Mathematical Optimization Model

(Toward a New Transactive Energy System with Distributed Energy Resources in Brazil: A Real Case Application)

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***Abstract*— In this document is presented the mathematical model of the work Toward a transactive energy system after the law 14.300/2023 in Brazil: a real case application.**

***Index Terms*—Transactive energy, Regulated market in Brazil, Mathematical optimization model, Coordination of Distributed Energy Resource.**

NOMENCLATURE

Uppercase characters

C_E^p	Cost of on-peak energy consumption [R$]
C_E^{op}	Cost of off-peak energy consumption [R$]
C_E^{mid}	Cost of mid-peak energy consumption [R$]
C_ϕ^p	Cost of on-peak losses [R$]
C_ϕ^{op}	Cost of off-peak losses [R$]
C_ϕ^{mid}	Cost of mid losses [R$]
C_{pv}	Cost per unit of panel [R$]
C_{inv}	Cost per unit of inverter [R$]
C_{bat}	Cost per unit of battery [R$]
$C_{I_{pv}}$	Investment cost in panels [R$]
$C_{I_{bat}}$	Investment cost in batteries [R$]
$C_{I_{inv}}$	Investment cost in inverters [R$]

C_{Annual}	Average annual cost [R$]
C^{noDER}	Operating cost without DERs
$C^{DER}_{operation}$	Operating cost with DERs
$E^{G,p}_{b,d,h}$	On-peak energy credit at bus b, for day/scenario d and hour h [kWh]
$E^{G,op}_{b,d,h}$	Off-peak energy credit at bus b, for day/scenario d and hour h [kWh]
$E^{G,mid}_{b,d,h}$	Mid-peak energy credit at bus b, for day/scenario d and hour h [kWh]
$E^{D,op}_{b,d,h}$	Off-peak energy debit at bus b, for day/scenario d and hour h [kWh]
$E^{D,mid}_{b,d,h}$	Mid-peak energy debit at bus b, for day/scenario d and hour h [kWh]
$E^{C(V),p}_{d,\mathrm{h}}$	Energy contract for on-peak green tariff [kWh]
$E^{C(V),op}_{d,\mathrm{h}}$	Energy contract for off-peak green tariff [kWh]
$E^{C(B),p}_{d,\mathrm{h}}$	Energy contract for peak white tariff [kWh]
$E^{C(B),op}_{d,\mathrm{h}}$	Energy contract for off-peak white tariff [kWh]
$E^{C(B),mid}_{d,\mathrm{h}}$	Energy contract for mid-peak white tariff [kWh]
$E^{C(C),op}_{d,\mathrm{h}}$	Energy contract for off-peak conventional tariff [kWh]
DOD	Battery depth of discharge [%]
$G^{p}_{b,d,h}$	Photovoltaic energy generated on-peak at bus b, for day/scenario d and hour h [kWh]
$G^{op}_{b,d,h}$	Photovoltaic energy generated off-peak for day/scenario d and hour h [kWh]
$G^{mid}_{b,d,h}$	Photovoltaic energy generated mid-peak for day/scenario d and hour h [kWh]
H	Set of defined hours for a day [H = {1:24}]
H^p	Set of on-peak hours [$H^p = \{19,20,21\}$]
H^{op}	Set of off-peak hours [$H^{op} = \{1,2,\cdots,17,23,24\}$]
H^{mid}	Set of mid-peak hours [$H^{mid} = \{18,22\}$]
P_{bat}	Battery power [kW]
N	Total system lifetime
M	High-value auxiliary parameter
$T^{p(V)}_{E}$	On-peak green energy tariff [R$/kWh]
$T^{op(V)}_{E}$	Off-peak green energy tariff [R$/kWh]
$T^{(C)}_{E}$	Conventional energy tariff [R$/kWh]
$T^{p(B)}_{E}$	On-peak white energy tariff [R$/kWh]
$T^{op(B)}_{E}$	Off-peak white energy tariff [R$/kWh]

$T_E^{mid(B)}$	Mid-peak white energy tariff [R$/kWh]
$\Delta G_{b,d,h}$	Auxiliary variable for energy credit at bus b, for day/scenario d and hour h [kWh]
$\Delta D_{b,d,h}$	Auxiliary variable for energy debit at bus b, for day/scenario d and hour h [kWh]
$P_{b,d,h}$	Net power (credit - debit) on-peak at load bus b, for day/scenario d and hour h [kW]
$\phi_{b,h}$	Loss factor at load bus b, for day/scenario d and hour h
E_{total}	Overall energy consumed in the entire system
$E_{d,h,b}$	Energy consumed on day d and hour h at load bus b

Lowercase characters

d_b^{FIO}	TUSD Fio discount rate
ef	Battery efficiency
k	Discount rate
n	System lifetime
n_1	Battery lifetime
n_2	Panel lifetime
n_3	Inverter lifetime
n_{bat}	Total Number of Batteries in the System
n_b^{bat}	Variable for the number of batteries at the load bus
n_{pv}	Total Number of Panels in the System
n_b^{pv}	Variable for the number of panels at the load bus
n_{inv}	Total Number of Inverters
n_b^{inv}	Variable for the number of inverters at the load bus
$x_{h,b}$	Vector indicating battery charging operation at hour h, load bus b
$y_{h,b}$	Vector indicating battery discharging operation at hour h, load bus b
α_{h}	Contract percentage for different on-peak tariffs
β_{h}	Contract percentage for different off-peak tariffs
b	Load Bus Position

MATHEMATICAL MODEL

In this section, the mathematical model for optimizing the use of DERs considering price arbitrage and the sensitivity of the electrical grid will be presented. The model can calculate optimal energy contracts for different tariff types. The modeling involves tariff arbitrage, as well as electrical loss and voltage constraints. The proposed optimization model to minimize the system cost considering tariff arbitrage can be presented as follows:

$$C_{Annual} = \min_{x_h, E_{d,h}^{C,p(V)}, E_{d,h}^{p(B)}, E_{d,h}^{op(B)}, E_{d,h}^{mid(B)}, E_{d,h}^{C}} C_E^p + C_E^{op} + C_E^{mid} + C_\phi^p + C_\phi^{op} + C_\phi^{mid} + M \cdot \sum_{b\in B}\sum_{d\in D}\sum_{h\in H} \Delta G_{b,d,h} + \Delta D_{b,d,h} \tag{1}$$

s.a.

$$E_{b,d,h}^{G,p} = n_b^{pv} \cdot G_{b,d,h}^{p} + y_{h,b} \cdot n_b^{bat} \cdot DOD \cdot P_{bat} \cdot ef + \Delta G_{b,d,h} \quad \forall\, d \in D, h \in H^p, b \in B \tag{2}$$

$$E_{b,d,h}^{G,op} = n_b^{pv} \cdot G_{b,d,h}^{op} - x_{h,b} \cdot n_b^{bat} \cdot DOD \cdot P_{bat} + \Delta G_{b,d,h} \quad \forall\, d \in D, h \in H^{op}, b \in B \tag{3}$$

$$E_{b,d,h}^{G,mid} = n_b^{pv} \cdot G_{b,d,h}^{mid} - x_{h,b} \cdot n_b^{bat} \cdot DOD \cdot P_{bat} + \Delta G_{b,d,h} \quad \forall\, d \in D, h \in H^{mid}, b \in B \tag{4}$$

$$E_{b,d,h}^{G,p} \geq 0 \qquad \forall\, d \in D, h \in H^p, b \in B \tag{5}$$

$$E_{b,d,h}^{G,op} \geq 0 \qquad \forall\, d \in D, h \in H^{op}, b \in B \tag{6}$$

$$E_{b,d,h}^{G,mid} \geq 0 \qquad \forall\, d \in D, h \in H^{mid}, b \in B \tag{7}$$

$$\Delta G_{b,d,h} \geq 0 \qquad \forall\, d \in D, h \in H \;\;, b \in B \tag{8}$$

$$E_{b,d,h}^{D,op} = x_{h,b} \cdot n_b^{bat} \cdot DOD \cdot P_{bat} - n_b^{pv} \cdot G_{b,d,h}^{op} + \Delta D_{b,d,h} \quad \forall\, d \in D, h \in H^{op}, b \in B \tag{9}$$

$$E_{b,d,h}^{D,mid} = x_{h,b} \cdot n_b^{bat} \cdot DOD \cdot P_{bat} - n_b^{pv} \cdot G_{b,d,h}^{mid} + \Delta D_{b,d,h} \quad \forall\, d \in D, h \in H^{mid}, b \in B \tag{10}$$

$$E_{b,d,h}^{D,op} \geq 0 \qquad \forall\, d \in D, h \in H^{op}, b \in B \tag{11}$$

$$E_{b,d,h}^{D,mid} \geq 0 \qquad \forall\, d \in D, h \in H^{mid}, b \in B \tag{12}$$

$$\Delta D_{b,d,h} \geq 0 \qquad \forall\, d \in D, h \in H \;\;, b \in B \tag{13}$$

$$P_{b,d,h} = E_{b,d,h}^{G} - E_{b,d,h}^{D} \qquad \forall\, d \in D, h \in H \;\;, b \in B \tag{14}$$

$$C_\phi^p = \sum_{\substack{d\in D \\ h\in H^p \\ b\in B}} -\phi_{b,h} \cdot P_{b,d,h} \cdot T_E^{p\,(V)} \tag{15}$$

$$C_\phi^{op} = \sum_{\substack{d\in D \\ h\in H^{op} \\ b\in B}} -\phi_{b,h} \cdot P_{b,d,h} \cdot T_E^{op\,(V)} \tag{16}$$

$$C_\phi^{mid} = \sum_{\substack{d\in D \\ h\in H^{mid} \\ b\in B}} -\phi_{b,h} \cdot P_{b,d,h} \cdot T_E^{op\,(V)} \tag{17}$$

$$C_E^p = \sum_{\substack{d\in D \\ h\in H^p}} \left(E_{d,h}^{C(V),p} - \alpha_h \cdot d_b^{FIO} \cdot \sum_{b\in B} E_{b,d,h}^{G,p} \right) T_E^{p\,(V)} + \left[E_{d,h}^{C(B),p} - (1-\alpha_h) \cdot \sum_{b\in B} d_b^{FIO} \cdot E_{b,d,h}^{G,p} \right] \cdot T_E^{p\,(B)} \tag{18}$$

$$C_E^{op} = \sum_{\substack{d\in D \\ h\in H^{op}}} \left(E_{d,h}^{C(B),op} - \beta_h \cdot \sum_{b\in B} (d_b^{FIO} \cdot E_{b,d,h}^{G,op} - E_{b,d,h}^{D,op}) \right) \cdot T_E^{op\ (B)}$$
$$+ \left(E_{d,h}^{C(C),op} - (1-\beta_h) \cdot \sum_{b\in B} (d_b^{FIO} \cdot E_{b,d,h}^{G,op} - E_{b,d,h}^{D,op}) \right) \cdot T_E^{(C)} \tag{19}$$

$$C_E^{mid} = \sum_{\substack{d\in D \\ h\in H^{mid}}} \left(E_{d,h}^{C(B),mid} - \sum_{b\in B} (d_b^{FIO} \cdot E_{d,h}^{G,mid} - E_{d,h}^{D,mid}) \right) \cdot T_E^{mid\ (B)} \tag{20}$$

$$\sum_{\substack{d\in D \\ h\in H^{p}}} E_{d,h}^{C(V),p} \geq \sum_{\substack{d\in D \\ h\in H^{p} \\ b\in B}} (\alpha_h) \cdot d_b^{FIO} \cdot E_{b,d,h}^{G,p} \tag{21}$$

$$\sum_{\substack{d\in D \\ h\in H^{p}}} E_{d,h}^{C(B),p} \geq \sum_{\substack{d\in D \\ h\in H^{p} \\ b\in B}} (1-\alpha_h) \cdot d_b^{FIO} \cdot E_{b,d,h}^{G,p} \tag{22}$$

$$\sum_{\substack{d\in D \\ h\in H^{op}}} E_{d,h}^{C(B),op} \geq \sum_{\substack{d\in D \\ h\in H^{op} \\ b\in B}} \beta_h \cdot (d_b^{FIO} \cdot E_{b,d,h}^{G,op} - E_{b,d,h}^{D,op}) \tag{23}$$

$$\sum_{\substack{d\in D \\ h\in H^{mid}}} E_{d,h}^{C(B),mid} \geq \sum_{\substack{d\in D \\ h\in H^{mid} \\ b\in B}} (d_b^{FIO} \cdot E_{b,d,h}^{G,mid} - E_{b,d,h}^{D,mid}) \tag{24}$$

$$\sum_{\substack{d\in D \\ h\in H^{op}}} E_{d,h}^{C(C),op} \geq \sum_{\substack{d\in D \\ h\in H^{op} \\ b\in B}} (1-\beta_h) \cdot (d_b^{FIO} \cdot E_{b,d,h}^{G,op} - E_{b,d,h}^{D,op}) \tag{25}$$

$$\sum_{\substack{d\in D \\ h\in H^{op} \\ b\in B}} d_b^{FIO} \cdot E_{b,d,h}^{G,op} \geq \sum_{\substack{d\in D \\ h\in H^{op} \\ b\in B}} E_{b,d,h}^{D,op} \tag{26}$$

$$\sum_{\substack{d\in D \\ h\in H^{mid} \\ b\in B}} d_b^{FIO} \cdot E_{b,d,h}^{G,mid} \geq \sum_{\substack{d\in D \\ h\in H^{mid} \\ b\in B}} E_{b,d,h}^{D,mid} \tag{27}$$

$$x_{h,b} = 0 \qquad \forall\, h \in H^p, b \in B \tag{28}$$

$$x_{h,b} \geq 0 \qquad \forall h \in H^{op}, b \in B \tag{29}$$

$$x_{h,b} \geq 0 \qquad \forall h \in H^{mid}, b \in B \tag{30}$$

$$y_{h,b} = 1 \qquad \forall\, h \in H^p\ , b \in B \tag{31}$$

$$y_{h,b} = 0 \qquad \forall\, h \in H^{op}, b \in B \tag{32}$$

$$y_{h,b} = 0 \qquad \forall\, h \in H^{mid}, b \in B \tag{33}$$

$$x_{h,b} \leq 0{,}5 \quad \forall\, h \in H, b \in B \tag{34}$$

$$\sum_{h \in H^p} y_{h,b} = \sum_{h \in H^{op}} x_{h,b} + \sum_{h \in H^{mid}} x_{h,b} \qquad \forall\, b \in B \tag{35}$$

The objective function (1) can be divided into three parts: the first presents the cost balances between energy and contract for the different tariff periods (peak, off-peak, and mid-peak); the second part presents the costs generated by electrical losses; and the third part presents penalties for the auxiliary variables $\Delta G_{b,d,h}$ and $\Delta D_{b,d,h}$.

Expressions (2)-(4) represent the energy credit for different tariff periods. During peak hours, as represented by constraint (2), the energy credit is equal to the energy generated by photovoltaic panels and discharged from the batteries. During off-peak and mid-peak periods (constraints (3) and (4)), the energy credit is equal to the energy generated by the panels minus the battery load. The auxiliary variable $\Delta G_{b,d,h}$ ensures that these constraints have values greater than or equal to zero. That is, if the battery load is greater than the energy generated by the photovoltaic panels at any hour of the day, the energy credit will be equal to zero. Expressions (5)-(8) ensure that the energy credit and auxiliary variable $\Delta G_{b,d,h}$ are greater than or equal to zero for the entire period.

Expressions (9)-(13) represent the energy debit when the battery load is higher than the photovoltaic panel generation. The auxiliary variable $\Delta D_{b,d,h}$ serves the same function as $\Delta G_{b,d,h}$, preventing the energy debit from being less than zero.

Expression (14) represents the hourly net power. Expressions (15)-(17) present the cost of electrical losses, considering the A4 green tariff for peak and off-peak time periods. The sensitivity factor $\phi_{b,h}$ is calculated by the energy extraction performed by the loads. As the optimization model considers generation, i.e., power injection into the system, $\phi_{b,h}$ receives a negative sign in the constraints in which it appears.

Expressions (18)-(20) represent the energy and contract cost balance. For the peak and off-peak periods, there may be contracts for different tariff types. Thus, the model determines these contracts through a factor α_h(peak) and β_h (off-peak). For instance, for an α_h = 0.7, 70 % of the energy contract for peak hours will be for the green tariff, while 30 % will be for the white tariff.

Expressions (21)-(25) ensure that the energy contracts are greater than or equal to the net energy. The energy credit will always be greater than the debit, as determined by expressions (26) and (27).

Expressions (28)-(34) are used to establish the charging and discharging operation of the batteries throughout the day and expression (35) establishes that the total discharge must be equal to the total charge. For voltage control and lifespan preservation, the batteries cannot be charged with a load greater than 50 % of their nominal capacity.

CALCULATION OF OPERATING COST AND GAIN PROVIDED BY DERS.

The operating cost using DERs is expressed as:

$$C_{operation}^{DER} = C_E^p + C_E^{op} + C_E^{mid} + C_\phi^p + C_\phi^{op} + C_\phi^{mid} \tag{36}$$

To determine the gain provided by the installation and operation of DERs, it is necessary to calculate the operating cost without DERs. For this purpose, the following expression is used:

$$C^{noDER} = T_E^{p\,(V)} \cdot \sum_{d \in D\; h \in H^p} E_{d,h}^{C(V),p} + T_E^{p\,(B)} \cdot \sum_{d \in D\; h \in H^p} E_{d,h}^{C(B),p} + T_E^{p\,(B)} \cdot \sum_{d \in D\; h \in H^{op}} E_{d,h}^{C(B),op} + T_E^{mid\,(B)} \cdot \sum_{d \in D\; h \in H^{mid}} E_{d,h}^{C(B),mid} + T_E^{(C)} \cdot \sum_{d \in D\; h \in H^{op}} E_{d,h}^{C(C)} \tag{37}$$

TARIFFS

In this analysis, The DERs were distributed in 10 different buses, with 4 PV systems, 2 hybrids (PV and BESS) and 2 buses with BESS. The energy tariff for each modality and the specification and costs of the equipment are presented in table I and II, respectively, as follows:

Table 1: Energy tariffs.

Green Tariff	
Peak	R$ 2,32 /kWh
Off-peak	R$ 0,34 /kWh
White Tariff	
Peak	R$ 1,82 /kWh
Off-peak	R$ 0,63 /kWh
Intermediate	R$ 1,14 /kWh
Conventional tariff	
	R$ 0,76 /kWh