

# Série dos Seminários de Acompanhamento à Pesquisa

**DEI**  
DEPARTAMENTO  
DE ENGENHARIA  
INDUSTRIAL

Número 20 | 09 2021

Data-driven joint chance-constrained optimization for  
the workover rig scheduling problem

Autor:

Iuri Martins Santos



# Série dos Seminários de Acompanhamento à Pesquisa

Número 20 | 04 2021

## Data-driven joint chance-constrained optimization for the workover rig scheduling problem

Autor:

Iuri Martins Santos

Orientador: Prof. Silvio Hamacher

Coorientador: Prof. Fabricio Oliveira (Aalto University)

### CRÉDITOS:

SISTEMA MAXWELL / LAMBDA

<https://www.maxwell.vrac.puc-rio.br/>

Organizadores: Fernanda Baião / Soraida Aguilar

Layout da Capa: Aline Magalhães dos Santos

# Agenda

Personal presentation

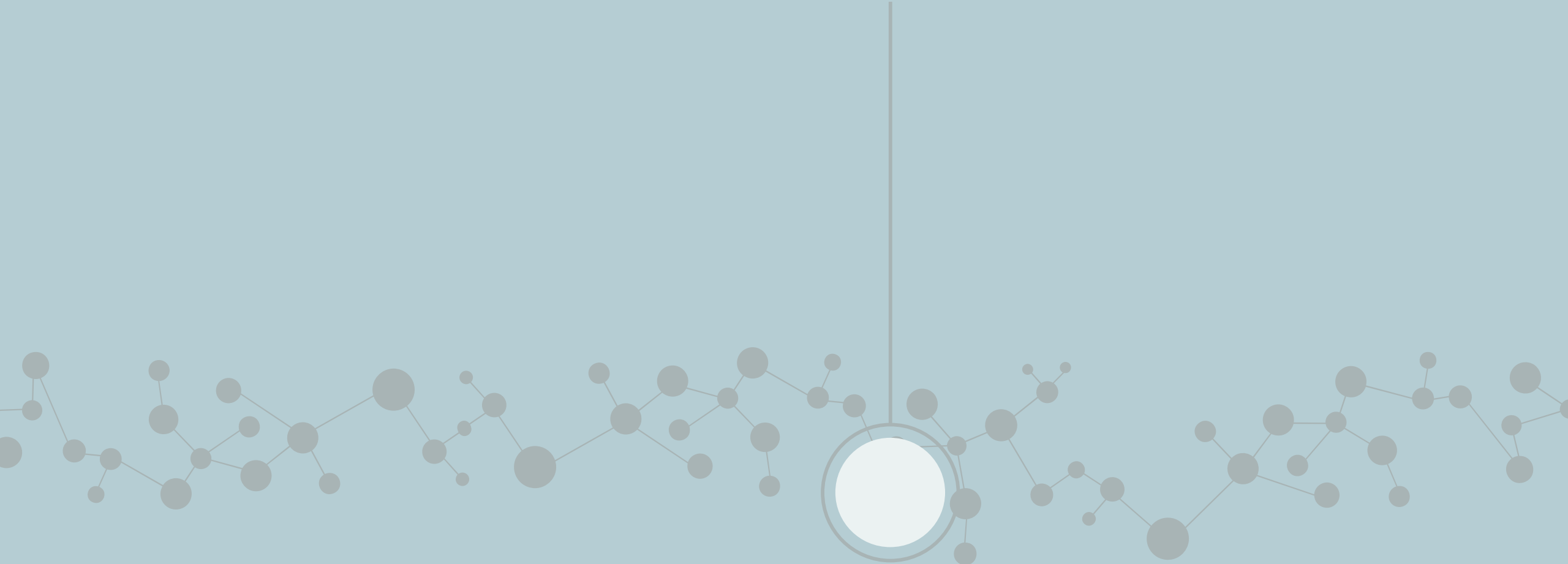
Introduction

Systematic Literature Review

Workover Rig Scheduling Problem

- Deterministic model
- Data preparation and prediction
- Chance-constrained model

Next steps



# Personal presentation



# Iuri Santos – PhD Candidate at PUC-Rio



## Academic Background

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- Bachelor and Master in Industrial Engineering (PUC-Rio).
- PhD Candidate in Industrial Engineering (PUC-Rio) – 4th year
- Visiting Researcher at Aalto University



## Professional Background

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- Researcher and Intern at Tecgraf Institute (2016-).
- Intern at the Petrobras' department of Intelligence and Strategy of Maritime Transportation (2014-2015).



## Iuri Santos – PhD Candidate at PUC-Rio

### Maritime Fleet Size and Mix


- Stochastic and deterministic optimization models for the *cabotage* tanker fleet size and mix decisions.
- 1 final project (Santos, 2015), 2 conference papers (Vieira et al., 2016; Santos et al., 2017), 1 journal article (Vieira et al., 2017) and 1 prize (Vieira et al., 2016).

### PLSVs Scheduling

- Heuristics and optimization models for ship scheduling.
- 1 conference paper (Cunha et al., 2017) and 1 journal article (Cunha et al., 2018).

### Oil Rigs Scheduling

- Simulation, heuristics and optimization model for offshore rigs scheduling.
- 1 conference paper (Santos et al., 2017) and 1 master thesis (Santos, 2018)



# Data-driven joint chance- constrained optimization for the workover rig scheduling problem

# Introduction - The Rig Scheduling Problem

Oil rigs are the most important resources in the Exploration and Production of Oil and Gas.

Use mainly in **drilling, completion, workover and abandonment**.  
(complex, expensive and risky operations)



The **Rig Scheduling Problem (RSP)** emerges as the decision-making process to determinate **which rigs** will attend **which wells** and **when**.







# Systematic Literature Review The Rig Scheduling Problem

Article “A Systematic Literature Review for the Rig Scheduling Problem: classification and state-of-the-art” currently in revision (R1) for the *Computers and Chemical Engineering*.  
(Qualify paper)

Analysis of 128 papers from Scopus (67/3248), WOS (35/551) and Google Scholar (citation links).  
Proposes a classification and taxonomy.

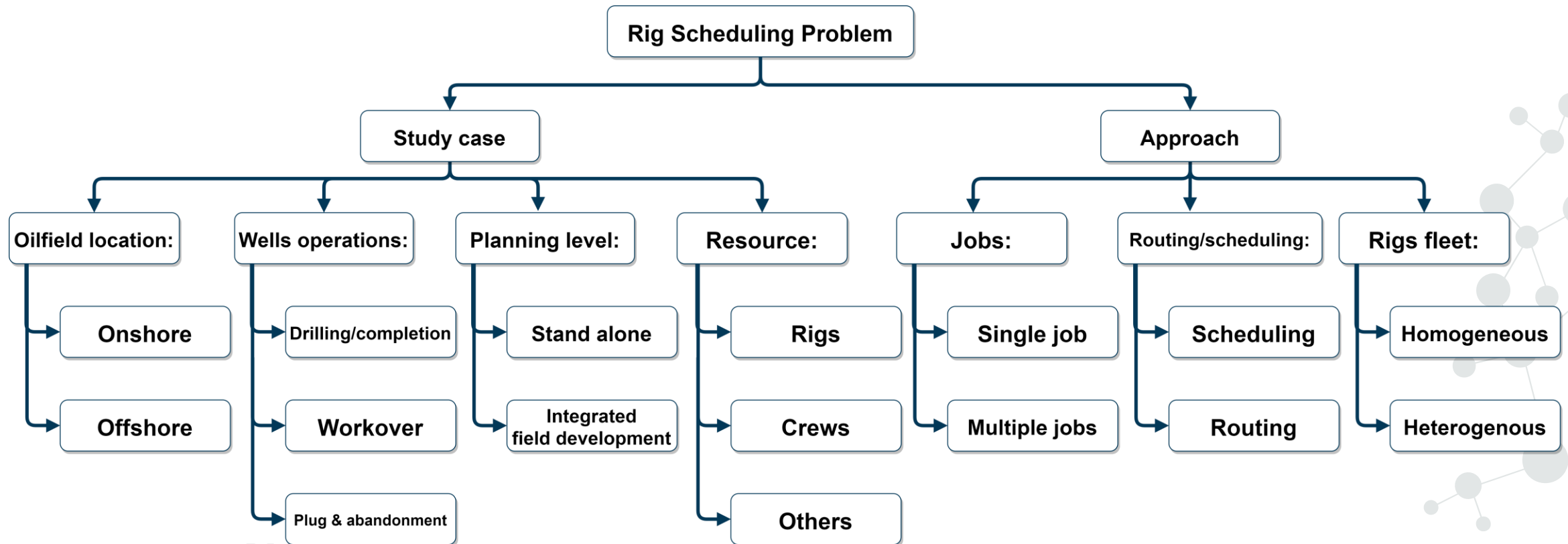
## Main findings:

- Trend for models considering the **uncertainty** of the rig scheduling problems:  
**Stochastic/robust models**, simulation-optimization, dynamic programming and **data-driven optimization**.
- Need for models closer to the demands of the industry:  
**Realistic objective functions**, **heterogeneous fleets**, use of **real data** and **validation/implementation** of results.

# Systematic Literature Review

## The Rig Scheduling Problem – New Classification

The **Rig Scheduling Problem (RSP)** main attributes:

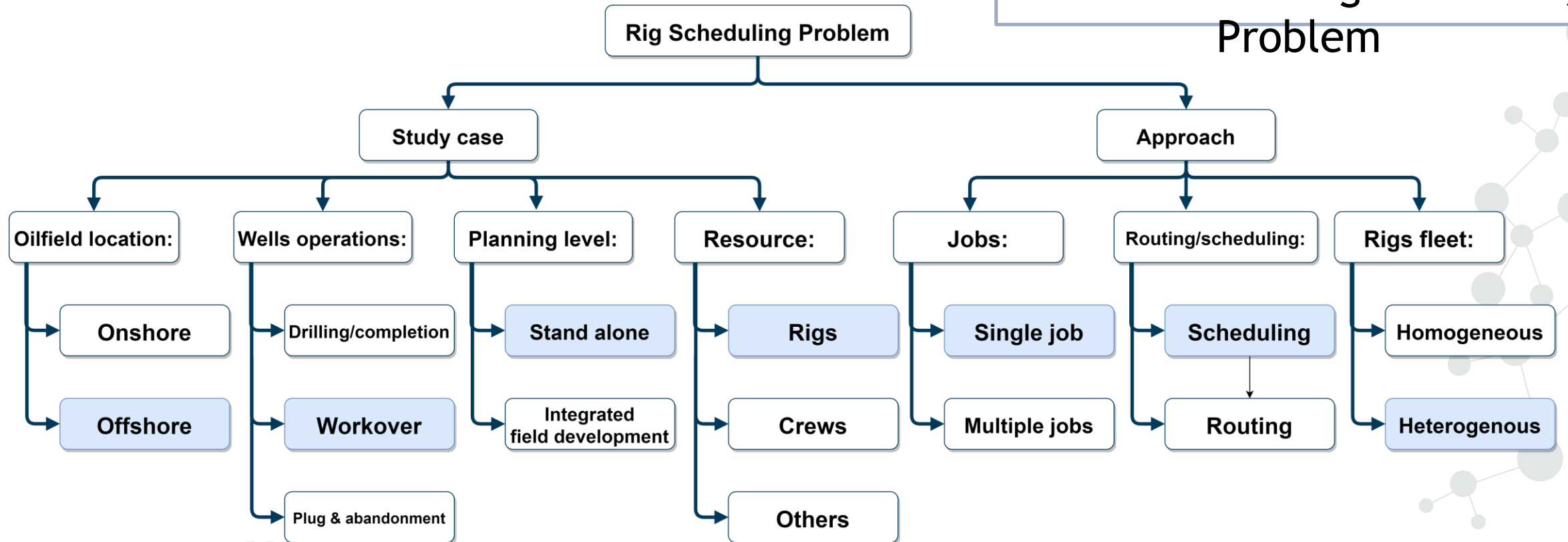


# Systematic Literature Review

## The Rig Scheduling Problem – New Classification

The **Rig Scheduling Problem (RSP)** main attributes:

Focus of this study:  
**The Workover Rig Scheduling Problem**





# Workover Rig Scheduling Problem

## Assumptions

A set of **offshore wells**, each one requiring a specific workover operation with a release date.  
A set of **heterogenous rigs** is available for hiring with eligibilities and different durations.

### Objectives:

- **Fleet size:**  
Select rigs to hire minimizing the fleet costs.
- **Wells service:**  
Select wells to served minimizing the oil production loss.
- **Scheduling:**  
Allocate a well to a rig that can serve it.  
Select when the well will be served by the rig minimizing the oil production loss.

# Workover Rig Scheduling Problem

## Deterministic Model

- Several formulations were developed and tested for the deterministic model.
- We propose a formulation based in a routing model.

### Sets:

- $i, j \in J$ : wells
- $k \in K$ : wells

### Parameters:

- $l_i$ : oil production loss
- $a_i$ : release date.
- $d_i^k$ : processing time.

### Main Variables:

- $X_{ij}^k$ : If rig  $k$  goes from well  $i$  to well  $j$ .
- $S_i$ : Starting time of task  $i$ .
- $Z^k$ : If rig  $k$  is hired or not.

### Auxiliary Variables:

- $x1_i^k$ : If rig  $k$  enters well  $i$ .
- $x2_i^k$ : If rig  $k$  leaves well  $i$ .
- $w_i$ : If well  $i$  is served.

$$\begin{aligned}
 & \text{Min} \quad \sum_{i \in J | i \neq 0} l_i \left[ S_i + \sum_{k \in K} (d_i^k - a_i) X1_i^k + H(1 - W_i) \right] + \sum_{k \in K} c^k Z^k \quad (11) \\
 & \text{Subject to:} \quad X1_i^k = X2_i^k \quad \forall i, k \quad (12) \\
 & \text{Flow-balance} \quad \left\{ \begin{aligned} X1_i^k &= \sum_{j \in J} X_{ji}^k \\ X2_i^k &= \sum_{j \in J} X_{ij}^k \end{aligned} \right. \quad \forall i, k \quad (13) \quad (14) \\
 & \text{Well service} \quad \left\{ \begin{aligned} W_i &= \sum_{k \in K} X1_i^k \\ W_i &= \sum_{k \in K} X2_i^k \end{aligned} \right. \quad \forall i | i \neq 0 \quad (15) \quad (16) \\
 & \text{Sequence timing} \quad S_i - d_j^k \geq S_j - M(1 - X_{ji}^k) \quad \forall i, j, k | i \neq j \quad (17) \\
 & \text{Release dates} \quad S_i \geq a_i W_i \quad \forall i | i \neq 0 \quad (18) \\
 & \text{Rigs hiring} \quad X1_i^k \leq Z^k \quad \forall i, k \quad (19) \\
 & \text{Variable domains} \quad \left\{ \begin{aligned} X_{ij}^k &\in \{1, 0\} \\ X1_i^k &\in \{1, 0\} \\ X2_i^k &\in \{1, 0\} \\ W_i &\in \{1, 0\} \\ S_i &\in \mathbb{Z}^+ \\ Z^k &\in \{1, 0\} \end{aligned} \right. \quad \begin{aligned} &\forall i, j, k | i \neq j \quad (20) \\ &\forall i, k \quad (21) \\ &\forall i, k \quad (22) \\ &\forall i | i \neq 0 \quad (23) \\ &\forall i \quad (24) \\ &\forall k \quad (25) \end{aligned}
 \end{aligned}$$

# Workover Rig Scheduling Problem

## Deterministic Model

- Several formulations were developed and tested for the deterministic model.
- We propose a formulation based in a routing model.



Sets:

- $i, j \in J$ : wells
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- $w_i$ : If well  $i$  is served.

$$\text{Min} \quad \sum_{i \in J | i \neq 0} l_i \left[ S_i + \sum_{k \in K} (d_i^k - a_i) X1_i^k + H(1 - W_i) \right] + \sum_{k \in K} c^k Z^k \quad (11)$$

Oil production loss (tardiness and well service)      Rig hiring cost

$$\text{Subject to: } X1_i^k = X2_i^k \quad \forall i, k \quad (12)$$

$$\text{Flow-balance } \left\{ \begin{array}{l} X1_i^k = \sum_{j \in J} X_{ji}^k \\ X2_i^k = \sum_{j \in J} X_{ij}^k \end{array} \right. \quad \forall i, k \quad (13) \quad (14)$$

$$\text{Well service } \left\{ \begin{array}{l} W_i = \sum_{k \in K} X1_i^k \\ W_i = \sum_{k \in K} X2_i^k \end{array} \right. \quad \forall i | i \neq 0 \quad (15) \quad (16)$$

$$\text{Sequence timing } S_i - d_j^k \geq S_j - M(1 - X_{ji}^k) \quad \forall i, j, k | i \neq j \quad (17)$$

$$\text{Release dates } S_i \geq a_i W_i \quad \forall i | i \neq 0 \quad (18)$$

$$\text{Rigs hiring } X1_i^k \leq Z^k \quad \forall i, k \quad (19)$$

$$\text{Variable domains } \left\{ \begin{array}{l} X_{ij}^k \in \{1, 0\} \\ X1_i^k \in \{1, 0\} \\ X2_i^k \in \{1, 0\} \\ W_i \in \{1, 0\} \\ S_i \in \mathbb{Z}^+ \\ Z^k \in \{1, 0\} \end{array} \right. \quad \forall i, j, k | i \neq j \quad (20) \quad (21) \quad (22) \quad (23) \quad (24) \quad (25)$$

Can be infeasible  
when simulating the  
deterministic model  
decisions for the  
rigs hired and wells  
served

Rig hiring cost

$$\text{Subject to: } X1_i^k = X2_i^k \quad \forall i, k \quad (12)$$

$$\left[ \begin{array}{l} X 2_i^k = \sum_{j \in J} X_{ij}^k \end{array} \right] \quad \forall i, k \quad (14)$$

$$W_i = \sum_{k \in K} X 2_i^k \quad \forall i | i \neq 0 \quad (16)$$

$$\text{Release dates} \quad S_i \geq a_i W_i \quad \forall i | i \neq 0 \quad (18)$$

$$\text{Rigs hiring} \quad X1_i^k \leq Z^k \quad \forall i, k \quad (19)$$

$$\left[X_{ij}^k \in \{1, 0\} \quad \forall i, j, k | i \neq j \quad (20)\right.$$

$$X1_i^k \in \{1, 0\} \quad \forall i, k \quad (21)$$

$$\text{Variable } \boxed{X2_i^k \in \{1, 0\}} \quad \forall i, k \quad (22)$$

$$\text{domains} \quad | \quad W_i \in \{1, 0\} \quad \quad \quad \forall i | i \neq 0 \quad (23)$$

$$S_i \in \mathbb{Z}^+ \quad \forall i \quad (24)$$

$$\left[ \begin{array}{l} Z^* \in \{1, 0\} \\ \end{array} \right] \quad \forall k \quad (25)$$



# Workover Rig Scheduling Problem

## Deterministic Model

Solution: Data-driven joint chance constrained approach

Can be infeasible  
when simulating the  
deterministic model  
decisions for the  
rigs hired and wells  
served

developed and tested for the  
ed in a routing model.

## Parameters:

- Can be infeasible  
when simulating the  
deterministic model  
decisions for the  
rigs hired and wells  
served

Oil production loss (tardiness and well service)

Rig hiring cost

$$\text{Min} \quad \sum_{i \in J | i \neq 0} l_i \left[ S_i + \sum_{k \in K} (d_i^k - a_i) X 1_i^k + H(1 - W_i) \right] + \sum_{k \in K} c^k Z^k \quad (11)$$

$$\text{Subject to: } X1_i^k = X2_i^k \quad \forall i, k \quad (12)$$

$$\begin{aligned} \text{Subject to: } & X1_i^k = X2_i^k & \forall i, k & \quad (12) \\ \text{Flow-balance } & \left\{ \begin{aligned} & X1_i^k = \sum_{j \in J} X_{ji}^k & \forall i, k & \quad (13) \end{aligned} \right. \end{aligned}$$

$$X2_i^k = \sum_{j \in J} X_{ij}^k \quad \forall i, k \quad (14)$$

$$W_i = \sum_{k \in K} X1_i^k \quad \forall i | i \neq 0 \quad (15)$$

$$W_i = \sum_{k \in K} X 2_i^k \quad \forall i | i \neq 0 \quad (16)$$

$$\text{Sequence timing} \quad S_i - d_j^k \geq S_j - M(1 - X_{ji}^k) \quad \forall i, j, k | i \neq j \quad (17)$$

$$\text{Release dates} \quad S_i \geq a_i W_i \quad \forall i | i \neq 0 \quad (18)$$

$$\text{Rigs hiring} \quad X1_i^k \leq Z^k \quad \forall i, k \quad (19)$$

$$\left[ X_{ij}^k \in \{1, 0\} \quad \forall i, j, k | i \neq j \right. \quad (20)$$

$$X1_i^k \in \{1, 0\} \quad \forall i, k \quad (21)$$

Variable	$X2_i^k \in \{1, 0\}$	$\forall i, k$	(22)
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$$\text{domains} \quad W_i \in \{1, 0\} \quad \forall i | i \neq 0 \quad (23)$$

$$S_i \in \mathbb{Z}^+ \quad \forall i \quad (24)$$

$$Z^k \in \{1, 0\} \quad \forall k \quad (25)$$

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# Workover Rig Scheduling Problem

## Data-driven joint chance constrained approach

### Methodology:

#### Data preparation

- Data cleaning
- Text Mining to treat qualitative data
- Task Classification using clustering methods
  - Tool: R.



#### Predictive Models

- Task Duration estimation (duration, log, norm)
- Regressions models (GLM and Ridge)
- Best distribution for residuals
  - Tool: R.



#### Joint Chance-Constrained Model

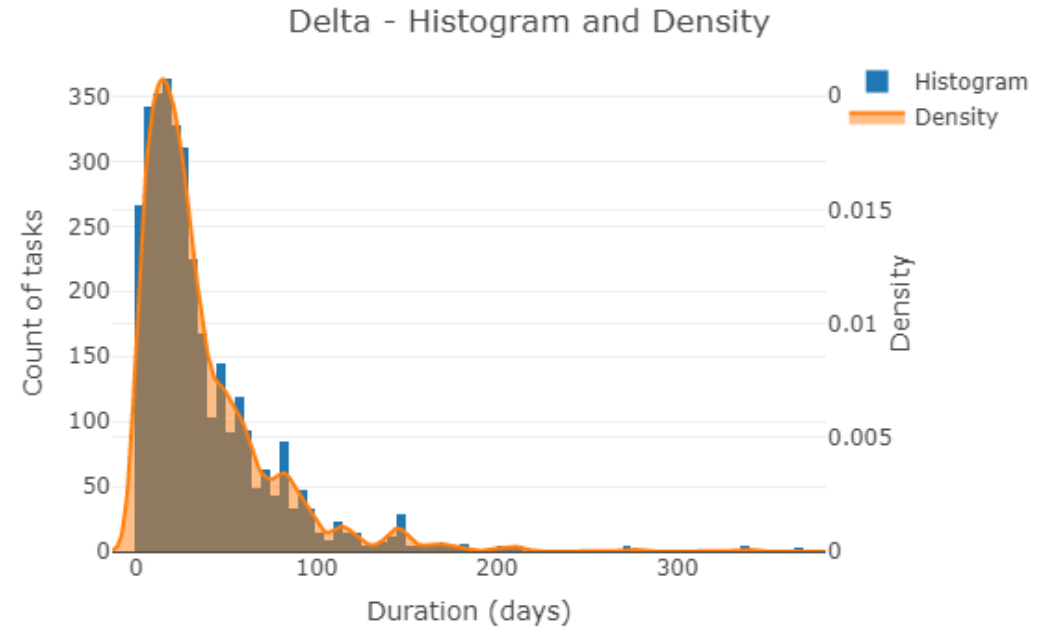
- Representation of the probability
- Non-linear deterministic equivalent
- Stochastic programming with linear model.
- Scenario Generation techniques
  - Tools: Julia and Gurobi.

# Workover Rig Scheduling Problem

## Data preparation and prediction

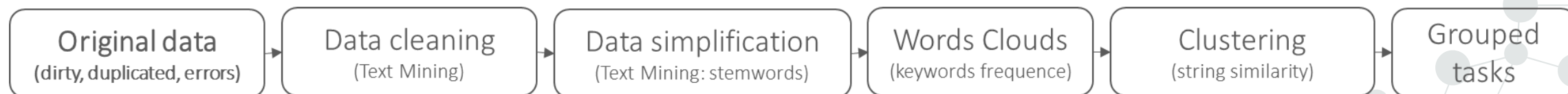
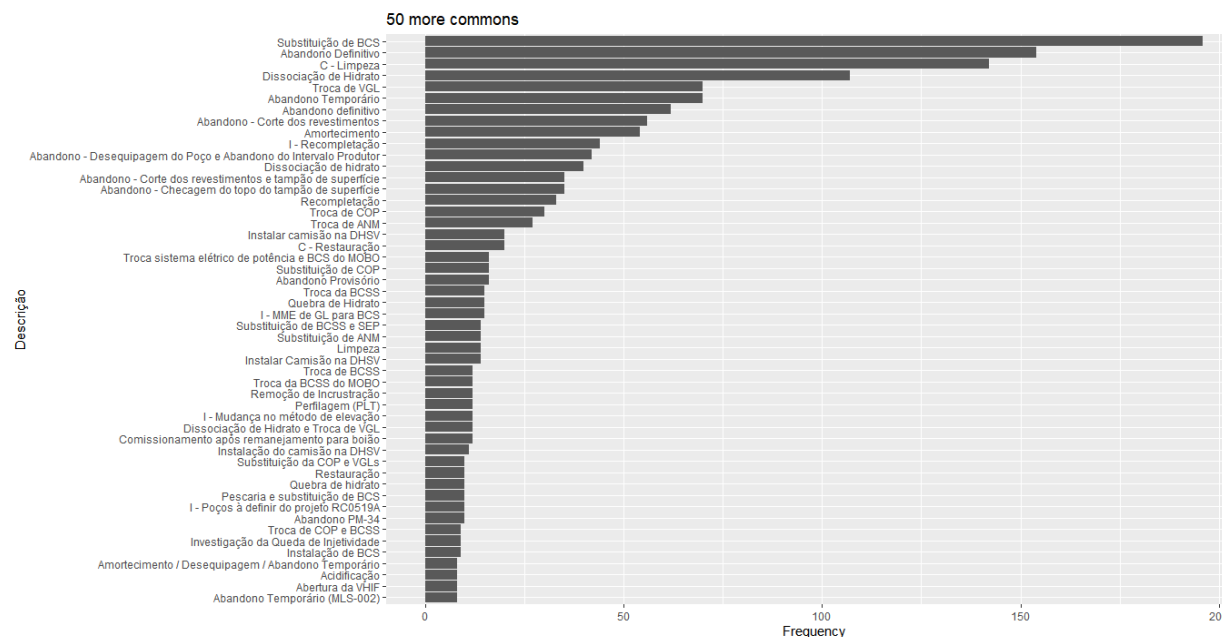
The workover rig scheduling environment is full of uncertainties (durations, dates, occurrence, workover properties).

How can we predict the workover durations?



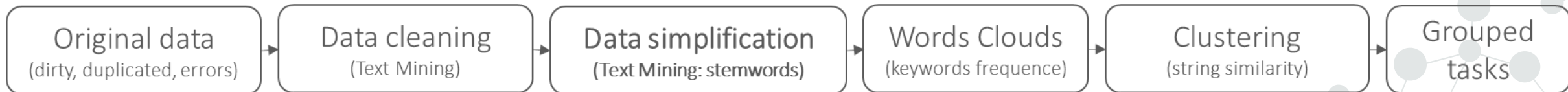
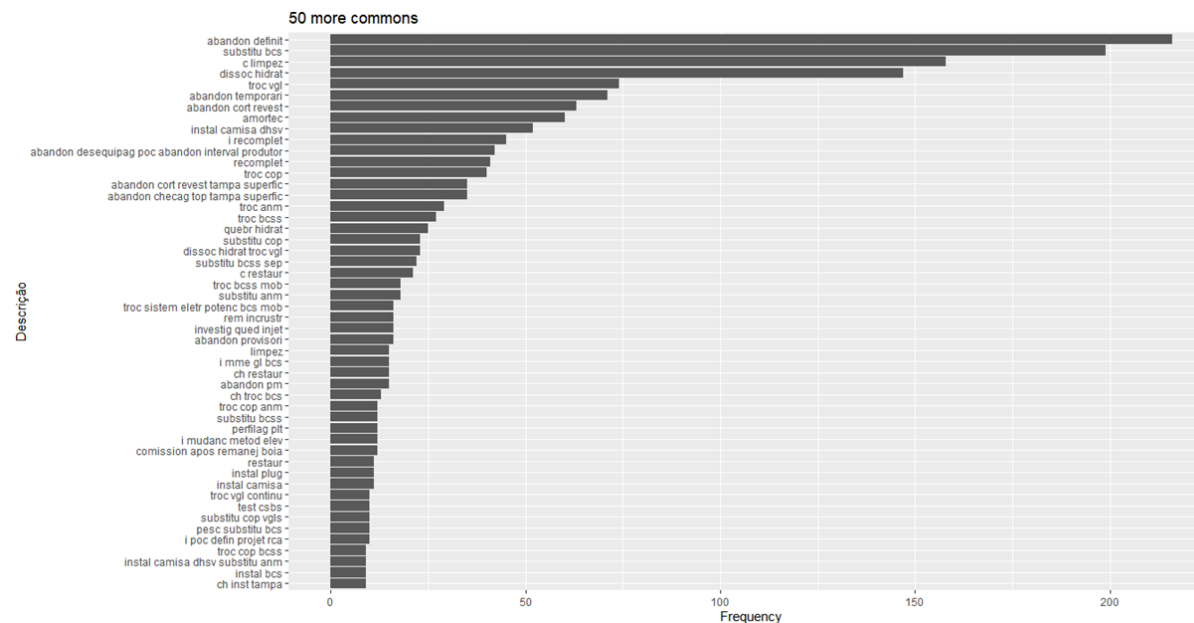
# Workover Rig Scheduling Problem

## Data preparation



# Workover Rig Scheduling Problem

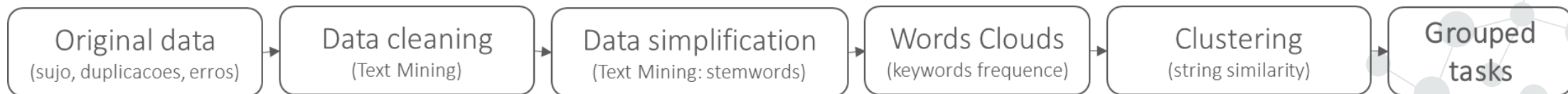
## Data preparation



# Workover Rig Scheduling Problem

## Data preparation

CLUSTER	Original data	Number of obs.
A	Substituição de BCS	100
A	Substituição de ANM	3
A	Substituição de BCSS + SEP	2
A	Substituição da COP e VGLs	2
A	Substituição de BCSS e SEP	2
A	Substituição da BCSS	2
B	Abandono Definitivo	32
B	Abandono Temporário	14
B	Abandono - Corte dos revestimentos	9
B	Abandono - Checagem do topo do tampão de superfície	5
B	Abandono - Corte dos revestimentos e tampão de superfície	5
B	Abandono Permanente	2
B	Abandono Definitivo (Interrompido)	1
B	Abandono Definitivo (não finalizado)	1
B	Abandono - Concluir recuperação do revestimento de 30"	1
B	Abandono Temporário (MLS-002)	1
C	Dissociação de Hidrato	37
C	Dissociação de Hidrato e Troca de VGL	4
C	Dissociação de Hidrato + Troca de VGL	3
C	Dissociação de hidrato (Retorno)	1
C	Dissociação de Hidrato e Abandono Temporário	1
C	Dissociação de hidrato nas LGL e LPO	1
C	Dissociação de Hidrato + Desincrustração	1
D	C - Restauração	7
D	Restauração	6
D	Teste de estanqueidade	2
D	BG-16 - Restauração	2
D	CH-32 - Restauração	2
D	CH-27 - Restauração	1





# Workover Rig Scheduling Problem

## Data prediction

Objective: Predict the duration of the tasks.

Regression models:

- Generalized Linear Model Regression
- Ridge Regression

Variations of the dependent variable:

- Duration.
- $\log(\text{Duration})$ .
- Normalized Duration.

Cross-validation with several distributions (gaussian, poisson and gamma).

Samples sizes: 479 (in) and 103 (out).



# Workover Rig Scheduling Problem

## Data prediction

Final regression model:

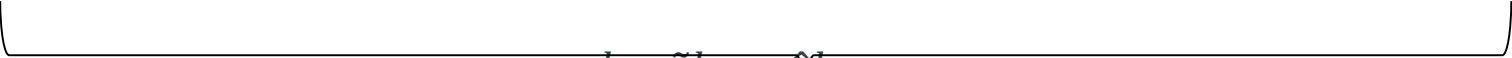
- Ridge Regression:

$$\log(\text{Duration}) \sim \text{WellDepth} + \text{Subpool} + \text{Basin} + \text{Clusters45} + \text{RigType}$$

- Reformulating the regression to WRSP notation:

$$\log(d_i^k) \sim \alpha \cdot \text{Depth}_i + \beta_i \cdot \text{Pool}_i + \gamma_i \cdot \text{Basin}_i + \delta_i \cdot \text{Clusters}_i + \varphi^k \cdot \text{Type}^k$$

$$d_i^k \sim \exp(\alpha \cdot \text{Depth}_i + \beta_i \cdot \text{Pool}_i + \gamma_i \cdot \text{Basin}_i + \delta_i \cdot \text{Clusters}_i + \varphi^k \cdot \text{Type}^k) + \varepsilon$$


$$d_i^k \sim \tilde{d}_i^k = \hat{d}_i^k + \varepsilon$$

Where:  
 $\varepsilon$  can be  
estimated

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model



### Basic representation

$$\text{Min} \quad \mathbb{E} \left( \sum_{i \in J | i \neq 0} l_i \left[ S_i + \sum_{j \in J} \sum_{k \in K} (\tilde{d}_i^k - a_i) X_{ji}^k \right] \right) + H \sum_{i \in J | i \neq 0} l_i (1 - W_i) + \sum_{k \in K} c^k Z^k \quad (55)$$

Subject to

$$\sum_{j \in J} X_{ji}^k = \sum_{j \in J} X_{ij}^k \quad \forall i, k \quad (56)$$

$$\sum_{k \in K} \sum_{j \in J} X_{ji}^k = W_i \quad \forall i | i \neq 0 \quad (57)$$

$$\longrightarrow \Pr(S_j + H(1 - X_{ij}^k) \geq S_i + \tilde{d}_i^k \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (58)$$

$$S_i \geq a_i * W_i \quad \forall i | i \neq 0 \quad (59)$$

$$\sum_{j \in J} X_{ij}^k \leq |J| Z^k \quad \forall i, k \quad (60)$$

$$X_{ij}^{k\omega} \in \{1, 0\} \quad \forall i, j, k | i \neq j \quad (61)$$

$$S_i \geq 0 \vee S_i \in \mathbb{Z}^+ \quad \forall i | i \neq 0 \quad (62)$$

$$W_i \in \{1, 0\} \quad \forall i \quad (63)$$

$$Z^k \in \{1, 0\} \quad \forall k \quad (64)$$

Joint chance-constrained  
Probability  $1-\alpha$  of the well  $i$   
duration respects the start of  
the next task start in the same  
rig.



# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model



### Basic representation

$$\text{Min} \quad \mathbb{E} \left( \sum_{i \in J | i \neq 0} l_i \left[ S_i + \sum_{j \in J} \sum_{k \in K} (\tilde{d}_i^k - a_i) X_{ji}^k \right] \right) + H \sum_{i \in J | i \neq 0} l_i (1 - W_i) + \sum_{k \in K} c^k Z^k \quad (55)$$

Subject to

$$\sum_{j \in J} X_{ji}^k = \sum_{j \in J} X_{ij}^k \quad \forall i, k \quad (56)$$

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Joint chance-  
constrained

Probability  $1-\alpha$  of the well  $i$   
duration respects the start of  
the next task start in the same

How to represent this  
probability?

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



$$P(S_j + H(1 - X_{ij}^k) \geq S_i + \hat{d}_i^k \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (46)$$

Initial representation:  $P(S_j + H(1 - X_{ij}^k) \geq S_i + \tilde{d}_i^k + \epsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (47)$

$$P(S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k \leq -\epsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (48)$$

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



$$P(S_j + H(1 - X_{ij}^k) \geq S_i + \hat{d}_i^k \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (46)$$

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MINLP deterministic  
equivalents:

$$\prod_{j \in J, k \in K | i \neq j} P(S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k \leq -\epsilon) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (49)$$

$$\prod_{j \in J, k \in K | i \neq j} P(-\epsilon \geq S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (50)$$

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



$$P(S_j + H(1 - X_{ij}^k) \geq S_i + \hat{d}_i^k \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (46)$$

Initial representation:  $P(S_j + H(1 - X_{ij}^k) \geq S_i + \tilde{d}_i^k + \varepsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (47)$

$$P(S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k \leq -\varepsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (48)$$

MINLP deterministic  
equivalents:

$$\prod_{j \in J, k \in K | i \neq j} P(S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k \leq -\varepsilon) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (49)$$

$$\prod_{j \in J, k \in K | i \neq j} P(-\varepsilon \geq S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (50)$$

$$\prod_{j \in J, k \in K | i \neq j} P(\varepsilon' \geq g_{ij}^k(X)) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (51)$$

$$\prod_{j \in J, k \in K | i \neq j} P\left(\frac{\varepsilon' - \mu'}{\sigma} \geq \left(\frac{g_{ij}^k(X) - \mu'}{\sigma}\right)\right) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (52)$$

If  $\varepsilon \sim N(\mu, \sigma)$ , still MINLP:

$$\prod_{j \in J, k \in K | i \neq j} \left[1 - \Phi\left(\frac{g_{ij}^k(X) - \mu'}{\sigma}\right)\right] \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (53)$$

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



$$P(S_j + H(1 - X_{ij}^k) \geq S_i + \hat{d}_i^k \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (46)$$

Initial representation:  $P(S_j + H(1 - X_{ij}^k) \geq S_i + \tilde{d}_i^k + \varepsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (47)$

$$P(S_i - S_j - H(1 - X_{ij}^k) + \tilde{d}_i^k \leq -\varepsilon \quad \forall j, k | i \neq j) \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (48)$$

MINLP deterministic  
equivalents:

$j \in J, k \in K$

$j \in J, k \in K$

Solution for a MILP representation of the  
Joint Chance-constrained model via  
regression?

Stochastic programming / Scenarios

If  $\varepsilon \sim N(\mu, \sigma)$ , still MINLP:

$$\prod_{j \in J, k \in K | i \neq j} \left[ 1 - \Phi \left( \frac{g_{ij}^k(X) - \mu'}{\sigma} \right) \right] \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (53)$$

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



Two-stages stochastic programming approach:



# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



Two-stages stochastic programming approach:

New set:

- $\omega \in \Omega$  (Scenarios)

First stage variables:

- $W_i$ : wells attended
- $Z^k$ : rigs hired

Second stage variables:

- $X_{ij}^{k\omega}$ : “travels”
- $S_i^\omega$ : well start
- $V_{ij}^{k\omega}$ : slack variable for constraint relaxation.
- $Y_i^\omega$ : well i feasibility.

+ Auxiliary variables used for better relaxation

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model



Two-stages stochastic programming approach:

$$\text{Min} \underbrace{\sum_{\omega \in \Omega} \sum_{i \in J | i \neq 0} \pi^\omega l_i \left[ S_i + \sum_{j \in J} \sum_{k \in K} (\hat{d}_i^k + \varepsilon_i^{k\omega} - a_i) X_{ji}^{k\omega} \right]}_{\text{Second stage}} + \underbrace{H \sum_{i \in J | i \neq 0} l_i (1 - W_i) + \sum_{k \in K} c^k Z^k}_{\text{First stage}} \quad (66)$$

Subject to **Second stage** **First stage**

$$\sum_{j \in J} X_{ji}^{k\omega} - \sum_{j \in J} X_{ij}^{k\omega} = 0 \quad \forall i, k, \omega \quad (67)$$

$$\sum_{k \in K} \sum_{j \in J} X_{ji}^{k\omega} = W_i \quad \forall i, \omega | i \neq 0 \quad (68)$$

Slack variable relax the uncertainty  $\rightarrow S_j^\omega + H(1 - X_{ij}^{k\omega}) \geq S_i^\omega + \hat{d}_i^k + \varepsilon_i^{k\omega}(1 - V_{ij}^{k\omega}) \quad \forall i, j, k, \omega | 0 \neq i \neq j \quad (69)$

Slack variable only for when X=1  $\rightarrow V_{ij}^{k\omega} \leq X_{ij}^{k\omega} \quad \forall i, j, k, \omega | 0 \neq i \neq j \quad (70)$

Slack variable use counts as Infeasible  $\rightarrow V_{ij}^{k\omega} \leq Y_i^\omega \quad \forall i, j, k, \omega | 0 \neq i \neq j \quad (71)$

Infeasible probability less the 1- $\alpha$   $\rightarrow \sum_{\omega \in \Omega} \pi^\omega Y_i^\omega \leq 1 - \alpha \quad \forall i | i \neq 0 \quad (72)$

$$S_i^\omega \geq a_i * \sum_{j \in J} \sum_{k \in K} X_{ji}^{k\omega} \quad \forall i, \omega | i \neq 0 \quad (73)$$

$$\sum_{j \in J} X_{ij}^{k\omega} \leq Z^k \quad \forall i, k, \omega | i \neq 0 \quad (74)$$

$$X_{ij}^{k\omega} \in \{1, 0\} \quad \forall i, j, k, \omega | i \neq j \quad (75)$$

$$V_{ij}^{k\omega} \in [1, 0] \quad \forall i, j, k, \omega | 0 \neq i \neq j \quad (76)$$

$$Y_i^\omega \in \{1, 0\} \quad \forall i, \omega | i \neq 0 \quad (77)$$

$$S_i^\omega \in \mathbb{Z}^+ \quad \forall i, \omega | i \neq 0 \quad (78)$$

$$W_i \in \{1, 0\} \quad \forall i \quad (79)$$

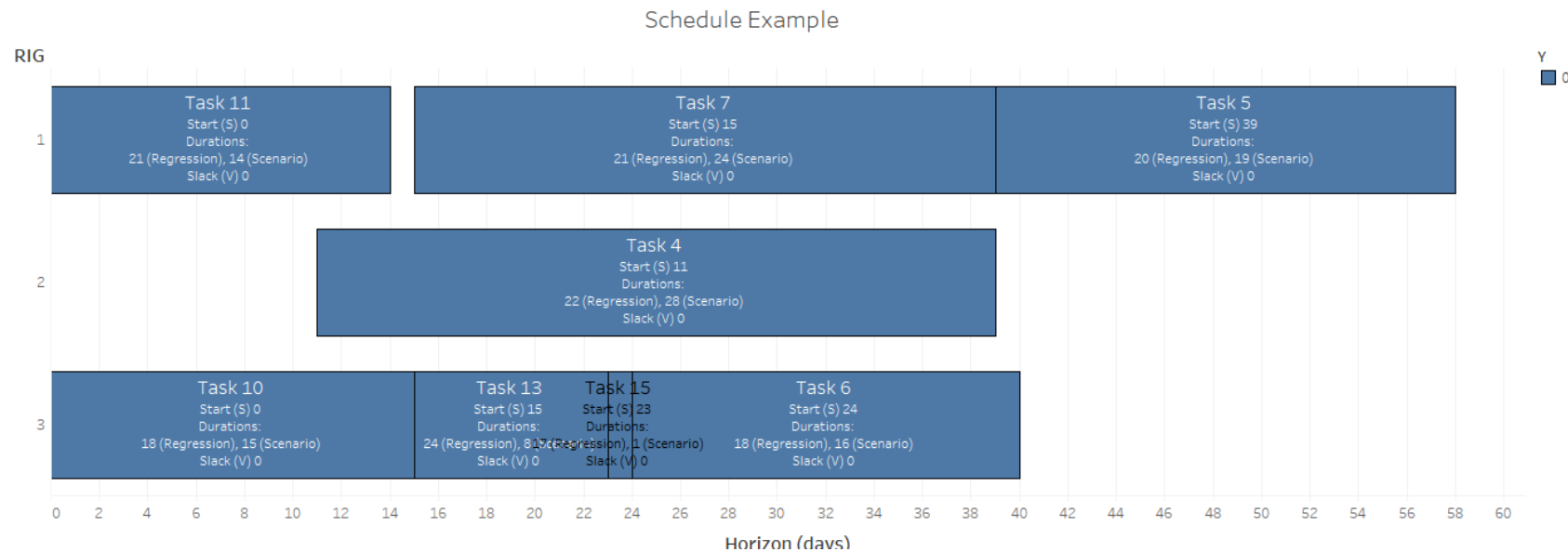
$$Z^k \in \{1, 0\} \quad \forall k \quad (80)$$

Joint  
Chance-  
constrained



# Workover Rig Scheduling Problem

Joint Chance-Constrained Model via Regression Model: stochastic approach

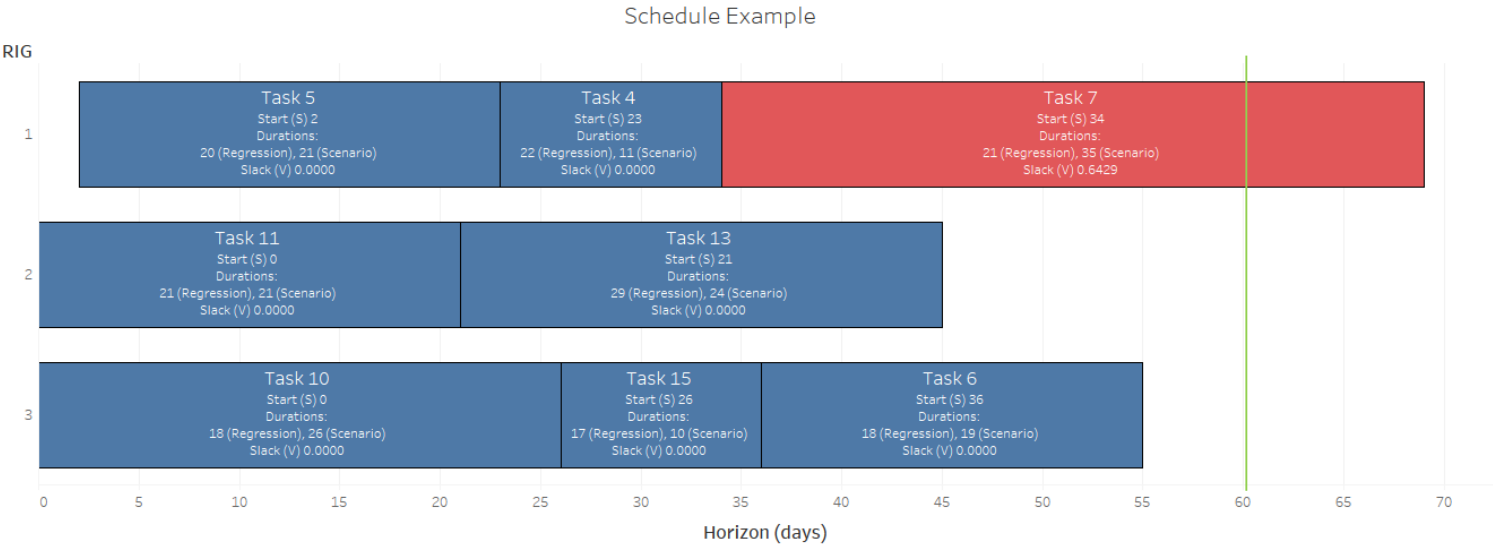
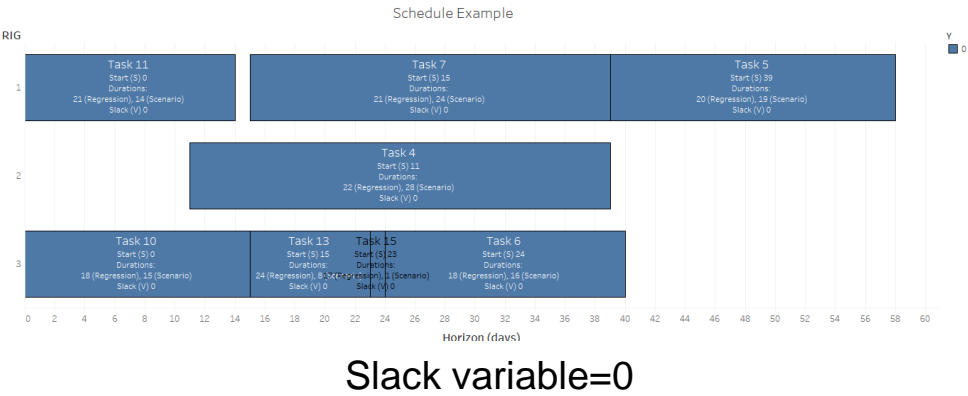


$$V \text{ (slack variable)} = 0$$

$$Y \text{ (infeasible)} = 0$$

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model: stochastic approach

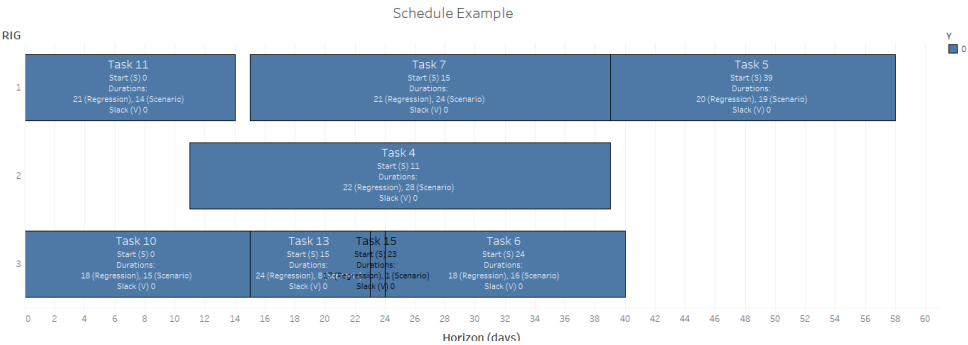


$V \text{ (slack variable)} = 0.6429$

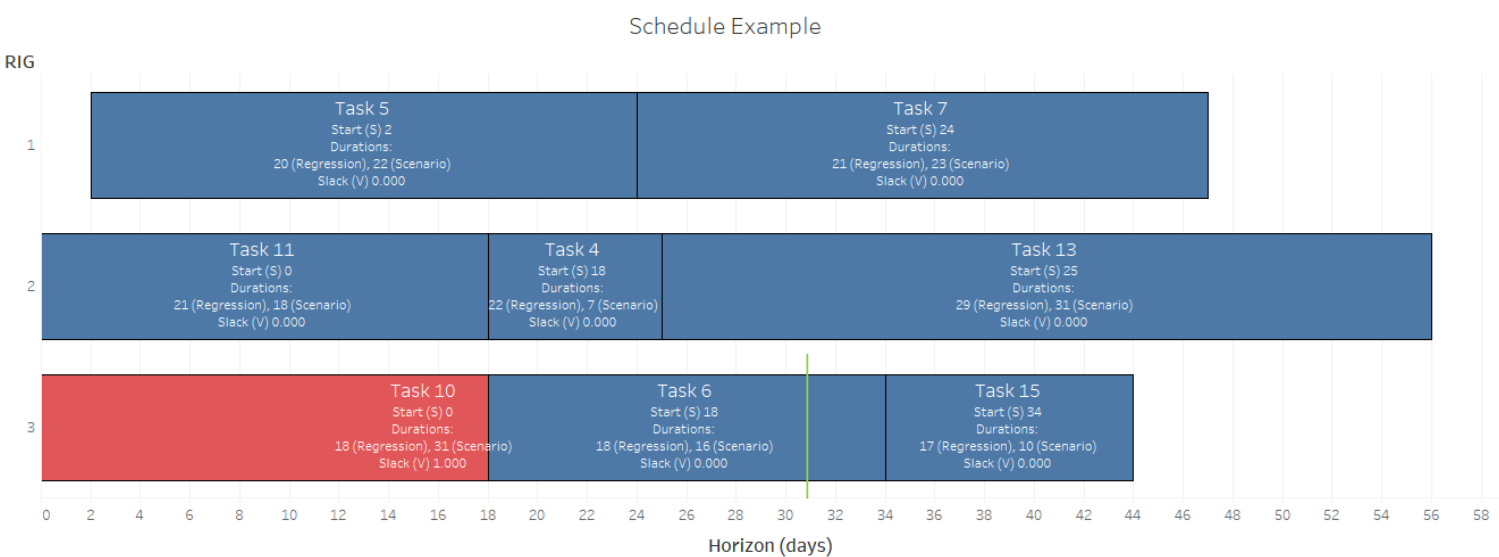
$Y \text{ (infeasible)} = 1$

# Workover Rig Scheduling Problem

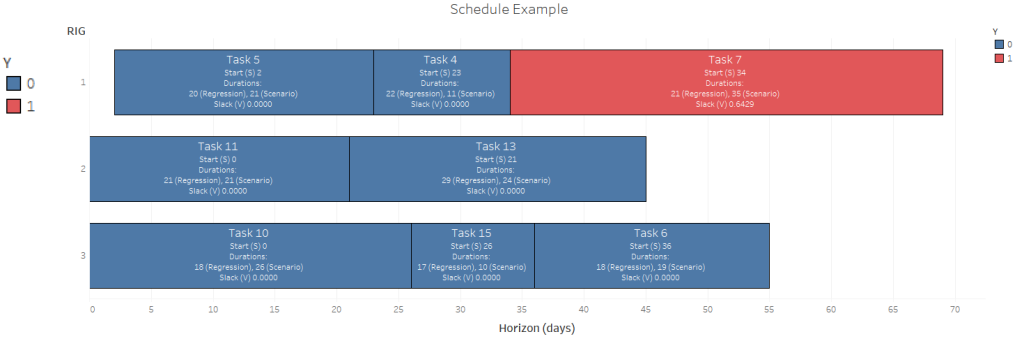
## Joint Chance-Constrained Model via Regression Model: stochastic approach



Slack variable=0



V (slack variable) = 1  
Y (infeasible) = 1

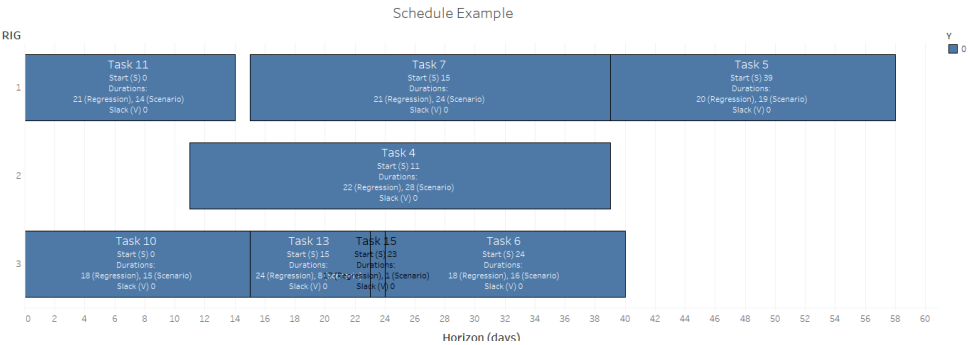


V (slack variable) = 0.6429  
Y (infeasible) = 1

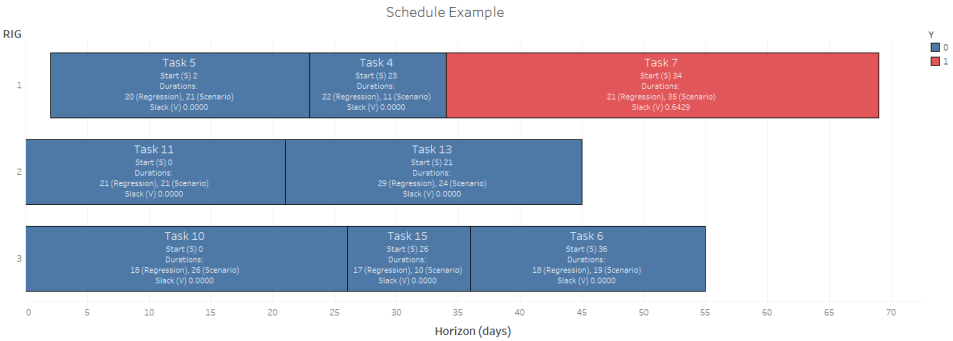
# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model: stochastic approach

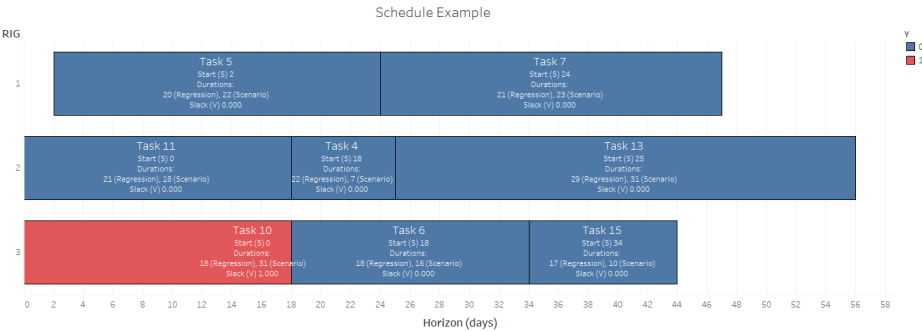
Penalties are used in the objective function to assure that  $V_{ij}^{k\omega}$  and  $Y_i^\omega$  are minimum.



Slack variable=0



V (slack variable) = 0.6429  
Y (infeasible) = 1



V (slack variable) = 1  
Y (infeasible) = 1

# Workover Rig Scheduling Problem

## Joint Chance-Constrained Model via Regression Model: stochastic approach

Model is able to optimize in reasonable time different sizes of instances with 30 scenarios.

Scenarios	Jobs	Rigs	Horizon	Seed	Density	Status	Gap	Time (s)	UB (M)	LB (M)
									111.8	
30	21	5	360	1234	0.3	OPTIMAL	0.919%	55	1	110.78
									108.0	
30	21	5	360	1234	0.7	OPTIMAL	0.954%	23	7	107.04
30	21	10	360	1234	0.3	OPTIMAL	0.000%	49	93.01	93.01
30	21	10	360	1234	0.7	OPTIMAL	0.126%	49	94.84	94.72
									271.5	
30	31	5	360	1234	0.3	OPTIMAL	0.650%	301	8	269.82
									200.0	
30	31	5	360	1234	0.7	OPTIMAL	0.955%	456	5	198.14
									176.8	
30	31	10	360	1234	0.3	TIME_LIMIT	4.793%	900	0	168.32
									161.4	
30	31	10	360	1234	0.7	OPTIMAL	0.647%	196	9	160.45

\*30 scenarios seems enough for Wasserstein Distance.

Instances x Computational efforts: More tests are still needed!



## Next steps (thesis)

### Instances Classification:

- Several instances were generated, yet they aren't grouped according to its properties.

Exhaustive experiments with these instances.

### Scenario generation and reduction:

- Monte Carlo Simulation
- Wasserstein distance

Simulation-optimization to check if confidence level is respected.



## Next steps (collaboration with Aalto)

Deterministic formulations for the WRSP.

- Several formulations were developed. Exhaustive tests with them is possible.

Branch-price-and-cut formulation that allows to solve the MINLP joint chance-constrained models.



# Future studies



Testing others data classification and prediction methods with the joint chance-constrained model.

- Neural networks, machine learning...

Simu-heuristics approaches for the problem.

Insertion of the regression optimization models in the joint chance-constrained model.

Closed-loop data-driven optimization under uncertainty.





# Future studies

Testing others data classification and prediction methods with the joint chance-constrained model.

- Neural networks and machine learning.

Simu-heuristics approaches for the problem.

Insertion of the regression optimization models in the joint chance-constrained model.

Closed-loop data-driven optimization under uncertainty.




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# Thank you

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