Pontifícia Universidade Católica do Rio de Janeiro

Naielly Lopes Marques

Essays in Infrastructure Economics

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Administração de Empresas of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Administração de Empresas.

Advisor: Prof. Luiz Eduardo Teixeira Brandão

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To my family.

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Abstract

Marques, Naielly Lopes; Brandão, Luiz Eduardo Teixeira (Advisor). **Essays in Infrastructure Economics**. Rio de Janeiro, 2023. 181p. Tese de Doutorado - Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

After a careful review of academic works that apply the real options approach to the evaluation of infrastructure concession projects and identifying the main gaps in this literature, we developed this Thesis, composed of four independent studies. The first explains the binomial model and shows how to incorporate the project cash flows using the cash flow dividend rate to create the project value lattice. We develop a R code, provide a tutorial on how to use this model and show how the code can be customized for particular applications. The second shows why additional investments in expansion as firm obligations in concession contracts are suboptimal and proposes a real options model that combines flexible capacity expansion decisions with conditional term extensions. Using a typical toll road project in Brazil, we show how this kind of flexibility can be useful for policy development to attract private investment in public infrastructure projects. The third evaluates the concession of a Light Rail Vehicle in Brazil. We adopt the real options approach to model the different flexible clauses embedded in this contract and analyze whether they conflict with each other and how they impact the overall project evaluation. Finally, the fourth uses Unit Root, Variance Ratio tests, and the Parameter Approach Measure to evaluate the most appropriate stochastic process to model the uncertainty of passenger demand in airport concessions in Brazil. We analyze samples ex-ante and ex-post covid-19 and show that both seasonality and the pandemic significantly impact the stochastic diffusion model.

Keywords

Infrastructure; Concession; Real options; Stochastic process; Flexible clauses.

Resumo

Marques, Naielly Lopes; Brandão, Luiz Eduardo Teixeira. **Ensaios em Economia da Infraestrutura**. Rio de Janeiro, 2023. 181p. Tese de Doutorado - Departamento de Administração, Pontifícia Universidade Católica do Rio de Janeiro.

Após uma criteriosa revisão de trabalhos acadêmicos que aplicam a abordagem de opções reais para a avaliação de concessões de infraestrutura e identificação das principais lacunas dessa literatura, desenvolvemos esta Tese, composta por quatro estudos independentes. O primeiro explica o modelo binomial e mostra como incorporar os fluxos de caixa do projeto usando a taxa de dividendos para criar a treliça de valor do projeto. Desenvolvemos um código em R, fornecemos um tutorial sobre este modelo e mostramos como o código pode ser personalizado para aplicações específicas. O segundo mostra porque cláusulas obrigatórias de investimentos adicionais em contratos de concessão são subótimos e propõe um modelo de opções reais que combina decisões flexíveis de expansão de capacidade com extensões condicionais de prazo. Usando um projeto de rodovia no Brasil, mostramos como essa flexibilidade pode ser útil para atrair investimentos privados em projetos públicos de infraestrutura. O terceiro avalia a concessão de um Veículo Leve sobre Trilhos no Brasil. Adotamos a abordagem de opções reais para modelar as cláusulas flexíveis embutidas neste contrato e analisamos se elas são conflitantes entre si e como impactam a avaliação geral do projeto. Por fim, o quarto utiliza testes de Raiz Unitária, Razão de Variância e Medida de Abordagem de Parâmetros para avaliar o processo estocástico mais adequado para modelar a incerteza de demanda de passageiros em concessões aeroportuárias no Brasil. Analisamos amostras *ex-ante* e *ex-post* covid-19 e mostramos que tanto a sazonalidade quanto a pandemia impactam significativamente o modelo de difusão estocástica.

Palavras-chave

Infraestrutura; Concessão; Opções reais; Processo estocástico; Cláusulas flexíveis.

Table of contents

List of figures

List of tables

1
Introduction

Given the limited financing capacity of governments and the worldwide trend to grant to private investors projects that can provide an adequate return, from the 1990s onwards, there was significant growth in concession projects in the infrastructure sector (Narbaev et al., 2020). The method of choice for the valuation of this class of projects, encompassing P3 (Public Private Partnerships) and BOT (Build-Operate-Transfer) projects is the Discounted Cash Flow (DCF) method. Although robust, this method considers only information known at the initial moment and does not capture the value of any managerial flexibilities that may be embedded in the project. Thus, this approach ignores the dynamic environment of the market and the value of changing the operational strategy of the project as new information is revealed.

As there are many risks and managerial flexibilities involved in infrastructure concession projects, their value cannot be determined through traditional project valuation methods and the use of option pricing tools, such as the Real Options Approach (ROA), becomes necessary. Several academic publications address the use of ROA to evaluate infrastructure projects (Carbonara & Pellegrino, 2018; Chiara et al., 2007; Liu et al., 2017; Shan et al., 2010; Rose, 1998). In addition, some literature review articles provide valuable insights into the trajectory of ROA emergence, its main characteristics, similarities and differences from financial options, and the models and techniques of option pricing used in this field of study (Pellegrino et al., 2013; Martins et al., 2015). However, these articles do not provide a detailed review of the literature and give an incomplete picture of this topic by focusing only on the specific type of option priced in the article or limiting the analysis to particular aspects of real options valuation.

In this sense, we present a more thorough review of academic papers that apply ROA to infrastructure concession project valuation. By combining a procedure for literature review, the strengths of existing studies in the field, and an exhaustive data sample, we develop a more comprehensive framework for this topic to support researchers, project developers, and policymakers in planning and evaluating infrastructure concession projects. This analysis was fundamental for the development of this thesis, as it identified the main gaps in this literature, providing a series of ideas for the formulation of studies and public policies, which became the objectives of this research.

1.1. Related work

ROA arose from the need to consider managerial flexibility in evaluating projects, which is not contemplated by traditional techniques, such as the DCF method (Copeland & Tufano, 2004; Trigeorgis & Tsekrekos, 2018). This innovative approach adapts the pricing models for options on financial instruments developed by Black and Scholes (1973) and Merton (1973) to evaluate options on real assets, allowing analysis of capital investments under uncertainty and flexibility.

A financial option is a derivative that provides the holder the right, but not the obligation, to buy or sell the underlying asset, which may be a stock, index, or future contract, for a predetermined price, known as the exercise price. According to Hull (2003), the right to buy an asset is known as a Call Option, while the right to sell the asset is known as a Put Option. An option that can only be exercised on its maturity date is known as a European option, while one that can be exercised at any time up to its due date is known as an American option.

In the case of real options, the underlying asset can be an investment project or any real asset. The exercise price of the option is the capital investment required to purchase or implement the project in the case of a call option or the value to be received in case of abandonment for a put option. While financial options typically involve a detailed contract between the parties involved, real options are characterized as investment strategies. This is because real options represent the managerial flexibility that managers have to change and adapt the operating strategy of a project as it evolves in time and new information becomes available.

Triantis (2005) states that, when using ROA, a company's managers can respond to market changes more easily, be proactive, and generate new flexibilities in the projects. Therefore, the theory of real options allows the evaluation of companies or projects with managerial flexibility and uncertainty (Trigeorgis, 1996).

There are many distinct types of real options, the most common in the infrastructure literature being (Martins et al., 2015):

a) Option to defer – refers to postponing or delaying the investment to obtain more information (Cruz & Marques, 2013). This option can benefit infrastructure projects when investment capital is extremely high, expensive, or scarce. This option can also be understood as a timing option, enabling waiting until the appropriate time to start the investment (Kozlova, 2017).
b) Option to abandon – implies an option to stop and exit the investment. In infrastructure projects, this option increases the flexibility of the concessionaire regarding the decision to invest (Huang & Chou, 2006).
c) Option to expand – considers the possibility of expanding the project at favorable times. Expansion is based on new investments. Therefore, the exercise price of this option must be equal to the present value of the total additional assets (Marques, Brandão, & Gomes, 2019; Polat & Battal, 2021).
d) Guarantee option – one of the traditional forms of risk mitigation in infrastructure projects is the government guarantees, where public authorities undertake to compensate the concessionaire if the demand, traffic, or revenue falls below a pre-established level by establishing a lower boundary or floor. This option is modeled as a series of European puts and is known as MDG (Minimum Demand Guarantee), MTG (Minimum Traffic Guarantee), or MRG (Minimum Revenue Guarantee).
e) Collar option – in addition to the floor of the guarantee option, it is common in infrastructure projects to establish a demand ceiling, above which the concessionaire passes on to the public agent any extraordinary gains if there is an excess of demand, traffic, or revenue. This combination of put and call options is known as a demand collar, traffic collar, or revenue collar.

Several textbooks (Trigeorgis, 1996; Dixit & Pindyck, 1994) and review papers (Garvin & Ford, 2012; Martins et al., 2015) focus on the modeling and pricing approaches of real options. In this article, we focus on showing commonly used techniques already in the literature rather than discussing the different valuation methods.

The choice of the real options pricing model depends not only on the type of option intended to be modeled but also on other factors, such as the number of uncertain variables, the diffusion processes they follow, how these options are exercised, and the boundary conditions involved. Table 1.1 presents the real options pricing models and approaches found in the reviewed literature, as well as the characteristics and limitations of each one.

Valuation Approach	Continuous Models	Numerical Methods	Simulation Models
Model	Differential Equation and Boundary Conditions	CRR Binomial Model	Monte Carlo Simulation
		Finite Differences	Least Square Model
Flexibility	Only one Option	Multiple Options	Only one Option
	European Option	European/American Options	European/American Options
	Simple Option	Composite Options	Simple Option
	Without Dividends	With Dividends	With Dividends
Uncertainty	One Source	Multiple Sources	Multiple Sources
Seminal Articles	Black & Scholes (1973) and Merton (1973)	Cox, Ross & Rubinstein (1979) and Brennan & Schwartz (1978)	Boyle (1977), Longstaff & Schwartz (2001) and Fu, Laprise, & Madan (2001)

Table 1.1 – Real options pricing models

Another essential part of real options valuation is defining the sources of uncertainty and their modeling. The single-factor stochastic diffusion processes most used for real options valuation are the Geometric Brownian Motion (GBM) and Mean Reversion Processes (MRP). Choosing the appropriate process to model asset price dynamics is still one of the main challenges for researchers and practitioners in the field (Bastian-Pinto et al., 2021; Collan et al., 2016).

This literature review analysis follows the method proposed by Kozlova (2017) and incorporates the strengths of previously published literature reviews in the field (Garvin & Ford, 2012; Pellegrino et al., 2013; Martins et al., 2015; Zhang et al., 2016; Bao et al., 2018; Song et al., 2019; Akomea-Frimpong et al., 2021). This process is done in three phases, as shown in Figure 1.1: i) Database search, ii) Filter and review, and iii) Analysis of the final set of selected articles. The initial search in the SCOPUS and Google Scholar databases was limited to articles written in English that use ROA to evaluate at least one type of infrastructure concession project. The search was also restricted to articles published between January 2000 and March 2022. This period was chosen as it is sufficiently long to identify current literature trends (Trigeorgis & Tsekrekos 2018).

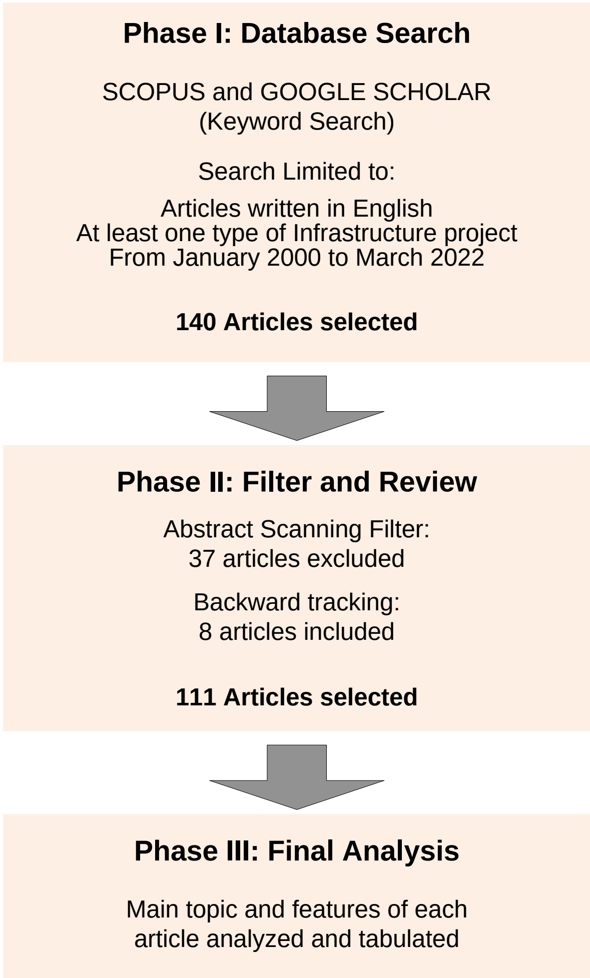

Figure 1.1 – Literature selection process

To identify the papers that adopt the real options approach to assess infrastructure projects, we used the following keywords as a search criterion: (a) “infrastructure project” and “real option”; (b) “concession” and “real option”; (c) “PPP” and “real option”; (d) “P3” and “real option”; and (e) “public-private partnership” and “real option”. The search returned one hundred and forty (140) results. Thirty-seven (37) papers were excluded based on abstract scanning, resulting in one hundred and three (103) candidates for the review. From these selected papers, backward tracking was performed by analyzing their bibliographic references. We found eight (8) more articles related to the topic. Therefore, one hundred and eleven (111) articles were selected for this study. Next, for a more detailed and exhaustive analysis, in each of the selected papers, the following features were extracted: (a) year of publication; (b) country for which the research is conducted; (c) project type; (d) uncertainty sources; (e) stochastic process; (f) real option type; and (g) valuation approach. Detailed results of this analysis are presented in Appendix I. The reliability of the research was assured by only considering academic articles from indexed scientific journals. The following section summarizes and discusses the results obtained.

Given the increasing interest in infrastructure concession project valuation in recent years, several reviews addressing this issue have been published (Ke et al., 2009; Song et al., 2016; Cui et al., 2018; Zhang et al., 2020). In this sense, before considering the results of this research, we present a summary of these works. A review conducted by Garvin and Ford (2012) shows the potential for real options to enhance project value by managing uncertainty through investment, structuring, and design decisions. The authors analyze how ROA can be applied to infrastructure projects and identify that a viable way to disseminate this valuation method is to improve the understanding of the environment and managerial behavior in projects of this type. Pellegrino et al. (2013) perform a more extensive review of the use of ROA in the infrastructure sector. Considering that infrastructure concession projects require analyzing and allocating a broad spectrum of risks, they claim that the risk management process must consider the managerial flexibility present in these projects. So, they review the literature in this field to identify the key risks and related mitigation strategies and model them as real options that naturally exist or are built artificially in contractual conditions and clauses. Their results allowed the development of an option-based risk management framework.

More recently, researchers have presented limited studies, mainly emphasizing and discussing a particular aspect of ROA valuation. Martins et al. (2015), for example, provide an overview of the academic literature on ROA in infrastructure, highlighting the importance of this approach to project design. They address the main types of options, valuation mechanisms, and fields of application. On the other hand, Akomea-Frimpong et al. (2021) present a systematic review of 49 relevant and available studies on financial risk management of P3 projects from

1995 to 2019. Their results show that high-interest charges, increased construction costs, and increased market risks are some of the key financial risks in P3 projects. Besides, they find that financial risk control adopted by project managers includes MRG and real option pricing.

Although these reviews provide valuable results on the topic, their scope is limited to several studies that focus on specific aspects of ROA in infrastructure concession project valuation. Therefore, to broaden research, we: i) expand the sample size, ii) present the results in a quantitative and summarized form, and iii) develop a more detailed and comprehensive analysis of the topic to support researchers, project developers, and policymakers.

Figure 1.2 illustrates the growing research devoted to real options valuation of infrastructure concession projects. The sample of 111 papers is distributed along a timescale based on the publication year. It can be noted that there is a positive trend with more than seven papers per annum in recent years. As this review covers only the first months of 2022, there are few papers for that year.

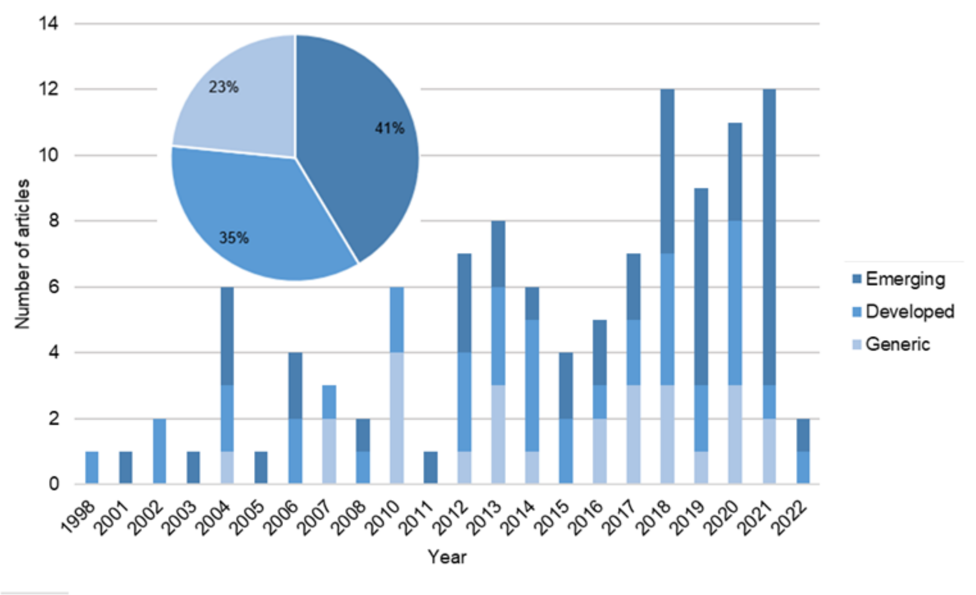

Figure 1.2 – Research trend and country focus

By dividing the data into articles focusing on developed and emerging economies, we note that since 2018, increasing attention has been given to infrastructure concession project valuation in emerging countries. Publications from a developing/emerging country already account for forty papers, representing 41% of the sample, covering countries such as China, India, Colombia, Brazil, Taiwan, Turkey, Indonesia, and Egypt.

Research focusing on developing countries will likely continue the positive trend of the last years because where transportation infrastructure is lacking or underdeveloped, newly built facilities tend to attract and increase demand, which may require further investments in capacity expansion. In our research, we note that this possible need for expansion has been included in several concession agreements through mandatory or flexible contract clauses (Marques et al., 2019; Polat & Battal, 2021).

We perform the same analysis to investigate which infrastructure projects are evaluated in these reviewed papers. As we expected, most of the sample (47%) evaluates road projects, such as toll roads (Blank et al., 2016; Chiara et al., 2007; Liu et al., 2020); bridges (Carbonara & Pellegrino, 2018); and motorways (Colín et al., 2017).

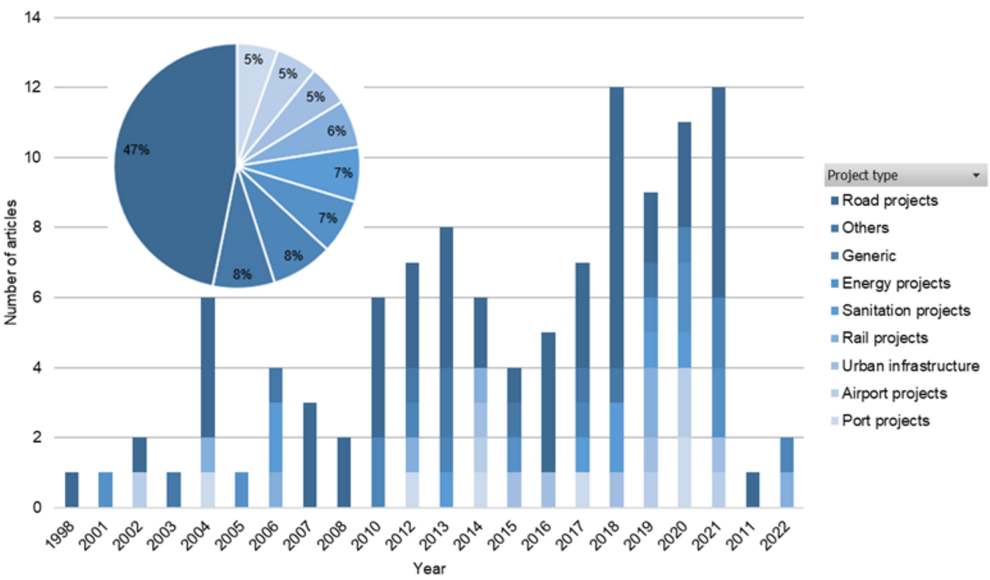

Figure 1.3 – Research trend and country focus

We now examine the following components of real options valuation: (a) identification of the sources of uncertainty; (b) recognition of the real options; (c) modeling of the uncertain variables; and (d) valuation of the real options.

Table 1.2 shows the sources of uncertainty found in the reviewed papers. Percentage values indicate the proportion of studies identifying a particular source of uncertainty. However, as many articles analyze multiple sources, the total value does not sum up to 100%. More than a quarter of the reviewed papers incorporate traffic uncertainty into the valuation design, which is crucial for the infrastructure sector, mainly when dealing with highway and road concession projects. Several other sources of uncertainty are often identified in infrastructure projects, for example: the value of the project itself; revenue; and demand, which is a common uncertainty in airport, port, and subway concessions. Other uncertainty sources, such as inflation, interest rates, and O&M cost, are examined less often. In the literature reviewed, 85% of studies focus on a single uncertainty source in their valuation model, most commonly traffic or project value. A maximum of four sources of uncertainty are identified in the individual studies.

Uncertainty	Number of publications	Share of sample
Traffic	29	26%
Project value	23	21%
Revenue	18	16%
Demand	18	16%
Cash Flow	9	8%
IR	6	5%
O&M cost	6	5%
Toll	2	2%
Investment	2	2%
Cost	2	2%
Capacity	2	2%
Natural gas prices	1	1%
Naphta	1	1%
Urban residents	1	1%
Electricity quantity	1	1%
Timber prices	1	1%
Failure cost	1	1%
Land price	1	1%
Highway deterioration	1	1%
Quantity	1	1%
Inflation	1	1%
Climate change	1	1%
Energy saving amount	1	1%
Energy price	1	1%
Tariff	1	1%
Price	1	1%
Emergency incident	1	1%

Table 1.2 – Uncertainty sources in infrastructure concession valuation

Note: IR = Interest rate; O&M cost = Operation and Maintenance cost.

Regarding uncertainty modeling, the most used techniques are stochastic processes such as GBM or MRP. Most of the reviewed papers use GBM for uncertainty modeling (69%), whereas MRP is a much less popular choice (5%). This is because ROA derives from financial option theory, where GBM is commonly used to model the evolution of stock prices. Researchers also use GBM because uncertainties such as rates, demand, revenues, and costs can never be negative.

Finally, we analyze the real option types identified in the reviewed papers. Table 1.3 shows that the option to defer (timing option) is the most common in infrastructure concession project valuation studies. This type of real option provides the flexibility to postpone the investment, allowing the investor to identify the most favorable moment to invest. This option is generally addressed with continuous models (Adkins & Paxson, 2017; Saito et al., 2001) and simulation methods (Jeong et al., 2016; Gaudard, 2015).

The second most analyzed option is the option to abandon a project. In many cases, it is often reasonable to terminate a concession agreement early in the face of various uncertainties. This option can be modeled by binomial trees (Kitabatake, 2002; Rakić & Rađenović, 2014; Rodrigues et al., 2019), by Monte Carlo simulation (Colín et al., 2017), or by continuous models (Huang & Pi, 2014). We also note that the option to abandon appears as a single option in 8% of the reviewed papers.

Another option that caught our attention in our analysis was the MRG. This form of risk mitigation has been used in infrastructure concession contracts in several countries, such as Brazil (Brandão et al., 2012), China (Jin et al., 2021), and Egypt (Marzouk & Ali, 2018). Since MRG is considered to be a series of European put options, this mechanism is primarily modeled by Monte Carlo simulation (Kim et al., 2019; Zapata Quimbayo et al., 2019; Zhang et al., 2010). Unlike the abandonment option, the MRG appears as a single option in 12% of the reviewed papers.

The fourth type of real option that appears most frequently in these studies is the expansion option, commonly addressed with binomial, scenario, or decision trees (Krüger, 2012; Marques et al., 2019). The expansion option is usually evaluated to be exercised during the project's operation phase. Nonetheless, in cases where the expansion is triggered at the end of the concession period, it will most likely not be conducted because there would be no time to obtain profitability unless the government offers additional benefits (Marques et al., 2021).

RO type	Number of publications	Share of sample
Timing	33	30%
To abandon	23	21%
MRG	20	18%
To expand	18	16%
Length of concession term	13	12%
Guarantee	8	7%
Revenue collar	6	5%
MTG	4	4%
MDG	2	2%
MIR	2	2%
To transfer	2	2%
To extend	2	2%
To switch	2	2%
To renegotiate	2	2%
Traffic collar	2	2%
Buyout	1	1%
Conditional buyout	1	1%
Revenue-sharing	1	1%
Demand collar	1	1%
Investment subsidy	1	1%
Revenue subsidy	1	1%
To rescue	1	1%
MRL	1	1%
MEL	1	1%
MCFG	1	1%
MROIG	1	1%
Fiscal support	1	1%
Tariff guarantee	1	1%
Debt guarantee	1	1%
PARG	1	1%
PCRG	1	1%
To rehabilitate	1	1%
To contract	1	1%
Expenditure ceiling guarantee	1	1%
TAM	1	1%

Table 1.3 – Real option types in infrastructure concession valuation

Note: PARG = Payment-based annual revenue guarantee; PCRG = Period-extension-based cumulative revenue guarantee; TAM = Toll-adjustment mechanism; MCFG = Minimum cash flow guarantee; MRL = Maximum revenue limit; MEL = Maximum expense limit; MIR = Maximum interest rate; MROIG = Minimum return on investment guarantee; and, IR = Interest rate.

Figure 1.4 and Figure 1.5 complement this analysis by showing the number of real options identified within one case and the valuation techniques used in the reviewed papers. It can be noted that 72% of the sample considers only one real option in their analysis. The MRG is the option that most appears to be evaluated solely, followed by the abandonment and expansion options. Our search also shows that the choice of valuation approach is linked to the types of real option identified.

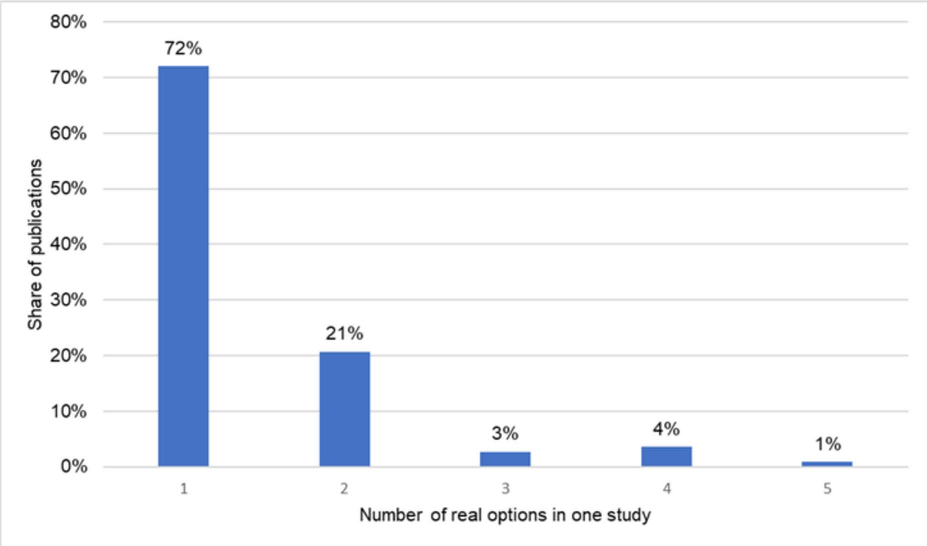

Figure 1.4 – Number of real options identified and their valuation methods

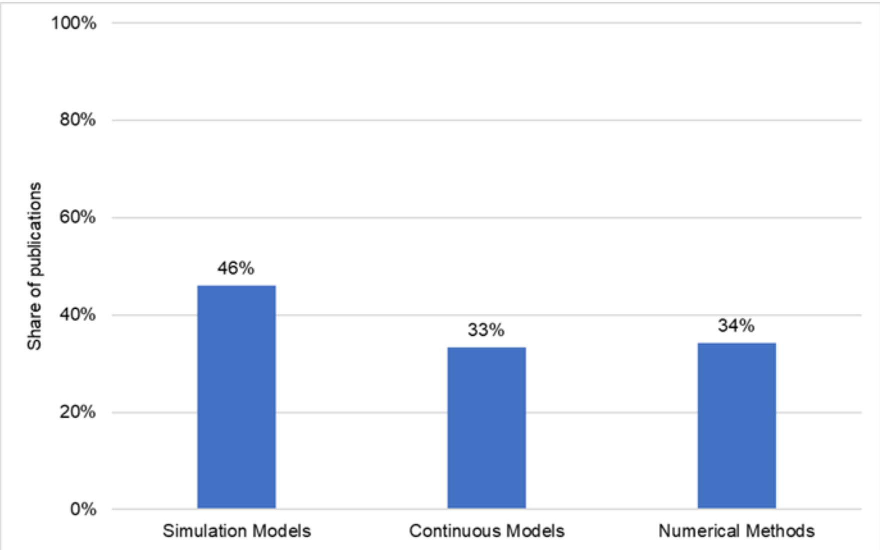

Figure 1.5 – Valuation methods identified in the sample

These results illustrate the relevance and superiority of ROA over traditional investment valuation techniques, highlighting its ability to value managerial flexibility and capture uncertainty. The research also shows that over the years, there has been an increase in the number of articles published in this area and the interest in infrastructure projects developed in emerging economies.

We also show that most of the articles reviewed focus on the evaluation of road projects. Still, we highlight the growing interest in evaluating sanitation, rail, airport, and port projects. As far as valuation techniques are concerned, we find a connection between the chosen methodology and the real options identified for valuation. The most common real options recognized in the analysis of such projects are the abandonment, MRG, timing, and expansion options. However, other types of options are currently gaining space in this literature, such as MTG, MDG, and collar options.

In addition, we show that more than a quarter of the reviewed papers consider traffic uncertainty in their analyses. But, other sources of uncertainty are also frequently identified in infrastructure projects, such as the value of the project itself, revenue, and demand. According to our review, most researchers chose the stochastic processes of GBM or MRP to model these uncertainties.

This review contributes to the real options literature by providing a more detailed overview of the works that use this approach to evaluate infrastructure concession projects, showing trends and aspects that still need to be addressed and highlights the differential that ROA offers relative to other methods. Our findings suggest that there is significant potential for future research in this field for other types of infrastructure projects, such as ports, airports, and sanitation, which may involve different types of managerial flexibilities and uncertainty. The choice of stochastic process to model the uncertainties in these projects is also challenging and could provide interesting insights.

1.2. Research objectives

Given these aspects that still need to be addressed in the real options literature applied to the evaluation of infrastructure concession projects, the objectives of this thesis are:

a) Propose a code in an open-source software with intuitive guidelines to help researchers and practitioners model real options lattices from project cash flows.
b) Show why additional investments in expansion as firm obligations in concession contracts are suboptimal and propose a real options model

that combines flexible capacity expansion decisions with conditional term extensions.

c) Apply the real options approach to analyze the case of the Salvador Light Rail Vehicle (LRV) concession project and investigate how the different flexible clauses embedded in the contract interact and impact the overall valuation of the project.

d) Use unit root and variance ratio tests and the Parameter Approach Measure (PAM) to evaluate which would be the most appropriate stochastic process to model this uncertainty in real cases of airport concessions, considering samples ex-ante and ex-post covid-19 pandemic and the effect of seasonality.

1.3. Main contributions

Thus, we believe that the main contributions of this thesis are:

a) First, we propose a tutorial that provides a simple mechanism for analyzing investment opportunities in projects that have uncertainty and flexibility.

b) Second, we consider the fact that concession revenues are bounded by the current traffic capacity of the road, which represents an upper absorbing barrier that has implications for the expansion decision.

c) Third, we evaluate how the option to expand capacity coupled with a term extension increases the probability of a timely and voluntary expansion, allowing the granting authority to elaborate low-cost contractual clauses that align the objectives of both public and private agents.

d) Fourth, we evaluate the interaction of a bundle of European call and put options created by the cap and floor mechanism, with an American call option arising from the flexible expansion and term extension clauses.

e) Fifth, we propose a model based on the Cox, Ross, and Rubinstein (1979) lattice approach, rather than on simulation methods, which are more common for valuing the interaction of a bundle of European call and put options with an American call option.

f) Sixth, we show how the clauses that govern the managerial flexibilities in contracts must be carefully designed to achieve the objectives of both government and private investors.
g) Seventh, we show that the choice of stochastic process is not as straightforward as the extant literature in the field of infrastructure concession projects may suggest.

1.4. Organization of the thesis

After this introduction, derived from a literature review article, which is currently under review in the Transportation Research Record, we present the structure of this doctoral thesis.

This thesis is organized into four central chapters in the form of complete and independent articles covering each of the topics discussed in this introduction. The articles presented in chapters 2 and 3 have been published in the Journal of Contemporary Administration (https://doi.org/10.1590/1982-7849rac2021200093) and in the Construction Management and Economics journal (https://doi.org/10.1080/01446193.2020.1863439). The article presented in chapter 4 will be submitted to Case Studies on Transport Policy and the article presented in chapter 5 is under review in Journal of Contemporary Administration.

In the second chapter, I explain the CRR binomial model, its parameters and show a direct application with the corresponding R code for a simple application of option valuation. Then, I explain the issues and characteristics of a project cash flow model and show how to implement the CRR approach in such a model. Next, I show how to incorporate the project cash flows into the CRR lattice using the cash flow dividend rate, as suggested by Copeland & Antikarov (2001) to create the value lattice of the project. Following that, I show how to model the real options as decision nodes in the project value lattice and determine the expanded value of the project with a backward maximization framework. Finally, I develop an additional R language code specifically to run this model and provide a detailed step-by-step tutorial on parameter determination, use of the model and show how the code can be customized for particular applications.

In chapter three, I show why additional investments in expansion as firm obligations in concession contracts are suboptimal and propose a real options model that combines flexible capacity expansion decisions with conditional term extensions. As a novel contribution, I model the project value uncertainty during the life of the concession as a Brownian Bridge and consider the fact that concession revenues are bounded by the current traffic capacity of the road, which represents an upper absorbing barrier that has implications for the expansion decision. Using a typical toll road project in Brazil, I show how this kind of flexibility can be useful for government officials involved in policy development to attract private investment in public infrastructure projects.

In the fourth chapter, I evaluate the 20-year concession contract for the construction and operation of an LRV connecting the district of Comércio, in Salvador, to Ilha de São João, in the district of Simões Filho in the state of Bahia, Brazil. I adopt the real options approach to model the different flexible clauses embedded in this contract and to analyze how they impact the overall valuation of the project.

In chapter five, I use unit root and variance ratio tests and the Parameter Approach Measure (PAM) to evaluate which would be the most appropriate stochastic process to model the passenger demand uncertainty in real cases of airport concessions.

Finally, the highlight and the final comments are presented the chapter six.

2 A Tutorial for Modeling Real Options Lattices from Project Cash Flows

Several methods for evaluating real options have been extensively studied and published. But, recombining binomial trees, known as lattices, are perhaps one of the most practical and intuitive approaches to model uncertainty and price project managerial flexibilities for real option applications. Although the Cox, Ross and Rubinstein (1979) lattice model is simple to implement for financial options, modeling real options lattices requires a different approach such as the one proposed by Copeland and Antikarov (2001), which considers project cash flows as dividends in the lattice model. In this tutorial, we propose a code in an open-source software with intuitive guidelines to help researchers and practitioners model real options lattices from project cash flows. Our code considers the correct project's volatility estimation, dividend yield modeling and lattice building. The results show how real options can affect the value of projects. As a contribution, this tutorial provides a simple mechanism for analyzing investment opportunities in projects that have uncertainty and flexibility.

2.1. Introduction

Real Options Approach (ROA) was developed to overcome the limitations of the Discounted Cash Flow method by using option-pricing methods to capture the value of any managerial flexibility that may be embedded in a project subject to future uncertainty.

It is a well-established principle in Finance and Economics that the future cash flows generated by an asset, discounted to the present time at an appropriate risk-adjusted rate is the correct measure of this value. Nevertheless, in the case of real assets, this principle does not account for the uncertainty over the future behavior of the cash flows nor for the value generated by the flexibility some projects have to react to future events. In order to value project cash flows which can flexibly change their trajectory as future uncertainties are resolved, it is necessary to use a more adequate approach. Given that the flexibility to adapt to changes in expected future cash flows has option-like characteristics, they can only be valued with option-pricing methods.

The Real Options Approach fulfills these conditions and has been widely discussed in the academic as well as the practitioner literature. Although several option-pricing methods are well known and widely used, probably the most intuitive and flexible method is the Binomial Lattice option-pricing model originally developed by Cox, Ross and Rubinstein (1979) for financial options (CRR model) and then extended by Copeland and Antikarov (2001) for the pricing of real options.

However, even though the CRR binomial lattice model is widely used in the real options literature (Brandão & Dyer, 2005; Lee & Shih, 2010; Ashuri et al., 2012; Lin & Wesseh, 2013; Kim, Park & Kim, 2017; Bastian-Pinto, Brandão & Hahn, 2009; Jang, Lee & Oh, 2013), there are many obstacles and difficulties in its implementation for real options valuation, as its principles are sometimes not very well understood by practitioners. In this tutorial, we guide the user through the sequence of steps necessary for the correct implementation of a real options model based on project cash flow estimation.

We begin by explaining the simplest CRR binomial model, its parameters and show a direct application with the corresponding R Code for a simple application of option valuation. Then, we explain the issues and characteristics of a project cash flow model and show how to implement the CRR approach in such a model. We do that by demonstrating the procedure required to correctly estimate the volatility of the project value (V) from the project uncertainties. Next, we show how to incorporate the project cash flows into the CRR lattice using the cash flow dividend rate, as suggested by Copeland & Antikarov (2001) to create the value lattice of the project. Following that, we show how to model the project flexibilities, or real options, as decision nodes in the project value lattice and determine the expanded value of the project with a backward maximization framework. Finally, we develop an additional R language code specifically to run this model and provide a detailed step-by-step tutorial on parameter determination, use of the model and show how the code can be customized for particular applications.

2.2. Valuing derivatives with the CRR Lattice Model

Financial options are classified as derivative securities since their value derives in part from the price of another marketed financial asset, known as the underlying asset.

The Cox, Ross & Rubinstein (1979) binomial tree, or lattice model, emulates the Black & Sholes (1973) (B&S) option valuation approach. One advantage of this model is that it allows for the pricing of American type options, which can be exercised at any time before expiration, which is not possible under the B&S model. Although it is a discrete approach, it is accurate enough for most real asset valuation applications.

To implement the CRR model on a derivative of an asset whose current price is S_0, and has a volatility of σ, at each time step the asset value (S) is multiplied by a random variable that can take two values, u or d. For this representation to emulate a lognormal distribution, the values for u, d and the risk neutral probability p must be as shown in equation (2.1) where σ is the asset volatility and r is the risk-free discount rate.

$$u = e^{\sigma\sqrt{\Delta t}},\ d = \frac{1}{u} \text{ and } p = \frac{(1+r)^{\Delta t} - d}{u - d} \qquad (2.1)$$

where u and d are respectively the up and down multipliers of the lattice nods, and p is the risk-neutral probability which will be used in discounting the lattice nods.

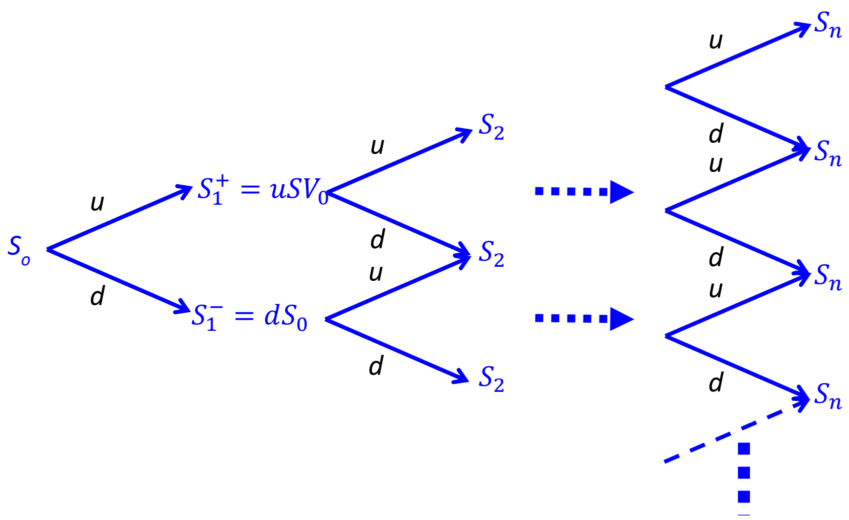

Figure 2.1 – The CRR (1979) Binomial Lattice Model

Note: This is a discretization of a Geometric Brownian Motion (GBM) diffusion process for price *S* using the volatility σ of this price through the multipliers *u* and *d* from equation (2.1). The recombining feature of the lattice is important since at the final step *n*, it will have $n + 1$ nods, instead of 2^n in the case of a non-recombining tree.

With these parameters, we can implement the lattice shown in Figure, which is a discretization of a Geometric Brownian Motion (GBM) diffusion process for S. In the last period *n*, which is the time of expiration of the option or last period of the lattice, the options, which are a maximization process, are exercised on the values of S_n at each node. At the nodes where the options are exercised, the values of S_n, change to $S_n^{'}$. These can be Call options or Put options. After this step, we move to the previous period $(n - 1)$ and perform the same maximization process at each node, but now also considering the value of continuation, which takes in account the present value of the expected future nodes discounted at the risk-free rate and weighted by the probabilities *p* and $(1 - p)$. This process of backward maximization is summarized by equation (2.2).

$$\max\left[S_{t-1}^{'};\left(S_t^{'+}p+S_t^{'-}(1-p)\right)/(1+r)\right] \tag{2.2}$$

where S_t is the asset value at time *t*, before the exercise of any option, and of $S_t^{'}$ after the exercise of an option.

The whole process is shown in Figure 2.2, where at the initial step, the bold red arrow indicates the value of the options exercised along the whole lattice.

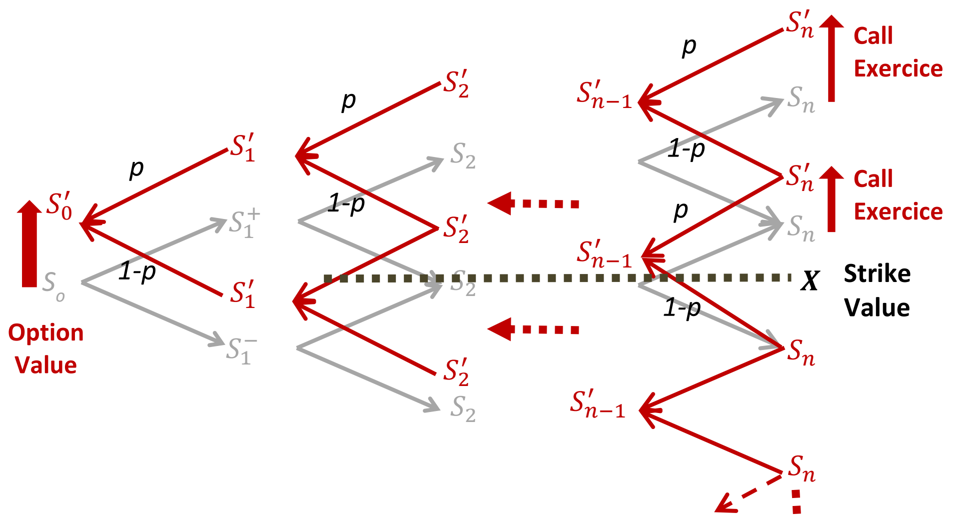

Figure 2.2 – Diagram showing Backwards Discounting of the CRR Lattice for a Call Option

Note: This figure shows that the call option is exercised above the strike value *X*. Starting from the last period n after exercising the options, we move backwards at each node, weighting the future values by the risk neutral probabilities *p* and (1-*p*) and discounting with the risk free rate r using equation (2.2), up to the initial step.

In order to help researchers and practitioners understand how this CRR binomial model works for the calculation of financial options, we provide a simple code with the open software R (www.r-project.org), which uses the available fOptions package developed by Wuertz, Setz, and Chalabi (2017). This code, described in Appendix III (Code I), evaluates call and put options considering the following parameters: n = 1.5 years (18 months); S_0 = \$100; σ = 30% per year; r = 6% per year; call strike price of \$120; and, put strike price of \$90. After running this code, considering dt = 2 months, we find that the call value is \$11.00 and the put value is \$6.71.

Note that the fOptions package is very well suited for this type of calculation and for plotting the binomial lattices. However, it does not allow us to determine the value of the real options, whose main characteristics are having investment projects as underlying assets and more complex exercise rules. Given this, we develop in this article a specific R code for modeling real options applications based on Project Cash Flows. This code is described in section 2.4 and presented in Code II of Appendix III. Other packages such as the DerivaGem software that accompanies Hull (2003) publication and that can be found online free of charge are also a good solution for calculating options with CRR lattices.

2.3.
From Cash Flows to a Real Option Lattice Model

While the same principles of financial options apply to real options, in this case the underlying asset is an investment project, which as a real asset, is not traded in the market and thus does not have their market price determined. Additionally, exercise rules for real options are typically significantly more complex than those for financial options, as they may involve multiple exercise opportunities, combinations of simultaneous distinct option types and multiple uncertainties.

2.3.1.
Market Asset Disclaimer – MAD

Given that the underlying real asset, such as an investment project, is not traded in the market, it is impossible to determine its true value and risk-return characteristics. A simple solution to this problem is to assume that the underlying asset is the project itself and that the true market value of the project is the present value (V_0) of the project. This assumption implies that the traditional present value of the project´s cash flows without flexibility is the best non-biased estimator of the market value of the project if it were a traded asset. Copeland & Antikarov (2001) refer to this hypothesis as the Market Asset Disclaimer (MAD).

The first step in the model is to determine the cash flow structure F_t of the project, as shown in equation (2.3):

$$F_t = \left[R_t \left(1-\gamma\right) - \lambda_t - \Gamma \right]\left(1-\pi\right) + \lambda_t \quad (2.3)$$

where R_t is the total revenue in year t; γ represents variable costs; π is the income tax λ_t is the depreciation in year t, and Γ represents fixed costs. These are projected for a number of years (n), after which we consider a continuation value (CV). The project value at time t = 0, V_0, can be determined with equation (2.4).

$$V_0 = \sum_{t=1}^{t=n} \frac{F_t}{\left(1+\mu\right)^t} + \frac{CV_n}{\left(1+\mu\right)^n} \quad (2.4)$$

where:

$$CV_n = \frac{F_n\left(1+g\right)}{\left(\mu - g\right)} \quad (2.5)$$

and μ is the risk-adjusted discount rate of the project and g is the cash flow perpetuity growth rate.

The purpose is to model the project's value V, with the CRR binomial approach, which allows the real options to be exercised by retroactive induction, or backward maximization, maximizing V along the binomial nodes. When the starting point of the lattice is reached, we will have a V increased by the optimal exercise of real options (RO), which we call expanded present value (V_0^*), that is: $V_0^* = V_0 + RO$.

2.3.2. Estimating the Project´s Volatility from the Income Variables

To model CRR´s lattice for a given project, we must estimate the volatility σ_V of the project value V. We assume that the revenue R_t is the product of a Price and a Quantity, where Q is deterministic but the price $\tilde{P}$ is stochastic (indicated by the ~ sign above the variable) with a growth rate of α and volatility σ_P. The project revenues are described by equation (2.6).

$$\tilde{R}_t = Q\tilde{P}_t \qquad (2.6)$$

The price $\tilde{P}$ is assumed to follow a Geometric Brownian Motion (GBM) type of stochastic diffusion process, which can be represented by the differential equation: $dP = \alpha P dt + \sigma_P P dz$, where $dz = \varepsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$ is the standard Wiener process.

To estimate the two parameters necessary to model this GBM (α and σ_P), given a historical time series of prices P_t, with n events, these can be calibrated using the following procedure: first, calculate the log return series of the (n -1) events of the price series with $\ln\left(P_t / P_{t-1}\right)$. The growth rate, or drift, α can be estimated by calculating the mean of this log return series, and the volatility parameter σ_P by the standard deviation of the same series. These must be in the same time increment, which for project cash flows is in years. If the time series of P_t, is provided in a different time interval such as monthly, then the values of α and σ_P must be converted to yearly values. This is done by multiplying the drift parameter by 12 (12 months per year) and the volatility parameter by $\sqrt{12}$. The values thus obtained can now be used in modeling the GBM for the prices in the process described in equations (2.8), (2.9) and (2.10).

For such a stochastic process, the expected value equation is represented by (2.7).

$$P_t = P_0 e^{\alpha(t-t_0)} \tag{2.7}$$

and the simulation equation is shown in (2.8).

$$\tilde{P}_t = P_{t-1} e^{\left[\left(\alpha-\sigma_P^2/2\right)\Delta t+\sigma_P N(0,1)\right]} \tag{2.8}$$

where *N(0,1)* is the normal distribution with mean 0 and standard deviation 1.

Using the proof of Samuelson (1965) according to whom the return rate of a financial asset will follow a random walk, independently of its future cash flows as long as investors have access to all the asset's information, we assume that *V* will also follow a GBM process. Therefore, future cash flows dependent on multiple uncertainties, even with autoregressive processes, can be combined into a single multiplicative binomial lattice.

To estimate the volatility of *V*, we use the approach suggested by Copeland & Antikarov (2001), but with the correction made by Brandão, Dyer & Hahn (2012). After estimating the stream of n cash flows with equation (2.3), and the deterministic initial project value $\bar{V}_0$, we calculate the project value in t = 1, with equation (2.9).

$$\tilde{V}_1 = \sum_{t=1}^{t=n} \frac{\tilde{F}_t}{(1+\mu)^{t-1}} + \frac{CV_n}{(1+\mu)^n} \tag{2.9}$$

As $\tilde{P}$ follows a GBM stochastic process, $\tilde{F}$ and $\tilde{V}$ will also be the result of this GBM diffusion process. We define the variable $\tilde{Z}$ with equation (2.10):

$$\tilde{Z} = \ln\left(\frac{\tilde{V}_1}{\bar{V}_0}\right) \tag{2.10}$$

Running a Monte Carlo Simulation (MCS) we use the standard deviation of the variable $\tilde{Z}$ as the volatility of the stochastic project (σ_V) value $\tilde{V}$. Note that $\bar{V}_0$ is a static value, while $\tilde{V}_1$ is stochastic and will have a new value with every simulated trajectory of $\tilde{F}$. The modification of Brandão et al. (2012) to this procedure points that for the MCS on equation (2.9), it is necessary to attribute a stochastic value from equation (2.8), only in the first time period t = 1 of the simulation, with the subsequent increments calculated by the expected value of *P* from equation (2.7). Otherwise, the yearly volatility estimate will be overestimated and will monotonically increase with the number of cash flow periods of the project.

2.3.3. CCR Lattice applied to Real Options from Cash Flow Projection

The lattice structure shown in section 2.2 prices options on assets that do not pay dividends or cash flows. In the case of assets such as stocks, ongoing projects, and firms that generate a continuous stream of cash flows to the shareholders, some adjustments must be made.

After estimating the Cash Flows stream of the project in question and using equation (2.9) it is straightforward to calculate the stream of V_t values for t = 1 to n, before the subtraction of cash flow at this period t, and we call it $V_{ex\ ante}$. Consequently, after subtraction of the cash flows F_t we will have the stream of $V_{ex\ post}$, or the values of the project ex-post cash flows, or dividends. From these, we estimate the stream, or vector of dividend yield δ_t from t = 1 to n, as defined in equation (2.11).

$$\delta_t = F_t / V_{t\ ex-ante} \qquad (2.11)$$

Using this vector of dividend yield, Copeland & Antikarov (2001) proposed a scheme that builds a project value lattice that incorporates the dividends, or cash flows paid out at each step of the process. At each time step t, the values at each node ex-ante subtracting of cash flows (V_a) are multiplied by (1 – δ_t), yielding V_p, or the ex-post project values: $V_p = V_a (1 - \delta_t)$. This model is shown in Figure 2.3, where the red arrows represent the Cash flows subtracted from the project value V_a at each time step. The value of the dividends (D) is calculated by equation (2.12) at every node of the ex-ante values lattice V_a.

$$D_t = V_{at}\delta_t \qquad (2.12)$$

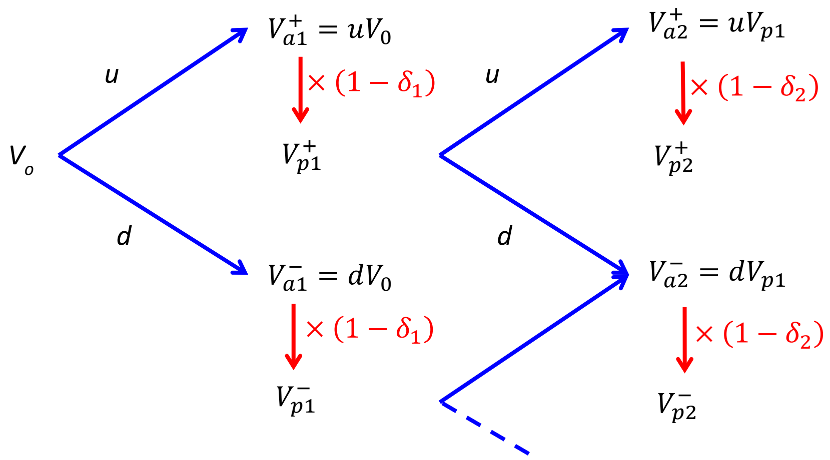

Figure 2.3 – CCR Lattice with dividend payment at each node

Note: This figure presents the values of *V* ex-ante (V_a) and ex-post (V_p). The lattice thus "penalized" by the dividend yields δ_t still is recombining as the one of Figure.

As this scheme will produce, at every node, two values (ex-ante and ex-post), this will yield two lattices (one for values of V_a and another for V_p), which are interdependent. As this can encumber the lattice model, we will simplify the above scheme with the model displayed in Figure 2.4, where the V_a lattice is bypassed and only the V_p lattice remains, using at each time step the dividend yield δ_t stream, or vector, of the cash flow model to do this.

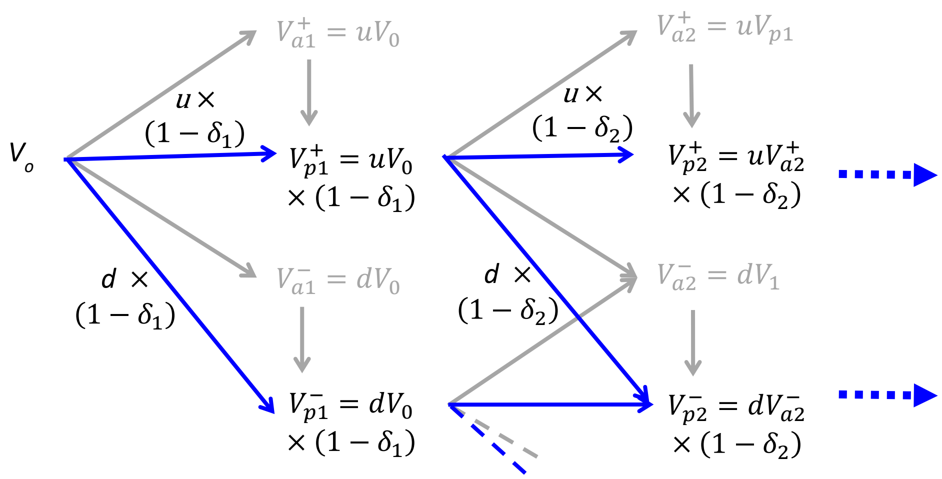

Figure 2.4 – CRR lattice for ex-post values of V_p

Note: This lattice, simpler and more straightforward than that of Figure 2.3, provides all the necessary values for the real options estimation (V_p). Note that this lattice remains recombining.

In Figure 2.4, the V_a values of Figure 2.3 appear only in light grey and the values of V_p are apparent. As stated, our tutorial model uses this approach as it has fewer steps to implement. Yet it brings another complication when discounting the lattice backward from the last step of the model.

At this last step, the real options are exercised on the values of V_{pn}, at each node (maximization process) on the ex-post values, that is, after the cash flow (dividends) has been paid. At the nodes where the options are exercised, the values of V_{pn}, change to $V_{pn}^{'}$. As with the normal CCR lattice (without dividends), we move to the previous time period (n – 1), and perform the same maximization process at each node, also considering the value of continuation, as done also with the simple CRR lattice in section 2.2. But this value, previously calculated with equation (2.10), now uses the ex-ante values $V_{an}^{'}$ of the last step n, which is equal the dividends D_n added to $V_{pn}^{'}$. And as these dividends are the ones already paid by the project, they are the values of $V_{an}^{'}$ multiplied by δ_n. But as the model of Figure 2.4 has bypassed the estimation of V_a lattice, we must use another approach to calculate D_t. We do this using the following algebra: $D_t = V_{at} - V_{pt} = V_{pt} \times \delta_t / (1-\delta_t)$.

The vector of values is then calculated with equation (2.13), which multiplied to V_{pt} yields the dividend values needed to discount the values lattice. Instead of equation (2.10), the maximization process is the one shown in equation (2.14).

$$\delta_t^{'} = \frac{\delta_t}{(1-\delta_t)} \tag{2.13}$$

$$\max\left[V_{p(t-1)}^{'};\left(\left[V_{pt}^{'+} + V_{pt}^{+} \times \delta_t^{'}\right]p + \left[V_{pt}^{'-} + V_{pt}^{-} \times \delta_t^{'}\right](1-p)\right)\Big/(1+r)\right] \tag{2.14}$$

This discounted process is displayed in Figure 2.5.

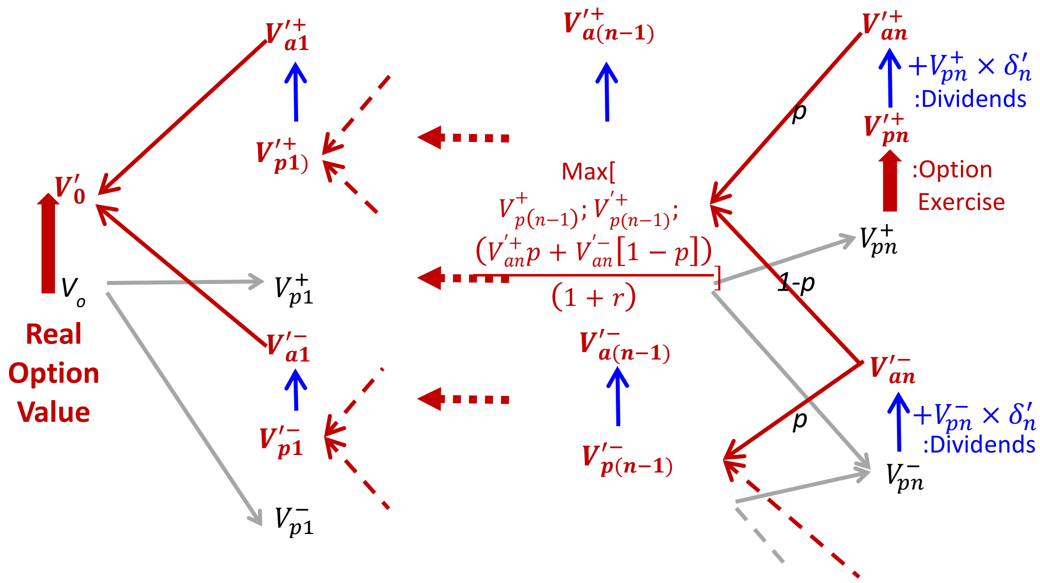

Figure 2.5 – Backwards discounting of the Lattice for ex-post values of V_p

Note: Now, as we don't have the values of the ex-ante lattice, we must estimate these by calculating the vector δ'_t. As shown in Figure 2.2, starting from the last period n after exercising the options, we move backwards at each node adding the value of dividends estimated by δ'_n, weighting the future values by the risk neutral probabilities *p* and $(1 - p)$ and discounting with the risk-free rate *r*, with equation (2.14) up to the initial step.

The full understanding of the particulars of the cash flow lattice model shown in this section is relevant for real options researchers and practitioners, as this model is widely used in many real options applications. Several authors have applied this model to infrastructure projects (Garvin & Cheah, 2004; Marques, Brandão & Gomes, 2019; Iyer & Sagheer, 2011; Rakić & Rađenović, 2014; Oliveira, Couto, & Pimentel, 2020), renewable energy (Dalbem, Brandão & Gomes, 2014; Santos et al., 2014; Wesseh & Lin, 2015; Zhang, Zhou & Zhou, 2014), mining (Miranda, Brandão & Lazo Lazo, 2017) and in other fields of research.

2.4. R Language Model

2.4.1. Guidelines for the Model Routine and Numerical Example

In order to assist researchers and practitioners model project cash flows under the real options approach considering the correct volatility estimation and lattice implementation, we propose an intuitive R code. In this code, we exemplify the model described in section 2.3 through a numerical example.

In this sense, we assume a hypothetical project requiring an initial capital expenditure of $1.5 million, which will depreciate in 10 years, which also is the projection horizon of the project (n). In addition, it will have a fixed output (Q) of 10,000 units which will sell at a price (P) assumed today of 100 $/unit, but which is expected to grow at a rate α per year: from t = 1 to t = 10.

As we assume that price (P) is the main source of uncertainty in this project, we will treat it as a stochastic variable that follows a GBM, as mentioned in section 2.3. Thus, the code first inputs the parameters shown in Table 2.1 to simulate the price using the Monte Carlo simulation technique. Note that users can change any of these parameters to adapt the code to their project.

Inputs		
Depreciation duration time (n)	*n*	10 years
Number of time intervals	*i*	10
Price growth rate (α)	*a*	3% (per year)
Price volatility (σ_P)	vol	15% (per year)
Price at t = 0 (P_0)	P_0	$100
Number of simulations	*nt*	10,000

Table 2.1 – Price Simulation Parameters

Note: This table presents all the necessary parameters for price simulation.

This simulation results in a matrix (X) of dimension nt x 11, where the first column represents the prices at t = 0 (P_0 = 100) and the other columns the prices simulated at each time point until the tenth year of the project.

After simulating price, the code estimates the project's revenue using equation (2.6) and assuming a production (Q) of 10,000 units. Then, through equation (2.3), the code starts the process of calculating the project's cash flows (F_t), considering fixed costs (Γ) of $300,000 per year, variable costs (γ) of 55% of the project's income (R), an income tax rate (π) of 34% of EBIT (Earnings Before Interest and Taxes), and an annual investment to maintain the project (EI) of $50,000. Table 2.2 summarizes all of these parameters that need to be entered in the code to calculate the cash flows (F_t) and the project value (V_0). As well as the parameters mentioned in Table 2.1, these can also be changed in the code so that the user can adapt it to the characteristics of their project.

Inputs		
Risk-free rate (r)	r	6% (per year)
Perpetuity growth rate (*g*)	g	3% (per year)
Discount rate (μ)	k	12% (per year)
Production (Q)	prod	10,000 units
Variable costs (γ)	VC	55% of revenues
Fixed costs (Γ)	FC	$300,000
Investment	I	$1,500,000
Extra investments	EI	$50,000
Income tax (π)	IT	34% (per year)

Table 2.2 – Cash Flow and Project Value Parameters

Note: This table shows all the parameters required to estimate the cash flows and the project value.

As the simulated prices are arranged in a matrix, we need to calculate the cash flow for each year and each price trajectory. In this step, we exclude the first column of the price matrix, because at t = 0 the project does not generate cash flow. In this way, we will have a cash flow matrix (FCF) of dimensions nt x 10. In addition, since we consider that this project has a continuation value (CV), we have included a column in this matrix that represents the perpetual cash flows of that project. To calculate perpetuity, the code uses equation (2.5).

Considering a risk-adjusted discount rate of k = 12% per year, a perpetuity growth rate of g = 3% per year, equation (2.4) and the NPV equation of the package developed by Signorell et al. (2016), we find that the project value is V_0 = $1,661, yielding a Net Present Value (NPV) of $161,549. From this, we can also estimate the dividend yield and, consequently, the present values ex-ante and ex-post, which allow us to find the variable $\tilde{Z}$, as well as the project volatility (σ_V).

Thus, using Monte Carlo Simulation as described in section 2.3.2, the project volatility is estimated as σ_V = 33% per year. This value can vary slightly as it is the result of a MCS. Given the volatility of the project, we can determine the parameters of the binomial cash flow tree (u = 1.39, d = 0.72 and p = 0.51). With this, the code calculates both the Value lattices, using the approach of section 2.3.3. The Value lattices are displayed in Appendix II with both the ex-ante and ex-post values (Figures A.1 and A.2).

After this, we model two real options on the project lattice. First, an expansion option that is modeled as a call option on the value lattice. It considers that at any time for the next 10 years (t = 1 to 10) the project value can be augmented by 80% (multiplied by 1.8) at a cost of $1,200,000. Second, an abandonment option, modeled as a put option, that considers the project can be sold at a value of its total depreciated investment, minus a discount of 20%.

To incorporate these managerial flexibilities, we need to consider the inputs listed in Table 2.3. As with the other inputs, these can also be changed so that the code adapts to the project that each user is analyzing.

Inputs		
Abandonment factor	abandf	0.8 (per year)
Expansion factor	expf	1.8 (per year)
Expansion cost	expc	$1,200,000

Table 2.3 – Cash Flow and Project Value Parameters

Note: These are the parameters required to calculate the abandonment and expansion options values.

In addition to the inputs mentioned in Table 2.3, to determine the project value considering the options, we need to find the residual value of the project in case of abandonment. For this, first, we calculate the depreciated asset value each year until year 10 by discounting the depreciation amount from the investments. Then, we multiply these values by the abandonment factor.

Given these parameters, the code calculates the backward discounted lattices for both managerial flexibilities. In summary, following the model of Figure 2.5, the code evaluates backwards the maximum value between maintaining, abandoning and expanding the project each year until year 10. Appendix II also presents these lattices (Figures A.3 and A.4). At the starting step of this lattice, the project value is now: $2,109,671, compared to $1,661,448 for the project without options. This yields an incremental value of $448,223 derived from both expansion and abandonment options.

Finally, the code plots the ex-post dividends lattices with and without options so that we can have a better understanding of the results, as shown in Figure 2.6:

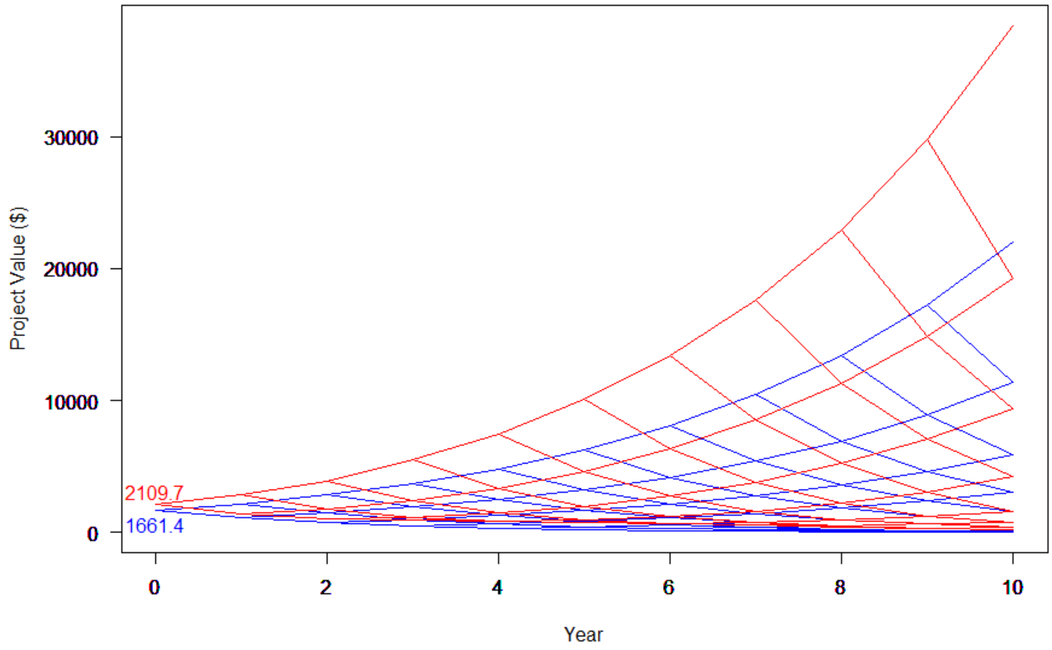

Figure 2.6 – Ex post lattices with and without abandonment and expansion options

Note: This figure shows the lattice ex-post dividends considering the abandonment and expansion options (*) and the lattice ex-post dividends without options (*). The values are in $ thousands.

In addition, we also provide the command to plot the same lattices, but on a log scale. This allows better visualization of the results found especially the exercise boundaries of the Expansion (Call) and Abandonment (Put) options, as shown in Figure 2.7:

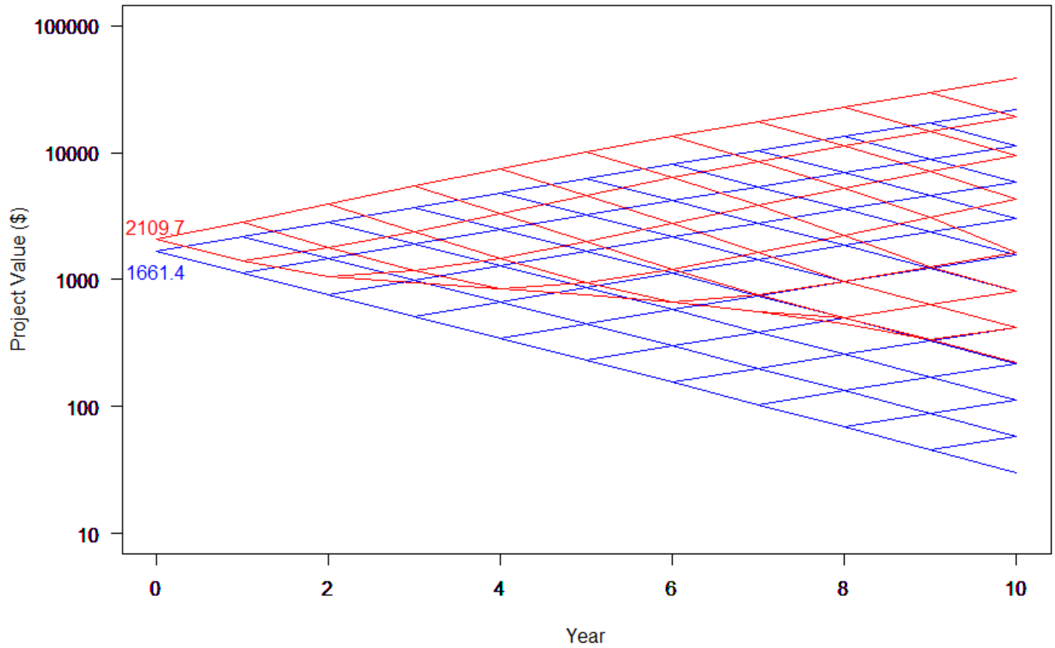

Figure 2.7 – Ex post lattices with and without abandonment and expansion options in log scale

Note: This figure shows, in log scale, the lattice ex-post dividends considering the abandonment and expansion options (*) and the lattice ex-post dividends without options (*). The values are in $ thousands.

2.4.2. Discussing the Proposed Code

The proposed code was built in a very intuitive and simple way so that even beginning researchers can use it in their analysis. However, we believe it is important to provide a general guideline for its correct use. Thus, we draw the reader's attention to a few points. First, the code only models annual cash flows, that is, it only allows the time interval to be equal to 1 ($dt = 1$). Another point is that only the values listed in Tables 2.1, 2.2 and 2.3 can be changed by the users to adjust them to the reality of the projects they are modeling. To stress this point, the following comment: "Parameters - Here, you can change the input values to suit your project" was included in the appropriate places in the code. Finally, we emphasize that readers should not be concerned if they use the same input values as ours and find output values slightly different from those presented in our article. This variation is common, as the output values are the result of a Monte Carlo simulation.

2.5. Conclusions

This tutorial provides a guide on the sequence of steps required to implement a real options model, based on the estimation of project cash flows. It presents students, researchers and practitioners the correct real option procedures to calculate the volatility of a project's value; incorporate the project's cash flows into the CRR binomial model using the cash flow dividend rate; and model the managerial flexibilities (real options) of the project. It uses the Copeland & Antikarov (2001) scheme, incorporating the dividends or cash flows paid out, into a CCR Lattice modeling, and is adaptable enough to reproduce a great number of managerial flexibilities available to managers.

We believe this tutorial to be relevant for real options students, researchers and practitioners, as it contributes to the understanding of project cash flow lattice modeling. It provides a simple and practical method for the pricing of real options that can assist decision-makers to analyze investment opportunities in projects where there is uncertainty and flexibility.

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3
Crossing the Brownian Bridge: valuing infrastructure capacity expansion policies as real options

In countries where transportation infrastructure is underdeveloped, newly built facilities tend to attract and increase demand. This can lead to situations where future traffic levels exceed the concession capacity limit, and additional investments in expansion is required. One common solution is to mandate this investment as a firm obligation in concession contracts, either after a set number of years or when demand reaches capacity. In this article, we show why these policies are suboptimal and propose a model that combines flexible capacity expansion decisions with conditional term extensions. We model this flexibility under the real options approach and the project value uncertainty during the life of the concession as a Brownian Bridge. As a novel contribution, we take into account the fact that concession revenues are bounded by the current traffic capacity of the road, which represents an upper absorbing barrier that has implications for the expansion decision. As a numerical application, this model is applied to a typical toll road project in Brazil. The results show that flexible expansion policies, coupled with conditional term extensions, have significant advantages. These findings can be of use to government officials involved in developing policies to attract private investment in public infrastructure projects.

3.1.
Introduction

The process of granting public infrastructure projects to the private sector is a worldwide trend since this spares the government from having to allocate scarce public resources to projects that have the potential to be privately funded (Takashima, Yagi, and Takamori, 2010). Government concessions, encompassing both PPPs (Private Public Partnerships) and BOT (Build-Operate-Transfer) projects, are usually granted in the form of inflexible contracts with a pre-established time term during which the objective of the private party is to earn a return on the invested capital, while the government's objective is to increase the value of the concession, inducing the anticipated investment in expansion when the capacity limit is exceeded to improve the quality of services rendered to the public. Although these objectives may differ, both parties can profit from this type of arrangement. The private party will seek to maximize profits within the limits allowed by the contract. At the same time, the government must regulate and verify the quality of the services that are being provided. Therefore, a well-designed contract should be able to align the incentives of all stakeholders in favor of the successful development of the concession.

Designing such a contract is challenging. While the rules and obligations of each party must be contractually defined at the start, it is difficult to foresee all possible future scenarios that may occur during the tenure of a typical 25-year or longer concession. Additionally, long-term future traffic levels are particularly difficult to predict, and there are risks derived from lower than expected traffic levels (Adkins, Paxson, Pereira and Rodrigues, 2019; Galera, Soliño and Abad, 2018; Brandão and Saraiva, 2008). On the other hand, in countries where transportation infrastructure is lacking or underdeveloped and comprised mostly of simple two-lane undivided roads, newly built facilities tend to attract and increase demand (Iyer and Sagheer, 2011). This can lead to situations where future traffic levels exceed the capacity limit of the concession, which may require further investments in capacity expansion that typically involve doubling the capacity by transforming the original two-lane road into a four-lane divided highway.

It may seem reasonable to include this capacity expansion as a hard obligation in concession contracts. On the other hand, investment in capacity expansion under uncertainty is one of the most critical decisions for a firm (Lu and Meng, 2017; Trigeorgis and Tsekrekos, 2018). Thus, given that future demand is uncertain, it is unlikely that the concessionaire will proceed with the expansion if the expected demand does not materialize, as it will not be able to earn a return on this investment. This has been the case of several Brazilian federal road concessions totaling 2,683 km auctioned in 2013 (ANTT, 2019; Valor Econômico, 2019). Thus, traffic risk considerations may preclude the government from achieving its stated goals. In addition, even if demand does increase in time, the likelihood of the investment in capacity expansion decreases as the concession term approaches the end, as the remaining cash flow streams may be insufficient to cover the investment costs.

The value of a concession project subject to a fixed contract term *n* under demand uncertainty can be modeled as a Brownian Bridge, which is a stochastic process that is pinned at both ends (Metwally and Atiya, 2002). At time $t = 0$, V_0 represents the expected value of the project, while the value V_n at the end of contract term at time $t = n$ is zero, as no more cash flows are expected after this time.

The fact that the Brownian Bridge converges to zero at the end of the contract is one of the reasons why investment in capacity expansion during the concession term tends not to add value to the concession project. Expansion typically does not occur during the early years of the project as demand is still low. Then, in the later years, when demand eventually does exceed capacity, the number of years remaining until the end of the contract term may be insufficient to justify the investment as the Brownian Bridge value goes to zero.

In this article, we analyze the problem of capacity expansion in concession projects considering term extensions, which allow the concessionaire to “cross” the Brownian Bridge and propose a model that allows the testing of different public policies. We adopt the real options approach to determine optimal policies for the timely expansion of the concession facilities by the private investor. Our model also takes into account the fact that concession revenues are bounded by the traffic capacity of the road, which represents an upper absorbing barrier and limits the potential revenues of the project, with implications for the expansion decision.

This research contributes to the literature on real options valuation of concession contracts in several ways. First, we incorporate into the real option valuation model the significant impact that capacity limits have on the value of the project, which cannot be determine under traditional DCF methods. Second, we show how the option to expand capacity coupled with a term extension increases the probability of a timely and voluntary expansion. Finally, we show that this model allows the granting authority to design low-cost contract clauses that align the incentives of all stakeholders so that an early capacity expansion is in the best interest of the private investor. To the best of our knowledge, neither the real options traffic capacity limit model nor the issue of capacity expansion policies linked to the concession period extension has been addressed in the literature.

This article is organized as follows. After this introduction, we present a brief review of the related literature in the field. Then, we develop a real options model to evaluate different policies for capacity expansion with and without term extensions in concession projects. Afterward, we present a numerical application of this model to a road concession project. Finally, we discuss the results and conclude.

3.2. Literature review

A. The Real Options Approach (ROA) arose as a response to the limitations of the traditional Discounted Cash Flow model (DCF). This approach builds on the financial options pricing models initially developed by Black and Scholes (1973) and Merton (1973) to expand the use of these models to the treatment of investments in real assets under uncertainty and flexibility.

In the literature, much is still discussed about the practical value of ROA (Smith and Nau, 1995; Ford and Lander, 2011). Garvin and Ford (2012) demonstrated the potential of real options regarding the management of uncertainties and flexibilities in infrastructure projects. On the other hand, they also highlighted the existence of barriers that prevent or limit the widespread adoption of ROA, such as project managers' risk aversion, lack of adequate resources to fully explore the real options, and agency problem.

Triantis (2005) reaffirmed that there are barriers between theory and practice in the use of real options models. The author also pointed out that for ROA to become the main tool for evaluating large capital investment projects, it is necessary: on the theory side, to use more realistic models and easy-to-implement heuristic methods; and, on the practical side, to spread the knowledge of real options in all organizations, to align management incentives and to develop user-friendly real options software, as the one proposed by Marques, Bastian-Pinto, and Brandão (2020).

Despite the limitations of this method and the existing barriers to its wide dissemination, Fleten, Linnerud, Molnár, and Nygaard (2016) showed that even without using the ROA, energy project managers in Norway intuitively make the same investment decisions as indicated by the real options models. Their findings showed that ROA is able to define the optimal decision, which is different from deterministic models (DCF), even when the manager does not use it explicitly. Given this evidence, the authors concluded that ROA has practical relevance.

Besides, over the years, ROA has found several applications in infrastructure projects, such as transportation, roads, airports, and highways. Bowe and Lee (2004) used ROA to analyze the construction of the Taiwan High-Speed Rail project, considering expansion, deferral, abandonment, and contraction options. Their results suggested that managerial flexibility in the face of unexpected market developments is relevant in determining the economic feasibility of the project. Huang and Chou (2006) complemented the analysis performed by Bowe and Lee (2004), considering the Minimum Revenue Guarantee (MRG) risk mitigating mechanism and the option to abandon during the project pre-construction phase.

Attarzadeh, Chua, Beer, and Abbott (2017) also addressed the issue of MRG by calculating equitable bounds for the guaranteed revenue and used fuzzy logic tools to model the cash flow uncertainty. In the same line, Carbonara, Costantino, and Pellegrino (2014) developed a model to assist the government in setting a fair revenue guarantee level which balances the private sector’s profitability needs and the public sector’s fiscal management interests and applied it to the Camionale di Bari toll road in Italy.

Feng, Zhang, and Gao (2015) developed a model to evaluate a Minimum Traffic Guarantee (MTG) and price compensation guarantee. In addition, the authors determined the optimal toll price on highway projects and verified the impact of government guarantees on toll collection, highway capacity, and road quality. On the other hand, instead of proposing a revenue guarantee put option, which has a limitation due to an upfront premium payment requirement, Shan, Garvin, and Kumar (2010) suggested a collar option to improve the effectiveness of risk management in a real toll project and to redistribute downside losses and upside profits to each stakeholder.

Another article that proposed a model to evaluate a collar option in a toll road project was that of Buyukyoran and Gundes (2018). They considered future traffic demand as the most critical risk factor to affect the financial viability of the project and used a simulation model to identify the optimum upper and lower boundaries of options. Their results showed that these optimum values contribute to limit the government's contingent liabilities. Zhao, Sundararajan, and Tseng (2004) also used ROA to evaluate highway planning policies and proposed a model that considers traffic demand, land price, and highway deterioration as uncertainties. Their findings indicated that the model supports optimal decision-making regarding the development, operation, expansion, and rehabilitation of highways.

Several authors analyzed the issue of flexible contract terms. Engel, Fischer, and Galetovic (2001) were the first to show that fixed-term concession contracts do not allocate demand risk optimally and proposed the Least Present Value of Revenue auction model (LPVR). This model optimally hedges the revenue uncertainty faced by the concessionaire through a flexible contract term where the concession term is extended if demand is lower than expected and decrease otherwise. Contreras and Angulo (2018) used ROA to determine the opportunity cost to the government of concession term extensions and concluded that these costs might be high in some cases.

Lv, Ye, Liu, Shen, and Wang (2015) proposed a real options model to determine the ideal concession term, considering the investment in transportation projects under dynamic uncertainty. They argued that their model can determine the optimal interval for the length of a concession period, leaving the specific concession term period to be negotiated between the parties involved in the project. In a similar vein, Wu, Wing Chau, Shen, and Yin Shen (2012) also considered that the concession period is a key decision variable in the arrangement of concession projects. The authors argued that, under a concession agreement, the project´s net assets may have significant value at the time of their transfer to the government. Therefore, developing a model that takes this into account may be interesting for the optimal investment decision.

Gryglewicz, Huisman, and Kort (2008) studied investment projects with a finite project life to revisit the outcome of the ROA to investment under uncertainty, which states that increased uncertainty raises the value of waiting and, therefore, decelerates investment. Their results suggested that an investment project with a finite life in combination with a risk premium on expected rates of return can reverse the usual effect of uncertainty on irreversible investments. This finding is relevant since finite terms are one of the main characteristics of infrastructure concession projects. In a more recent study, Jin, Liu, Sun, and Liu (2019) not only addressed the problem of optimizing the revenue guarantee level but also the length of the concession period to meet the interest of public and private parties in concession contracts. Through an imperfect information trading model and ROA, they showed that the length of the concession period is inversely proportional to the revenue guarantee level, and this correlation is influenced by the likelihood of reaching the equilibrium return rate of the investment.

Sánchez-Silva (2019) considered the capacity expansion of a project, arguing that flexibility is a central element for the successful development and operation of infrastructure projects. The purpose of this article was to use a ROA model to determine the optimal time to expand a project to maximize its value. This model was applied to two numerical examples, and the results showed that incorporating flexibility into infrastructure projects is beneficial for public and private agents, as this can generate a significant increase in the value of these projects. While their focus was on the need to incorporate managerial flexibility into these projects to enhance value, their findings indicated that expansion is more likely to occur with long contract terms of more than 30 years.

In a study closer to ours, Krüger (2012) analyzed the option to expand an existing two-lane road in Sweden in a concession and examined how ownership affects the decision to expand. They conclude that when the value of the option to expand is considerable, it is optimal for the concessionaire to delay the expansion, which may require public ownership to ensure a social-optimal outcome.

This article differs from the literature in the following ways. First, we develop a real options model that allows governments to determine low-cost optimal expansion policies that consider term extensions. Second, we show that capacity limitations in infrastructure projects have a significant negative impact on the value of the project, which is not captured by current models. By incorporating both of these features into our model, we show why current expansion policies for concession projects involving term extensions are suboptimal and how they can be modeled in order to achieve the desired outcome, which, to the best of our knowledge, is an original contribution.

3.3. Model

We propose a model for the investment decision in BOT road concessions that considers a combination of different road capacity expansion policies. As is standard in the literature, we consider that the primary source of uncertainty that affects the private agent investment returns, and investment decision is the traffic demand (Lu and Meng, 2017).

When determining total revenues in road concessions, it is common to assume that the toll is charged in both directions. A multiplying factor (EVM – Equivalent Vehicle Multiplier) is used to normalize the traffic data among cars and freight vehicles. The total revenues in year *t* are then defined by equation (3.1):

$$R_t = D_t \times T \times EVM \tag{3.1}$$

where R_t is the total revenues in year *t*; D_t is the traffic demand in year *t*; *T* represents the toll rate, which we assume constant, and *EVM* is the Equivalent Vehicle Multiplier Factor. From these definitions, we can determine the cash flows in each year with (3.2):

$$F_t = \left[R_t \left(1-\gamma_t\right) - \delta - \Gamma \right]\left(1-\pi\right) + \delta \tag{3.2}$$

where γ_t represents the variable cost ratio related to R_t; π is the income tax; Γ represents the fixed costs and δ is the depreciation, which is an annual capital expenditure for the operational maintenance of the infrastructure. To simplify the cash flow equation, we can express the project cash flows as a function of the demand *D*, as shown in equation (3.3):

$$F_t = f\left(D_t\right) \tag{3.3}$$

The value of a concession project that has a demand *D* as the main uncertainty can be determined with equation (3.4) as shown:

$$V_0 = \int_{t=1}^{n} f\left(D_t\right) e^{-kt} dt \tag{3.4}$$

where *V* is the present value of the concession project at time $t = 0$; *k* is the cost of capital (WACC), and n is the concession term.

As demand for infrastructure capacity tends to be correlated to GDP growth in developing countries (Brandão, Bastian-Pinto, Gomes, and Labes, 2012; Irwin, 2007), we assume that the demand follows a Geometric Brownian Motion (GBM), as shown in equation (3.5).

$$dD_t = \mu D_t dt + \sigma_D D_t dz_t \tag{3.5}$$

where dD_t is the incremental variation of demand in the time interval *dt*; μ represents the expected growth rate of demand; σ_D is the demand volatility; and $dz_t = \varepsilon\sqrt{dt}$ represents the standard increment of Wiener, where $\varepsilon \approx N(0,1)$.

Although the use of GBM is standard in the literature of stochastic traffic modeling, Zapata Quimbayo, Mejía Vega, and Marques (2019) argued that road traffic demand may show a mean reversion behavior. However, in their case, this occurs because the data was presented on a monthly basis, which shows a seasonal characteristic of the series and, consequently, a mean reversion behavior. Since we assume that the seasonal characteristic is not present when considering yearly time increments, we assume that we can model this uncertainty as a GBM diffusion process.

Then, we consider the opportunity to expand capacity as a non-perpetual American option, as the exercise period is limited to the concession contract term, and use the discrete binomial tree model of Cox, Ross, and Rubinstein (1979) (CRR) to price this option. We initially model a demand lattice and then determine the equivalent cash flow lattice with equation (3.3). The basic nodes of these lattices are shown in Figure 3.1.

While one would expect that the investment in expansion will occur when traffic demand reaches roadway capacity, this is not the case as the trigger for expansion is not the traffic level but the economic feasibility of this investment, which is driven by the project cash flows. Thus, both the demand and the cash flow lattices are needed to determine the optimal investment decision.

The model parameters for the CRR model are presented in equation (3.6):

$$u = e^{\sigma_D \sqrt{\Delta t}},\ d = \frac{1}{u} \text{ and } p = \frac{\left(1+r_f\right)^{\Delta t} - d}{u-d} \tag{3.6}$$

where u and d are, respectively, the upside and downside multiplying factors; p is the risk-neutral probability; σ_D is the demand volatility; r_f is the risk-free rate; and Δt the discrete-time increment.

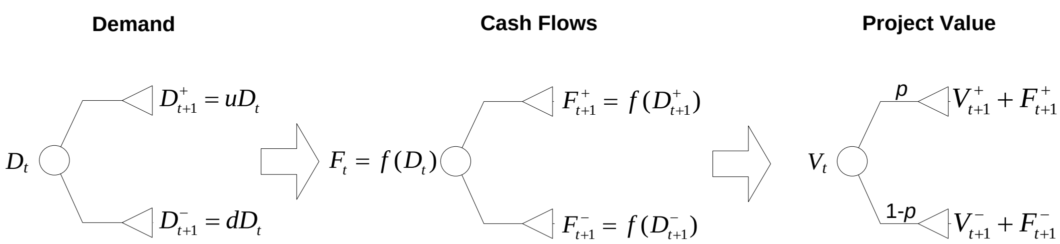

Figure 3.1 – Demand, Cash Flows and Project Value lattices

The CRR model considers that, at each decision node, the stochastic variable $\tilde{D}$ is modeled by equation (3.7).

$$D_t^+ = D_{t-1} e^{+\sigma_D \sqrt{\Delta t}} \\ D_t^- = D_{t-1} e^{-\sigma_D \sqrt{\Delta t}} \tag{3.7}$$

where D^+ and D^- are the ascending and descending random values that are modeled within the cash flow tree.

Additionally, this option pricing model requires the use of the risk-neutral measure that can be determined by deducting the risk premium from the asset's rate of return and then discounting cash flows at the free risk rate. Thus, the risk-neutral process of demand is defined by equation (3.8):

$$dD_t^R = \left(\mu - \zeta_D\right) D_t^R dt + \sigma_D D_t^R dz_t \tag{3.8}$$

where ζ_D represents the demand risk premium; μ is the rate of return of the demand; and, dD_t^R is the incremental variation of the risk-neutral demand in the time interval *dt*.

Following Freitas and Brandão (2010), the demand risk premium ζ_D is determined by considering that the present value of project without any options under a risk-neutral valuation must be strictly equal to the expected present value provided by the traditional risk-adjusted static valuation, as shown by equation (3.9) :

$$\int_{t=1}^{n} f\left(\tilde{D}_t\right) e^{-\mu t} dt = \int_{t=1}^{n} f\left(\tilde{D}_t^R\right) e^{-(\mu - \zeta_D)t} dt \tag{3.9}$$

Thus, this model will consider the risk-neutral probability p^* defined in equation (3.10):

$$p^* = \frac{\left(1 + \mu - \zeta_D\right)^{\Delta t} - d}{u - d} \tag{3.10}$$

and equation (3.11) to determine the risk-neutral present value of the project.

$$V_t^R = \int_{t=1}^{n} E\left[f\left(\tilde{D}_t^R\right)\right] e^{-r_f t} dt \tag{3.11}$$

The lattice that represents the project value (V) uncertainty over time is determined by discounting the probability-weighted future cash flows of the cash flow lattice. Thus, if the concession term is n, then the project value at time $n - 1$ can be determined by equation (3.12):

$$V_{n-1} = \left[F_n^+ p^* + F_n^- \left(1 - p^*\right)\right] \Big/ \left(1 + r_f\right) \tag{3.12}$$

Generalizing this process for $t < n - 1$, we arrive at equation (3.13).

$$V_{t-1} = \left[\left(F_t^{+} + V_t^{+}\right)p^{*} + \left(F_t^{-} + V_t^{-}\right)\left(1 - p^{*}\right)\right] \Big/ \left(1 + r_f\right) \qquad (3.13)$$

This process can be observed in Figure 3.1. As the value lattice ends at time n with no continuation value, it will converge to zero in the end in the case of a fixed time term grant, and the Brownian Bridge will be pinned at that value.

We use this framework to develop a valuation model, described in the next subsection, for the traditional flexible capacity expansion policy where the concessionaire receives no additional incentives to expand, which effectively restricts the duration of the contract to the original term represented by the Brownian Bridge.

3.3.1. Flexible Capacity Expansion

Consider a fixed-term concession with a demand capacity limit D_{max1}, where the concessionaire has the option to expand capacity at any time. There will be an absorbing barrier or cap, for $\tilde{D}$ should demand rise above D_{max1}. As the cash flows generated by the project are a direct function of $\tilde{D}$, these will also be limited to an upper level: $F_{\max 1} = f\left(D_{\max 1}\right)$. So, if $F_t > F_{max1}$, then $F_t = F_{max1}$.

Denoting the time t project value with an expansion option by $V_{t_{\exp}}$, the value of this option is conditioned to the optimal exercise of the expansion. In this sense, the expansion option value is expressed by equation (3.14):

$$V_{t_{\exp}} = \max\left[V_t^{R}; V_t^{*} - I_t\right] \qquad (3.14)$$

where V_t^{R} represents the risk-neutral present value of the project at t without considering any options; V_t^{*} is the present value of the project cash flows after expansion at time t, and I_t represents the expansion *CAPEX* (Capital Expenditure). The project cash flows, after expansion, are no longer limited to the original maximum road capacity limit Dmax1, but to a new higher capacity limit (D_{max2}), which depends on the expansion *CAPEX*. This new cap will limit the expanded cash flows F_t^{*} to: $F_{\max 2} = f\left(D_{\max 2}\right)$, and $F_{\max 1} < F_t^{*} < F_{\max 2}$. We will also consider that there is a time to build for the expansion of one period. Therefore, cash flows of the expansion will be “delayed” for one period. The value of the expanded project at time *t* can be determined by equation (3.15).

$$V_t^* = \int_{\tau+1}^{n} E\left[f\left(\tilde{D}_t\right)\right] e^{-kt} d\tau + F_{\max 1} \tag{3.15}$$

Under the discrete model, we use equation (3.16) to determine the present value of the expanded cash flows at time *t*. These will grow at a rate μ for $n - (t + 1)$ years and are discounted at k up to time *t*. Equation (3.16) already considers that there is a one year time to build up the expansion capacity, during which the project will receive the maximum level of cash flow prior to expansion: F_{max1}.

$$V_t^* = \frac{F_t^*}{(k-\mu)(1+k)}\left[1-\left(\frac{1+\mu}{1+k}\right)^{n-(t+1)}\right] + F_{\max 1} \tag{3.16}$$

To estimate the expanded value of the project at time $t = 0$, we calculate the demand and cash flow lattices following Figure 3.1 up to time *n*, then discount the present values with equation (3.13) and exercising the options with equation (3.14) , up to time $t = 0$.

Next, we extend this framework to optional capacity expansion policies with term extension incentives where the Brownian Bridge is crossed.

3.3.2. Crossing the Brownian Bridge: Flexible Capacity Expansion with Term Extension

We now consider a roadway concession where the concessionaire earns a term extension if it decides to invest in capacity expansion. To model this policy, as previously, we adopt the CRR binomial approach, considering this as an American call option. In order to price this option, we consider again equation (3.14), where the present value of the expanded cash flows at time *t* is changed from equation (3.16) to (3.17):

$$V_t^* = \frac{F_t^*}{(k-\mu)(1+k)}\left[1-\left(\frac{1+\mu}{1+k}\right)^{\varpi-1}\right] + F_{\max 1} \tag{3.17}$$

where ϖ represents a number of years that varies according to the policy adopted. As described in the previous section, we consider that there is a time to build for the expansion of one period.

3.4. Numerical Application: Highway Concession Projects in Brazil

A concession agreement is a contract between a public sector authority and a private entity through which a project is constructed to provide a service directly to the public or a public authority. Concessions include services in several sectors, such as highways, railways, hospitals, airports, and ports. In Brazil, the practice of granting highway concessions became more common in the 1990s when the Federal Highway Concession Program was created (ANTT, 2019).

The National Land Transportation Agency (ANTT) currently manages 20 highway concessions, totaling approximately 9,697 km. The first round of road concessions began in 1993 and consisted of six road sections, covering 1,315.9 km. The second round began in 2007. In its first phase, seven lots of federal highways totaling 2,624.4 km were granted, while the second phase involved a single lot of federal highways with 680.6 km. The third round began in 2012 and granted eight highways. However, one of them was canceled due to the concessionaire's failure to fulfill its commitments. The fourth and final round occurred in January 2019 and granted a single road section of 473.4 km. Table 3.1 summarizes the main information on granted highways in Brazil during this period.

Round	Roads	Extension (km)	Contract Start	Concession Term (years)	Contract Termination
1	Ponte Rio-Niterói	13.2	1995	20	2015[a]
1	Osório – Porto Alegre	121.0	1997	20	2017[a]
1	BR-040/MG/RJ	179.9	1996	25	2021
1	BR-116/RJ	142.5	1996	25	2021
1	BR-116/293/RS	457.3	1998	28	2026
1	BR-116/RJ/SP	402.0	1996	25	2021
2	BR-381/MG/SP	562.1	2008	25	2033
2	BR-101/RJ	320.1	2008	25	2033
2	BR-376/PR - BR-101/SC	405.9	2008	25	2033
2	BR-116/PR/SC	412.7	2008	25	2033
2	BR-116/SP/PR	401.6	2008	25	2033
2	BR-393/RJ	200.4	2008	25	2033
2	BR-153/SP	321.6	2008	25	2033
2	BR-116/324/BA	680.6	2009	25	2034
3	BR-060/153/262/DF/GO/MG	1,176.5	2014	30	2044
3	BR-050/GO/MG	436.6	2014	30	2044
3	BR-101/ES/BA	475.9	2013	25	2038
3	BR-101/RJ	13.2	2015	30	2045
3	BR-153/TO/GO	624.8	2014	30	2017[b]
3	BR-163/MS	847.2	2014	30	2044
3	BR-163/MT	850.9	2014	30	2044
3	BR-040/DF/GO/MG	936.8	2014	30	2044
4	BR-101/290/386/448/RS	473.4	2019	30	2049

Table 3.1 – Highways Granted in Brazil (1993 – 2019)

Note: [a]concession contracts already terminated; [b]concession contract canceled.
Source: ANTT (2019).

Three of the contracts established in the first round had a 25-year term and, consequently, will be terminated in 2021. One of these concessions is the BR-116/RJ/SP highway that was granted in March 1996 to CCR Nova Dutra, which is responsible for 402 km and six toll plazas (CCR, 2019). This highway links São Paulo to Rio de Janeiro, the two largest metropolitan regions of the country. Currently, it presents significant congestion along most of its route, which will require future capacity expansion. Another concession that will end in 2021 is the

BR-040/MG/RJ highway, which links Rio de Janeiro to Minas Gerais, two of the most important Brazilian Southeast states. This highway was granted in March 1996 to concessionaire CONCER, which is responsible for 179.9 km and three toll plazas (CONCER, 2019), but ran into severe problems after expansion works in a new section were interrupted in 2016.

In addition to the end of these early concessions, it is expected that several new concessions will become available in the country in the coming years, such as the concession of the 250 km Rio-Santos highway. According to ANTT (2019), since the concession of the BR-116/RJ/SP highway will end in 2021, one of the ideas under consideration is to combine these two roads that connect São Paulo and Rio de Janeiro into a single concession contract. In this sense, when renewing the concession contract with CCR Nova Dutra concessionaire, the government would require counterpart investments in the duplication of the Rio-Santos highway.

There is still much discussion on whether it is best to extend the term of these concessions or grant them again through a new bid process (Rocha and Marques, 2023). Nonetheless, many of these concessions will demand capacity increases during their tenure, and the issue of how to best incorporate this requirement in the concession contracts remains open.

3.4.1. Base Case

For our base case scenario, we consider a hypothetical 25-year road concession project using typical industry parameters in Brazil, which are listed in Table 3.2.

Contract term (*n*)	25 years
CAPEX	USD 300 million
Expansion CAPEX	USD 150 million
Fixed cost (Γ)	USD 32 million per year
Variable cost (γ)	35% of revenues
Tax rate (π)	34%
Depreciation (δ)	USD 12 million
Tariff	USD 8.20 per vehicle
Risk-free rate (r_f)	4.08% per year
Risk-adjusted rate (*k*)	7.25% per year
Daily maximum road capacity	20,000 vehicles
Yearly Maximum Road capacity (D_{max1})	7,300,000 vehicles

Table 3.2 – Concession parameter and data based on a typical 400 km toll road in Brazil

As explained earlier, we model traffic demand as a GBM diffusion process. Figure 3.2 exhibits binomial lattice representation of this uncertainty as well as the road capacity limitation, using the values and parameters defined in Table 3.3, where we assume that road capacity is approximately double the current traffic levels.

Initial daily demand (in t = 0)		10,000 vehicles
Initial Annual demand	D_0	3,650,000 vehicles
Equivalent Vehicle Multiplier	*EVM*	2.2
Demand drift (growth)	μ	5% (per year)
Demand Volatility	σ_D	13.38% (per year)
Demand risk premium	ζ_D	1.88% (per year)

Table 3.3 – Stochastic Demand values and parameters

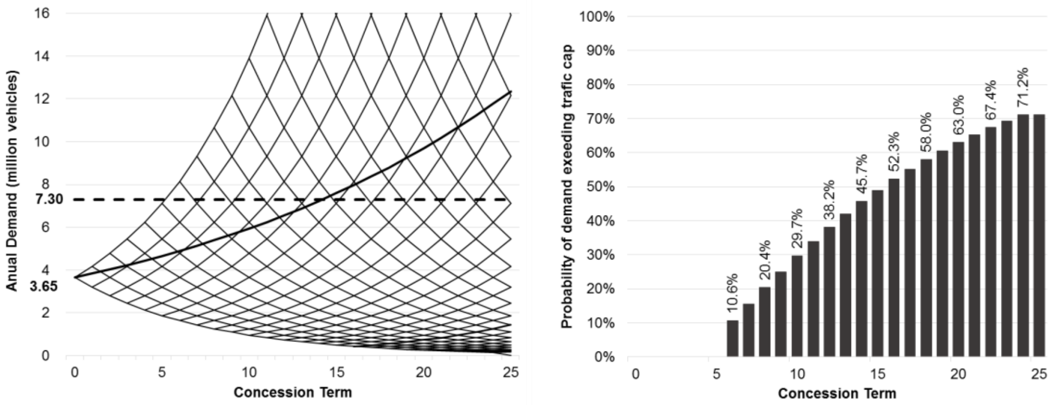

Figure 3.2 – Annual stochastic demand projection

Note: Figure 3.2 shows expected demand starting at 3.65 million vehicles and capacity limit of the roadway of 7.3 million vehicles (left); and the probability in each year that demand exceeds the road capacity limit of 7.3 million vehicles (right).

Note that after the fifth year of the concession, there is a probability that demand exceeds the road capacity limit of 7.3 million vehicles.

The values and parameters of Table 3.2 and Table 3.3 are used in equation (3.4) to determine project cash flow estimates. Following the DCF method, we discount these future cash flows at the risk-adjusted rate k of 7.25%, which provides an expected value of the project at time $t = 0$ of \$363.4 million. The Net Present Value (NPV) considering the *CAPEX* of \$300 million is then \$63.4 million.

This value can also be found by modeling the traffic demand as a GBM with a CRR lattice, determining the corresponding project cash flows for all demand levels using equation (3.4), and then discounting these at the project risk-adjusted rate. Alternatively, this procedure can also be done under the risk-neutral measure using risk-neutral probability defined in equation (3.10) and discounting the cash flows at the risk-free rate. As expected, both approaches provide the same project value as the DCF method as this model presents no flexibility, but only the risk-neutral approach can be used for option-pricing purposes. The corresponding project value lattice is shown on the left of Figure 3.3. This type of project value lattice is the discretization of a Brownian Bridge, as the concession has a term limit, after which there are no more cash flows. Thus, the project value will forcefully become zero at the expiration of the term (year 25) after the last cash flows of the project are paid out.

On the other hand, this analysis assumes that all future traffic demand will be fully met by the concession. Given that actual traffic is limited to the demand capacity of the roadway, as shown in Table 3.2 and Figure 3.2, total yearly traffic is limited to 7.3 million vehicles per year. Thus, when we consider this absorbing barrier and equation (3.13), the value and the NPV of the base case scenario is reduced to $301.4 million and $1.4 million, respectively. Like the previous results, these can also be found both through the DCF method and the CRR lattice approach since we have not yet considered any managerial flexibility.

The evolution of the project value lattice, considering this capacity limitation, is shown on the right of Figure 3.3. One can note that there is a significant difference in value when the absorbing barrier of the capacity limit is taken into account. This effect with be considered in all our subsequent analysis.

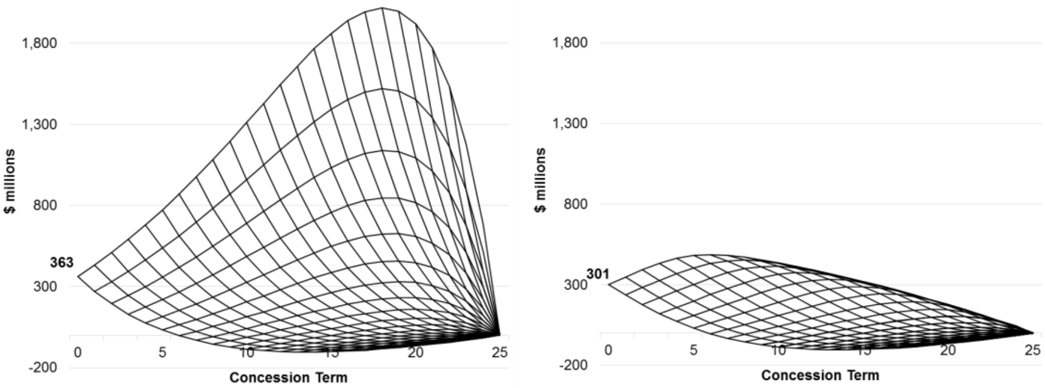

Figure 3.3 – Project value lattice

Note: Figure 3.3 shows the project value lattice without demand capacity limit (left) and the project value lattice with demand capacity limit (right).

As was the norm for most concessions in the 1990s and 2000s, the base case scenario does not incorporate any expansion of the capacity of the roadway. Next, we will analyze different policies that include the requirement for expansion.

3.4.2. Firm Mandatory Capacity Expansion

Seven federal highways concessions totaling over 3,500 km of roadways were auctioned in 2013 in Brazil with the obligation of doubling the capacity of the existing two-lane roads within five years (ANTT, 2019) independent of the traffic level. As this was a mandatory contractual clause with a fixed exercise date, we denote this as a European obligation. We add this obligation to our base case scenario and determine the project value lattice and the project NPV for this case.

We assume that the investment required to increase the daily roadway capacity limit to 40,000 vehicles is $150 million, therefore doubling the capacity at the cost of 50% of the original investment, which can be considered a conservative estimate. Using the same approach described previously provides the lattice shown on the left side of Figure 3.4, where the evolution of the project value considering a mandatory expansion in year 5 is presented. It shows that with this obligation, the project becomes less attractive, and the project value decreases by $60.6 million to $231.2 million. Note that this policy does not have any managerial flexibility, as it is an obligation with a fixed exercise date. In this sense, the traditional DCF method would report the same results found by ROA.

The same basic parameters shown in Table 3.2 and Table 3.3 were used, including the expected traffic growth rate of 5% per year. However, if the growth rate is lower, the results will decrease even further. This was the case of the 2013 auction, where only 618 km, or 17% of the total, had been duplicated by the 2019 deadline. In five of these concessions, the expansion works have been suspended, and one of the contracts has been revoked for non-performance (Miranda, 2019).

3.4.3. Conditional Mandatory Capacity Expansion

We now consider a policy where the concessionaire is only required to commit to a capacity expansion if traffic levels reach or surpass roadway capacity. The investment trigger is then the traffic level, and once this trigger is activated, the expansion is mandatory.

We define this arrangement as an American obligation. An American Obligation is equivalent to a short Put where the government has the right to hold the concessionaire responsible for the CAPEX costs or exercise price if traffic demand exceeds the exercise level, which is the road capacity. Thus, while the decision to exercise is based on traffic levels, the exercise cost is based on project cash flows.

Considering the same parameters and procedure as before, the present value of the concession project using the deterministic cash flow estimates assuming that the expansion will take place when traffic demand reaches capacity in year 14, we arrive at a project value of $291.8 million. Nonetheless, given that future traffic levels are uncertain, there is no guarantee that the capacity limit will be achieved by year 14. Thus, the exact expansion date or hitting time will vary. The stochastic valuation model (CRR lattice), which takes into account the uncertainty in the traffic levels, provides the correct project value of $256.3 million, which is $35.5 million lower than the deterministic value of $291.8 million provided by the DCF method. This is due to the fact that the distribution of the hitting times is not symmetric, and while the exercise date is uncertain, the expansion is mandatory when it occurs. This also represents a reduction compared to the base case scenario, although less so than in the previous inflexible case. This suggests that the concessionaire will only make this investment if it is mandatory, in which case it must factor in these costs in its bid price during the auction process. The stochastic evolution of the project value, also considering the upper absorbing barrier (cap), is shown on the right side of Figure 3.4.

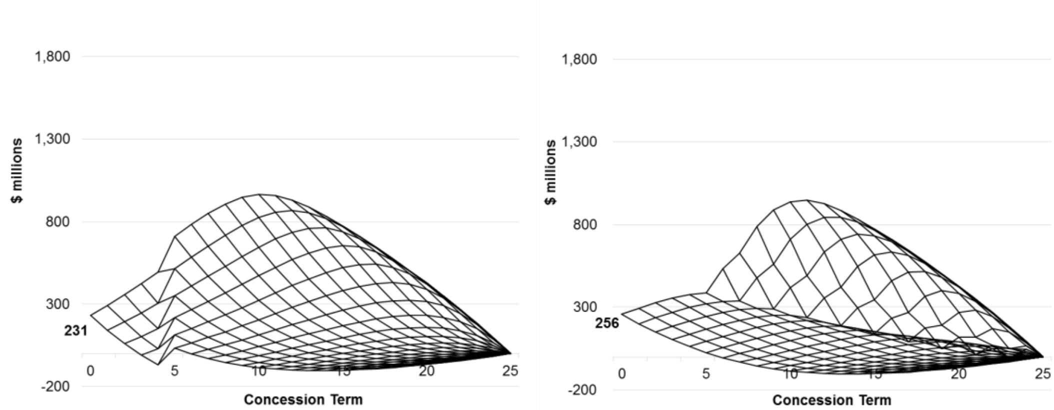

Figure 3.4 – Project value lattices with a mandatory duplication

Note: Project value lattice with demand capacity limit considering a mandatory duplication of the roadway in year 5 (left) and project value lattice with demand capacity limit considering a mandatory duplication of the roadway only when traffic levels reach roadway capacity (right).

3.4.4. Optional Capacity Expansion

The mandatory expansion policies, which are standard contract clauses that have been widely used in toll road concession contracts in Brazil, suggest that the project value will decrease under these policies. While these results may vary for different parameters, the assumptions adopted in our model, such as traffic growth rate and expansion costs, are conservative and likely to favor the expansion, but even so, are shown to be insufficient.

We now consider the case where capacity expansion is optional rather than mandatory. We evaluate this policy using the CRR option pricing approach, considering that the concessionaire´s investment decision will be based simply on whether the expansion will create value for the firm, as described in equation (3.14) . This implies that from the point of view of the concessionaire, the decision trigger is whether the project value increases with the expansion, and not if traffic demand had reached roadway capacity. Thus, the underlying asset of the option to expand capacity is now the project value, not traffic levels.

Considering the same parameters as before, for the values used, the option to expand increases the project value from $301.4 for the base case scenario to $310.4 million, an increase of approximately 3%, which indicates that the probability that the expansion option will be exercised is low, as shown in Figure 3.5.

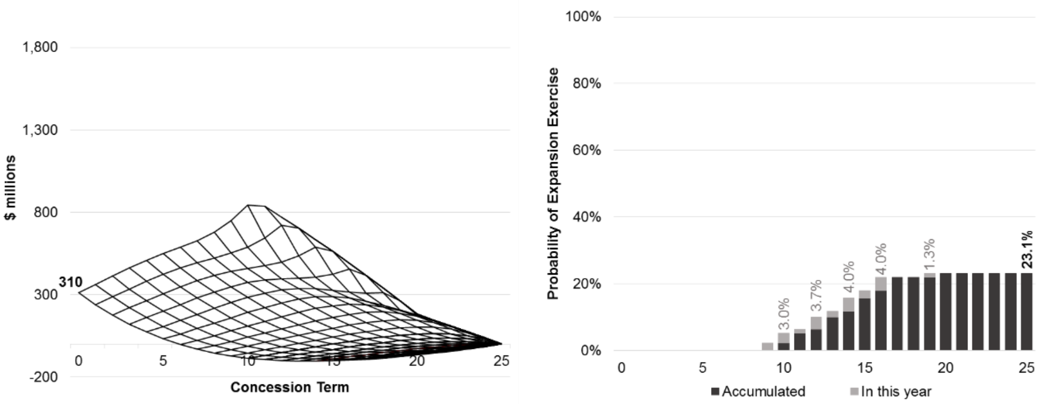

Figure 3.5 – Project value lattice and expansion exercise probabilities

Note: Project value lattice for optional capacity expansion during the original concession term (left) and expansion exercise probabilities (right), per year and accumulated.

It is also apparent that the probability that capacity expansion will occur in any year decreases rapidly after year 16 of the concession, as the concessionaire will not have enough years of cash flow to earn a return on the costs incurred for the expansion. Even with the expansion events during the original concession, these will only occur with an accumulated probability of 23.1%, compared to 71.2% probability in year 25 that demand exceeds the road capacity, as shown in Figure

3.2. Thus, the value of any option exercised during the original term grant is limited. These results show that this scenario does not guarantee that the expansion will take place within the concession contract term. This indicates that additional benefits must be provided to the concessionaire for this investment to take place. One possible solution is a conditional extension of the contract term to encourage the investment in capacity expansion. In the next section, we will analyze alternatives that involve the extension of the contract term, therefore effectively crossing the Brownian Bridge.

3.4.5. Optional Capacity Expansion with Term Extension Policies

We now analyze three possible zero-cost policies for capacity expansion with term extension, where the decision to invest in expansion is optional for the concessionaire. As the contract will no longer be bound to the original term time limit, the project value lattice will continue beyond that point, implying the crossing of the Brownian Bridge. We compare the results obtained in each policy in order to determine which is better capable of simultaneously increasing the value of the project and encouraging the earliest investment in expansion.

3.4.5.1. Fixed Term Extension

First, we assume that the concessionaire will automatically receive a 25-year extension from the time the investment in capacity expansion is made. Thus, if the expansion occurs in year t, the contract term is extended to $t + \omega$ years, where $\omega = 25$. This assumption was adopted to consider the possibility of extending the term from 1 to 25 years. This allow us to evaluate how the option is exercised, given the two variables under analysis: term and traffic. Besides, it is worth mentioning that we consider 50 years as the maximum concession term, as this is the case that would replace the government's need to carry out a new bid in the same proportion as the previous one.

Under this policy and using equation (3.17), the value of the concession increases to \$510.8 million, or 69.5% over the base case value. The value of the option to expand is \$209.4 million. While this policy significantly increases the value of the project, the effect on the decision for an early expansion is limited, and the probability that the roadway will be expanded before year 15 decreases to less than 15%, which corroborates the findings of Krüger (2012). This is because the concessionaire has the incentive to defer the expansion decision to the latest date possible, as this strategy maximizes the total length and value of the contract. In this sense, while the expansion is almost guaranteed, it will most likely occur in year 25. Given that an early expansion is optimal for the users of the roadway, this is an undesirable side effect of this policy.

Figure 3.6 shows the project value lattice and the expansion exercise probabilities.

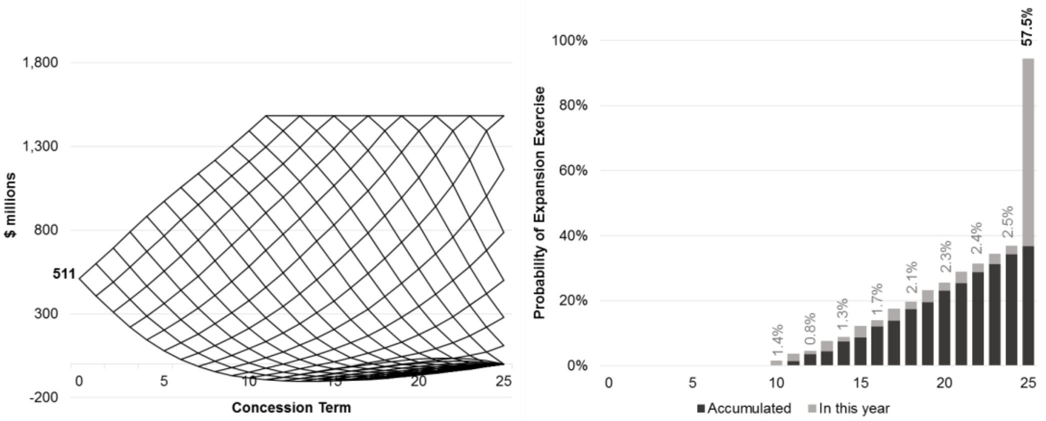

Figure 3.6 – Project value lattice with a fixed-term extension and expansion exercise probabilities

Note: Project value lattice with a fixed-term extension of 25 years at the time of capacity expansion (left) and expansion exercise probabilities (right), per year and accumulated.

3.4.5.2.
Doubling of the Concession Term

After, we consider a policy that provides the concessionaire a fixed 25-year extension of the contract term, which is doubled to 50 years if the concessionaire invests in the expansion at any time during the first 25 years. Unlike the premise of the previous policy, this one was adopted with the hope of generating an incentive to anticipate investment in expansion, as this would provide the private agent with a greater number of years with higher cash flows, which, consequently, would increase the concession value to the public agent.

Using equation (3.17) and considering $\omega = 50 - t$, the results indicate that the value of the concession increases to \$540.6 million, which is higher than the result of the first policy, representing an increase of 79.4% over the base case scenario and an option value of \$239.2 million. Figure 3.7 shows the evolution of the project value under this policy.

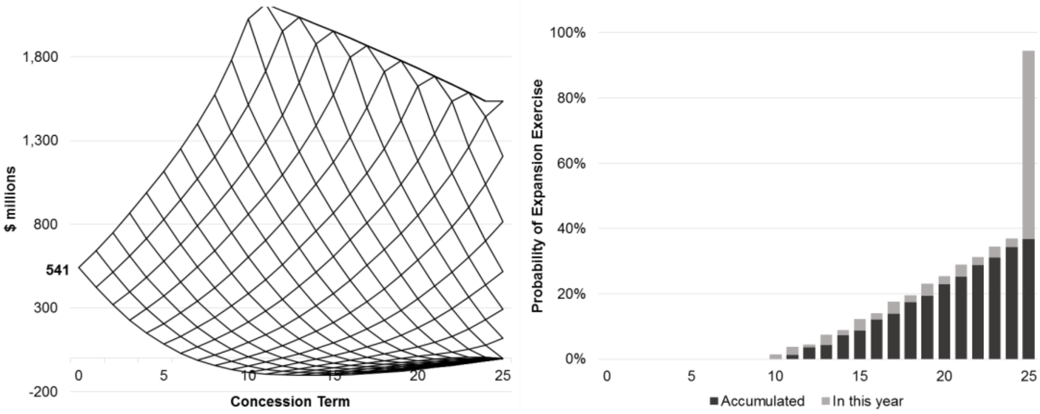

Figure 3.7 – Project value lattice with a fixed-term extension conditional on the roadway duplication and expansion exercise probabilities

Note: Project value lattice with a 25-year fixed term extension conditional on the roadway duplication within the first 25-year term, totaling 50 years (left) and expansion exercise probabilities (right), per year and accumulated.

Although we expect this policy to encourage early investment in expansion, our results show that this does not happen due to the high relative cost of the expansion. The probability of an early expansion remains unchanged, which indicates that this increase in value does not have any measurable effect on the likelihood of expansion.

While these two policies add substantial value to the project, neither is effective in anticipating the expansion of the roadway, which means that part of the government's objective is not achieved.

3.4.5.3. Penalized Term Extension

Given that the previous policies significantly increased the project value but were unable to guarantee an early expansion, we analyze the impact of a condition term extension policy that penalizes the concessionaire for late investment in expansion. This is achieved by considering, for example, that the 25-year term extension will be reduced by one year for every year the concessionaire fails to expand roadway capacity. Thus, when the concessionaire decides to expand, it will receive a term extension equal to the time remaining in the original term. If the firm expands in year t, it will earn a total contract term or $25 + (25 - t)$, which makes it clear that as the concession approaches the end of the original term, the length of the extension decreases, thus penalizing the concessionaire for investing late and creating an incentive to expand as soon as possible.

Note that the proposed model allows us to use different ways to penalize the deferment of the expansion, such as tariff reduction, compensation in the amount of the grant installments due by the concessionaire, and insertion of traffic guarantee mechanisms. However, as we are dealing with the problem of capacity expansion in concession projects considering term extensions and the possibility of crossing the Brownian Bridge, we chose to project the penalty on this. Thus, using equation (3.17) and assuming $\omega = 50 - 2t$, we find that the project value increases to $372.6 million, which is 23,6% above the base case value, but significantly lower than the $540.6 million of the full term extension case, as shown in Figure 3.8.

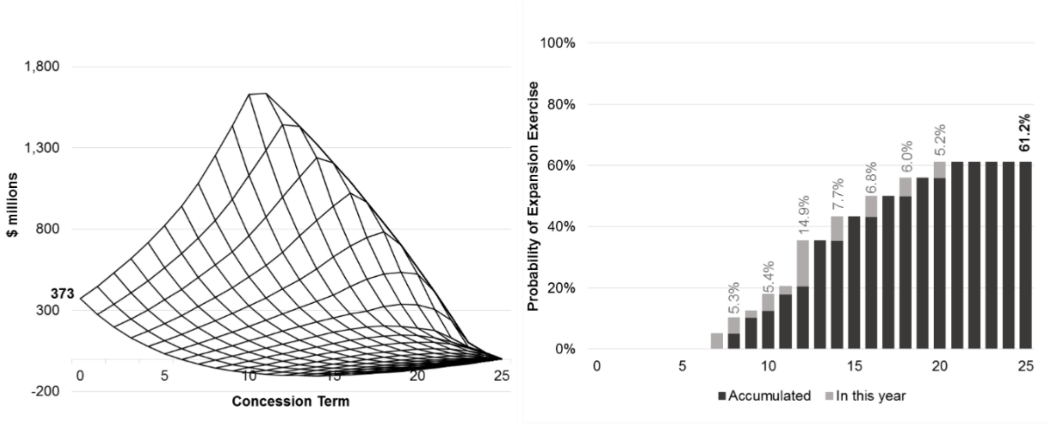

Figure 3.8 – Project value lattice with reducing fixed-term extension additional to concession term and expansion exercise probabilities

Note: Project value lattice with reducing fixed-term extension additional to concession term (left) and expansion exercise probabilities (right), per year and accumulated.

This policy is the most effective in creating incentives for an early expansion of the roadway with a 43.2% probability that it will occur before year 15, which is a threefold increase over the other policies, and a 61.2% probability that the expansion will voluntarily take place before the end of the concession term. Given that there is a 28.8% probability that demand will not exceed road capacity by year 25 (Figure 3.2), this implies that in only slightly over 10% of the cases, demand will exceed capacity. In this case, there will be no expansion, as this will tend to occur towards the end of the term.

3.5. Analysis of Results

Our results show that mandatory expansion policies, which are standard contract clauses in toll road concessions, are suboptimal. In addition, after analyzing the case where capacity expansion is optional, our findings show that the value of any option exercised during the original term grant is limited and that this flexible policy does not guarantee that the expansion occurs within the concession contract term.

By providing a term extension as an incentive to encourage the investment in capacity expansion, we guarantee that the expansion will take place voluntarily, but one that will most probably occur towards the end of the 25-year term. We correct this problem by introducing a penalty for late expansion, which shows that expansion will occur 60% of the time. Considering that there is approximately a 30% probability that no expansion will be needed until the end of the term due to weak demand, there is only a 10% probability that demand will be greater than capacity at any time. The results of all policies are summarized in Table 3.4.

Scenarios	Project PV ($ million)	Increment over Base Case (with cap)
Base case	363.4	-
Base case with absorbing barrier (with cap)	301.4	-
Firm Mandatory Capacity Expansion	231.2	-23.3%
Conditional Mandatory Capacity Expansion (American Obligation)	256.3	-15.0%
Optional Capacity Expansion	310.4	3.0%
Optional Capacity Expansion with Fixed Term Extension	510.8	69.5%
Optional Capacity Expansion with Doubling of Concession Term	540.6	79.4%
Optional Capacity Expansion with Penalized Term Extension	372.6	23.6%

Table 3.4 – Model results of the base case and all the policies considered

3.6. Discussion

In this article, we focus only on the assessment of schemes that allow the crossing of the Brownian Bridge, proposing a flexible model for decision-making on capacity expansion with term extension incentives that can benefit the government and private investors by aligning the objectives of both parties towards an early expansion and minimizing the uncertainties of an adverse economic environment. The general framework can assist government policymakers to determine optimal concession policies for capacity expansion.

Alternative road expansion incentive mechanisms, such as increases in toll rates and contingent payments, can be easily incorporated into our framework, as we show. Consider, for instance, a policy that increases the tariff by 30% if the concessionaire invests in the expansion at any time during the 25 years of concession. Using equation (3.16), we find that this policy results in a project present value of $331.9 million and a probability that the option be exercised of 39.9%, which are values lower than those found in the term extension policies.

Besides, it is worth mentioning that, although these mechanisms stimulate investment in capacity expansion and increase the present value of the concession, they are limited by the length of the Brownian Bridge. This mechanism, unlike the others proposed in this article, generates a cost from the increase in toll rate that is fully passed on to the public. Similarly, government contingent payouts also involve a cost to the public.

To demonstrate the robustness of our model and that the extension of the concession period does not affect the choice of policies, we perform a sensitivity analysis of the result to the term extension length ranging from 0 to 25 years for all three policies, as shown in Figure 3.9. In this figure, we refer to the optional capacity expansion with term extension policies as Policy 1 (“Fixed Term Extension”), Policy 2 (“Doubling of the Concession Term”) and Policy 3

(“Penalized Term Extension”), as the second naming does not make sense for periods other than the original 25 years.

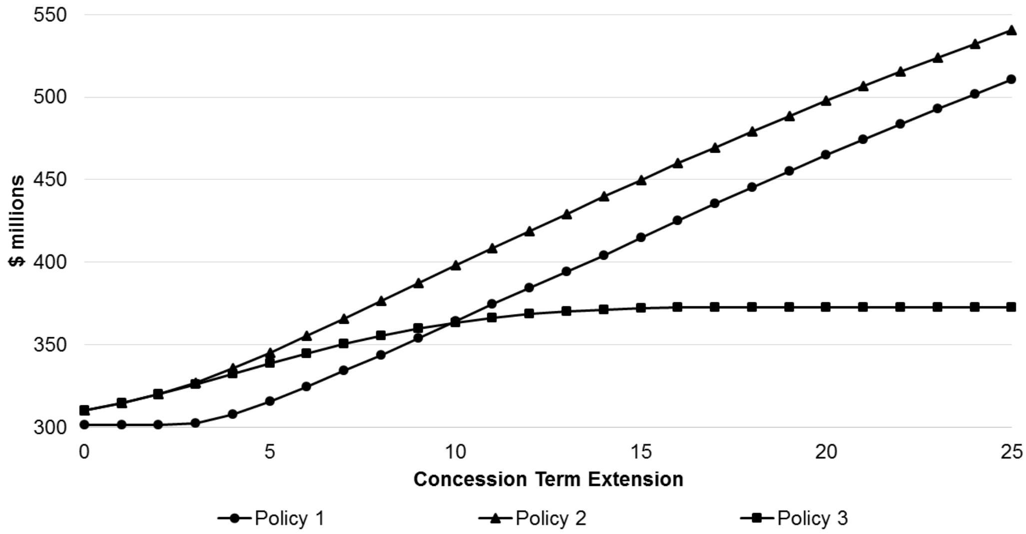

Figure 3.9 – Sensitivity analysis

Note: Sensitivity analysis of the project value to the term extension from 0 to 25 years for all three policies.

Note that Policies 1 and 2 show dependence of the option value to the extension time, as expected. In Policy 3, this dependence of the extension is reduced as this policy uses this period as the variable to which it is measured. Yet, for values of extension years lower than ten years, Policy 3 shows better results than Policy 1. Therefore, these results confirm that the third policy appears to be the most coherent. It is also interesting to note that with Policies 2 and 3, as the extension time goes to zero, these converge on the value of the optional capacity expansion case depicted in Figure 3.5 (310.4 M$). This is consistent as these policies without extension time replicate that case. While with Policy 1, as the extension time goes to zero, this value goes to the base case without any expansion (301.4 M$) since exerting this policy with low values of extension, close to the original term, would, in fact, be a reduction of this original term.

3.7.
Conclusion

In many developing countries, the majority of the highways are single undivided two-lane roads. In Brazil, there has been an effort to upgrade and expand these roads into four-lane divided highways in the past decade by including mandatory expansion clauses in public concessions to private investors, which have met with limited results. Thus, adequate and effective expansion policies that align the incentives of both the government and the concessionaire may assist in overcoming this problem.

In this article, we show why mandatory clauses, be it in the form of a European or an American obligation, are ineffective and significantly reduce the value of the project. In addition, we show that investments in capacity expansion tend not to add value to concession projects subject to a fixed contract term due to the Brownian Bridge.

Given that, we propose policies based on contract term extensions that create incentives for the concessionaire to flexibly and timely invest in capacity expansion. The results indicate that these policies allow the concessionaire to cross the Brownian Bridge, significantly increasing the project value by providing the firm with a valuable proprietary call option. We also find that the policy for optional capacity expansion with penalized term extension is the only one that allows generating a return on capital invested by the concessionaire; maximizing the value of the concession by inducing anticipated investment in expansion when the highway capacity limit is exceeded; and improving the quality of services provided to the public.

As one of the contributions to the literature, our model takes into account the fact that traffic demand is limited to road capacity, which represents an upper absorbing barrier or cap, and reduces the value of the project. Other contribution is that it evaluates how the option to expand capacity coupled with a term extension increases the probability of a timely and voluntary expansion. Finally, it also allows the granting authority to elaborate low-cost contractual clauses that align the objectives of both public and private agents.

This article focuses on evaluating policies that allow the crossing of the Brownian Bridge. Future work may extend the proposed model to other policies or verify the interaction between optional capacity expansion policies and other managerial flexibilities commonly found in concession contracts, such as MRG, MTG, and collar and abandonment options. Limitations of the model include estimation of the main parameters such as traffic volatility, risk-adjusted discount rate, and expansion costs.

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4
When Contract Clauses Conflict: the Case of the Salvador Light Rail Vehicle Concession

Infrastructure concession contracts may include clauses that allow for capacity expansion, term extension, or even risk management strategies such as cap and floor schemes. These clauses provide project managers with the flexibility required to adapt their operational strategies as new information is revealed in time and limit investor risk. However, the combination of these clauses is often done without consideration for how they may affect each other and the overall project objectives. We adopt the real options approach and analyze the interaction between distinct flexibility clauses and their impact on the overall value of the project. Using the Salvador Light Rail Vehicle (LRV) concession project as a numerical example, our results show that this interaction can have negative effects on both the public and private sector´s objectives. We also show how such clauses can be effectively designed to increase value and improve the project's risk and return profile.

4.1. Introduction

Given the limited financing capacity of the government, there is a worldwide tendency to grant infrastructure projects to private investors (Zhang, 2005). These projects are characterized by long maturity periods and high capital investments, which create significant risks and may make it difficult to secure private capital in some cases. Due to this, risk mitigation mechanisms may be used to make these investments more attractive. One of the traditional forms of risk mitigation in infrastructure projects is the Minimum Demand Guarantee (MDG), where the granting authority agrees to compensate the concessionaire if demand falls below a pre-established level (Rocha Armada, Pereira, & Rodrigues, 2012; Song, Yu, Jin, & Feng, 2018). Another traditional scheme is the cap and floor mechanism, where in addition to an MDG, a demand ceiling is also established, above which the concessionaire reimburses the granting authority for any gains earned above the cap level (Vasudevan, Prakash, & Sahu, 2018). These two forms of risk mitigation have been used in infrastructure concession contracts in several countries, such as Brazil (Brandão, Bastian-Pinto, Gomes, & Labes, 2012), China (Zhang, Li, Li, & Zhang, 2021), and India (Iyer & Sagheer, 2011).

Another recent trend is the consideration of the capacity expansion potential of infrastructure projects, which can be contemplated in the concession contract through mandatory or flexible clauses (Sismanidou & Tarradellas, 2017; Liu, Wang, Li, & Zhou, 2020). In the former, the concessionaire typically must invest in expansion after a set number of years or when demand reaches a pre-established level (Marques, Brandão, & Gomes, 2019). In the latter, the government may provide incentives such as a contract term extension or an increase in tariff in case the expansion is undertaken (Sun, Wang, & Meng, 2019; Polat & Battal, 2021; and, Balliauw et al., 2020). Marques, Bastian-Pinto, and Brandão (2021) also show that flexible clauses that associate a term extension with capacity expansion can add additional value to all project stakeholders.

Nonetheless, MDG or cap and floor schemes may conflict with expansion incentives, contrary to the intended result of the government, as caps on excess demand may render useless any incentive to expand the project. As these mechanisms have option like characteristics, we evaluate the combined effect of these clauses on the project's value and risk and their impact on the government and private agent's goals and objectives under the real options approach. This is the case of the Light Rail Vehicle (LRV) contract for the city of Salvador, Brazil, where demand risk is shared between the concessionaire and the government under a cap and floor contractual mechanism. This concession also includes flexible clauses that allow the concessionaire to expand capacity and extend the concession term (SEDUR, 2019).

In this article we apply a real options model to analyze the 20-year concession contract for the construction and operation of an LRV connecting the district of Comércio, in Salvador, to Ilha de São João, in the district of Simões Filho in the state of Bahia, Brazil.

This article contributes to the literature on real options applications in infrastructure projects in two ways. First, by analyzing the joint impact on project value of the simultaneous adoption of a cap and floor mechanism and expansion and term extension clauses. Second, by showing how the clauses that govern the managerial flexibilities in contracts must be carefully designed to achieve the objectives of both government and private investors.

This article is organized as follows. After this introduction, we present a review of the related literature in the field, and in section three we develop a real options model to evaluate the joint impact of these clauses. Next, in section four, we apply this model to the Salvador LRV concession contract. In section five we discuss the results and in section 6 we propose alternative approaches for combining flexible clauses. Finally, we conclude.

4.2. Literature review

Infrastructure investment analysis has been an active field of research in the last decades. The Discounted Cash Flow (DCF) is the most common method to evaluate an infrastructure project. However, thanks to the seminal work developed by Black and Scholes (1973) and Merton (1973) for the pricing of financial options, new methods that applied these concepts to the valuation of real assets under uncertainty and flexibility were developed, such as the Real Options Approach (ROA).

In recent years, ROA has found many applications in infrastructure projects. Alonso-Conde, Brown, and Rojo-Suarez (2007), for example, use this valuation tool to calculate government guarantees set in the Melbourne CityLink project and to analyze whether these guarantees affect investment incentives and whether the public sector may be transferring considerable value to the private sector. Brandão and Saraiva (2008) also evaluate government guarantees in a PPP project. For this, the authors consider market data from the BR-163 highway project and propose a Minimum Traffic Guarantee (MTG) model to assess the impact of government guarantees on project risk and the expected value of the resulting government liability.

Huang and Chou (2006) complement the analysis of the Taiwan High-Speed Rail project performed by Bowe and Lee (2004), considering the Minimum Revenue Guarantee (MRG) risk mitigating mechanism and the option to abandon during the project pre-construction phase. Their results show that when the MRG level increases, the value of the abandon option decreases, and that at a certain MRG level, the option to abandon will be rendered worthless. Chiara, Garvin, and Vecer (2007) analyze a BOT (Build-Operate-Transfer) project, considering that the government guarantee is a Bermudan or a simple multiple-exercise real option, depending on the number of exercise opportunities offered. They use the multi-least-squares Monte Carlo technique and find interesting results to improve risk mitigation and facilitate contractual and financial negotiations of BOT projects.

Attarzadeh, Chua, Beer, and Abbott (2017) are also concerned with the issue of effectively mitigating the impact of revenue uncertainty on BOT projects. In this sense, they propose a model for calculating equitable limits for guaranteed revenue for the private agent. The authors apply their model to a freeway PPP project and a power plant PPP project in Iran. Their findings show that the proposed systematic negotiating mechanism provides benefits to both the public and private sectors. Brandão et al. (2012) evaluate another kind of government guarantee, mainly, the Minimum Demand Guarantee (MDG). The authors study the Line 4 of the São Paulo Metropolitan Subway System and determine how different guarantee levels for each demand bands impact the risk and value of the project.

Buyukyoran and Gundes (2018) propose a model to evaluate the MRG in a BOT toll road project, considering that future demand is the most critical risk factor that affects the financial viability of the project. In this sense, they combine an optimization approach with Monte Carlo Simulation to identify the optimum upper and lower boundaries of guarantees. Analogously Carbonara and Pellegrino (2018) develop a model to calculate the optimal revenue floor and ceiling values in a way that creates a win-win condition for the concessionaire and the government. The authors apply this model to the Strait of Messina Bridge case and conclude that this mechanism can support the decision-making process of the government in assessing the values of public subsidies necessary to make the project attractive to private investors.

Shi, An, and Chen (2020) propose a method to determine the optimal capacity, toll, and subsidy for a BOT road contract, considering the MTG mechanism, and examine the impact of this mechanism on social welfare. Their findings show that the government sector will specify a lower toll and a higher capacity for a BOT road with a paid MTG, which results in a social welfare loss. More generally, Rouhani et al. (2018) review major revenue risk-sharing approaches developed worldwide that are designed to mitigate concessionaire risk and thus encourage private participation in concessions. These approaches depend on the level of demand risk, the risk-taking preferences of both partners, and the nature of the project. With this, they provide recommendations on how revenue risk-sharing strategies should be targeted under alternative economic and social conditions and specific project configurations.

Shan, Garvin, and Kumar (2010) argue that a limitation of an MRG is its requirement for an upfront premium payment and that a cap and floor mechanism, which is a collar option or a combination of put and call options, could be a better tool to share the risks in a concession because it overcomes this barrier. Through a numerical example, they show that the collar option improves the effectiveness of risk management in concession projects and redistributes downside losses and upside profits between the government and the concessionaire. Adkins, Paxson, Pereira, and Rodrigues (2019) also analyze collar arrangements and extend the model for determining the pre-investment and post-investment values of infrastructure projects with a perpetual collar-style incentive. Their analytical solution shows that there are many differences between perpetual and finite or retractable collars and that it is necessary to consider the current price level of the output and its expected volatility over the life of the contract when negotiating the floors, ceilings, and duration of a finite or a retractable collar.

Zhang et al. (2021) evaluate a Chinese road concession that has in its contractual clauses an MRG and an excess revenue sharing mechanism (ERS). Thus, they use ROA to model this flexibility as a swing option, which can hedge the underlying risk in two directions above and below expectations. The results show that this revenue risk allocation mechanism developed by the swing option method can help negotiations between the government and the concessionaire. Jin, Liu, Sun, and Liu (2019) address not only the problem of optimizing the level of MRG but also the issue of the concession period length. They propose an imperfect information trading model based on ROA and show that the length of the concession period is inversely proportional to the MRG level, and this correlation is influenced by the likelihood of reaching the equilibrium return rate of the investment.

Regarding infrastructure projects with expansion options, Ashuri, Lu, and Kashani (2011) use ROA to evaluate investments in toll road projects delivered under the two-phase development plan. The authors apply the risk-neutral binomial lattice model to analyze the demand uncertainty and to find the optimal time for the toll road expansion. Their results show that a flexible two-phase development plan can improve the investor's financial risk profile in the toll road project. Marques et al. (2019) also use the binomial lattice model to evaluate the expansion options present in the concession contract of Rio de Janeiro International Airport. Their results show that even considering the value of expansion options they are not sufficient to justify the bid premium offered by the concessionaire.

Sun et al. (2019) develop a trading and pricing method for expansion option model to solve expansion problems of BOT freeway projects and avoid contractual renegotiations. Considering a BOT freeway in Liaoning province, the authors find that there is a minimum price at which the government can sell the expansion option and a maximum price that the private sector is willing to pay. Also, they suggest that their method can be used by the government to manage its resources efficiently. Xiao, Fu, Oum, and Yan (2017) model the choice of airport capacity when a real option for expansion can be purchased. They also analyze how this option affects the efficiency of airports, considering not only profit maximization but also welfare maximization. Their analytical results show that if demand uncertainty is low and capacity and reserve costs are relatively high, the expansion option will not be exercised.

In a paper that specifically analyzes contract term extensions, Contreras and Angulo (2018) use ROA to assess the impact of this flexibility on the public budget. Through a case study of a hypothetical highway project, the authors show that the opportunity cost of term extensions is mainly affected by the extension period itself, the base interest rates, and the government's risk premium. Marques et al. (2021) propose a strategy that combines capacity expansion decisions with conditional term extensions and models this flexibility under the ROA and the project value uncertainty as a Brownian Bridge. Their results show that this strategy can be useful in attracting private investment in public infrastructure projects and that flexible infrastructure contracts can overcome the difficulty of accurately forecasting how market conditions and demand may evolve over the concession term.

Our article differs from the extant literature as none of these works addresses how distinct flexible clauses may interact with each other and affect public and private agents. In this study, we show how the clauses that govern these managerial flexibilities in the contract must be carefully designed to add value to the project and avoid non-intended consequences.

4.3. Model

4.3.1. Base Case

We propose a model to evaluate an infrastructure concession project that contains distinct managerial flexibilities in its contract. As is standard in the literature, we consider that the main source of uncertainty that affects the private agent investment returns and investment decision is the demand D_t which we assume follows a Geometric Brownian Motion (GBM), as shown in equation (4.1) :

$$dD_t = \mu D_t dt + \sigma_D D_t dz_t \qquad (4.1)$$

where dD_t is the incremental variation of demand in the time interval dt; μ represents the expected growth rate of demand; σ_D is the demand volatility; and $dz_t = \varepsilon\sqrt{dt}$ represents the standard increment of Wiener, where $\varepsilon \approx N(0,1)$.

To model this uncertainty, we use the discrete binomial tree approach proposed by Cox, Ross, and Rubinstein (1979) (CRR). Since this option pricing model requires the use of the risk-neutral measure, we deduct the risk premium from the asset's rate of return. Thus, the risk-neutral process of demand is defined by equation (4.2):

$$dD_t^R = (\mu - \zeta_D) D_t^R dt + \sigma_D D_t^R dz_t \qquad (4.2)$$

where ζ_D represents the demand risk premium; μ is the return rate of the demand; and dD_t^R is the incremental variation of the risk-neutral demand in the time interval *dt*. Following the steps proposed by Freitas and Brandão (2010), we estimate numerically the value of ζ_D and determine the CRR parameters and the risk neutral probability p using equation (4.3).

$$u = e^{\sigma_D\sqrt{\Delta t}}, \quad d = \frac{1}{u} \quad \text{and} \quad p = \frac{\left(1+\mu-\zeta_D\right)^{\Delta t} - d}{u-d} \qquad (4.3)$$

where *u* and *d* are, respectively, the upside and downside multiplying factors; r_f is the risk-free rate; and Δt the discrete-time increment.

After modeling the demand lattice, we calculate the equivalent cash flow lattice using equation (4.4):

$$F_t = \left\{\left[D_t^R \times \pi\left(1+\nu\left(1-\varphi\right)\right)+\chi\right]\left(1-\varpi\right)-\left(\phi+\delta\right)\right\}\left(1-\iota\right)+\delta \qquad (4.4)$$

where π is the tariff; ν represents the non-tariff revenues, which mainly derive from the commercial exploitation of public spaces; φ is the non-tariff revenues tax; χ represents the public financial subsidy; ϖ represents the variable costs, which are a percentage of total revenues (tariff and non-tariff); ι is the income tax; δ is the depreciation and ϕ represents fixed costs.

Note that the project value lattice V^R can be determined by discounting the future cash flows of the cash flow lattice. In this sense, under the discrete model, if the concession term is *n*, the project value at time *t* can be defined by equation (4.5) :

$$V_t^R = \left[\left(F_{t+1}^+ + V_{t+1}^+\right)p + \left(F_{t+1}^- + V_{t+1}^-\right)\left(1-p\right)\right] / \left(1+rf\right) \qquad (4.5)$$

This base case scenario considers the demand uncertainty but no flexibility. Next, we show how the flexibility clauses embedded in the concession contract can be modeled.

4.3.2. Demand Cap and Floor Mechanism Clause

As shown by Brandão & Saraiva (2010), the demand cap and floor mechanism (collar option) can be modeled as a bundle of European options with maturities between 1 and *n* years. Due to the MDG (floor), the concessionaire holds a series of put options against the government, while the cap provides the government a series of call options against the concessionaire. The valuation process assumes that at each time *t* the optimal decision will be made considering the cap and floor demand mechanism shown in Table 4.1. In addition, like any infrastructure project, we should consider that the Salvador LRV has a natural demand capacity limit (D_{max1}), which can be modeled as a demand-absorbing barrier and represented by a percentage ψ of the initial demand D_0. Thus, to evaluate this flexibility clause, we use equation (4.6) in the modeling of the demand binomial tree.

$$D_t^{'} = \min\left[D_t^{A*};\psi\left(D_0\right)\right] = \min\left[D_t^{A*};D_{\max 1}\right] \qquad (4.6)$$

As the cash flows generated by the project are a direct function of demand, considering $D_t^{'}$, the new cash flow lattice can be calculated by using equation (4.4) and, in a simplified way, be expressed by $F_t^{'} = f\left(D_t^{'}\right)$. Consequently, the project value $V_t^{'}$, considering the demand cap and floor mechanism (demand collar option) and the demand capacity limit, can be estimated through equation (4.7):

$$V_t^{'} = \left[\left(F_{t+1}^{'+} + V_{t+1}^{'+}\right)p + \left(F_{t+1}^{'-} + V_{t+1}^{'-}\right)(1-p)\right] / (1+rf) \qquad (4.7)$$

4.3.3. Capacity Expansion with Term Extension Clause

Unlike the cap and floor mechanism, the flexible capacity expansion with term extension clause must be modeled as an American call option, as the concessionaire has the option to expand capacity at any time during the concession term.

Since this clause allows a flexible capacity expansion conditioned to the extension of the concession term, the value of this option $V_{t_{exp}}$ can be defined by equation (4.8):

$$V_{t_{\exp}}^{'} = \max\left[V_t^{'};V_t^{'''} - I\right] \qquad (4.8)$$

where $V_t^{'}$ represents the present value of the project at t considering the demand collar option and the demand capacity limit; $V_t^{''}$ is the present value of the project cash flows after capacity expansion and term extension in time t; and I represents the expansion CAPEX (Capital Expenditure).

Note that, if the concessionaire chooses to expand capacity, the project cash flows are no longer limited to D_{max1}, but to a new higher capacity limit D_{max2}, which is represented by a percentage ψ of D_{max1}. To evaluate the flexible capacity expansion with term extension clause, we use equation (4.9) in the modeling of the demand binomial tree.

$$D_t^{"} = \left(\min\left[D_t^{A*};\psi D_{\max 1}\right]\right) = \min\left[D_t^{A*};D_{\max 2}\right] \quad (4.9)$$

Then, using equation (4.4) and considering $D_t^{"}$, we determine the expanded cash flow lattice $F_t^{"} = f\left(D_t^{"}\right)$. Assuming that there is a time to build for the expansion of one period, during which the project will receive the maximum level of cash flow prior to expansion $F_{\max 1} = f\left(D_{\max 1}\right)$, we determine the present value of the expanded cash flows at time t using equation (4.10). Note that these will grow at μ for $(n + \omega) - (t + 1)$ years and are discounted at k up to time t, where ω is the term extension $(n + \omega)$ provided for in this flexible clause.

$$V_t^{"} = \frac{F_t^{"}}{(k-\mu)(1+k)}\left[1-\left(\frac{1+\mu}{1+k}\right)^{(n+\omega)-(t+1)}\right] + F_{\max 1} \quad (4.10)$$

4.4. Numerical Application

We apply this model to the Salvador Light Rail Vehicle (LRV) project. This project consists of a 20-year concession contract for the construction and operation of a 20 km long LRV linking the district of Comércio, in Salvador, to Ilha de São João, in the district of Simões Filho in the State of Bahia, Brazil. The objective of this concession project is to improve the urban railway system in Salvador, which currently operates in poor conditions, through the construction of twenty-one stations, benefiting the more than 600,000 residents of the region.

In February 2019, the Salvador LRV concession contract between the government and the concessionaire Skyrail Bahia, a consortium composed of BYD Brazil and Metrogreen was celebrated (ANTT, 2019). Like most infrastructure concessions, the Salvador LRV concession contract allows the concessionaire to earn a return from a mix of tariff and non-tariff revenues and public subsidies. In addition, to make the project more attractive to the concessionaire, the contract included incentives and managerial flexibility clauses.

One of these clauses is a risk mitigation scheme known as the cap and floor demand mechanism. The Salvador LRV concession contract provides for the sharing of the risk of variation in projected demand between the public and private agents. These clauses state that the concessionaire will receive compensation whenever the Actual Demand in year *t*, D_t^A, falls below a certain percentage of the Projected Demand, D_t^P, for the same year. Likewise, if D_t^A is above a certain percentage of D_t^P, the concessionaire will hand over part of this gain to the government. Table 4.1 and Figure 4.1 present the cap and floor demand mechanism described in the Salvador LRV concession contract.

Actual Demand – D_t^A	Adjusted Demand – D_t^{A*}
$D_t^A < 75\%\ D_t^P$	*Contract renegotiation*
$75\% D_t^P \leq D_t^A < 90\% D_t^P$	$D_t^{A*} = D_t^A + 0.7\left(0.9 \times D_t^P - D_t^A\right)$
$90\% D_t^P \leq D_t^A \leq 110\% D_t^P$	$D_t^{A*} = D_t^A$
$110\%\ D_t^P < D_t^A \leq 125\% D_t^P$	$D_t^{A*} = 1.1 \times D_t^P - 0.3\left(1.1 \times D_t^P - D_t^A\right)$
$D_t^A > 125\%\ D_t^P$	*Contract renegotiation*

Table 4.1 – Cap and floor demand mechanism described in the Salvador LRV concession contract

Source: SEDUR (2019).

Note: Note: As in Brandão et al. (2012), we assume that an eventual renegotiation will necessarily result in an additional burden for the government, as services cannot be interrupted, and the limitation of passenger capacity ensures that levels above 125% of expected demand will rarely be reached. This implies that it is unlikely that a renegotiation will be favorable to the government. Thus, the mitigation rules were applied to all possible ranges of traffic demand up to the 75% and 125% barriers. When these limits are reached, fixed demand levels of 75% and 125% are assumed, as shown in Figure 4.1.

Thus, according to Table 4.1:

a) if D_t^A is below 75% of D_t^P, the contract will be renegotiated as explained in the note in Table 4.1.

b) if D_t^A is between 90% and 75%, inclusive, of D_t^P, the concessionaire will be entitled to receive a compensation of 70% of the volume shortfall, up to the limit of the risk fully assumed by the concessionaire, that is, 90% of D_t^P ;

c) if D_t^A is between 90%, inclusive, and 110%, inclusive, of D_t^P, there will be no demand compensation;

d) if D_t^A is between 110% and 125%, inclusive, of D_t^P, the concessionaire will be entitled to receive the revenues from 110% of D_t^P and from 30% of the volume of passengers transported above 110% of D_t^P; and,

e) if D_t^A is above 125% of D_t^P, the contract will be renegotiated as explained in the note in Table 4.1.

An additional flexibility present in the Salvador LRV concession contract is the option to expand capacity. The development plan for this project is divided into two phases, where the first phase is mandatory and the second optional. The exercise of the latter will depend on the results found in the feasibility studies performed by the concessionaire. In addition, a term extension clause linked to capacity expansion is also included in this concession contract, allowing for a term extension of 15 years, which may result in a contract term of 35 years if the concessionaire decides to invest in capacity expansion (SEDUR, 2019).

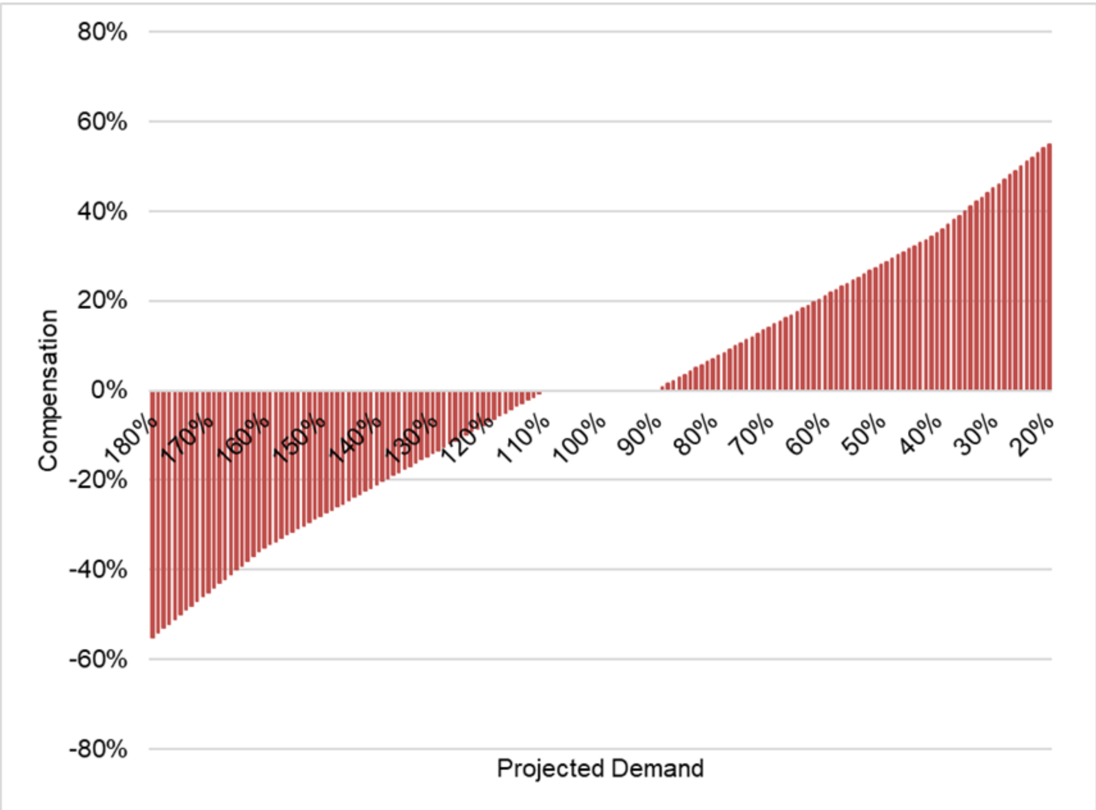

Figure 4.1 – Cap and floor demand mechanism described in the Salvador LRV concession contract

4.5. Results and Discussion

Our numerical application considers the data of the Salvador LRV concession project summarized in Table 4.2.

Grant term	n	20 years
CAPEX	I_0	USD 450.0 million
Expansion CAPEX	I	USD 225.0 million
Fixed costs	ϕ	USD 30.0 million per year
Variable costs	ϖ	25.6% of total revenues
Income tax	ι	34.0%
Depreciation	δ	USD 22.5 million per year
Tariff	π	USD 2.0
Non-tariff revenues	ν	10.0% of tariff revenues
Risk-free rate	r_f	4.1% per year
Risk-adjusted rate	k	6.0% per year
Initial Annual demand	D_0	22.2 million
First Yearly Maximum Demand Capacity	$D_{\max1}$	33.3 million
Second Yearly Maximum Demand Capacity	$D_{\max2}$	50.0 million
Capacity Limit Factor	ψ	150%
Public financial subsidy	χ	USD 47.8 million
Non-tariff revenues tax	φ	14.3%

Table 4.2 – Salvador LRV concession data

Source: SEDUR (2019).

As suggested in section 4.3.1, we first model the demand uncertainty as a GBM, using the values and parameters presented in Table 4.3.

Initial Annual demand	D_0	22.2 million
Demand drift (growth)	μ	1.9% (per year)
Demand Volatility	σ_D	12.0% (per year)
Demand risk premium	ζ_D	2.7% (per year)

Table 4.3 – Stochastic demand values and parameters

Figure 4.2 shows demand stochastic behavior, expected demand, demand capacity limit and the risk sharing mechanism described in Table 4.1.

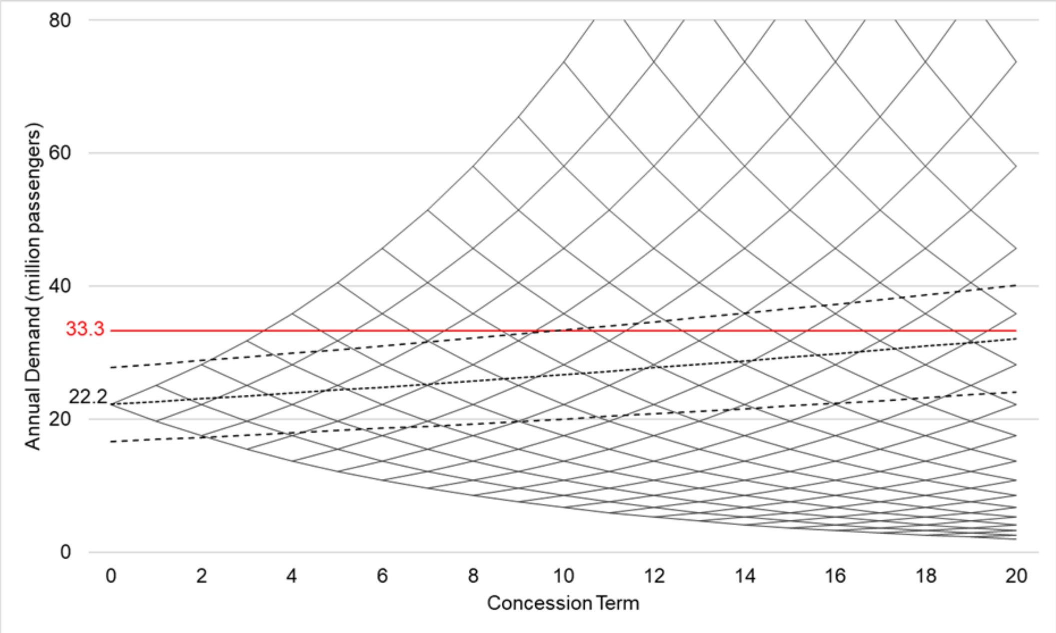

Figure 4.2 – Annual stochastic demand projection

Note: Annual stochastic demand projection (lattice in gray) showing expected demand starting at D_0 = 22.2 million passengers; capacity limit of D_{max1} = 33.3 million passengers (red line); and the demand cap and floor mechanism (dashed lines).

Considering the demand binomial tree shown in Figure 4.2 (lattice in gray) and using equation (4.5), we determine that the value of the Salvador LRV concession project at time $t = 0$ is V_0^R = \$450.0 million, which yields a Net Present Value (NPV) equal to zero, as shown in Figure 4.3.

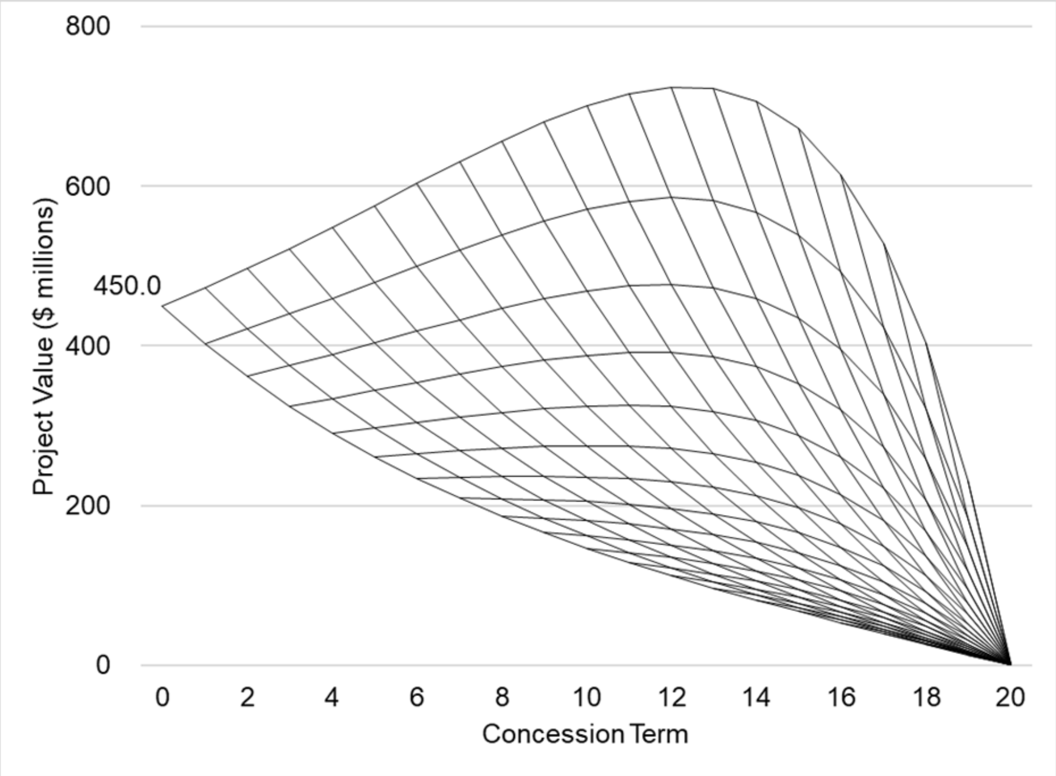

Figure 4.3 – Project value lattice (base case)

Note: Project value lattice without demand capacity limit (D_{max1}) and flexible clauses (base case scenario).

Note that Figure 4.3 presents the project value lattice without considering the demand capacity limit (D_{max1}) and the flexible clauses included in the Salvador LRV concession project. For comparison purposes, we call this first result our base case scenario.

By including the demand capacity limit (D_{max1}) in our analysis, the demand binomial tree assumes the behavior shown in Figure 4.4. With this limitation, we find that the value of the Salvador LRV concession project at time $t = 0$ is $V_0^R =$ \$444.1 million, as shown in Figure 4.5.

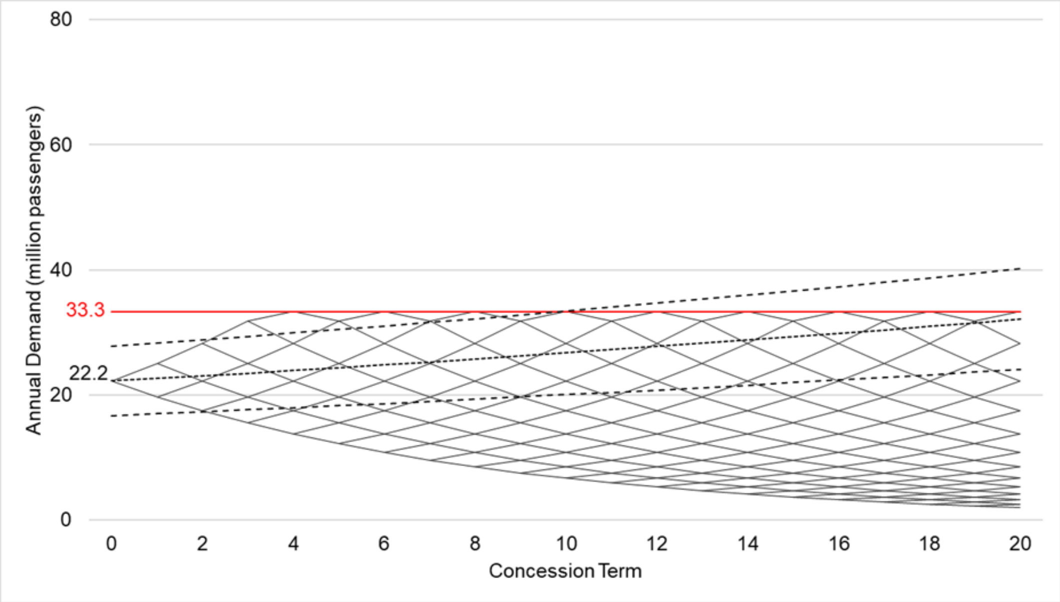

Figure 4.4 – Demand lattice with demand capacity limit (D_{max1})

Note: Annual stochastic demand projection with an absorbing barrier of D_{max1} = 33.3 million passengers (red line) showing expected demand starting at D_0 = 22.2 million passengers and the demand cap and floor mechanism (dashed lines).

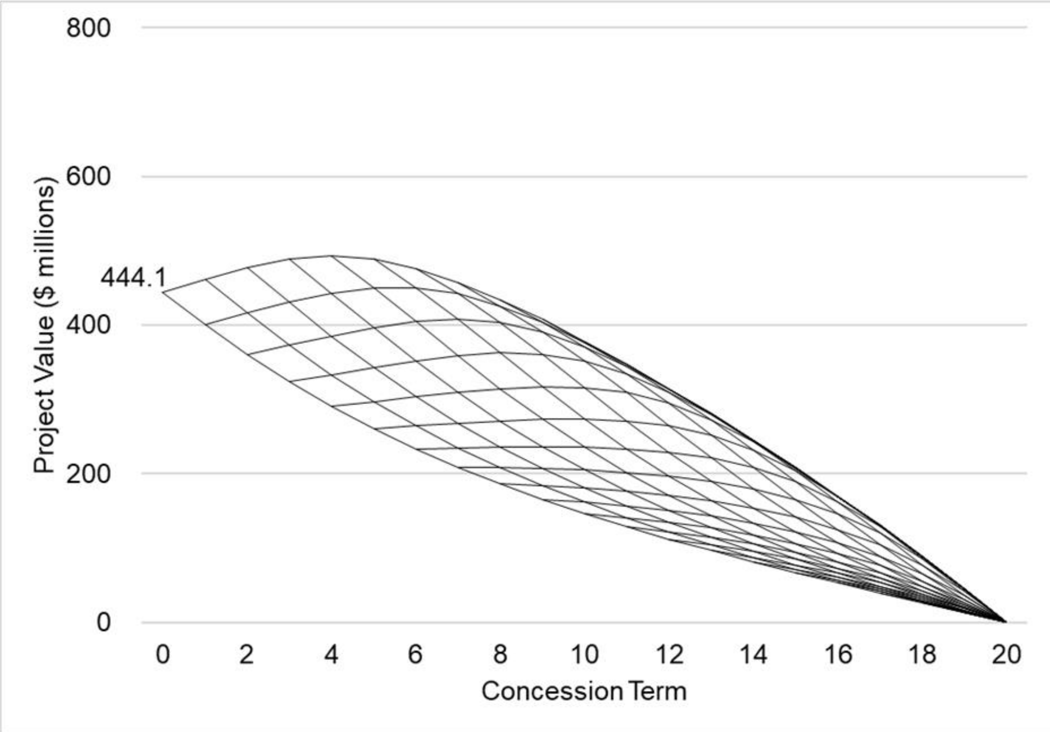

Figure 4.5 – Project value lattice with demand capacity limit (D_{max1})

To determine the project value considering the demand cap and floor mechanism (demand collar option), we should not only consider the demand lattice with the limitations imposed by the demand capacity limit (D_{max1}) but also the cap and floor demand mechanism described in Table 4.1. With this, the demand binomial tree assumes the behavior shown in Figure 4.6.

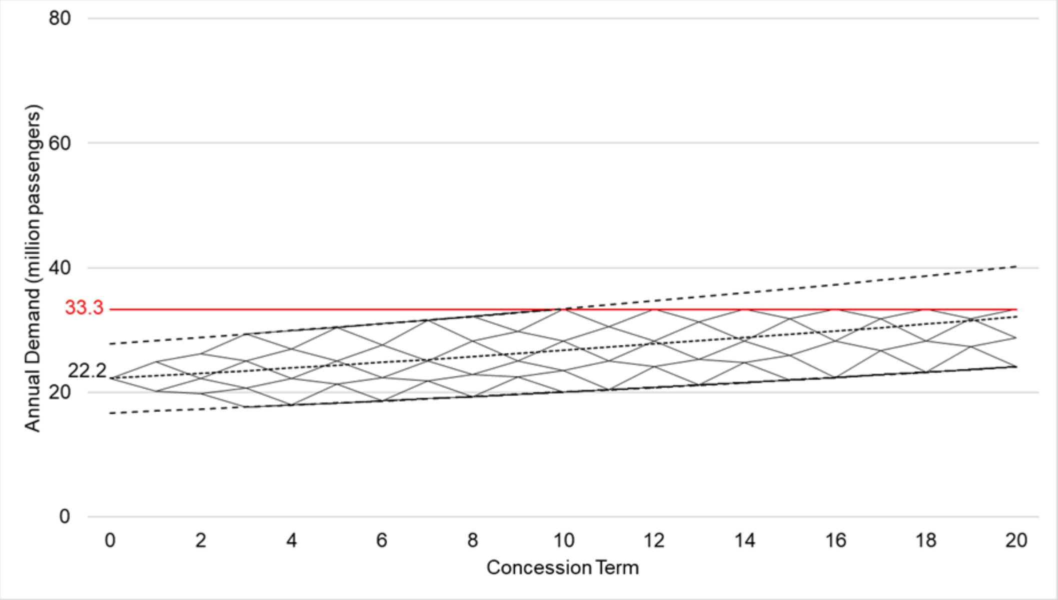

Figure 4.6 – Demand lattice with demand capacity limit (D_{max1}) and cap and floor mechanism

Note: Annual stochastic demand projection with the limitations imposed by the demand capacity limit – D_{max1} (red line) and by the cap and floor mechanism. The irregularity of the nodes and branches of the binomial tree is due to the fact that we are using a discrete model that assumes a time interval of 1 year.

Considering the demand lattice presented in Figure 4.6 and using equation (4.7), we find that the adoption of the demand cap and floor mechanism mitigates the risk of both the concessionaire and the government, as it eliminates part of the effect of demand volatility, as shown in Figure 4.7. Thus, when we consider this clause, the project value and the NPV of the base case scenario increases to $486.1 million $\left(V_t^{'}\right)$ and $36.1 million, respectively.

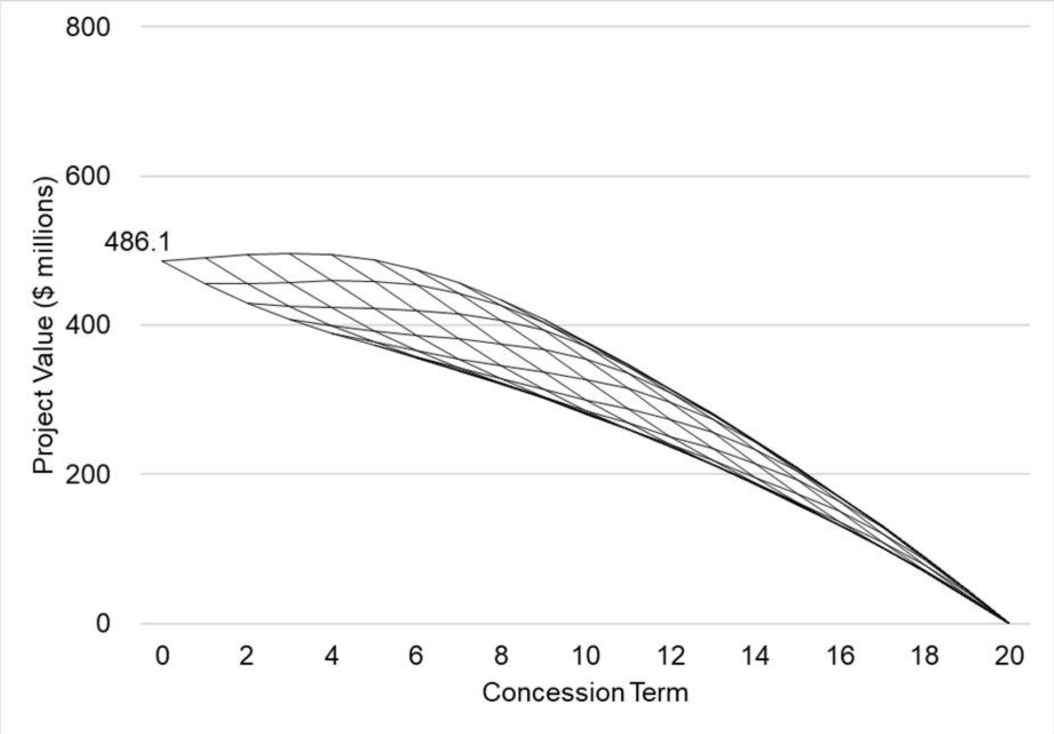

Figure 4.7 – Project value lattice with demand capacity limit and cap and floor mechanism

Note: Project value lattice after incorporating the demand capacity limit D_{max1} and the demand cap and floor mechanism (demand collar option).

Note that the demand cap and floor mechanism (demand collar option) not only reduces the risk for the concessionaire but also increases the value of the project itself. This is an effect of the risk reduction and asymmetry of returns from the demand uncertainty.

Then, we add to our analysis the flexible capacity expansion with term extension clause. For this, as described in equation (4.9), we reallocate the demand capacity limit when the maximization equation (4.8) suggests that the concessionaire should invest in capacity expansion considering a term extension.

The demand binomial tree that should be considered in case of expansion is shown in Figure 4.8.

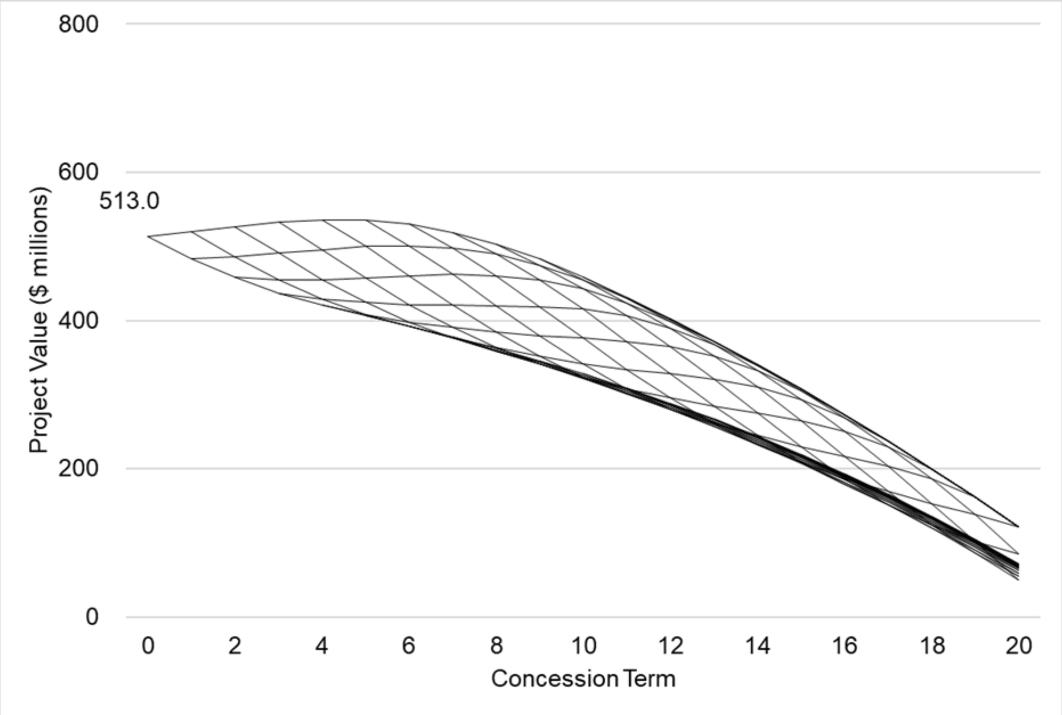

Figure 4.8 – Project value lattice with demand collar option and capacity expansion with term extension option

Figure 4.8 shows that the expansion option linked to the incentive of term extension adds value to the project even when there is a demand risk-sharing mechanism, since the value of the concession increases to $513.0 million. During the original term of the concession, the cap and floor mechanism inhibit the exercise of this option. However, note that this option is always exercised in the last year (year 20). This occurs because the lower band of the demand collar option (floor) serves as a subsidy to the private investor, thus guaranteeing at least a minimum amount of cash flow in the 15-year term extension, and yielding a positive NPV for the expansion option, even at very low values of verified traffic demand. Therefore, such a clause will lead the concessionaire to implement the expansion of the system, even if there is no traffic demand to justify this.

In this sense, despite adding value to the project, the capacity expansion with term extension option together with the demand collar option is conflicting for both parties involved in this project. Under these conditions, the government encourages private investment in capacity expansion and term extension in the 20th year of the concession by guaranteeing a minimum demand (floor) in the 15 years of extension, increasing its contingent liabilities. In addition, these clauses are also conflicting for the private investor, because even if the demand suggests exercising the capacity expansion with term extension option before year 20, it will not be optimal to carry out the investment due to the barrier imposed by the upper band of the demand collar option (cap).

4.6. Discussion

An interesting aspect of the Salvador LRV contract is that the inclusion of these conflicting clauses in the auction documents, which went unnoticed by the interested parties. This raises concerns regarding the level of due diligence performed by the potential concessionaires during the auction process, as this suggests either a lack of analysis or unawareness of the potential impacts and risks of these clauses. This has the potential to result in scenarios where the concessionaire may seek to renegotiate the contract during the implementation phase, leading to ex-post alterations of the original terms (Xiong & Zhang, 2016). Thus, the failure to properly assess and account for the implications of such clauses during the auction process can have far-reaching implications for the project´s risk-return matrix.

Additionally, the granting authority may accede to such demands, as they are interested in implementing these changes. However, this could result in objections and legal action from other bidders who did anticipate such modifications to the base rules and thus failed to factor them into their bid offers.

In light of these considerations, we propose several measures to mitigate the potential conflicts arising from the interaction between flexible clauses in infrastructure concession contracts.

4.6.1. Capacity expansion with term extension and reallocation of the upper band of the demand collar option (cap)

One solution to this problem is to reallocate the upper band (cap) of the demand collar option when the expansion option is exercised. To simplify the analysis, we assume that the new cap can be found through the product of the percentage ψ (which represents how much capacity has been expanded) and the original cap described in Table 4.1. Thus, the demand binomial tree assumes the behavior shown in Figure 4.9.

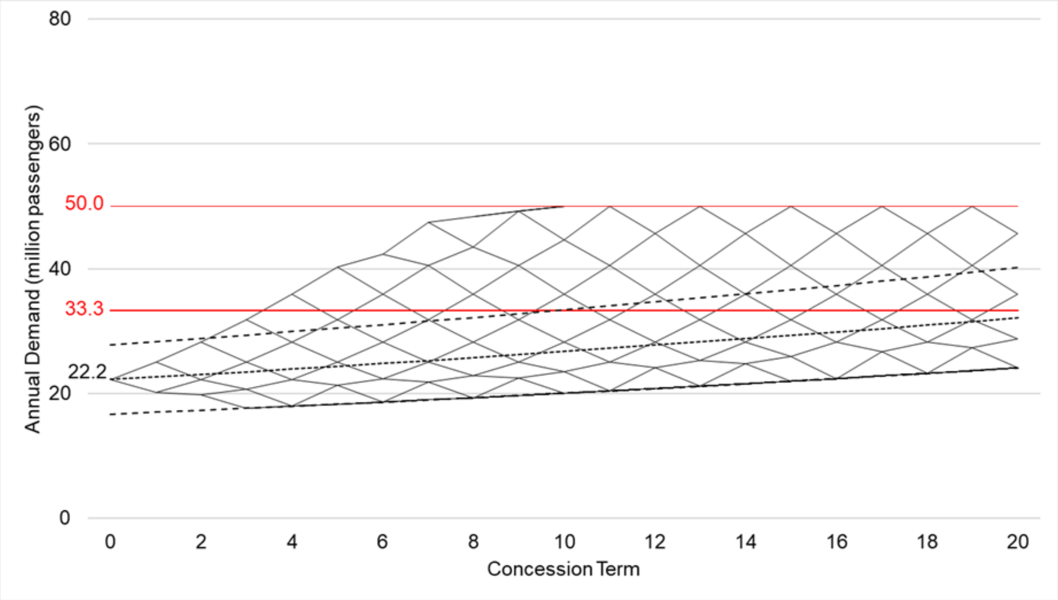

Figure 4.9 – Demand lattice with demand capacity limit (D_{max2}) and new cap

Note: Demand lattice with the limitations imposed by D_{max2} = 50.0 million passengers (red line) and by the new cap that should be considered when exercising the expansion option. The irregularity of the nodes and branches of the binomial tree is due to the fact that we are using a discrete model that assumes a time interval of 1 year.

Then, we follow the scheme described in section 4.3.3, but consider the cap reallocation and, consequently, the demand lattice shown in Figure 9. The new project value lattice is presented in Figure 4.10.

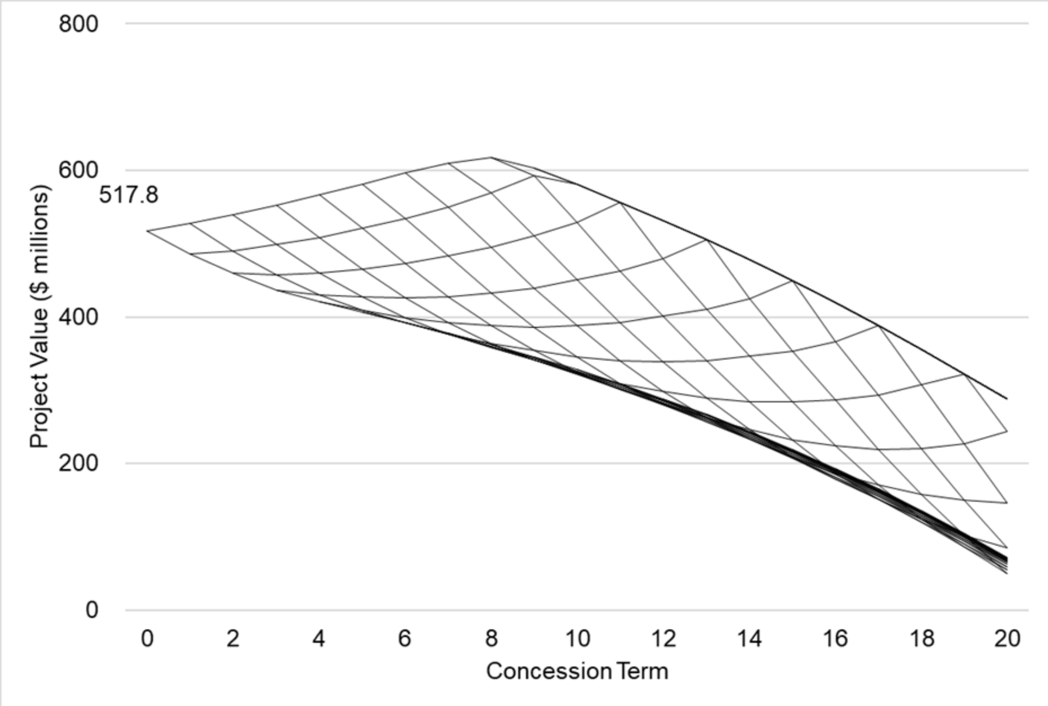

Figure 4.10 – Project value lattice with demand collar option with cap reallocation and the term extension linked to a capacity expansion option

After reallocating the level of the upper band (cap) of the demand collar option, Figure 10 shows that the expansion option linked to the incentive of term extension adds value to the project, since the value of the concession increases to $517.8 million. We can observe that this option is always exercised in the last year (year 20), but also at the top of the lattice in previous years (up to year 10). As shown above, the option exercise in the last year occurs because the lower band of the demand collar option (floor) serves as a subsidy to the private investor. On the other hand, the option exercise in previous years is encouraged by relocating the demand collar option's upper band (cap).

Through this proposal, we partially resolved the conflicts presented in the previous section. We resolved the conflict for the private investor, however the conflict remained for the public agent.

4.6.2.
Capacity expansion with term extension and abandonment of the demand collar option

In the second proposal, we assume that the cap and floor mechanism is eliminated when the model suggests exercising the capacity expansion with the term extension option. For this, we consider a backward maximization between the cash flows generated by each flexible clause.

Figure 4.11 shows that the exercise of the capacity expansion with term extension option does not always occur, not even in the last year of the original concession term (year 20) since there is no longer any subsidy derived from the demand risk sharing mechanism. The exercise of capacity expansion with the term extension option and the abandonment of the demand collar option occurs until year

7, always close to the top of the lattice, increasing the value of the concession to $502.7 million.

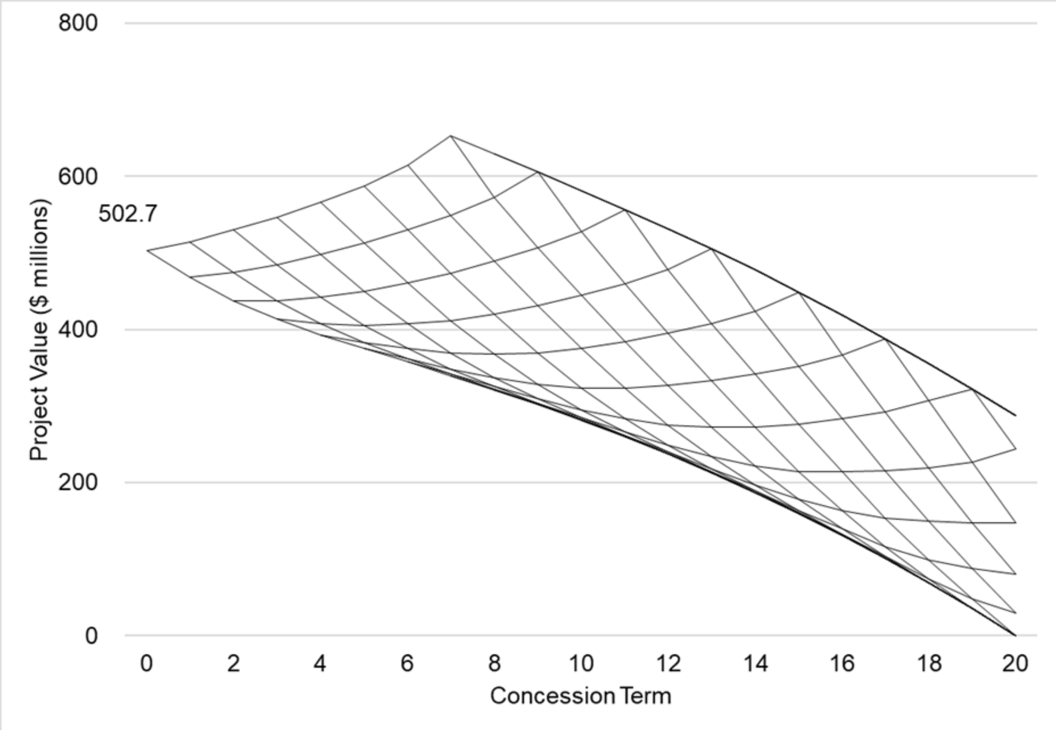

Figure 4.11 – Project value lattice with term extension linked to a capacity expansion option and the abandonment of the demand collar option

Therefore, this proposed flexible clause proves to be the best alternative for the government and the private investor, as it guarantees the achievement of the objectives of both agents. This proposal allows the private sector to monetize the capital invested and the government to increase the value of the concession, inducing early investment in expansion when the capacity limit is exceeded to improve the quality of services provided to the public.

4.7. Conclusion

This article develops a model to determine the value and impact of conflicting risk-mitigating clauses in infrastructure concession contracts. As these flexible clauses affect the risk and return matrix of the project and depend in a non-linear way on some uncertain state variable of the project, such as demand for the service, we use a real options model to evaluate their effect on the value and risk of the project and investigate whether they are beneficial for the government and private agent. We apply this model to the case of a Light Rail Vehicle (LRV) concession project in Brazil which was auctioned in 2019 that includes a demand cap and floor mechanism and flexible clauses for expansion and term extension in its contract.

Although these managerial flexibilities are already common in concession contracts for some time, especially the demand collar option, they are usually not included together and when they are, they are not correctly valued. In the case of the Salvador LRV project, it appears that neither the granting authority nor the concessionaire correctly analyzed these clauses. The approach used here of modeling a CRR adapted binomial lattice to value a bundle of European call and put options and an American call option allows us to quantify their effect and their interaction, which, as in this case, can present conflicting results given the objectives envisaged by the granting authority and the private agent.

This approach is not only simple and practical but also has the benefit of being visual and intuitive, allowing not only the understanding of its implications but the easy adaptation to other similar cases. This article also points to concession contracts, which are sometimes ineffective and poorly designed, and proposes flexible clauses to public policymakers so that both public and private parties get the best of them.

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5
Stochastic processes for modeling uncertainty in airport concessions in Brazil

Determining the stochastic process to model demand uncertainty in infrastructure concession projects may not be as simple a task as the literature suggests. Most papers in this field assume, without any further analysis, that demand follows a Geometric Brownian Motion (GBM). This is because it is a much simpler process and, in many cases, allows for an analytical solution. Modeling with mean reversion requires more complex and computer intensive methodologies, especially if approaches such as real options are used. In this article, we evaluate the most appropriate stochastic process to model the uncertainty of passenger demand in airport concessions in Brazil. We use Unit Root, Variance Ratio tests, and the Parameter Approach Measure to analyze samples ex-ante and ex-post covid-19 pandemic. In the cases analyzed, the Mean Reversion Model (MRM) presents the greatest suitability for modeling passenger demand, contrary to what is generally assumed in the literature. Our results also suggest that both seasonality and the covid-19 pandemic significantly impact the stochastic diffusion model. After correcting the demand series for the seasonality effect, most of them tend to show a GBM behavior. However, when analyzing the same series encompassing the effect of the covid-19 pandemic, these continue to demonstrate MRM behavior even after correcting for seasonality. These findings have important consequences for contract clauses that may be embedded in the concession agreement. In many infrastructure concessions, especially in transportation projects, minimum demand guarantee clauses are significantly affected when demand has a mean reverting component.

5.1. Introduction

Infrastructure concessions are subject to uncertainty over future traffic demand. For dynamic valuation models, this uncertainty must be modeled as a stochastic diffusion process. Since the adoption of different processes may provide distinct valuation results, the choice of process is fundamental to developing a correct valuation model.

Several stochastic models are available, the most common being the Geometric Brownian Motion (GBM) and the Mean Reverting Models (MRM). The Geometric Brownian Motion (GBM) is a popular model and the most widely used stochastic process in economic finance theory and practice due to its appropriateness for a wide range of cases and ease of use. From a practical point of view, it is the model of choice for diffusion processes that are assumed to grow exponentially following a lognormal probability distribution with normally distributed returns such as stock prices (Dixit & Pindyck, 1994). Its application is also straightforward as it is mathematically tractable and can be defined with only two parameters.

On the other hand, for some commodities and financial indexes, such as interest rates that are assumed to revert to a long-term mean, Mean Reverting Models (MRM) provides are more appropriate and provide a better fit than the GBM (Bastian-Pinto, Brandão, Ozorio, & do Poço, 2021).

Government concessions frequently include risk-sharing clauses, such as minimum demand guarantees, which are managerial flexibilities with option-like characteristics. These clauses, therefore, must be valued using option pricing methods, such as the Real Options Approach, which models the underlying uncertainty of these projects stochastic processes. Thus, the choice of the type of stochastic process is fundamental for the correct modeling of the uncertainty and, therefore, of the value of the real option present in a given process.

The vast majority of the works in the evaluation of infrastructure projects adopt the GBM to model demand (Song, Yu, Jin, & Feng, 2018; Marques, Brandão, & Gomes, 2019) or traffic (Brandão & Saraiva, 2008; Liu, Bennon, Garvin, & Wang, 2017; Soliño, Galera, & Colín, 2018; Martins, Marques, & Cruz, 2014) without further analysis of the appropriateness of this process. We found only one paper in the literature that does this analysis and chooses the MRM over the GBM to model traffic demand in a road project in Colombia (Zapata Quimbayo, Mejía Vega, & Marques, 2019).

In this article, we use historical traffic data to analyze whether the GBM is the best process to model demand uncertainty for this class of projects. We test this hypothesis by applying the Unit Root and Variance Ratio tests, as suggested by Bastian-Pinto, Brandão, and Hahn (2009), and the Parameter Approach Measure (PAM) proposed by Bastian-Pinto et al. (2021), on historical demand series of five airport concession projects in Brazil. Our research was limited to these five airports since they are the only ones for which data is publicly available.

We show that GBM may not always be the best fit for demand uncertainty since demand in transportation projects is strongly affected by seasonality, which is determinant for the choice of the correct diffusion model. We also investigate the effect of the covid-19 pandemic on the choice of process, and find that when incorporating events such as this, a MRM provides a better fit for modeling passenger demand for these airports. This shows that the choice of stochastic process is not as straightforward as the extant literature in infrastructure concession projects may suggest.

This article is organized as follows. After this introduction, we provide a revision of the related literature in the field. In section 5.3, we preset the tests that support the choice of the stochastic process in infrastructure concession projects. In section 5.4, we describe the projects analyzed in this study and model their historical data series. In section 5.5, we present the results, which we discuss in section 5.6, and finally, we conclude.

5.2.
Literature review

Cox and Ross (1976) were one of the first researchers to question the assumption that stock prices follow a GBM. They state that the type of stochastic process that determines stock movements is of prime importance in valuing financial options. They suggested alternative diffusion and jump processes and provide solutions for these cases. They argue that these solutions have potential empirical applications and suggest that a comparative study should provide additional information on the approach of option valuation and which stochastic process is most appropriate for each type of stock.

According to Cortazar, Milla, and Severino (2008), there is an extensive and evolving literature on how to model the stochastic behavior of commodity futures prices. The authors claim that defining the most appropriate stochastic process for modeling commodity futures prices is relevant, as it allows estimating contract prices for which there are no market prices and provides an estimate of the volatility term structure, which is required to value option-like derivatives or to estimate risk exposures.

Stochastic modeling of commodities has evolved in relation to the number of risk factors (Brennan & Schwartz, 1985; Cortazar & Naranjo, 2006; Cortazar & Schwartz, 1994; Gibson & Schwartz, 1990); the way the drift and the factors are modeled, including seasonality, time-varying risk premiums, and mean reversion (Casassus & Collin-Dufresne, 2005; Schwartz, 1997; Sørensen, 2002); estimation procedures, including simple cross-section model calibration, Kalman filtering (Cortazar & Naranjo, 2006; Cortazar & Schwartz, 2003; Sørensen, 2002); and volatility specification (Trolle & Schwartz, 2009).

Watling and Cantarella (2015) evaluate a particular class of stochastic approaches to modeling transport systems first proposed by Cascetta (1989). They point out that this choice of stochastic processes must consider three elements: how travelers learn from their travel experiences in past times, how travelers make decisions based on their experiences, and the experiences of travelers at a given time. The authors also state that decision-makers can deal with dynamic change and uncertainty when transport systems are modeled with adequate stochastic processes.

Garvin and Cheah (2004) state that the selection of a valuation model depends critically upon the characteristics of a project's variables. For infrastructure projects, researchers often select the present value of cash flows derived from the completed project or specific operating assets as the variable of interest (Ho & Liu, 2002). However, they also may decide to model cash flow components at a more detailed level, so the value of the underlying project/asset is further decomposed into variables such as price, costs, demand, and traffic (Carbonara, Costantino & Pellegrino, 2014; Colín, Soliño & Galera, 2017). As these variables may present different behaviors and underlying distributions, this will impact the choice of the most appropriate stochastic process for their modeling and, consequently, the project valuation model.

In addition, Garvin and Cheah (2004) affirm that arguing that traffic follows a GBM or an MRM is quite a stretch. They believe that a stochastic process incorporating multi-stage growth with jumps would better represent traffic evolution. However, its mathematical complexity might not warrant the effort needed once all the required assumptions are made. They conclude that decision-makers should properly appraise project variables and carefully consider the assumptions underlying valuation methods, particularly real option models, since inappropriate applications can misinform and result in wrong choices.

However, despite the importance and contributions of this work, Garvin and Cheah (2004) do not statistically analyze the historical series of these main stochastic variables present in infrastructure projects. To the best of our knowledge, Zapata, Quimbayo, et al. (2019) are the only authors that analyze historical traffic demand data in an infrastructure project to determine the most suitable diffusion model and conclude that, for this case, traffic demand shows a mean reversion behavior. Marques, Bastian-Pinto, and Brandão (2021) argue that this occurs in that case because the data is presented on a monthly basis, which shows a seasonal characteristic of the series and, consequently, a mean reversion behavior.

Even though most authors adopt a particular stochastic process to model demand in infrastructure projects, to the best of our knowledge, there are no studies in the literature that provide support for whatever process is chosen. Thus, this article contributes to the literature by showing through different tests that the choice of stochastic process is not as straightforward as these authors may suggest.

5.3. Choice of Stochastic Process

Dixit and Pindyck (1994) have suggested that the best approach to selecting an appropriate stochastic process for modeling a variable is to rely on theoretical considerations, such as equilibrium mechanisms and statistical tests. This section presents three tests that may be used for this.

5.3.1. Unit Root Test

A frequently used test to statistically assess if a series is non-stationary is the Dickey-Fuller (DF) Unit Root test (Dickey & Fuller, 1981; Enders, 2008; Wooldridge, 2015), as shown by Bastian-Pinto, Brandão & Hahn (2009). As shown in equation (5.1), the DF test estimates a first-order autoregressive model for the lagged log of the variable under analysis and tests whether the autoregressive coefficient (*b*) is 1. Confirmation of this hypothesis implies that a non-stationary process drives the variable.

$$\ln\left(S_t\right) = a + b\ln\left(S_{t-1}\right) + \varepsilon_t \qquad (5.1)$$

where S_t is the variable under analysis (demand or traffic) in time *t*.

DF test works with the hypothesis H_0: $(b - 1) = 0$, or H_0: $b > 1$, which posits that a unit root exists and that the time series is not stationary (Dickey & Fuller, 1981). In this sense, failure to reject the null hypothesis using a *t-test* should be taken as evidence of a GBM component. However, if the null hypothesis can be rejected, there will be support for the claim of stationarity and a mean-reversion pattern in the time series. The critical values of the DF test are standardized and dependent on the number of values in the time series tested and can be obtained from Wooldridge (2015), as shown in Table 5.1.

The use of the Dickey-Fuller test (ADF) is recommended (Said & Dickey, 1984) if there is autocorrelation between the log of the variable under analysis and residues of the regression presented above. The ADF test adds lags of $\Delta \ln(S_t) = \ln(S_t) - \ln(S_{t-1})$ as explanatory variables to equation (5.1) and considers the same critical values of the DF test (Table 5.1). Furthermore, if the log-returns of $\Delta \ln(S_t)$ do not present stationarity, the literature also suggests including a determinist trend (c_t), as shown in equation (5.2). Only in this scenario (ADF with linear trend) will the critical values change, as shown in Table 5.1.

$$\ln\left(S_t\right) = a + b\ln\left(S_{t-1}\right) + c_t + \varepsilon_t \qquad (5.2)$$

Level of significance	DF & ADF (without Trend) Intercept	ADF (with linear Trend) Trend & Intercept
1%	-3.43	-3.96
2.5%	-3.12	-3.66
5%	-2.87	-3.41
10%	-2.57	-3.12

Table 5.1 – Critical values for unit root test analysis

Source: Wooldridge (2015).

5.3.2. Variance Ratio Test

Another approach that can be used to define the most appropriate stochastic process to model a variable is the variance ratio test suggested by Pindyck (1999), who states that investigating the extent to which price shocks are permanent can be more informative than looking for a unit root in the random walk or mean reversion check. This test verifies whether the variance of the logarithm of the variable under analysis increases proportionally over time, that is, the persistence of stochastic shocks.

$$R_k = \frac{1}{k}\frac{Var\left[\ln\left(S_{t+k}\right)-\ln\left(S_t\right)\right]}{Var\left[\ln\left(S_{t+1}\right)-\ln\left(S_t\right)\right]} \tag{5.3}$$

where R_k is the variance ratio, and *Var*[.] terms indicate the variance of the series of lagged (by *k* periods) differences in the natural logarithm of the variable under analysis (demand/traffic).

In the case of MRM, shocks tend to dissipate due to the reversal trend and we expect the variance to be bounded as *k* increases, tending to zero as *k* tends to infinity. On the other hand, in the case of GBM, price shocks are persistent and we expect the variance to increase linearly with *k* and the ratio R_k to approach 1 as *k* increases.

5.3.3. Parameter Approach Measure

Although the unit root and variance ratio tests help verify the behavior of the series, they do not allow a clear indication of the adequacy of a given stochastic model. The ADF tests, for example, considers the persistence, or stationarity, of a series, which is only one of the characteristics of a GBM. This can lead to an incorrect judgment on the adequacy of the stochastic process (Pindyck, 1999).

On the other hand, the Variance Ratio test is an indicator of the behavior of the variance of a time series and tests whether the variance of a series grows continually with *t* or is bounded within a certain limit (Pindyck, 1999). Bastian-Pinto et al. (2021), however, showed that a significant number of trajectories generated by a GBM simulation were identified as a MRM in the variance ratio test, which indicates a limitation of this test.

Given this, Bastian-Pinto et al. (2021) develop a parameter based approach to stochastic process selection that overcomes some of the limitations of these traditional tests. The Parameter Approach Measure (PAM), as opposed to the usual approach of assuming GBM behavior and testing its robustness, involves determining a suitable MRM model, choosing a parameter calibration procedure for this particular MRM and for the time series involved.

Considering the characteristics of a GBM and a MRM modeling and examining equations (5.4) and (5.5), we can see that the main difference between them rests in the presence of the mean reversion speed (η).

$$dS_t = \mu S_t dt + \sigma S_t dz \tag{5.4}$$

where S_t is the asset price at time t; μ is the drift; σ is the volatility; dt is the time increment, and dz is the standard Wiener increment, where $dz = \varepsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$, and

$$dS = \eta\left[\ln \bar{S} - \ln S\right] S dt + \sigma S dz \tag{5.5}$$

where η is the speed of reversion; σ the volatility; $\ln \bar{S}$ is a measure of the equilibrium level, $\bar{S}$ is the long-term equilibrium level in the same unit as S.

Note that an MRM with $\eta = 0$, or even a significantly small one, is equivalent to a GBM with a drift of zero ($\mu = 0$). The mean reversion speed can be estimated from time series data, but this value alone cannot indicate its intensity. Bastian-Pinto et al. (2021) solve this problem by converting the reversion speed to the half-life ($T_{1/2}$) of the process, as shown in equation (5.6):

$$T_{1/2} = \frac{\ln 2}{\eta} \tag{5.6}$$

Bastian-Pinto et al. (2021) also propose the Normalized Variance (NVar) estimation, as shown in equation (5.7). The authors explain that the reason for the use of the NVar is that the higher this equilibrium value is, the longer it takes for the variance of the series to stabilize under the mean reversion effect, thus the more it resembles a GBM where the variance grows indefinitely with t.

$$NVar = \frac{\sigma^2}{2\eta} \tag{5.7}$$

Finally, considering these both measures described in equations (5.6) and (5.7), Bastian-Pinto et al. (2021) propose the PAM, as shown in equation (5.8):

$$PAM = \frac{\ln 2}{2}\left(\frac{\sigma}{\eta}\right)^2 \tag{5.8}$$

The PAM is the product of these measures and considers the two dimensions of the stochastic behavior of the process rather than a single one, as in the case of the ADF and Variance Ratio tests. Thus, the lower the PAM value, the stronger the MRM characteristics present in the series, which can be a clear indication of the behavior of the MRM in a time series as opposed to the GBM (Bastian-Pinto et al., 2021).

5.4. Airport Concession Projects in Brazil

In this section, we present the concession projects that are analyzed in this research. We only consider Brazilian airport concessions for which data is publicly available.

Since Zapata Quimbayo et al. (2019) found a significant effect from seasonal demand on road traffic in Colombia, it seems that passenger traffic demand is frequently subject to seasonality, which may affect the uncertain behavior model of these variables. As it is possible that this may also be the case with airport passenger demand, we analyzed all airports with historic traffic series and adjusted for seasonality using EViews® software, as done in Bastian-Pinto et al. (2021).

Also, because of the significant effect of the covid-19 pandemic on traffic demand globally, we analyzed two samples of data to account for the effect of the covid-19 pandemic on the results analysis of best fit of stochastic process for the periods studied. The first period encompasses the data available from each airport up to February 2020, therefore, ex-ante the covid-19 pandemic effect, and the second up to July 2022, ex-post the covid-19 pandemic effect.

Table 5.2 shows that 44 airports were granted in Brazil since 2012 (ANAC, 2022). However, given the availability of passenger demand data, we will focus our study on just five of these airports (Viracopos, Natal, Guarulhos, Brasília and Confins).

Airports / Blocks	Contract Start	Concession Term (years)	Contract Termination
Viracopos	2012	30	2042
Natal	2012	28	2040
Guarulhos	2012	20	2032
Brasília	2012	25	2037
Galeão	2014	25	2039
Confins	2014	30	2044
Salvador	2017	30	2047
Porto Alegre	2017	25	2042
Fortaleza	2017	30	2047
Florianópolis	2017	30	2047
Central West (4 airports)	2019	30	2049
Northeast (6 airports)	2019	30	2049
Southeast (2 airports)	2019	30	2049
South (9 airports)	2021	30	2051
North (7 airports)	2021	30	2051
Central (6 airports)	2021	30	2051

Table 5.2 – Airports Granted in Brazil (2012 – 2022)

Source: ANAC (2022).

The first airports granted were Viracopos (VCP), Natal (SBNT), Guarulhos (GRU), and Brasília (BSB), and in 2013 it was the turn of the airport of Belo Horizonte (CNF – Confins). In all five cases, Infraero retained a 49% share in the concession. The Guarulhos airport was awarded to the Invepar consortium for US$ 7,791 million, representing a premium of 374% over the minimum price established by ANAC at the time. The bid for Viracopos airport was won by the Aeroportos Brasil consortium, which offered US$ 1,836 million, a premium of 160%, wh. In contrast, Brasília and Natal airports were awarded to Inframérica Aeroportos consortium, with winning bids of US$ 2,163 million and US$ 79 million, respectively. The Confins airport was awarded to the BH Airport consortium for US$ 847 million, which represented a premium of 66%.

Despite the success in the auctions, some of these concessions went into default in the following years. The political and economic crisis in Brazil significantly reduced passenger demand and some concessionaires had problems meeting their financial commitments (Marques et al., 2019). In July 2018, of these five airports, only Guarulhos (GRU) was up to date with its obligations, and Viracopos (VCP) was under threat of bankruptcy (ANAC, 2022).

As the financial result of these contracts basically depends on passenger demand, we begin our analysis of which stochastic process is most appropriate to model this variable by presenting in Figure 5.1 and Figure 5.2 the number of passengers per month in each of these five airports for the two time frames analyzed, which we name the ex-ante and ex-post pandemic cases. The data were obtained from the ANAC website (ANAC, 2022) and comprise the following time periods. For the ex-ante period: Viracopos (VCP, January 2013 – February 2020), Natal (SBNT, June 2014 – February 2020), Guarulhos (GRU, January 2013 – February 2020), Brasília (BSB, January 2013 – February 2020) and Confins (CNF, August 2014 – February 2020). For the ex-post period, the starting dates for all five airports is the same, but all ending in July 2022, therefore after the covid-19 pandemic effect.

A visual observation of the time series of traffic demand would suggest a strong seasonal behavior of these but with different degrees of intensity: SNBT appears to be much more subject to seasonality than the others possibly due to the tourism characteristic of Natal – RN. This could affect a mean reversion intensity of this airport traffic. In addition, some airports appear to have regained pre-covid-19 levels of traffic faster than others, such as VCP. Again, this might be due to the intensity of a stationary effect of the demand in this airport, as its main characteristic is to be a hub for domestic and international air traffic in Brazil and this could be the reason for such a rapid return to normal operation levels compared to the others.

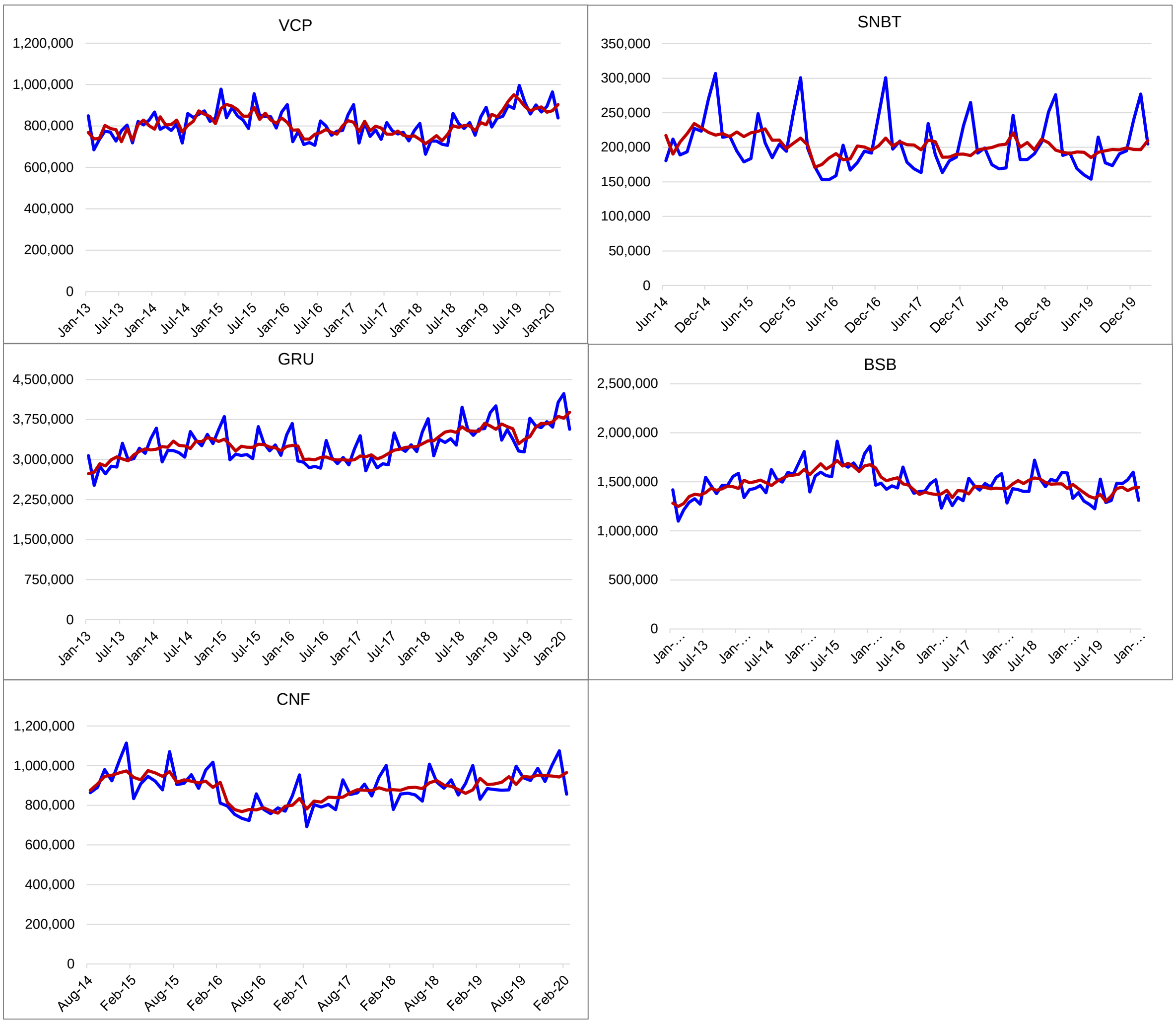

Figure 5.1 – Number of passengers per month for the ex-ante pandemic case

Note: The dataset includes the number of passengers per month from the following Brazilian airports: Viracopos (VCP, January 2013 – February 2020), Natal (SBNT, June 2014 February 2020), Guarulhos (GRU, January 2013 – February 2020), Brasília (BSB, January 2013 – February 2020) and Confins (CNF, August 2014 – February 2020). Source: ANTT (2022). In blue, we present the original demand series, and in red the demand series adjusted for seasonality in EViews®.

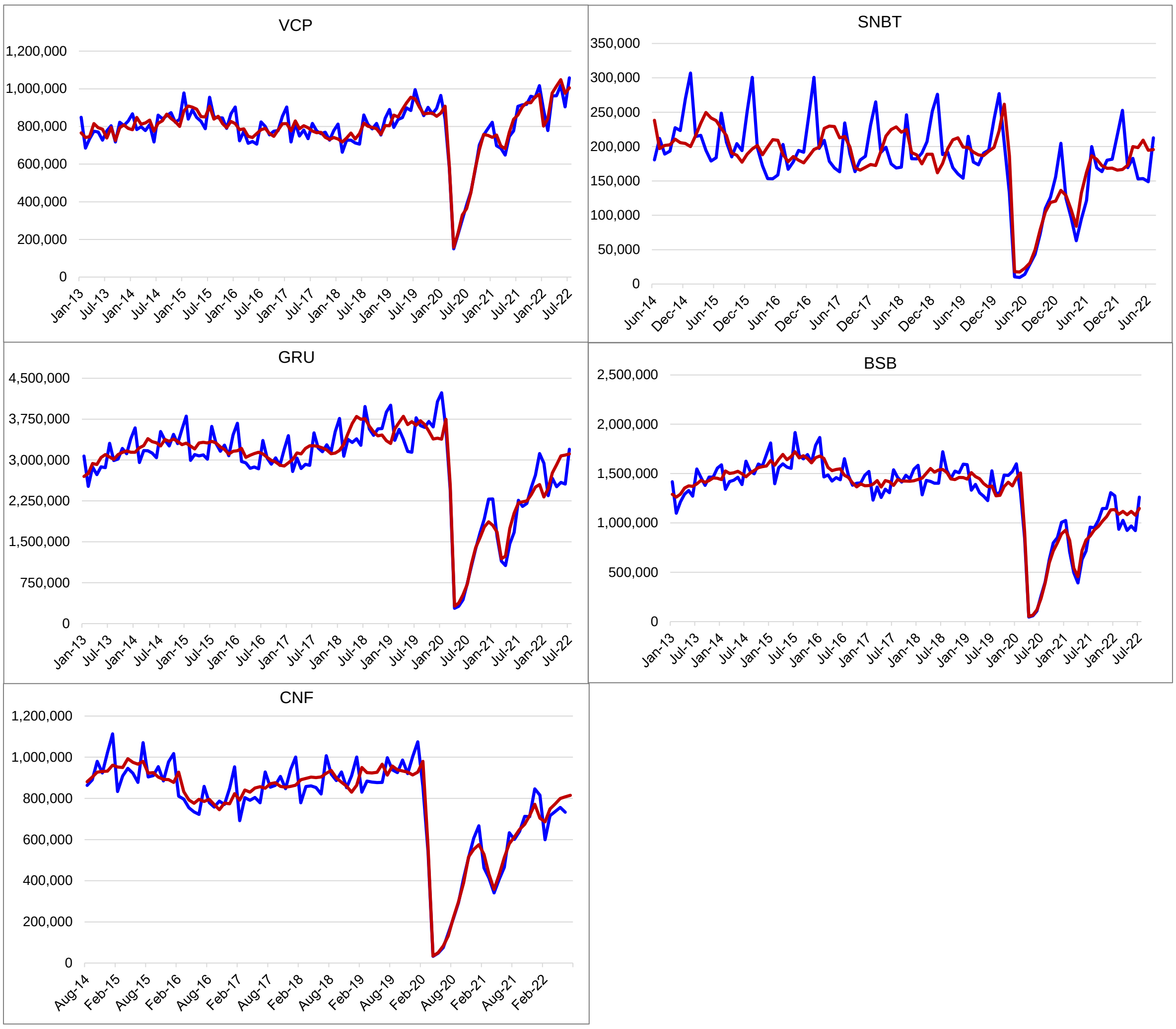

Figure 5.2 – Number of passengers per month for the ex-post pandemic case

Note: The dataset includes the number of passengers per month from the same Brazilian airports as in Figure 5.1, with the same starting dates. Ending dates are July 2022, for all airports. Source: ANTT (2022). In blue, we present the original demand series, and in red the demand series adjusted for seasonality in EViews®.

5.5. Results

5.5.1. Passenger Demand Series ex-ante covid-19 pandemic

All five series (log) were tested for unit root presence, using the ADF in EViews® software: first, an ADF test is applied considering trend (linear) and intercept. In these results, we check the trend significance to determine if we can use the t-statistic to verify the rejection of a unit root. If trend does not show statistical significance, we test again only with intercept and verify the t-statistic again. The results for ex-ante pandemic series are shown in Table 5.3.

	Trend (linear) & Intercept			Intercept		ADF
	Trend Prob.	t-Statistic	Prob	t-Statistic	Prob	Analysis
VCP	0.4927	-4.962059	0.0007	-3.48531	0.0107	MRM
SNBT	0.4391	-4.654971	0.0019	-4.91831	0.0001	MRM
GRU	0.0207	-4.400044	0.0037			MRM
BSB	0.2432	-4.214778	0.0066	-4.13381	0.0015	MRM
CNF	0.7399	-3.478547	0.0502	-3.48992	0.0113	MRM

Table 5.3 – Unit root ADF tests for the five airports ex-ante pandemic with original data

Trend only shows significance in the demand series of GRU, which presents a t-Statistic value that keeps it within the significance value of 1%, indicating a MRM behavior. As the trend did not show statistical significance in the other four series, we test again only with intercept and verify that demand series of VCP, SNBT, BSB, and CNF reject the presence of a unit root at around 1% level of significance, which also suggests an MRM behavior.

We then perform the same test considering the series corrected for the seasonality effect. Now, only the demand series of BSB shows a significant trend and a *t*-Statistic value that rejects the presence of a unit root, indicating a GBM behavior. Considering only the intercept, we find that all airport series reject the presence of a unit root, indicating a GBM behavior, therefore diverging from the results of previous analysis. Only the demand series of VCP rejects the presence of a unit root at a 7.5% level of significance, indicating some level of MRM behavior without the seasonal effect.

	Trend (linear) & Intercept			Intercept		ADF
	Trend Prob.	t-Statistic	Prob	t-Statistic	Prob	Analysis
VCP	0.4927	-2.056709	0.5618	-1.99776	0.2874	GBM
SNBT	0.1206	-3.176894	0.0979	-2.72867	0.0745	Undefined
GRU	0.2105	-1.879944	0.6561	-1.38812	0.5844	GBM
BSB	0.0450	-2.893887	0.1698			GBM
CNF	0.3407	-1.227935	0.8961	-1.18750	0.6751	GBM

Table 5.4 – Unit root ADF tests for the five airports ex-ante covid-19 after correcting for seasonality

The Variance Ratio test is also applied to the original demand series (VCP, BSB, CNF, SNBT, and GRU) and to the demand series corrected for seasonality (VCP(S), BSB(S), CNF(S), SNBT(S), and GRU(S)) for the ex-ante covid-19 situation. Figure 5.3 shows the results of this test, considering 50 monthly lags, using the EViews® Variance Ratio function, which corrects the bias of the test for limited size samples. To verify stochastic process adequacy from this test, we proceeded as follows: we considered that if after the first few months, the variance ratio is either bellow or close to 1, and in the subsequent months, it permanently decreases bellow 1, we assume that the variance ratio indicates the presence of a mean reversion. For the series that clearly do not decrease with lag increment or do not behave as described, we assume that the test does not reject the presence of GBM. For the remaining cases, we assume the conclusions are undefined.

In the cases of the five airports series of ex-ante pandemic original demand, the results indicate that all series can be modeled with MRM-type dynamics. Yet when corrected for seasonality, only SNBT maintains a MRM behavior, and VCP only partially so. The other three demonstrate a clear GBM behavior, partially confirming the results obtained with the unit root analysis.

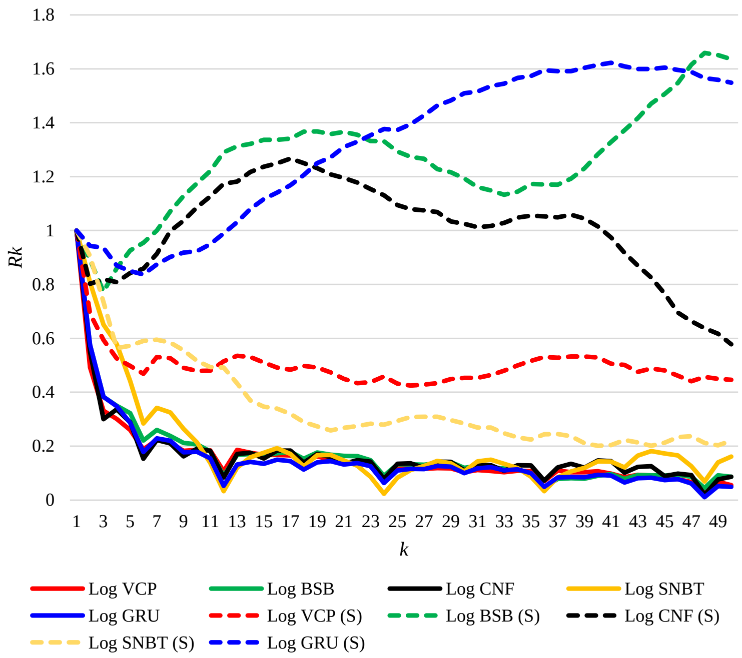

Figure 5.3 – Variance ratio tests of demand series in monthly data ex-ante covid-19

Note: Variance ratio tests of demand series in monthly data ex-ante covid-19 pandemic – for original data (solid lines) and adjusted for seasonality (dashed lines).

Table 5.5 presents a summary of the results of PAM analysis for the five original demand series. Like the variance ratio test, the results indicate that all the five series are clearly a MRM, since all of them present a PAM value below 0.2, which is the critical value defined by Bastian-Pinto et al. (2021) in this approach.

	GBM		MRM		Half-Life $T_{1/2}$	NVar	PAM
	Drift μ	Vol σ	Rev Sp η	Vol σ	Years	$\sigma^2/2\eta$	$T_{1/2}$ x Nvar
VCP	-0.17%	31.1%	9.93	38.1%	0.07	0.007	0.001
SNBT	2.22%	63.8%	10.41	79.1%	0.07	0.030	0.002
GRU	2.11%	33.2%	7.18	38.7%	0.10	0.010	0.001
BSB	-1.11%	35.0%	8.58	42.1%	0.08	0.010	0.001
CNF	-0.15%	38.2%	11.28	48.2%	0.06	0.010	0.001

Table 5.5 – Results of PAM considering the original demand series ex-ante covid-19

Table 5.6 presents a summary of the results of PAM analysis for the five demand series corrected for the seasonality effect. As all of them have a PAM lower than 0.2, the most appropriate stochastic process for modeling the analyzed variables is the MRM, contrary to the analysis with ADF and Variance Ratio tests. The intensity of the reversion parameters causes these divergent results. The reversion speed parameter (η), which is exceptionally high in the original series, is reduced when excluding the seasonality effect, but not so much as to change significantly the Half-Life measure. In Table 5.6, we can see that these values are still significantly small, which is a characteristic of mean reversion. The decrease in volatility, due to the exclusion of the seasonality, also reduces the normalized variance measure, maintaining the PAM values well below the threshold for MRM.

	GBM		MRM		Half-Life $T_{1/2}$	NVar	PAM
	Drift μ	Vol σ	Rev Sp η	Vol σ	Years	$\sigma^2/2\eta$	$T_{1/2}$ x Nvar
VCP	2.29%	13.6%	2.25	14.5%	0.31	0.005	0.001
SNBT	-0.61%	17.1%	4.28	18.9%	0.16	0.004	0.001
GRU	4.95%	7.8%	0.62	8.0%	1.11	0.005	0.006
BSB	1.69%	9.2%	1.20	9.4%	0.58	0.004	0.002
CNF	1.78%	10.3%	1.04	10.7%	0.67	0.006	0.004

Table 5.6 – Results of PAM considering the original demand series corrected for seasonality ex-ante covid-19

We estimated the correlation between traffic demand for all six possible pairs of the five airports for the ex-ante pandemic period original series and corrected for seasonality. These are shown in Table 5.7. We can observe that for the original series, the correlation for all ten pairs is exceptionally high, especially between CNF x GRU and GRU x SNBT. All pairs demonstrate an almost full correlation of traffic demand. However, when the same factor is estimated after correction for seasonality, these are reduced significantly, and for one pair (GRU x VCP), being almost zero. These results point to the fact that the correlation of airport traffic is primarily due to the seasonal characteristic of this type of passenger traffic demand.

Original					
Correlation	VCP	SNBT	GRU	BSB	CNF
VCP	1.000				
SNBT	0.829	1.000			
GRU	0.893	0.935	1.000		
BSB	0.898	0.887	0.939	1.000	
CNF	0.923	0.881	0.949	0.922	1.000
Corrected for Seasonality					
Correlation	VCP (S)	SNBT (S)	GRU (S)	BSB (S)	CNF (S)
VCP (S)	1.000				
SNBT (S)	0.023	1.000			
GRU (S)	0.003	0.483	1.000		
BSB (S)	0.424	0.230	0.350	1.000	
CNF (S)	0.414	0.236	0.472	0.388	1.000

Table 5.7 – Correlation of demand series ex-ante covid-19

5.5.2. Passenger Demand Series ex-post covid-19 pandemic

We proceeded with the same sample of tests as with the ex-ante demand series for the ex-post period series in their original data and corrected for seasonality. First, we test the series with ADF tests, with results shown in Tables 5.8 (original) and Table 5.9 (corrected for seasonality). We then apply the variance ratio tests for both versions of the data. These are plotted in Figure 5.4. As done with the ex-ante series, we proceeded with the PAM approach, and results are shown in Tables 5.10 and 5.11. Finally, we estimate the correlation factors for the ex-post series as done with the ex-ante series and list these in Table 5.12.

	Trend (linear) & Intercept			Intercept		ADF
	Trend Prob.	t-Statistic	Prob	t-Statistic	Prob	Analysis
VCP	0.7915	-3.789365	0.0007	-3.80126	0.0039	MRM
SNBT	0.2252	-3.991276	0.0121	-3.80074	0.0040	MRM
GRU	0.1241	-4.023471	0.0105	-3.68980	0.0055	MRM
BSB	0.0501	-4.376973	0.0035			MRM
CNF	0.173	-3.620576	0.0334	-3.33668	0.0160	MRM

Table 5.8 – Unit root tests for the five airports ex-post pandemic with original data

	Trend (linear) & Intercept			Intercept		ADF
	Trend Prob.	t-Statistic	Prob	t-Statistic	Prob	Analysis
VCP	0.7682	-4.027485	0.0104	-4.03915	0.0018	MRM
SNBT	0.1862	-3.737195	0.0245	-3.480666	0.0106	MRM
GRU	0.1264	-3.866771	0.0166	-3.866771	0.0166	MRM
BSB	0.0432	-4.424522	0.003			MRM
CNF	0.1884	-3.690037	0.0278	-3.435627	0.0121	MRM

Table 5.9 – Unit root tests for the five airports ex-post covid-19 after correcting for seasonality

These new analyses show that even after correcting for seasonality, all series of traffic maintain an MRM behavior in the ADF tests, Variance ratio tests, and PAM values. Also, the correlation factors for all pairs of airports are mostly unaffected by the correction for seasonality, contrary to what occurred in the same analysis with the ex-ante series.

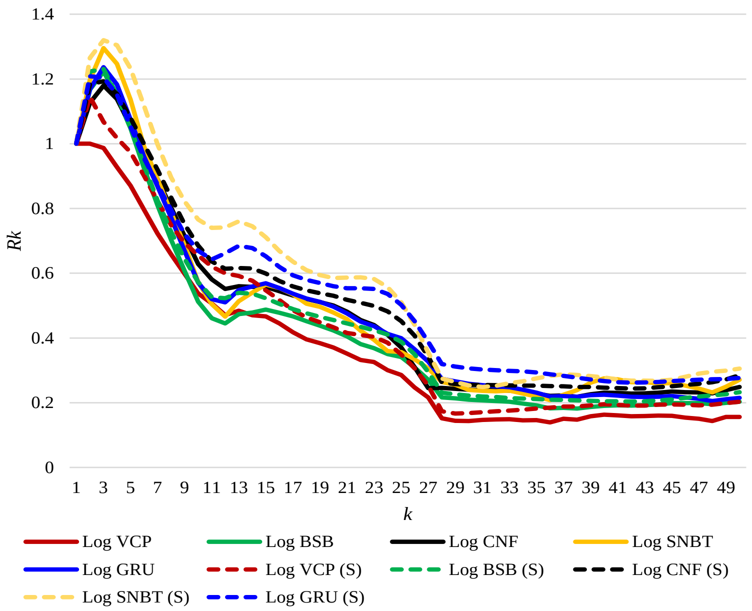

Figure 5.4 – Variance ratio tests of demand series in monthly data ex-post covid-19

Note: Variance ratio tests of demand series in monthly data ex-post covid-19 pandemic – for original data (solid lines) and adjusted for seasonality (dashed lines).

	GBM		MRM		Half-Life $T_{1/2}$	NVar	PAM
	Drift μ	Vol σ	Rev Sp *η*	Vol σ	Years	$\sigma^2/2\eta$	$T_{1/2}$ x Nvar
VCP	2.32%	58.7%	2.94	62.94%	0.24	0.067	0.016
SNBT	2.03%	120.3%	2.24	127.30%	0.31	0.361	0.111
GRU	0.42%	85.5%	1.95	89.77%	0.35	0.206	0.073
BSB	-1.24%	113.2%	2.11	119.26%	0.33	0.337	0.111
CNF	-2.07%	115.8%	2.14	122.23%	0.32	0.348	0.113

Table 5.10 – Results of PAM considering the original demand series ex-post covid-19

	GBM		MRM		Half-Life $T_{1/2}$	NVar	PAM
	Drift μ	Vol σ	Rev Sp *η*	Vol σ	Years	$\sigma^2/2\eta$	$T_{1/2}$ x Nvar
VCP	2.86%	52.0%	2.44	55.18%	0.28	0.062	0.018
SNBT	-2.33%	95.2%	1.73	99.63%	0.40	0.288	0.116
GRU	1.50%	74.7%	1.67	78.06%	0.41	0.182	0.076
BSB	-1.23%	104.4%	1.91	109.54%	0.36	0.314	0.114
CNF	-0.98%	109.7%	2.00	115.53%	0.35	0.334	0.116

Table 5.11 – Results of PAM considering the original demand series corrected for seasonality ex- post covid-19

Original					
Correlation	VCP	SNBT	GRU	BSB	CNF
VCP	1.000				
SNBT	0.895	1.000			
GRU	0.934	0.956	1.000		
BSB	0.928	0.941	0.985	1.000	
CNF	0.951	0.933	0.979	0.977	1.000
Corrected for Seasonality					
Correlation	VCP (S)	SNBT (S)	GRU (S)	BSB (S)	CNF (S)
VCP (S)	1.000				
SNBT (S)	0.894	1.000			
GRU (S)	0.932	0.964	1.000		
BSB (S)	0.926	0.962	0.983	1.000	
CNF (S)	0.950	0.953	0.980	0.983	1.000

Table 5.12 – Correlation of demand series ex-post covid-19

5.6. Discussion

We analyzed and ran statistical tests on historical data series of demand for a sample of five major Brazilian airports that were granted to private investors through concession contracts since 2010. The results indicate that airport traffic demand displays a significant seasonal behavior. This introduces a strong mean reverting component in the stochastic demand model, especially for time frames of less than one year. This fact has important consequences to contract clauses that may be part of the concession agreement. Although this has not been the case with airport concessions, minimum demand guarantee clauses in transportation infrastructure projects are significantly affected when traffic demand has a mean reverting component rather than the GBM model that is usually assumed.

As shown in the literature review section, most studies concerning real options applied to infrastructure concessions assume that demand follows a GBM stochastic diffusion process. The observed preference for GBM in the literature is because it is a much simpler process to model and, in many cases, allows for an analytical solution. Modeling real options with mean reversion requires significantly more complex and computer intensive methodologies, such as those used by Bastian-Pinto et al. (2009). Nonetheless, the value of the real option in a GBM model is typically significantly greater than with an MRM model for similar values of volatility (Bastian-Pinto et al., 2021), which suggests that the results of some of these works may be overestimated if the main source of uncertainty does not follow a GBM as assumed.

After correcting for seasonality, the tests indicate that demand does behave as a GBM, although some series still display some degree of mean reversion. For the ADF tests, the ex-ante original series indicate MRM. However, when corrected for seasonality, the unit root for most airport series cannot be rejected, therefore suggesting that the series follow a GBM. The Variance Ratio analysis shows a similar effect: ex-ante series for most airport changes from MRM to GBM when corrected for seasonality, exception of VRC and NBT, which are inconclusive. Therefore, the same care must be taken to correctly estimate the diffusion process to be used. We also show that removing the seasonality effect changes the correlation between passenger demand for different airports. This allows a better understanding of the relationship between the demand for different routes and regions in the country and to better plan incentives and strategic decisions. Correlation analysis of the ex-ante series shows a very high correlation between demand in all airports and a sharp reduction (almost to zero for VRC x SBNT and GRU) when corrected for seasonality, implying that the microeconomic drivers for these airports are quite different.

Our results indicate that the covid-19 pandemic caused a disruptive effect that significantly altered statistical readings. Such a disruptive event on global demand affects most statistical analyses. The negative spike in the year 2020, which may be compared to a downwards Poisson jump with the rapid almost return to normality in 2022, changes the modeling of any tool for reading such an event. It introduces a "reversion" behavior in the time series in virtue of the "return to normality" after the event that is present in the ADF indicators of ex-post series, both in the original as well as the corrected for seasonality and all airports maintain their MRM behavior. The same happens in the Variance Ratio analysis. The correlation analysis of the series ex-post shows almost no change in correlation measures when correcting for seasonality. Again, a pandemic effect is worth considering.

It would appear that disruptive events such as the covid-19 pandemic, which caused a temporary but reversible effect on global demand, can change the approach for stochastically modeling traffic demand uncertainty. Another such event could also be the financial crisis of 2008, which, although of a very different nature than the covid-19 pandemic, also had important yet temporary effects on all business aspects of the global economy. Incorporation of a jump factor may undoubtedly be an interesting approach for dealing with such events but including more than a single factor in stochastic modeling exponentially increases the difficulty of valuation with real options, for instance, not to mention the difficulty in estimating the correct parameters for such a jump factor. Even with this approach, the main model to be used should still be an MRM since after the disruptive event has passed, the microeconomic factor would bring back demand to its original level.

5.7. Conclusion

In this paper, we studied the appropriateness of using the GBM stochastic diffusion process to model demand uncertainty in airport concessions in Brazil. Contrary to the widespread practice of modeling demand as a GBM for real options applications in infrastructure concessions, our results suggest that effects such as seasonality or disruption events can make an auto-regressive model such as MRM more appropriate than a GBM.

This can be important to government and concessionaire alike since it may substantially change the valuation of contract clauses embedded in the concession terms and substantially impact the return and the risk of the concession.

For the government, an incorrect stochastic model of the future demand may over or undervalue the cost to the government of any risk-mitigating mechanisms that may be granted to the concessionaire. For the concessionaire, the wrong choice of stochastic process may not result in the expected risk reduction in such a capital investment venture.

Although the results presented do share some light on the matter, the objective of this work is to bring attention to such a matter of importance in modeling real options applied to concessions around the globe. Our results are limited, as only a few airport concessions have traffic data available publicly in Brazil. Expansion of this work could involve comparing our results to those of other countries, expanding the number of cases comparing domestic and international demand, and applying the framework shown here to other types of infrastructure concessions such as roads, trains, urban trains, ports, and others.

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6
Final Remarks

After a careful review of academic works that apply the real options approach to the evaluation of infrastructure concession projects and identifying the main gaps in this literature, we developed this doctoral thesis, composed of four independent studies. The objective of the first study was to propose a code in an open-source software with intuitive guidelines to help researchers and practitioners model real options lattices from project cash flows. The objective of the second study was to show why additional investments in expansion as firm obligations in concession contracts are suboptimal and propose a real options model that combines flexible capacity expansion decisions with conditional term extensions. The objective of the third study was to apply the real options approach to analyze the case of the Salvador Light Rail Vehicle (LRV) concession project and investigate how the different flexible clauses embedded in the contract interact and impact the overall valuation of the project. Finally, the objective of the fourth study was to use unit root and variance ratio tests and the Parameter Approach Measure (PAM) to evaluate which would be the most appropriate stochastic process to model this uncertainty in real cases of airport concessions, considering samples ex-ante and ex-post covid-19 pandemic and the impact of seasonality.

In this sense, we believe that this thesis contributed to the development of the field of study of infrastructure economics in several ways. First, we proposed a tutorial that provides a simple mechanism for analyzing investment opportunities in projects that have uncertainty and flexibility. Second, we considered the fact that concession revenues are bounded by the current traffic capacity of the road, which represents an upper absorbing barrier that has implications for the expansion decision. Third, we evaluated how the option to expand capacity coupled with a term extension increases the probability of a timely and voluntary expansion, allowing the granting authority to elaborate low-cost contractual clauses that align the objectives of both public and private agents. Fourth, we evaluated the interaction of a bundle of European call and put options created by the cap and floor mechanism, with an American call option arising from the flexible expansion and term extension clauses. Fifth, we proposed a model based on the Cox, Ross, and Rubinstein (1979) lattice approach, rather than on simulation methods, which are more common for valuing the interaction of a bundle of European call and put options with an American call option. Sixth, we showed how the clauses that govern the managerial flexibilities in contracts must be carefully designed to achieve the objectives of both government and private investors. Seventh, we showed that the choice of stochastic process is not as straightforward as the extant literature in the field of infrastructure concession projects may suggest.

7
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8 Appendix

Appendix I – Summary of reviewed articles

Authors	Year	Country	Project type	Option	Valuation approach	Uncertainty	Uncertainty modeling
Adkins & Paxson	2017	-	-	Timing	Continuous	Project	GBM
Aldrete et al.	2012	US	Toll road	MRG	Simulation	Revenue	Generic
Almassi et al.	2013	-	-	Guarantee	Continuous Numerical	Demand	Markov process
Alonso-Conde et al.	2007	Australia	Toll road	Timing Abandon	Simulation	IR	GBM
Arboleda & Abraham	2006	US	Water and sewer systems	Timing Abandon	Numerical Simulation	Project	GBM
Asao et al.	2013	Philippines	Toll road	PARG, PCRG	Simulation	Revenue	GBM
Ashuri et al.	2012	Korea	Highway	MRG	Numerical Simulation	Traffic	GBM
Attarzadeh et al.	2017	Iran	Freeway Power plant	Revenue collar Guarantee	Continuous Simulation	Revenue	Triangular fuzzy
Balliauw et al.	2020	-	Port	Timing	Continuous	Demand	GBM
Balliauw & Onghena	2020	Belgium	Airport	Expand	Continuous	Demand	GBM
Bian et al.	2021	China	Retrofit project	Timing	Simulation	Energy amount and price	GBM
Blank et al.	2016	Brazil	Toll road	Traffic collar Abandon	Continuous Simulation	Traffic	GBM
Bowe & Lee	2004	Taiwan	High-speed rail	Expand Timing Abandon Contract	Numerical	Project value	GBM
Brandão & Saraiva	2008	Brazil	Toll road	MTG	Simulation	Traffic Revenue	GBM
Brandão et al.	2012	Brazil	Subway	Demand collar	Simulation	Demand	GBM
Buso et al.	2021	-	-	Timing	Continuous	Cash Flow	GBM
Buyukyoran & Gundes	2018	-	Toll road	Revenue collar	Simulation	Traffic	GBM
Cabral & Silva Jr	2013	Brazil	Sporting event facilities	MCFG	Numerical	Project	GBM

Carbonara & Pellegrino	2018	Italy	Bridge	Revenue collar	Simulation	Traffic	GBM
Carbonara et al.	2014	Italy	Port	Term	Simulation	Traffic	GBM
Carbonara et al.	2014	Italy	Toll road	MRG	Simulation	Traffic	GBM
Cerqueti & Ventura	2020	-	Oil Exploitation	Timing	Continuous	Cash Flow	GBM
Chavanasporn & Ewald	2010	-	-	Timing Abandon	Continuous	Project	MRP GBM
Cheah & Liu	2005	India	Power station	Expand, Switch Abandon Guarantee Extend	Numerical	Natural gas prices	GBM
Chen et al.	2018	China	Toll road	TAM	Numerical	Toll	Generic
Chiara & Garvin	2007	-	Toll road	MRG	Simulation	Revenue	Generic
Chiara & Kokkaew	2013	US	Toll road	MRG	Simulation	Revenue	GBM
Chiara et al.	2007	-	Toll road	MRG	Simulation	Revenue	Generic
Colín et al.	2017	Spain	Motorway	Abandon	Simulation	Traffic	GBM
Cruz & Marques	2013	Portugal	Hospital	Switch	Numerical Simulation	Demand	GBM
Cui et al.	2008	US	Highway	Guarantee	Numerical	Cost	GBM
Cui et al.	2004	US	Highway	Guarantee	Continuous	Failure cost	GBM
D'Alpaos & Marella	2014	Italy	Urban infrast.	Timing	Continuous	Cash Flow	GBM
D'Alpaos et al.	2006	Italy	Water Abstraction Plant	Term Timing	Continuous	O&M cost	GBM
Defilippi, E.	2004	Peru	Port	Timing	Simulation	Tariff, costs	Normal dist.
Doan & Menyah	2013	-	Toll road	Timing	Numerical	Traffic O&M cost	MRP GBM
Di Maddaloni et al.	2022	Italy	Subway	Expand	Continuous	Project value	GBM
Fitch et al.	2018	-	Toll road	Abandon MRG	Continuous Simulation	Project Revenue	GBM
Galera & Soliño	2010	Spain	Highway	MTG	Continuous	Traffic	GBM
Galera et al.	2018	Spain	Toll road	MTG	Continuous Simulation	Traffic	GBM
Gao & Liu	2020	-	-	Abandon	Numerical Simulation	Project	-
Garvin & Cheah	2004	US	Toll road	Timing	Numerical	Traffic	GBM

Gaudard	2015	Switzerland	Hydroelectric energy storage	Timing Abandon	Simulation	Cash Flow	GBM
Geng et al.	2022	China	-	Abandon	Continuous	Cash Flow	GBM
Guo et al.	2021	China	Tunnel	Term	Numerical	Project	GBM
Ho & Liu	2002	Canada	Airport	Guarantee	Numerical	Project O&M cost	GBM
Huang & Chou	2006	Taiwan	High-speed rail	MRG Abandon	Continuous	Revenue	GBM
Huang & Pi	2014	Taiwan	High-speed rail	Abandon	Continuous	Project	GBM
Huang et al.	2021	China	Highway	MRG	Simulation	Demand	GBM
Iyer & Sagheer	2011	India	Highway	Traffic collar	Numerical	Traffic	GBM
Jeong et al.	2016	South Korea	Highway	Timing	Simulation	Investment Revenue Risk-free IR	Log-logistic Logistic Triangular dist.
Jin et al.	2021	China	Highway	MRG Term	Simulation	Invest, O&M cost Traffic	Beta, Even dist., GBM
Kauppinen et al.	2018	-	Innovation	Timing	Continuous Numerical	Project O&M cost	GBM
Kim et al.	2019	Korea	Railway	MRG	Simulation	Traffic	GBM
Kim & Li	2020	Canada	Toll road	Timing	Numerical	Climate change	GBM
Kitabatake	2002	Japan	Road	Abandon	Numerical	Project	GBM
Kokkaew & Chiara	2013	-	Highway	MRG	Simulation	Revenue	Variance model
Krüger	2012	Sweden	Road	Expand	Numerical	Traffic	GBM
Li et al.	2016	China	Public rental housing	Abandon Transfer Expand	Numerical	Project	GBM
Liang & Ashuri	2020	China	Highway	MRG	Numerical Simulation	Traffic	GBM
Liu et al.	2020	China	Road	Expand	Continuous	Project	GBM
Liu et al.	2019	China	Treatment plant	Guarantee	Simulation	Revenue	GBM MRP
Liu et al.	2018	Australia	Public rental housing	Term	Continuous	Demand	GBM
Liu et al.	2017	-	Toll road	MRG, Term Fiscal support	Simulation	Traffic	GBM
Lomoro et al.	2020	Italy	Treatment plant	Guarantee	Simulation	Quantity	MRP

Lv et al.	2015	China	Transportation	Term	Continuous	Traffic	GBM
Ma et al.	2018	China	Treatment plant	Guarantee Term	Simulation	Capacity	GBM
Man et al.	2016	China	Road	MROIG	Continuous	Project	GBM
Marques et al.	2021	Brazil	Toll road	Expand, Extend	Numerical	Traffic	GBM
Marques et al.	2019	Brazil	Airport	Expand	Numerical	Demand	GBM
Martins et al.	2014	Portugal	Airport	Timing	Numerical Simulation	Traffic	GBM
Martins et al.	2017	Spain	Port	Expand	Numerical Simulation	Demand	GBM
Marzouk & Ali	2018	Egypt	Treatment plant	MRG	Simulation	Revenue	GBM
Melese et al.	2017	-	CO2 transport infrastructure	Expand, Term MRG	Simulation	Demand	GBM
Nguyen & Sun	2021	US	Toll road	Renegotiate	Continuous	Cash Flow	-
Novaes et al.	2012	Brazil	Port	Abandon	Continuous	Demand	GBM
Oliveira et al.	2020	Portugal	Airport	Expand	Numerical	Demand	GBM
Park et al.	2013	US	Water and sewer systems	MRL, MEL	Simulation Numerical	O&M cost	GBM
Pellegrino et al.	2019	Italy	Toll road	MIR	Simulation	Traffic IR	GBM
Pimentel et al.	2020	Portugal	Port	Expand	Numerical	Demand	GBM
Polat & Battal	2021	Turkey	Airport	Timing, Expand	Continuous	Revenue	GBM
Power et al.	2016	-	Toll road	Buyout, MRG Revenue-sharing	Simulation	Traffic	GBM
Rakić & Radenović	2014	-	Toll road	Abandon	Numerical	Project	GBM
Rocha Armada et al.	2012	-	-	Subsidies, MDG Rescue	Continuous	Demand	GBM
Rocha et al.	2006	Brazil	Forest concession	Timing	Continuous	Timber prices	GBM MRP
Rodrigues et al.	2019	Brazil	Forest concession	Abandon	Numerical	Cash Flow	GBM
Rose S.	1998	Australia	Toll road	Abandon Timing	Simulation	IR Traffic	GBM
Saito et al.	2001	Brazil	Oil and gas	Timing	Continuous	Project	GBM
Sang et al.	2019	-	Rail and property	Timing	Continuous	Urban residents	GBM, GBM with Poisson

Scandizzo & Ventura	2010	Italy	Toll motorway	Timing, Abandon	Continuous	Cash Flow	GBM
Shan et al.	2010	-	Toll road	Revenue collar	Continuous	Revenue	GBM
Shi et al.	2019	China	Public Rental Housing	Timing, Expand Abandon	Numerical	Project	GBM
Silaghi & Sarkar	2021	-	-	Timing, Exit	Continuous	Demand	GBM
Soliño et al.	2018	Spain	Road	Abandon	Simulation	Traffic	GBM
Song et al.	2018	China	Expressway	MDG, Abandon	Simulation	Demand	GBM
Suttinon et al.	2012	Thailand	Industrial water infrast.	Timing	Numerical	Demand	GBM
Swanson & Sakhrani	2020	Zambia Zimbabwe	Hydropower Dam	Expand	Numerical	Project	GBM
Takashima et al.	2010	-	-	MRG, Transfer	Continuous	Revenue	GBM
Vahdatmanesh et al.	2021	Iran	Urban infrastructure	Revenue collar	Numerical	Revenue	GBM
Vasudevan et al.	2018	India	Highway	Revenue collar	Numerical	Traffic IR	GBM
Wang et al.	2017	China	Treatment plant	Timing	Continuous	Demand	GBM
Wang et al.	2015	Canada	Urban infrast.	Timing	Simulation	Project	Triang. fuzzy
Wang et al.	2021	China	Expressway	Timing	Numerical Simulation	Project value	GBM
Wibowo	2004	Indonesia	Toll road	MRG, MTG, MIR, Tariff and Debt guarantees	Simulation	Inflation rate, IR Traffic, Toll	Normal dist. Markov process, GBM
Xiong & Zhang	2016	-	Toll road	Renegotiate	Numerical	Cash Flow	GBM
Yeo & Qiu	2003	China	Automobile	Expand	Continuous	Project	GBM
Zapata Quimbayo et al.	2019	Colombia	Toll road	MRG	Simulation	Traffic	MRP
Zeng & Chen	2019	China	Solar photovoltaic power	Timing Term	Continuous	Electricity quantity	GBM
Zhang et al.	2010	-	Road	MRG	Simulation	Revenue Incidents	GBM, Poisson
Zhao et al.	2004	-	Highway	Expand Rehabilitate	Simulation	Traffic Land price Deterioration	GBM, Markov chain
Zhu et al.	2021	China	Offshore wind power	Timing	Simulation	Capacity Price	GBM

Appendix II – Project Lattices (ex ante & ex post)

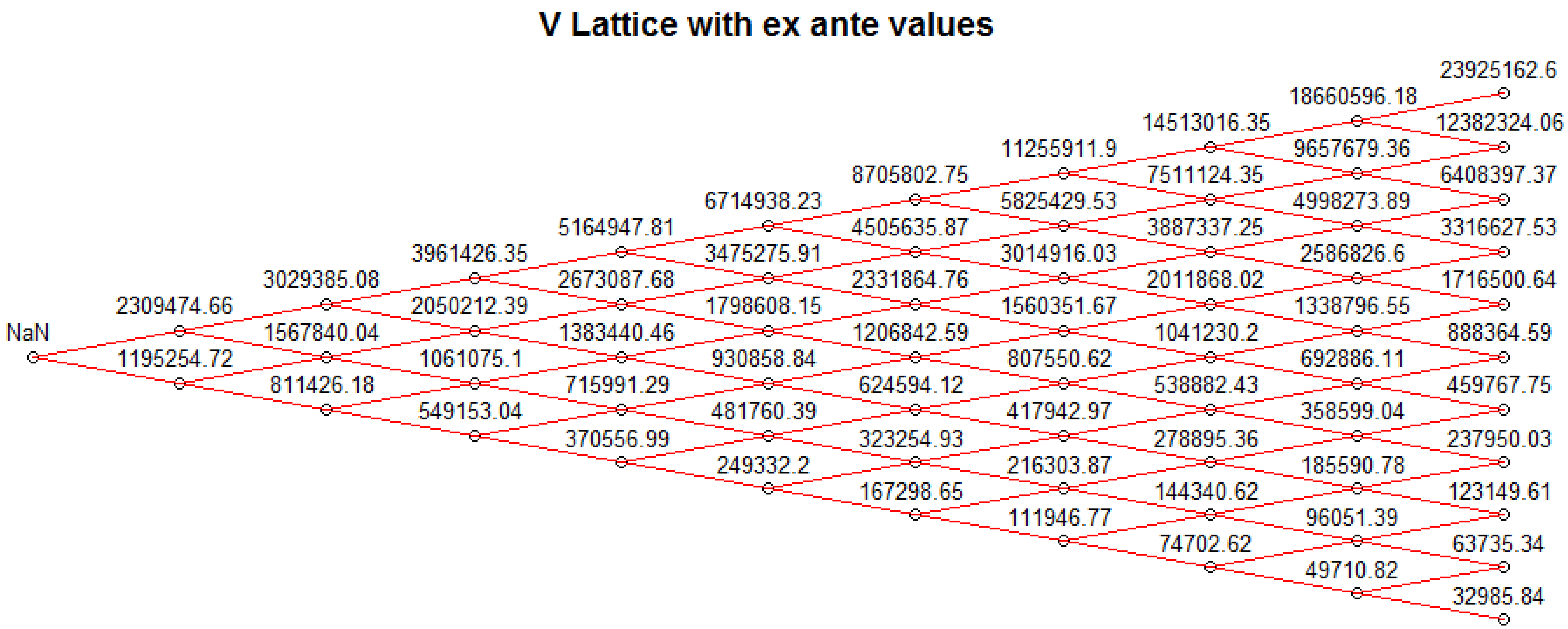

Figure A.1 – Ex ante Project Value Lattice

Note: This is one of the outputs of our R code (Code II) that shows the ex-ante project value lattice. To estimate this, the code uses the approach described in section 2.3.3. Observe that no managerial flexibility was considered in this calculation.

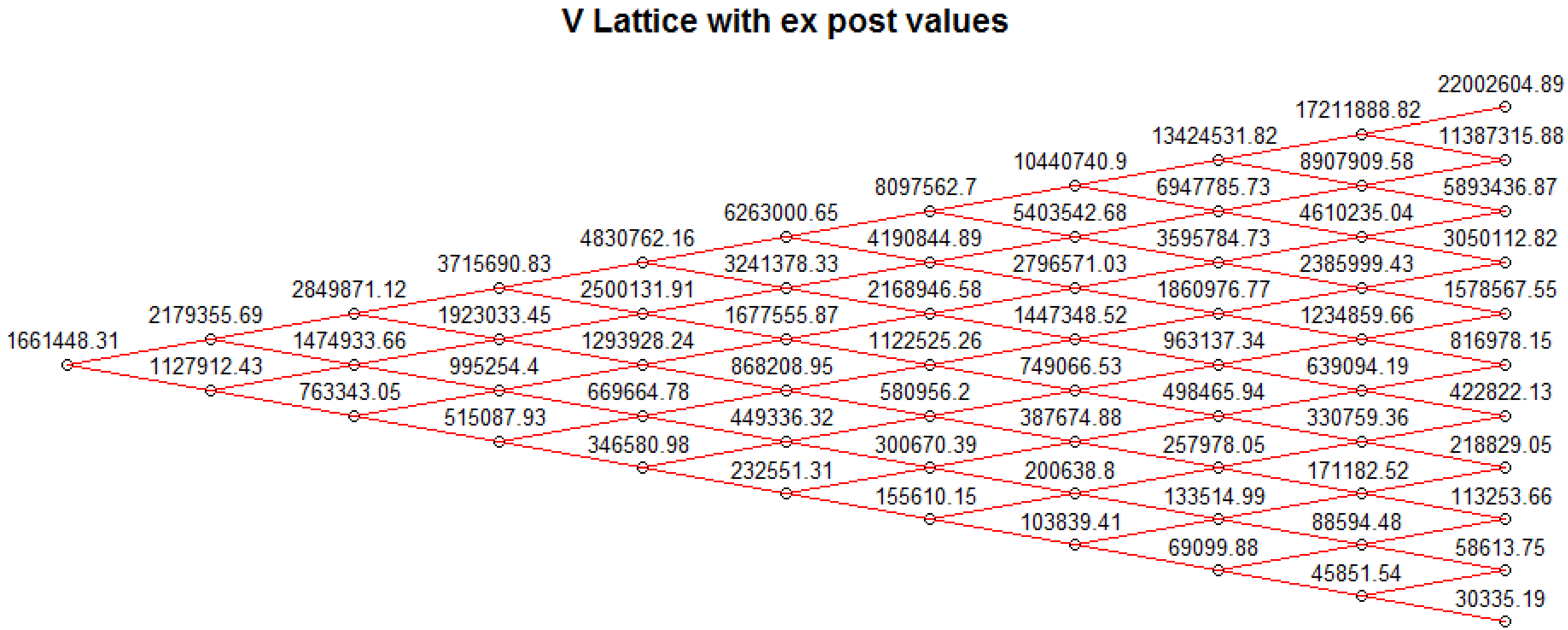

Figure A.2 – Ex post Project Value Lattice

Note: This is one of the outputs of our R code (Code II) that shows the ex post project value lattice. To estimate this, the code uses the approach described in section 2.3.3. Observe that no managerial flexibility was considered in this calculation.

Expansion Option: 80% expansion, at a cost of: 1,200,000 $

Abandoment option: project can be sold at a value of its depreciated investments, minus: 20%

	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
Abandonment value:	1,120,000	1,036,000	948,000	856,000	760,000	660,000	556,000	448,000	336,000	220,000

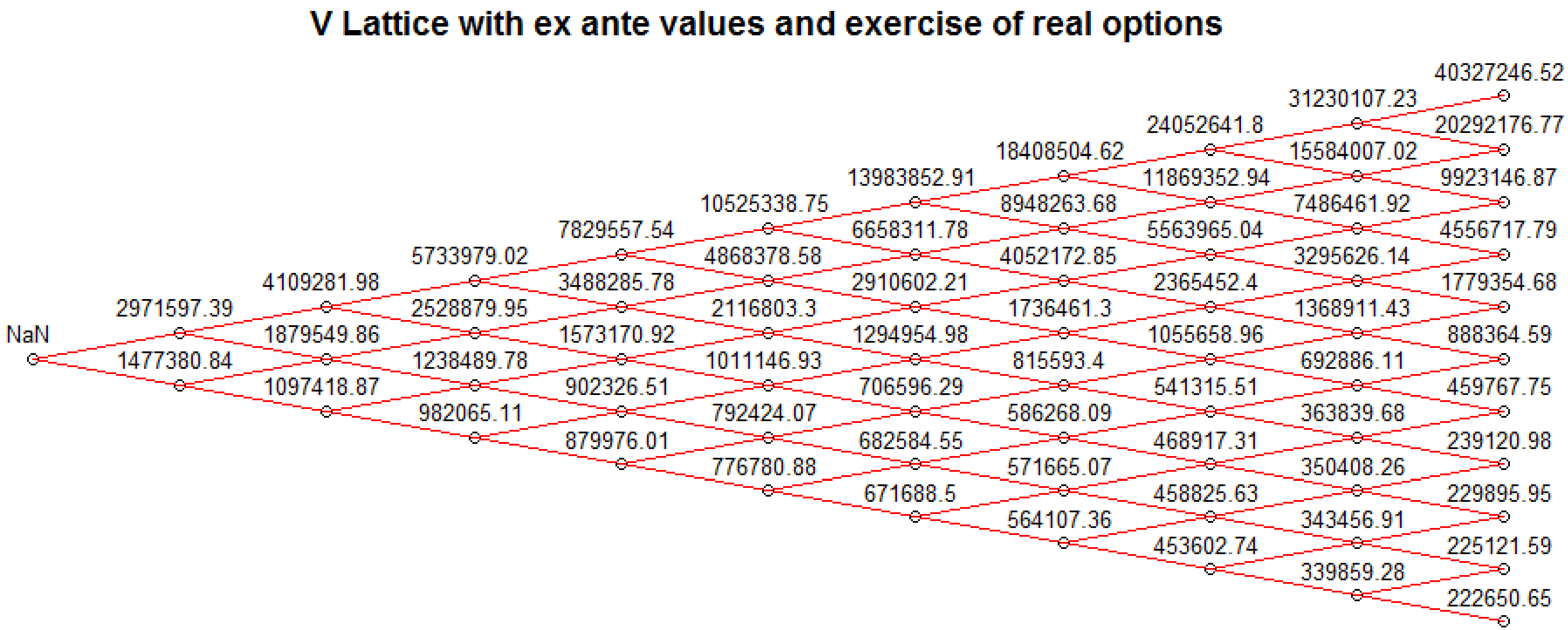

Figure A.3 – Ex ante Project Value Lattice with exercise of abandonment and expansion options

Note: This is one of the outputs of our R code (Code II) that shows the ex-ante project value lattice with exercise of abandonment and expansion options. To estimate this, the code evaluates backwards the maximum value between maintaining, abandoning and expanding the project each year until year 10.

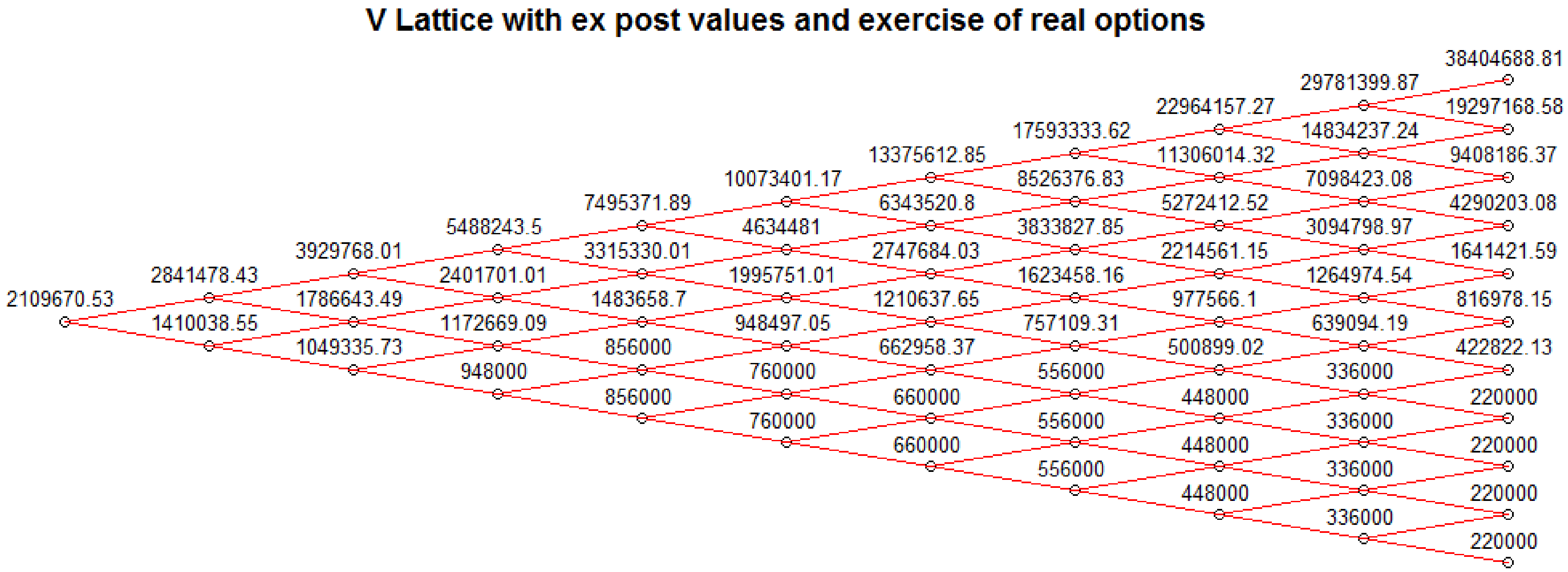

Figure A.4 – Ex post Project Value Lattice with exercise of abandonment and expansion options

Note: This is one of the outputs of our R code (Code II) that shows the ex post project value lattice with exercise of abandonment and expansion options. To estimate this, the code evaluates backwards the maximum value between maintaining, abandoning and expanding the project each year until year 10.

Appendix III

Code I – CRR Binomial Tree Model

```
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##Purpose: This R code helps researchers and practitioners calculate the value of financial options using the CRR Binomial Model
##Link to published paper: A Tutorial for Modeling Real Options Lattices from Project Cash Flows
##Link to code: https://doi.org/10.5281/zenodo.3885925
##Last update: June 9th, 2020

##Package needed for this code
if (!require(fOptions)) {
  install.packages("fOptions") #We will use this package to calculate the values of financial options and to plot the lattices
}

##Parameters - Here, you can change the input values to suit the financial options you want to calculate
n <- 1.5 #Time to maturity in years, which is equivalent to 18 months
i <- 9 #Number of time steps. Here, we would like to have a dt = 2 months. In this sense, we choose a number of intervals equal to 9.
vol <- 0.30 #Price volatility
P0 <- 100 #Price at t = 0
r <- 0.06 #Risk free rate
C <- 120 #Call exercise price
P <- 90 #Put exercise price

##CRR Binomial Model
#Call Option
A<-BinomialTreeOption(TypeFlag = "ca", S = P0, X = C,
                      Time = n, r = r, b = r, sigma = vol, n = i)
```

```
A[1,1] #At the starting step of this lattice, we can verify that the value of the call option
is: $11.00
#Saving this lattice in a pdf file
pdf(file = "CRR Binomial Model (call).pdf", #You can name the file and change the
directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(A, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ")
title(main = "CRR Binomial Model (call)")
dev.off()

#Put Option
B<-BinomialTreeOption(TypeFlag = "pa", S = P0, X = P,
                      Time = n, r = r, b = r, sigma = vol, n = i)
B[1,1] #At the starting step of this lattice, we can verify that the value of the put option
is: $6.71
#Saving this lattice in a pdf file
pdf(file = "CRR Binomial Model (put).pdf", #You can name the file and change the
directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(B, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ")
title(main = "CRR Binomial Model (put)")
dev.off()
```

Code II – CCR Lattice applied to Real Options from Cash Flow Projection

```
##Authors and contact: Naielly Lopes Marques (naielly.lopes@iag.puc-rio.br), Carlos
de Lamare Bastian-Pinto (carbastian@gmail.com), and Luiz Eduardo Teixeira Brandão
(brandao@iag.puc-rio.br). Institution: Pontifical Catholic University of Rio de Janeiro
(PUC-RJ). Department: IAG Business School
```

```
##Purpose: This R code helps researchers and practitioners model project cash flows for real option applications considering the correct volatility estimation (Brandão et al., 2012), dividend yield modeling and lattice building
##Link to published paper: A Tutorial for Modeling Real Options Lattices from Project Cash Flows
##Link to code: https://doi.org/10.5281/zenodo.3885925
##Last update: June 9th, 2020

##Package needed for this code
if (!require(DescTools)) {
  install.packages("DescTools") #We will use the NPV function of this package to calculate the Net Present Values
}
if (!require(fOptions)) {
  install.packages("fOptions") #We will use the BinomialTreePlot function of this package to plot the lattices
}

##Modelling price
#Parameters - Here, you can change the input values to suit your project
n <- 10 #Depreciation duration time
a <- 0.03 #Price growth rate
vol <- 0.15 #Price volatility
P0 <- 100 #Price at t = 0
nt <- 10000 #Number of simulations
#Calculations derived from the input values
i <- n #Number of time intervals
dt <- n/i #Time interval
t <- seq(from=dt,to=n,by=dt)
l <- length(t)
at <- a*dt
volt <- vol*sqrt(dt)
as <- at-(volt^2)/2
#Simulating price
```

```
#Creating a function to simulate a GBM process
GBM_function <- function(start, nsim, n, drift, volatility, growth) {
  x = matrix(NA, nrow=nsim, ncol=(n+1))
  x[,1] <- start
  for (j in 1:nsim) {
    x[j,2] <- start*exp(drift+volatility*rnorm(1))
  }
  for(j in 1:nsim){
    for (i in 3:(n+1)) {
      x[j,i] <- x[j,(i-1)]*exp(growth)
    }
  }
  return(x)
}
X <- GBM_function(P0,nt,l,as,volt,at)
X #This will generate matrix X of dimension nt x (l+1), where the first column
represents the prices at t = 0 (P0 = 100) and the other columns the prices simulated at
each time point until the tenth year of the project

##Calculating Cash Flow
#Parameters - Here, you can change the input values to suit your project
r <- 0.06 #Risk free rate
k <- 0.12 #Discount rate
g <- 0.03 #Perpetuity growth rate
prod <- 10000 #Production
VC <- 0.55 #Variable costs
FC <- 300000 #Fixed costs
I <- 1500000 #Investment
EI <- 50000 #Extra investments
IT <- 0.34 #Income tax
#Calculations derived from input values
rt <- r*dt
kt <- k*dt
gt <- g*dt
```

```
R <- prod*X[,-1] #Revenue
FC <- matrix(rep(FC),nrow=nt,ncol=l) #Fixed costs matrix
RO <- R-R*VC-FC #Operating revenue
Imatrix <- matrix(rep(I),nrow=nt,ncol=l) #Investment matrix
EImatrix <- matrix(rep(EI),nrow=nt,ncol=l) #Extra investments matrix
Dep0 <- Imatrix[,1]/l #Depreciation at t = 1
Dep <- matrix(rep(0),nrow=nt,ncol=(l-1)) #Depreciation
for (i in 1:(n-1)) {
  Dep[,i] <- (Imatrix[,i]+EImatrix[,i]*i)/l
}
Dep <- cbind(Dep0,Dep)
colnames(Dep) <- NULL
EBIT <- RO-Dep #Earnings Before Interest and Taxes
#Cash Flow Matrix
FCF <- EBIT-IT*EBIT-EImatrix+Dep #Free Cash Flow
Perp <- FCF[,l]/(kt-gt)*(1+gt)*(1+kt) #Perpetuity
FCF <- cbind(FCF,Perp)
colnames(FCF) <- NULL #This will generate a cash flow matrix (FCF) of dimensions nt x l. In addition, since we consider that this project has a continuation value (perpetuity), we have included a column in this matrix that represents the perpetual cash flows of that project

##Calculating PV ex ante and ex post
PV <- matrix(rep(0),nrow=nt,ncol=1) #Present Value
for (i in 1:nt) {
  PV[i,] <- NPV(kt,FCF[i,],seq(along=FCF[i,]))
}
NPV <- PV-Imatrix[,1] #Net Present Value
PVa <- matrix(rep(0),nrow=nt,ncol=l) #Ex ante Present Value
for(j in 1:nt){
  for (i in 1:l){
    PVa[j,i] <- NPV(kt,FCF[j,((i+1):(l+1))],seq(along=FCF[j,((i+1):(l+1))]))+FCF[j,i]
    #This will generate a matrix of Ex ante Present Values
  }
```

```
}
PVp <- matrix(rep(0),nrow=nt,ncol=l) #Ex post Present Value
for (i in 1:l) {
  for(j in 1:nt){
    PVp[j,i] <- NPV(kt,FCF[j,((i+1):(l+1))],seq(along=FCF[j,((i+1):(l+1))]))
    #This will generate a matrix of Ex post Present Values
  }
}

##Calculating the project volatility
PVd <- mean(PV)
PVd #Here, we find that the average project value is: V0 = $1,661,448, yielding a Net
Present Value (NPV) of $161,448
#This value can vary slightly as it is the result of a Monte Carlo Simulation
Ret <- (PVa[,1]/PVd) #Return
lRet <- log(Ret)
sig <- sd(lRet,na.rm = TRUE)
sig #We find that the project volatility is 33%
#This value can vary slightly as it is the result of a Monte Carlo Simulation

##Ex ante and ex post Lattices
u <- exp(sig) #Upside multiplying factor
d <- 1/u #Downside multiplying factor
p <- (1+rt-d)/(u-d) #Probability
divr <- FCF[,-(l+1)]/PVa #Dividend rate
div <- cbind(1,(1-divr)) #Dividends
Latta <- matrix(NaN,(l+1),(l+1)) #Ex ante Lattice, see Apendix I of the published paper
Lattp <- matrix(NaN,(l+1),(l+1)) #Ex post Lattice, see Apendix I of the published paper
Lattp[1,1] <- PVd
Latta[1,2] <- PVd*u
for (a in 2:(l+1)) {
  for (b in 3:(l+1)) {
    Lattp[1,a] <- Latta[1,a]*div[1,a]
    Latta[1,b] <- Lattp[1,(b-1)]*u
```

```
  }
}
for (b in 2:(l+1)) {
  for (a in 2:(l+1)) {
    Latta[b,a] <- Lattp[(b-1),(a-1)]*d
    Lattp[b,a] <- Latta[b,a]*div[1,a]
  }
}
#Saving ex ante and ex post lattices in a pdf file
pdf(file = "V Lattice with ex ante values.pdf", #You can name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(Latta, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ") #We use this function of fOptions package to plot the ex ante lattice
title(main = "V Lattice with ex ante values")
dev.off()
pdf(file = "V Lattice with ex post values.pdf", #You can name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(Lattp, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ") #We use this function of fOptions package to plot the ex post lattice
title(main = "V Lattice with ex post values")
dev.off()

##Incorporating abandonment and expansion options
#To find the residual value of the project in case of abandonment, we calculate the depreciated asset value in each year until year n by discounting the depreciation amount from the investments and we multiply these values by the abandonment factor
Dep0 <- c(I/l) #Depreciation at t = 1
Dep <- matrix(rep(0,(l-1)),nrow=1) #Depreciation
```

```
for (i in 1:(l-1)) {
  Dep[,i] <- (I+EI*i)/l
}
Dep <- c(Dep0,Dep)
EIvector <- (rep(EI,l))
Depasset <- (rep(NaN,(l-1))) #Depreciated asset
for (i in 2:n) {
  Depasset[(i-1):(l-1)] <- (I+sum(EIvector[-c(i:n)])-sum(Dep[-c(i:n)]))
}
Depasset <- cbind(0, t(as.matrix(Depasset, ncol = (l-1), nrow = 1)), (I +
sum(EIvector[1:n]) - sum(Dep[1:n])))
#Parameters - Here, you can change the input values to suit your project
abandf <- 0.8 #Abandonment factor
expf <- 1.8 #Expansion factor
expc <- 1200000 #Expansion cost
#Calculation derived from input values
Residual <- Depasset*(abandf) #Residual value

##Ex ante and ex post Lattices with options
#Here, the code evaluates backwards the maximum value between maintaining,
abandoning and expanding the project each year until year n
Lattpo <- matrix(NaN,(l+1),(l+1)) #Ex post Lattice with Options, see Apendix I of the
published paper
for (i in 1:(l+1)) {
  Lattpo[i,(l+1)] <- max(Lattp[i,(l+1)],Lattp[i,(l+1)]*expf-expc,Residual[,(l+1)])
}
Lattao <- matrix(NaN,(l+1),(l+1)) #Ex ante Lattice with Options, see Apendix I of the
published paper
for (i in 1:(l+1)) {
  Lattao[i,(l+1)] <- Lattpo[i,(l+1)]+(Latta[i,(l+1)]-Lattp[i,(l+1)])
}
for (j in l:1) {
  for (i in 1:l) {
```

```
    Lattpo[i,j] <- max(Lattp[i,j]*expf-
expc,Residual[,j],(Lattao[i,(j+1)]*p+Lattao[(i+1),(j+1)]*(1-p))/(1+rt))
    Lattao[i,j] <- Lattpo[i,j]+(Latta[i,j]-Lattp[i,j])
  }
}
Lattpo[1,1] #At the starting step of this lattice (Lattpo), we can verify that the project
value considering the expansion and abandonment options is: $2,109,671
#This value can vary slightly as it is the result of a Monte Carlo Simulation
#Saving ex ante and ex post lattices with exercise of real options in a pdf file
pdf(file = "V Lattice with ex ante values and exercise of real options.pdf", #You can
name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(Lattao, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ") #We use this function of fOptions package to plot the
ex ante lattice with options
title(main = "V Lattice with ex ante values and exercise of real options")
dev.off()

pdf(file = "V Lattice with ex post values and exercise of real options.pdf", #You can
name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
BinomialTreePlot(Lattpo, dy = 1, cex = 0.8, axes = FALSE, ylim = c(-15, 15),
                 xlab = " ", ylab = " ") #We use this function of fOptions package to plot the
ex post lattice with options
title(main = "V Lattice with ex post values and exercise of real options")
dev.off()

##Graphs of ex post lattices with and without options
#First, we create a function to mirror any lattice so that we can plot it
latt_inverter <- function(vec){
  vec2 <- vec[!is.na(vec)]
  vec3 <- c(rep(NaN,length(vec)-length(vec2)),vec2)
```

```
  vec3
}
Lattpb <- apply(Lattp,2,latt_inverter) #Mirrored Ex post Lattice
x <- c(0:l) #x-axis corresponds to the project's duration years
pdf(file = "Ex post lattices with and without abandonment and expansion options.pdf",
#You can name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
par(mar=c(5.5,5.5,5.5,5.5)) #You can choose the plot margins
for (b in 1:(l+1)) {
  y <- (Lattp[b,]/1000) #y-axis corresponds to the ex post lattice without options

plot(x,y,type="l",ylim=c(min(Lattp/1000,na.rm=TRUE),max(Lattpo/1000,na.rm=TRU
E)),las=1,col="blue",ann = FALSE)
  par(new=T)
}
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpb[b,]/1000) #y-axis corresponds to the mirrored ex post lattice without
options

plot(x,y,type="l",ylim=c(min(Lattp/1000,na.rm=TRUE),max(Lattpo/1000,na.rm=TRU
E)),las=1,col="blue",ann = FALSE)
}
par(new=T)
value <- round(y[1],digits=1)
text(0,rep(value-1000,1),(value),col=4)
Lattpob <- apply(Lattpo,2,latt_inverter) #Mirrored Ex post Lattice with Options
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpo[b,]/1000) #y-axis corresponds to the ex post lattice with options

plot(x,y,type="l",ylim=c(min(Lattp/1000,na.rm=TRUE),max(Lattpo/1000,na.rm=TRU
E)),las=1,col="red",ann = FALSE)
```

```
}
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpob[b,]/1000) #y-axis corresponds to the mirrored ex post lattice with options

plot(x,y,type="l",ylim=c(min(Lattp/1000,na.rm=TRUE),max(Lattpo/1000,na.rm=TRU
E)),las=1,col="red",ann = FALSE)
}
par(new=T)
value <- round(y[1],digits=1)
text(0,rep(value+1000,1),(value),col=2)
mtext(side = 1,text = "Year",line = 3) #x-axis label
mtext(side = 2, text = "Project Value ($)",line = 4) #y-axis label
dev.off()

##Plotting ex post lattice with and without options in log scale
pdf(file = "Ex post lattices with and without abandonment and expansion options in log
scale.pdf", #You can name the file and change the directory you want to save it
    width = 13.00, #You can choose the width of the plot in inches
    height = 10.00) #and the height of the plot in inches
par(mar=c(5.5,5.5,5.5,5.5)) #You can choose the plot margins
marks <- c(10,100,1000,10000,100000)
for (b in 1:(l+1)) {
  y <- (Lattp[b,]/1000) #y-axis corresponds to the ex post lattice without options
  plot(x,y,type="l",log="y",ylim=c(10,100000),yaxt="n",col="blue",ann=FALSE)
  axis(2,at=marks,labels=format(marks,scientific=FALSE),las=2)
  par(new=T)
}
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpb[b,]/1000) #y-axis corresponds to the mirrored ex post lattice without
options
  plot(x,y,type="l",log="y",ylim=c(10,100000),yaxt="n",col="blue",ann = FALSE)
  axis(2,at=marks,labels=format(marks,scientific=FALSE),las=2)
```

```
}
par(new=T)
value <- round(y[1],digits=1)
text(0,rep(value-500,1),(value),col=4)
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpo[b,]/1000) #y-axis corresponds to the ex post lattice with options
  plot(x,y,log="y",type="l",ylim=c(10,100000),yaxt="n",col="red",ann = FALSE)
  axis(2,at=marks,labels=format(marks,scientific=FALSE),las=2)
}
for (b in 1:(l+1)) {
  par(new=T)
  y <- (Lattpob[b,]/1000) #y-axis corresponds to the mirrored ex post lattice with options
  plot(x,y,log="y",type="l",ylim=c(10,100000),yaxt="n",col="red",ann = FALSE)
  axis(2,at=marks,labels=format(marks,scientific=FALSE),las=2)
}
par(new=T)
value <- round(y[1],digits=1)
text(0,rep(value+500,1),(value),col=2)
mtext(side = 1,text = "Year",line = 3) #x-axis label
mtext(side = 2, text = "Project Value ($)",line = 4) #y-axis label
dev.off()
```

Appendix III – Summary of variables used in the R code

Description	Variable
Time interval	dt
Price simulation	X
Revenue	R
Operating revenue	RO
Depreciation (λ_t)	Dep
Earnings Before Interest and Taxes	EBIT
Free Cash Flow (F_t)	FCF
Perpetuity (CV)	Perp
Present Value (V_0)	PV
Net Present Value	NPV
Ex ante Present Value	PVa
Ex post Present Value	PVp
Average Present Value	PVd
Return ($\tilde{Z}$)	lRet
Project volatility (σ_V)	sig
Upside multiplying factor (u)	u
Downside multiplying factor (d)	d
Probability (p)	p
Dividend rate	divr
Dividends	div
Ex ante Lattice	Latta
Ex post Lattice	Lattp
Depreciated asset	Depasset
Residual value	Residual
Ex ante Lattice with Options	Lattao
Ex post Lattice with Options	Lattpo

Note: This table describes all the variables used in the proposed R code (Code II).