Pontifícia Universidade Católica
do Rio de Janeiro

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Fiscal Risk in an Emerging Open Economy: the Brazilian case

Dissertação de Mestrado

Thesis presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC–Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Carlos Viana de Carvalho

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To my family, specially grandmother Sandra,
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Abstract

Perrupato Mendonça, Marina; Viana de Carvalho, Carlos (Advisor). **Fiscal Risk in an Emerging Open Economy: the Brazilian case**. Rio de Janeiro, 2022. 55p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

What if the fiscal risk is not negligible? Could the Central Bank continue effectively bringing inflation to the target when it ignores the default risk? To address those questions, we propose a small open economy DSGE model with an endogenous fiscal limit, where the government can default on its domestic bonds, and monetary authority may account for that. We evaluate dynamics under two different Central Bank decision rules: when (i) it wrongly tracks that risk, and (ii) it perfectly tracks default risk. The model is calibrated based on Brazilian data, as its recent budgetary deterioration makes the country an ideal case to be studied. We find that high inflation and depreciated currency coexist with a high interest rate when the monetary authority does not fully account for the default risk. The higher the default probability, the greater the differences across the effects of the two types of policy rules that we analyzed. For a central banker to restore the inflation target, she must fully track default risk in its decision rule. In addition, our model generates an endogenous premium across countries' interest rates due to differences in sovereign default risk.

Keywords

Monetary policy; Fiscal Limit; DSGE; Small Open Economy; Endogenous Regime Switching.

Resumo

Perrupato Mendonça, Marina; Viana de Carvalho, Carlos. **Risco Fiscal em uma Economia Emergente: o caso do Brasil**. Rio de Janeiro, 2022. 55p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

E se o risco fiscal não for desprezível? O Banco Central poderia continuar efetivamente trazendo a inflação para a meta ao ignorar o risco de default? Para responder a essas questões, propomos um modelo DSGE de pequena economia aberta com limite fiscal endógeno, onde o governo pode dar calote em seus títulos domésticos, e a autoridade monetária pode responder por isso. Avaliamos a dinâmica sob duas regras de decisão do Banco Central: quando (i) rastreia erroneamente esse risco e (ii) rastreia perfeitamente o risco de inadimplência. O modelo é calibrado com base em dados brasileiros, dado que a recente deterioração orçamentária do país faz dele um caso ideal a ser estudado. Constatamos que inflação alta e moeda desvalorizada coexistem com uma alta taxa de juros quando a autoridade monetária não leva em conta integralmente o risco de inadimplência. Quanto maior a probabilidade de default, maiores as diferenças entre os efeitos dos dois tipos de regras de política que analisamos. Para uma banqueira central restaurar a meta de inflação, ela deve incorporar perfeitamente toda a dinâmica do risco de inadimplência em sua regra de decisão. Além disso, nosso modelo gera um prêmio endógeno entre as taxas de juros dos países devido às diferenças no risco de default soberano.

Palavras-chave

Política monetária; Limite Fiscal; DSGE; Pequena Economia Aberta; Mudança de Regime Endógena.

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List of Abreviations

CB - Central Bank

CDS - Credit Default Swap

COPOM - Monetary Policy Committee

DSGE - Dynamic Stochastic General Equilibrium

GDP - Gross Domestic Product

NFA - Net Foreign Asset Position

O correr da vida embrulha tudo, a vida é assim: esquenta e esfria, aperta e daí afrouxa, sossega e depois desinquieta. O que ela quer da gente é coragem. O que Deus quer é ver a gente aprendendo a ser capaz de ficar alegre a mais, no meio da alegria, e inda mais alegre ainda no meio da tristeza! Só assim de repente, na horinha em que se quer, de propósito — por coragem.

João Guimarães Rosa, *Grande Sertão: Veredas.*

1
Introduction

Just over 20 years ago, the inflation targeting regime was adopted in Brazil when the fiscal risk was high, and the feasibility of continuing with the peg currency regime was questioned. From that after, the Brazilian economy experienced a period with considerable growth levels but ended up in a huge recession with a combination of inflation and high interest rates.

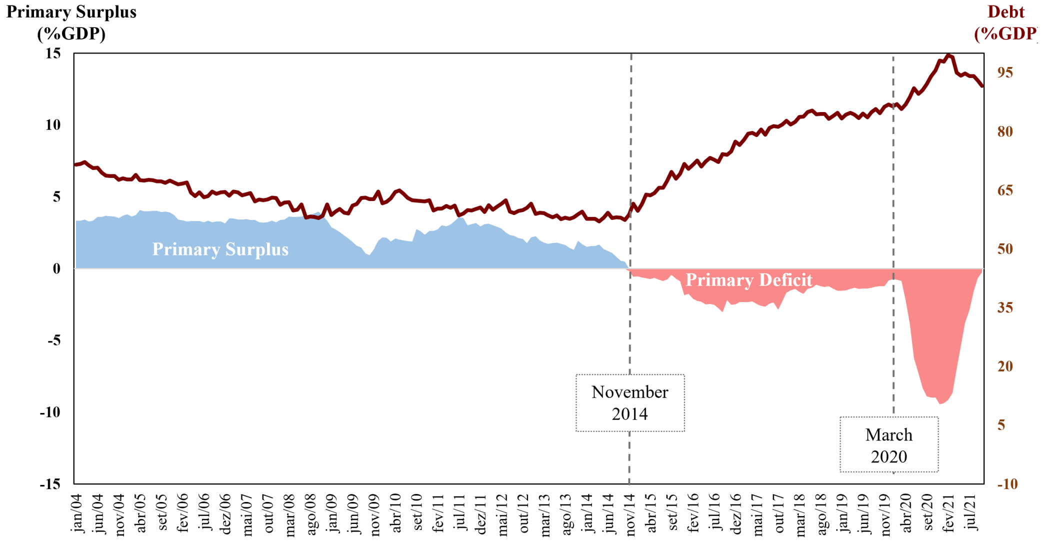
Although the monetary policy conducted through the inflation target regime was well established during this period, fiscal policy issues were not totally solved and played out generating inflationary pressures. The Law of Fiscal Responsibility (LRF), published in 2000 in order to establish norms of good fiscal management practices - as targets between revenues and expenses, transparency with public accounts, budget plans, and indebtedness limits - represented signaling toward a real concern with fiscal policy conducting. During Fernando Henrique Cardoso's (FHC) second presidential term and Lula's first, the Brazilian monetary policy was mainly anchored in the inflation targeting regime, the fulfillment of primary surpluses, and the floating exchange rate. Thus, as Werneck (2014b) points out, the first government of the Workers' Party represented a continuation of that of FHC and a consolidation of economic stability.

Nonetheless, after a period of huge growth and favorable external economic conditions, the Brazilian economy suffered the effects of the global financial crisis of 2008, and the government pushed some expansionary fiscal policies even further. If, on the one hand, these measures helped to contain the effects of the international crisis, on the other, they also served as a pretext[1] for the government to continue its expansionist fiscal policy, which involved everything from tax waivers to large volumes of credit granted through public banks. Furthermore, according to Werneck (2014a), one significant aspect of the regression of fiscal regime regards institutional descomissing of the separation of financial and non-financial public sector presented in LRF. In addition, the parallel budget between the Brazilian Treasury and BNDES was used as a mechanism to fake primary surplus as a fiscal measure. That practice turned out

[1]As argued in Werneck (2014a).

to entail the impeachment process of then president Dilma Rousseff(Werneck (2014a) and Ayres et al. (2019)).

Figure 1.1: Primary Surplus (%GDP) 12-Months accumulated and Debt/GDP (%)

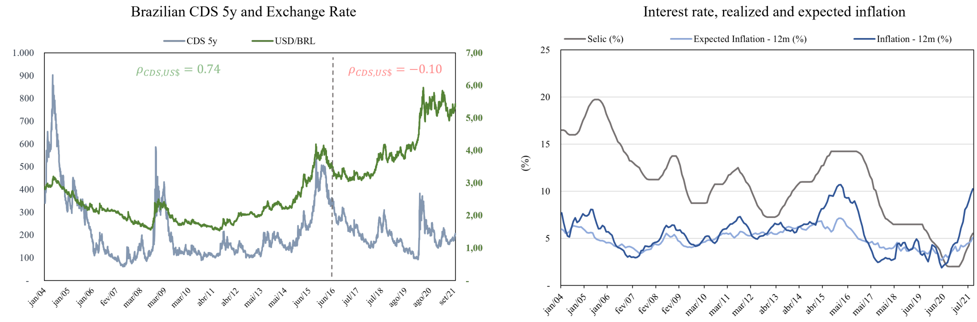


The abandonment of a responsible fiscal policy resulted in credibility crises and record debt levels (Figure 1.1). Some political reforms in fiscal field were done in order to contain the advance of fiscal deterioration, like the adoption of a limit of public expenses growth and the reform in pension system. While the former has some effects in the present, the latter will only impact the evolution of debt in the long term. Nevertheless, it is indeed a meaningful reform, as highlighted in Leeper (2013) countries with pay-as-you-go pension system can achieve an unstable path of government transfers due to the demographic consequences of the growth of the aging population besides stability in the number of active workers. A low number of pension system contributors relative to a vast number of beneficiaries can translate into a high level of unfunded liabilities, thus in more debt.

Still, the COVID-19 pandemic hit the world when the primary deficit was improving. This current crisis claimed expansionary fiscal policies to contain the economic effects of social distancing policies and huge public health spending. As a result, Brazilian indebtedness has achieved its highest level in more than 15 years.

Indeed, the deterioration of public accounts has already seemed to affect other economic variables since 2015. By then, inflation and its expectations were above the monetary policy target, and interest rates were elevated to tackle that. We can also observe a comovement of exchange rate and risk premium (measured by five-year Credit Default Swap (CDS)) - with a spike when Brazil was downgraded. However, from the end of 2016 until now, the risk premium is

Figure 1.2: Deterioration of expectations



Source: BCB and Ipeadata

presenting a downslope tendency, while the dollar is increasing its value against the real even with the high interest rates. During part of that time, inflation was also high, and agents' expectations were unanchored.

We can conjecture that, since Brazilian debt is almost only denominated in domestic currency, a fiscal risk has been mainly captured by exchange rate instead of CDS. And also that the persistence of inflation at high levels, even with huge contractionary monetary policy, can rely on fiscal imbalances' effects on expectations. Some monetary policy statements can give us more evidence on that latter. Since 2015, 95%[2] of Copom Meetings Minutes enumerated fiscal policy conduction as one of the sources of adverse risk. In the middle of 2015, they highlighted that new trajectories of fiscal variables affected expectations and asset prices.[3] While from 2016 to ownwards was common to emphasize that fiscal recovery and reforms of that nature were important to bring inflation to target.[4] After the social security and pension reform approval, that type of citation ceased. Although, as fiscal imbalances worsened after the pandemic, the Central Bank (CB) has returned to place the fiscal policy in its risk balance.[5]

For that matter, it is crucial to understand the effects of fiscal and monetary policy in an open economy if fiscal risks are not negligible. If the Central Bank accounts for those risks in its policy rule, the effects on exchange rates, inflation, and output would be different? For this purpose, we will extend the small-open economy DSGE model of Carvalho and Vilela (2015) by including a fiscal authority that will partially default on its bonds when

[2]Calculated based on the number of Meeting's Minutes that cite fiscal policy conduction as a source of risk for monetary goals.

[3] *"However, the new trajectories regarding fiscal variables affected the expectations and asset prices".* - 192nd Meeting (jul/2015)

[4] *"The process of approval and implementation of the necessary adjustments in the economy, including those of fiscal nature, presents itself as both a risk and an opportunity [...]"* - 202nd Meeting (out/2016)

[5] *"[...] recent questioning regarding the fiscal framework increased the risk of deanchoring inflation expectations, raising the upward asymmetry in the balance of risks"* - 242nd Meeting (out/2021)

its debt level surpasses the fiscal limit and modeling a monetary policy that is conducted with risky assets. This model was chosen because it features important characteristics of the Brazilian economy, such as high indexation, the presence of risk premium to foreign securities, and the imperfect passthrough from importing prices and nominal exchange rates to consumer prices. The fiscal limit approach of Bi (2012) turns out to be a consistent framework for analyzing the economic behavior of Brazil since our high tax burden can be seen as a constraint in the Treasury's capability to raise revenues (Ayres et al. (2019)). In terms of the literature, the proposed model can fill a gap, since it has not yet been developed a work based on Bi's theory of fiscal limits for an open economy where the monetary authority conduces open market operations with a risky asset and a default occurs in terms of domestic government debt.

Our main findings point out that when the monetary authority does not incorporate the dynamics of fiscal risk by adjusting its decision rule properly, the economy can present high inflation and depreciated exchange rates, even in the presence of high interest rates. In addition, these adverse results can be even more pronounced when the government is under budgetary instability. On top of that, the Central Bank can only bring inflation to target when it perfectly tracks default's dynamic. Our proposed model can also account for an endogenous risk premium over international interest rates. By observing that the Treasury can declare a haircut in its liabilities, investors require a premium to absorb domestic bonds. Even the most remote probability of default is able to generate a wedge over international rates that developed into depreciation if the CB does not match the requirement of the household. Besides, comparing our estimation for Brazil's fiscal limit, we conclude that it still has considerable fiscal space, since we find a default probability that is close to 0%.

We can say that our work builds on three complementary literature: small open economy New Keynesian models, sovereign default, and endogenous regime switching methods. The model of Enrique Alberola-Ila and Mirkov (2021) for Brazil is the small open economy model that better approximates to ours. They rely on the continuous time version of Galí and Monacelli (2005) and on fiscal limit approach of Uribe (2006) to determine the endogenous default probability. The main differences from our model are that the switching probabilities are exogenous in their work, the fiscal limit calculation is more ad hoc, monetary policy is based on a risk-free rate, and foreign bonds and international investors play an important role in default risk. Arellano et al. (2020) also constructed a quantitative sovereign default model for a small open economy that approximates ours in terms of the New Keynesian framework based on Galí and Monacelli (2005). Although they calibrated the model for

Brazil to analyze the recent crises that began in 2015, they supposed that government engages in international borrowing and modeled the default event in following Arellano (2008) - i.e., as a strategic choice of an optimizing government. Since Brazilian indebtedness is mainly denominated in local currency, that model does not seem ideal for understanding the recent comovements of high nominal interest rates, high inflation and activity deepening.

We based on Maih (2015) to solve our model, mainly because it allows for nonlinearities and switching parameters while solving the model with perturbation methods. This framework differentiates from the linearized MS-DSGE models[6] to the extent that it does not require the structural model to be linear and, in that way, allows steady state to be different across regimes. In addition, it does not incur in the curse of dimensionality problem of global methods[7] - an essential advantage since we are working with a model with many state variables.

Concerning the adoption of fiscal limit approach a la Bi (2012), the conduction of a monetary policy with risky assets, and the model's solution using the algorithm of Maih (2015), our work is very similar to Amaral (2021). He proposed a DSGE model for the Brazilian economy, with two types of agents where the government can default, and Central Bank should account for that risk. Therefore, the main difference is that we are proposing an open economy model with a representative agent.

This paper proceeds as follows. Section 2 presents the proposed small open economy DSGE model. The solution steps and the estimation of the fiscal limit are detailed in Section 3. Section 4 contains the information regarding our baseline calibration for a Brazilian economy. We present our results in Section 5, and finally concludes in the last section.

[6]Farmer et al. (2011)
[7]Davig et al. (2011),Bi and Traum (2014)

2 Model

We propose an extension of the small open-economy New Keynesian model of Carvalho and Vilela (2015), including a fiscal block and the presence of a fiscal limit in terms of Bi (2018). Following Amaral (2021), we allow the monetary authority to target a risky interest rate. We also adopt a different utility specification, since GHH[1] function enables a faster model's computation.

Consumers and the government have access to two different types of imperfect substitute goods: domestic and imported products. The domestic producer firms operate under monopolistic competition, employ labor, are subject to the same technology shock, and own an equal amount of capital. The retail importing firms also operate under imperfect competition and differentiate the goods bought abroad without cost. All firms readjust prices infrequently, and under other conditions, they follow an indexation rule.

The government finances itself by leaving distortionary income taxes or issuing debt, while spending its resources paying interests, consuming, and making fixed lump-sum transfers to the households. As in Bi (2018), the taxation evolves according to a rule that takes into account the level of indebtedness and output gap. The Treasury is not allowed to increase taxes indefinitely. It can do that until it achieves the peak of the Laffer curve. At the same time, its indebtedness is restricted by the fiscal limit, once it occurs a partial default is declared. The Central Bank follows an inflation target regime, and its nominal interest rate rule may account for the fiscal risk.

On the other side, domestic consumers are supposed to absorb all government bonds. The remaining probability of default leads households to charge a default premium over the domestic interest rate. At the same time, they can also smooth consumption through foreign bonds that yield an interest rate determined in the international market plus a risk premium that depends on Brazil's foreign asset position.

[1]Greenwood (1988)

2.1 Households

There is a continuum of households indexed by $j \in [0,1]$, who chooses how to compound their consumption basket with $i \in [0,1]$ substitute goods and services, that can be produced by domestic firms or be imported (Carvalho and Vilela (2015)). Thus, the consumption basket of the representative household can be defined as:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{D,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{1-\eta}} \tag{2-1}$$

where α corresponds to the share of imported goods in total consumption and η is the elasticity of substitution between domestic and imported goods. The composites of domestic and foreign goods - $C_{D,t}$ and $C_{F,t}$, respectively - are obtained using Dixit-Stiglitz aggregator:

$$C_{D,t} = \left[\int_0^1 C_{D,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the elasticity of substitution between varieties with the same origin. As goods and services are imperfect substitutes, the firms face a downward sloped demand curve for each variety:

$$C_{D,t}(i) = \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\varepsilon} C_{D,t} \quad \text{and} \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}$$

Hence, the demand for each origin type is obtained in analogous way:

$$C_{D,t} = (1-\alpha)\left(\frac{P_{D,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2-2}$$

The price indexes accordingly to goods and services origin are:

$$P_{D,t} = \left(\int_0^1 P_{D,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

implying in the following price index for domestic economy:

$$P_t = \left[(1-\alpha)P_{D,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2-3}$$

Besides that, the representative household chooses consumption and savings level intertemporally by maximizing her expected utility, subject to a nominal budget constraint, and avoiding Ponzi scheme.

$$max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(C_t - \varphi_N \frac{N_t^{1+\varphi}}{1+\varphi}\right)^{1-\sigma} \tag{2-4}$$

s.t:

$$P_t C_t + \frac{P_t B_t^g}{R_t} + S_t P_t^* B_t^f \leq (1-\delta_t) P_{t-1} B_{t-1}^g + S_t P_{t-1}^* B_{t-1}^f R_{t-1}^* \Phi_{t-1}(V_{t-1}) + (1-\tau_t)(W_t N_t + D_t) + P_t(Z_t + T_t^{LS})$$

$$\lim_{T\to\infty}\mathbb{E}_t\left[\beta^{T-t}\frac{\left(C_T-\varphi_N\frac{N_T^{1+\varphi}}{1+\varphi}\right)^{-\sigma}}{\left(C_t-\varphi_N\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{-\sigma}}\left(\frac{(1-\delta_T)\,B_T^g}{P_T}+\frac{B_T^f}{S_T P_T^*}\right)\right]\geq 0 \tag{2-5}$$

where: $\beta\in(0,1)$ is the subjective time discount factor; σ is the inverse elasticity os intertemporal substitution; φ is the inverse Frisch elasticity of labor supply; φ_N is a degree of labor disutility; S_t is the nominal exchange rate quoted in Brazilian Real per Dollar; R_t and R_t^* are domestic and foreign gross interest rates; δ_t is a time-varying function, to be defined, that indicates the default rate over domestic debt; $\tau_t\in[0,1]$ is the distorting income tax; W_t is the nominal wage; D_t is the dividends earned from domestic and importing firms; $(Z_t+T_t^{LS})$ are real lump-sum transfers from government and $\Phi_{t-1}(V_{t-1})$ is the risk premium.

Likewise Carvalho and Vilela (2015), the risk premium is defined as a function of Brazil's net foreign asset position and is supposed to be taken as given by the representative Brazilian:

$$\Phi_t(V_t)=e^{-\chi V_t+\phi_t}=e^{\left[-\chi\left(\frac{S_t B_t^f}{P_t Y}\right)+\phi_t\right]}$$

where ϕ_t is a risk-premium shock and Y is the steady-state level of output. Besides its interpretability as a deviation from uncovered interest parity condition, that hypothesis is important to guarantee the stationary of the open economy model.[2]

We can interpret the equation above in the following way: if the Brazilians are lenders ($V_t>0$, i.e. they buy foreign assets), they receive a lower remuneration than the market rate; on the contrary, when $V_t<0$ and households are borrowers they need to pay an endogenous premium over the foreign interest rate.

Therefore, the remaining first-order and the transversality condition for representative Brazilian's problem are:

$$\frac{W_t}{P_t}=\frac{\varphi_N N_t^{\varphi}}{(1-\tau_t)} \tag{2-6}$$

$$\left(C_t-\varphi_N\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{-\sigma}=\beta\mathbb{E}_t\left[(1-\delta_{t+1})\frac{R_t}{\Pi_{t+1}}\left(C_{t+1}-\frac{\varphi_N N_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\sigma}\right] \tag{2-7}$$

[2]For more information regarding stability of small-open economy models, see Schmitt-Grohe and Uribe (2003).

$$\left(C_t - \varphi_N \frac{N_t^{1+\varphi}}{1+\varphi}\right)^{-\sigma} = \beta \mathbb{E}_t \left[R_t^* \Phi_t \left(\frac{S_t B_t^f}{P_t Y}\right) \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \left(C_{t+1} - \frac{\varphi_N N_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\sigma} \right] \tag{2-8}$$

$$\lim_{T\to\infty} \mathbb{E}_t \left[\beta^{T-t} \frac{\left(C_T - \varphi_N \frac{N_T^{1+\varphi}}{1+\varphi}\right)^{-\sigma}}{\left(C_t - \varphi_N \frac{N_t^{1+\varphi}}{1+\varphi}\right)^{-\sigma}} \left(\frac{(1-\delta_T) B_T^g}{P_T} + \frac{B_T^f}{S_T P_T^*} \right) \right] = 0 \tag{2-9}$$

By the labor supply equation (2-6), it is possible to notice that the income tax distorts the total amount of optimal labor supply: higher taxation leads to lower net real return per hour worked, and so to fewer labor hours. As will be shown further, this effect generates the Laffer curve: a concave government revenue function, and it will be important to define the debt limit in the model. In addition, if we compare the domestic bonds Euler equation (2-7) with the one in Carvalho and Vilela (2015) it shows that the representative household will incorporate the fiscal risk in her portfolio decision, which is made by charging the Treasury a default premium.

If we combine the Euler equations (2-7 and 2-8), we end up with a risk-premium uncovered interest parity condition (UIP):

$$\mathbb{E}_t \left[\left(C_{t+1} - \frac{\varphi_N N_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\sigma} \frac{1}{\Pi_{t+1}} \left(R_t^* \Phi_t(V_t) \Delta S_{t+1} - (1-\delta_{t+1}) R_t\right) \right] = 0 \tag{2-10}$$

As we can see, the non-zero default probability induces an extra premium over the international market rate. Addressing the effects of that endogenous wedge over interest rates, all other things being equal, the greater the domestic indebtedness, the higher the probability of a debt haircut. Hence that model is able to capture the fact that, the sovereign's capability to honor its liabilities is a driven factor of the interest rate differential across countries. Further we are going to explore more how the presence of such risk can generate different results in terms of exchange rates and domestic nominal interest rate levels.

2.2 Firms

As in Carvalho and Vilela (2015), there are two types of firms: domestic producers and importing retailers, which operates under monopolist competition and set their prices as in Calvo (1983).

2.2.1 Domestic firms

There is a continuum of domestic producers - $i \in [0,1]$ - who employ the national labor force $N_t(i)$ in order to produce imperfect substitute goods and services that are purchased by domestic and foreign consumers. We define the production technology as a linear function of labor; suppose that all firms are subject to a common productivity shock A_t and own an equal amount of capital stock that is fixed at $\bar{K}$ in every period.

$$y_{D,t}(i) = A_t \bar{K} N_t(i) \tag{2-11}$$

Since we assumed constant returns to scale, the input level will be the same across firms, leading to an equal marginal cost for all domestic producers:

$$MC_{D,t} = \frac{W_t}{A_t P_{D,t}} \tag{2-12}$$

The firm i 's profits is, thus, given by:

$$\Pi_{D,t}(i) = y_{D,t}(i)\left(P_{D,t}(i) - P_{D,t} MC_{D,t}\right) \tag{2-13}$$

The price is adjusted infrequently, like in Calvo (1983), and in each period only a fraction $(1-\theta_D)$ of firms reoptimizes their prices, while the remaining follows an indexation rule:

$$P_{D,t}(i) = P_{D,t-1}(i)\left(\frac{P_{D,t-1}}{P_{D,t-2}}\right)^{\delta_D} \tag{2-14}$$

where δ_D is a parameter that determines the degree of indexation to past inflation.

The fraction of firms that revise their prices in t choose the same price $X_{D,t}(i) = X_{D,t}$, as they solve the same intertemporal problem: maximize the present discounted value of its expected profits subject to the sequence of demands.

$$E_t \sum_{\tau=0}^{\infty} \theta_D^{\tau} \Theta_{t,t+\tau} y_{D,t+\tau|t}\left[X_{D,t}\left(\frac{P_{D,t+\tau-1}}{P_{D,t-1}}\right)^{\delta_D} - P_{D,t+\tau} MC_{D,t+\tau}\right] \tag{2-15}$$

s.t:

$$y_{D,t+\tau|t} = \left(\frac{X_{D,t}}{P_{D,t+\tau}}\left(\frac{P_{D,t+\tau-1}}{P_{D,t-1}}\right)^{\delta_D}\right)^{-\varepsilon}\left(C_{D,t+\tau} + G_{D,t+\tau} + C^*_{D,t+\tau}\right) \tag{2-16}$$

where it was assumed that the foreign demand $(C^*_{D,t})$ has the same functional form as the domestic one (2-2), $(G_{D,t+\tau})$ is the government spending with domestic goods and $\Theta_{t,t+\tau} = \beta^{\tau}\frac{P_t}{P_{t+\tau}}\frac{U_{c,t+\tau}}{U_{c,t}}$ is the nominal stochastic discount factor of consumers.

It is possible to define the price index for domestic products as:

$$P_{D,t} = \left[(1-\theta_D)\, X_{D,t}^{1-\varepsilon} + \theta_D \left(P_{D,t-1} \left(\frac{P_{D,t-1}}{P_{D,t-2}} \right)^{\delta_D} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{2-17}$$

2.2.2 Importing firms

Importing firms buy goods and services from abroad and, at no cost, transform them into differentiated products sold in the domestic market. Considering that they are also under monopolist competition and periodic pricing adjustment, it is possible to capture an imperfect passthrough from international prices and nominal exchange rate variations to consumers' prices.

Their optimization problem is analogous to that of domestic producers, and all reoptimizing firms in t choose the same price $X_{F,t}(i) = X_{F,t}$:

$$E_t \sum_{\tau=0}^{\infty} \theta_F^{\tau} \Theta_{t,t+\tau} M_{t+\tau|t} \left[X_{F,t} \left(\frac{P_{F,t+\tau-1}}{P_{F,t-1}} \right)^{\delta_F} - P_{t+\tau}^{*} S_{t+\tau} \right] \tag{2-18}$$

s.t:

$$M_{t+\tau|t} = \left(\frac{X_{F,t}}{P_{F,t+\tau}} \left(\frac{P_{F,t+\tau-1}}{P_{F,t-1}} \right)^{\delta_F} \right)^{-\varepsilon} (C_{F,t+\tau} + G_{F,t+\tau}) \tag{2-19}$$

where P_t^{*} is the price of imported products in international market, θ_F is the price rigidity $G_{F,t+\tau})$ is the government spending with foreign goods and δ_F is the indexation parameters.

So, the aggregate price index for imported goods sold in domestic economy is:

$$P_{F,t} = \left[(1-\theta_F)\, X_{F,t}^{1-\varepsilon} + \theta_F \left(P_{F,t-1} \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\delta_F} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{2-20}$$

Moreover, it is important to define some variables of interest, that express the relation between the different prices of that economy:

- the real interest rate $\mathbf{Q_t}$ is given by the ratio between foreign and domestic prices expressed in the same unit: $Q_t = \frac{S_t P_t^{*}}{P_t}$;

- the terms of trade $\mathbf{ToT_t}$ are the relative price of Brazil's exports and imports: $ToT_t = \frac{P_{D,t}}{P_{F,t}}$;

- the deviations from Law of One Price for imported goods $\psi_{F,t}$ is the ratio between international prices in Reais and the price of imported goods in domestic market: $\psi_{F,t} = \frac{S_t P_t^{*}}{P_{F,t}}$.

2.3
Fiscal Policy

Adapting Bi et al.(2018) and Bi (2016), the government is represented by a budget constraint (2-21) and a fiscal policy rule (2-23):

$$\frac{P_t B_t^g}{(1+i_t)} + \tau_t(W_t N_t + D_t) \geq (1-\delta_t)\, P_{t-1} B_{t-1}^g + P_t G_t + P_t(Z_t + T_t^{LS}) \quad \text{(2-21)}$$

where δ_t represents the partial default on the interests that the Treasury should pay to all investors. As we can see it is the same for all Brazilians that hold the bonds. We assume that transfers[3] are fixed in its steady state level ($Z_t + T_t^{LS} = Z + T^{LS}$) and government expenses follows an exogenous AR(1) process that also depend on past output gap:

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \rho_Y \log\left(\frac{Y_{t-1}}{\bar{Y}}\right) + \sigma_G \epsilon_t^g \quad \text{(2-22)}$$

such that $|\rho_G| < 1; |\rho_Y| < 1; \sigma_G$ is the standard deviation of the shocks; and ϵ_t^g are i.i.d and follows a standard Normal distribution.

Also, the government follows a fiscal policy rule, where taxation responds to past deviations of tax rate and debt from its steady state level:

$$\log\left(\frac{\tau_t}{\bar{\tau}}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right) + \rho_B \log\left(\frac{B_{t-1}^g}{\bar{B}^g}\right) \quad \text{(2-23)}$$

The real debt law of movement is easily obtained from the budget constraint holding in equality:

$$B_t^g = (1+i_t)\left[(1-\delta_t)\,\frac{P_{t-1}}{P_t} B_{t-1}^g + G_t + Z_t + T_t^{LS} - T_t\right] \quad \text{(2-24)}$$

where: $T_t = \tau_t(\frac{W_t N_t}{P_t} + \frac{D_t}{P_t})$ is the real tax revenue.[4]

2.4
Monetary Policy

The Central Bank follows an interest rate rule in order to lead inflation to its target $\bar{\Pi}$, which is done by reacting not only to the deviations of inflation, but also responding to output gap and variations in nominal exchange rate.[5] As in Amaral (2021) the open market operations are made through risky Treasury

[3] T^{LS} account for the residual that appears when we calibrate the model to fit Brazilian data, and for the subsidy required to eliminate steady-state distortion induced by imperfect competition in domestic and imported markets as in Justiniano and Preston (2010). Since we attribute sample values to all fiscal variables steady state it is necessary to introduce a lump-sum transfer to guarantee that the equation will be satisfied in equality in the steady state.

[4] $T_t = \tau_t\left(C_t + G_t + \frac{P_{D,t}}{P_t} C_{D,t}^* - Q_t C_{F,t}\right)$

[5] Following Carvalho and Vilela (2015).

bonds due to the possibility of government default. The presence of a fiscal risk entails a new challenge for the monetary authority: it needs to neutralize the presence of a risk premium over the risk-free real interest rate so it can achieve its inflation goal. We evaluate the model under two different monetary policy intercepts $\bar{\iota}_t$ to investigate the results that Central Bank's negligence concerning that risk can spawn.

$$(1+i_t) = (1+\bar{\iota}_t)\left[\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\lambda_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\lambda_y}(\Delta S_t)^{\lambda_s}\, e^{\varepsilon_R}\right] \tag{2-25}$$

Before presenting the two cases, it is important to understand how the non-zero probability of a default event translates in a risk premium over the risk-free rate. Likewise Amaral (2021), we define the net risk-free nominal interest rate i_t^{RF} as:

$$\left(1+i_t^{RF}\right) = \Pr^Q\left(B_t \leq \mathcal{B}_{t+1}\right)(1+i_t) + \Pr^Q\left(B_t > \mathcal{B}_{t+1}\right)\left[\left(1-\delta_{t+1}\right)(1+i_t)\right]$$

where $\Pr^Q(E)$ is the risk-neutral probability that an event (E) happens and $\mathcal{B}_{t+1}$ is the fiscal limit - a main concept in the model that will be addressed further. As the events are complementary, we can rewrite the definition in terms of the indicator function of a default event $1_{(B_t>\mathcal{B}_{t+1})}$.

$$\mathbb{E}_t^Q(1+i_t) = \left(1-\mathbb{E}_t^Q 1_{(B_t>\mathcal{B}_{t+1})}\delta_{t+1}\right)(1+i_t)$$

$$\Rightarrow \quad \Omega_t \equiv \frac{1}{\left(1-\mathbb{E}_t^Q 1_{(B_t>\mathcal{B}_{t+1})}\delta_{t+1}\right)} = \frac{(1+i_t)}{\mathbb{E}_t^Q(1+i_t)}$$

Thus, Ω_t is the time-varying gross default premium that agents request to finance the government debt. Since even in the steady state the probability of a haircut is non-zero, that fiscal risk will lead to a premium over the real natural interest rate $\bar{r}^n$. It means that the intercept of the monetary policy rule will be greater than in the canonical case - when it always coincides with the risk-free rate.

To illustrate that fact, we suppose that we are in the steady state and evaluate the Central Bank's rule. As $\Pi_t = \bar{\Pi}, Y_t = \bar{Y}$ and $\Delta S_t = 1$:

$$(1+\bar{i}) = (1+\bar{r})\bar{\Pi} \xrightarrow[(1+\bar{r})=\frac{1}{\beta(1-\bar{\delta})}]{\text{Euler eq. in SS}} (1+\bar{i}) = (1+\frac{1}{\beta\left(1-\bar{\delta}\right)})\bar{\Pi} = \left(1+\bar{r}^n\frac{1}{1-\bar{\delta}}\right)\bar{\Pi}$$

and using the approximation for the default premium $\bar{\Omega} \approx -log(1-\bar{\delta})$:

$$(1+\bar{i}) = (1+\bar{r}^n+\bar{\Omega})\bar{\Pi}$$

It is possible to notice that the real interest rate in the steady state is now the natural real interest rate plus the default premium. Hence, for the monetary authority to bring inflation to its target, it is also necessary to neutralize the fiscal risk, which implies higher interest rate levels in the steady state than in the risk-free canonical case. To appraise the importance of a Taylor rule that accounts for that risk we simulate the model under two different intercepts.

Case 1: Central Bank wrongly tracks fiscal risk

Here, the CB tries to incorporate the fiscal risk in its monetary policy rule, but the default dynamic is ignored. Therefore, for all t only the mean level of the default premium is accounted. In spite of correctly setting the interest rate at the natural rate level in the steady state, in periods where the fiscal risk is outstanding, the correct adjustment of interest rates is underestimated.

$$\bar{\iota} = r_t^{RF} + \bar{\Omega} \tag{2-26}$$

Case 2: Central Bank perfectly tracks default risk

The last case is the ideal one. The monetary authority is able to fully track the fiscal risk dynamics and to adjust its rule to neutralize the default premium. Regardless of not being a trustworthy hypothesis when considering the CB's information set, it sheds light on how important it is to incorporate those risks in interest rate decisions.

$$\bar{\iota} = r_t^{RF} + \Omega_t \tag{2-27}$$

2.5 Foreign Economy

As in Carvalho and Vilela (2015), the world economy is assumed to behave according to a first-order vector autoregressive model, and as domestic economy is assumed to be small, it does not affect the international market dynamics. To estimate the model we used US data provided by FRED Saint Louis and Federal Reserve Bank of Atlanta. As in the period of analysis the main economies of the world reached the zero lower bound for the interest rate, we use as external interest rate the shadow rate for federal funds calculated following Wu and Xia (2016).[6]

It was imposed the usual Cholesky ordering - Y_t^* is the foreign output, π_t^* is external inflation, i_t^* the international nominal interest rate. The representation of VAR(1) is described as:

6

$$A_0 \begin{bmatrix} y_t^* \\ \pi_t^* \\ i_t^* \end{bmatrix} = A_1 \begin{bmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ i_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_y^* \\ \varepsilon_\pi^* \\ \varepsilon_i^* \end{bmatrix} \tag{2-28}$$

with coefficient matrices given by:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 \\ a_{0,\pi y} & 1 & 0 \\ a_{0,iy} & a_{0,i\pi} & 1 \end{bmatrix} A_1 = \begin{bmatrix} a_{1,yy} & a_{1,y\pi} & a_{1,yi} \\ a_{1,\pi y} & a_{1,\pi\pi} & a_{1,\pi i} \\ a_{1,iy} & a_{1,i\pi} & a_{1,ii} \end{bmatrix}$$

Table 2.1: Data Source used in VAR(1) Estimation

	Data	**Source**
y_t^*	EUA: Real GDP - Quarterly, Seasonally Adjusted	FRED St Louis
π_t^*	EUA CPI: Total All Items - Quarterly	FRED St Louis
i_t^*	Wu-Xia Shadow Federal Fund Rate	FRB (Atlanta)

2.6 Shocks

The economy faces seven structural shocks, where three of them are related to the international block. Those related to domestic dynamics corresponds to monetary policy (ε_R), government expenses,[7] technology (A), and risky premium (ϕ). All perturbations that comes from foreign economy are supposed to be i.i.d $\mathcal{N}(0,1)$.

$$\varepsilon_{R,t} = \sigma_R \epsilon_R, \quad \text{where } \epsilon_R \text{ is i.i.d } \mathcal{N}(0,1)$$

$$\log\left(\frac{A_t}{\bar{A}}\right) = \rho_A \log\left(\frac{A_{t-1}}{\bar{A}}\right) + \sigma_A \epsilon_{A,t}$$

$$\phi_t = \rho_\phi \phi_{t-1} + \sigma_\phi \epsilon_{\phi,t}$$

$$\varepsilon_{y^*,t} = \sigma_{y^*}\epsilon_{y^*,t}, \quad \varepsilon_{\pi^*t} = \sigma_{\pi^*}\epsilon_{\pi^*,t}, \quad \varepsilon_{i^*,t} = \sigma_{i^*}\epsilon_{i^*,t}$$

[7]Presented in subsection 2.3, equation 2-22.

2.7 Equilibrium

Equilibrium conditions implies representative household acting optimally, maximizing her discounted present value of utility. Domestic and importing firms solve their problems, setting the price that maximizes expected discounted profit or readjusting prices according to the indexation rule. Also, the government budget constraint is satisfied in all t.

The market clearing conditions hold for all markets:

- Labor: $N_t = \int_0^1 N_t(j)dj = \int_0^1 N_t(i)di, \quad \forall t$
- Domestic Bonds: $B_t^{g,S} = \int_0^1 B_t^g(j)dj = B_t^{g,D}, \quad \forall t$
- Domestic goods: $Y_{D,t} = C_{D,t+\tau} + G_{D,t+\tau} + C^*_{D,t+\tau}, \quad \forall t$

where $C^*_{D,t} = \left(\frac{P_{D,t}}{S_t P_t^*}\right)^{-\eta} Y_t^*$

3
Model Solution

We follow the solution strategy of Amaral (2021). In the first step, we solve the model as a function of τ_t and δ_t - variables directly related to our model's two discontinuities: the peak of the Laffer curve and the fiscal limit. The previously derivated solution is then used to simulate the fiscal limit distribution and obtain the default probability. In the last step, each nonlinearity is attributed to a different regime of a DSGE regime-switching model. For that, we use the endogenous switching method of Maih (2015), which is implemented through RISE toolbox.

3.1
First step: single regime and inflation at its target

The fiscal limit theory firstly presented in Bi (2012) was done for a straightforward real business cycle model, with productivity and government expenses shocks. The non-rigidity model makes it possible to obtain an algebraic solution and a less computationally demanding simulation of the fiscal limit. Notwithstanding, it is relevant to evaluate the interactions between active monetary policy and passive fiscal policy in an environment where the probability of default differs from zero. According to Bi (2018), the presence of sovereign default risk exposes the possibility that a contractionary monetary policy ends up in a sustained increase in inflation path, even though the first is active and the fiscal policy passively adjusts taxes to stabilize debt.

Even so, we follow Bi (2018) to reconcile the computationally faster simulation with the rigidity prices. First, we suppose that the monetary authority always maintains the inflation rate at its target. Then, since this is an open economy model, we avail of other simplified hypotheses: prices of domestic and imported products are the same ($P_t = P_{D,t} = P_{F,t}$) and the law of one price (LPU) holds $Q_t = 1$. Hence, it is like the economy is under a flexible price and it will be possible to easily obtain the peak of the Laffer curve as a function of only exogenous state variables (A_t, G_t).

First of all, we obtain an expression for labor quantity by substituting the marginal cost expression (2-12) in the labor supply (2-6), and then replacing the marginal cost by its value in the flexible prices environment $MC_{D,t} = \frac{\varepsilon-1}{\varepsilon}$:

$$N_t = \left[\frac{(\varepsilon - 1)}{\varepsilon} \frac{(1 - \tau_t)}{\varphi_N} A_t \bar{K} \right]^{\frac{1}{\varphi}} \tag{3-1}$$

By setting $MC_{D,t} = \frac{\varepsilon - 1}{\varepsilon}$ in the marginal cost expression (2-12), it is possible to find the nominal wage in terms of exogenous variables and domestic prices:

$$W_t = \left(\frac{(\varepsilon - 1)}{\varepsilon} \right) A_t \bar{K} P_{D,t} \tag{3-2}$$

Substituting theses expressions in the real government revenue equation:

$$T_t = \tau_t \left[\frac{(\varepsilon - 1)}{\varepsilon} \frac{(1 - \tau_t)}{\varphi_N} \right]^{\frac{1}{\varphi}} \left[\bar{K} A_t \right]^{\frac{\varphi + 1}{\varphi}} \tag{3-3}$$

The maximum tax rate is obtained by deriving the government revenue with respect to τ_t:

$$\frac{\partial T_t}{\partial \tau_t} = \left[\left(\frac{(\varepsilon - 1)}{\varepsilon} \frac{(1 - \tau_t)}{\varphi_N} \right)^{\frac{1}{\varphi}} - \frac{(\varepsilon - 1)}{\varepsilon} \frac{1}{\varphi_N} \frac{1}{\varphi} \tau_t \left(\frac{(\varepsilon - 1)}{\varepsilon} \frac{(1 - \tau_t)}{\varphi_N} \right)^{\frac{1}{\varphi} - 1} \right] = 0$$

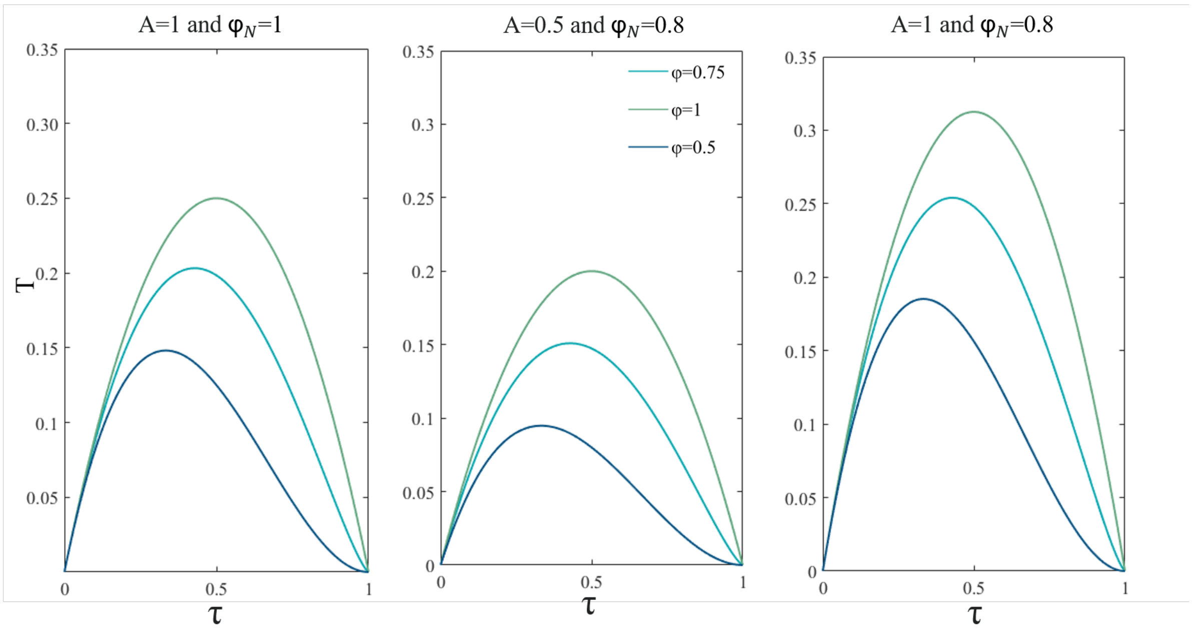
$$\tau_t^{\max} = \frac{\varphi}{1 + \varphi} \tag{3-4}$$

As follows, in order to maximize its revenue, government would always set $\tau_t = \frac{\varphi}{1+\varphi}$, for any state of economy. Nevertheless, the maximum revenue that the Treasury can receive from household's income is state dependent:

$$T_t^{\max}(A_t) = \frac{\varphi}{1 + \varphi} \left[\frac{(\varepsilon - 1)}{\varepsilon} \frac{1}{(1 + \varphi)\varphi_N} \right]^{\frac{1}{\varphi}} \left[\bar{K} A_t \right]^{\frac{\varphi + 1}{\varphi}} \tag{3-5}$$

One of the most important determinants of the government ability to levy taxes is the economy's productivity level and the elasticity of labor supply. Supposing that the firms are under perfect competition ($\varepsilon \to \infty$), we evaluate the possible Laffer curves shape for different productivity, inverse Frisch elasticity (φ) and labor disutility (φ_N). The results, presented in (Figure 3.1) are in accordance with the ones of Bi (2017). *Ceteris paribus* the greater the productivity of an economy, the higher the peak of the curve and the greater the level of the maximum tax rate. Also, a more elastic labor supply - higher disutility of labor and/or inverse Frisch elasticity - reduces the ability of the State to raise taxes, while workers respond more to tax changes either for preferring more leisure or due to a lack of tax compliance, as more people can substitute formal jobs position by informal ones.

Figure 3.1: Simulated Laffer Curves for $\varepsilon \to \infty$



3.2 Second step: fiscal limits and default probability

From the derivation above, it is possible to notice that, as in Bi (2018) there is a unique mapping between exogenous state vectors and $\tau_t^{\max}$, as well as the maximum government revenue $T_t^{\max}$. Accordingly, it is possible to obtain the distribution of fiscal limit ($\mathcal{B}_t^*$) as a function of economic states through Monte Carlo simulation. Besides that, the conditional default probability, $P\left(B_t > \mathcal{B}_{t+1} | A_t, G_t\right)$, can be obtained at any point in time, which will be used in the endogenous Markov switching approach of the model.

Following the definition of Bi (2012), the fiscal limit is the present value of maximum discounted primary surpluses conditional on the existing state. The maximum surplus is characterized by income taxation at the peak of the Laffer curve, so from that point ahead, it is not possible for the government to finance higher debt levels by raising taxes. As stated before, the income tax distorts labor supply, and after-tax real wage is lower than in an environment without tributes. Thus tax burden can increase the government revenues until a certain point ($\tau^{max}(A_t)$). After that point, the decrease in labor income more than compensates for the rate's increase, deepening the tax collection. Moreover, the Laffer curve's shape also depends on the state of the economy: for each level of productivity there exists a maximum level of revenues.

On the grounds that the economy is subject to multiple shocks in each period, the concept of an upper bound for government indebtedness is not a single point. Instead, it is state-dependent and stochastic. Thence, at each period, an effective fiscal limit is drawn from the conditional distribution:

$$\mathcal{B}^* (A_t) \sim \sum_{t=0}^{\infty} \underbrace{\beta^t \frac{U_c^{\max}(A_t, G_t)}{U_c^{\max}(A_0, G_0)}}_{\substack{\text{stochastic discount factor} \\ \text{at the peak of Laffer curve}}} \underbrace{\left(T^{\max}(A_t, G_t) - G_t - Z_t - T_t^{LS}\right)}_{\text{primary surplus}} \tag{3-6}$$

For each debt level that can be supported by taxing income at the peak of Laffer curve, the distribution shows the probability of incurring in partial default. Therefore, at any point of $\mathcal{B}^*$, default is possible. In the right tail, where the probability is high, it can happen even in the presence of good shocks. While in the left tail, it requires a run of nasty shocks. Economic randomness and policy conditions change the shape of that distribution and then impact on economic decisions of households.

Whereas part of sovereign default literature makes it a strategic choice of an optimizing government, as Arellano (2008), in the Bi's (2012) characterization, the ability to service its debt emerges endogenously from dynamic Laffer curves. In addition, another advantage is the fact that the limit concept depends on agents' expectations about government streams of primary surplus. The discount rate is the household's stochastic discount factor, which means that the notion of a fiscal limit refers to the private sector's perception of government solvency and bond value. As found by Bi (2017), compared to advanced economies, emerging markets investors tend to be more impatient and then they discount future surpluses more - implying one reason for lower fiscal limits in that type of economy.

As in Amaral (2021), we use the effective fiscal limit $\mathcal{B}_t$ - that is drawn from the fiscal limit density ($\mathcal{B}^* \sim \mathcal{N}(\bar{\mathcal{B}}^*, \sigma^2_{\mathcal{B}})$) - as the upper bound level of debt in the following default rule: if the real debt level exceeds the fiscal limit at the beginning of period t, then the government defaults on a fraction δ_t of outstanding debt. Otherwise, the Treasury fulfills its obligations.

$$\delta_t = \begin{cases} \delta \in (0,1], & \text{if } B^g_{t-1} > \mathcal{B}_t \text{ (Above Effective Fiscal Limit)} \\ 0, & \text{if } B^g_{t-1} \leq \mathcal{B}_t \text{ (Below Effective Fiscal Limit)} \end{cases} \tag{3-7}$$

To obtain the fiscal limit distribution, we follow the Bi (2012)[1] and Bi (2018) algorithms:

1. For each simulation k, we randomly draw the shocks for productivity (A_{t+j}) and government purchases (G_{t+j}) for 200 periods conditional on the

[1]The main difference is that in our model, as we fixed transfers in its steady state level, there are two exogenous state variables instead of three. In Bi (2012), transfers follow a Markov switching regime.

starting state (A_t, G_t). Assuming that the tax rate is always at the peak of the Laffer curve, we compute the paths of all other variables following the household first-order conditions, the budget constraints, and the discounted sum of maximum fiscal surplus:

$$\mathcal{B}_k^*(t) = \sum_{j=0}^{\infty} \beta^j \frac{U_c^{\max}(A_{t+j}, G_{t+j})}{U_c^{\max}(A_t, G_t)} \left(T^{\max}(A_{t+j}, G_{t+j}) - G_{t+j} - Z - T^{LS}\right)$$

2. We repeat the simulation for 150,000 times and obtain the conditional distribution of $\mathcal{B}^*(A_t, G_t)$ using the $\mathcal{B}_k^*(t)(k = 1, \ldots, 150,000)$. Next, as in Bi (2018), we approximate the distribution to a Normal: $\mathcal{N}(\bar{\mathcal{B}}^*, \sigma_{\mathcal{B}}^2)$. In that way, we obtain the mean $(\bar{\mathcal{B}}^*)$ and standard deviation $(\sigma_{\mathcal{B}}^2)$ related to each combination of states (A_t, G_t).

3. Finally, we repeat the first and second steps for all possible exogenous states (A_t, G_t) within the discretized[2] state space.

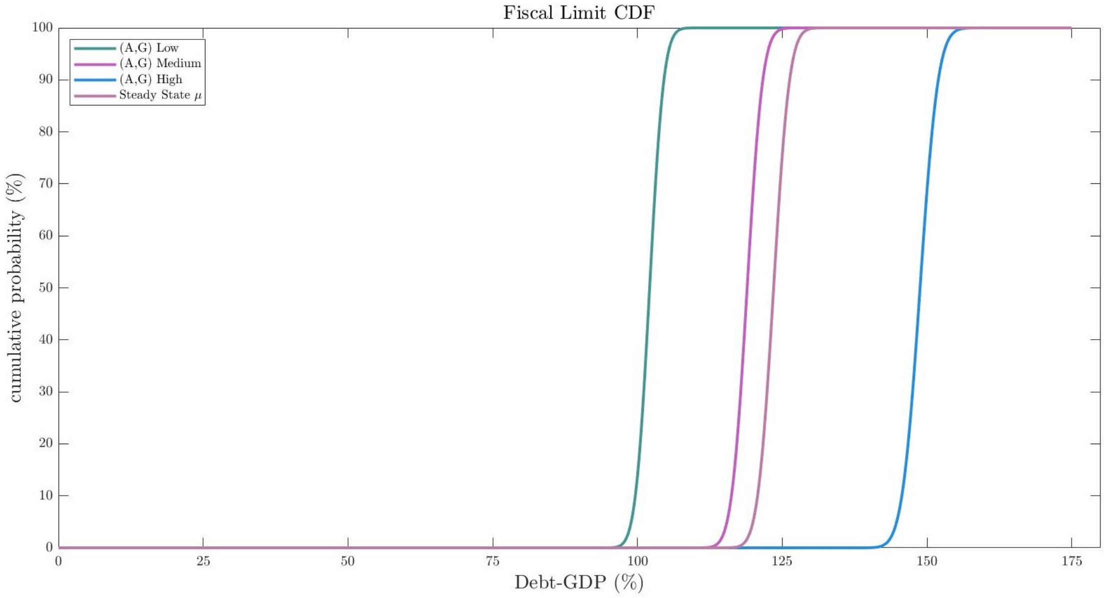
4. At each period, the effective fiscal limit $\mathcal{B}_t^*$ is a random draw from the distribution $\mathcal{N}(\bar{\mathcal{B}}^*, \sigma_{\mathcal{B}}^2)$.

Figure 3.2 illustrates how the fiscal limit accumulated distribution function can change depending on the state variables. The productivity (A_t) and government expenses (G_t) levels tamper with the distribution in opposite directions. Assuming that tax income and government transfers remain the same, an increase in public spending would result in a lower primary surplus - or even a deficit - which in turn increases the amount of expenses that need to be financed with bonds issuance. A higher productivity level is related to a greater production capacity and hence to a higher income; if everything remains equal, this results in larger tax collection and lower debt stock. Yet (A_t) responds to the foremost effect, as we can see high productivity levels more than compensates the government spending pressures, resulting in broad fiscal space. The opposite is also verified; a contractionary fiscal policy G_t is insufficient to reduce the probability of a default if productivity is low.

Table 3.1 gives us more evidence of how the probability of a default evolves concerning state variables level. Our baseline calibration is when the TFP and government spending are at a steady state. In that case, when debt to GDP level achieves 123.5%, there is a 50% probability of a partial default event. That estimation is in line with Bi (2017), whose debt-GDP median for Brazil was 125%. By fixing A_t in its steady state level and varying the Treasury's spending, we can see that even a more austere fiscal policy corresponds to a higher fiscal limit median (131%). When we repeat the exercise but let the productivity vary, the indebtedness representing a 50% default probability

[2]For this purpose, we use Tauchen (1986) method.

Figure 3.2: Fiscal Limit Distribution by (A_t, G_t) levels



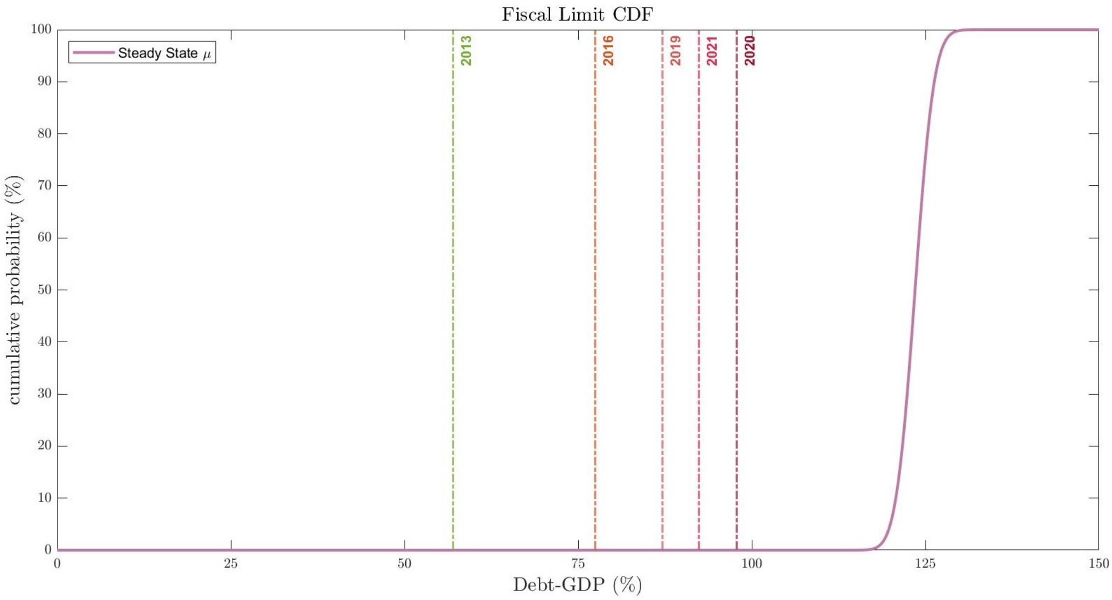
jumps 33.5 p.p in relation to our baseline calibration. The sensitivity of fiscal limit to A_t underlines one of the reasons advanced economies - such as Japan and USA - can keep issuing debt even with a higher debt/GDP. Emerging markets usually present lower productivity and more volatile ones (Bi (2017)); hence, agents' attributes to them a tight fiscal space are expected.

Table 3.1: Fiscal Limit Median over State Variables

State levels	**(A,G) Low**	**(A,G) Medium**	**(A,G) SS**	**(A,G) High**
Median	102%	119%	123.50%	149%
State levels	$\bar{\mathbf{A}}, \mathbf{G}^{\mathbf{low}}$	$\bar{\mathbf{A}}, \mathbf{G}^{\mathbf{med}}$	**(A,G) SS**	$\bar{\mathbf{A}}, \mathbf{G}^{\mathbf{high}}$
Median	131%	125%	123.50%	115.83%
State levels	$\mathbf{A}^{\mathbf{low}}, \bar{\mathbf{G}}$	$\mathbf{A}^{\mathbf{med}}, \bar{\mathbf{G}}$	**(A,G) SS**	$\mathbf{A}^{\mathbf{high}}, \bar{\mathbf{G}}$
Median	95.50%	117%	123.50%	157%

We can also compare how close was Brazilian economy to its fiscal limit in the past years. In Figure 3.2 the dashed vertical lines correspond to the gross general government debt over GDP for the respective years. In 2013, government indebtedness achieved its lowest level (around 59%) due to the remarkable sequence of primary surpluses in the previous years. But it is not all good, the fiscal results have been getting worse since 2011, and by the end of 2014 the surpluses turned into primary deficits. At that time, the economic debate refocuses on finding manners to contain the fiscal deterioration. Few measures, such as the limit for government spending and the reform in the pension system, were adopted. But, when the budgetary background seems to be improving, the COVID-19 pandemic calls upon government intervention to invest in healthcare and social security politics to alleviate the economic effects. Like in the rest of the world, the result was massive public spending, and in 2020 the debt level achieved 97.8%. In the last 15 years, that was the highest it became, yet the default probability still being virtually zero. It is important to mention that the interpretation of those results must be taken in a parsimonious way, as they are very sensitive to parametrization. Even so, the shape of fiscal limit distribution shows an important fact: the probability of default increases heavily when debt levels are high.

Figure 3.3: Fiscal Limit Distribution and Brazilian Debt-GDP



3.3 Third step: endogenous regime switching method

Finally, the complete model is solved by applying the endogenous regime switching method proposed in Maih (2015).[3] A proper reason for us to choose that approach is that it allows for the transition probabilities to be endogenous and and for steady states to be different across regimes. Furthermore, it also represents a promising strategy regarding the problems involved in the alternatives like: the global solution methods,[4] that are very computationally demanding, and the linearized Markov Switching DSGE models,[5] which require the structural model to be linear.

As presented in the former section, the occurrence of default is related to a debt level that surpasses the fiscal limit. That being the case, we need to

[3]For that purpose, we implemented the model using the Matlab toolbox RISE (Rationality In Switching Environments), also developed by Junior Maih.
[4]Davig et al. (2011),Bi and Traum (2014)
[5]Farmer et al. (2011)

first obtain at each period a drawn from the distribution $\mathcal{N}(\bar{\mathcal{B}}^{*}, \sigma^2_{\mathcal{B}})$ and then to obtain the endogenous default probability for the current time.

Following Amaral (2021), we suppose that the state variables - namely (A_t, G_t) - have the same steady state across regimes and obtain a reduced form equation for the mean of the fiscal limit. It is straightforward; we must regress the mean on its respective state-vector deviations from the steady state. The coefficients are saved and compound the set of DSGE model's parameters, being used at each t to define the fiscal limit value according to the realizations of A_t, G_t.

The probability of default is defined as the one of reaching the fiscal limit. We adopt a logistic function (3-8), following Amaral (2021). With the main of recovering the parameters $\gamma_0, \gamma_B, \gamma_A$ and γ_G, we implement a logistic approximation of $P\left(B_t > \mathcal{B}_{t+1}\right)$ to the 3-tuple $(\mathcal{B}_t, A_t, G_t)$. According to Bi and Traum (2014), this functional form has at least two advantages: the associated parameters can be uniquely determined and it is able to capture the strong nonlinear behavior of fiscal limits - once the default probability starts to rise it achieves high levels very fast (Figure 1.2).

$$\Pr\left(B_t > \mathcal{B}_{t+1}\right)_t = \frac{1}{1+\exp\left[\gamma_0 + \gamma_b\left(B_t - \mathbb{E}_t\mathcal{B}_{t+1}\right) + \gamma_a\left(\mathbb{E}_t A_{t+1} - \bar{A}\right) + \gamma_g\left(\mathbb{E}_t G_{t+1} - \bar{G}\right)\right]} \tag{3-8}$$

We define the second nonlinearity likewise Amaral (2021), imposing an occasionally-binding constraint whenever the tax rule leads to an $\tau_t \geq \tau_t^{max}$. At the end of each period, the transition probability for the next one has to be known. That being the case, we also define a shadow rate (τ_t^{sdw}) which anticipates the level of tax rate if the constraint would not be in place. Hence, at each period t, we compare the shadow income tax aliquot to the maximum one to estimate if the binding probability in $t+1$ is 1 or 0.

$$\tau_t = \begin{cases} \log\left(\frac{\tau_t}{\bar{\tau}}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right) + \rho_\tau \log\left(\frac{B_{t-1}}{\bar{B}}\right), & \text{if } \tau_t^{sdw} < \tau^{\max} \\ \tau_t^{\max}, & \text{if } \tau_t^{sdw} \geq \tau_t^{\max} \end{cases} \tag{3-9}$$

Since we have two nonlinearities - defaulting on government bonds and reaching the peak of the Laffer curve - they can be translated into a model with four regimes whose transition probability will be a function of state variables. Along these lines, the model with nominal rigidities is solved through a perturbation strategy, in which first or second-order approximations are made around each regime-specific steady state.[6]

[6]The Maih (2015)'s algorithm is summarized in Appendix A.2

	Below $\tau^{\max}$	**At $\tau^{\max}$**
Below fiscal limit	Regime 1	Regime 2
Reached fiscal limit	Regime 3	Regime 4

The first regime is the baseline one. Although Brazil has been experiencing a fiscal deterioration, it continues to be far from its fiscal limit. However, some expansionary public spending politics that started to contain the economic effects of the COVID-19 pandemic are still in place; investors continue to absorb the issued bonds, and the indebtedness level seems stable - though at a high level. Here, monetary and fiscal policies continue operating like the standard New Keynesian models. Still, their effects can be a little distinct as households are forwarding looking and incorporate in their expectations the probability of a default and thus of a regime-switching.

The second occurs when the debt grows faster and surpasses its steady-state level but does not reach its limit. By the taxation rule, the government will increase the aliquot until it reaches the peak of the Laffer curve. That austere response reduces the debt; time by time, the default probability also falls. The monetary policy remains capable of affecting prices and consumption decisions. Though an interest rate increase in that environment represents a pressure over-indebtedness that cannot be responded to through tax burden. As a result, increases in interest rates are fully converted to more debt.

The third regime is a very unlikely one, in view of the fact that it is necessary that large shocks affect that economy in a way that debt increases very fast and the fiscal limit is reached, while the taxes remain below the peak of the Laffer curve. As the default represents a debt haircut, the tax burden will fall in the following periods. As in regime 1, Central Bank and Treasury can conduct their economic policies. Also, the last one can respond to an interest rate increase with higher taxes in order to reduce debt accumulation. Nonetheless, the government's leeway can be short if its revenues are already close to the maximum.

The fourth, and last one, happens when the peak of the Laffer curve is reached, and the debt level is so high that the Treasury ends up defaulting. In that regime, even the maximum government revenues are not able to handle the increase in debt. This probably occurs due to a sequence of unfortunate shocks that pushes indebtedness to high levels. After the default, there are two possibilities depending on how outstanding is the country's debt stock. If the haircut is sufficient to reduce debt below the fiscal limit and to imply a tax burden lower than the maximum, the regime switches to another one as time goes by. On the other hand, if the debt level is so high that the haircut is insufficient to alleviate the debt constraint, a sequence of default events arises. The series of regime changes will then depend on how fast income tax or indebtedness moves far from its upper limits.

4
Calibration

We calibrated the model for Brazil, given that the country has experienced a deterioration of its public account in the past few years while interest rates and inflation achieved high levels. That whole scenario makes the Brazilian economy an exciting case to be studied in light of our proposed framework. We rely on works that implemented Bayesian estimations for Brazil and some databases to define parameter values. It is essential to mention that our calibration generates default probabilities that approximate 0%, which is consistent with our fiscal limit estimations illustrated in Figure 3.3.

The parameters related to nominal rigidities, monetary policy, and risk premium were settled following the posterior mode of Carvalho and Vilela (2015). Table 4.1 resume their values in detail, but it is important to emphasize that for all regimes, monetary policy is active[1]: interest rates reaction to inflation deviations from its target was set to 1.99, while responses to output gap and exchange rate variations were set to 0.2 and 0.31, respectively.

The discount factor (β) is set at 0.987, implying a real interest rate of approximately 5.5% per year in steady-state. We follow Amaral (2021) for the other parameters related to preferences, as the inverse elasticity of intertemporal substitution (σ), the inverse of Frisch elasticity (φ) and the degree of labor disutility (φ_N). The latter is calculated so that labor supply equals 1/3 at the steady state.

The quarterly debt to output ratio at the steady state is calibrated to 258%, corresponding to the mean of Central Bank of Brazil (BCB) data for annual gross public debt over GDP of 64.5%. We define the steady state value for the other fiscal variables to be the ones that match the Brazilian data available on Government Finance Statistics (GFS) from 2006 to 2021. Proceeding as Amaral (2021) we also estimate the fiscal policy parameters using linear regression with the GFS and BCB database and get very similar results: $\rho_\tau = 0.86$ and $\rho_B = 0.11$. The partial default rate is defined as in Bi (2018) to be 5%.

All steady-state variables are defined in terms of GDP, as we imposed it to be 1 in the steady state. In that way, $\bar{A}$ and $\bar{C}$ are the residuals that

[1] In the terminology of Leeper (1991).

guarantee the output to be unitary. The inflation target is defined in 1.011 to match the 4.5% annual rate, since it was the one that prevailed in Brazil from 2005 to 2018. Likewise Amaral (2021) we define the fixed capital to be 18; this result was obtained from Penn World Table using constant national prices data for capital stock and GDP.

Regarding the parameters related to the exogenous process, we opt to follow Amaral (2021) to the ones presented in the domestic sector: fiscal, monetary, and technology shocks. For the AR(1) parameter of risk premium shock, we follow Carvalho and Vilela (2015). Still, the variance was arbitrarily defined at the same level as the monetary shock to avoid higher perturbation during simulations.

Table 4.1: Calibration

Symbol	**Parameter Description**	**Value**	**Sources**
β	Discount factor	0.987	Real interest rate of 5.5%
σ	Inverse EIS	1	Amaral (2021)
φ	Inverse Frisch elasticity	1	Amaral (2021)
χ	Elast. risk premium - NFA	0.03	Carvalho and Vilela (2015)
Nominal rigidities			
ϵ	Demand elasticity	5	Galí (2015)
δ_D	Domestic indexation	0.07	Carvalho and Vilela (2015)
θ_D	Domestic price rigidity	0.82	Carvalho and Vilela (2015)
δ_F	Imported indexation	0.13	Carvalho and Vilela (2015)
θ_F	Imported price rigidity	0.92	Carvalho and Vilela (2015)
η	Subst. elast. domestic and imported	1	Carvalho and Vilela (2015)
α	Economic openness	0.25	Carvalho and Vilela (2015)
Monetary Policy			
λ_y	Output feedback Taylor Rule	0.2	Carvalho and Vilela (2015)
λ_π	Inflation feedback Taylor Rule	1.99	Carvalho and Vilela (2015)
λ_s	Devaluation feedback Taylor Rule	0.31	Carvalho and Vilela (2015)
Fiscal Policy			
δ	Default rate	5%	Bi (2018)
ρ_τ	Autocorrelation Income tax	0.86	Brazilian data
ρ_B	Income tax response to debt	0.11	Brazilian data
Shocks			
ρ_G	Autocorrelation fiscal shock	0.79	Amaral (2021)
ρ_Y	Expense response to product	0.13	Amaral (2021)
ρ_A	Autocorrelation technology shock	0.93	Amaral (2021)
ρ_ϕ	Autocorrelation risk premium shock	0.5	Carvalho and Vilela (2015)

Table 4.1 – Continued

Symbol	Parameter Description	Value	Sources
σ_G	Government expenses shock std dev.	0.013	Amaral (2021)
σ_A	Technology shock std dev.	0.004	Amaral (2021)
σ_R	Monetary policy shock std dev.	0.004	Amaral (2021)
σ_ϕ	Risk premium shock std dev.	0.004	Author's choice
Steady state			
$\bar{Y}$	Steady state output	1	Author's choice
$\bar{C}$	Steady state consumption	0.867	Calculated based on $\bar{Y} = 1$
$\bar{\Pi}$	Steady state inflation	1.011	Inflation target 4.5%
$\bar{G}$	Steady state government expenses	0.216	Brazilian data
$\bar{\tau}$	Steady state income tax	0.394	Brazilian data
$\bar{B}$	Steady state real debt	2.58	Brazilian data
$\bar{Z}$	Steady state lump sum transfers	0.142	Brazilian data
$\bar{N}$	Steady state labor	1/3	Amaral (2021)
$\bar{K}$	Fixed capital stock	18	Amaral (2021)
$\bar{A}$	Steady state technology	1.337	Calculated based on $\bar{Y} = 1$

5
Simulation

We conduct a few simulations to evaluate if there exists some difference regarding the inflation, output gap, and exchange rate levels when the Central Bank incorporates the fiscal risk in its policy rule.[1] Most of the particularities of each monetary policy rule are only observable in terms of variable levels instead of in their dynamics. We proceed as follows: (i) draw all stochastic shocks for the four regimes with 110,000 periods each, where the first 10,000 are thrown out; (ii) set all observations to their steady state level and simulate the model again with the selected shock values obtained in the first step. Then, we start simplifying the monetary policy to focus on the differences that the incorporation of fiscal risk can generate. After that, we compare the unconditional distributions when the default probability is close to 0% to the case where the small open economy is closer to its fiscal limit.

5.1
Monetary policy efficacy under fiscal risk

To illustrate how important is the incorporation of fiscal risk in monetary policy conduction, we simplify the Taylor rule - by setting $\lambda_y = \lambda_s = 0$ - and turning off monetary shocks. Since our baseline calibration results in a low probability of default, we adjust the debt to GDP ratio to be the one that is equivalent to a 1% chance for the economy to surpass its fiscal limit. Proceeding in that way, the relevance of a Central Bank capable of neutralizing fiscal risk becomes more evident. The exercise is first conducted only with domestic shocks activated, and further, we present the unconditional distribution when all shocks but the monetary are in place.

In Figure 5.1 we can observe that inflation can remain at its target only when Central Bank perfectly incorporates the default risk in its decision rule. This result shows how relevant it is to incorporate the dynamic of fiscal risk so that the monetary authority can achieve its primary goal. The simple fact that it does not revise its intercept, as it understands that the fiscal dynamics have already been incorporated in the natural rate, results in an inflation 1.5pp above the target. Furthermore, exchange rate stability is only observed when

[1]The ergodic distribution for more selected variables are presented in the Appendix A.1

the CB is able to neutralize all default risk. Despite showing less volatility, the domestic currency is depreciated under a monetary rule inconsistent with the adversity of the fiscal environment.

Concerning interest rates, we can see that, on average, their levels are not significantly different. However, the distribution in the case where the intercept varies over time has heavier tails, mainly on its right side. It is an evidence that it is costly to control inflation and exchange rate in an open economy that is subject to many international shocks. In terms of output, it is well above its potential level when the Central Bank does not account for risk and slightly below when it is fully incorporated.

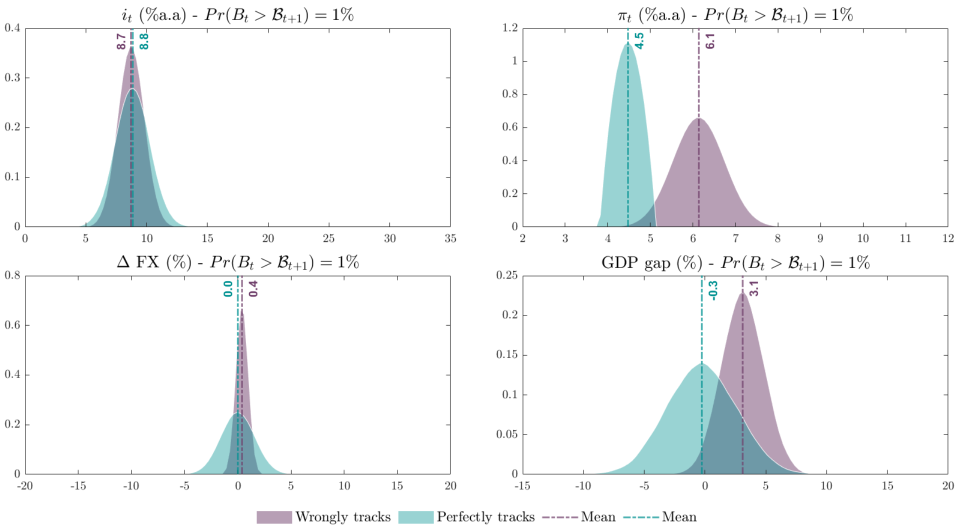


Figure 5.1: Ergodic distribution under different monetary policy rules - Simulation for all domestic shocks less monetary and $\Pr\left(B_t > \mathcal{B}_{t+1}\right) = 1\%$

Our results are close to those of Amaral (2021), given that in their work the only way for the Central Bank to fulfill its mandate was to fully incorporate the dynamics of default in its decision. However, we do not verify that there is no trade-off between price and output stability. In our case, the target is recovered at the cost of a lower level of economic activity. In some extension we can infer that the unpleasant coincidence of high interest rates and high inflation in an environment where the Central Bank neglects fiscal risks is valid here. Almost the same level of interest rate leads to very different levels of inflation.

Regarding references that also sought to analyze the Brazilian case using a small open economy model, we can say that our results corroborate some previous conclusions. As in Arellano et al. (2020), in an environment where the

Treasury can dishonor its liabilities, exchange rates and inflation can remain high even though interest rates are at a high level.

The main conclusions remain when we analyze the case where we allow the economy to be hit by an international shock (Figure 5.2). However, it is possible to notice that the exchange rate becomes more volatile when the Central Bank does not adjust its intercept correctly. On average, the interest rates required to stabilize inflation are now even higher, showing the adverse effects that international shocks can generate in the small open economy. There is still high volatility in interest rates, which can still reach harmful levels since we did not impose any zero lower bound condition here. Many of these results are related to periods in which adverse shocks in international interest rates and risk premiums coexist.

Furthermore, the bimodal character of the output gap distribution is related to the regime changes that the economy has undergone. By allowing the presence of international shocks, the domestic economy reached the fiscal limit more often throughout the simulation.

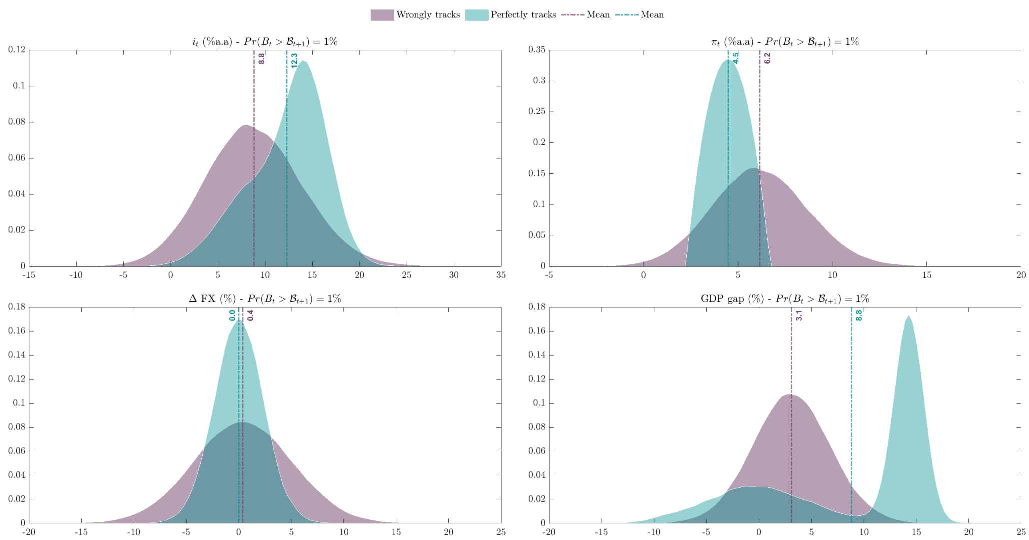


Figure 5.2: Ergodic distribution under different monetary policy rules - Simulation for all shocks less monetary and $\Pr(B_t > \mathcal{B}_{t+1}) = 1\%$

5.2 Risk differentials

This second exercise aims to analyze the particularities that arise when a small open economy approaches its fiscal limit. To compare how different the results can be, we adjust the steady state debt to GDP to be the one that corresponds to a 5% probability of default and simulate the model under the same shock sequence as the ones of our baseline case - where the default

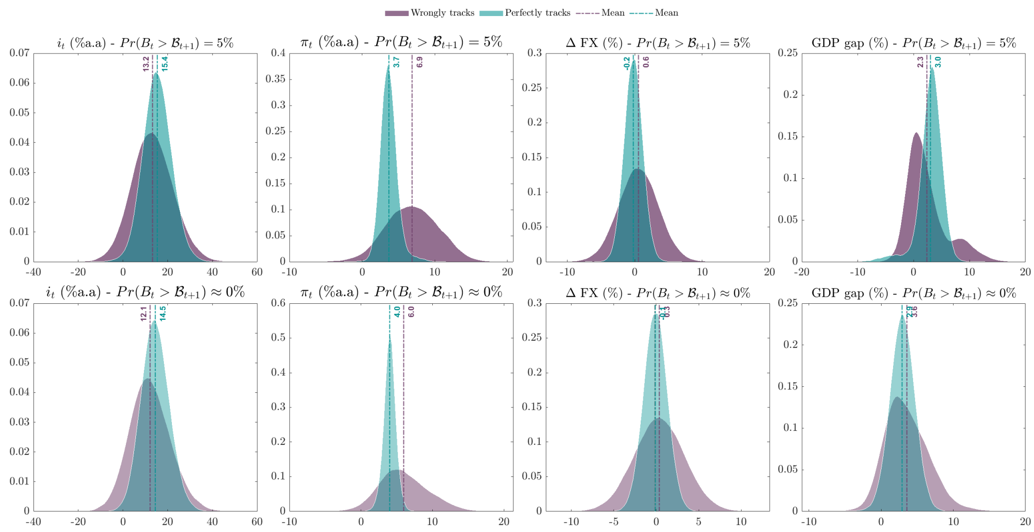


Figure 5.3: Ergodic distribution under different monetary policy rules - Simulation for all shocks

probability is close to 0%. Even though this increase in probability may seem modest, it refers to a debt-to-GDP ratio about 30 p.p higher than the historical average of Brazilian gross debt (64.5%).[2]

In general, we observe that, regardless of default probability, the monetary policy rule that incorporates all the fiscal risk is the one that, on average, results in lower inflation levels and a stable or appreciated exchange rate, guaranteed via a higher interest level. By contrasting the two cases, the importance of a decision rule that allows the Central Bank to adjust its intercept becomes even more evident. When the government is closer to its fiscal limit, there is an increase in the variance of inflation, interest, exchange rate, and product. Those findings align with the ones of Arellano et al. (2020).

As regime changes become more recurrent, the product has a bimodal distribution in cases where monetary policy cannot compensate for all the default risk. As a result, inflation becomes even higher than in the standard case, and the averages between inflation become more distant, even though the interest differential is the same. On average, it is observed that the domestic currency's depreciation is greater when the monetary authority insufficiently monitors the fiscal risk.

[2]It is our baseline steady state level, calculated using General Government Gross Debt data for 2004 to 2019

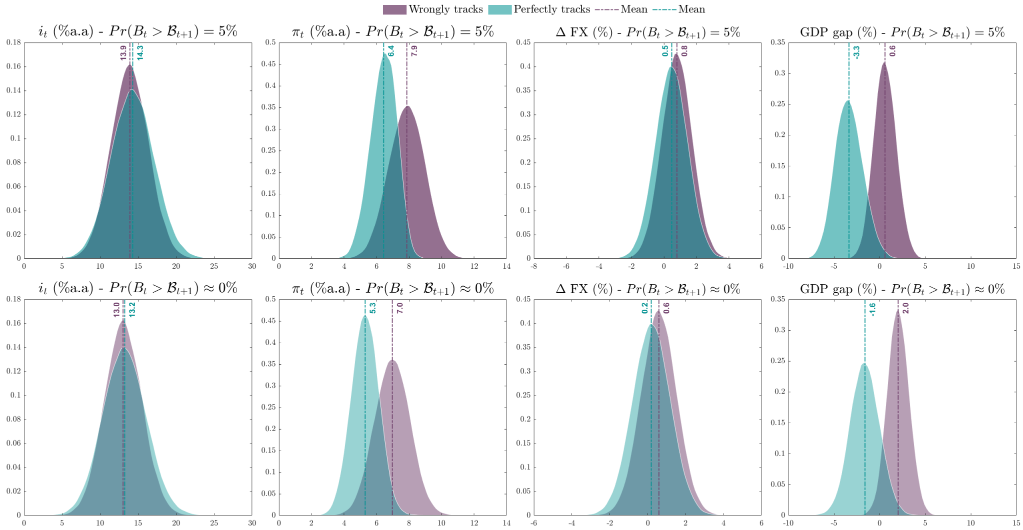


Figure 5.4: Ergodic distribution under different monetary policy rules - Simulation for all domestic shocks

In addition, another finding is the coexistence of high interest rates and depreciated currency. That one is in line with the conclusions of Enrique Alberola-Ila and Mirkov (2021). By their evaluation of the economy's behavior in the face of fiscal and monetary shocks in different regimes, they show that a contractionary monetary (expansionary fiscal) shock can lead to a depreciation - rather than an appreciation as predicted in standard international macro models - if fiscal fundamentals are deteriorating. Despite the differences in analysis, here we can see that under the same sequence of shocks, the simple fact that an economy is closer to its fiscal limit implies a depreciation of the exchange rate when monetary policy cannot incorporate such a change in its rule. The explanation relies on the uncovered interest parity equation (2-10): in the presence of default risk, the investors require an additional premium over the international interest rate to absorb domestic bonds. Therefore, only when such a premium is incorporated into domestic rates can we observe a more heightened attractiveness of the domestic currency. Figure 5.5 shows that there is a persistent difference in total risk premium $(\Phi_t(V_t) + \Omega_t)$, but this is even higher when there is a greater chance of the Treasury declaring a default.

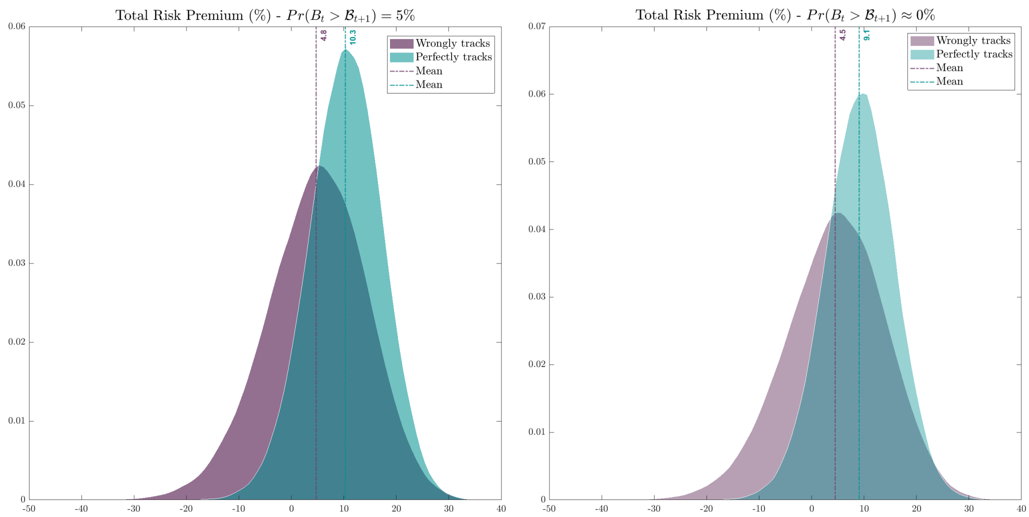


Figure 5.5: Ergodic distribution of Total Risk Premium under different monetary policy rules - Simulation for all shocks

6
Conclusion

In recent years, Brazil has been showing deterioration in public accounts, at the same time that high interest rates, above-target inflation and a depreciated exchange rate coexist. In numerous Monetary Policy Minutes, the Central Bank of Brazil points out fiscal risk as one of the factors that may hinder the convergence of inflation and expectations to the target.

This scenario makes the country suitable for our investigation. We calibrate a DSGE model of a small open economy, where the monetary authority operates at risky rates, and the government may incur a partial default. Given that country's tax burden vis-à-vis other emerging economies,[1] incorporating the fiscal limit model a la Bi (2012) is appropriate. The presence of two nonlinearities made us resort to endogenous regime switching methods to solve the model.

Our findings are in line with what is predicted in the literature and, in a way, reproduce the Brazilian economic scenario in recent years. When the Central Bank does not correctly incorporate the risk of default in its monetary policy rule, the result is a higher level of inflation, interest rates, and a depreciated exchange rate - even though the country is far from its fiscal limit. In addition, differences in results concerning a monetary authority ideally incorporating such risk into its decisions are even more relevant when the probability of default increases. The inflation target goal can only be reached when the whole default dynamic is neutralized in CB's decision rule.

Our proposed model can also account for an explanation for interest rate differentials across countries. Even the most remote probability of default is able to make agents require a wedge over risk free domestic rate to invest in domestic bonds. Therefore, countries presenting more fiscal instability are expected to present higher interest rates when compared to a similar market that does not incur in that type of problem. Moreover, if the monetary authority neglects that risk dynamics the country can present high interest rates and depreciated currency. This result shed some light in the Brazilian scenario in the past years: Brazilian Real remained more depreciated than a basket of emerging market

[1]https://observatorio-politica-fiscal.ibre.fgv.br/politica-economica/outros/tributacao-equidade-e-crescimento-economico

currencies, even with high interest rates.

Finally, it is essential to highlight some limitations of the model. The distribution of the fiscal limit and, consequently, the probability of default is extremely sensitive to calibration. Furthermore, although the trend of the variables reflects the Brazilian context, the moments of simulations are not sufficiently close to those of the data. Thus, the numbers portrayed here should be viewed with parsimony. A possible extension of the work to better capture the dynamics of fiscal risk for Brazil would be the model's Bayesian estimation and account for foreign investors buying Brazilian bonds.

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A
Appendix

A.1
Graph Appendix

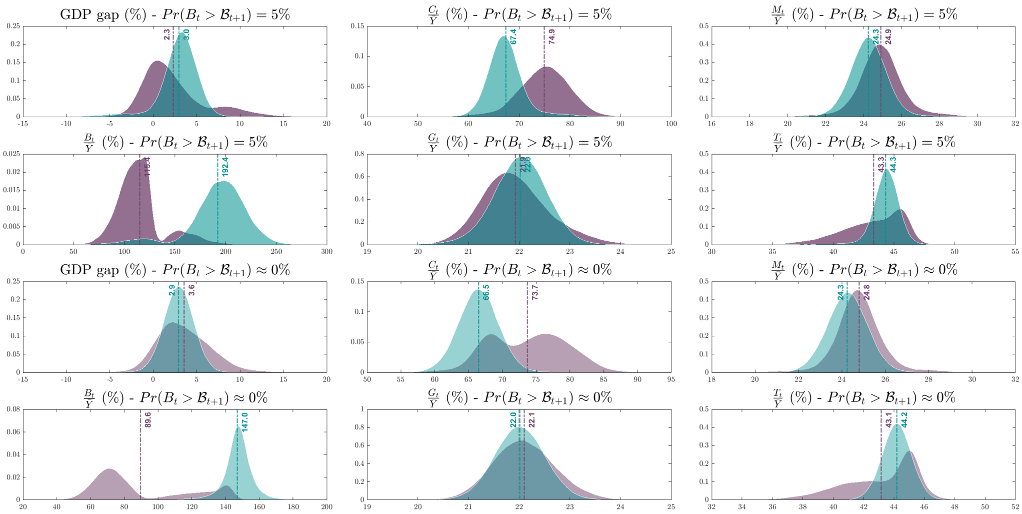

Figure A.1: Ergodic distribution under different monetary policy rules - Simulation for all shocks

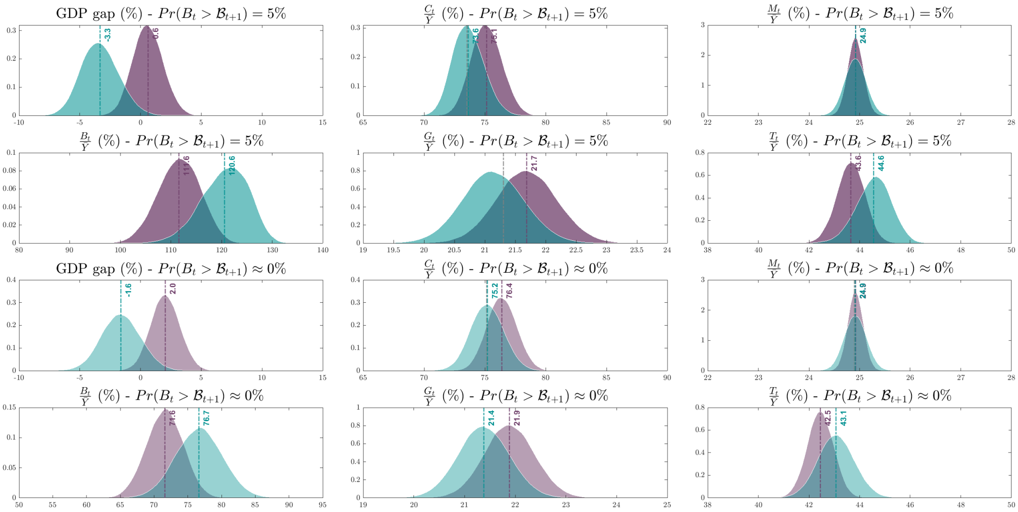

Figure A.2: Ergodic distribution under different monetary policy rules - Simulation for all domestic shocks

A.2 Endogenous Markov Switching Algorithm

We follow the method described in Maih (2015) to solve our regime switching open economy DSGE model, and implement it through the MatLab toolbox RISE - which was also developed by Maih. A proper reason for us to choose that approach is that it allows for the transition probabilities to be endogenous and for steady states to differ across regimes.

In this appendix, we summarize[1] the algorithm, following the same notation of Maih (2015). We start by stating the problem to be solved:

$$E_t \sum_{r_{t+1}=1}^{h} \pi_{r_t,r_{t+1}} (\mathcal{I}_t)\, \tilde{d}_{r_t}(v) = 0 \tag{A-1}$$

where E_t is the expectation operator; $\tilde{d}_{r_t} : \mathbb{R}^{n_e} \longrightarrow \mathbb{R}^{n_d}$ is a $n_d \times 1$ vector of functions (maybe nonlinear) of v; $r_t = 1, 2, .., h$ is the regime at time t; $\pi_{r_t,r_{t+1}} (\mathcal{I}_t)$ is the transition probability for going from regime r_t to r_{t+1} in the next period. In our case, this probability is endogenous and - as required by the method - only depends on constant parameters and unique steady state variables.

In the sense that it is a function of $\mathcal{I}_t$, the information set at time t. The only restriction imposed on the endogenous switching probabilities is that the parameters affecting them do not switch over time and that the variables entering

[1]For more details, we recommend referring Maih (2015) and his directory on Github (`https://github.com/jmaih/RISE_toolbox`).

those probabilities have a unique steady state. To remember, the default probability is a function of debt, the mean of fiscal limit, government expenses and productivity. In turn, the probability of reaching the peak of the Laffer curve is a function only of the shadow tax rate, whose steady state is also the same across regimes.

The $n_v \times 1$ vector v is given by:

$$v \equiv \left[b_{t+1}(r_{t+1})' \quad f_{t+1}(r_{t+1})' \quad s_t(r_t)' \quad p_t(r_t)' \quad b_t(r_t)' \quad f_t(r_t)' \quad p'_{t-1} \quad b'_{t-1} \quad \epsilon'_t \quad \theta_{r_{t+1}} \right]' \tag{A-2}$$

where : - s_t is a $n_s \times 1$ vector of static variables. - f_t is a $n_f \times 1$ vector of forward-looking variables. - p_t is a $n_p \times 1$ vector of predetermined variables, the ones that appears in the model at time t and at time $t-1$. - b_t is a $n_b \times 1$ vector of variables that are predetermined and forward-looking. - ε_t is a $n_\varepsilon \times 1$ vector of shocks with $\varepsilon_t \sim N(0, I_{n_\varepsilon})$ - $\theta_{r_{t+1}}$ is a $n_\theta \times 1$ vector of switching parameters appearing with a lead.

The parameters of each regime are implicitly attached to $\tilde{d}_{r_t}$; hence the model equations can also switch across regimes.

Maih (2015) also assumes that the agents have information for future shocks - the anticipated shock approach. Hence they define the vector of state variables that depends on a perturbation parameter σ, as:

$$z_t \equiv \left[\begin{array}{ccccccc} p'_{t-1} & b'_{t-1} & \sigma & \varepsilon'_t & \varepsilon'_{t+1} & \cdots & \varepsilon'_{t+k} \end{array} \right]' \tag{A-3}$$

Let $y_t(r_t)$ be the vector of all endogenous variables and $\mathcal{T}^{r_t}(z_t)$ their decision rules. The general solution follows:

$$y_t(r_t) \equiv \begin{bmatrix} s_t(r_t) \\ p_t(r_t) \\ b_t(r_t) \\ f_t(r_t) \end{bmatrix} = \mathcal{T}^{r_t}(z_t) \equiv \begin{bmatrix} \mathcal{S}^{r_t}(z_t) \\ \mathcal{P}^{r_t}(z_t) \\ \mathcal{B}^{r_t}(z_t) \\ \mathcal{F}^{r_t}(z_t) \end{bmatrix}$$

As a whole, even if $\tilde{d}_{r_t}$ or $d_{r_t, r_{t+1}}$ is linear, there is no guarantee of an analytical solution to A-2. Therefore, the author proposed a perturbation method that allows us to approximate the decision rules.

In order to do the approximation, he defines some matrices (λ_g) that will select a g-type variables in $\mathcal{T}$ or y: $\lambda_x \equiv \begin{bmatrix} \lambda_p \\ \lambda_b \end{bmatrix}$: is the selector of predetermined and both variables;

$\lambda_{bf} \equiv \begin{bmatrix} \lambda_b \\ \lambda_f \end{bmatrix}$: selects both and forward variables.

Also, he defines for all $g \in \{p_{t-1}, b_{t-1}, \sigma, \varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+k}\}$, a matrix m_g that selects the g-type variables in the state vector z_t.

To obtain the solution in terms of the state vector, we proceed by expressing all variables as a function of z_t:

$$z_{t+1} = h^{r_t}(z_t) + uz_t \tag{A-4}$$

where:

$$h^{r_t}(z_t) \equiv \left[(\lambda_x \mathcal{T}^{r_t}(z_t))' \quad (m_\sigma z_t)' \quad \ldots \quad (m_\epsilon z_t)' \quad (0_{n_\epsilon \times 1}) \right]$$

$$u \equiv \begin{bmatrix} 0_{(n_p+n_b+1+kn_\varepsilon)\times n_z} \\ \varepsilon_{t+k+1} m_\sigma \end{bmatrix}$$

Regarding the switching parameters, he proposed an auxiliary variable: $\theta_{r_{t+1}} = \bar{\theta}_{r_t} + \sigma\hat{\theta}_{r_{t+1}}$. In our case, $\bar{\theta}_{r_t}$ is the steady state of regime 1. Now we can express v in terms of all state variables:

$$v = \begin{pmatrix} \lambda_{bf} \mathcal{T}^{r_{t+1}}(h^{r_t}(z_t) + uz_t) \\ \mathcal{T}^{r_t}(z_t) \\ m_p z_t \\ m_b z_t \\ m_{\varepsilon,0} z_t \\ \bar{\theta}_{r_t} + \hat{\theta}_{r_{t+1}} m_\sigma z_t \end{pmatrix}$$

and the objective function (A-2) becomes:

$$E_t \sum_{r_{t+1}=1}^{h} d_{r_t, r_{t+1}}(v(z_t, u)) = 0$$

Finally, with the problem expressed in terms of z_t we can solve it by applying successive Taylor approximations around our chosen reference point, the regime-specific steady state. Maih (2015) emphasizes that, regarding the possibility of an unstable system, he assumes its stability if the solution reaches one of its regime-specific means. In that way, the system must stay there without any further shocks. In our case, we just conducted a first-order approximation around the regime-specific steady state.