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8. Apêndices

8.1. Introdução

Nos apêndices a seguir encontram-se exemplos dos programas utilizados ao longo desta Dissertação, desenvolvidos com o programa de álgebra simbólica MAPLE.

Os programas apresentados a seguir referem-se à coluna bi-apoiada, com fundação até a metade da sua altura total e coeficiente de rigidez linear $K = 100$, conforme representado na Figura 8.1.

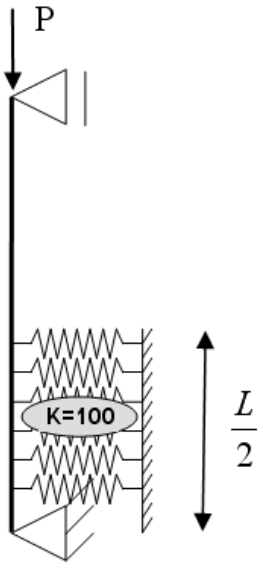

Figura 8.1: Situação analisada nos apêndices.

No Apêndice B, é mostrado o programa no qual a função de aproximação utilizada possui 8 termos.

8.2. Apêndice A

Programa para o Cálculo das Cargas Críticas e Modos Críticos

```
>restart:
>with(DEtools):with(LinearAlgebra):
>Digits:=28:
```

Equação diferencial do trecho sem fundação:

```
>eq1:=diff(w1(x),x,x,x,x)+(lambda^2*Pi^2*diff(w1(x),x,x))=0:
>sol1:=dsolve(eq1):
```

```
>w[1]:=rhs(sol1):
>w[1,x]:=diff(w[1],x):
>w[1,xx]:=diff(w[1],x,x):
>w[1,xxx]:=diff(w[1],x,x,x):
```

Equação diferencial do trecho com fundação:

```
>eq2:=diff(w2(x),x,x,x,x)+(lambda^2*Pi^2*diff(w2(x),x,x))+(K
*w2(x))=0:
>sol2[inicial]:=dsolve(eq2):
>sol2a:=subs(_C1=_C5,_C2=_C6,_C3=_C7,_C4=_C8,sol2[inicial]):
> sol2:=rhs(sol2a):
>w[2]:=sol2:
>w[2,x]:=diff(w[2],x):
>w[2,xx]:=diff(w[2],x,x):
>w[2,xxx]:=diff(w[2],x,x,x):
```

Condições de contorno definidas pelos apoios:

```
>exp1:=eval(w[1],x=1/2):
>exp2:=eval(w[1,xx],x=1/2):
>exp3:=eval(w[2],x=0):
>exp4:=eval(w[2,xx],x=0):
```

Condições de continuidade:

```
>w[1,x=0]:=eval(w[1],x=0):
>w[2,x=a]:=eval(w[2],x=1/2):
>w[1,x_x=0]:=eval(w[1,x],x=0):
>w[2,x_x=a]:=eval(w[2,x],x=1/2):
>w[1,xx_x=0]:=eval(w[1,xx],x=0):
>w[2,xx_x=a]:=eval(w[2,xx],x=1/2):
>w[1,xxx_x=0]:=eval(w[1,xxx],x=0):
>w[2,xxx_x=a]:=eval(w[2,xxx],x=1/2):
>exp5:=w[1,x=0]-w[2,x=a]:
>exp6:=w[1,x_x=0]-w[2,x_x=a]:
>exp7:=w[1,xx_x=0]-w[2,xx_x=a]:
>exp8:=w[1,xxx_x=0]-w[2,xxx_x=a]:
```

Solução trivial:

```
>solve({exp1,exp2,exp3,exp4,exp5,exp6,exp7,exp8},{_C1,_C2,_C
3,_C4,_C5,_C6,_C7,_C8}):
```

Cálculo dos elementos da matriz dos coeficientes:

```
>a11:=coeff(exp1,_C1):
>a21:=coeff(exp2,_C1):
>a31:=coeff(exp3,_C1):
>a41:=coeff(exp4,_C1):
>a51:=coeff(exp5,_C1):
>a61:=coeff(exp6,_C1):
>a71:=coeff(exp7,_C1):
>a81:=coeff(exp8,_C1):
>a12:=coeff(exp1,_C2):
>a22:=coeff(exp2,_C2):
>a32:=coeff(exp3,_C2):
>a42:=coeff(exp4,_C2):
>a52:=coeff(exp5,_C2):
>a62:=coeff(exp6,_C2):
>a72:=coeff(exp7,_C2):
>a82:=coeff(exp8,_C2):
```

```
>a13:=coeff(exp1,_C3):
>a23:=coeff(exp2,_C3):
>a33:=coeff(exp3,_C3):
>a43:=coeff(exp4,_C3):
>a53:=coeff(exp5,_C3):
>a63:=coeff(exp6,_C3):
>a73:=coeff(exp7,_C3):
>a83:=coeff(exp8,_C3):
>a14:=coeff(exp1,_C4):
>a24:=coeff(exp2,_C4):
>a34:=coeff(exp3,_C4):
>a44:=coeff(exp4,_C4):
>a54:=coeff(exp5,_C4):
>a64:=coeff(exp6,_C4):
>a74:=coeff(exp7,_C4):
>a84:=coeff(exp8,_C4):
>a15:=coeff(exp1,_C5):
>a25:=coeff(exp2,_C5):
>a35:=coeff(exp3,_C5):
>a45:=coeff(exp4,_C5):
>a55:=coeff(exp5,_C5):
>a65:=coeff(exp6,_C5):
>a75:=coeff(exp7,_C5):
>a85:=coeff(exp8,_C5):
>a16:=coeff(exp1,_C6):
>a26:=coeff(exp2,_C6):
>a36:=coeff(exp3,_C6):
>a46:=coeff(exp4,_C6):
>a56:=coeff(exp5,_C6):
>a66:=coeff(exp6,_C6):
>a76:=coeff(exp7,_C6):
>a86:=coeff(exp8,_C6):
>a17:=coeff(exp1,_C7):
>a27:=coeff(exp2,_C7):
>a37:=coeff(exp3,_C7):
>a47:=coeff(exp4,_C7):
>a57:=coeff(exp5,_C7):
>a67:=coeff(exp6,_C7):
>a77:=coeff(exp7,_C7):
>a87:=coeff(exp8,_C7):
>a18:=coeff(exp1,_C8):
>a28:=coeff(exp2,_C8):
>a38:=coeff(exp3,_C8):
>a48:=coeff(exp4,_C8):
>a58:=coeff(exp5,_C8):
>a68:=coeff(exp6,_C8):
>a78:=coeff(exp7,_C8):
>a88:=coeff(exp8,_C8):
```

Montagem da matriz dos coeficientes:

```
>A:=Matrix([[a11,a12,a13,a14,a15,a16,a17,a18],[a21,a22,a23,a
24,a25,a26,a27,a28],[a31,a32,a33,a34,a35,a36,a37,a38],[a41,a
42,a43,a44,a45,a46,a47,a48],[a51,a52,a53,a54,a55,a56,a57,a58
],[a61,a62,a63,a64,a65,a66,a67,a68],[a71,a72,a73,a74,a75,a76
,a77,a78],[a81,a82,a83,a84,a85,a86,a87,a88]]):
>det:=Determinant(A):
>K:=100:
```

Plotagem do gráfico do determinante (determinante x lambda):

```
>plot(Re(det),lambda=0..5.0,y=-1..1):
```

Definição do intervalo de variação de lambda para observação do ponto onde ocorre troca de sinal (repetir este procedimento até que o determinante atinja a ordem de cinco casas decimais):

```
>for lambda from 1.2 by 0.1 to 1.4 do
determ[lambda]:=print(simplify(det)) end do:
```

Definição do valor obtido para a primeira carga crítica:

```
>lambda:=1.222755491:
```

Cálculo dos novos elementos da matriz dos coeficientes:

```
>b11:=simplify(a11):
>b12:=simplify(a12):
>b13:=simplify(a13):
>b14:=simplify(a14):
>b15:=simplify(a15):
>b16:=simplify(a16):
>b17:=simplify(a17):
>b18:=simplify(a18):
>b21:=simplify(a21):
>b22:=simplify(a22):
>b23:=simplify(a23):
>b24:=simplify(a24):
>b25:=simplify(a25):
>b26:=simplify(a26):
>b27:=simplify(a27):
>b28:=simplify(a28):
>b31:=simplify(a31):
>b32:=simplify(a32):
>b33:=simplify(a33):
>b34:=simplify(a34):
>b35:=simplify(a35):
>b36:=simplify(a36):
>b37:=simplify(a37):
>b38:=simplify(a38):
>b41:=simplify(a41):
>b42:=simplify(a42):
>b43:=simplify(a43):
>b44:=simplify(a44):
>b45:=simplify(a45):
>b46:=simplify(a46):
>b47:=simplify(a47):
>b48:=simplify(a48):
>b51:=simplify(a51):
>b52:=simplify(a52):
```

```
>b53:=simplify(a53):
>b54:=simplify(a54):
>b55:=simplify(a55):
>b56:=simplify(a56):
>b57:=simplify(a57):
>b58:=simplify(a58):
>b61:=simplify(a61):
>b62:=simplify(a62):
>b63:=simplify(a63):
>b64:=simplify(a64):
>b65:=simplify(a65):
>b66:=simplify(a66):
>b67:=simplify(a67):
>b68:=simplify(a68):
>b71:=simplify(a71):
>b72:=simplify(a72):
>b73:=simplify(a73):
>b74:=simplify(a74):
>b75:=simplify(a75):
>b76:=simplify(a76):
>b77:=simplify(a77):
>b78:=simplify(a78):
>b81:=simplify(a81):
>b82:=simplify(a82):
>b83:=simplify(a83):
>b84:=simplify(a84):
>b85:=simplify(a85):
>b86:=simplify(a86):
>b87:=simplify(a87):
>b88:=simplify(a88):
```

Montagem da nova matriz dos coeficientes:

```
>B:=Matrix([[b11,b12,b13,b14,b15,b16,b17,b18],[b21,b22,b23,b
24,b25,b26,b27,b28],[b31,b32,b33,b34,b35,b36,b37,b38],[b41,b
42,b43,b44,b45,b46,b47,b48],[b51,b52,b53,b54,b55,b56,b57,b58
],[b61,b62,b63,b64,b65,b66,b67,b68],[b71,b72,b73,b74,b75,b76
,b77,b78],[b81,b82,b83,b84,b85,b86,b87,b88]]):
```

Redução de ordem da matriz B através da eliminação da sua terceira coluna e última linha:

```
>W:=Matrix([[b11,b12,b14,b15,b16,b17,b18],[b21,b22,b24,b25,b
26,b27,b28],[b31,b32,b34,b35,b36,b37,b38],[b41,b42,b44,b45,b
46,b47,b48],[b51,b52,b54,b55,b56,b57,b58],[b61,b62,b64,b65,b
66,b67,b68],[b71,b72,b74,b75,b76,b77,b78]]):
>V:=Vector([_C1,_C2,_C4,_C5,_C6,_C7,_C8]):
```

Montagem do sistema matricial:

```
>Q:=W.V:
```

Vetor correspondente à coluna eliminada da matriz B sem o elemento da sua última linha:

```
>R:=Vector([-b13,-b23,-b33,-b43,-b53,-b63,-b73]):
```

Resolução do sistema matricial:

```
>sols:=solve({Q[1]=R[1], Q[2]=R[2], Q[3]=R[3], Q[4]=R[4],
Q[5]=R[5], Q[6]=R[6], Q[7]=R[7]},
{_C1,_C2,_C4,_C5,_C6,_C7,_C8}):
>sols[1]:
```

```
>sols[2]:
>sols[3]:
>sols[4]:
>sols[5]:
>sols[6]:
>sols[7]:
>V:=subs(sols[1],sols[2],sols[3],sols[4],sols[5],sols[6],sol
s[7],V):
```

Cálculo da amplitude normalizada dos elementos do vetor das constantes:

```
>ampnorm:=sqrt(((V[1])^2)+((V[2])^2)+((V[3])^2)+((V[4])^2)+(
(V[5])^2)+((V[6])^2)+((V[7])^2)+1):
>Vnorm:=V/ampnorm:
>C3:=1/ampnorm:
```

Montagem do vetor normalizado das constantes:

```
>C:=Vector([Re(Vnorm[1]),Re(Vnorm[2]),Re(C3),Re(Vnorm[3]),Vn
orm[4],Vnorm[5],Vnorm[6],Vnorm[7]]):
>ee1:=_C1=C[1]:
>ee2:=_C2=C[2]:
>ee3:=_C3=C[3]:
>ee4:=_C4=C[4]:
>ee5:=_C5=C[5]:
>ee6:=_C6=C[6]:
>ee7:=_C7=C[7]:
>ee8:=_C8=C[8]:
>w[1]:
>w[2]:
```

Autofunção w1(x) após a substituição das constantes:

```
>w[1]:=subs(ee1,ee2,ee3,ee4,ee5,ee6,ee7,ee8,w[1]):
```

Autofunção w2(x) após a substituição das constantes:

```
>w[2]:=subs(ee1,ee2,ee3,ee4,ee5,ee6,ee7,ee8,w[2]):
```

Plotagem dos gráficos das funções w1(x) e w2(x):

```
>with(plottools):plot([w[1],x,x=0..0.5]):plot([w[2],x,x=0..0
.5]):
```

Comandos para exportação dos dados dos gráficos das funções w1(x) e w2(x):

Para o gráfico de w1(x):

```
>inf:=0:
>inter:=0.01:
>sup:=0.5:
>cont:=0:
for x from inf by inter to sup do
   cont:=cont+1
end do: cont:
>W1:=array(1..cont):
>X:=array(1..cont):
>cont:=0:
>fd:=fopen("u:\\saidaw1.dat",WRITE,BINARY):
>for x from inf by inter to sup do
    cont:=cont+1:
    X[cont]:=x;
    W1[cont]:=simplify(w[1]):
    fprintf(fd,"%f  , %f \n",X[cont],W1[cont]):
end do:
> fclose(fd):
```

Para o gráfico de w2(x):

```
>inf:=0:
>inter:=0.01:
>sup:=0.5:
>cont:=0:
for x from inf by inter to sup do
   cont:=cont+1
end do: cont:
>W2:=array(1..cont):
>X:=array(1..cont):
>cont:=0:
>fd := fopen("u:\\saidaw2.dat",WRITE,BINARY):
>for x from inf by inter to sup do
    cont:=cont+1:
    X[cont]:=x;
    W2[cont]:=simplify(w[2]):
    fprintf(fd,"%f  , %f \n",X[cont],Re(W2[cont])):
end do:
>fclose(fd):
```

8.3. Apêndice B

Programa para o Cálculo das Cargas Críticas através do Método Aproximado de Ritz

```
>restart;
>with(DEtools):with(LinearAlgebra):
>Digits:=16:
```

Função aproximada para os deslocamentos transversais:

```
>w(x):=A1*sin(Pi*x)+A2*sin(2*Pi*x)+A3*sin(3*Pi*x)+A4*sin(4*P
i*x)+A5*sin(5*Pi*x)+A6*sin(6*Pi*x)+A7*sin(7*Pi*x)+A8*sin(8*P
i*x):
```

Equação parametrizada de energia:

```
>eq:=int((diff(w(x),x,x)^2)-
(lambda^2*Pi^2)*(diff(w(x),x)^2),x=0..1)+int(K*w(x)^2,x=0..1
/2):
```

Equações de equilíbrio:

```
>deq1:=diff(eq,A1):
>deq2:=diff(eq,A2):
>deq3:=diff(eq,A3):
>deq4:=diff(eq,A4):
>deq5:=diff(eq,A5):
>deq6:=diff(eq,A6):
>deq7:=diff(eq,A7):
>deq8:=diff(eq,A8):
```

Solução trivial do sistema:

```
>solve({deq1,deq2,deq3,deq4,deq5,deq6,deq7,deq8},{A1,A2,A3,A
4,A5,A6,A7,A8}):
```

Cálculo dos elementos da matriz dos coeficientes:

```
>a11:=coeff(deq1,A1):
>a12:=coeff(deq1,A2):
>a13:=coeff(deq1,A3):
>a14:=coeff(deq1,A4):
```

```
>a15:=coeff(deq1,A5):
>a16:=coeff(deq1,A6):
>a17:=coeff(deq1,A7):
>a18:=coeff(deq1,A8):
>a21:=coeff(deq2,A1):
>a22:=coeff(deq2,A2):
>a23:=coeff(deq2,A3):
>a24:=coeff(deq2,A4):
>a25:=coeff(deq2,A5):
>a26:=coeff(deq2,A6):
>a27:=coeff(deq2,A7):
>a28:=coeff(deq2,A8):
>a31:=coeff(deq3,A1):
>a32:=coeff(deq3,A2):
>a33:=coeff(deq3,A3):
>a34:=coeff(deq3,A4):
>a35:=coeff(deq3,A5):
>a36:=coeff(deq3,A6):
>a37:=coeff(deq3,A7):
>a38:=coeff(deq3,A8):
>a41:=coeff(deq4,A1):
>a42:=coeff(deq4,A2):
>a43:=coeff(deq4,A3):
>a44:=coeff(deq4,A4):
>a45:=coeff(deq4,A5):
>a46:=coeff(deq4,A6):
>a47:=coeff(deq4,A7):
>a48:=coeff(deq4,A8):
>a51:=coeff(deq5,A1):
>a52:=coeff(deq5,A2):
>a53:=coeff(deq5,A3):
>a54:=coeff(deq5,A4):
>a55:=coeff(deq5,A5):
>a56:=coeff(deq5,A6):
>a57:=coeff(deq5,A7):
>a58:=coeff(deq5,A8):
>a61:=coeff(deq6,A1):
>a62:=coeff(deq6,A2):
>a63:=coeff(deq6,A3):
>a64:=coeff(deq6,A4):
>a65:=coeff(deq6,A5):
>a66:=coeff(deq6,A6):
>a67:=coeff(deq6,A7):
>a68:=coeff(deq6,A8):
>a71:=coeff(deq7,A1):
>a72:=coeff(deq7,A2):
>a73:=coeff(deq7,A3):
>a74:=coeff(deq7,A4):
>a75:=coeff(deq7,A5):
>a76:=coeff(deq7,A6):
>a77:=coeff(deq7,A7):
>a78:=coeff(deq7,A8):
>a81:=coeff(deq8,A1):
>a82:=coeff(deq8,A2):
>a83:=coeff(deq8,A3):
>a84:=coeff(deq8,A4):
```

```
>a85:=coeff(deq8,A5):
>a86:=coeff(deq8,A6):
>a87:=coeff(deq8,A7):
>a88:=coeff(deq8,A8):
```

Montagem da matriz dos coeficientes:

```
>A:=Matrix([[a11,a12,a13,a14,a15,a16,a17,a18],[a21,a22,a23,a
24,a25,a26,a27,a28],[a31,a32,a33,a34,a35,a36,a37,a38],[a41,a
42,a43,a44,a45,a46,a47,a48],[a51,a52,a53,a54,a55,a56,a57,a58
],[a61,a62,a63,a64,a65,a66,a67,a68],[a71,a72,a73,a74,a75,a76
,a77,a78],[a81,a82,a83,a84,a85,a86,a87,a88]]):
```

Cálculo do determinante da matriz dos coeficientes:

```
>det:=Determinant(A):
```

Definição do valor de K:

```
>K:=100:
```

Plotagem do gráfico do determinante em função de lambda:

```
>plot(det,lambda=0..3):
```

Cálculo dos "zeros" da equação do determinante, ou seja, os valores de lambda crítico:

```
>sols:=fsolve(det,lambda=0..3):
```

8.4. Apêndice C

Programa para a Determinação do Caminho Pós-Crítico pelo Método de Ritz

```
>restart:
>with(DEtools):with(LinearAlgebra):
>Digits:=28:
```

Equação não-linear de energia:

```
>eq:=(1/2)*(int((diff(w1(x),`$`(x,2))^2+diff(w1(x),`$`(x,2))
^2*diff(w1(x),x)^2+1/4*diff(w1(x),`$`(x,2))^2*diff(w1(x),x)^
4)-
(lambda^2*Pi^2)*(diff(w1(x),x)^2+1/4*diff(w1(x),x)^4),x=0..1
/2)+(int((diff(w2(x),`$`(x,2))^2+diff(w2(x),`$`(x,2))^2*diff
(w2(x),x)^2+1/4*diff(w2(x),`$`(x,2))^2*diff(w2(x),x)^4)-
(lambda^2*Pi^2)*(diff(w2(x),x)^2+1/4*diff(w2(x),x)^4),x=0..1
/2)+int(K*w2(x)^2,x=0..1/2))):
>K:=100:
```

Soluções analíticas do primeiro modo para os dois trechos da coluna, multiplicadas pela deformação máxima, eta:

```
>w1(x):=eta*(.34779196320845193242595489 76-
.69558392641690386485190979 52*x+.25525171589436987466908282 5
2*sin(1.222755491*Pi*x)+.69947471101138142730540966 96*cos(1.
222755491*Pi*x)):
>w2ssimp(x):=eta*((-.93725496132670571118673461 26e-1-
.22042710397923083808973925 62*I)*exp(-1/2*(-
2.990261981541302162*Pi^2-
2*(2.23541667956282872795410346 9*Pi^4-
400)^(1/2))^(1/2)*x)+(.93725496132670571118673461 26e-
1+.22042710397923083808973925 62*I)*exp(1/2*(-
2.990261981541302162*Pi^2-
2*(2.23541667956282872795410346 9*Pi^4-
400)^(1/2))^(1/2)*x)+(-.93725496132670571118673461 26e-
1+.22042710397923083808973925 62*I)*exp(-1/2*(-
```

```
2.990261981541302162*Pi^2+2*(2.235416679562828727954103469*P
i^4-400)^(1/2))^(1/2)*x)+(.937254961326705711186734612 6e-1-
.220427103979230838089739256 2*I)*exp(1/2*(-
2.990261981541302162*Pi^2+2*(2.235416679562828727954103469*P
i^4-400)^(1/2))^(1/2)*x)):
>w2(x):=simplify(w2ssimp(x)):
```

Parte da equação da energia correspondente ao trecho sem fundação, sem as integrais (apenas o integrando):

```
>eq1:=(1/2)*((diff(w1(x),`$`(x,2))^2+diff(w1(x),`$`(x,2))^2*
diff(w1(x),x)^2+1/4*diff(w1(x),`$`(x,2))^2*diff(w1(x),x)^4)-
(lambda^2*Pi^2)*(diff(w1(x),x)^2+1/4*diff(w1(x),x)^4)):
```

Parte da equação da energia correspondente ao trecho com fundação, sem as integrais (apenas o integrando):

```
>eq2:=(1/2)*((diff(w2(x),`$`(x,2))^2+diff(w2(x),`$`(x,2))^2*
diff(w2(x),x)^2+1/4*diff(w2(x),`$`(x,2))^2*diff(w2(x),x)^4)-
(lambda^2*Pi^2)*(diff(w2(x),x)^2+1/4*diff(w2(x),x)^4)):
>eq3:=(1/2)*(K*w2(x)^2):
```

Integração das equações nos limites correspondentes:

```
>result1:=evalf(int(eq1,x=0..1/2)):
>result2:=evalf(int(eq2,x=0..1/2)):
>result3:=evalf(int(eq3,x=0..1/2)):
```

Soma das expressões resultantes das integrações parciais da equação de energia:

```
>result:=result1+result2+result3:
```

Diferenciação da equação de energia com relação à constante eta:

```
>deq:=diff(result,eta)=0:
```

Obtenção das soluções da equação:

```
>sols:=solve(deq):
```

Equação do caminho pós-crítico obtida no passo anterior, com a exclusão dos eventuais termos imaginários:

```
>lambda:=10./(.749360322242585124642922 2776e61+.310399519184
6389273196978993e62*eta^2)*(-
1.*(.749360322242585124642922 2776e59+.3103995191846389273196
978993e60*eta^2)*(-.429274258547840491415 7583436e62*eta^2-
.930786068502076212035251 9378e62*eta^4-
.112039184221941146789822 2441e62))^(1/2):
```

Carga crítica já obtida anteriormente:

```
>lcr:=1.222755491:
```

Plotagem do gráfico do caminho pós-crítico:

```
>plot([lambda/lcr],eta=-0..0.5):
```

Exportação dos dados do gráfico do caminho pós-crítico:

```
>inf:=0:
>inter:=0.0002:
>sup:=0.2:
>cont:=0:
for eta from inf by inter to sup do
   cont:=cont+1
end do: cont:
>L:=array(1..cont):
>ETA:=array(1..cont):
>cont:=0:

>fd := fopen("u:\\camposcr.dat",WRITE,BINARY):
>for eta from inf by inter to sup do
    cont:=cont+1:
```

```
    ETA[cont]:=eta;
    L[cont]:=simplify(lambda):
    fprintf(fd,"%f  , %.7f \n",ETA[cont],L[cont]):
end do:
>fclose(fd);
```