call options to evaluate its performance. Formally, in the context of this work, for a given amount q_i of put-and-call option negotiated by a renewable agent $i \in \mathcal{I}$, the payoff function at a given hour $t \in \mathcal{T}$ within the maturity of analysis is given by

$$\left(\max\left\{\left(S - \tilde{\pi}_t\right), \left(\tilde{\pi}_t - S\right)\right\} - \lambda\right) q_i, \tag{9-1}$$

with S and λ representing the put-and-call option strike price and premium, respectively. It is thus noteworthy that the benchmark derivative features an exercise rule and associated payoff function, which solely rely on the prevailing spot price conditions.

9.0.1
Case Study I: Single WPC

In Table 9.1, all details of each renewable power plant considered in this Case Study I are presented. Column 1 and Column 2 indicate, respectively, the name of the power plants and their source type; Column 3 and Column 4 present, respectively, the PPA volume (average MW) and sales price ($/MWh) of each renewable agent; Column 5 and Column 6 display the minimum and maximum amount that each agent is able to negotiate; Column 7 depicts the risk-averse level of each power company; and Column 8 displays the region in which each agent is located. Note that, to correctly characterize the trading environment in which the WPC is the holder (buyer) of the derivative and the SPCs are the underwriters (sellers), the minimum level for the former and the maximum level for the latter are set to zero. Also, for illustrative purposes, we assume that each SPC has different attitudes towards risk, with *Lapa* exhibiting a low risk-averse level, *São Pedro* with a medium risk-averse level, and *Bom Jesus* with a high risk-averse level.

Table 9.1: Data and details of each renewable power plant considered in this Case Study I.

Power Plant	**Source**	$\boldsymbol{V}$	$\boldsymbol{P}$	$\underline{\boldsymbol{q}}$	$\overline{\boldsymbol{q}}$	$\boldsymbol{\beta}$	**Region**
Brotas de Macaúbas	Wind	35.7	192	0.0	35.7	0.95	NE
Lapa	Solar	17.0	192	-17.4	0.0	0.10	NE
São Pedro	Solar	16.0	192	-16.0	0.0	0.50	NE
Bom Jesus	Solar	16.8	192	-16.8	0.0	0.90	NE

In Figure 9.1, the willingness-to-contract curve for the WPC (demand curve) and the aggregated willingness-to-contract curve for the SPCs (offer curve) are presented, with the intersection between demand and offer curves indicating the equilibrium for the premium[1]. Firstly, note that the supply (selling) curve starts at (roughly) 45 $/MWh and follows the risk-averse profile of each solar power company. More specifically, the SPC with the lowest risk-averse level, *Lapa* (in orange), comes first in the "merit order", followed by *São Pedro* (in green) and *Bom Jesus* (in pink). Furthermore, from the buying counterpart (in blue), the maximum price the WPC is willing to buy the instrument is close to 100 $/MWh. The equilibrium premium is settled at

[1]We refer to (KRISHNA, 2009) for further discussion, formal analysis, and interpretation related to, economic equilibrium, uniform pricing, and willingness-to-contract curves.

$\lambda^* = 78$ \$/MWh. We highlight that these values are significantly below the PPA sales price. Thus, the derivative can be classified as a relatively cheap product for trading.

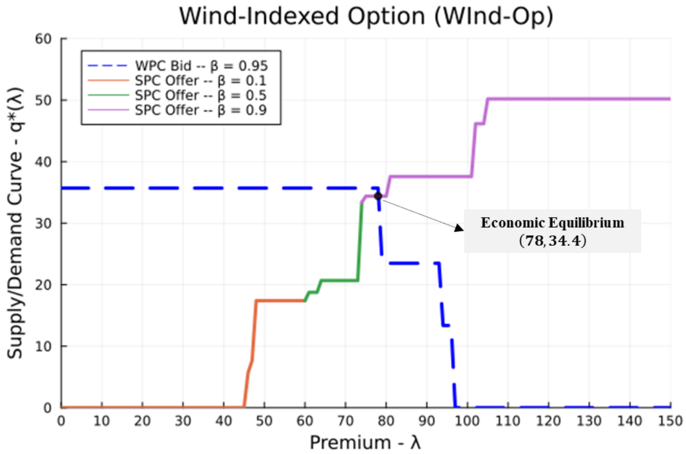


Figure 9.1: WPC's willingness-to-demand curve and the SPCs' willingness-to-supply curves for the proposed WInd-Op.

To illustrate the benefits of the proposed WInd-Op, in Table 9.2, we display the premium at equilibrium (Column 2), the contracting level of each renewable company (Column 3), and the variation of the Expected Value (Column 4), CVaR (Column 5), and Certainty Equivalent (Column 6) with respect to not trading the hedging instrument (thus committed only in the long-term contract). Firstly, note that, in the equilibrium, all renewable companies increase their certainty equivalent metrics with respect to the context of not trading the hedging instrument, indicating an increase in the overall welfare value. In this context, all agents, according to their risk profiles, are better off in the case where they can trade the proposed derivative. Interestingly, we highlight that the CVaR metric for the WPC (buyer counterpart) improved against a decrease in the Expected Value, whereas the reverse condition is observed for all SPCs. This happens because the buyer, who is purchasing a hedging instrument, does so to reduce risk, thereby seeking a better risk metric in exchange for a fixed payment, which decreases the expected value. On the other hand, sellers are adding to their revenue function a negative payoff, which was set by the equilibrium to cover the PQ-Risk by maximizing total welfare. So, it is expected that their risk would increase, yet only at a given price (premium) that compensates in terms of their certainty equivalent. So, the obtained equilibrium satisfies both the willingness to hedge of the buyers and the expected gain of the sellers as a reward.

Table 9.2: Equilibrium results for the proposed derivative and relative performance metrics with respect to not trading the derivative

Power Plant	$\boldsymbol{\lambda}^*$	$\boldsymbol{q}^*$	Δ	Δ CVaR	$\Delta\rho$
Brotas de Macaúbas	78.0	34.0	$-213,978$	$81,260$	$66,498$
Lapa	78.0	-17.0	$108,268$	$-64,275$	$91,014$
São Pedro	78.0	-16.0	$99,557$	$-61,457$	$19,050$
Bom Jesus	78.0	-1.0	$6,152$	-42	577

Finally, in order to evaluate the effectiveness of the proposed product, we benchmark the equilibrium results with another equilibrium where we replace the proposed derivative with the standard put-and-call option. Table 9.3 displays the same result structure compared to Table 9.2, but for the standard put-and-call option, with μ^* representing the premium of the derivative and $\boldsymbol{z}^*$ the respective amount traded at equilibrium. Firstly, note that, in the benchmark equilibrium, the total volume traded in standard put-and-call options (Column 3 of Table 9.3) is significantly lower than the volume traded in the proposed WInd-Op (Column 3 of Table 9.2). This happens because the standard derivative delivers a payoff proportional to the full amount contracted, q_i, whereas the proposed derivative delivers only the parcel of q_i under the PQ-Risk, i.e., q_i is adjusted by $\Delta\left(\tilde{\mathbb{G}}_t, F\right)$. Additionally, the put-and-call option premium at equilibrium (Column 4 of Table 9.3) is higher when compared to the premium at the equilibrium of the proposed instrument (Column 4 of Table 9.2), indicating a higher hedging cost in the benchmark case. Furthermore, the increase in the Certainty Equivalent level (Column 6 of Table 9.3) of all renewable companies is lower when compared to the WInd-Op (Column 6 of Table 9.2), except for the SPC with the highest risk-aversion level (*Bom Jesus*) which had a slightly superior increase, which indicates that the proposed derivative provides higher gains to the buyers and inframarginal sellers in comparison to the benchmark. These results indicate that, by adjusting the payoff according to a representative volumetric index, the proposed derivative is capable of reducing the hedging cost, increasing the market liquidity, and improving the benefits for most of the market players (buyer and inframarginal sellers).

Table 9.3: Equilibrium results for a standard put-and-call option and relative performance metrics with respect to not trading the option

Power Plant	$\boldsymbol{\mu}^*$	$\boldsymbol{z}^*$	Δ	Δ CVaR	$\Delta\rho$
Brotas de Macaúbas	124.0	15.0	$-43,783$	$36,757$	$32,730$
Lapa	124.0	-13.0	$39,376$	$-232,460$	$12,193$
São Pedro	124.0	-1.0	$2,184$	$-1,461$	361
Bom Jesus	124.0	-1.0	$2,223$	917	$1,047$

9.0.2
Case Study II: Multi-vRES Market

In this second case study, the attractiveness of the proposed instrument is evaluated in a wider environment comprising 26 agents, namely, 15 WPC and 11 SPC. In Table 9.4, the specific data and details for each renewable power plant considered in this case study are presented. Column 1, Column 2, and Column 3 indicate the name of each power plant, the source type, and the individual firm energy certificates (FEC)[2], respectively; Column 4 and Column 5 express, respectively, the PPA volume and sales price due to each renewable agent. We assume a long-term contracting level equal to 90% of the FEC of each agent. Column 6 and Column 7 display the minimum and maximum amount of the instrument each vRES is able to negotiate; Column 8 presents the risk-averse level of each power company; and Column 9 and Column 10 display, respectively, the State and Region the vRES are located. Note that, similar to Case Study I (Section 9.0.1), we also assume that the SPCs are the

[2]FECs are issued to each power plant in Brazil by the Ministry of Mines and Energy and, for the purposes of this paper, it is considered as the maximum regulatory contracting amount. See (RIBEIRO et al., 2023) for further details.

sellers of the WInd-Op; thus, their maximum trading levels are set to zero. Nevertheless, in this case study, we relax the condition over the WPCs and allow them to both buy and sell the derivative. Furthermore, for illustrative purposes, we consider that each SPC has different attitudes towards risk, with risk parameters displayed in Column 7 of Table 9.4.

Table 9.4: Data and details of each renewable power plant

Power Plant	**Source**	**FEC**	V	P	$\underline{q}$	$\overline{q}$	β	**State**	**Region**
Brotas de Macaúbas	Wind	35.70	32.13	192.00	− 35.70	35.70	0.95	BA	NE
Calango 1	Wind	27.80	25.02	192.00	− 27.80	27.80	0.95	RN	NE
Calango 2	Wind	40.00	36.00	192.00	− 40.00	40.00	0.95	RN	NE
Chapada I	Wind	110.00	99.00	192.00	−110.00	110.00	0.95	PI	NE
Curva dos Ventos	Wind	27.70	24.93	192.00	− 27.70	27.70	0.95	BA	NE
Caetés II	Wind	95.00	85.23	192.00	− 94.70	94.70	0.95	PE	NE
Pelourinho	Wind	23.60	21.24	192.00	− 23.60	23.60	0.95	BA	NE
Serra de Santana 1 e 2	Wind	47.30	42.57	192.00	− 47.30	47.30	0.95	RN	NE
Serra de Santana 3	Wind	52.50	47.25	192.00	− 52.50	52.50	0.95	RN	NE
Cristal	Wind	47.70	42.93	192.00	− 47.70	47.70	0.95	BA	NE
Caetité 123	Wind	38.90	35.01	192.00	− 38.90	38.90	0.95	BA	NE
Brisa Potiguar I	Wind	89.40	80.46	192.00	− 89.40	89.40	0.95	RN	NE
Pedra Cheirosa	Wind	27.50	24.75	192.00	− 27.50	27.50	0.95	CE	NE
Trairí	Wind	97.20	87.48	192.00	− 97.20	97.20	0.95	CE	NE
Icaraizinho	Wind	20.80	18.72	192.00	− 20.80	20.80	0.95	CE	NE
Lapa	Solar	17.00	15.66	192.00	− 17.40	0.00	0.30	BA	NE
São Pedro	Solar	16.00	14.40	192.00	− 14.40	0.00	0.50	BA	NE
Juazeiro Solar	Solar	34.80	31.32	192.00	− 31.32	0.00	0.70	BA	NE
Bom Jesus	Solar	17.00	15.12	192.00	− 16.80	0.00	0.30	BA	NE
Horizonte	Solar	25.00	22.05	192.00	− 24.50	0.00	0.50	BA	NE
Ituverava	Solar	58.80	52.92	192.00	− 58.80	0.00	0.70	BA	NE
Calcário	Solar	35.00	31.32	192.00	− 34.80	0.00	0.30	CE	NE
Nova Olinda	Solar	61.60	55.44	192.00	− 61.60	0.00	0.50	PI	NE
Assú V	Solar	9.20	8.28	192.00	− 9.20	0.00	0.70	RN	NE
Floresta	Solar	25.00	22.59	192.00	− 25.10	0.00	0.30	RN	NE
Sol do Futuro	Solar	16.00	14.58	192.00	− 16.20	0.00	0.50	CE	NE

In this case study, the WPP-I is written over public data from WPCs in the state of Bahia, NE of Brazil. So, in this case study, the spot price remains the same as in the previous one, but the WPP-I reflects the overall wind power production of the 21 power plants already in operation in the state of Bahia (state of the NE region of Brazil). To emulate a realistic case where the total PPA volume would not be precisely calibrated to a given WPC, we build the WPP-I with the forward involvement reference F equal to the overall FEC amount of the 21 power plants comprising the generation profile. Additionally, to add another layer of reality, we considered an instance where not all generators composing the generation index participate in the equilibrium, and we also permit other generators from the NE region to participate. This scenario explores an interesting reality in which the WPP-I would not be perfectly designed for any generator buying the derivative, but in the equilibrium, the attractiveness of the proposed derivative will be reflected by each generator traded amount and the equilibrium price. Figure 9.2 showcases the WPP-I hourly distribution over the week of which the derivative is valid. We highlight the seasonal-like dynamics of the index, typically observed in wind production worldwide: a high generation level during the night followed by a decrease in production in daylight periods. The general

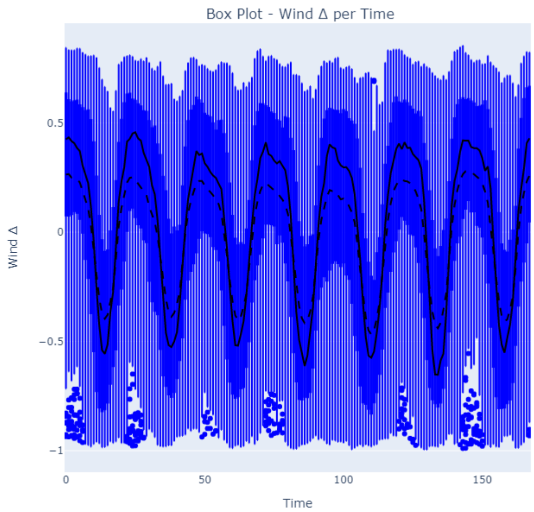


Figure 9.2: WPP-I hourly distribution over the week (derivative horizon).

equilibrium results and relative gain metrics (benefits compared to the base case, where WInd-Op is not available and agents' revenues are based only on the forward and spot markets) are presented in Table 9.5. This table follows Table 9.2, with additional percentage information about the traded amounts with respect to the FEC of each unity and certainty equivalent variation. First, we highlight the existence of an equilibrium in this market between wind and solar companies with a total of 355.00 avgMW negotiated at an equilibrium premium of 66.00 R$/MWh. Furthermore, we observe an increase in each agent's certainty equivalent level with respect to the base case, thus indicating the attractiveness of this hedging instrument for the selected set of agents. Note that we are excluding many other actors that could be participating, such as trading companies, hydro generators, and banks, just to mention a few. In fact, note that its measured benefits can reach values higher than 100% (e.g., *Brotas de Macaúbas* and *Calango*) with an increase of 281% for *Caetité 123*. An increase in the CVaR level is observed, in the majority of WPCs, with a decrease in the Expected Value, highlighting the hedging characteristic of this instrument.

Table 9.5: Equilibrium results and relative performance metrics with respect to not trading the hedging instrument

Power Plant	$\boldsymbol{\lambda}^*$	$\boldsymbol{q}^*$	$\lvert \boldsymbol{q}^* \rvert$/**FEC**	Δ	Δ CVaR	$\Delta\rho$	$\Delta\rho(\%)$
Brotas de Macaúbas	66.00	34.00	96%	−218,858	87,805	72,472	132%
Calango 1	66.00	26.00	93%	−164,291	51,070	40,302	127%
Calango 2	66.00	20.00	50%	−126,483	36,880	28,712	11%
Chapada I	66.00	94.00	85%	−597,546	290,921	246,498	50%
Curva dos Ventos	66.00	27.00	99%	−173,941	67,299	55,237	69%
Caetés II	66.00	−95.00	100%	602,857	386,571	397,385	51%
Pelourinho	66.00	17.00	72%	−108,790	42,365	34,807	30%
Serra de Santana 1 e 2	66.00	23.00	48%	−143,264	38,070	29,003	8%
Serra de Santana 3	66.00	− 7.00	13%	41,987	−134	1,972	0%
Cristal	66.00	31.00	64%	−194,285	119,594	103,900	23%
Caetité 123	66.00	28.00	72%	−178,316	44,331	33,198	281%
Brisa Potiguar I	66.00	56.00	62%	−354,225	58,780	38,129	13%
Pedra Cheirosa	66.00	0.00	0%	−	−	−	0%
Trairí	66.00	−35.00	36%	220,669	37,976	47,111	120%
Icaraizinho	66.00	− 4.00	19%	25,216	4,311	5,356	104%
Lapa	66.00	−17.00	100%	110,768	−55,642	60,845	13%
São Pedro	66.00	−16.00	100%	101,856	−55,236	23,310	6%
Juazeiro Solar	66.00	−14.00	40%	89,295	11,184	34,617	4%
Bom Jesus	66.00	−17.00	100%	106,948	−60,619	56,678	11%
Horizonte	66.00	−25.00	100%	155,966	−77,888	39,039	10%
Ituverava	66.00	−26.00	45%	167,329	−10,294	42,993	5%
Calcário	66.00	−35.00	100%	221,536	−119,429	119,246	12%
Nova Olinda	66.00	−23.00	37%	146,303	−137,536	4,383	1%
Assú V	66.00	− 1.00	11%	6,360	274	2,100	1%
Floresta	66.00	−25.00	100%	159,786	−93,651	83,755	13%
Sol do Futuro	66.00	−16.00	100%	103,129	−10,276	46,426	9%

In Table 9.6, we present the aggregated traded volumes (avgMW), equilibrium price premium ($/MWh), and the welfare gain with respect to the base case (only forward and spot markets) when considering the proposed WInd-Op and the benchmark derivative, i.e., the standard put-and-call derivatives. Similarly to Case Study I, we highlight the following points from the results of Table 9.6: 1) the aggregated volume of energy traded considering the proposed WInd-Op is significantly higher than the volume negotiated considering the benchmark derivative, 2) a smaller equilibrium price (premium) is obtained with the WInd-Op in comparison to the equilibrium price obtained with the benchmark derivative, and 3) the overall welfare gain is significantly higher when considering the proposed WInd-Op in comparison to the welfare gain obtained considering the benchmark derivative.

Table 9.6: Aggregated Equilibrium results and relative performance metrics with respect to not trading the hedging instrument

		Total Traded (avgMW)	**Eq. Premium** ($/MWh)	**Total Δ** ($)	**Total Δ** CVaR ($)	**Total $\Delta\rho$** ($)
WInd-OP buyers/sellers	WPCs Buying	355	66	$-2,260,005$	$837,115$	$682,259$
	WPCs Selling	-140	66	$890,730$	$428,724$	$451,824$
	SPCs Selling	-215	66	$1,369,276$	$-609,112$	$513,393$
WInd-OP aggregated	(summary)	355	66	146	$657,727$	$1,647,476$
Put-and-Call buyers/sellers	WPC Buying	111	146	$-735,777$	$316,517$	$263,903$
	WPCs Selling	-37	146	$243,599$	$77,983$	$86,264$
	SPCs Selling	-74	146	$492,179$	$-78,266$	$251,316$
Put-and-Call aggregated	(summary)	111	146	0	$316,234$	$601,482$

To further illustrate the impact of the instruments in the key performance metrics (Expected Value, CVaR, and Overall Welfare), the results in Table 9.6 are disaggregated per group, namely, WPCs and SPCs, and buyers and sellers. Note that the sum of the ΔCVaR metrics of the WPCs significantly increases for both buyers and sellers when considering the proposed derivative in comparison to the case where the benchmark derivative is considered. From the perspective of the SPCs, on the other hand, although we have a decrease in the total ΔCVaR, an overall higher increase in the Certainty Equivalent value is observed when considering the proposed derivative.

To showcase this effect by renewable agent, Figure 9.3 presents the relative change in Expected Value and Risk (valued by the difference between the Expected Value and the CVaR) for each renewable agent considered in this case study when trading the proposed WInd-Op. Similarly, Figure 9.4 depicts the same context but for the benchmark derivative. The square marker indicates the risk and return metrics when considering only the PPA and spot, while the round marker indicates the same metrics, adding the effect of the hedging instrument. The arrow connects the square marker and the round marker for each generator. With respect to Figure 9.3, by the direction of the arrows, it is possible to identify that most of the renewable agents gave up part of the Expected Value in favor of a risk reduction. Nevertheless, it is also observed that some agents (most of them playing the role of sellers) are willing to slightly increase the risk to obtain higher Expected Values. When comparing the results in Figure 9.3 with the ones in Figure 9.4, we can observe the higher benefits of the proposed instrument compared to the benchmark in terms of risk reduction or expected value gain. Finally, we conduct a sensitivity

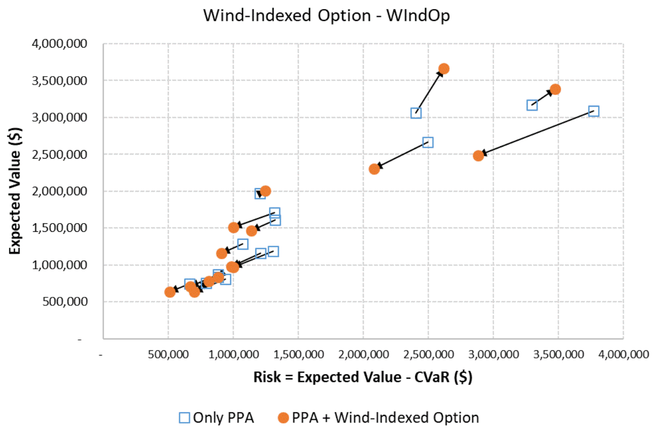


Figure 9.3: Relative change in risk and return (expected value) for each renewable agent in the equilibrium considering the proposed derivative.

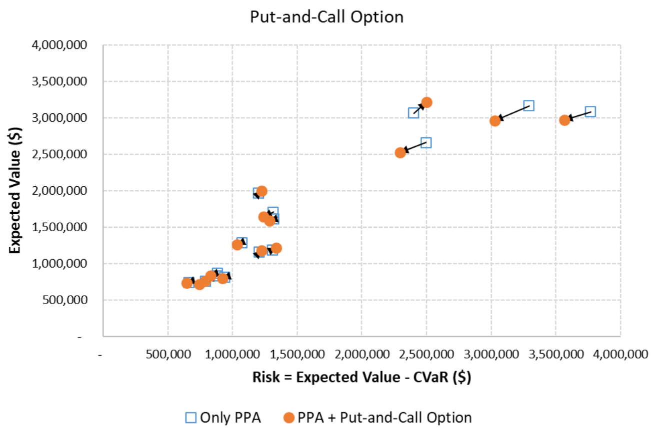


Figure 9.4: Relative change in risk and return (expected value) for each renewable agent in the equilibrium considering the benchmark (put-and-call) derivative.

analysis of the total welfare with respect to the forward involvement, i.e., with respect to a $\gamma 100\%$ of the total FEC amount, when considering the proposed derivative. Structurally, we parameterize the contracted PPA volume of each renewable agent as: $V_i = \gamma \text{FEC}_i, \ \forall \ i \in \mathcal{I}$, and vary $\gamma \in \{0.0, 0.1, \ldots, 1.0\}$. Thus, $\gamma = 0.0$ represents a market with no long-term contracts, only spot, and, on the one hand, $\gamma = 1.0$ indicates that all renewable agents sell their maximum regulatory limit in PPAs. Table 9.7 showcases the resulting equilibrium price λ^* (Column 2), the total amount of energy negotiated at the equilibrium (Column 3), the sum of the FEC of all WPCs that purchases the instrument (Column 4), and the welfare gain (Column 5) with respect to the base case, not trading the WInd-Op for each value of $\gamma \in \{0.0, 0.1, \ldots, 1.0\}$. Firstly, note as the higher the long-term contracting level, the more the proposed derivative is negotiated at the equilibrium. As a consequence, the equilibrium price (premium) and the total welfare increase as the forward involvement increases.

Interestingly, a consequence of the increase of the overall welfare with the forward involvement is that for a given welfare, the total forward involvement can be higher in the presence of the proposed derivative. To quantify this relationship, Figure 9.5 presents the overall welfare (horizontal axis) and the forward involvement (in % of FEC) for each equilibrium. In this figure, the orange line depicts the equilibrium data when considering the proposed derivative and the blue line depicts the welfare when not considering the derivative. Note that, for the same overall welfare level, it is possible to sustainably increase the forward involvement in at least 8% of the renewable agent's FEC.

Table 9.7: Equilibrium results for each value of $\gamma \in \{0.0, 0.1, \ldots, 1.0\}$.

γ	λ^*	**Total Traded Volume**	**Sum FEC (Buyers)**	**Total Traded Volume (% Sum FEC)**	**Total $\Delta\rho$**
0.0	11.00	105.00	379.00	28%	106,990
0.1	11.00	75.00	362.00	21%	132,352
0.2	13.00	95.00	352.00	27%	235,712
0.3	28.00	125.00	539.00	23%	294,163
0.4	34.00	166.00	562.00	29%	398,998
0.5	36.00	178.00	539.00	33%	611,325
0.6	44.00	227.00	594.00	38%	912,592
0.7	53.00	284.00	634.00	45%	1,149,080
0.8	55.00	287.00	536.00	54%	1,394,351
0.9	66.00	355.00	488.00	73%	1,647,476
1.0	71.00	362.00	568.00	64%	1,786,477

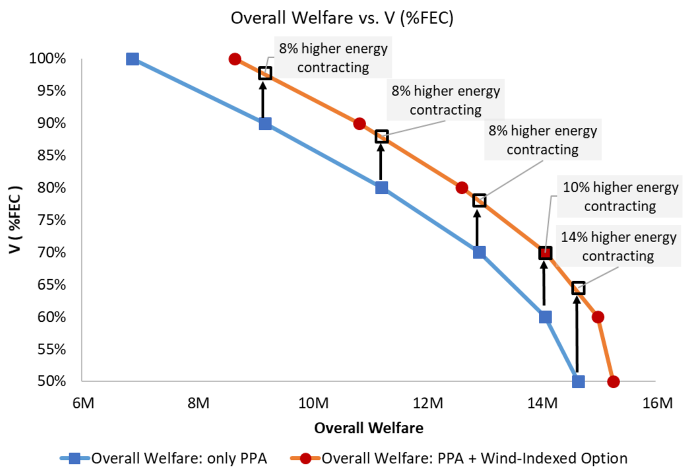


Figure 9.5: PPA volume vs total Certainty Equivalent value when considering (orange curve) and not considering (blue curve) the proposed derivative.

10
Practical Implementations in Brazil

The proposed derivative is feasible to be implemented in practice in Brazil. As outlined in chapter 3, two platforms have been authorized by the CVM to facilitate transactions involving purely financial derivatives in the country, namely B3 and BBCE. These entities currently offer derivatives tied to spot price or marginal operational expense. For the WInd-Op derivative to become viable for trading on these platforms, it is imperative that the WPP-I is calculated in a manner that is both transparent and amenable to auditing. Furthermore, its publication on a publicly accessible website and subsequent approval by the CVM are prerequisites. We recognize that an esteemed university, a reputable bank, or a respected trading firm could assume the responsibility of publishing the index based on officially endorsed data. Hence, once a well-defined index methodology is established and CVM endorsement is obtained, Wind-Op could seamlessly integrate into these pre-existing platforms for trading.

An alternative, less intricate method could involve customized bilateral negotiations. In this context, we recognize that trading firms could hold a substantial position in supplying this product to power generators, acting as intermediaries between generators with complementary hourly profiles.

11
Conclusion

In this work, a new financial hedging instrument to mitigate the double-sided price-and-quantity risk faced by Wind Power Companies (WPCs) committed to long-term forward contracts is proposed. The proposed instrument, named Wind-Indexed Option (WInd-Op), is based on a Wind Power Performance Index (WPP-I), which adjusts the payoff of the proposed WInd-Op to the proportion of generation deficits and surpluses that is representative of a set of wind power generators. This allows the derivative to reduce unnecessary payments to mitigate the price and quantity risk of these generators. Two numerical experiments based on the maximum welfare equilibrium approach were conducted to test the effectiveness and attractiveness of the proposed hedging instrument using real data from the Brazilian power system. From the results of our case study, we can draw the following conclusions and observations:

1. The proposed WInd-Op is effective in reducing the price-and-quantity risk of contracted WPCs. This is observed when comparing the performance metrics to the base case, where only the forward and spot markets are considered.

2. The proposed WInd-Op is efficient in reducing the price and quantity risk and increasing the total welfare in comparison to the benchmark, the call-and-put derivative. The proposed derivative exhibits a lower premium price (cheaper), higher improvements in the risk metric (better hedging instrument), higher traded values (higher liquidity), and higher risk-adjusted welfare metrics.

3. The proposed WInd-Op allows to sustainably increase the long-term PPA contracted volumes to consumers or utilities without jeopardizing the overall market welfare levels.

For future research endeavors, it would be valuable to conduct a sensitivity analysis on the risk aversion parameters employed. Furthermore, incorporating generators from different sources, banks, and trading companies into the equilibrium model is worth considering. Additionally, exploring the potential impacts of the proposed derivative within a hybrid (solar and wind) power plant presents a highly promising study path. Such hybrid power plants are becoming more and more common, as they reduce project costs and make it more profitable. In this context, the inherent complementarity of energy generation between solar and wind resources could partially mitigate the PQ-Risk. The final adjustment could then be achieved through the implementation of the proposed derivative.

12
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