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Quantum Computers: An Overview of Their Market and Physical Realisations

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Abstract

Quantum computing has been a topic of research in academia for well over three decades. However, only in the past ten years did global markets follow that trend, investing millions into research. This is due to the belief that quantum computers could harness the power to solve problems that today's supercomputers can barely simulate.

While some progress has been made, we are still years away from quantum computers reaching their full potential. Bulding them is an intricate task, and operating on them isn't further behind: decoherence, error correction and scalability are three of the many challenges the field of quantum computing faces.

The aim of this work is to introduce quantum computing from a theoretical point of view, and examine the market share of the field given the recent increase in investments. It also discusses three of the most popular methods of fabricating quantum computers: photonic, superconducting and electron spin quantum dot computers. We also discuss advantages each has over the other, acknowledging that we are still in the noisy intermediate scale quantum era.

Keywords: quantum computing, qubits, quantum information

Computadores Quânticos: Uma Visão Global de Seu Mercado e Implementações Físicas

Resumo

A computação quântica tem sido um tópico de pesquisa acadêmica por mais de três décadas. No entanto, apenas nos últimos dez anos os mercados globais seguiram essa tendência, investindo milhões em pesquisa. Isso se deve à crença de que os computadores quânticos poderão futuramente resolver problemas que os supercomputadores de hoje são incapazes de simular.

Embora algum progresso tenha sido feito, o potencial dos computadores quânticos ainda se encontra a décadas de distância. Isso porque tanto construí-los quanto operá-los é uma tarefa complexa: descoerência, correção de erros e escalabilidade são três dos muitos desafios que o campo da computação quântica enfrenta.

O objetivo deste trabalho é apresentar a computação quântica de um ponto de vista teórico e examinar seu efeito no mercado, dado o recente aumento exponencial de investimentos na área. Também serão discutidos três dos métodos mais populares de fabricá-los: computadores fotônicos, supercondutores e de electron spin. Também discutimos as vantagens que cada um tem sobre o outro, reconhecendo que ainda estamos na era *noisy intermediate scale quantum*.

Palavras-chave: computação quântica, qubits, informação quântica

Summary

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1 Introduction

Quantum computing is a technology that takes advantage of quantum mechanics to provide information processing that surpasses that of their classical counterparts [2]. According to physicist John Preskill, we are at the entanglement frontier: learning to build computers that can interact with highly entangled quantum states of many particles in ways classical computers are unable to simulate. What makes quantum computers fascinating is their ability to store complex probabilities (i.e. quantum states) in a single bit. This however is also what makes them so difficult to design, build, and verify. As quantum states are fragile, several challenges are separating us from precise measurements.

While Richard Feynmann and Yuri Manin proposed such devices in the 1980s [3], their commercial value was not significant due to many factors, including their difficulty in fabrication [4]. Even then, the computational potential of QC has attracted the attention of major industries such as chemistry, pharmaceuticals, finance, medicine and even cybersecurity [5]. In recent years, there has been a surge of investments in the field during what Nature deemed the quantum gold rush [6]. The global market of quantum computing was valued at over USD 490Mi in 2021, and projections expect it to reach a whopping USD 2,930.67Mi by 2028 [7].

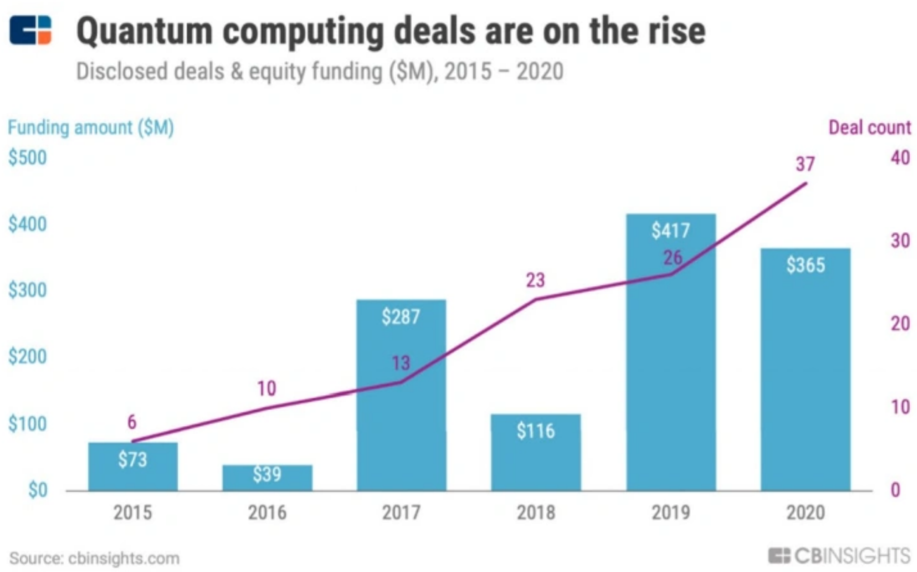

Figure 1: Investment deals in quantum computing between 2015 and 2020.

Source: Charles O'Brien [8]

This has come as a surprise even to academics such as John Preskill, who has been in the field for decades. This is because the unprecedented power universal quantum computers are likely to hold is still years away in research and improvement [9]. We are currently in an era describable by the valuable acronym NISQ (Noisy Intermediate Scale Quantum). The word "noisy" comes from the fact that the control we have over qubits is imperfect, imposing some serious limitations on the system. The largest quantum computer currently available, Osprey, was demonstrated by IBM in 2022, and it has 433 qubits [10]; it is still intermediate in size. Most companies, such as Google and IBM, believe that quantum computers will only be commercially useful when they reach 1,000 or more qubits [11].

Even then, research has recently shown that even with a small number of qubits, quantum computers can already be utilised for interesting applications [9]. According to Kane, this is due to two important insights: quantum algorithms that outperform classical computers have already been discovered and quantum error correction can allow for the computers to perform despite a small degree of decoherence [2].

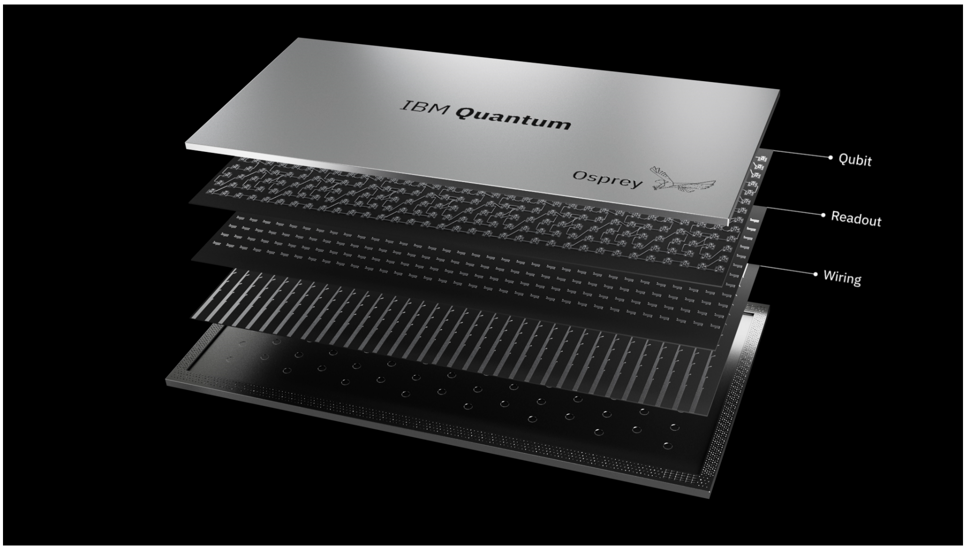

Figure 2: Current largest computer, IBM Osprey, at 433 qubits.

Source: IBM [10]

There are still, however, many challenges to be overcome when it comes to the fabrication of a universal quantum computer. Firstly, there is a need for the system to be scalable, as many qubits are required for the computer to operate [4]. Qubit quality is an important additional factor, especially as quantum logic gates that operate on them must be lossless to avoid rapid decoherence [12]. For the same reason, operations must occur rapidly on a large number of qubits [4]. Another challenge is the low temperature, usually in the range of milli-Kelvins, that most current-day quantum computers have to operate in [13]. Cryogenic systems are expensive to maintain, and refrigerator sizes can deter the construction of large computers. Lastly, while most of today's systems rely solely on error mitigation techniques, true quantum error correction is also necessary for the longevity of the information harnessed by the system [14].

In order to overcome the aforementioned challenges, companies and universities have invested in researching multiple methods of building quantum computers. Both Google's and IBM's efforts have been focused on superconducting quantum computers [10, 15]. Canadian start-up Xanadu, on the other hand, believes photonic devices should be the go-to when it comes to quantum computing [16]. Electron spin qubits have also been targeted by multiple researchers, as atoms are prototypical quantum systems [17].

The purpose of this work is to expand on quantum computing and its principles, seeking to identify why it has become such an important technology. We aim to illustrate the challenges in the field by describing three methods of QC fabrication: photonic computers, superconducting computers, and quantum dot electron spin computers.

2 Literature Review

Before exploring more complex aspects of quantum computing, it is necessary to lay the foundations and understand what is driving and motivating the technology before diving into more complex aspects of the technology.

a The Qubit

As far as classical computing is concerned, a bit is the smallest unit of data, which is usually expressed as states 0 or 1. According to Chris Bernhardt , "it can be represented by anything that has two mutually exclusive states. The standard example is a switch that can be either the on or off position." [18].

We can translate this concept to quantum computing, in which we study quantum bits or qubits. Unlike classical bit, quantum bits, or qubits, work with quantum physics concepts, being representations of particle states. The computational basis we usually use to describe the states of qubits is comprised of $|0\rangle$ and $|1\rangle$. Even then, qubits cannot be seen as being in an "on or off" state; they can be in a superposition of the two states. This means that these quantum states need to be measured, which makes them more complex to work with.

The representation of the states in the mathematical scenario is defined by qubit being a unit vector in C^2. The equation below describes the states that may fill a two-dimensional complex vector space for a single qubit [19]:

$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

The state of the qubit remains in superposition until it is measured. We cannot know for certain in which state a qubit lies, but only the probability that it lies in it. In the orthonormal basis shown in Equation 1, the probability for the qubit to be in state $|0\rangle$ is given by $|\alpha|^2$, while the probability for it to be in state $|1\rangle$ is known to be $|\beta|^2$. Since we are dealing with probabilities, it must be emphasised that $|\alpha|^2 + |\beta|^2 = 1$, since no probability can exceed 1. Therefore, the state must always be represented as a normalised vector.

The power of quantum computers can be expressed when the number of qubits is compared to the number of bits and bytes in the random access memory (RAM) of a classical computer. While according to Adrian Cho building a current-day 16GB RAM is complicated, in QC terms, only 37 qubits would be necessary for a quantum computer to have the same power. IBM has already built a 433-qubit computer and is aiming to put out an 1121-qubit one by the end of 2023 [20].

N Qubits	N bits	RAM size
1	2	2 bits
2	4	4 bits
3	8	1 byte
4	16	2 bytes
5	32	4 bytes
6	64	8 bytes
7	128	16 bytes
8	256	32 bytes
9	512	64 bytes
10	1024	128 bytes
11	2048	256 bytes
12	4096	512 bytes
13	8192	1kB
14	16384	2kB
15	32768	4kB
16	65536	8kB
33	8589934592	1GB

1 Multi-qubit States

We have already seen how the state of one qubit can be described by computational basis $|0\rangle$ and $|1\rangle$. What happens if there are multiple qubits in the system? The computational basis that a multi-qubit system spans over is of the form $|x_1, x_2, ..., x_n\rangle$, and each term is a different combination of $|0\rangle$ and $|1\rangle$. That means that, for a two-qubit system, the computational basis becomes $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$. It must be stated, however, that this is only a way to simplify the notation, which would otherwise be $|0\rangle \bigotimes |0\rangle$. Therefore, in $|00\rangle$, the first zero represents the state of the first qubit and the same holds true for the second.

The power of these systems is clear in that the two qubit system can be a superposition of the four states; this means that there are four complex constants needed to fully describe the system. As we increase the number of qubits in the system, n, the harder it is for it to be described, as the 2^n coefficients must be known. This altogether shows how interesting a quantum computer can be, but also how hard it is to model [4].

2 Bell States

Some of the most important multi-qubit states in quantum computing are the Bell states. They are mathematically represented by

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \tag{2}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \tag{3}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \tag{4}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \tag{5}$$

While at first look they seem like any other state, they hold some consequences that mould quantum computing. Let us analyse the first state, $|\Psi^+\rangle$. If a measurement is done on the first qubit, there is a $1/2$ probability that it will be in state $|0\rangle$. After measurement, then, the system will be in state $|00\rangle$. The same probability is true for the first qubit being $|1\rangle$, leaving the system in state $|11\rangle$. This means that the second qubit will always be in the same state as the first after the measurement is performed on the system. Thus, there is some sort of correlation between both qubits [21].

The name of these states comes from physicist John Bell, who proved that this correlation between the states is stronger than any in classical systems [21].

3 Bloch Sphere

Albeit complicated, the mathematical concept of a qubit can be visually represented by what is known as the Bloch sphere. Named after physicist Felix Bloch, it is often used to represent operations on single qubits. When referring to a state represented by the Bloch sphere, we usually use the following equation, which is derived from Equation 1:

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{j\varphi}\sin\frac{\theta}{2}|1\rangle \tag{6}$$

In the equation above, "θ represents the angle from the positive z-axis to the positive x-axis on the x-y plane" [22]. Angle φ on the other hand is between the "positive x-axis and the positive y-axis on the x-y plane" [22].

While the Bloch sphere makes itself useful for simpler, one qubit systems, a way to generalise it to multiple qubit systems is not known [21].

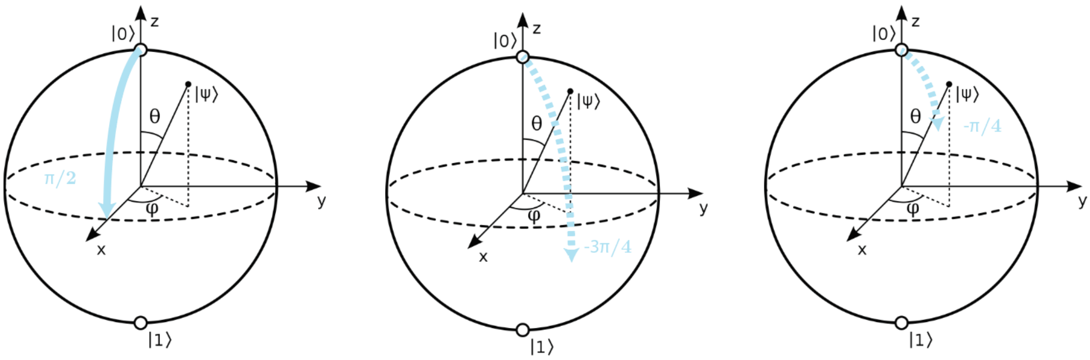

Figure 3: Bloch sphere representing changes in two-level quantum mechanical systems.

b Overview of Quantum Computing

This section aims to give a brief overview of quantum computing concepts.

1 Operations on Qubits

How do quantum computers work? While this is a complicated question that cannot be answered in a brief overview of the topic, some aspects must still be introduced. There are two main approaches to quantum computing; the first is a gate-based approach, in which a problem is broken down into primitive gates with well-defined outputs for the given input state [4]. The second, quantum annealers, relies on the initialisation of the qubit and further manipulation of the Hamiltonian of the system to achieve the desired answer [4]. While quantum annealer computers of around 5000 qubits have been made, they are not interesting in the search for universal quantum computers, as they are only useful for some optimisation applications. This dissertation will focus on the former, as the types of quantum computers used for the latter are not of interest to our study of physical realisations.

Operations must be made on qubits, the same way they are on classical bits, to perform calculations. Therefore, quantum logic gates are the quantum counterpart to the classical gates used in current-day computing. To keep the state vector normalised, every operation done on a single qubit must be described by a 2x2 unitary matrix [21]. There are three quantum gates that, in spite of being simple, are quite notable, since they are derived from the well-known Pauli matrices. Known as gates X, Y and Z, they are represented by the following matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{7}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{8}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9}$$

If we wish to interpret these quantum gates geometrically, we can visualise them on the Bloch sphere. Each operator describes some sort of reflection about its axis. Operator X causes a reflection about the x-axis; the same holds true for operators Y and Z, only about their respective axis.

Another important gate is the Hadamard gate. Its visualisation would be "a rotation of the sphere about the y-axis by $90\circ$, followed by a rotation about the x-axis by $180\circ$." [21]. Its unitary matrix is given by

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{10}$$

The importance of this state comes from its ability to put a qubit initialised in $|0\rangle$ to an equal superposition of $|0\rangle$ and $|1\rangle$. The same holds true for a state initialised in $|1\rangle$, but with opposite phases [4].

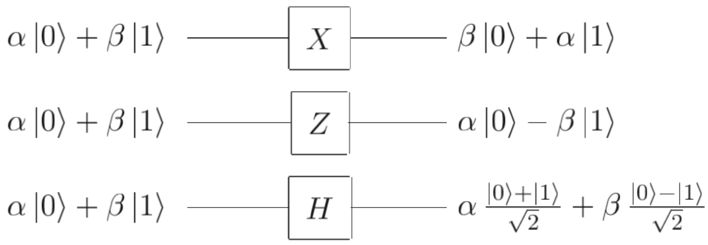

Figure 4: Qubit gates and their effect on the state vector.

Source: Nielsen and Chuang [21]

Multiple qubit systems can also be operated on by logic gates. A notable two-qubit operation known as CNOT is the quantum equivalent of logical gate XOR. It flips the target qubit if and only if the control qubit is $|1\rangle$. Thus, this quantum gate is capable turning non-correlated states into an entangled state.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{11}$$

The previously introduced Bell states can be acquired by using the CNOT gate. An example would be creating state $|\Phi^+\rangle$ by applying H to a qubit q_0 initialised in $|0\rangle$ and then applying CNOT to q_0 and another qubit q_1 also initialised in $|0\rangle$. The states, initially separable, become maximally entangled.

The physical implementation of quantum gates requires the Hamiltonian of the desired qubits to be precisely changed for the time needed by the operation [4]. There are, however, a few known constraints to the operations. Since noise is a factor for decoherence, it is required for the operations to be lossless. This also has another consequence for the gates: since there is energy dissipation in the loss of information, quantum gates must also be reversible. Additionally, to make sure the quantum states are always normalised, all quantum gates must be a simple rotation of the state vector to a new position [4].

Usually, due to the complexity of their construction, quantum gates only operate on a maximum of three qubits.

2 Quantum Measurements

The second postulate of quantum mechanics states that every closed quantum system is described by a unitary transformation [21]. Nonetheless, when an experiment is being done, the experimentalist is likely to observe the system, which consequently ceases to be closed. What happens, then, when a measurement is performed?

As seen when qubits were introduced, what we know about quantum systems are probabilities. When a measurement is made, the wave function collapses, and the system is no longer in a superposition of multiple states. From then on, there is a well-defined state that the system is in. This means that additional measurements cannot provide any information on the previous state of the system. This in itself exemplifies why working with quantum systems is so hard: any interaction with them can lead to the collapse of the wave function.

3 Quantum Circuits

Complications from working with quantum gates arise from the several steps that must be taken to achieve a given state. In addition to that, since quantum states are so fragile, there are also intricacies involved with keeping the states intact. In quantum information theory, quantum circuits are well-known and well-established models of how to realise quantum computation [21]. They consist of a routine of steps of coherent operations on quantum data using quantum gates and measurements [23].

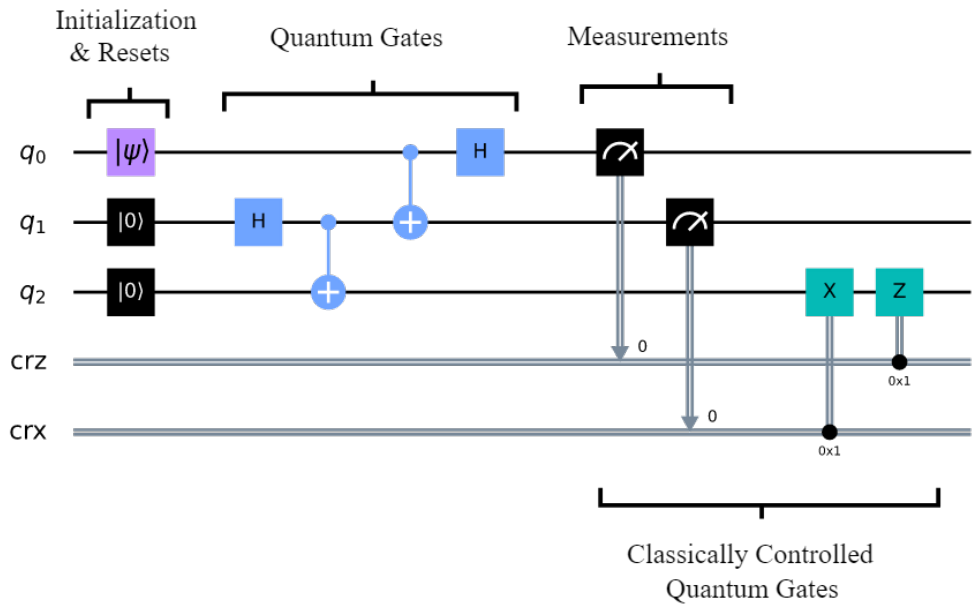

Figure 5: Qubit gates from IBM Qiskit Example.

Source: Qiskit [23]

The schematic notation for quantum circuits usually consists of one line for each qubit, and symbols to represent logic gates and measurements. While each line in the schematic does represent a wire, such wires are not necessarily physical wires; they can for instance mean the passage of time or movement through space [21]. Another convention in the space of quantum computing is that all qubits are initialised to state $|0\rangle$ unless otherwise stated [21]. To illustrate this, Figure 5 shows qubits q_1 and q_2 initialised to the conventional state $|0\rangle$, while q_0 is initialised to state $|\Psi\rangle$.

The significance of these circuits is more evident when observing user-machine interactions in which the user determines what will be run. This involves creating a quantum program, which consists of instructions that initialise the particles, create quantum circuits, and work non-concurrently with classical computing to measure and process data [23].

As previously said, quantum logic gates are the quantum counterpart to the classical gates used in current-day computing [21]. Gates can be understood as the fundamental block of every quantum operation [18]. While logic operations may sound simple in classical computing, where it is possible to work with bits 0 and 1, quantum computers use a more intricate system to perform their operations. This is particularly true when dealing with operations that involve multiple qubits [18].

4 Quantum Entanglement

The potential computational power of quantum computers is primarily a consequence of their foundation in quantum physics. This is why they hold the ability to handle multiple entangled particles. In quantum physics, quantum entanglement occurs when two or more particles become connected so that their properties become strongly correlated, or "entangled". Thus, you cannot describe the state of an entangled particle independently from the state the other entangled particles are in. This remains true in spite of how far these particles might be [21].

Bell states are an example of quantum entanglement. While knowing the state of the second qubit after measuring the first was introduced as a correlation between both, these states are actually maximally entangled. Maximum entanglement simply means that knowledge about one state means the other is also completely known [21].

According to Chris Bernhardt's book "Quantum Computing for Everyone", when working with quantum circuits, we must understand how we can entangle particles. The most common method is called Spontaneous Parametric Down-Conversion (SPDC) [18]. SPDC is the optical process of hitting a photon using a laser beam, in order to convert one higher-energy photon into a pair of less-energetic ones. This results in the creation of a pair of entangled photons with the combined momentum and energy of the primary one, according to the laws of conservation and momentum [24].

As mentioned in Section 2.b.3, working with more than one qubit is complicated and tends to trigger a series of problems. There is a possibility that errors will occur as different processes and operations are conducted. It is imperative to know how to handle these errors in advance, just as with a conventional computer.

5 Quantum Algorithms

Despite fault-tolerant quantum computers not being currently available, quantum algorithms have been actively researched for over two decades [25]. These algorithms have been used to accelerate the runtime of operations when compared to classical computers.

In 1995, Peter Shor challenged the general assumption that factoring integers with over 1000 digits was impossible [23]. Shor created an algorithm based on the quantum Fourier transform that provided an exponential speedup over the best-known solution on a classical computer [21]. It is currently known that the quantum Fourier transform can be used to solve not only factoring problems, but also discrete logarithm problems; this could enable a quantum computer to break most of the cryptosystems in use today [21].

A further well-known quantum algorithm was developed by Lov Grover in 1996 [26]. It was able to deliver a quadratic speedup over classical algorithms for database searching [21]. Simple databases can easily be stored in a structured manner; using a quantum algorithm would most likely not be economically viable in this scenario. On the other hand, more sophisticated databases that might require unanticipated or complex queries could see a significant benefit from using Grover's technique [21]. The algorithm is, however, universal and has other use cases such as the travelling salesman and satisfiability problems [21].

6 Handling Errors

As opposed to conventional computers, which have a well-established method for handling errors, quantum error correction (QEC) is a growing area of research within quantum information.

For a computer to be considered reliable, it is imperative that the information received to be verified and errors to be corrected. A common way of checking for mistakes is to re-send the information as in redundancy protocols. However, in quantum computing, states are extremely fragile; sending the same message repetitively can lead to errors and other issues such as overheating.

The main thing that makes quantum error correction different from classical error correction is the no-cloning theorem. This important theorem states that " states that a set of pure states can be cloned if and only if the states are mutually orthogonal." [27]. Consequently, classical coding based on data-copying is not feasible in quantum computing architectures [28]. According to Devitt et al., since measurements interfere with a qubit's current state, any correction protocol must detect and correct errors without taking measurements or determining any information from the qubit. [28].

To understand the importance of quantum error correction, one must know the sources of errors in a quantum system [28]. One of the most common sources is the incorrect application of gates. Another, and perhaps the most well known, is environmental decoherence. This occurs when some sort of entanglement occurs between the state and the environment, inherently turning the system into a classical one. Other errors may come from qubit initialisation, loss, leakage and incorrect measurements.

The most basic quantum error correction algorithm is the bit flip code, which is a protocol to reinstate a previously flipped qubit. As an example, if a qubit $|0\rangle$ should have been $|1\rangle$, a simple three-qubit circuit is required to correct it. Using two auxiliary qubits to protect the original one, the bit-flip code can successfully identify and correct a single flipped qubit error. This is a simple method to identify and correct errors. However, it can't protect against two flipped bits or correct phase-flip errors; there are more complicated algorithms that provide that level of protection.

Quantum information theory introduces us to multiple sources of errors and how to correct them. While this is of extreme importance in reaching fault-tolerant quantum computers, error correction is not the main focus of this work.

7 Di Vincenzo's Criteria

In order to build a quantum computer, a few specifications must be met. Those are known as Di Vincenzo's criteria since they were introduced by physicist David Di Vincenzo in 2000 [19]. There are five conditions in total, and two extra ones are only made necessary in case quantum communication is also desired. They are as follows:

- Scalability and characterisation: Building quantum computers requires quantum systems in the form of qubits. As described in Equation 3, these systems must be quantum two-level systems. Additionally, they must be well characterised. To accomplish this, it is necessary to know the internal Hamiltonian of the qubit, as well as the interactions between it and other qubits and external fields. 1.
- Qubit must be able to be initialised to a simple fiducial state. The reason for this does not necessarily have to do with quantum computing, but rather with the fact that the initial state of each register must be known before any computation can begin [19]. Furthermore, quantum error correction requires a continuous supply of qubits with a low entropy level [19].
- Decoherence time longer than gate operation time. This allows quantum computations to be performed alongside quantum error correction before the behaviour of the quantum system turns borderline classical [19].
- Set of quantum gates that can be universally applied to qubits [19]. This is necessary so arbitrary operations can be made.
- Ability to measure specific qubits without altering nearby qubits' states [19].
- "Ability to interconvert stationary and flying qubits" [19].
- "Ability to faithfully transmit to flying qubits between specified locations" [19].

Trying to build a system that follows all of these conditions is highly complicated, and this can be taken as the reason why building a quantum computer is taking so long [29].

3 Market of Quantum Computing

a Why Quantum Computing is Important

The industry of quantum technologies is undoubtedly growing, most likely due to the highly-compelling computational power quantum computers might have in the future. Although we are yet to reach quantum supremacy, the potential for practical applications of QC is unprecedented. As the industry matures, new research brings us closer to the goal of building fault-tolerant universal quantum computers [4].

b Industrial Applications of Quantum Computing

It is nearly impossible to predict the extent of the impact quantum technologies will have, as their computational power is growing fast. QC can be thought of in the same manner as classical computers and the internet at the beginning of their development. Then, it was difficult to predict the direction they would take in the next 20 years. Despite assumptions that computers would be an integral part of people's lives in the future, no one predicted the extent at which they affect society today, with smartphones, automation and robotics [18]. We can however estimate the impact this innovative technology may have on several industries.

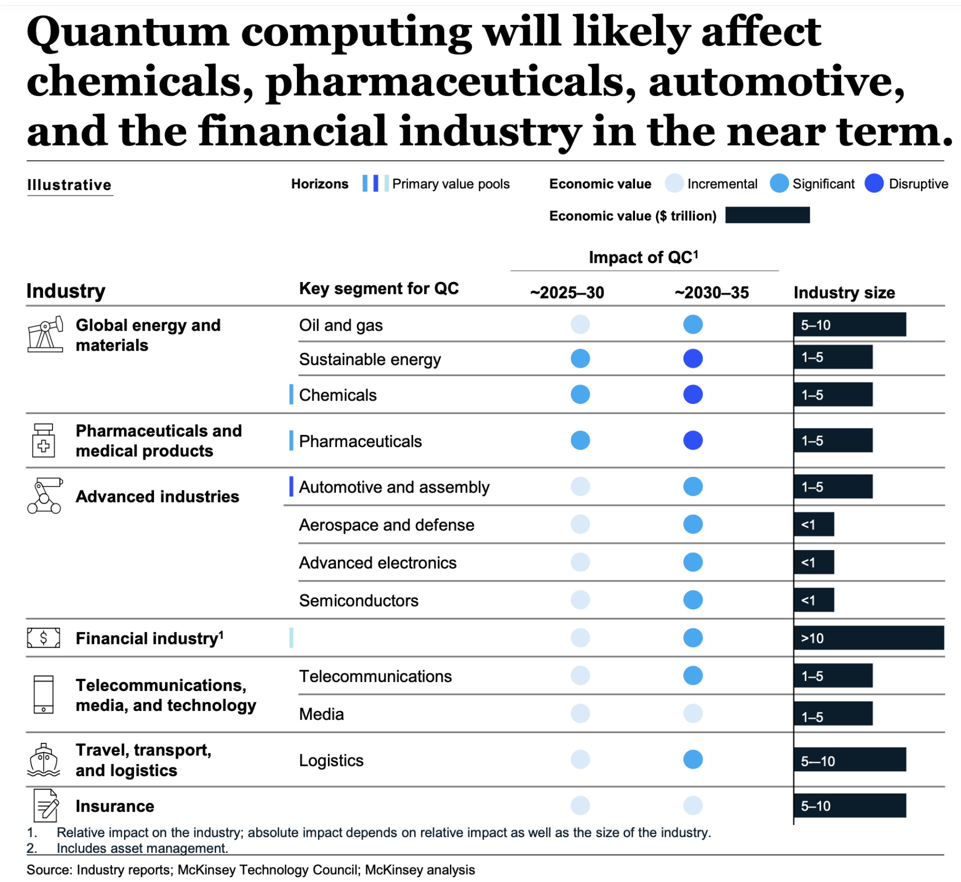

Figure 6: Predicted impact of quantum computing in important industries.

Source: McKinsey [30]

One of the biggest industries concerned with the development of quantum computers is cyber-security. This is because quantum computers might be able to process in seconds algorithms that would take months or years for the best classical computers to process. Consequently, QC can become the greatest ally and the greatest danger for cryptographic analysis [18]. An example of this is Shor's algorithm [18]. While it remains impossible by today's standards to break the 2048-bit RSA encryption, as it would take an average of quadrillions of years, it is likely that a general-purpose QC with a large number of qubits could take down the entire cryptographic public-key infrastructure in a few hours [4].

That is why post-quantum cryptography is now an active field of research, as there is a need to understand how cybersecurity will handle cryptoanalysis when quantum computers are available. This comes due to the fact that a quantum computer, when used with malicious intent, could be used to for instance exploit and crack a bank account's public keys [18]. This highlights the importance of developing new ways to encrypt private data [18].

1 Pharmaceutical industry

The pharmaceutical industry also shows great interest in quantum computing. This is because it could potentially cut down the development time of a market-ready product, which is known to take a minimum of 12 years [31]. Since chemistry involves theory related to the interactions of atoms, scientists tend to create mathematical approximations that can ignore minute details. With quantum computers, this could change quite drastically as they could be capable of performing exact calculations and simulating the system minutely [18]. This has the potential to improve drug discovery and pre-clinical research in pharmaceutical companies [31].

An interesting application of QC in drug discovery is the modelling of chemical reactions which would allow the study of how "drugs interact with proteins and enzimes in the human body." [31]. This may explain the emergence of numerous collaborations between big pharma and quantum computing companies [31]. British Cambridge Quantum Computing and Swiss multinational Roche are in a notable partnership that aims to collaborate on the design of quantum algorithms for drug discovery and development [31].

2 Finance industry

It is believed that the financial sector will be among the first to benefit from quantum computing, as financial problems are often more fault tolerant to imprecisions than those in the life sciences [32]. One of the most important techniques in stochastic process modelling in finance is the Monte Carlo method. A quantum approach named quantum Monte Carlo integration (QMCI) has been introduced, as well as quantum partial differential equation solvers [32]. They can be used to model derivative pricing and risk analysis, both of which are important factors in finance [32].

Since optimisation is also of great importance for this industry, adiabatic and variational quantum algorithms can also be of use [32]. In particular, credit scoring, financial crash prediction and portfolio optimisation are three of the most direct applications for quantum computing [32].

One last use-case of quantum technologies in finance is speeding up the training of machine learning algorithms. These are important for asset pricing and anomaly detection, among others [32].

3 Process and product design and logistics industries

The current pipeline of a product combines various computational models which could benefit from quantum computing [31]. This is expected as computational power is of extreme importance to logistics applications, as multiple variables must be considered and machine learning methods might have been applied. Rieffel et al. have used D-Wave quantum annealers to tackle optimisation problems in operational planning [33]. They have proven that the development of a framework that could apply direct mapping schemes for specific planning problems would be beneficial [31].

There is, however, a downside to using quantum computers in their current state for logistics. According to Ajagekar et al., the capacity of today's quantum computers hinders the quality of the solutions in problems with a larger scale [34]. The conclusion came from a study that "solved unit commitment, facility-location, and heat exchanger network problems using QC." [31]. They do however believe that the evolution of quantum error correction and the number of qubits in quantum computers will make QCs beneficial for this application [31].

c Market Share

According to McKinsey's last market report, the pharmaceutical, chemical, automotive and finance industries are the top four investors aiming to benefit from quantum technology. McKinsey predicts quantum technologies will have a market value of USD 700B by 2035, with their main impact being on financial services, cybersecurity and life sciences. The report also reveals the qualitative expectation of the value QC will unlock for industries. According to the report, the highest values are for optimisation for transport and logistics and simulations for life sciences [30].

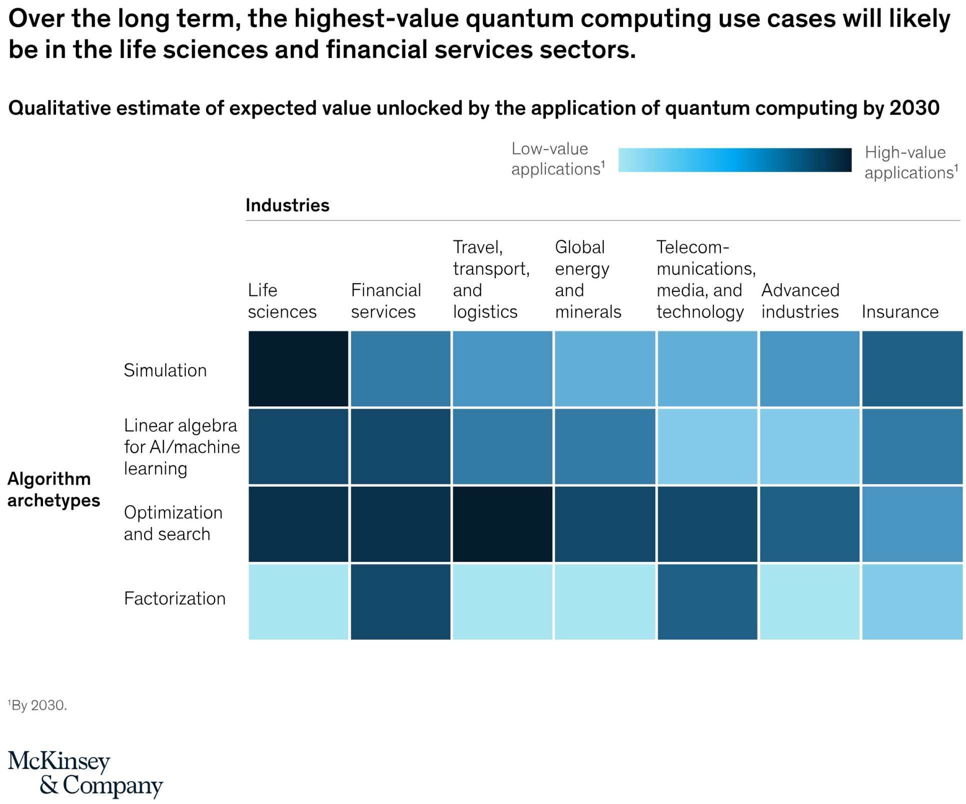

Figure 7: Most common use expectations for quantum technology.

Source: McKinsey [30]

In a recent trend, multiple governmental institutions in countries including China, the United States and Japan have been investing in the field. Aiming to reap the benefits of quantum computing, world governments planned a total of USD 31.3 Billion in public funding in 2021. The European Union and China planned to make the largest investments, with an estimate of USD 22.5 Billion.

Despite governmental funding, private capital investments in quantum technologies have shown no signs of slowing down. A great example of this is Honeywell, which invested USD 300 Million into a new quantum company called Quantinuum even after merging with Cambridge Quantum [30].

Every major industry needs to see investment in several areas before the entire sector shows signs of growth. Quantum computing is no different; the sector of quantum technologies still

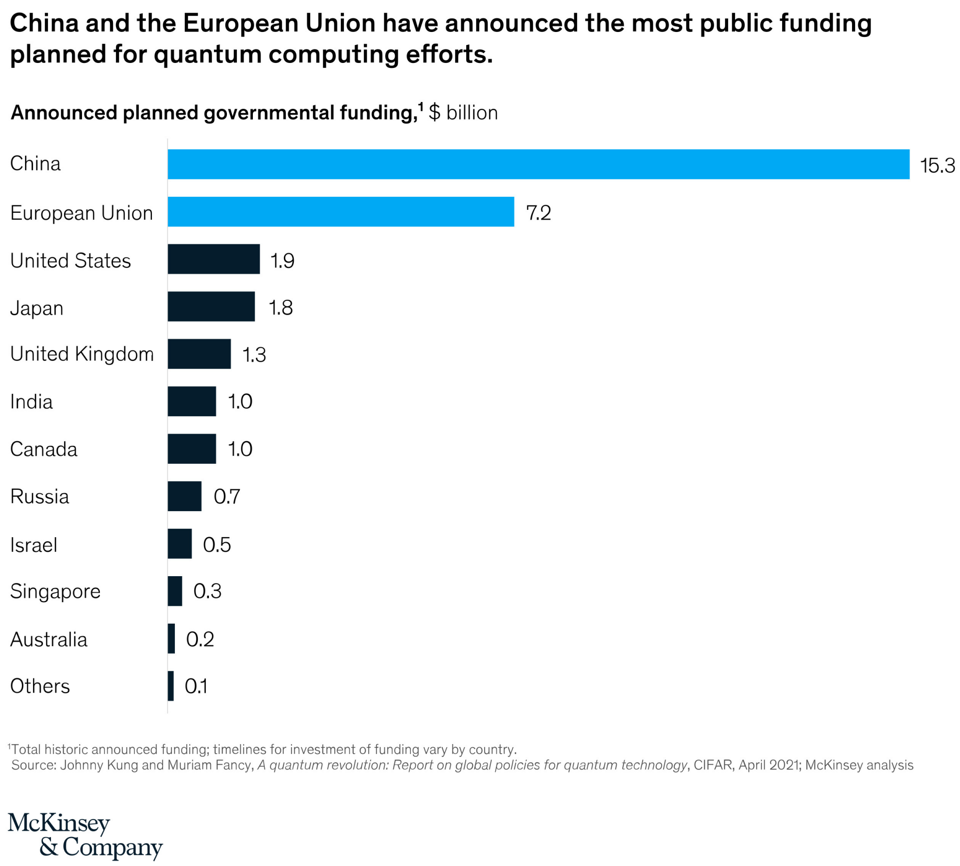

Figure 8: Planned Governmental Funding.

Source: McKinsey [30]

has multiple challenges to overcome before benefits are reaped by investors. The three main divisions of the sector are quantum computing, quantum communications and quantum sensing [30].

Quantum computing consists in the use of quantum hardware and software to create a computer with exponentially superior performance. Quantum communications, on the other hand, aims to deliver data securely across space. Lastly, quantum sensing is the development of the newest generation of sensors, with improved sensitivity over classical sensors [30].

Quantum computing is ahead of overall investments, with a focus on the hardware manufacturing segment [30]. However, quantum sensing and quantum communications had the highest increase in investors in the second half of 2021 [30].

In addition to Google, Microsoft and IBM, other leading technology companies are also striving to develop quantum technology and capitalise on investors' hype by improving their hardware and proving the return on investment quickly.

As previously mentioned, IBM unveiled they have been working with a 433-qubit machine and aims to build a 1121-qubit computer in 2023. This is just a step in their roadmap to build a 4128-qubit machine by the end of 2025 [20]. While IBM invests in Qiskit, cloud computing and pushing boundaries by increasing the number of qubits, Microsoft is focusing its research and resources on topological qubits. As Sandeep Pattathil points out in Venturebeat, topological qubits are less likely to lose quantum information [20]. By working on topological phases of matter, Microsoft's researchers aim to comprehend how smaller and faster qubits can help

Not exhaustive — Quantum computing; Quantum communication; (2021) Deal announced but not yet closed

Company	Country	Tech	Segment	Deal size, $ million	Deal year	Lead investor
1 PsiQuantum	United States	Quantum computing	Hardware manufacturing	450	2021	BlackRock
2 ArQit	United Kingdom	Quantum communication	Hardware manufacturing	400[2]	(2021)	Centricus Acquisition Corp
3 IonQ	United States	Quantum computing	Hardware manufacturing	350	2021	dMY Technology Group
4 CQC	United Kingdom	Quantum computing	Systems software	300[3]	(2021)	Honeywell
5 PsiQuantum	United States	Quantum computing	Hardware manufacturing	215	2020	Atomico
6 Xanadu	Canada	Quantum computing	Hardware manufacturing	100	2021	Bessemer Venture Partners
7 Rigetti	United States	Quantum computing	Hardware manufacturing	79	2020	Bessemer Venture Partners
8 CQC	United Kingdom	Quantum computing	Systems software	78	2015	Grupo Arcano; Stanhill Capital
9 Silicon Quantum Computing	Australia	Quantum computing	Hardware manufacturing	66	2017[4]	University of New South Wales
10 ID Quantique	Switzerland	Quantum communication	Hardware manufacturing	65	2018[4]	SK Telekom

Figure 9: Highest venture capital and private equity investments in quantum technology start-ups in 2021.

Source: McKinsey [30]

building a scalable quantum computer [20].

Quantum cloud computing has been a focus at Google since 2019, when their scientists claimed quantum supremacy by solving calculations which would allegedly take 10,000 for supercomputers to solve [35]. More recently, Google decided to work on noise mitigation and made consistent progress in developing new quantum chips [20].

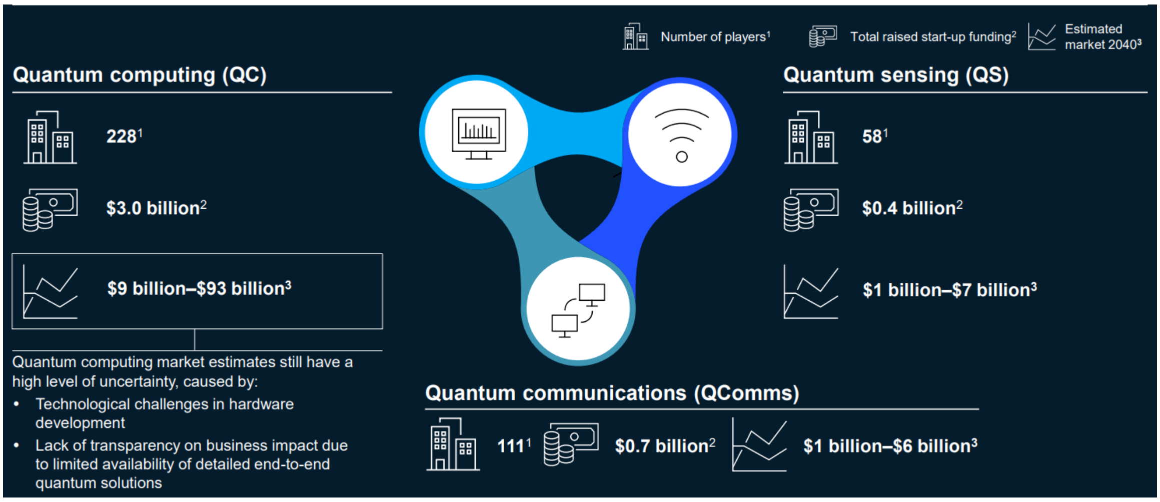

Figure 10: Market share for each sector in quantum technology.

Source: McKinsey [30]

4 Physical Implementations

There are multiple ways a quantum computer can be built. The first physical realisation of one was the work of K. Igeta and Y. Yamamoto in 1998, using a single atom and photon fields [36]. Since then, the massive influx of investments in the field of quantum computing has led to the discovery of multiple other methods of building quantum computers.

Three of the most common quantum computing realisations are photonic, superconducting and electron spin. The first mainly works with beam splitters, phase shifters and other kinds of linear-optical components [37]. The second harness the power of superconducting materials and especially the Josephson effect to create qubits [38]. The third, on the other hand, uses mainly quantum dot technology to trap electrons to be used as quantum bits. While all of these technologies are somewhat promising, there are many challenges ahead that researchers will need to face before reaching a universal quantum computer. In this section, we explain how these three approaches are used to build quantum systems, and how they fare when compared to one another.

a Photonic Quantum Computers

Despite the research into linear-optical quantum computers, they were not the initial choice for most companies that invested in the field. Zachary Vernon, head of hardware at Canadian photonics quantum computing start-up Xanadu claims it was too early in the timeline of photonic technology when Google and IBM started building their quantum computers [39]. According to Vernon, advancements made in the past few years in photonic chip fabrication, optical telecommunications technology and architecture scaling made photonics a solid choice in the search for a universal quantum computer [39]. The company, which has gathered over $235M in funding [40], seems to have made the right choice, as they recently claimed quantum advantage with their photonic processor [16]. Named Borealis, all its implemented gates can be programmed dynamically, allowing for the validation of photonic technologies in the search for a practical quantum computer [16]. In this section, we aim to introduce the research behind photonic quantum computers and detail their advantages as compared to other methods of quantum computation.

1 Boson Sampling

It seems to be indisputable in the literature that Aaronson et al. [37] led the research into linear-optical quantum computers by introducing boson sampling [41–43]. Even though their attempt was, according to themselves, rudimentary, it still proved that classical computers could not simulate quantum systems efficiently [37]. Quantum computers do not have to be universal in order to solve such a problem: as long as they can implement the necessary dynamics, they are sufficient [43]. In this specific scenario, the quantum system can be build using nought but linear-optic blocks, such as photon sources, beam splitters, phase shifters and photodetectors [37]. According to Brod et al., the idea of boson sampling "consists in sampling from the output distribution of n indistinguishable bosons that interfere during the evolution through a Haar-random-chosen linear network" [43]. The initial state is comprised of n photons and m modes, and each mode can have either one or zero photons [43]. The state then goes through an interferometer "described by an m-dimensional Haar-random unitary operator U." [43]. This randomness is what ensures U cannot be simulated by a classical computer, as it can't have a specific structure [43]. The following stage is comprised of photodetectors, which are placed at every output of the interferometer [43].

Hamilton et al. were the first to leverage the Gaussian nature of the states used for boson sampling, in a protocol they called Gaussian Boson Sampling (GBS) [41]. Although more complex, using the Gaussian states as inputs in the linear interferometer did prove to have some advantages [43]. The generation rate of photons could be increased as a result of the increase by a factor of $\binom{k}{n}, k >> n$ in the number of single photons per pulse [43]. The pump power could also be raised, with no detrimental effect on the system, as it uses Single Mode Squeezed States (SMSS) [41, 43]. The previously introduced Borealis processor focused on carrying out GBS "on 216 squeezed modes entangled with three-dimensional connectivity [...]" [16]. According to Madsen et al., only $36\mu s$ were necessary to produce a sample from the programmed

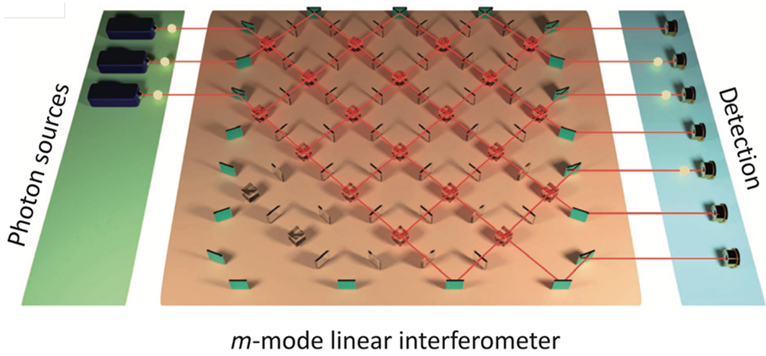

Figure 11: Photonic quantum device able to solve the aforementioned Boson Sampling problem.

Source: Brod et al. [43]

distribution, whereas over 9,000 years would be necessary for a classical supercomputer to perform the same calculation [16].

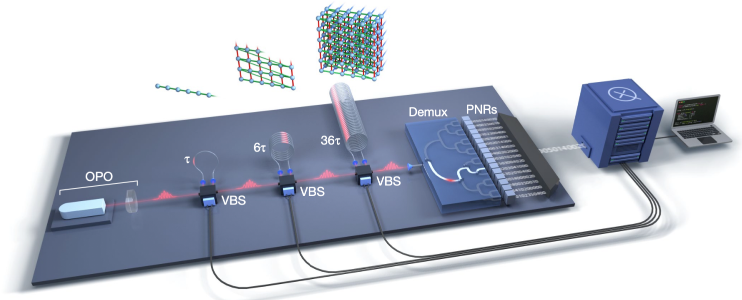

Figure 12: "High-dimensional GBS from a fully programmable photonic processor.".

Source: Madsen et al. [16]

2 Advantages of Photonic Quantum Computers

As compared to other forms of quantum computing, photonic quantum computation offers a number of advantages [16]. Due to the weak interactions between photons and the external environment, photonic quantum computers are not dependent on cryogenic temperatures or vacuum environments [13]. This enables quantum computers to operate at room temperature at a much lower initial cost [39]. Additionally, the large bandwidth of these systems allows for a high rate of data transmission, making the technology an attractive option for quantum communications [13]. This is especially true with the advancements made in optical fibre technology, which can be leveraged in photonic quantum computers by allowing them to coherently exchange information [39]. What allows for the component count and overall extent of a photonic quantum computer to be decoupled from its size is the possibility to multiplex optical channels allowing them to "process information on a large number of modes [...]" [16]. This in itself shows benefits of this technology when it comes to scalable quantum computers. We must however examine time-domain multiplexing under strenuous conditions to make sure there is a reasonable trade-off between hardware efficiency and computational errors [16].

Another benefit offered by the photonic approach is that it uses continuous degrees of freedom, making it the closest to many physical systems which are inherently continuous [44].

b Superconducting Quantum Computers

Superconducting qubits were used in some of the earlier physical implementations of quantum computing, as early as 1999 [45]. Although Nakamura et al. only developed one single qubit [46], research in superconducting quantum computers has rapidly developed. In 2019, quantum supremacy was demonstrated by Arute et al., when superconducting qubits were used to create quantum states on 53 qubits [15]. Consequently, this technology is a promising one when it comes to building scalable quantum computers. In the literature, superconducting qubits are sometimes referred to as Josephson junction qubits, as the Josephson junction is the main physical component that allows these man-made systems to exhibit quantum properties [47].

Superconducting qubits are usually classified as charge, flux or phase qubits, depending on their degree of freedom [47]. Nonetheless, our focus lies in one charge qubit-derived implementation: the Transmon qubit. The Transmon is currently the most popular superconducting qubit architecture [48], and it is also the one utilised by IBM in their quantum computers [49]. It distinguishes itself from the charge qubit by offering much lower charge noise sensitivity, as well as increased qubit-photon coupling [47].

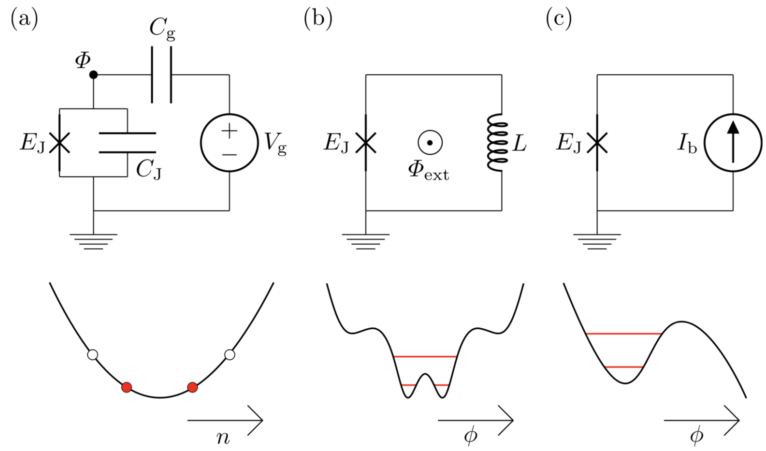

Figure 13: Josephson junction qubits. Each letter represents a different type in which names are derived from their respective degree of freedom. (a) Charge qubit. (b) Flux qubit. (c) Phase qubit.

Source: Kockum et al. [50]

1 Superconductivity

In order to comprehend how superconducting quantum computers work, one must have a basic understanding of the phenomenon of superconductivity. Since this dissertation aims to examine quantum computing, we are interested in studying how superconductivity applies to quantum effects. Transmon qubits are also known as circuit quantum electrodynamics (QED) devices, and they come in the same dimensions as regular circuits, meaning they are macroscopic in size. To observe quantum effects at a macroscopic level, one parameter must be altered: the superconductors must be cooled to exceptionally low temperatures [38]. For a system to have a low level of thermal noise, those temperatures should be in the range of 10mK [38].

As a result of such temperatures, Cooper pairs are formed in the system. These quasiparticles are not subject to the Pauli exclusion principle, and therefore can condense, i.e., exist in a single quantum ground state [38]. Moreover, these states tend to be stable since scattering events are not able to easily alter them [38]. By virtue of this, a small number of macroscopic quantum states can be defined for the system, which can be represented as a collective wave function [38].

A second relevant concept is the Josephson Effect. It happens when current tunnels through an insulating barrier between two superconducting metals [51]. Known as a supercurrent, it can flow continuously in spite of the barrier, even when there is zero or near zero voltage applied [51,52]. This occurs because the wave functions of the Cooper pairs can overlap, given that the insulating material is thin enough [38]. Consequently, the pairs tunnel coherently between the two superconducting materials [38]. This effect is what allows quantum behaviours to be observed macroscopically, as Josephson junctions can mimic the behaviour of a potential barrier [38].

An explanation for the operation of a Josephson junction qubit can usually be found by examining the relationship between two parameters: the Josephson energy E_J and the single electron charging energy, E_C. Sometimes referred to as simply charging energy, E_C relates to some capacitance in the circuit and is described by

$$E_C = \frac{e^2}{2C}. \tag{12}$$

The ratio E_J/E_C can show whether a qubit has a well defined charge number n and large or short quantum fluctuations. The charge qubit, for instance, has $E_J/E_C << 1$, which means that n is well defined and the quantum fluctuations, large. However, the opposite is true for both flux and phase qubits, as $E_J/E_C >> 1$.

2 Harmonicity and Anharmonicity

One of the most important quantum mechanical systems is the quantum harmonic oscillator. It is known for having equally spaced energy levels, i.e., the necessary energy to excite state $|0\rangle$ to state $|1\rangle$ is the same as the necessary energy to excite any state $|n\rangle$ to state $|n+1\rangle$ [50]. Consequently, when a system has energy distributions similar to that of the Harmonic Oscillator, we refer to that system as being harmonic. The closer the energy levels are to being the same, the greater the measure of harmonicity in the system [50].

When the opposite is true and each energy level is different from the next, the system is considered anharmonic [50]. Therefore, if the system is given enough energy to excite state $|0\rangle$ to state $|1\rangle$, that does not mean that state $|n\rangle$ could also be excited to state $|n+1\rangle$.

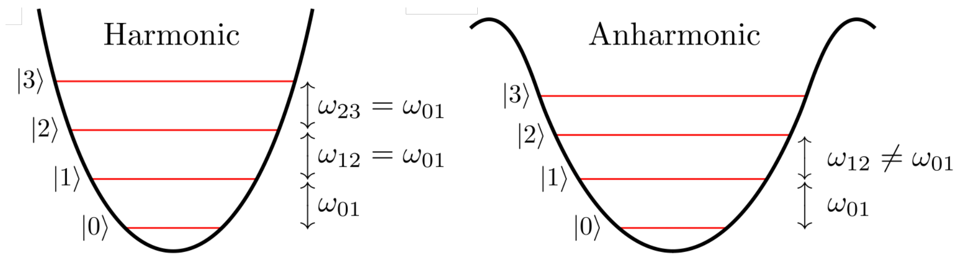

Figure 14: Energy distributions of harmonic and anharmonic quantum systems.

Source: Kockum et al. [50]

This relates to quantum computing because anharmonicity is necessary for the correct functioning of qubits [50]. If the energy levels are the same, an excitation in state $|\Phi\rangle$ could also cause an excitation in higher states outside the computational subspace of $|0\rangle$ and $|1\rangle$ [50]. Therefore, the energy distribution of qubits needs to be non-linear to allow for computational operations [50].

3 Transmon qubits

The Transmon qubit is a derivation of the charge qubit, also known as Cooper Pair Box (CPB) [38]. The CPB, which is represented by schematic (a) in Figure 13, is essentially a "small superconducting island [...], which is connected to a superconducting reservoir through a Josephson junction" [50]. This allows Cooper pairs to tunnel on and off the island due to the Josephson effect studied previously [50]. A voltage source V_g is also connected to the island through a gate capacitance C_g, which determines the background charge induced on the island [50].

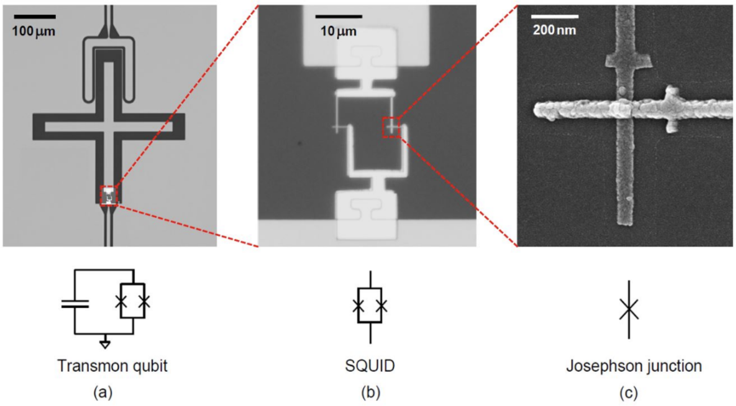

Figure 15: A frequency tunable transmon qubit can be seen in (a). (b) shows a zoomed-in superconducting quantum interference device (SQUID) and (c) a zoomed-in Josephson junction.

Source: Roth et al. [38]

What differs the CPB qubit from the Transmon is that the latter is "shunted by a capacitor that is large relative to the stray capacitance of the Josephson junction." [38]. Such capacitance, as expressed by Equation 12, changes the ratio E_J/E_C from being smaller than one in a CPB qubit to being greater than one in the Transmon. The increase in this ratio causes a reduction in sensitivity to charge noise, which consequently increases the coherence time of the Transmon an orders of magnitude over the Cooper Box model [38]. A known downside of this alteration is that it also lowers the anharmonicity of the system [50]. Thankfully, according to Kockum et al., that is a favourable trade-off [50]. That is because while the charge noise is reduced exponentially, the reduction in anharmonicity follows a weak power law [38].

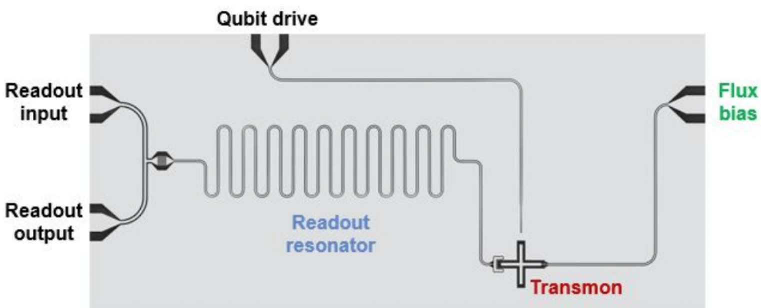

Figure 16: Single transmon device.

Source: Roth et al. [38]

A superconducting quantum computer, however, is not only made of Transmon qubits. Some more circuitry is necessary to build the system, and in the case of superconducting qubits, they tend to be coupled to transmission line structures [38]. Figure 16 shows a representation of a typical single transmon device. The main features shown in the image operate as follows:

> The flux bias line is used to apply a magnetic flux to the SQUID loop of the transmon to change its operating frequency. This can help correct for manufacturing variability and can also be a key capability in executing certain quantum information processing algorithms. The qubit drive line is used to modify the state of the qubit by applying a microwave drive pulse with a center frequency that matches the transmon's operating frequency. If the transmon starts in its ground state, an appropriately designed microwave drive pulse can be used to raise the transmon to its excited state or to place it in some superposition of the transmon's ground and excited states. Further microwave drive pulses can be applied along the qubit drive line to modify the transmon's state as needed throughout the course of executing a quantum algorithm. [38].

4 Advantages of Superconducting Qubits

Similarly to regular circuits, QED devices are highly customisable. The capacitance, inductance and energy of the Josephson Junction can be used to configure parameters such as the energy level of the qubit and its coupling strength [45]. That means that the Hamiltonian of the system can be adjusted by altering the circuitry [45]. Additionally, microwave control and operability can also be applied to these devices, since they are essentially the union of a Josephson junction and a system of planar microwave circuitry. This comes as an advantage as commercial microwave devices and equipment can then be repurposed.

These systems are also advantageous compared to other quantum computing techniques as superconducting qubits are built using the existing semiconductor micro-fabrication process [50]. The constant evolution of semiconductor fabrication is independent of quantum computing research, which can be exploited. Consequently, the manufacturing of transmon qubits is extremely scalable when compared to other devices, such as photonic quantum computers. They are therefore an obvious choice for large-scale quantum computing; however, they also present a number of challenges.

Since temperatures need to be so low for quantum properties to be observed in macroscopic systems, it is extremely expensive to cool down the environment in which the superconducting quantum computers must be kept [38]. That makes the initial cost high despite the ease of manufacture mentioned previously. Short decoherence times are also a challenge since superconducting qubits are not true two-level systems.

c Quantum Dot Quantum Computers

The idea of using electron spin and quantum dots for quantum computing was initially introduced by Loss, DiVincenzo [1] and Kane [2]. The fact that it happened over 20 years ago is not surprising: a single atom is an ideal candidate for the construction of quantum bits since it is a prototypical quantum system [17]. According to Trauzettel et al. [53], there have been major experimental breakthroughs using electron spins in quantum dots formed in semiconductors. Due to them being solid-state electrical devices [17], they are one of the few qubit realisations that are viable candidates for large-scale implementation [54].

1 Quantum Dots

Quantum dots (QD) are derived from the need for a platform in a laboratory that enables the isolation of a single charge or a few charges [55]. They are "semiconductor nanocrystals, roughly spherical and typically have unique optical, electronic and photophysical properties [...]" [56]. They behave similarly to transistors, except the gate above the channel is able to control the flow of one electron at a time [57]. Creating them requires that they be in a controlled environment, to control the charge's wave function and spin degree of freedom [55].

QD systems can usually be controlled by magnetic and electric fields, depending on whether they contain a single or multiple quantum dots [58].

According to Zwerver et al., fabrication of quantum dots has relied mostly on electron-beam lithography [57]. The low yeald and poor uniformity of the technique hinder the possibility of creating large-scale quantum computers with QDs [57]. Recent research has aimed to verify if the design rules of industrial patterning can be applied to qubit device layous [57]. If so, quantum dots could benefit from well-known semiconductor device fabrication methods [57].

The desire for controllability made semiconductors an appealing material for QD research, as they allow for the tuning of the electron density in a material system through the use of gates [55]. This happens due to the phenomenon of the Coulomb blockade [58]. Firstly, we connect one source electrode to one of the quantum dot's tunnel barriers, and one drain electrode to the other one [55]. Since the environment is cooled to around $8mV$, the temperature is much lower than the charging energy of the system [55]. However, when voltage is applied to the electrodes, electrons can be added or removed from the quantum dot. There is a specific gate voltage at which the electron charge state is degenerate, and oscillates as a function of time as we add and remove electrons from the dot [55, 58].

2 Multiple quantum dots

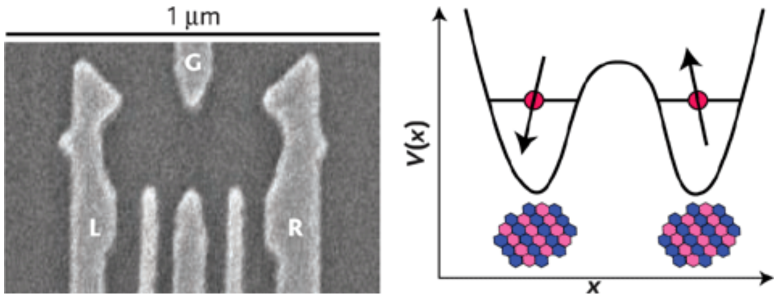

Figure 17: Realisation of a double quantum dot (DQD).

Source: DiVincenzo [59]

Multiple spin qubits are usually made utilising multiple quantum dots. The most common in this category are Double Quantum Dots (DQD), which are tunnel-coupled [58]. This makes the system much more complicated, as the quantum dots have cross capacitance [58]. The cross capacitance fortunately tends to have most of its consequences in the complexity of the equations that describe the system, rather than in its physics as a whole [58]. Chemical potentials in these systems are controlled by the gate electrodes [58]. The voltage difference between such gates is what controls the linear function that dictates the energy difference between the two states [58].

3 Single spin qubits

A single spin qubit is formed by the confinement of a single electron within a quantum dot. It's the simplest type of spin qubit, and its subspace is $|\uparrow\rangle$ and $|\downarrow\rangle$ [58]. There are usually magnetic fields applied to them to control their operation [58]. If a static magnetic field is being used, the energies of the spin states are split [58]. However, "an oscillating magnetic field perpendicular to the static one at a frequency close to qubit splitting, drives oscillations around the Bloch sphere." [58].

A recent partnership between Intel and QuTech has proved it possible to obtain over 10,000 arrays of QD single spin qubits using an optical lithography technique [60]. According to Koiller et al., this indicated the potential of spin-based qubits in silicon for large-scale quantum computing [60]. This is because we now know that industrial processing conditions not only allow for

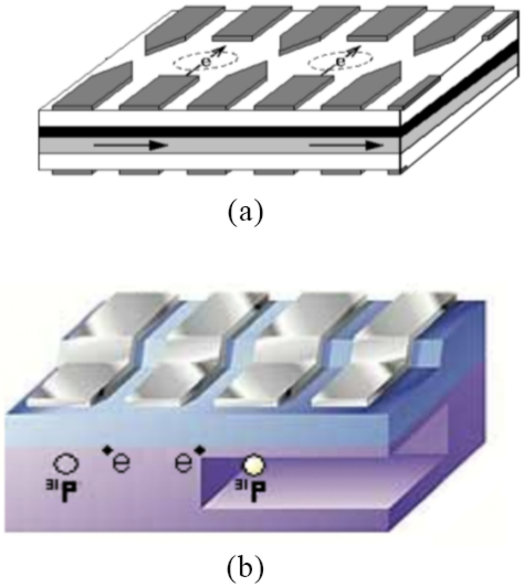

Figure 18: Representation of the earliest spin qubit quantum computer architectures. (a) Loss-DiVincenzo's qubits bound to quantum wells [1]. (b) Kane's P substitutional donors in Si [2].

Source: Koiller et al. [60]

good yield, but doesn't particularly affect key performance indicators [57]. However, challenges still remain, as this process has not yet been implemented in the production of two-dimensional spin arrays [60].

4 Charge qubits

The basis for the quantum state of charge qubits is the electron in a double quantum dot [58]. The electron can either be in the left or right QD, and the voltage detuning between the QDs is what controls the charge state [58]. This is a notable difference between charge qubits and single-spin qubits, since single-spin qubits also require the application of resonant pulses to be controlled [58]. They are also known for their short coherence times, which come as a consequence of their coupling to charge noise [58]. Consequently, they are not seen as suitable qubits for quantum computing applications [58].

5 Singlet-triplet qubits

The singlet-triplet qubit uses the charge qubit as its prototype, altering it to benefit from longer coherence times [58]. Another difference is that the splitting, which is the energy difference between the two possible quantum states of the qubit, is reduced to a near zero value [58]. This allows for the charge noise to have a lesser effect on the system, making high fidelity gates achievable [58].

6 Advantages of quantum dot quantum computers

There are a few advantages quantum dot quantum computers have over the previously introduced methods of QC realisation [61]. The first one, as demonstrated by Intel-QuTech [60], is scalability. Similarly to the superconducting qubits, spin qubits can be manufactured by the same methods as traditional transistor-based electronics [60]. Additionally, the use of silicon for the quantum dots also significantly improved their coherence time; according to Veldhorst et al., “over 100 two-qubit gates can be performed within a two-qubit coherence time of $8\mu s$”, which satisfies the condition for scalability [62].

Another benefit is the gate fidelity of single qubits, which is above 99% even when cross-talk and neighbouring qubit errors are taken into account [63]. This is important because the higher the fidelity of the system, the higher the feasibility of near-term quantum computing applications [63].

Although quantum dots are currently operated in a temperature range of milli-Kelvin, previous studies have also shown that electron spin qubit operation can occur in higher temperatures between 5-10 K [64]. Ono et al. claim this is made possible by the “spin-blockade effect based on the tunnelling transport via two impurities.” [64]. If spin qubits can operate at a higher temperature, small refrigerators will be enough to cool the system down [64]; furthermore, energy costs for the operation and maintenance of the system will be lower.

5 Conclusions & future work

Quantum computing has been proven to be a promising field in both academia and private capital markets. This is mainly due to the millions of dollars invested in the industry, especially throughout the past decade. The interest comes from the possibility to use quantum computers to explore problems that cannot be solved by current-day classical computing.

While multiple sectors of the economy would benefit from quantum technologies, emphasis must be given to pharmaceuticals and finance. For pharmaceuticals, large compounds that cannot be well described with approximations could potentially be simulated in a quantum computer. This would cut down the time needed to develop life-saving medications. In finance, stochastic process calculations and optimisations could be solved faster with the help of quantum algorithms.

As expressed throughout this work, quantum computers are complex systems to be designed, built and controlled. While there are multiple technologies being researched in the hopes of building a fault-tolerant quantum computer, there are specific benefits and disadvantages unique to each one.

Photonic quantum computers have been on the rise lately due to advancements in photonic technology. Weak interaction between photons and the external environment secures extremely long coherence times. This poses photonics as a candidate for fault-tolerant quantum computing. Another advantage it has over most other realisations is that cryogenic temperatures are not required for their operation. On the other hand, multi-qubit operations are hard to implement, and measurement destroys photonic qubits.

Google and IBM made their bet on superconducting computers, specifically with Transmon qubits. Josephson junctions can be utilised alongside planar microwave circuitry to build qubits that can be controlled using microwave drive pulses. The massive investments in superconducting quantum computing have made the technology the most advanced to date. Faster gate speeds and easy-to-reproduce qubits are two of their advantages; however, they have the shortest coherence times and qubit connectivity is poor.

Similarly to Transmon qubits, quantum dot electron spin qubits have also shown the potential to create scalable computers, as they can be manufactured using industrial semiconductor facilities. Although their coherence times are not as long as that of photonic qubits, they are still longer than the coherence times observed in transmon qubits. Since they are small in size, their industrial manufacture with good yield could lead to multiple advancements in large-scale computing.

As a whole, each technology presented here still requires a considerable amount of development. The advancements in quantum error correction are still to match those in quantum hardware, which currently focuses on quantum error mitigation. Despite some fault tolerance being acquired with the use of error mitigation, that won't be enough for universal quantum computers.

Overall, the field of quantum technologies has a lot of promise, but a quantum computer with real-life applications is still out of reach. Quantum decoherence, error correction, scalability and fidelity are only a few of the issues that require further research to allow for the realisation of the next generation of quantum computers.

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