Pontifícia Universidade Católica
do Rio de Janeiro

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Modeling the equilibrium in the free and regulated markets in the context of power sector liberalization: A MOPEC approach

Dissertação de Mestrado

Dissertation presented to the Programa de Pós-graduação em Engenharia Elétrica do Departamento de Engenharia Elétrica of PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

Advisor: Prof. Álvaro de Lima Veiga Filho

Co-Advisor: Bernardo Vieira Bezerra

Rio de Janeiro
April 2022

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Bibliographic data

Bastos, João Pedro Thimotheo

Modeling the equilibrium in the free and regulated markets in the context of power sector liberalization: A MOPEC approach / João Pedro Thimotheo Bastos ; advisor: Álvaro Veiga de Lima Filho ; co-advisor: Bernardo Vieira Bezerra. – 2022.

167 f.: il. color. ; 30 cm

Dissertação (mestrado)–Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Elétrica, 2022.

Inclui bibliografia

1. Engenharia Elétrica – Teses. 2. Preços de energia. 3. Liberalização do mercado. 4. Modernização do setor elétrico. 5. Problema de equilíbrio. 6. MOPEC. I. Veiga Filho, Álvaro de Lima. II. Bezerra, Bernardo Vieira. III. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Engenharia Elétrica. IV. Título.

CDD: 621.3

To Leonardo Faccini Tavares Bastos and Symone Thimotheo Faccini Bastos.

Acknowledgments

To all friends from PSR, who contribute to a positive and challenging environment, and incentivize my academic enhancement.

To Mario Veiga Pereira and Luiz Augusto Barroso, for the trust and for all opportunities conceded, which allow my personal and professional development.

To Bernardo Bezerra, for the advisory, mentorship and friendship offered throughout the development of this work and in other countless occasions.

To Prof. Álvaro de Lima Veiga Filho, for the advisory and trust conceded for the development of this work.

To Sergio Granville, for all contributions given in the development of this work and for his examples of humbleness and technical excellence.

To Bruno Peixoto and Mateus Cavaliere, for the friendship, contributions and discussions throughout the development of this work.

To Ana Sofia Aranha, Alessandro Soares, Matheus Gonçalves and Rodrigo Novaes for the contributions and collaborative spirit.

To other colleagues and friends who have greatly supported me: Ana Beatriz Werlang, Gabriel Cunha, Jairo Terra, Joaquim Dias Garcia and Juliana Xavier.

To Paula Peixoto, for her support, kindness and companionship during the development of this work.

To my parents, Leonardo and Symone Bastos, for their support throughout this trajectory and all others in my live.

To ANEEL's Research and Development Program and to CTG Brasil, whose contracting contributed for the development of this work (project code: PD-03637-0318/2018).

To CAPES and PUC-Rio, for the aid conceded and for the excellent study environment.

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

Abstract

Bastos, João Pedro Thimotheo; Veiga Filho, Alvaro de Lima (Advisor); Bezerra, Bernardo Vieira (Co-Advisor). **Modeling the equilibrium in the free and regulated markets in the context of power sector liberalization: A MOPEC approach**. Rio de Janeiro, 2022. 167p. MSc. Dissertation – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

The power sector is undergoing a series of transformations to accommodate the energy transition. Among these changes, the market liberalization stands out, with greater consumer autonomy and the possibility of migration between the regulated and free markets. This context adds complexity to the task of energy distribution companies to project their future demand and to contract energy to serve the captive market, as these contracts should serve increasingly dynamic consumers and, consequently, a more unstable portfolio. Moreover, the migration of consumers to the free market may generate over-contracting of distributors, incurring extra costs for it and for the remaining captive consumers. This work proposes an optimization model to determine the equilibrium prices in the free and regulated environments in a liberalized market. Given the interdependence of agents' decisions (distributors, captive consumers, free consumers and generators), their interactions are modeled as Multiple Optimization Problems with Equilibrium Constraints (MOPEC), in which each agent's revenue maximization problems are combined into a single optimization problem, and connected by equilibrium constraints. It is demonstrated that the proposed MOPEC can be represented by a linear programming problem. The agents are modeled as risk-averse, with their individual objective functions represented as the convex combination of the expected value and the Conditional Value at Risk (CVaR) of their revenues. Among the results of the model, we highlight the optimal levels of consumer migration, contracting decisions, and contract prices in the free market and tariffs in the regulated market, for a given system configuration. In addition, different regulatory proposals are presented and modeled for the treatment of liabilities associated with the over-contracting of distributors. The model is applied in a simplified case study and another one with realistic data of the Brazilian power system. Finally, the model is integrated in an iterative process that determines the optimal system expansion, so that the resulting contracting decisions and prices generate economic signals for investments in generation capacity expansion. This methodology is applied in a case study comprising generation expansion exercises of the Brazilian power system.

Keywords

Energy prices; Market liberalization; Power sector modernization; Equilibrium problem; MOPEC.

Resumo

Bastos, João Pedro Thimotheo; Veiga Filho, Alvaro de Lima; Bezerra, Bernardo Vieira. **Modelagem do equilíbrio dos mercados livre e regulado no contexto de liberalização do setor elétrico: uma abordagem por MOPEC**. Rio de Janeiro, 2022. 167p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

O setor elétrico está passando por uma série de transformações para acomodar a transição energética. Dentre essas mudanças, destaca-se a liberalização do mercado, com maior autonomia dos consumidores e possibilidade de migração entre os mercados regulado e livre. Este contexto adiciona complexidade à tarefa das distribuidoras de energia de projetar a sua demanda futura e realizar as contratações de energia para o mercado cativo, uma vez que devem servir consumidores cada vez mais dinâmicos e, consequentemente, um portfólio mais instável. Ainda, a migração de consumidores para o mercado livre tem o potencial de gerar sobrecontratação das distribuidoras, incorrendo em sobrecustos a elas e aos consumidores cativos remanescentes. Este trabalho propõe um modelo de otimização para determinar os preços de equilíbrio para os mercados livre e regulado em um mercado liberalizado. Dada a interdependência das decisões dos agentes (distribuidora, consumidores cativos, consumidores livres e geradores), as suas interações serão modeladas como Múltiplos Problemas de Otimização com Restrições de Equilíbrio (MOPEC), em que os problemas de maximização de receitas de cada agente são combinados em um único problema de otimização, e conectados por restrições de equilíbrio. Demonstra-se ainda que o MOPEC proposto pode ser representado por um problema de programação linear. Os agentes são modelados avessos a risco, sendo suas funções objetivos individuais representadas como a combinação convexa do valor esperado e do Conditional Value at Risk (CVaR) de suas receitas. Entre os resultados do modelo, destacam-se os níveis ótimos de migração dos consumidores, decisões de contratação, e preços de contratos no mercado livre e tarifas no mercado regulado. São também apresentadas e modeladas diferentes propostas regulatórias para o tratamento dos passivos associados à sobrecontratação das distribuidoras. O modelo é aplicado em um estudo de caso simplificado e outro com dados realistas do sistema elétrico brasileiro. Finalmente, acopla-se o modelo em um processo iterativo que determina a expansão ótima do sistema, de forma que as decisões de contratação e preços resultantes geram sinais econômicos aos investimentos em expansão da capacidade de geração. Essa metodologia é aplicada em um estudo de caso, com exercícios de expansão da geração do sistema elétrico brasileiro.

Palavras-chave

Preços de energia; Liberalização do mercado; Modernização do setor elétrico; Problema de equilíbrio; MOPEC.

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1
Introduction

The power sector is an investment-intensive one, in which new generation infrastructure is built to meet a growing demand and/or replace less efficient units, being remunerated by electricity consumers. Since the wave of deregulation in power markets worldwide (which took place largely between 1980s and 2000s), such investments, once exclusive responsibility of verticalized public utilities, started to be carried out by private (and public) players in a competitive fashion. Securing remuneration for such investments has been a challenge since the deregulation – several solutions have been proposed and implemented worldwide, one of them being contracting energy in long-term commitments to stabilize generators' revenues.

On the demand side, it has been historically treated in the literature (and in market practices) with an inelastic and growing behavior (in most cases, especially the ones of developing countries). The responsibility for meeting demand commercially has been mostly of regulated retailers (activity that in most countries, such as in Brazil, is embedded in the distribution companies' responsibilities), in such a way that this demand is subject to regulated tariffs. In several markets, large consumers can purchase energy directly from generators and/or retailers (free market). In these cases, contract prices tend to be driven by the expectancy of short-term market prices (spot prices) and of the regulated tariff, which is an opportunity cost for remaining a captive (regulated) consumer instead.

This endeavor of long-term contracting, in several cases treated as an obligation, such as in Brazil, where all consumers must be 100% contracted, has been successful to enable the construction of new generation capacity [1]. The most relevant risk has usually been errors in demand growth projections that could lead to contractual misbalances entailing exposures to the spot market and/or penalties. Specific mechanisms have been designed for short-term adjustments: e.g., auctions for contracting existing generation, mechanisms for the sale of surpluses by the distributors, etc.). Moreover, in most cases, distribution companies are allowed to pass-through at least some of their cost overruns. In Brazil, for instance, distributors can pass-through their contracting costs up to a contracting level of 105%, being responsible for whatever exceedance of this threshold.

In the last years, this logic has been increasingly challenged by the advancement of the market liberalization agenda, which is an ongoing trend in several countries. The market liberalization legitimately tackles a central assumption of the original model: that demand is mostly passive and inelastic. This is driven by the empowerment of consumers, enabled by more access to information, availability of service providers and technological disruption brought by distributed energy resources [2].

1.1 Motivation

The success of the abovementioned mechanisms of long-term contracts to enable system expansion has relied on two important assumptions: (i) the regulated demand grows, or at least is not expected to shrink, otherwise it would not make sense for distributors to make long-term commitments, and (ii) such long-term contracts enable the construction of cheap infrastructure – in particular, the most economic resources would be explored first [3].

However, the technological disruption in the power sector, largely driven by the cost decline of renewable energy sources, has struck this logic [3]. As new assets are increasingly cheaper than the existing ones, the long-term contracts committed previously become less attractive in comparison, contradicting point (ii) above. In turn, this incentivizes consumers to leave such high costs of the legacy contracts and migrate to the free market to negotiate new contracts. This strikes point (i), as the sustainable growth of regulated demand is jeopardized. In addition, the situation constitutes a vicious cycle, since the more consumers leave the distribution companies due to their costlier portfolio, the more over-contracted it becomes and the worse is the situation for the remaining consumers and for the distributor itself.

In addition to the lower generation costs, on the consumption side, consumers are increasingly more independent and willing to take their own decisions that could save them costs or be more aligned with their own risk profile. This is enabled by the wider access to information, digitalization, distributed generation, among other elements, that transform once inelastic consumers in active market participants (often referred to as "prosumers").

In this context, a global trend currently is an increasing liberalization of the power market, with an increasing number of consumers willing to harness the competitiveness of the free market in an independent way. And this will be enabled by a less restricted access to the free market: in several countries, there is a discussion for reducing the limits (demand levels) to be a free consumer. In Brazil, for instance, this is currently under discussion, being proposed in the Ordinance No. 465/2019 [4], as well as in draft Bills of Law for power sector modernization [5-6].

Given the above, the task of forecasting demand growth and of designing contracting strategies, by all agents involved in the market, gains a much higher complexity. The "old way" of projecting regulated and free consumption basically independently will be increasingly fragile and inaccurate, as consumers will be more and more dynamic and may migrate between those market environments depending on their attractiveness. Thus, this work has been motivated by the need to enhance the representation of a liberalized power market that would accurately capture the relationships among agents and their interconnected actions. This enables the appropriate modeling and projection of migration levels, contracting strategies and prices in each market environment.

1.2 Objectives and contributions of this dissertation

Given the challenging context to forecast the evolution of the power market and the optimal decisions to be taken in such a dynamic environment, this work proposes a methodology to model the equilibrium (optimal migration between markets, contracting decisions, contract prices) in the free and regulated markets in a liberalized electricity sector. The problem is modeled as Multiple Optimization Problems with Equilibrium Constraints (MOPEC), in which all agents (distribution company, regulated consumers, free consumers and generators) seek to maximize their individual revenues, and these individual maximization problems are linked with equilibrium constraints, having in mind that the agents' decisions are interconnected (i.e., a contract purchased by one agent must be sold by another one). It is noted as well that, by combining the maximization of revenues of all agents in the market, the model's optimal solution leads to the maximum Social Welfare that can be obtained in such environment.

The agents' objective functions incorporate their risk profile, which is an important parameter to determine the contracting strategies by each one of them. We do so by modeling the objective functions as the convex combination between the expected value and the Conditional Value at Risk (CVaR) of their revenues.

As it will be demonstrated further, the model provides the optimal decisions of migration and contracting by agent – and the contract prices are given by the dual variable of the equilibrium constraints that balance the contracts in the regulated and free markets (leading to regulated tariffs and free contract prices, in a theoretical view).

Nonetheless, there is an issue about market liberalization that is often overlooked and must be addressed in the methodology, which are the legacy contracts already assumed by the distribution companies with some of the generators. Not only these contracts impact the relative competitiveness between both markets (in general, leading to a costlier regulated environment), but also requires further treatment and implementations in the model, which are incorporated to address this issue. Still, the mere incorporation of such contracts (e.g., as a fixed portfolio owned by the distribution companies) is not enough: a relevant discussion in the context of market liberalization is the cost allocation resulting from the distribution companies' over-contracting. That is, the distributors took the decision to celebrate long-term contracts to serve a base of consumers under a set of rules (e.g., migration thresholds) that are changing only afterwards, opening the discussion about the responsible for such cost overruns. In this sense, when incorporating legacy contracts in the model, we also model different regulatory alternatives for its cost allocations, comparing its effects in the problem's final solution.

Furthermore, the results obtained by this model (migration between markets, contracting decisions by agents, tariffs and contract prices) are applicable to a certain configuration of the power system. That is, the inputs of the model (generators' portfolios, generation by agent, electricity spot price probability distribution etc.) are, in principle, fixed. But then the following questions arise: how is such market/system configuration determined? Wouldn't the results from the model generate market signals for a different system configuration? Determining long-term generation expansion resulting from economic signals is a topic widely assessed in the literature [7, 8]. However, in most applications, the short-term market price is the main driver for the system expansion (often complemented by other sources of revenues, modeled either as economic remuneration or as constraints in the model). Nonetheless, it becomes clear that the dynamic market environment modeled and its results such as the contract prices perceived by the generators should influence their investment decisions. In this sense, this work also incorporates the proposed equilibrium model into a generation expansion iterative process, integrating it with other tools (a generation expansion model and a portfolio optimization model, both developed outside the scope of this work). In this way, we seek to determine the optimal long-term equilibrium in an electricity market, not only for a fixed system configuration, but for an optimal one, which remunerates agents according to their decisions in the equilibrium model.

Finally, another ongoing discussion in electricity markets globally relates to the ability of the energy market alone to generate the appropriate incentives for investments. Typical solutions to address this issue are payments for contributions to system reliability (often through products such as firm energy and/or firm capacity). In Brazil, contracts embed both energy itself and the asset's physical contribution to the system's reliability (known in Brazil as physical guarantee, or "*garantia física*"). Currently, it is under discussion to separate the payment for this physical attribute in a separate contract, leaving the energy contract basically as a financial hedging mechanism dissociated with the system's supply reliability, as proposed in the draft Bill of Law No. 414/2021 [6]. In this context, the iterative expansion exercise proposed will be applied under different regulatory alternatives of market products, including firm energy and firm capacity products explicitly and assessing its impacts in the system expansion.

The main contributions of this work are:

- The development of a mathematical model to represent the dynamic environment of a liberalized market, whose methodology can support the decision-making process of market agents. This includes enhancing the distributor's ability to forecast its regulated demand and support its demand declaration for regulated auctions, as well as support all agents (distributors, consumers, generators) on their contracting (and migration) decisions, including the definition of the optimal contract prices and quantities.

- The assessment of different regulatory treatments for assigning the cost overruns resulting from the distribution company's over-contracting due to their legacy contracts and the migration of regulated consumers to the free market.
- The development of a long-term generation expansion methodology that encompasses the free and regulated markets contracting dynamics.
- The assessment of regulatory alternatives of market products (energy, firm energy, firm capacity) and their impacts on the system expansion (newbuilds and associated costs).

1.3 Literature review

The literature review of this work is centered on the developments of equilibrium models, which is the focus of the main contribution presented in this dissertation.

The classic concept of market equilibrium emerged at the end of the nineteenth century with the consolidation of the neoclassical school, also known as marginalist, and was the main pillar for the development of Microeconomics. Such development has arisen from the question of what would be the optimal allocation of scarce resources that had multiple purposes. The marginal analysis was then fundamental to develop tools based on the principles of maximization that would assist in this allocation process.

Game Theory development started in 1913, when Zarmelo developed the theorem that demonstrated the existence of a strategy for games such as chess that ensured victory, or minimally a tie, for the player who took it first, regardless of what the opponent's action [9]. This simple concept was fundamental for the development of other theorems such as minimax published by Von Neumann in [10] until the publication of the famous article by John Nash in 1950 [11], where he presents the concepts of Nash equilibrium in non-cooperative games and Nash's solution in cooperative bargaining games. This publication starts the modern era of

Game Theory, with a widespread use in several fields (sociology, law, finance, economics etc.). Over the past few years, Game Theory has been used to develop different applications within the context of energy markets (as will be detailed throughout this section), one of which is the study of market equilibrium, which is the main objective of this study.

1.3.1 Equilibrium models

With the development in the last years of the optimization solvers of systems of non-linear equations/inequalities, these have been an important tool in the solution of market equilibrium problems. With the existence of a wide variety of options, a deeper understanding of existing approaches and their relevance in different contexts is needed. In this sense, the literature indicates that the main framework in which most equilibrium problem modeling is inserted is the MOPEC (Multiple optimization problems with equilibrium constraints), as presented by [12-14]. Next, some relevant equilibrium models are presented.

1.3.1.1 Walras equilibrium

In the Walras equilibrium, there is a set of consumers, firms, intermediate and final goods. Consumers seek to optimize their utilities, which are functions of the consumption of final goods and employment of labor, subject to budgetary restrictions. Firms, on the other hand, seek to optimize their profits considering their respective production functions that involve the use of intermediate goods, capital and labor, and there is a balance constraint connecting the results of these optimizations [15]. For the calculation of Walras equilibrium it is considered that the agents do not exercise market power, i.e., it is assumed that the competition between the agents is perfect. This type of equilibrium corresponds to a MOPEC and can involve the entire economy (in this case it is called General Equilibrium) or a subset of sectors (in this case it is called Partial Equilibrium).

Applications of Walras equilibrium in the energy sector are done in [16] and [17]. In the latter, a macroeconomic model CGE (Computable General Equilibrium) involving twelve sectors (eight non-energy and four energy) of the Peruvian economy was integrated into an investment and operation model for the Peruvian energy sector.

1.3.1.2
Nash equilibrium

The Nash Equilibrium proposed originally in [11] portrays a set of agents that optimize their strategy considering the strategies of two other agents. The balance is such that no agent has an incentive to move to its decision, given that all the other agents will individually make their optimal decisions. Later, the Nash equilibrium concept was extended [18] to consider constraints involving two-agent strategies. With this extension, which later became known as the Generalized Nash Equilibrium (GNEP), it was shown that, under certain conditions, the Walras Equilibrium can be obtained from the Nash Equilibrium.

Subsequently, [19] demonstrates this same result considering less restrictive conditions. Besides constraints of two-agent strategies, or the Nash equilibrium concept, it was extended to consider constraints that involve the variances of two-agent optimization problems as presented in [20], giving rise to the concept of Variational Equilibrium. [21] applies this concept in the analysis of the decentralized energy market among prosumers in a distribution system. In this work, it is demonstrated that, if this market meets the conditions of Variational Equilibrium, this balance is equal to the optimization of the total welfare of the agents, and the Nash equilibrium corresponds to a MOPEC.

In the literature, there are various applications of Nash equilibrium applied to electricity, gas, and renewables markets, such as [22-29].

1.3.1.3
Stackelberg equilibrium

In Stackelberg's Equilibrium, described in [30], there is an agent, called a leader, who plays in the first round and seeks to optimize their outcomes based on the expectation of a response from two other agents, called followers. These follower agents move sequentially and seek to optimize their preferences based on the movement of the lead agent. Mathematically, it corresponds to a two-level optimization problem where optimization of the first-level problem is a constraint or set of solutions of the second-level problem, as presented in [31]. In certain applications, the Stackelberg equilibrium is combined with the Nash equilibrium.

Problems of these types are very common in the electricity sector, mainly those that relate to the expansion of generation and transmission and the operation of two non-market agents. [23] addresses a problem of a set of large hydroelectric generators (lead agents), a set of thermals (follower agents) and system operator, where the hydroelectric generators offer energy amounts and thermal ones meet the residual load. Through their offers, the hydroelectric generators affect spot prices with the objective of maximizing their revenues. Considering the performance strategy of each hydroelectric generator that takes into consideration the offers of two other hydroelectric generators and the resulting spot price, or the system tends to a Nash equilibrium. In turn, in [28] this formulation was extended for the stochastic multi-stage context.

Other examples of applications can be found in [32] and [33], where the formulation of bi-level problems is used, aiming at optimizing strategies for the action of generators in energy exchanges; in [34] where problems involving the dynamics between generators and consumers have been addressed and some case studies have been presented to better detail such dynamics; and in [35], where an approach has been presented via problem bi-level to find the balance of expansion considering liberalized energy markets.

1.3.2 Solution methods of equilibrium problems

There are different ways to calculate the different equilibrium approaches presented in the previous section. In this context, this section briefly describes the main algorithms used to solve equilibrium problems: fixed point search, algorithms based on the solution of complementarity problems, algorithm of mixed-integer programming and optimization of Welfare.

1.3.2.1 Fixed point search

In general germs, the solution $x^*$of an equilibrium model can be characterized as $F(x^*) = 0$ for a function F e vector x^* of given prices and quantities. Defining

$f(x) = F(x) + x$, the condition $F(x^*) = 0$ is equivalent to $f(x^*) = x^*$, which means that the solution of an equilibrium model corresponds to a fixed point search.

The fixed point concept, presented in [36], was developed by Brower in 1910, giving rise to Brower's Fixed Point Theorem, when it caused the existence of fixed points for continuous functions defined on sets compact and convex of Euclidean spaces. In 1941, Kakutani established this theorem of functions for correspondences, giving rise to Kakutani's Fixed Point Theory. Both are used to prove the existence of the Nash Equilibrium [11] and in [37-38] for economic equilibrium calculation.

The attractiveness of this type of procedure is that it can be applied directly to complex problems. For example, in [28] the above algorithm was used to calculate the Nash equilibrium in a multi-stage stochastic context.

1.3.2.2 Complementarity-based algorithm

This type of algorithm is applicable in MOPEC problems that satisfy the conditions of convexity and of being continuously differentiated. The Karush-Kuhn-Tucker conditions (first-order conditions) are established for each optimization problem and the resulting system of equations/inequations containing complementarity conditions is solved by specific algorithms. In this context, it is worth highlighting the algorithm developed in [39] and subsequently improved in [40]. Another possible strategy is to use non-linear programming solvers such as the one described in [41] to directly solve the resulting system of equations/inequations containing complementarity conditions.

1.3.2.3 Mixed-integer programming algorithm

This type of algorithm, or Nash equilibrium solution space, is discretized and the resulting problem is solved with mixed-integer programming solvers such. This procedure was adopted in [23].

1.3.2.4 Welfare optimization

Under certain conditions (e.g., integrability of demand), discussed in [42, 43], the market equilibrium can be obtained through the optimization of Welfare. One of the first applications to use optimization in calculating market equilibrium was in [44]. In this type of approach, the objective is to maximize the surplus of the society or a sector, optimally allocating the quantities of each of the agents and indirectly obtaining the equilibrium price. This can be compared to a central planner that solves a single optimization problem considering the variables, constraints, and preferences of each one of the market agents. Figure 1-1 illustrates the supply and demand surpluses as well as the equilibrium price obtained in a welfare maximization problem.

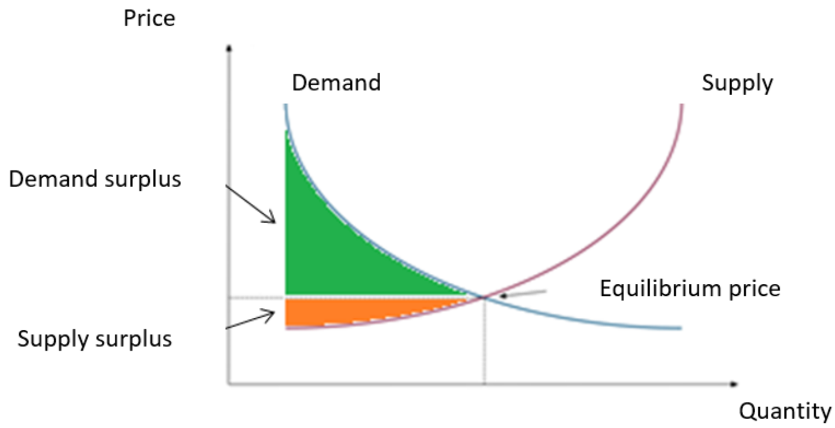

Figure 1-1 – Demand and supply surplus and equilibrium price in the Welfare maximization

Another example of the application of the Welfare optimization application in the calculation of equilibrium in energy markets was in USA's PIES energy model, documented in [45] and in [46]. In this specific case, the function demanded did not meet the integrability conditions and, therefore, a direct application of the Welfare optimization would not be possible. To circumvent this problem, the authors constructed a process of successive approximations of the demand function through an integrated demand function, with the Welfare being optimized at each iteration of the algorithm.

In a specific case in which demand is inelastic, this procedure corresponds to minimizing cost to serve the electricity market, which is usually done by centralized system operators and planners in the operation and expansion problems in power systems.

More recently, [47] adopted the same Welfare maximization procedure for the calculation of market equilibrium for electricity contracts. In this work, the Welfare was defined as the sum of risk-adjusted net revenues of the involved agents (generators and demand) – the present work uses a similar approach.

1.3.3 Conclusion and approach adopted

From what has been explained in the previous sections, it is clear the importance of Game Theory in the development of tools to represent the different dynamics existing in energy markets. In this context, the literature suggests that there are different approaches to represent similar problems, each one with its own advantages and disadvantages. However, it is highlighted that the approach to market equilibrium problems most used in the most recent works is the one inserted in the MOPEC framework.

Therefore, this work uses the MOPEC approach to represent the power sector dynamics in the Regulated Contracting Environment the Free Contracting Environment, including contracting and migration decisions, using the Welfare optimization as its solution method (this is addressed in detail in Chapter 2 and in Annex A).

1.4 Structure of the dissertation

The following chapters of this dissertations are organized as follows: Chapter 2 describes the problem of determining an optimal equilibrium between the free and regulated markets in a liberalized environment as well as the proposed methodology for doing so – through a Multiple Optimization Problems with Equilibrium Constraints (MOPEC) approach.

Chapter 3 presents case studies in which the market equilibrium model is applied for simplified systems. Using the outcomes of dispatch simulations as inputs, outputs such as contract prices in the free market, regulated tariffs and migration between markets are projected.

Chapter 4 presents an extension of the theoretical methodology presented in Chapter 2 – in particular, we extend the problem's formulation for the consideration of legacy contracts in the distributors' portfolios, which increases the realism of the problem and its applicability for the practical issues it is seeking to solve.

Chapter 5 presents a case study for the application of the extended methodology presented in Chapter 4, for a realistic configuration of the Brazilian electricity market. As outcomes, we present not only the ones mentioned in the case of Chapter 3 (migration, contracting, prices etc.), but we also propose and assess different approaches for allocating the costs associated with legacy contracts, which is a central issue in the context of market liberalization.

Chapter 6 addresses the optimal generation expansion problem. In particular, it is proposed an iterative methodology that incorporates: (i) the investor's perspectives into the expansion solution; and (ii) the outcomes of the market equilibrium model into the expansion exercise. To do so, we assure that the new entrants should be adequately remunerated, according to their risk profile, and taking into account the market conditions simulated by the equilibrium model.

Chapter 7 explores a generation expansion exercise by applying the methodology proposed in Chapter 6 for the Brazilian power system. Firstly, we incorporate information that are usually not encompassed by expansion models but that are ultimately perceived by investors in the expansion procedure (such as price floor and ceiling, possibility to celebrate contracts, risk aversion by investor) and assess the respective impacts in the system expansion. Finally, we incorporate the market equilibrium model proposed in this dissertation in the expansion approach and present expansion cases of the Brazilian power system taking into account the free and regulated market dynamics. The exercise is carried out for different regulatory assumptions for contracting mechanisms: (i) the current one, in which the product negotiated is an energy contract backed-up by firm energy ("physical guarantee"); (ii) separated contracts for energy and for firm energy; and (iii) separated contracts for energy, firm energy and firm capacity. The outcomes for these cases are assessed and compared.

Chapter 8 presents the conclusions and proposals for future works. Chapter 9 presents all the bibliographic references utilized throughout this dissertation. In addition, Chapter 10 (Annex A) complements the mathematical formulation presented in Chapter 2.

2
Problem's description

The main problem explored in this dissertation relates to the determination of an optimal equilibrium between the free and regulated electricity market environments in the context of a liberalized power market. This chapter presents the methodological approach for representing this problem and obtaining its solution.

2.1
Contextualization

In theory and most practical applications, electricity demand has been treated with a passive role, with its growth, consumption profile and participation at each market environment being a fixed input in long-term planning studies. In countries such as Brazil and many others, the regulated demand has been the major expansion driver, usually enabled by energy auctions to select the most competitive supply candidates and remunerate them in a stable fashion. The free market has been a separate environment, in which consumers large and organized enough (large industrial and commercial clients) can negotiate and contract their energy. Thus, there has been a clear division between the clients who could and who could not participate in the free market environment, usually set by a demand threshold high enough that would enable the participation of large companies only.

With the market liberalization agenda and the empowerment of consumers, the threshold of participation in the free market is expected to decrease in the next years, increasing competition not only in the free market but also between the contracting environments, as the decision of which market to participate will increasingly depend on economic incentives that each one can offer.

Thus, the projection of demand and market prices at each environment in a separate fashion becomes less and less accurate, as the price outcomes at each market generates economic incentives for migration and different arrangements, as the channel between the market environments and access to information are sharply increasing.

In this context, the problem that this dissertation intends to solve is precisely how to model this dynamic market environment, with possibility of transition of consumers between markets, and determine the equilibrium resulting from the economic incentives to which each market participant (regulated and free consumers, distribution company, generators) is subject. The expected results are the final participation of the regulated and of the free markets in total demand (optimal migration for each consumer), the contracting strategies of each agent and regulated tariff and free contract prices, taking into account that each of the agents' decision impact the supply-demand balance of each market and contract product and, thus, the final solution is an equilibrium of all agents' inter-dependent actions.

2.2 Proposed methodology

This section describes the modeling proposal for determining the equilibrium between the free and regulated markets in the context of market liberalization.

2.2.1 General view and definitions

Essentially, for modeling market equilibrium problems, one should define: who are the agents participating in such market, how is their interaction and other characteristics of the system in which they are inserted.

In this sense, the proposed methodology considers that the following agents are part of the market:

- Distribution company;
- Generators;
- Captive consumers, which are sub-divided into two groups:
    - Captive consumers type I, which cannot migrate between markets
    - Captive consumers type II, which can migrate between markets
- Free consumers.

In essence, captive consumers type I are the ones that are necessarily regulated (e.g. due to their small size, it may not make sense for most of them to participate actively in the market, and/or authorities may stipulate market limits for the participation in the free market. Captive consumers type II have the same rights of migrating between markets as the free ones – with the difference between them being their initial condition in the problem. It should also be highlighted that, in the proposed model, the distribution companies can be seen as last resource suppliers and all consumers may opt to purchase energy from them.

As mentioned, the interaction between agents is key for the construction of the model. In this sense, in the proposed model:

- Clients in the free market purchase energy through bilateral (forward) contracts, with varied prices and intervals.
- Distribution company purchases directly from generators, also through bilateral contracts.
- Captive consumers purchase energy from the distribution company, with a fixed tariff over the period of one year.

Finally, it is important to characterize the system in which these agents are inserted. In the proposed methodology, the equilibrium between the free and regulated markets will be studies considering that these agents are inserted in an existing system (at this point), in such a way that the following information is given:

- The physical power system configuration is already determined, in such a way that there is enough supply to meet the total demand.
- The operation of this system (dispatch) is carried out seeking to minimize operation costs, with no influence of contractual positions or any other commercial conditions of the agents other their operation costs as perceived by the system operator.

Having characterized the agents, their interactions, and the main characteristics of the system in which they are inserted, the next section presents the proposed formulation for the problem.

2.2.2 MOPEC formulation

This section presents the formulation proposed for the dynamics of the migration between the free and the regulated market environments. As thoroughly discussed in the literature review (section 1.3), a widely method in literature for equilibrium problems is Multiple Optimization Problems with Equilibrium Constraints (MOPEC).

For building the MOPEC, it is necessary to define the optimization problems of each one of the agents and the equilibrium constraints. However, as mentioned in the previous section, the agents studied have particular features, thus, it is needed to characterize individually the problem associated with each of them. These individual problems are described below, and, after these, the equilibrium constraints are also presented. The notations used in this chapter denote one agent per category for didactical reasons – however, the implementation allows the representation of several agents per category.

2.2.2.1 Optimization problem of the Distributor

The Distributor (Disco)'s optimization problem determines the energy amounts purchased from generators that maximize the net present value of their revenues, adjusted to its risk profile (R^D). Regarding sales, the Disco must meet its demand (d_t^D) at all stages t. The adjustment to the company's risk profile is done through the convex combination of the expected value and the Conditional Value at Risk (CVaR) of revenues in the problem's objective function (this approach is adopted for all agents). Revenues are stochastic because the agents are exposed to the system's spot price $\pi_{t,s}$, a stochastic variable, for contractual imbalances. Mathematically, this problem can be described as follows. The decision variable is the contractual amount purchased from generators q_t^D.

$$\underset{q_t^D}{\text{Maximize}}\, R^D \tag{2.1}$$

s.t.

$$R^D = \lambda^D \times E\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] + (1-\lambda^D) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] \tag{2.2}$$

$$R_{t,s}^D = \left(p^R - \pi_{t,s}\right) \times d_t^D - \left(p^C - \pi_{t,s}\right) \times q_t^D, \qquad \forall t, s \tag{2.3}$$

Where:

R^D: Present value of the Disco's risk-adjusted revenues

$R_{t,s}^D$: Disco's revenue at stage t, scenario s

λ^D: Weight given to the expected value of revenues in the Disco's optimization problem (in p.u.)

d_t^D: Disco's demand at stage t

q_t^D: Contractual amount purchased by the Disco

p^R: Price of the regulated tariff charged by the Disco at stage t

$\pi_{t,s}$: Spot price at stage t, scenario s

p^C: Price of contracts bilaterally negotiated, valid for time interval k_m.

2.2.2.2 Optimization problem of the Captive Consumer Type I

The Captive Consumer Type I cannot choose its energy supplier, and, therefore, purchases its energy mandatorily form the Disco. Thus, there is no problem of optimal allocation decision for this agent. In consequence, the net revenue (cost) of Captive Consumer Type I is not stochastic. Mathematically, it can be described as:

$$R^{CI} = \sum_t \frac{R_t^{CI}}{(1+r)^t} \tag{2.4}$$

$$R_t^{CI} = -p^R \times d_t^{CI}, \ \forall t \tag{2.5}$$

Where:

R^{CI}: Present value of the risk-adjusted revenues of captive consumer type I.

R_t^{CI}: Value of revenue of captive consumer type I at stage t, scenario s

d_t^{CI}: Demand of captive consumer type I at stage t

2.2.2.3 Optimization problem of the Captive Consumer Type II

From the prices of energy sales by distribution companies in the regulated market and the ones offered by generators in the free market, the optimization problem of Captive Consumer Type II seeks to determine the portion of its demand that should migrate to the free market (m_t^{CII}) and the contractual amounts to be celebrated (q_t^{CII}) in such a way to maximize the present value of its net revenues, adjusted to their risk profile (R^{CII}). Mathematically, this problem can be described as:

$$\underset{m_t^{CII},\, q_t^{CII}}{\text{Maximize}}\ R^{CII} \tag{2.6}$$

s.t.

$$R^{CII} = \lambda^{CII} \times E\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] + (1-\lambda^{CII}) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] \tag{2.7}$$

$$R_{t,s}^{CII} = -p^R \times\ (d_t^{CII} - m_t^{CII}) - p^C \times q_t^{CII} +\ \pi_{t,s} \times (q_t^{CII} - m_t^{CII}),\ \ \forall t, s \tag{2.8}$$

$$m_t^{CII} \leq d_t^{CII}, \qquad \forall t \tag{2.9}$$

Where:

R^{CII}: Present value of the revenues (risk adjusted) for captive consumer type II.

$R_{t,s}^{CII}$: Value of revenue of captive consumer type II at stage t, scenario s

λ^{CII}: Weight given to the expected value of revenues in the captive consumer type II optimization problem (in p.u.)

d_t^{CII}: Demand of captive consumer type II at stage t.

m_t^{CII}: Amount of captive consumer type II demand that migrates to the free market at stage t.

q_t^{CII}: Amount of energy contracted by captive consumer type II in the free market

The decision variables of the problem are the amount of load that migrates to the free market (m_t^{CII}) and the contractual amounts celebrated in the free market (q_t^{CII}).

Constraint (2.8) establishes that the net revenue of Captive Consumer II is composed by: (i) costs arisen by energy purchases from the Disco (associated with its share of demand that has not migrated to the free market); (ii) costs arisen by contracts purchased from generators (associated with its share of demand that has migrated to the free market); and (iii) cost/revenue associated with settlements of contractual imbalances in the short-term market at the spot price.

Constraint (2.9) establishes that the share of load that can migrate to the free market is smaller than the consumer's total load at each stage.

2.2.2.4 Optimization problem of the Free Consumer

From the prices of energy sales by distribution companies in the regulated market and the ones offered by generators in the free market, the optimization problem of Free Consumers seeks to determine the contractual amounts to be celebrated in the free market (q_t^L) and the portion of their demand that should migrate to the regulated market (m_t^L), in such a way to maximize the present value of its net revenues, adjusted to their risk profile (R^L). Mathematically, this problem can be described as:

$$\underset{m_t^L,\, q_t^L}{\text{Maximize}}\; R^L \tag{2.10}$$

s.t.

$$R^L = \lambda^L \times E\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] + (1-\lambda^L) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] \tag{2.11}$$

$$R_{t,s}^L = -p^R \times m_t^L - p^C \times q_t^L + \pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right), \qquad \forall t, s \tag{2.12}$$

$$m_t^L \leq d_t^L, \qquad \forall t \tag{2.13}$$

Where:

R^L: Present value of the free consumer's risk-adjusted revenues

$R_{t,s}^L$: Free consumer's revenue at stage t, scenario s

λ^L: Weight given to the expected value of revenues in the free consumer's optimization problem (in p.u.)

d_t^L: Demand of free consumer at stage t.

m_t^L: Amount of free consumer's demand that migrates to the regulated market at stage t.

q_t^L: Amount of energy contracted by the free consumer in the free market.

The decision variables of the problem are the amount of load that migrates to the regulated market (m_t^L) and the contractual amounts celebrated in the free market (q_t^L).

Constraint (2.12) establishes that the net revenue of the Free Consumer is composed by: (i) costs arisen by energy purchases from the Disco (associated with its share of demand that has migrated to the regulated market); (ii) costs arisen by contracts purchased from generators (associated with its share of demand that has remained in the free market); and (iii) cost/revenue associated with settlements of contractual imbalances in the short-term market at the spot price.

Constraint (2.13) establishes that the share of load that can migrate to the regulated market is smaller than the consumer's total load at each stage.

2.2.2.5 Optimization problem of the Generator

The optimization problem of the generator seeks to determine the contractual amounts for energy sales that maximize the risk-adjusted expected value of their revenues. Mathematically, this problem can be described as:

$$\underset{q_t^G}{\text{Maximize}}\, R^G \tag{2.14}$$

s.t.

$$R^G = \lambda^G \times E\left[\sum_t \frac{R_{t,s}^G}{(1+r)^t}\right] + (1-\lambda^G) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^G}{(1+r)^t}\right] \tag{2.15}$$

$$R_{t,s}^G = \pi_{t,s} \times g_{t,s} + (p - \pi_{t,s}) \times q_t^G, \qquad \forall t, s \tag{2.16}$$

Where:

R^G: Present value of the generator's risk-adjusted revenues

$R_{t,s}^G$: Generator's revenue at stage t, scenario s

λ^G: Weight given to the expected value of revenues in the generator's optimization problem (in p.u.)

$g_{t,s}$: Generation of generator k_G's portfolio at stage t, scenario s.

q_t^G: Amount of energy sold in contract by the generator.

Problem (2.14-2.16) has the following decision variable: amounts of energy sales in contracts by the generator (q_t^G).

2.2.2.6 Equilibrium constraints

The first equilibrium constraint concerns balancing the purchase and sale contracts of the distributor. Thus, the total amount of energy sold by the distributor must be equal to the sum of the amounts of energy purchased from the distributor by the Captive Consumers Type I, by the load share of the Captive Consumers Type II that remains in the regulated market and by the load share of the Free Consumers migrated to the regulated market. Mathematically:

$$d_t^D = d_t^{CI} + d_t^{CII} - m_t^{CII} + m_t^L, \qquad \forall t \tag{2.17}$$

The second equilibrium constraint relates to the balance of energy contracts in the (free) market. Thus, the amount of energy sold in contracts by the generator at a given validity period is equal to the sum of the amounts purchased by the Disco, by the Captive Consumers Type II that migrate to the free market and by the Free consumers that remained in the free market. Mathematically:

$$q_t^G = q_t^D + q_t^{CII} + q_t^L, \qquad \forall t \tag{2.18}$$

2.2.3 MOPEC solution

The MOPEC defined in the previous section can be rewritten by establishing the Karush–Kuhn–Tucker (KKT) conditions of each agent's optimization problem, which would lead to a system of non-linear equations/inequations containing the complementarity conditions, along with the equilibrium constraints, and solved using specific algorithms.

However, as discussed in the literature review, under certain conditions (e.g. demand integrability), the MOPEC solution can be obtained through Welfare optimization, which can be compared to a central planner that solves a single optimization problem considering the variables, constraints, and preferences of each one of the market agents.

In Annex A, it is formally demonstrated that the solution of the MOPEC defined in the previous section can be obtained, under certain conditions, through Welfare maximization. The objective function of the Welfare maximization is the sum of the risk-adjusted net revenues of each agent and the constraints correspond to the set of constraints of the individual optimization problem of each agent. Mathematically, this problem is described as follows:

$$\underset{\substack{q_t^D, m_t^{CII}, q_t^{CII} \\ m_t^L, q_t^L, q_t^G}}{\text{Maximize}} \; R^D + R^{CI} + R^{CII} + R^L + R^G \tag{2.19}$$

s.t.

$$R^D = \lambda^D \times E\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] + (1-\lambda^D) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] \tag{2.20}$$

$$R_{t,s}^D = \left(p^R - \pi_{t,s}\right) \times d_t^D - \left(p^C - \pi_{s,t}\right) \times q_t^D, \qquad \forall t, s \tag{2.21}$$

$$R^{CI} = \sum_t \frac{R_t^{CI}}{(1+r)^t} \tag{2.22}$$

$$R_t^{CI} = -p^R \times d_t^{CI}, \ \forall t \tag{2.23}$$

$$R^{CII} = \lambda^{CII} \times E\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] + (1-\lambda^{CII}) \times CVaR_\alpha \left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] \tag{2.24}$$

$$R_{s,t}^{CII} = -p^R \times \ (d_t^{CII} - m_t^{CII}) - p^C \times q_t^{CII} + \ \pi_{t,s} \times (q_t^{CII} - m_t^{CII}), \ \ \forall t, s \tag{2.25}$$

$$m_t^{CII} \leq d_t^{CII}, \qquad \forall t \tag{2.26}$$

$$R^{L} = \lambda^{L} \times E\left[\sum_t \frac{R_{t,s}^{L}}{(1+r)^t}\right] + (1-\lambda^{L}) \times CVaR_\alpha \left[\sum_t \frac{R_{t,s}^{L}}{(1+r)^t}\right] \tag{2.27}$$

$$R_{t,s}^{L} = -p^R \times m_t^L - p^C \times q_t^L + \pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right), \qquad \forall t, s \tag{2.28}$$

$$m_t^L \leq d_t^L, \qquad \forall t \tag{2.29}$$

$$R^{G} = \lambda^{G} \times E\left[\sum_t \frac{R_{t,s}^{G}}{(1+r)^t}\right] + (1-\lambda^{G}) \times CVaR_\alpha \left[\sum_t \frac{R_{t,s}^{G}}{(1+r)^t}\right] \tag{2.30}$$

$$R_{t,s}^{G} = \pi_{t,s} \times g_{t,s} + \left(p - \pi_{t,s}\right) \times q_t^G, \qquad \forall t, s \tag{2.31}$$

$$d_t^D = d_t^{CI} + d_t^{CII} - m_t^{CII} + m_t^L, \qquad \forall t \tag{2.32}$$

$$q_t^G = q_t^D + q_t^{CII} + q_t^L, \qquad \forall t \tag{2.33}$$

In the problem's optimal solution, the tariff in the regulated market and the contract prices in the free market can be obtained by the dual variables corresponding to the equilibrium constraints (2.32) and (2.33), respectively. This is intuitive, as they represent the incremental cost in increasing the contractual amounts (in the regulated or free market, respectively). Still, the calculation of the regulated tariff will be sophisticated further in this work (see Chapter 4).

The problem described in equations (2.19-2.33) is, in principle, non-linear, due to the multiplication of prices and quantities. However, it is possible to demonstrate that, under certain conditions – in particular, considering that the contract and tariff prices do not vary by scenario, the non-linear terms cancel out and the problem is reduced to a linear programming problem (see below for more details).

Thus, it is possible and desirable to separate the deterministic and stochastic components of each group of agents' revenues – so that the deterministic ones are eventually canceled out. This is done next.

2.2.3.1
Treatment of the Disco's problem

Separating the deterministic and stochastic components, the expression for the Disco's revenues can be rewritten as:

$$R_{t,s}^{D} = p^{R} \times d_{t}^{D} - p^{C} \times q_{t}^{D} - \pi_{t,s} \times d_{t}^{D} + \pi_{t,s} \times q_{t}^{D}, \qquad \forall t, s \tag{2.34}$$

Note that the first two products are deterministic (do not depend on the scenario s), while the last two are stochastic. Thus, the deterministic part can be separated from the stochastic one on the expression of the net present value of the Disco's revenues, which can be rewritten as:

$$\begin{aligned} R^{D} = & \sum_{t} \frac{p^{R} \times d_{t}^{D} - p^{C} \times q_{t}^{D}}{(1+r)^{t}} \\ & + \lambda^{D} \times E\left[\sum_{t} \frac{\pi_{t,s} \times (q_{t}^{D} - d_{t}^{D})}{(1+r)^{t}}\right] \\ & + (1-\lambda^{D}) \times CVaR_{\alpha}\left[\sum_{t} \frac{\pi_{t,s} \times (q_{t}^{D} - d_{t}^{D})}{(1+r)^{t}}\right] \end{aligned} \tag{2.35}$$

That is, once the prices and quantities associated with the regulated sales and bilateral purchases do not vary according to each scenario, the non-stochastic components of the Disco's net revenues and be separated from the expression that involves the linear combination between the Expected Value and CVaR (the stochastic one).

2.2.3.2
Treatment of the Captive Consumer Type I problem

In this case, there is no stochastic component, thus the revenue expression of these agents remains as:

$$R^{CI} = -\sum_{t} \frac{p^{R} \times d_{t}^{CI}}{(1+r)^{t}} \tag{2.36}$$

2.2.3.3
Treatment of the Captive Consumer Type II problem

Using the same procedure adopted for the separation of the deterministic and stochastic components of the Disco, we obtain the following for the Captive Consumers Type II:

$$\begin{aligned} R^{CII} = & \left[\sum_t \frac{-p^R \times \left(d_t^{CII} - m_t^{CII}\right) - p^C \times q_t^{CII}}{(1+r)^t}\right] \\ & + \lambda^{CII} \times E\left[\sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] \\ & + \left(1 - \lambda^{CII}\right) \times CVaR_\alpha\left[\sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] \end{aligned} \quad (2.37)$$

2.2.3.4
Treatment of the Free Consumer's problem

According to the same procedure applied previously, the revenues of the free consumer can be written as:

$$\begin{aligned} R^L = & \sum_t \frac{-p^R \times m_t^L - p^C \times q_t^L}{(1+r)^t} \\ & + \lambda^L \times E\left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - \left(d_t^L - m_t^L\right)\right)}{(1+r)^t}\right] \\ & + \left(1 - \lambda^L\right) \times CVaR_\alpha\left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - \left(d_t^L - m_t^L\right)\right)}{(1+r)^t}\right] \end{aligned} \quad (2.38)$$

2.2.3.5
Treatment of the Generator's problem

Analogously, the revenues of the generator can be written as:

$$\begin{aligned} R^G = & \sum_t \frac{p^C \times q_t^G}{(1+r)^t} + \lambda^G \times E\left[\sum_t \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t}\right] \\ & + \left(1 - \lambda^G\right) \times CVaR_\alpha\left[\sum_t \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t}\right] \end{aligned} \quad (2.39)$$

2.2.3.6 Sum of the components of the Social Welfare objective function

Combining the components described in the previous sub-sections, the objective function of the Social Welfare problem can be written as follows:

$$
\begin{aligned}
\underset{\substack{q_t^D, m_t^{CII}, q_t^{CII} \\ m_t^L, q_t^L, q_t^G}}{\text{Maximize}} \; & \sum_t \frac{p^R \times d_t^D - p^C \times q_t^D}{(1+r)^t} + \sum_t \frac{-p^R \times d_t^{CI}}{(1+r)^t} \\
& + \left[\sum_t \frac{-p^R \times (d_t^{CII} - m_t^{CII}) - p^C \times q_t^{CII}}{(1+r)^t} \right] \\
& + \sum_t \frac{-p^R \times m_t^L - p^C \times q_t^L}{(1+r)^t} + \sum_t \frac{p^C \times q_t^G}{(1+r)^t} \\
& + \lambda^D \times E\left[\sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} \right] \\
& + (1-\lambda^D) \times CVaR_\alpha \left[\sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} \right] \\
& + \lambda^{CII} \times E\left[\sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} \right] \\
& + (1-\lambda^{CII}) \times CVaR_\alpha \left[\sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} \right] \\
& + \lambda^L \times E\left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t} \right] \\
& + (1-\lambda^L) \times CVaR_\alpha \left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t} \right] \\
& + \lambda^G \times E\left[\sum_t \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t} \right] \\
& + (1-\lambda^G) \times CVaR_\alpha \left[\sum_t \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t} \right]
\end{aligned}
\tag{2.40}
$$

Still, it can be observed that, given the equilibrium constraints presented previously though the equations (2.32) and (2.33), the terms that multiply prices and quantities (denoted in the first part of the equation above) cancel out, as the quantities sold by some parties are equivalent to the ones purchased by the others.

Thus, canceling out the non-stochastic terms, using Rockafellar's representation for the CVaR [48], the Social Welfare optimization problem can be rewritten as:

$$
\begin{aligned}
\underset{\substack{q_t^D, m_t^{CII}, q_t^{CII} \\ m_t^L, q_t^L, q_t^G}}{\text{Maximize}} \quad & \lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] \\
& + (1-\lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] \\
& + \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] \\
& + (1-\lambda^{CII}) \times \left[a^{CII} + \frac{\sum_s y_s^{CII}}{S \times (1-\alpha)}\right] \\
& + \lambda^L \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] \\
& + (1-\lambda^L) \times \left[a^L + \frac{\sum_s y_s^L}{S \times (1-\alpha)}\right] \\
& + \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t}\right] \\
& + (1-\lambda^G) \times \left[a^G + \frac{\sum_s y_s^G}{S \times (1-\alpha)}\right]
\end{aligned}
\tag{2.41}
$$

s.t.

	dual variables	
$d_t^D \geq 0$	θ_t^D	(2.42)
$q_t^D \geq 0$	β_t^D	(2.43)
$y_s^D \leq 0$	γ_s^D	(2.44)
$y_s^D - \sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} + a^D \leq 0$	η_s^D	(2.45)
$m_t^{CII} \geq 0$	θ_t^{CII}	(2.46)
$m_t^{CII} \leq d_t^{CII}$	σ_t^{CII}	(2.47)
$q_t^{CII} \geq 0$	β_t^{CII}	(2.48)
$y_s^{CII} \leq 0$	γ_s^{CII}	(2.49)
$y_s^{CII} - \sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} + a^{CII} \leq 0$	η_s^{CII}	(2.50)

$$m_t^L \geq 0 \qquad \theta_t^L \tag{2.51}$$

$$m_t^L \leq d_t^L \qquad \sigma_t^L \tag{2.52}$$

$$q_t^L \geq 0 \qquad \theta_t^L \tag{2.53}$$

$$y_s^L \leq 0 \qquad \gamma_s^L \tag{2.54}$$

$$y_s^L - \sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t} + a^L \leq 0 \qquad \eta_s^L \tag{2.55}$$

$$q_t^G \geq 0 \qquad \beta_t^G \tag{2.56}$$

$$y_s^G \leq 0 \qquad \gamma_s^G \tag{2.57}$$

$$y_s^G - \sum_t \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t} + a^G \leq 0 \qquad \eta_s^G \tag{2.58}$$

$$d_t^D + m_t^{CII} - m_t^L = d_t^{CI} + d_t^{CII} \qquad \xi_t^R \tag{2.59}$$

$$q_t^G - q_t^D - q_t^{CII} - q_t^L = 0 \qquad \tau_t^C \tag{2.60}$$

3
Case study of the equilibrium model for simplified systems

In this chapter, the methodology presented in the previous one is applied to simplified systems – that is, configurations with limited number of agents. The goal of this application is to assess and interpret the functioning and results obtained by the model – which proved to be extremely useful to do prior to shifting to larger applications.

In the following sections, we briefly describe the implementation process, the case studies' configurations, the simulations carried out, and the results obtained.

3.1 Implementation in Julia language

The problem presented in Chapter 2 (denoted by equation 15 and its constraints) was implemented using Julia, an open source programming language widely used for mathematical programming and optimization problems due to its high efficiency [49]. Julia has as its main features: (i) dynamic typing, (ii) performance comparable to the traditional static languages; and (iii) just-in-time compiling. Additionally, this language counts with the JuMP package, which facilitates the specific modeling for mathematical programming, and supports several open source and commercial solvers.

3.2 Data description of the simplified case study

For the simplified case study, the agents were defined for a small hypothetical market, assigning features such as data on demand and generation, and risk profiles. The defined agents are described below:

- 1 Distribution company: represents 70% of total demand in t=0.
- 1 Captive Consumer Type I (cannot migrate): represents 40% of total demand

- 3 Captive Consumers Type II (can migrate): each of which represents 10% of the total demand
- 3 Free Consumers: each of which represents 10% of the total demand
- 1 or 2 Generators: two different system configurations are presented, as explained in more detail next.

For this study, we adopted a 1-year horizon, to simulate the static behavior of the system in an equilibrium situation. The source of generation and price data is a simulation from the Brazilian power system in year 2030, with 200 scenarios. For the purposes of this simplified case study, the detailed assumptions and modeling of this simulation will be suppressed at this point.

With respect to the contracts available in this market, one contracting window was considered, with 1-year contracts, which can be signed by agents at t=0. The choice for one contract window was made to help interpret the model's results and intuitions.

Regarding the generators, it is considered a 100% hydroelectric system, composed by one generator whose physical guarantee[1] is equal to the system's demand. In a second step, it is considered a 50% hydroelectric and 50% wind system, whose sum of physical guarantees equals demand.

The generation data (average and quantiles of the stochastic simulation) of each agent are presented next, as a percentage of their physical guarantee. The hydroelectric generation profile used is the one resulting from the combination of hydro plants in Brazil (in particular, the ones that integrate the Energy Reallocation Mechanism – MRE, for its Portuguese acronym). The wind power profile is one typical from the Brazilian Northeastern region. Once more, this is not critical for the purposes of the simplified case study but may help a reader with previous experience and intuitions about the Brazilian market to interpret some of the results obtained.

[1] Amount of energy that can be generated under scarcity conditions in one year by one generator, as defined in the Brazilian regulation.

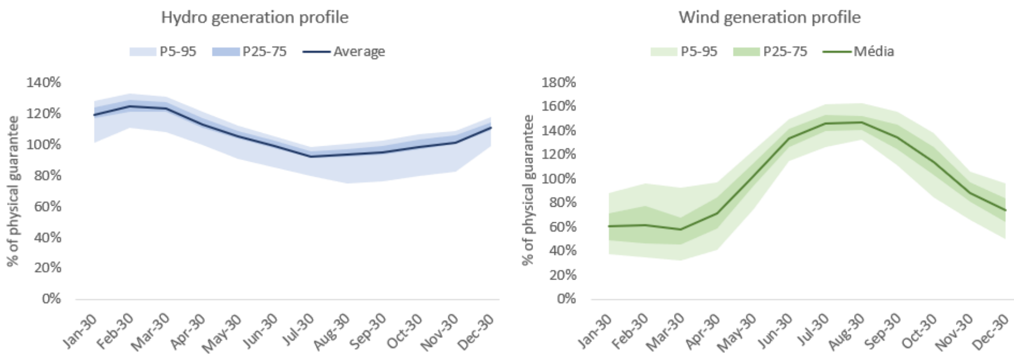

Figure 3-1 – Generation profile of the hydro and wind agents

In a first case, the system's demand is equal to the hydro's physical guarantee, and in a second one, hydro and wind contribute with 50% of the system's physical guarantee. As the generation and demand monthly profiles do not match, it is emphasized, without loss of generality, that in no case the system is physically isolated or self-sufficient, that is. This means that, physically, these agents can consume/generate from/to other non-modeled agents (that is, it is possible that there are physical exchanges of this system with others, valued at the short-term price). However, commercially, the only contracting option is the one with the represented agents. The figure below shows the configuration of the supply-demand balance in the 100% hydro case and in the hydro/wind case.

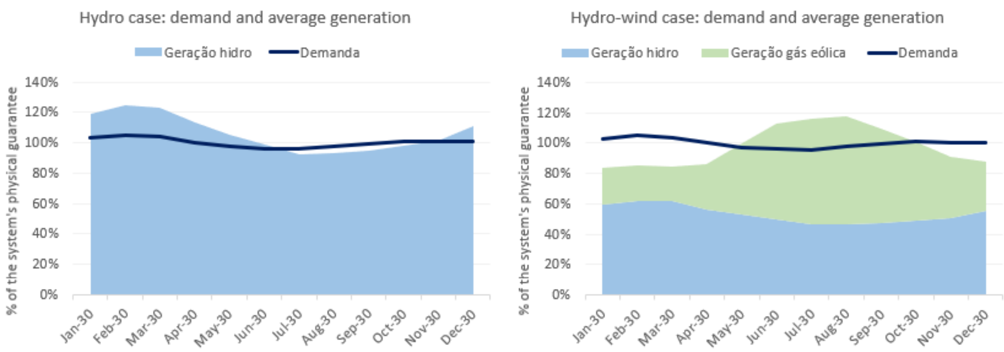

Figure 3-2 – Monthly average generation and demand in each of the two cases

For demand, a typical annual profile of the Brazilian system was considered, with higher values in the summer months. Finally, it should be noted once more that what was equaled in both cases was the physical guarantee of the system's generators and demand. While the average wind generation was practically equal to the plant's physical guarantee, the average hydroelectric generation was about 5% higher than the physical guarantee (which explains the visual excess of generation with respect to demand in the 100 % hydro graph).

Finally, the short-term prices to which all agents are subject are a stochastic variable, which varies in the same 200 generation scenarios (they come from the same simulation of the Brazilian power system) – this is highly desirable, as generation and prices are correlated). Short-term prices are shown below.

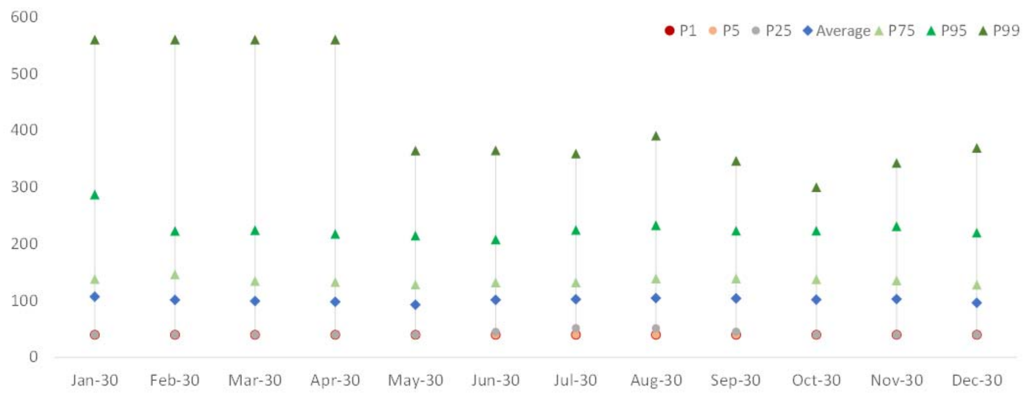

Figure 3-3 – Monthly spot price scenarios

The average spot price of the simulation is R$ 110,75/MWh. There is great variation in prices, as they can assume monthly values as low as the regulatory floor (circa R$ 40/MWh) up to levels higher than R$ 500/MWh.

3.3 Simulations, results, and analyses

This section discusses the results obtained when running the equilibrium model with the data presented above. The results for the two proposed system configurations are presented and sensitivity analyzes are carried out. The first three subsections assess the generators' risk aversion and its impact on the equilibrium. Then, section 3.3.4 explores constraints such as the limitation of sales to the physical guarantee (on the generation side) and contracting obligation (on the consumer side). Finally, section 3.3.5 addresses the consumers' side by assessing their risk aversion and responses to the imposed constraints.

3.3.1 Risk neutral agents

Firstly, all agents are considered risk neutral. The goal at this point is to verify whether agents would be willing to celebrate contracts and/or migrate between markets, and whether the average tariff and contract prices would be equivalent to the average spot price in each respective period. That is, performing an initial model fit test to confirm the occurrence of highly expected results.

As expected, the results mentioned above were found. There was no migration between markets and no contracts were signed. The figure below illustrates that the participation of agents in each category remained the same (with null migration variables).

Although no contract has been signed, it is possible to determine the prices at which each agent would be willing to sign contracts (dual variable of the respective equilibrium constraint). As expected, the equilibrium price of contracts and the price of regulated tariffs were equal to the average spot prices in each respective period (annual for the contract and monthly for the tariffs), thus resulting in zero risk premiums, as shown below.

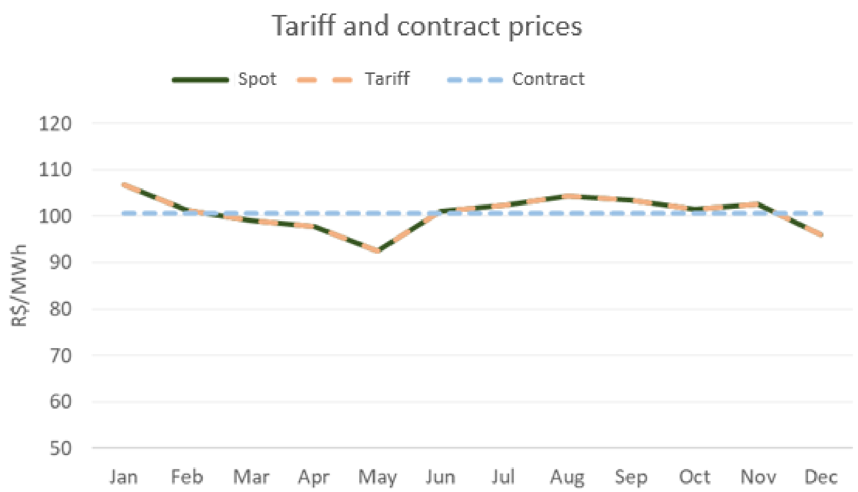

Figure 3-4 – Spot prices, regulated tariffs and contract prices with risk neutral agents

The same results were found for a 100% hydro market and for a 50% hydro, 50% wind one, as expected.

3.3.2 Risk aversion assessment

In this section, we assess the effect of the agents' risk aversion on contracting decisions and on free and regulated market prices. For this, we will consider the risk aversion parameter λ (the weight given to the expected value in the objective function, which is a combination of the expected value with the CVaR risk metric – that is, the lower the λ, the more risk averse), equal to 0.8 for all agents. This means that the objective function of all agents (distributor, type II captive consumers, free consumers and generators) has a weight of 80% to maximize the expected value and 20% to maximize the CVaR. In the previous case (without risk aversion), the λ of all agents was, evidently, equal to 1. Next, we show the results of this exercise for the hydro case and then for the hydro/wind case.

3.3.2.1
Hydro case

In this case, we assess the contracting level and contract price and regulated tariffs with the risk-averse generator in the 100% hydroelectric case. For price results, in this section and in the next ones, the contract/tariff results will be portrayed as the "risk premium", interpreted as the difference (markup) between its value and the average spot price in the same period.

The contracting level in this case was of 97% of the physical guarantee of the hydroelectric plant (and, therefore, of the annual demand). It is noteworthy that, in this phase of initial exercises, we are not imposing limitations such as that the demand must be 100% contracted, or that the generator can only sell up to its physical guarantee. This is because, with few agents and a physical equilibrium of supply and demand, this would force 100% contracting on both parties, making it inviable to investigate the balance sought by the model. Anyway, in future analyses, other constraints will be imposed.

With respect to prices, the graphs below show the risk premiums in the free and regulated markets in this case.

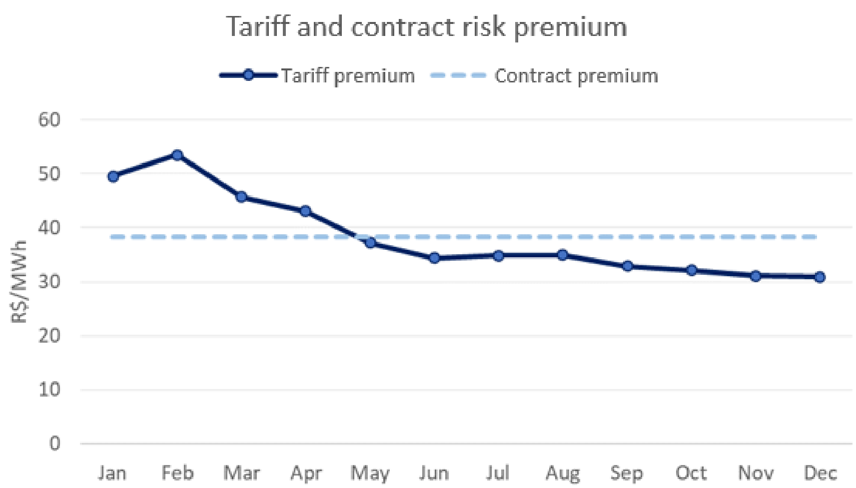

Figure 3-5 – Tariff and contract premiums with risk-averse generator: hydro case

The average contract price was R$138.9/MWh, with a risk premium of R$38.1/MWh with respect to the average spot price in the year. The reader should recall that the contract is has a 1-year tenure and is signed at t=0 (the contract's risk premium throughout its period was represented in a dotted line). The tariffs, on the other hand, have a monthly periodicity: in their case, the risk premium varied between amounts above R$50/MWh and a little above R$30/MWh, and had the same average of R$38.1/MWh. Therefore, the markups were solidly positive: a result compatible with a predominantly hydroelectric system and related to the Brazilian reality.

3.3.2.2
Hydro-wind case

In the hydro-wind case, a contracting level of 99% of the total demand was obtained. This contracted amount is divided into 43% for hydroelectric power plants and 56% for wind power plants. Therefore, the contracting of wind power exceeds its physical guarantee, which was 50% (as mentioned above, it was decided not to limit, at this early stage, the contracting to the physical guarantee, to explore the balance naturally achieved by the model – if we opted to limit, the balance would be 47% hydro and 50% wind).

As for the prices, they have reached an equilibrium at a level slightly lower than that obtained in the previous case. The main reason is the fact that wind generation is less correlated with spot prices than the hydroelectric one: hydro plants tend to demand slightly higher contract prices to enter into contracts, as they can be more exposed financially in dry periods of high prices. For wind plants, the lower correlation with spot prices reduces the severity of extreme events in which they would not generate enough to fulfill their contracts.

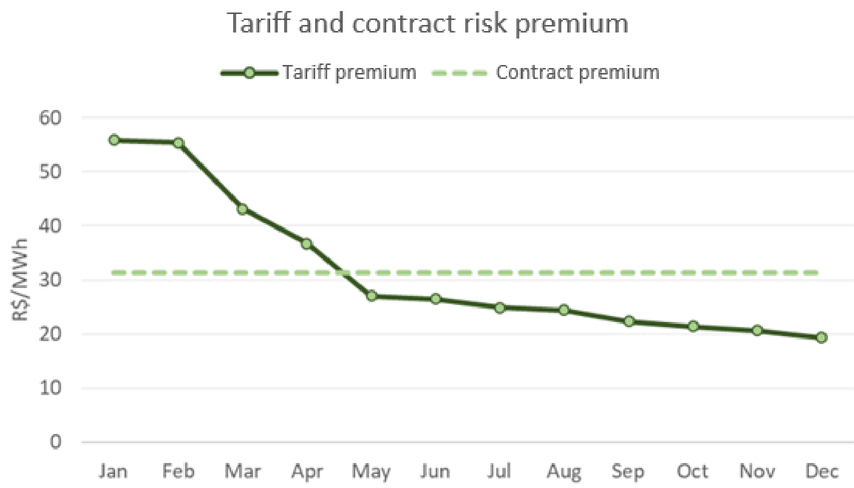

Figure 3-6 – Tariff and contract premiums with risk-averse generators: hydro-wind case

The contract price was R$132.1/MWh: a risk premium of R$31.3/MWh with respect to the average spot price in the year. In the case of tariffs, the risk premium varied between values close to R$ 60/MWh and below R$ 20/MWh, with the same average of the contracts. It should be noted once again that tariffs are modeled in a monthly fashion in the simplified case studies – sophistications in tariff modeling (such as yearly definition and several others are addressed further in this work).

3.3.3 Varying the risk aversion

In the previous section, it was shown that risk aversion (in addition to the supply composition) is a crucial element for the price equilibrium. In this one, sensitivity analyses are carried out in the risk aversion parameters. Previously, it was considered the same risk aversion level of $\lambda = 0.8$. Now, the generators' λ is varied from 0 to 1, in steps of 0.05, maintaining the other agent's λ fixed at 0.8.

3.3.3.1 Hydro case

Firstly, we analyze the systemic contracting level as a function of the generator's λ. It is shown that the more risk averse it is (lower λ), the less it is willing to celebrate contracts, once this contracting may generate higher exposures to honor the contract at the short-term market. With a null λ (i.e., when the generator's objective function is to maximize the CVaR), the contracting level gets slightly above 80%. This value increases slowly until 86% for λ = 0.8 (as found previously). As λ assumes values higher than 0.8, the contracting levels increase faster, until reaching 100% for λ values closer to 1. This is an intuitive result, once the risk neutral generator would be, in theory, willing to celebrate contracts at the expected value of the spot price, and the very fact that the other agents are risk averse tends to force them to accept such contracting as prices equal or higher than the spot average.

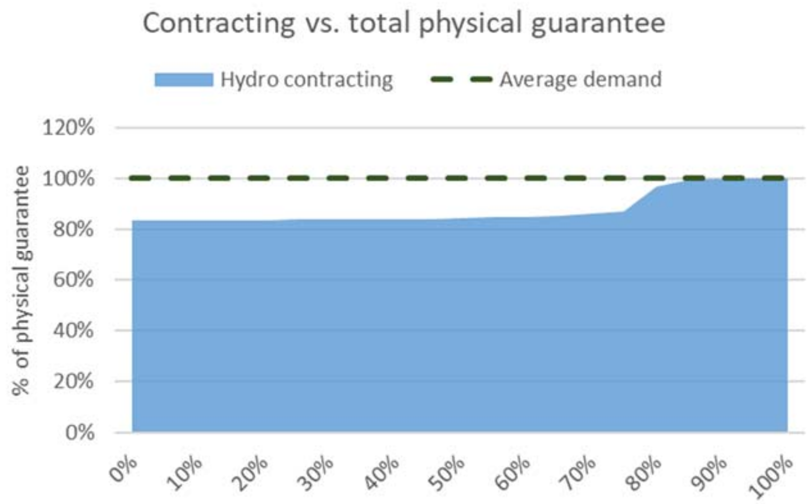

Figure 3-7 – Contracting levels as a function of the generator's λ: hydro case

With respect to contract prices, these remain constant, with a premium of R$ 38/MWh for values of λ between 0 and just before 0.8. This is what makes contracting smaller as you get closer to λ = 0, as seen above: given that demand is not willing to pay a premium greater than R$ 38/MWh, the solution is to reduce contracting for smaller λ values. On the other hand, as λ exceeds 0.8, the premium drops rapidly, reaching an equilibrium of R$ 16/MWh for λ close to 1. In theory, with λ = 1, the generator would accept an even lower premium (in limit, zero), but in this case, the risk aversion of demand, given the available supply, makes it accept to pay a premium of R$ 16/MWh, which becomes the equilibrium price.

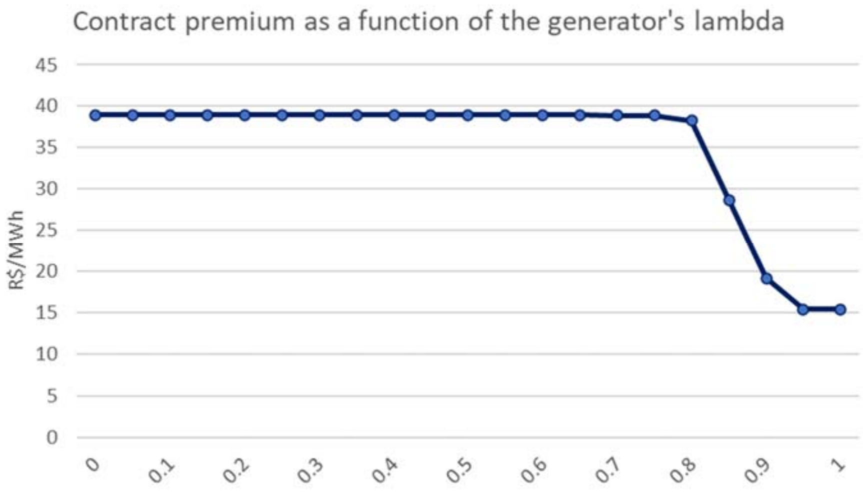

Figure 3-8 – Contract premium as a function of the generator's λ: hydro case

Finally, it is shown next the regulated tariffs' premium. Analogously to the contracts, this premium reduces as we increase the generator's λ of the generator – and much faster for values of λ between 0.8 and 1. Despite the monthly visualization, the average tariff premiums are equivalent to the annual ones found for the contracts.

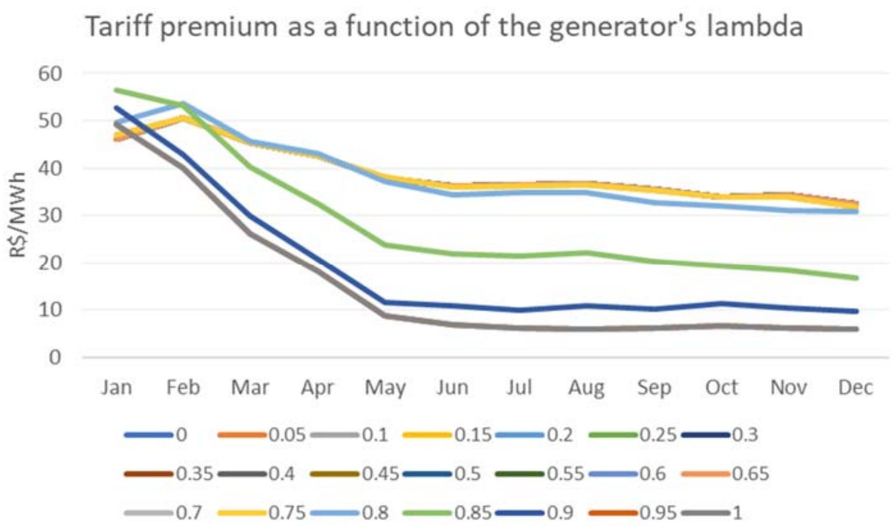

Figure 3-9 – Tariff premium as a function of the generator's λ: hydro case

3.3.3.2
Hydro-wind case

In the hydro-wind case, the agents' behavior for contracting is, in general, analogous to the previous one, with a slightly higher contracting level. In this case, we see agents more willing to sign contracts, especially given that wind farms do not have as great exposure as hydroelectric plants in the dry period. Thus, despite the same physical guarantee and the same risk aversion, the contracting of wind farms is always a little higher than that of hydroelectric plants.

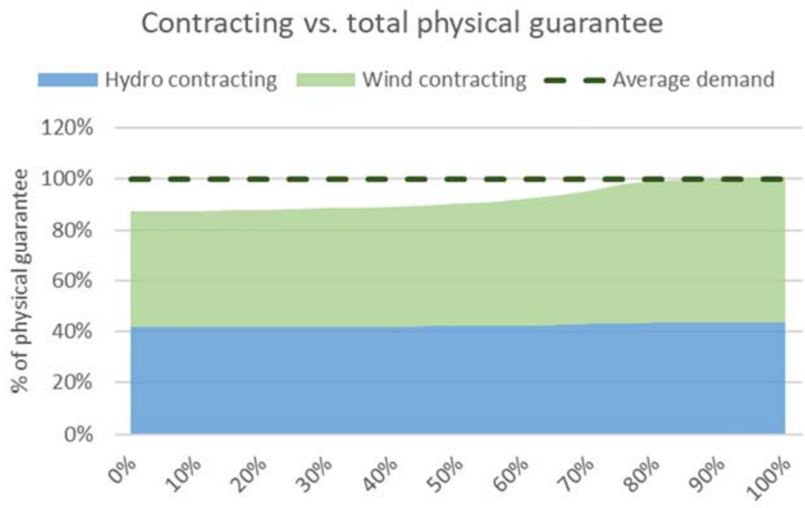

Figure 3-10 – Contracting levels as a function of the generators' λ: hydro-wind case

Regarding contract prices, these start from values virtually equal to the previous case, with a premium of around R$39/MWh but assume a more significant drop for values of λ between 0.8 and 1.0. In particular, for λ =1.0, the premium found reached zero, something that was not "accepted" by the generators in the previous case (the market equilibrium allowed a premium of R$ 16/MWh to be charged), but which, with this composition of supply-demand, the null risk aversion of the generating agents allowed them to enter into a contract with a null risk premium in this case.

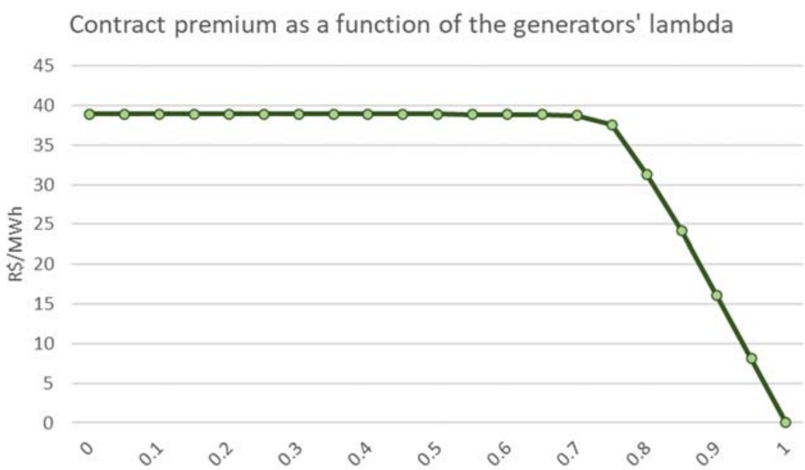

Figure 3-11 – Contract premium as a function of the generators' λ: hydro-wind case

As in the case of contracts, tariffs also reduce in value as the generators' λ increases, until they assume a virtually null average annual premium for λ = 1, even though there is a variation in their monthly values.

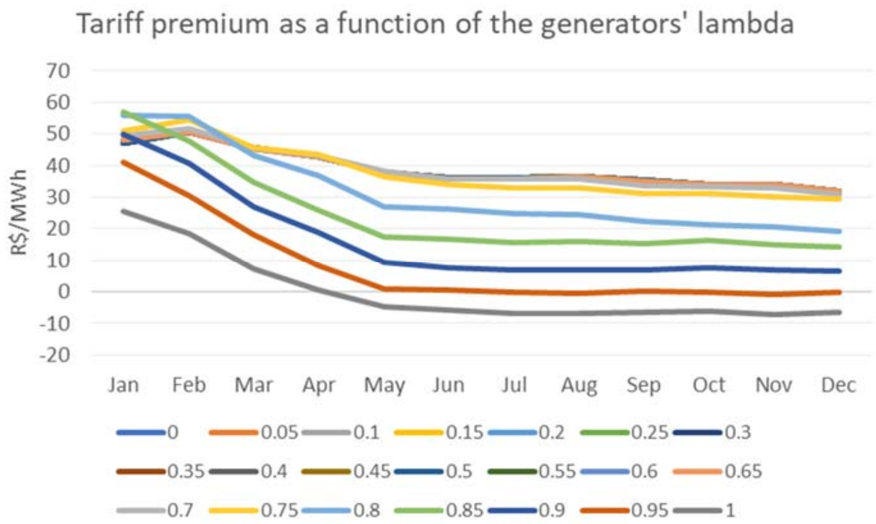

Figure 3-12 – Tariff premium as a function of the generators' λ: hydro-wind case

3.3.4 Introducing constraints: contracting limited to the physical guarantee and consumer contracting obligation

So far, the equilibrium problem has been presented in a very “loose” representation, without some of the “tiedowns” that occur in practice in several markets. This approach has been useful to interpret the contracting and price movements discussed in the previous sections, especially on the generators’ side. However, in order to continue advancing in the modeling, we find necessary to introduce into the problem some characteristics that are part of the reality of agents in the Brazilian and several other electricity sectors. In particular, this section will assess the impacts of introducing the following constraints in the model:

- Limitation of generators’ contracting up to their physical guarantee: in the previous cases, generators could take their contracting decisions from a financial point of view only. However, in several electricity market, the generators’ contracting level is limited to their ability to produce such amounts in scarcity conditions (in Brazil, this is measured as each project’s physical guarantee).

- Contracting obligation on the consumers' side: all consumers must have an annual contracted amount (in terms of physical guarantee) equal to or greater than their demand.

Next, we introduce these elements into the problem, which are modeled with the addition of mathematical constraints. Two subsections are elaborated next, testing each effect separately, and then a third, in which the effects are combined. It is noteworthy that, from this section onwards, in order to avoid the repetition of analyzes already carried out in previous sections and to focus on new analyses, the results presented will stick to the hydro-wind case.

3.3.4.1
Contracting limited to the physical guarantee

When presenting the hydro-wind case, it was described that the physical guarantee of each of the two projects corresponded to 50% of the demand. However, it was seen in the results of sections 3.3.2.2 and 3.3.3.2 that, given that the hydroelectric plant requires a higher risk premium for contracting, the results often pointed to a greater contracting of wind, even exceeding its 50% of physical guarantee, and smaller for hydro. This result was useful to understand the model's intuition, which captures these agents' preferences. However, in several markets such as the Brazilian, wind power could not contract more than its physical contribution (physical guarantee). Therefore, in this section, we introduce the constraint of contracting up to the physical guarantee of each project. The results are shown below and are compared to those obtained in section 3.3.3.2.

First, the figure below shows the contracting of the projects in each case:

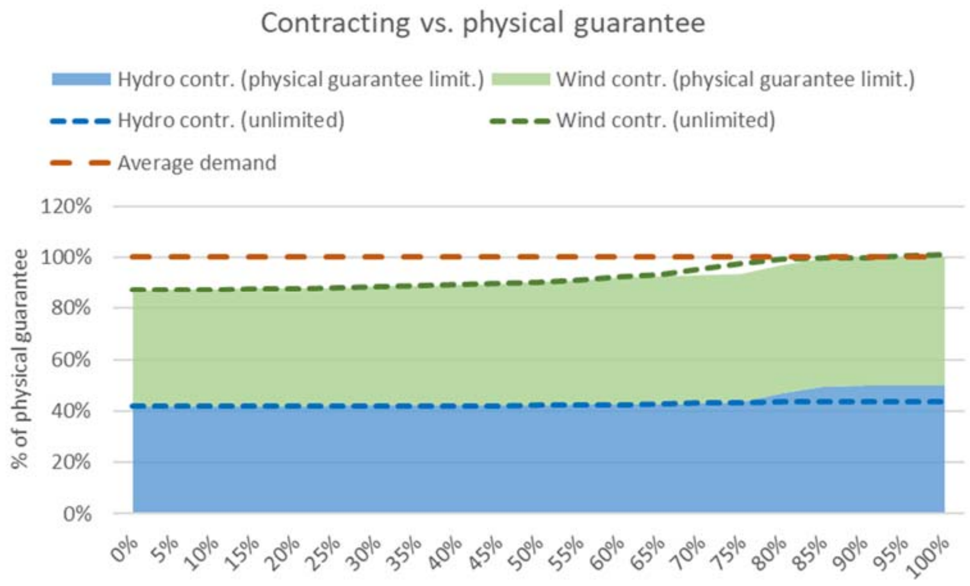

Figure 3-13 – Contracting levels as a function of the generators' λ: physical guarantee limitation

As it is possible to observe, hydroelectric contracting increases, especially when generators become less risk-averse, while wind power decreases. The hydro contracting evolves from the same 42% observed in the case without limitation but reaches 50% to values of λ close to 1.0 (while in the case without limitation, it reached only 44%. As for wind, we observe the opposite effect: now that the contracting is limited to the physical guarantee, the contracting of this source is limited to 50% for values of λ close to 1.0, whereas previously it reached 57%. This result is intuitive, since, while previously it was possible to carry out a larger number of contracts for one agent with lower downside in case of exposures (which is the case of wind), now, with the limitation, wind only reaches 50% and hydro contracts more (reaching the same 50%).

For the above effect to occur, that is, for hydros to accept to enter into contracts in an amount equal to the totality of their physical guarantee (50% of the total demand), there is a change in contract prices. This is shown in the chart below, with the contract premium with and without contracting limitation to physical guarantee.

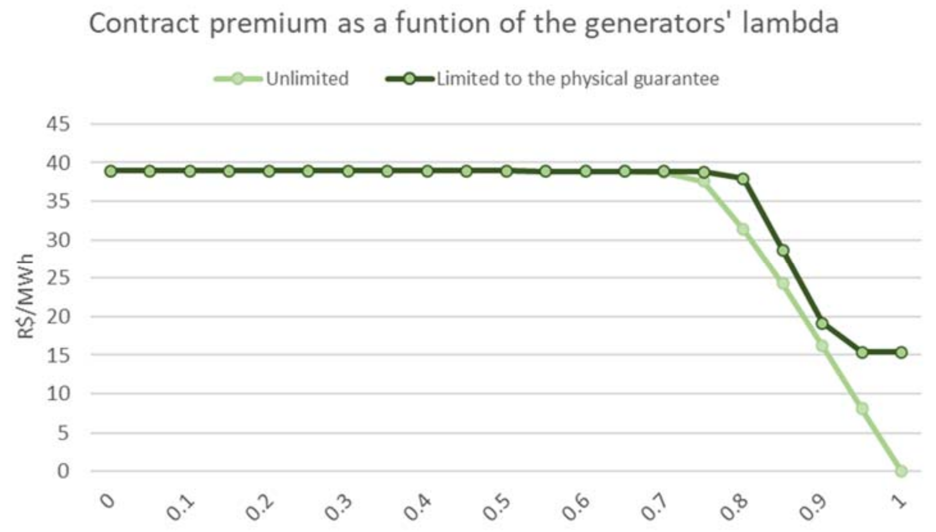

Figure 3-14 – Contract premium as a function of the generators' λ: physical guarantee limitation

As can be seen, the risk premium does not reduce to zero as it did in the "loose" hydro-wind case, motivated by the wind plant's willingness to contract. Instead, as hydro becomes the limiting agent, the premium becomes higher – similarly, by the way, to the purely hydroelectric case, with a balance of R$ 16/MWh. It is noteworthy, therefore, that the marginal generator in terms of contracting is the hydroelectric power plant, and that is why it was more determinant for contract prices.

3.3.4.2 Contracting obligation to Disco and free consumers

In this subsection, we analyze the results obtained by introducing the obligation to contract on the part of consumers. For this, we compare the unrestricted case, presented in section 3.3.3.2, with a case including such constraints, as done in the previous subsection for the constraint of contracting limited to physical guarantee for the generators.

We first compare the contracting of the projects in each case:

Figure 3-15 – Contracting levels as a function of the generators' λ: consumer's contracting obligation

As one could anticipate, the obligation to contract entails full contracting (amount equal to the system's total demand) for any value of λ. As the maximum contracting equal to the physical guarantee was not introduced as in the previous subsection, the contracting remains approximately constant at 56-57% for wind and 43-44% for hydro, which was basically the original result for risk neutral generators, when consumers were naturally willing to contract.

However, the introduction of this constraint causes a big change in contract prices. As generators become more risk-averse (lower values of λ), they become less willing to commit to large contractual quantities, as these can generate high exposures in scarcity scenarios. As such, a very risk-averse generator requires very high risk premiums to commit to a high contracted amount. This is shown in the figure below.

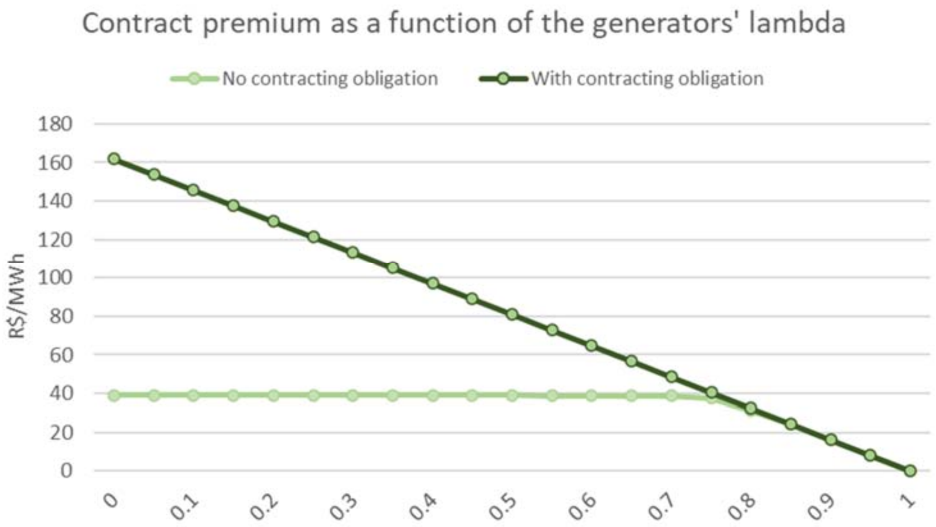

Figure 3-16 – Contract premium as a function of the generators' λ: consumer's contracting obligation

As mentioned, as the generator is more risk averse, with a greater weighting of the CVaR in its objective function (going to the left in the graph), it starts to demand higher risk premiums to celebrate contracts. In the unrestricted case, the optimal solution was a partial contracting, as consumers are naturally not willing to pay such high risk premiums for their contracts. However, as in this case the consumer, whether represented by the distributor or directly on the free market, is obliged to contract at least an amount equal to their annual consumption, they bear very expensive contracts as generators are more risk averse (which exceeds R$ 160/MWh in the case that the generator's objective function is to optimize only its CVaR).

Although this constraint is realistic in regarding the Brazilian market, the values on the left are too extreme, as they would assume that all generators are very risk-averse and would require very high contract premiums. Or even that generators, upon learning of the consumer's obligation, could charge very high risk premiums. However, in practice, there is also a natural competition in the generation segment, which tends to establish more competitive risk aversion levels, as well as contract equilibrium prices.

3.3.4.3
Combining both constraints: physical guarantee limitation and contracting obligation

In this subsection, the two constraints presented in the previous subsections are combined in the same case, that is, the contracting of generators is limited to the physical guarantee, and consumers, in turn, have a contracting obligation to, equal to their annual demand.

Regarding contracting, the following result was obtained:

Figure 3-17 – Contracting levels as a function of the generators' λ: physical guarantee limitation and consumer's contracting obligation

With the constraints, the contracting level of both agents is stable at 50%. This result is expected, given that generators cannot contract more than this, and consumers cannot contract less than this amount. This illustrates why, for the simplified case, it was important, in order to gain the desired intuitions about the model, to start working with the unrestricted model. When working with other cases, especially the real case of the Brazilian system, other combinations will be necessary – mainly due to the fact that the amount of physical guarantee available to enter into contracts is not exactly equal to the demand, as dimensioned in this example case.

Regarding prices, the graph below shows a combination of the effects obtained previously. Except that, as the generator is more risk-averse, the combination of effects resulted in even higher equilibrium prices: this is because, since wind cannot contract more than its physical guarantee, the equilibrium price is determined by the hydroelectric power plant, which is even more reluctant to sign such amount of contracts without a high reward. Thus, prices in the case where generators are totally risk-averse exceed R$ 190/MWh.

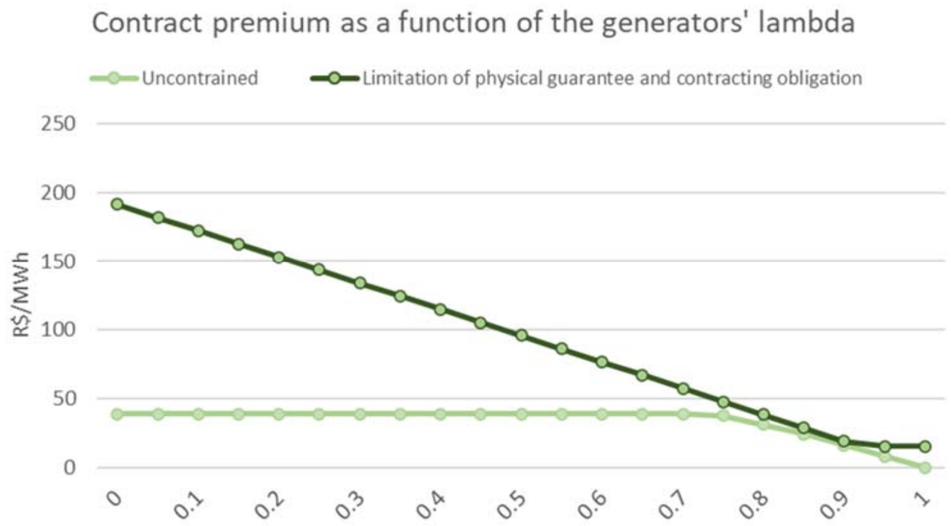

Figure 3-18 – Contract premium as a function of the generators' λ: physical guarantee limitation and consumer's contracting obligation

3.3.5 Assessing the consumer's side

The previous analyzes focused on the variations in equilibrium found, having as one of the main inputs the change in the risk aversion of generators. In this section, we evaluate the consumer side, showing to what extent the equilibrium depends on these agents' willingness to contract.

For the analyses in this section, we focus on the changes in preferences of the captive consumer who can migrate, keeping the risk aversion of other agents fixed. For this, we set the λ of the generators and distributor at 0.5; for the free consumer, a slightly lower risk aversion was adopted, with a λ of 0.6 – this was a convenient choice adopted, as shown later. Captive consumers who can migrate will have their risk aversion varied, and we assess the results in terms of contracting, pricing and migration. Also, at the end of the section, we show the impacts of variation in total demand.

3.3.5.1 Varying the risk aversion of the captive consumer with no contracting obligation

In this first subsection, we vary the risk aversion of the captive consumer from 0 to 1.0, in steps of 0.05, as we did for the generator before. The objective is to show the role of the consumer in pricing - primarily in a case where consumers are not required to purchase their energy through contracts, which allows us to have relevant insights, as evidenced in the generators' analyses.

Analogously to what was presented for the generators, we present the results of contracting and prices as a function of λ, this time of the captive consumer who can migrate.

Figure 3-19 – Contracting levels as a function of the captive consumers' λ

As it is possible to notice, in the first half of the graph, for smaller values of λ, the level of contracting is constant. As this agent becomes less risk-averse than the others (higher values of λ), the contracting with both generators drops – mainly with the wind generator, which was fully contracted in the first stretch. This happens because the less risk-averse consumer is the determinant of prices and, given their low aversion, is willing to contract at lower prices (like we see below), which in turn dissuades generators from taking on such contracting levels.

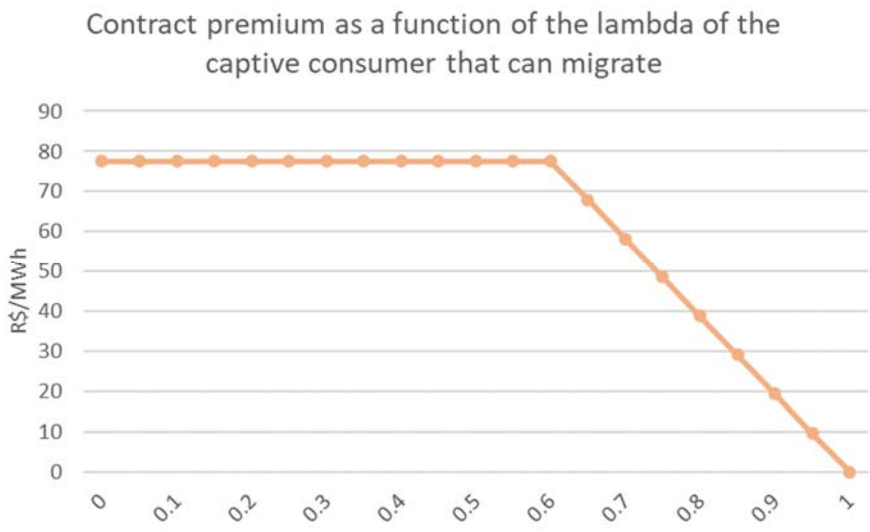

Figure 3-20 – Risk premium as a function of the captive consumers' λ

For low values of λ of the captive consumer, the market risk premium remains high, dictated by the risk aversion of other agents – in particular of the free consumer. As the captive consumer becomes the least risk-averse agent, it becomes decisive in the final price of free market contracts. Thus, the price substantially reduces as the risk aversion of this agent is lower – until, for $\lambda = 1$, this consumer is willing to contract with a null risk premium with respect to the spot, determining this price for the contracts.

Another interesting result to be analyzed is the migration of these captive consumers. The graph below shows that, as expected, this migration grows as this consumer is less averse to risk and seeks his own contracts in the free market, being decisive in the formation of prices in this market. The graph shows this result as a percentage of captive demand able to migrate (which in turn is 30% of total demand).

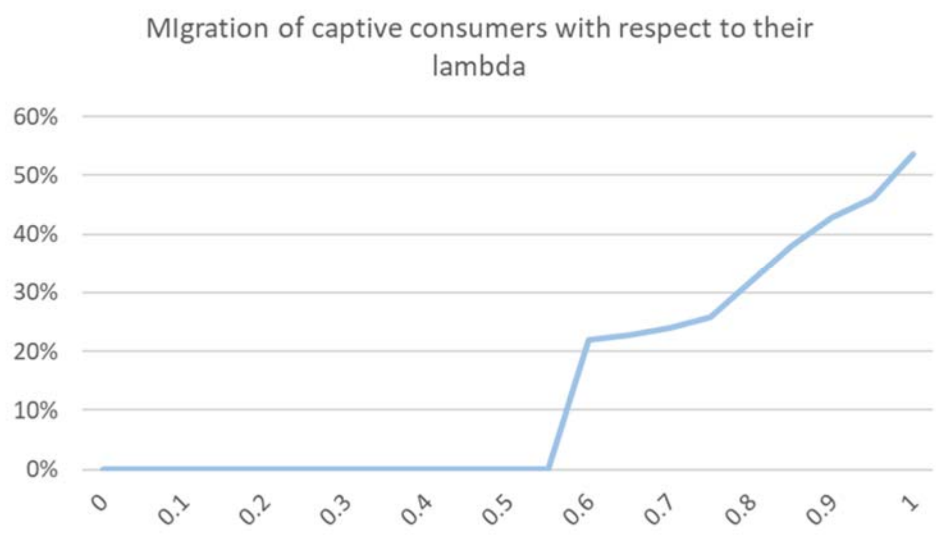

Figure 3-21 – Migration of captive consumers to the free market in function of their λ

The reader may question why this migration was not complete (e.g. 100% of their demand as soon as they became the most risk averse agents in the market). The interpreted reason for this is that the migration's magnitude is the one enough so set the prices accepted by these consumers at each step of λ. Once the price is reached, the consumer is indifferent between staying in the regulated or migrating to the free market – as, as shown in previous sections, the average regulated tariffs are equal to the free contract prices given the problem's parameters so far.

3.3.5.2
Varying the risk aversion of the captive consumer with no contracting obligation and without the possibility to migrate

In the previous subsection, it was concluded that, when the regulated consumer (who can migrate) is the least risk-averse agent, he migrates and is largely responsible for the equilibrium prices obtained in the free market (as evidenced by price variations as this agent has λ higher than the others and, in particular, because the contract price assumes the spot average in case this agent is risk neutral). This conclusion is very important for this work, which has as one of its main points of interest the assessment of this possibility of migration and influence of the migrating agents on free contract prices.

In order to demonstrate this, in this subsection, we vary the risk aversion of the regulated consumer in the same way, but not allowing their migration – and we compare the results with those obtained previously.

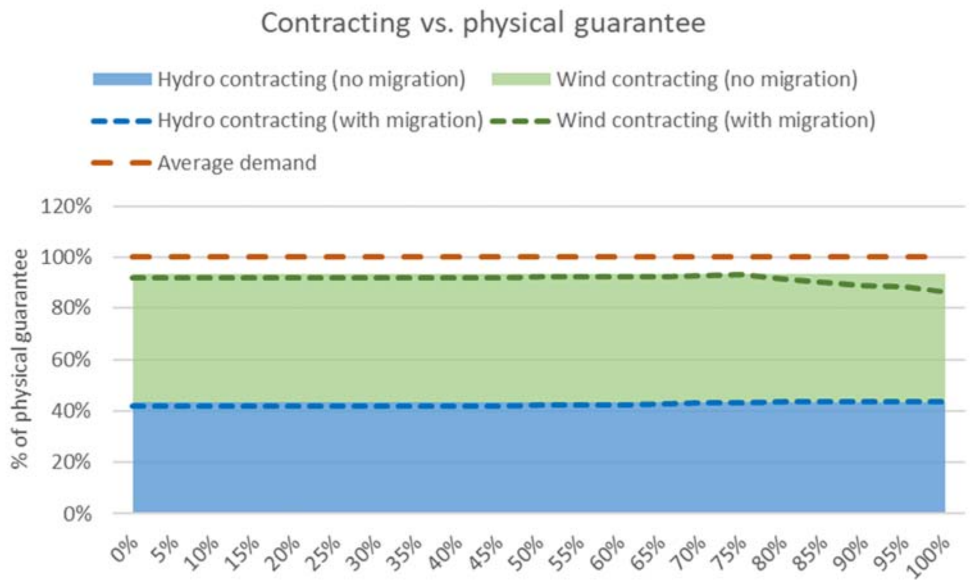

Figure 3-22 – Contracting levels as a function of the captive consumers' λ: with and without the possibility to migrate

Above we can note that, if captive consumers cannot migrate, they cannot exercise their preference in entering into contracts, which remain at prices determined by other agents – in particular, free consumers, who have greater λ than the others. In this way, the contracted quantity follows its equilibrium determined by the other agents, who have a fixed λ (0.6 for free consumers and 0.5 for distributor and generators).

The same is reflected in prices. Below, we see that, without the possibility of captive consumer migration, the risk premium obtained initially (R$ 78/MWh) for the balance determined by the other agents is maintained, regardless of the reduction in the captive consumer's risk aversion. In this case, they become a complete price taker – just like the captive consumers who cannot migrate in the original problem ("captive consumer type I").

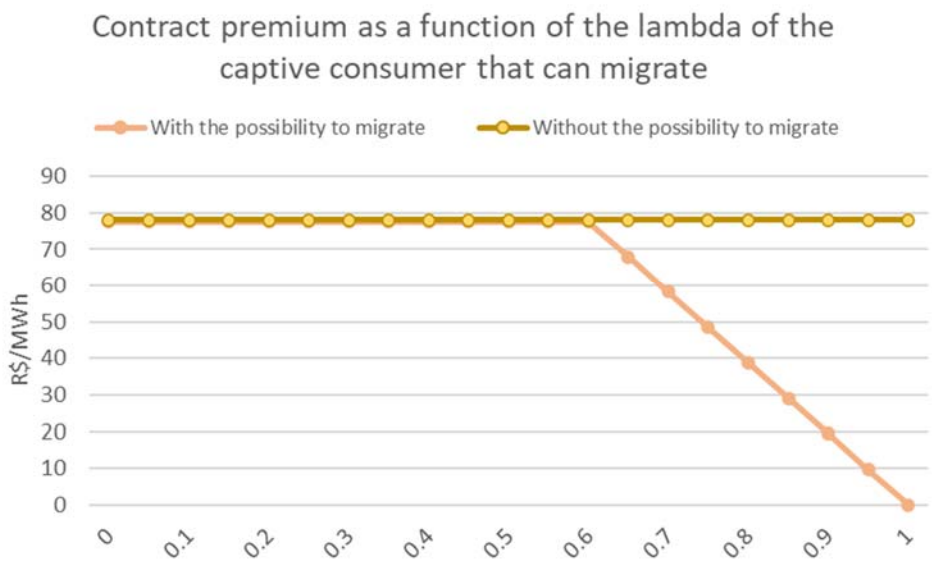

Figure 3-23 – Risk premium as a function of the captive consumers' λ: with and without the possibility to migrate

We conclude this subsection by highlighting the importance of modeling the possibility of migration, given its impact on contracting and prices – which is one of the central points of this work. That said, we will include this possibility again, as in all other cases, from the following analyses onwards.

3.3.5.3
Varying the risk aversion of the captive consumer with contracting obligation

In section 3.3.4.2 we had explored the concept of contracting obligation, and how it has an (upward) impact on contract prices - since the consumer is not able to impose his bargaining power and not contract from such contract prices. This was clear in that case, where we varied the risk aversion of generators and saw a large increase in the price of contracts.

On the other hand, this result raises the question of what happens to contract prices if the consumer has different preferences (risk aversions), but is obliged to contract. Thus, this subsection compares the results of subsection 3.3.5.1 (i.e., variation in consumer risk aversion without contracting obligation) with a new case in which we reproduce the same variation in captive consumer risk aversion but considering now that all consumers are obliged to have their load contracted.

The figures below show the contracting levels and prices obtained in each case.

Figure 3-24 – Contracting levels as a function of the captive consumers' λ: contracting obligation

The interpretation of the graph above is immediate, as it basically demonstrates that the contracting level was 100% for all values of λ of the captive consumer, as in this new case there is the obligation of contracting the entire demand.

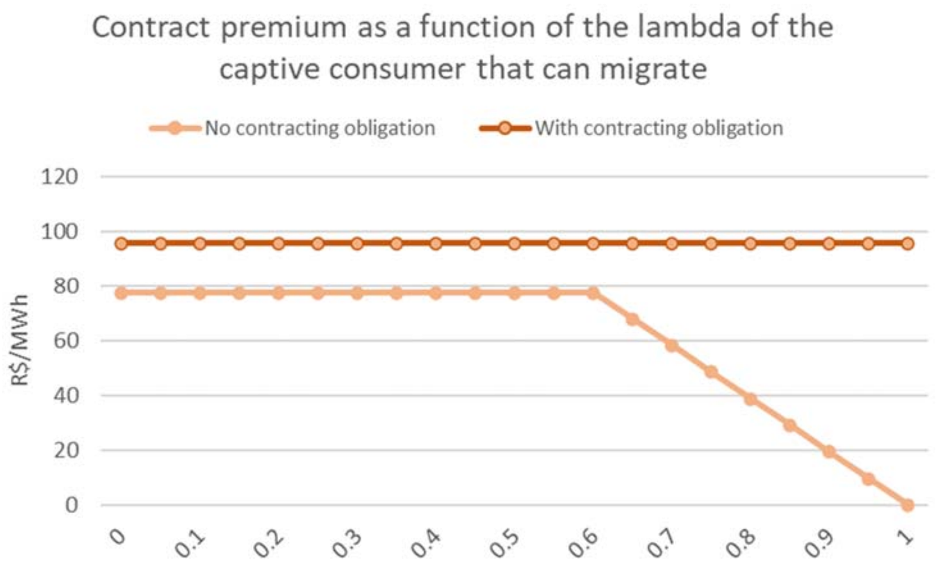

Figure 3-25 – Risk premium as a function of the captive consumers' λ: contracting obligation

The graph shows that the contract price is fixed throughout the horizon and, furthermore, that its value (R$ 96/MWh) is higher than the value originally obtained, even in the first part of lower λ (R$ 78/MWh) - which had also been the constant value obtained for the case with no possibility of migration.

The constant price results from the fact that the consumer is forced to contract, and the price is defined by the risk aversion of the generator. And this equilibrium price is higher because the risk aversion of the generator ($\lambda = 0.5$) is greater than that of the free consumer ($\lambda = 0.6$), who determines the prices in the original case when the captive consumer has lower λ (or when it cannot migrate). This was the reason for conveniently choosing the λ of the free consumer a little different from the generator – to show price differences from the previous cases and from this one, in which prices are defined exclusively by the generator.

This result leads to the strong conclusion that, under the assumptions considered, if there is an obligation to contract, and this constraint is active, the risk aversion of the consumer is irrelevant for the final price, which is determined purely by the generator's aversion. This conclusion was also reached by [47]. This result is relevant when we think about the formation of contract prices in Brazil and other countries where the consumer has this obligation, being, therefore, contract prices resulting from the profile and competition in the generation segment. On the other hand, it is important to emphasize that different compositions of supply and demand change this equilibrium price. Furthermore, if the contracting obligation is not an active constraint (that is, if the consumer is willing to contract an amount equal to or even greater than his demand regardless of the constraint), the consumer's preference influences the balance. This is shown in the next subsection.

3.3.5.4
Varying demand with contractual obligation

In the previous section we concluded that a consumer who contracts due to an obligation does so at a price determined by the risk aversion of the generator. However, these analyses were made for a predetermined supply-demand balance. Of course, were the supply and demand balance different, the equilibrium price would change. This has already been shown to a certain extent when we consider the supply side formed by a hydro plant or by a hydro-wind portfolio. However, it is also possible to vary demand – and, thus, different balances are reached.

Therefore, the purpose of this subsection is to show that the result of the previous one is valid, but varies according to the supply-demand balance and whether the contracting constraint is active or not. The case was then simulated, considering lower demand levels, from 100% (previous case), with gradual decreases of 5% until reaching 70% of the original demand (the reduction was applied proportionally, equally to all consumers). It was decided not to reduce more than this value, so as not to portray an exaggeratedly unbalanced system. As for the values of λ, the same 0.5 was adopted for generators and distributor, 0.6 for the free consumer, and 1.0 was chosen for the captive consumer that can migrate (risk-neutral).

Below, we show the contracting level by generating agent and the equilibrium prices.

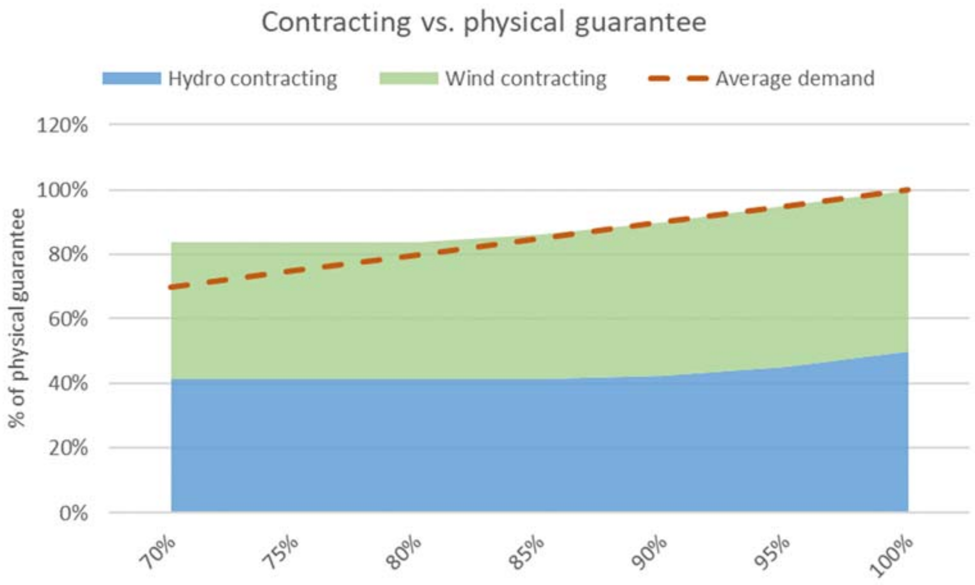

Figure 3-26 – Contracting level as a function of demand (percentage of the original demand) with contracting obligation

The graph above shows that, for higher demand levels, contracting was equal to demand. That is, the contracting obligation constraint is active, the consumer contracts according to his obligation, and he is a price taker. For lower levels of demand (less than 85%), the contracting obligation is not an active constraint - and the balance is a contracting level even higher than demand itself (the consumer is not forbidden to over-contract, which ends up being the problem’s equilibrium).

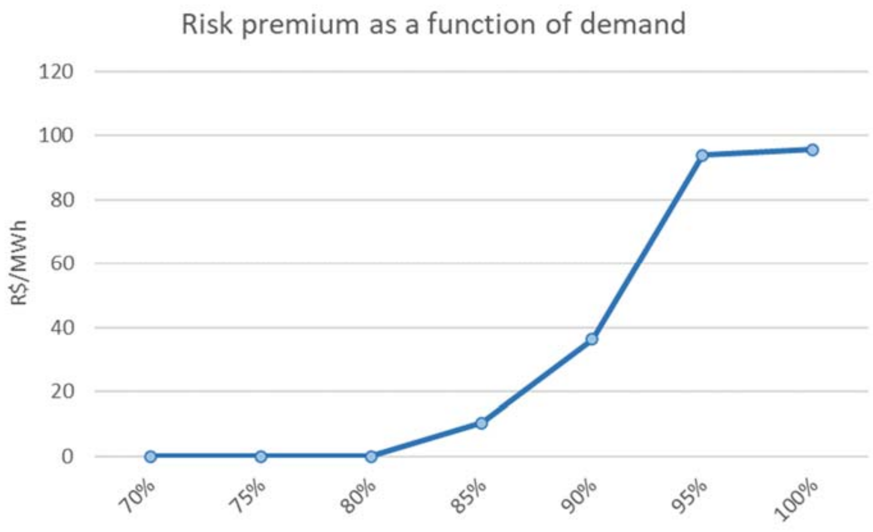

Figure 3-27 – Risk premium as a function of demand (percentage of the original demand) with contracting obligation

In terms of prices, the chart above shows some interesting results. The first one is that, even though on the far right of the graph (close to 100%, when the contract obligation restriction is active) the consumer is a price taker (i.e., the price does not change according to their risk aversion), this price varies with the quantity (total load). The other conclusion is that, for looser supply-demand balances, the constraint of the contracting obligation is not active, and, as in previous cases, the risk aversion of the captive consumer who may migrate has an influence on the final price - and as this was considered risk-neutral in this example, a null risk premium was reached in this stretch.

3.4 Final notes

In this simplified case study, several market equilibria obtained for small hypothetical systems were explored, varying the agents' preferences, obligations, and the supply and demand characteristics of this system. It was shown that the composition of the generation mix, as well as the load, have a direct influence on the contracting levels of agents and on market prices. It was also shown that the agents' risk aversion is key to determines prices, both on the generation and demand sides. Also, topics were explored such as the limitation of contracting to the physical guarantee of the generators, the obligation to contract demand, and the possibility of consumer migration, as well as its impacts on the final contracting and on contract prices. On the generator side, it was seen that limiting contractual sales by their physical guarantee increases the risk premium of agents with lower risk aversion, and the contracting obligation increases the premium of more risk-averse generators. On the consumer side, it was seen that, if consumers are forced to contract and this constraint is active, the equilibrium price depends on the supply-demand balance and on the competitiveness of generating agents, but not on the consumer's preferences.

4
Incorporating legacy contracts

This chapter proposes an extension of the methodology presented in Chapter 2, incorporating the legacy contracts. This topic is very relevant for the discussion that this work addresses, as the current contractual situation of the distribution companies and the prices in the regulated market (higher than the ones in the free market) incentives the migration of regulated consumers – as discussed in more detail in Chapter 1. So, although the formulation presented in Chapter 2 and the case study presented in Chapter 3 have been instrumental for, respectively, developing and understanding the functioning of the equilibrium model, in practice, the model should start from an unbalanced situation and move towards a different equilibrium. In this chapter, we show that, to incorporate this, some adjustments should be done to the formulation presented in Chapter 2.

An important aspect of this extension that differs from the previous approach is that the distributor's tariff must be calculated outside the equilibrium problem, weighting in the cost of legacy contracts to determine the final tariff. In this context, the problem will be composed by two sub-problems: (i) one equilibrium problem in which the distributor competes with free consumers in the purchase of energy from generators, and (ii) a problem of optimizing the welfare of Captive Consumers Type II and Free Consumers where the distributor competes with generators in the sale of energy, which will ultimately determine the consumers' migration.

The proposed extended methodology is presented next. The same notation as in Chapter 2 is adopted and, depending on the case, new quantities will be defined as indicated in the text.

4.1 Proposed iterative methodology

As previously mentioned, the problem is now divided into two sub-problems. After each one, there are some data treatments to insert results in the following problem, until a convergence is reached. Thus, the methodology is composed by four major steps (plus a "step zero", called "initialization"), as presented next:

0. Initialization: start the counter of iterations (i = *1*). Define:
    a. Initial load of free consumers: $d_{1,t}^{L} = d_t^L$
    b. Initial load of Captive Consumers Type II that have migrated to the free market: $m_{1,t}^{L} = 0$
    c. Initial load of the Disco, as the sum of the loads of Captive Consumers Type I and II that it represents: $d_{1,t}^{D} = d^{CI} + d^{CII}$
    d. Initial amount of legacy contracts by Disco: q_1^{LEG}
1. In this step, we calculate the equilibrium of (i) free consumers with their loads (that remained in the free market), (ii) the part of the Captive Consumers Type II loads that have migrated to the free market, (iii) the Disco with the demand they represent as well as the amount already contracted through legacy contracts, and (iv) generators. In this problem, the Disco operates as an energy buyer for the load it represents in the given iteration, and the migration between markets is not considered.
2. Calculation of Disco's tariff: given the cost of the legacy contracts, and the ones resulting from the decisions in step one, that is, the cost of contracts purchased from the generators and settlements in the free market, as well as the regulated demand that it is representing, we determine the tariffs (assuming it equal to these costs divided by the consumer's base).
3. Optimization of the Welfare of free consumers and captive consumers that can migrate: given the contract prices obtained in step 1 and the tariffs calculated in step 2, we maximize the Welfare of free and regulated consumers by allowing them to select their perceived optimal migration and contractual portfolio. Results of this step include the amounts contracted from generators, purchased from the Disco and the migrations between the free and the regulated markets. In this step, the Disco operates as a seller with a fixed tariff, calculated in step 2 – and so do the generators, at fixed contract prices calculated in step 1.
4. Update of Disco' loads, of the loads in the free market, and of the amount of legacy contracts (if applicable): based on the migration results in step

3, we update the Disco's and free loads, as well as the amount of legacy contracts based on the migration of regulated consumers to the free market (in case they are allowed to be broken – which is another discussion addressed further in this work). In case the results (such as the Disco's and consumer loads, tariffs, contract prices) do not vary significantly with respect to the previous iteration, the iterative process should stop, on the contrary, the iteration counter is updated ($i = i+1$) and we return to step 1.

One important aspect in the iterative process is that the Disco's tariff is going to be calculated in step 2 and thus not as the dual variable of the equilibrium problem.

4.2 Mathematical formulation associated with each step

This section presents the mathematical formulation of each step. The optimization problems of each module and the welfare optimization for calculating the associated equilibrium for Steps 1 and 3 of the iterative process are similar to those of the original methodology. Thus, the modules developed and adopted in the previous chapters have been adapted and reused in the implementation of the new methodology. The same notation as in Chapter 2 will be adopted and, as appropriate, new quantities will be defined.

4.2.1 Formulation of Step 1

Step 1 corresponds to an equilibrium problem involving the Disco, free consumers (including originally regulated that have migrated), and generators.

4.2.1.1 Optimization problem of the Disco

In this step, the distributor is an energy buyer. Its optimization problem calculates the amounts to be purchased from the generators through energy contracts, based on their prices, which maximize the present value of their risk-adjusted net revenues. Mathematically it can be described as:

$$\underset{q_t^D}{\text{Maximize}}\ R^D \tag{4.1}$$

s.t.

$$R^D = \lambda^D \times E\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] + (1-\lambda^D) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^D}{(1+r)^t}\right] \tag{4.2}$$

$$R_{t,s}^D = p_i^R \times d_{i,t}^D - p_i^{LEG} \times q_{i,t}^{LEG} - p^C \times q_t^D + \pi_{t,s} \times \left(q_{i,t}^{LEG} + q_t^D - d_{i,t}^D\right), \quad \forall t,s \tag{4.3}$$

Where:

$p_i^{LEG}, q_{i,t}^{LEG}$: price and quantity of the Disco's legacy contracts at iteration i (input data for Step 2, and updated at Step 4)

$d_{i,t}^D$: Disco's load at iteration i, stage t (input data for Steps 2 and 3, and updated at Step 4).

The decision variable is the amount of purchased contracts q_t^D. Constraint (4.3) establishes that the distributor's net revenue is composed of two components: expenses with contracts (legacy and new ones), energy purchases and revenue/expenses associated with settlements at the short-term market. The difference with respect to the corresponding problem presented in Chapter 2 is that here the Disco's load ($d_{i,t}^D$) is fixed and considers the migrations occurred in previous iterations (or defined for the first one).

4.2.1.2
Optimization problem of the Captive Consumers Type II

Based on prices of energy contracts purchased from generators, the optimization problem of Captive Consumers Type II who migrated to the free market aims to determine the amounts of energy contracts to be purchased that maximize the present value of their risk-adjusted net revenues. In this problem, the load of the consumer who migrated to the free market is fixed, based on the result of Step 3 of the previous iteration. Note that in the first iteration this issue this amount is null because the migration has not yet occurred. Mathematically it is described as:

$$\underset{q_t^{CII}}{\text{Maximize}}\ R^{CII} \tag{4.4}$$

s.t.

$$R^{CII} = \lambda^{CII} \times E\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] + (1-\lambda^{CII}) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] \tag{4.5}$$

$$R_{t,s}^{CII} = -p_i^R \times \left(d_t^{CII} - m_{i,t}^{CII}\right) - p^C \times q_t^{CII} + \pi_{t,s} \times \left(q_t^{CII} - m_{i,t}^{CII}\right), \qquad \forall t,s \tag{4.6}$$

Where:

$m_{i,t}^{CII}$: load of Captive Consumer Type II that has migrated to the free market (defined only from the second iteration of the algorithm – being null in the first one)

The decision variable is the amount of contracts purchased. Note that in this case, the revenue expression only includes revenues from contract purchases and settlements in the short-term market because the load that migrated to the free market is fixed in this problem and does not affect the optimization result.

4.2.1.3
Optimization problem of the Free Consumers

Based on prices of energy contracts purchased from generators, the Free Consumer optimization problem aims to determine the amount of energy contracts to be purchased that maximizes the present value of their risk-adjusted net revenues. In this problem, the free consumer's load is fixed, based on its original load subtracted from the part that migrated to the distributor in the previous iteration. Mathematically this problem is described as:

$$\underset{q_t^L}{\text{Maximize}}\, R^L \tag{4.7}$$

s.t.

$$R^L = \lambda^L \times E\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] + (1-\lambda^L) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] \tag{4.8}$$

$$R_{t,s}^L = -p_i^R \times m_{i,t}^L - p^C \times q_t^L + \pi_{t,s} \times \left(q_t^L - \left(d_t^L - m_{i,t}^L\right)\right), \qquad \forall t,s \tag{4.9}$$

Where:

$m_{i,t}^L$ = load of the free consumer that migrated to the regulated market at iteration i, at stage t (from iteration 2, if so determined in Step 4 of the previous iteration).

The decision variable in this problem is the amount contracts purchased. Constraint (4.9) establishes that the net revenue of the Free Consumer is composed of two components: expenses on the purchase of energy contracts and revenues/expenses associated with settlements at the short-term market.

4.2.1.4
Optimization problem of the Generators

From the prices of energy contract, the generator optimization problem seeks to determine the amount of energy contracts to be sold in contracts that maximizes the present value of their risk-adjusted net revenues. Mathematically this optimization problem is described as:

$$\underset{q_t^G}{\text{Maximize}}\; R^G \tag{4.10}$$

s.t.

$$R^G = \lambda^G \times E\left[\sum_t \frac{R_{t,s}^G}{(1+r)^t}\right] + (1-\lambda^G) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^G}{(1+r)^t}\right] \tag{4.11}$$

$$R_{t,s}^G = p_i^{LEG} \times q_{i,t}^{LEG} + p^C \times q_t^G + \pi_{t,s} \times \left(g_{t,s} - q_{i,t}^{LEG} - q_t^G\right), \qquad \forall t, s \tag{4.12}$$

The decision variable associated with this problem is the amount of energy contracts sold.

4.2.1.5
Equilibrium constraint

The total amount of energy sold through energy contracts in a given validity period by generators is equal to the sum of the amount of energy contracts purchased by the distributor, and by consumers in the free market, in the same validity period. Mathematically:

$$q_t^G = q_t^D + q_t^{CII} + q_t^L, \qquad \forall t \tag{4.13}$$

4.2.1.6
Welfare optimization problem

As in the previous methodology presented in Chapter 2, the equilibrium of this market can be obtained through the maximization of welfare given by Equations 2.19-2.33. The only difference is that, here, the migration between markets is not a decision variable, but an input for the given iteration. The representation of the MOPEC is suppressed for the sake of conciseness.

As developed in Chapter 2, the problem corresponds to a linear programming problem and the contract's equilibrium price will be given by the dual variable associated with the equilibrium constraint. Note that the legacy contracts are also fixed quantities, purchased by the Disco and sold by the generators, and so could be cancelled, analogously to the regular contract defined in the model.

Finally, we define p_i^C and $q_{i,t}^D$ as the contract prices and the contractual amounts celebrated by the Disco in the optimal solution of the problem (these will be needed in the next step).

4.2.2
Formulation of Step 2

$$p_i^R = \sum_{t,s}[p_i^{LEG} \times q_{i,t}^{LEG} + p_i^C \times q_{i,t}^D + E\left[\pi_{t,s} \times \left(d_{i,t}^D - q_{i,t}^{LEG} - q_t^D\right)\right]]/d_{i,t}^D \tag{4.14}$$

Where:

p_i^R = Disco's tariff at stage t (calculated)

$d_{i,t}^D$ = Disco's load at iteration $iter$, which results from the original load and the possible migrations between markets determined in the previous iteration

$p_i^{LEG}, q_{i,t}^{LEG}$ = price and quantity of Disco's legacy contract k_m at iteration $iter$

p_i^C = contract price at iteration i, resulting from Step 1

$q_{i,t}^D$ = amount of new contracts k_m celebrated by the Disco at iteration $iter$, resulting from the equilibrium obtained in Step 1

Equation (4.14) establishes that the distributor's tariff in stage t is equal to the sum of expenses with legacy and new contracts, and in the short-term market, divided by the load of the distributor in step t.

4.2.3 Formulation of Step 3

This step carries out the Welfare maximization of captive consumers that can migrate to the free market, and of the free consumers (that in turn may become captive). Next, we show the optimization problem of each of these two types of consumers, followed by the Welfare optimization problem, which is basically the combination of both in one maximization problem.

4.2.3.1 Optimization problem of the Captive Consumer Type II

Based on the distributor's energy sales prices (calculated in Step 2) and the free market contracts (determined in Step 1), the Captive Consumer Type II optimization problem aims to determine the portion of the load that must migrate to the free market and the and the contractual amount to be purchased in order to maximize the present value of their risk-adjusted net income. Mathematically this problem can be described by:

$$\underset{m_t^{CII}}{\text{Maximize}}\, R^{CII} \tag{4.15}$$

s.t.

$$R^{CII} = \lambda^{CII} \times E\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] + (1-\lambda^{CII}) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] \tag{4.16}$$

$$R_{t,s}^{CII} = -p_i^R \times (d_t^{CII} - m_t^{CII}) - p_i^C \times q_t^{CII} + \pi_{t,s} \times (q_t^{CII} - m_t^{CII}), \qquad \forall t,s \tag{4.17}$$

$$m_t^{CII} \leq d_t^{CII}, \ \forall t,s \tag{4.18}$$

Where:

p_i^R: Disco's regulated tariff at iteration *iter*, calculated at Step 2

p_i^C= contract price at iteration *i*, resulting from Step 1

Constraint (4.17) establishes that the net revenue of the Captive Consumer Type II is composed of three components: expense with the purchase of energy from the distributor associated with part of the load that did not migrate to the free market, expense with energy purchase contracts and income/expense associated with settlements at the short-term market.

Constraint (4.18) establishes that the share of the load that can migrate to the free market is lower or equal to the customer's total load.

The following decision variables are associated with Problem (4.15 – 4.18): amount of load that migrates to the free market and amount of energy contracts purchased in the free market.

4.2.3.2
Optimization problem of the Free Consumers

Based on the distributor's energy sales prices (calculated in Step 2) and the free market contracts (determined in Step 1), the Free Consumer’s optimization problem aims to determine the portion of the load that must migrate to the regulated market and the and the contractual amount to be purchased in the free market in order to maximize the present value of their risk-adjusted net income. Mathematically, this problem can be described by:

$$\underset{m_t^L}{\text{Maximize}}\; R^L \tag{4.19}$$

s.t.

$$R^L = \lambda^L \times E\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] + (1-\lambda^L) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] \tag{4.20}$$

$$R_{t,s}^L = -p_i^R \times m_t^L - p^C \times q_t^L + \pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right), \qquad \forall t,s \tag{4.21}$$

$$m_t^L \leq d_t^L,\ \forall t,s \tag{4.22}$$

Analogously to the case of the captive consumer, Constraint (4.21) establishes that the net revenue of the Free Consumer is composed of three components: expense with the purchase of energy from the distributor associated with part of the load that migrated to the regulated market, expense with energy purchase contracts in the free market and income/expense associated with settlements at the short-term market.

Constraint (4.22) establishes that the share of the load that can migrate to the regulated market is lower or equal to the customer's total load.

The following decision variables are associated with Problem (4.19 - 4.22): amount of load that migrates to the regulated market and amount of energy contracts purchased in the free market.

4.2.3.3
Welfare maximization problem of the Captive Consumers Type II and of the Free Consumers

Given the individual optimization problems presented in the last two sub-sections, below it is presented the Welfare optimization problem, which is basically a combination of both problems (sum of their objective function and combination of their constraints):

$$\underset{m_t^{CII},\, m_t^L}{\text{Maximize}}\; R^{CII} + R^L \tag{4.23}$$

s.t.

$$R^{CII} = \lambda^{CII} \times E\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] + (1-\lambda^{CII}) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^{CII}}{(1+r)^t}\right] \tag{4.24}$$

$$R_{t,s}^{CII} = -p_i^R \times (d_t^{CII} - m_t^{CII}) - p^C \times q_t^{CII} + \pi_{t,s} \times (q_t^{CII} - m_t^{CII}), \qquad \forall t,s \tag{4.25}$$

$$m_t^{CII} \le d_t^{CII}, \ \ \forall t,s \tag{4.26}$$

$$R^L = \lambda^L \times E\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] + (1-\lambda^L) \times CVaR_\alpha\left[\sum_t \frac{R_{t,s}^L}{(1+r)^t}\right] \tag{4.27}$$

$$R_{t,s}^L = -p_i^R \times m_t^L - p^C \times q_t^L + \pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right), \qquad \forall t,s \tag{4.28}$$

$$m_t^L \le d_t^L, \ \forall t,s \tag{4.29}$$

The problem above corresponds to a linear programming problem, because both the Disco's tariffs and the contract prices are fixed (calculated in previous steps).

The reader may have noticed that, in the optimization problem above, the agents' migration and contracting decisions do not affect prices, which are the ones established in the previous steps, differently to what happens in the equilibrium problems of Step 1, or in the initial formulation presented in Chapter 2. Thus, a natural solution to the problem of Step 3 tends, in principle, to be a massive migration by all agents to the cheapest market environment – as the price of this market is not responsive to this migration. This requires strategies for reducing the possibility of massive migrations throughout the iterations and for making the algorithm move towards the optimal solution. To do so, we limit the maximum migration amounts that can be obtained at each iteration. This limitation imposes that the migration in the current iteration cannot deviate from the one obtained in the previous by more than a share (δ) of the consumer's original demand. This is done by adding the following constraints to the model:

$$m_{i-1,t}^{CII} - \delta \cdot d_t^{CII} \leq m_{i,t}^{CII} \leq m_{i-1,t}^{CII} + \delta \cdot d_t^{CII} \tag{4.30}$$

$$m_{i-1,t}^{L} - \delta \cdot d_t^{L} \leq m_{i,t}^{L} \leq m_{i-1,t}^{L} + \delta \cdot d_t^{L} \tag{4.31}$$

This parameter δ is a smoothing factor in the iterative algorithm and is updated (reduced) in Step 4 until the problem's convergence, as explained next.

4.2.4 Formulation of Step 4

In Step 4, migration of type II captive consumers and free consumers and the Disco's loads are updated, to be applied in the next iteration. Also, it is possible to update the amounts of legacy contracts (this depends on a definition of whether the Disco is able get rid of these contracts or not – to be addressed further).

4.2.4.1 Update of Captive Consumers Type II migration

The Captive Consumer Type II migration amounts are updated for the next iteration, assuming the results obtained in Step 3.

$$m_{i+1,t}^{CII} = \overline{m}_t^{CII} \tag{4.32}$$

Where:

$\overline{m}_t^{CII}$: migration of Captive Consumers Type II to the free market found in Step 3 of the current iteration

The reader should notice that the variable updated is the migration, not the load itself, whose initial value remains fixed in the model.

4.2.4.2 Update of Free Consumers' migration

Analogously, the Free Consumers' migration amounts are updated for the next iteration, assuming the results obtained in Step 3.

$$m_{i+1,t}^{L} = \overline{m}_t^{L} \tag{4.33}$$

Where:

$\overline{m}_t^L$: migration of Free Consumers to the free market found in Step 3 of the current iteration

As in the case of the Captive Consumers, the variable updated is the migration, not the load itself, whose initial value remains fixed in the model.

4.2.4.3
Update of Disco's load considering the migrations

Given the updates in the migration quantities of the regulated and free consumers, the load represented by the Disco, which depends on these variables, is updated as follows:

$$d_{i+1,t}^D = d_t^{CI} + d_t^{CII} - \overline{m}_t^{CII} + \overline{m}_t^L \tag{4.34}$$

4.2.4.4
Possible update of the amounts of legacy contracts (optional)

With respect to the legacy contracts, these are in principle fixed commitments that the Disco has with some of the generators. When the migration occurs, the Disco may hold such contracts, even if this means facing over-contracting. Under this more stable assumption, the amount of legacy contracts would remain fixed in all iterations.

However, it is also possible to assume that the Disco are able to get rid of these contracts – for instance, through mechanisms for selling surpluses, or simply by considering a regulation that would protect Disco from such over-contracting risk and assign it to generators. In this sense, the model allows for the possibility of adjusting the amount of legacy contracts when regulated consumers migrate to the free market. This is shown in the formulation below.

$$q_{i+1,t}^{LEG} = \max\left(q_{1,t}^{LEG} + d_{i+1,t}^D - d_{1,t}^D, 0\right) \tag{4.35}$$

The "maximum" notation in the formula above means that the amount of legacy contracts cannot become negative in case the migration of consumers is higher than the initial legacy contracts' amount.

Despite not being the most common scheme for migration, the model allows this representation if the user so desires. For the purposes of this dissertation, this possibility, which basically allows the migration risk to the generator, will serve as illustration in the case study presented in the next chapter.

4.2.4.5
Update of the smoothing factor δ and convergence check

As explained at the end of Step 3, the migration at each iteration is limited by a deviation to the one obtained in the previous one, given by a share δ of the consumer's original demand. In Step 4, this parameter is reduced (is divided by half) whenever a solution found in a previous iteration is reached again. This prevents Step 3 to present repetitive extreme solutions (usually a pair) without convergence – on the contrary, it allows the algorithm to gradually narrow down the search for the optimal solution.

Not only the δ factor is important for the model to gradually find its solution, but also, given its diminishing behavior, it was selected as the criterion to determine the convergence of the algorithm. That is, when δ is small enough, this means that the migration in the current iteration cannot vary more than this very small factor of the original demand – which means that the problem has already found its optimal solution (given a tolerance level).

Based on empirical experience with the developed model, the initial δ has been selected as 40% (whose "high value" allows for rapidly moving towards the interesting magnitude of migration – e.g., in a case whose optimal result is total migration, a low initial δ would take the model long to approach the interesting "zone" where the optimal solution is). As for the minimum δ, i.e., the one that once reached determines the algorithm's convergence, it was defined, for the most complex cases explored, as 0.001%.

From the previous, we highlight that the factor δ is a tool for the algorithm to move towards the optimal solution, and the selection of its initial value is made only to speed up the convergence – and does not influence the solution itself.

5
Case study of the equilibrium model for the Brazilian market

In this chapter, we present a case study of the methodology presented in Chapter 4, for the Brazilian electricity market. For this, we use real data from a long-term simulation of the Brazilian system, model system agents and emulate their contracting decisions through simulations of the equilibrium model developed, under different assumptions.

The system configuration chosen for the case study is the year 2030, in which there is expansion of the system with respect to the current configuration, as well as updates provided for in the agents' contract portfolios and, especially, when it is expected that there will be a high level of liberalization in the market. Next, we present the system projected for the year 2030, followed by the characterization of the modeled agents. Then we present simulations and case study results.

5.1 Characterization of the case study

5.1.1 System configuration for year 2030

In this section, the demand and supply conditions of the Brazilian electricity market for year 2030 are presented. The selection of this year was made to enable reaching a supply-demand balance, preventing any conjunctural short-term characteristics (such as oversupply, over-contracting in the regulated market etc.).

The system configuration in 2030 is the one projected by the consulting firm PSR in October 2020. To do so, the company uses its internal market expertise and inhouse computational tools OptGen (optimization of the system expansion) and SDDP (optimization of the system's dispatch), for a stochastic representation of the power system operation (200 scenarios were used in this case). For more details on the models' methodologies, see [50, 51] and [52, 53] ,respectively. Although the system configuration for 2030 was confectioned outside the scope of this work, its features are presented next. In Chapters 6 and 7, more detail on optimal expansion planning will be presented and similar exercises will be carried out.

5.1.1.1
Demand in 2030

One of the key points of expansion studies in electrical systems is demand projection. As a developing country, Brazil's evolution of demand is very dependent on the evolution of its economy. The figure below shows the evolution of the historic of GDP growth in Brazil compared to the growth in demand between 2002 and 2019. In the graph, it is possible to observe the strong positive correlation between the variables, for example in recession periods, in 2009 and between 2013 and 2016.

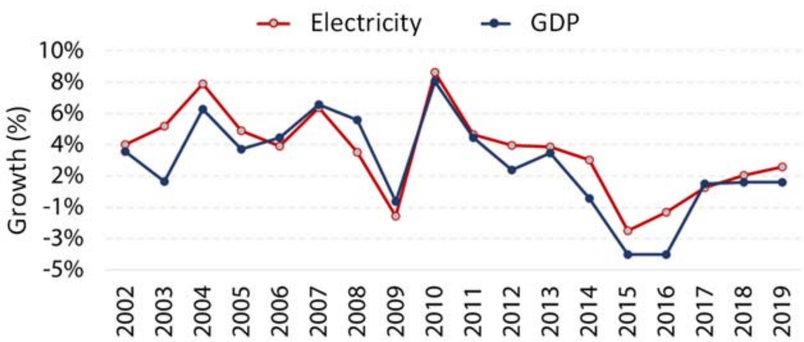

Figure 5-1 – Historic GDP and electricity demand growth in Brazil

Taking 2020 as a starting point, there is an estimated drop in demand of approximately 2% compared to 2019, caused by the pandemic. For the next few years, the average GDP evolution presented by the Brazilian Central Bank is used as a basis for calculating demand, yielding an average growth of 3% per year between 2021 and 2030, with an increase of almost 25 GW-average[2] in demand, as shown below.

[2] GW/MW-average are energy units widely used in Brazil. They are obtained by dividing an energy amount by the corresponding period when it was generated/consumed. For example, 8760 MWh in one year is equal to 1 MW-average.

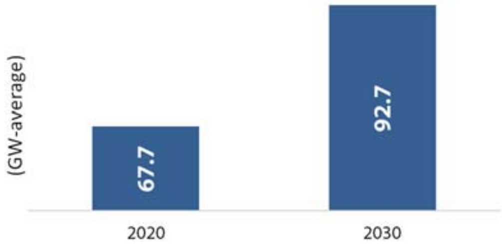

Figure 5-2 – Yearly electricity demand in 2020 and 2030 (projected)

5.1.1.2
Generation mix in 2030

The Brazilian power system is majorly composed by hydroelectric plants, with a share, in terms of installed capacity, of approximately 66% of the mix in 2020. Yet, this share has been reducing over the last 20 years, with the entry of other technologies in the mix, such as natural gas, biomass, wind and solar plants.

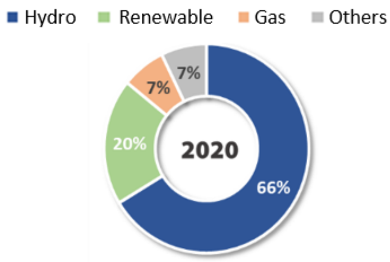

Figure 5-3 – Composition of the Brazilian power mix by technology in 2020

During this decade, no large hydroelectric addition is considered. The expectation is that most of the expansion will be carried out via unconventional renewable plants, especially wind and solar. These technologies have gained ground in the Brazilian market due to the competitive costs they have been showing both in energy auctions to meet demand in the regulated market and in meeting the demand of consumers in the free market. In addition, many consumers have opted for solar categorized as distributed generation.

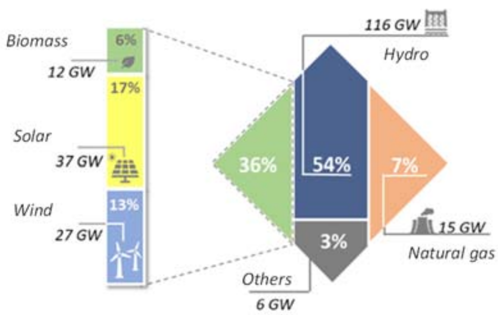

Figure 5-4 – Composition of the Brazilian power mix by technology projected in 2030

5.1.1.3
Generation and spot prices

Through the SDDP model it is possible to simulate the dispatch under uncertainty for the projected expansion. When analyzing the participation of each technology to meet demand, it is observed that the system still has a relevant share of renewable generation, mainly from hydroelectric plants. Even with a reduction in their participation in the energy mix, in 2030 hydroelectric power plants are still responsible, on average, for 65% of the total energy generated according to the projection made.

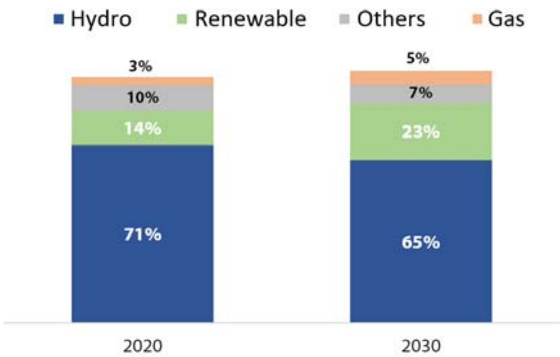

Figure 5-5 – Composition of the yearly average generation in the Brazilian power system by technology in 2020 and projected in 2030

For the system projected in the year 2030, it is possible to extract the spot prices for each of the four regions in Brazil (Southeast, South, Northeast and North). Due to the high participation of hydro plants in the system, the spot prices may have a large dispersion with respect to the average value, since the supply-demand balance is greatly affected by the availability of hydro (and other natural) resources. The figure below shows the yearly spot price for 2030, in the Southeast region, where there is an average equal to 123.5 R$/MWh, reaching 239 R$/MWh in more critical scenarios and 44 R$/MWh in scenarios with greater water availability. For the sake of simplicity, prices in this graph and in the study in general will be the ones from the Southeastern region, which is the one with greater demand in the Brazilian system.

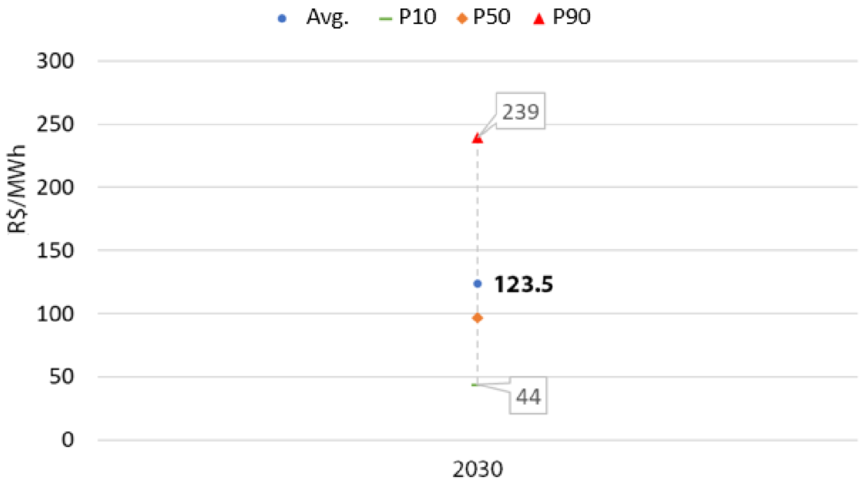

Figure 5-6 – Dispersion of yearly spot prices in 2030

In addition to the yearly dispersion, prices also have a monthly profile, with lower levels in the first semester (wet period), and higher ones in the second (dry period). This is a characteristic still noticeable in 2030, but it is gradually changing with the increase in the participation of wind power plants, which have a different (complementary) seasonality than the hydro, usually with greater generation in second half of the year.

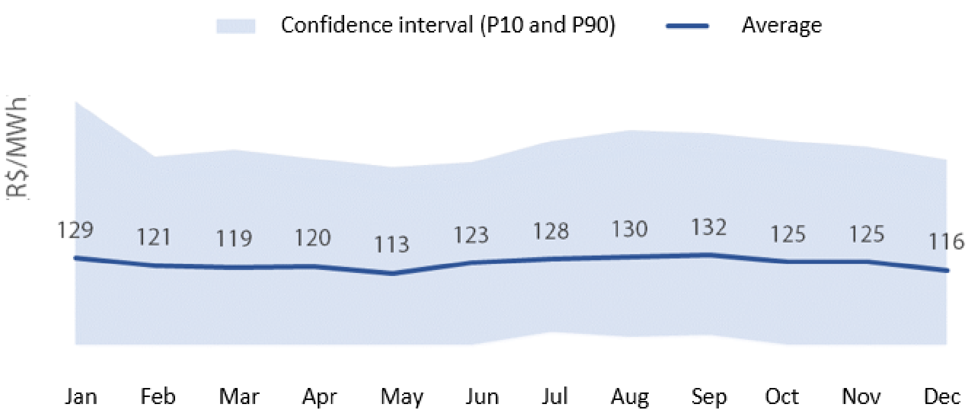

Figure 5-7 – Monthly spot prices (average and confidence interval formed by the 10th and 90th percentiles)

5.1.2 Characterization of the agents in the equilibrium problem

An important feature of the system concerns the physical guarantee of the plants. The amount of physical guarantee for each generator will determine the amount of energy that can be sold. According to the expansion presented above, the system will start from a supply of 87.0 GW-average in 2020 to 99.3 GW-average in 2030. The projected demand for these years is respectively 67.7 GW-average and 92.6 GW-average, the which demonstrates a reduction in the system's oversupply.

For the equilibrium model, the ten largest generators were selected for individual representation, according to their respective capacities installed in 2020. Subsequently, the portfolio of each generator was defined, based on their current and planned plants. For all plants, their physical guarantee for the initial condition of 2020 was defined, as well as the changes in physical guarantee foreseen throughout the study horizon. The reason for individually representing the large generators of the system is to bring realism to the modeling, capturing the portfolio effect of its plants and portraying its decision-making process more accurately. By this, it is understood that the "generators" in the equilibrium model are not individual plants, but agents in control of their own portfolio of generation assets (and contracts). The other agents were separated by blocks of technology. In total, 17 generators were represented in the model, as shown in the figure below.

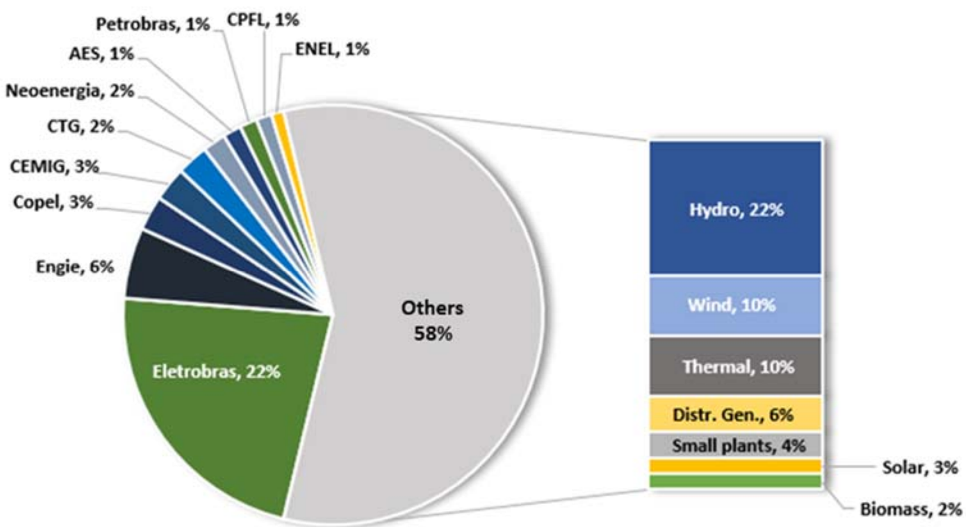

Figure 5-8 – Participation of generators in the system's total physical guarantee projected for 2030

Moreover, the "generic plants" included in the system expansion (i.e., additions there are not yet planned nor owned by specific agents) were also included in the "Others" group, as it is not possible to infer *a priori* which company will be responsible for projects in a distant future.

A single Distributor agent ("Disco Brasil") was modeled, which holds an initial portfolio of legacy contracts that will influence the decision-making of the other agents. It was decided to consider one single distributor for reasons of simplicity and to facilitate the analysis of results. Also for the sake of simplicity, all legacy contracts were registered at the same price, defined based on the Disco's average price mix considered for the year studied (R$ 187.1/MWh). The contracts were assumed with a flat hourly profile (one constant value for all hours), and the quantities allocated to each generator are presented in the table below. These legacy contracts for 2030 total 48.5 GW-average. Their distribution among the modeled agents is shown below.

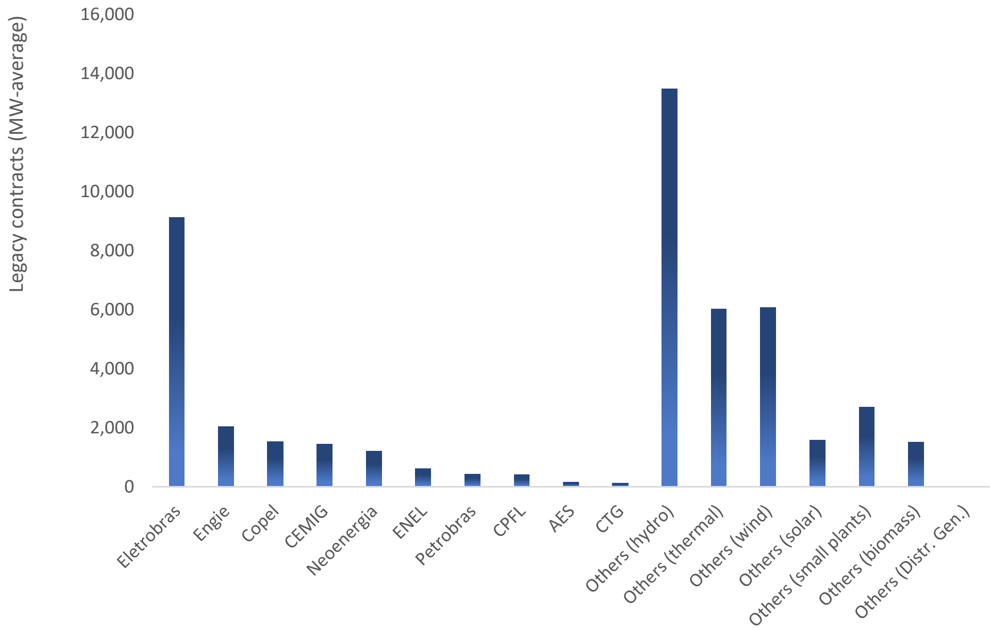

Figure 5-9 – Amounts of legacy contracts by company/groups considered in 2030

For the new free market contracts that will be optimized by the equilibrium model, one annual validity was considered, for the entire year 2030. The contracting obligation for demand (distributor or agents in the free market) was also considered, based on the average annual demand of these consumers. Such contracting levels must be supplied through modeled contracts, whether legacy ones or those determined by the model's solution, also respecting the limitation of contracting to the physical guarantee on the generator's side.

As explained above, the Disco's demand is defined as the sum of the demands of the Captive Consumers Types I and II, which represent the initial demand of regulated market. In recent years, some of these consumers have been migrating to the free market, which is expected to keep happening until 2030.

Based on the level of tension and the growth in consumer demand, for the year 2030 the ratio of Captive Consumer I, Captive Consumer Type II and Free

Consumers will initially be 46%, 14% and 40%, respectively – this implies a free market potential of 54% of total demand (estimated as the total high and medium voltage groups' demands). So, it is assumed that, initially, the free market is responsible for 40% of the load and the regulated for 60% (out of which 14% can still migrate to the free market). This is the starting point of the equilibrium model: despite having a proportionally smaller regulated market than observed historically (about 70% in 2020), it is considered conservative enough, in the sense that it is possible that there is a relatively greater demand in the free market by the beginning of 2030 – however, this was a conscious choice to allow the model to determine eventual migrations. The figure below shows the demand for the three types of consumers on a monthly scale, along with the system's physical guarantee supply.

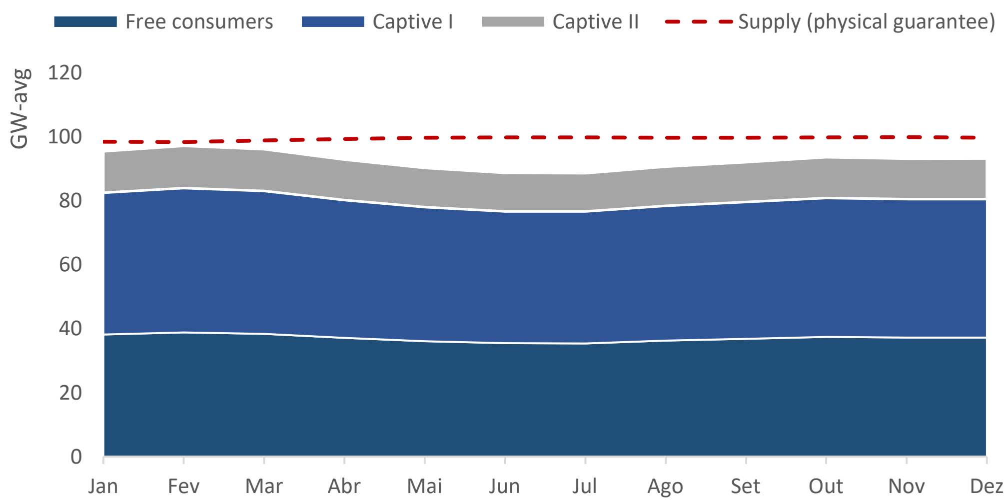

Figure 5-10 – Monthly demand by consumer type and supply of physical guarantee in the system projected for 2030 (initial condition for the model)

Regarding the risk aversion parameter λ, it was considered equal to 0.8, for all agents. This means that the objective function of all agents (Disco, regulated consumers, free consumers and generators) has a weight of 80% to maximize the expected value and 20% to maximize the CVaR. In the simplified case study, several analyses were carried out conceptually showing the impacts of the variation of this parameter. For the case study of the Brazilian market, it was decided to fix this value in a level interpreted as reasonable (i.e., a typical agent would have a greater focus on its average result, but would also consider the importance of protecting themselves from extremely negative scenarios), and focus the analysis on the dynamics of the Brazilian market.

5.2 Simulations and analyses

In this section, we present simulated case studies for the Brazilian system. We present several types of simulations for the system described above, which vary according to the regulatory treatment given to legacy contracts and the liabilities associated with them.

As regulated consumers already have a considerable amount of these contracts and it is natural that part of consumers would want to migrate to the free market, in case it has lower contract prices than the Disco's tariff, an important design element to be considered in the model is how consumer migration will be handled in contractual terms. The options covered in the case study are described below and their results will be presented in the following subsections:

1. Inclusion of a legacy contract adjustment mechanism. At the time of a migration to the free market, a corresponding volume of legacy contracts is undone, proportionally to the total amount of these contracts, and this offer (physical guarantee) automatically returns to the free market. This option allocates the risk of migration to the generators, who must seek to sell this offer in the free market, when having contracts broken in the regulated one.

2. Maintenance of legacy contracts with the Distributor. At the time of a migration, the Distributor retains the legacy contracts, which contributes to a more expensive regulated tariff, especially if this migration makes it over-contracted. This option allocates the risk of migration to the distributor, which passes on to captive consumers who cannot migrate (Type I) and to those who have not yet migrated, being increasingly harmful to these agents the greater the migration. An increase in the distributor's over-contracting limit is assumed to guarantee the transfer of these costs to the tariff.

3. Alternatives for socializing the liabilities associated with migration. In this case, the contractual surplus is sold by the Disco, keeping the payment to generators with legacy contracts unchanged. The liability left by the consumer who migrated to the free market (due to the commitment assumed by the Distributor corresponding to its demand), whose cost is equivalent to the difference between the price of the legacy contract and the market price, is divided in different ways between market agents, through a specific charge created for this purpose. In this

third section, we propose and model different regulatory alternatives for the allocation of such costs: (i) division by all demand, that is, captive and free consumers; (ii) allocation to the regulated consumers and to the consumer who decided to migrate, exempting from payment only consumers originally in the free market; (iii) payment of this liability by free consumers only (old and newly migrated), removing this burden from the regulated market; and (iv) the consumer who opted for the migration takes with him the costs associated with this decision, offsetting the potential benefits of closing contracts that are less costly than the tariff. At the end of the section, a fifth option is shown, which is basically a particular case of option (iii) of allocation to the free consumers.

5.2.1 Inclusion of a mechanism for adjusting the legacy contracts

When running the model for this case, it was observed that all captive consumers type II decided to migrate to the free market. Due to this, there is a reduction in the distributor's demand, reducing the amount of legacy contracts and freeing the generators to negotiate the corresponding volumes in the free market.

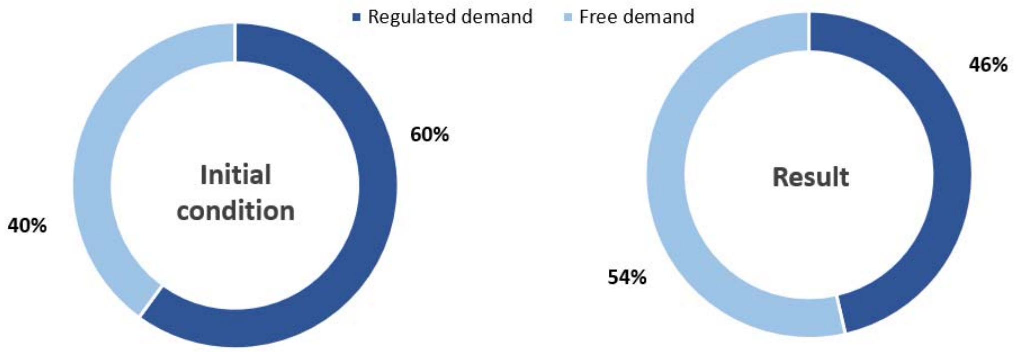

Figure 5-11 – Shares of regulated and free markets in total demand with adjustment in legacy contracts

Note, therefore, that demand, which started out divided into 60% regulated and 40% free, assumes a condition of 46% regulated and 54% free, since the since all eligible regulated consumers decide to migrate to the free market, which totals 12,573 MW-average (14% of the system's total demand).

It is important to recall that, in this case, at the time of migration, the corresponding supply of legacy contracts is made available on the free market. For this, it was considered that each legacy contract is terminated in the same proportion until the total amount of migration (see sub-section 4.2.4.4 in Chapter 4). As a result, the number of legacy contracts with the distributor also decreases, from 47,972 MW-average to 35,399 MW-average, as shown below.

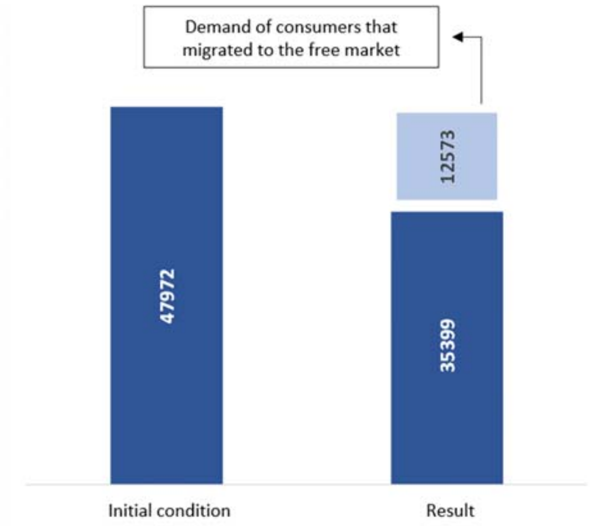

Figure 5-12 – Amounts of legacy contracts with the Disco (MW-average)

Such consumer movements occur due to equilibrium prices in the markets. The average price of contracts in the free market was R$ 140.8/MWh, presenting a risk premium of R$ 17.3/MWh with respect to the average price of the spot price in the year (R$ 123.5/MWh). In the regulated market, the final tariff is R$ 178.9/MWh, a premium of R$ 55.4/MWh. It is this greater attractiveness of the free market that generates migration.

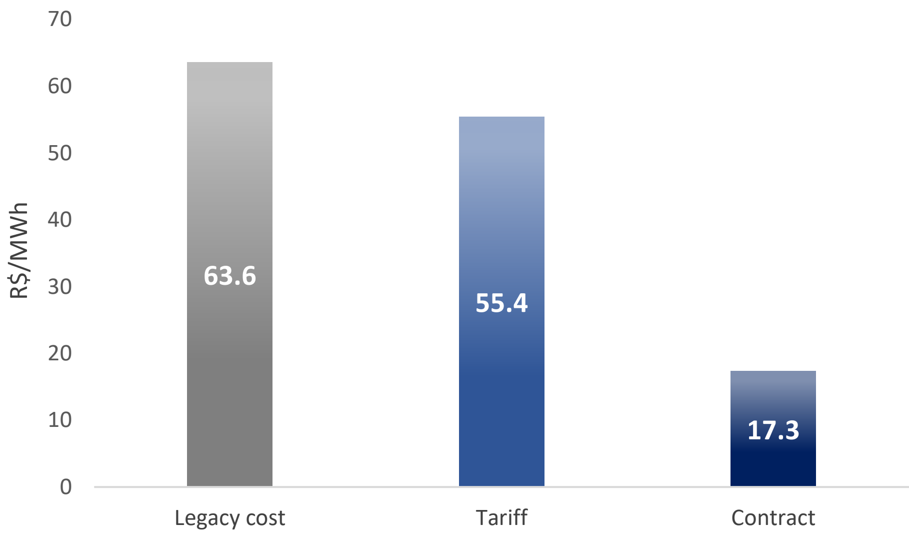

Figure 5-13 – Risk premium with adjustment in legacy contracts

Looking in more detail at market prices, it is noted that the free market presented a positive risk premium. One of the reasons that contributes to the premium is the migration of consumers which, despite being accompanied by a corresponding availability of supply in this market, leaves the free market always tight with respect to its supply-demand balance, since the projected system is well adjusted, as described in section 5.1.1.

In the case of tariffs, the final price obtained is basically a weighted average of legacy contracts, which had a price of R$ 187.1/MWh, as shown in the assumptions section (premium of R$ 63.6/MWh), for the price of the contracts themselves determined by the balance of the model (R$ 140.8/MWh), which are purchased by the distributor to complete its demand, generating a final tariff of R$ 178.9/MWh (premium of 55.4 R$/MWh), having, therefore, a considerable reduction in its value, precisely because the Distributor, in Step 1, fulfills is portfolio with contracts at the prices set by the model.

Therefore, we have a case in which the free market has more attractive prices than the distributor's tariff, which generates a total migration of regulated consumers who can do so. This migration proportionally undoes legacy contracts, which makes the Disco's final portfolio cheaper than the initial legacy contracts.

5.2.2 Maintenance of contracts with the Disco

In this case, consumer migrations do not entail changes in the Disco's legacy portfolio, which continue with this more expensive contractual amount in hand. This is the situation that occurs in most migration situations in the market nowadays, as distributors usually have long-term contracts that cannot be changed.

For the presentation of this option, three cases are shown, mainly due to the fact that, when retaining legacy contracts, there is a tendency to over-contracting in the regulated market and lack of supply in free market to absorb all consumers who would like to migrate. Therefore, simulated three cases are simulated, in which the amount of physical guarantee that could absorb the migration of initially captive consumers type II was varied. The intention to increase the corresponding physical guarantee is to emulate a greater availability of offer in the free market if the market shows signs of such need. In practice, such signals of needs for more supply in the free market would incentivize the entry of new generation (this is a topic addressed in Chapters 6 and 7).

5.2.2.1
Base Case: standard system configuration

In this first simulation, called the Base Case, we consider exactly the supply and demand configurations described in section 6.1, but with the maintenance of legacy contracts in case of migration (unlike in the previous section). In this case, the division between the free and regulated markets ends at 48% and 52%, respectively, that is, the regulated market remains larger.

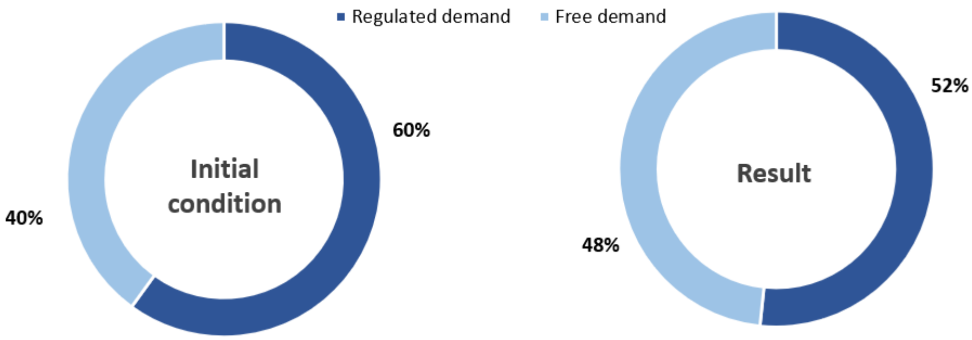

Figure 5-14 – Shares of regulated and free markets in total demand with maintenance of legacy contracts (Base Case)

This distribution results from a migration of 7,716 MW-average, substantially lower than the previously found (12,573 MW-average). However, this only partial migration of type II captive consumers does not occur due to a price balance between the market. On the contrary, the difference between the distributor's tariff and the price of contracts in the free market increased, as shown below.

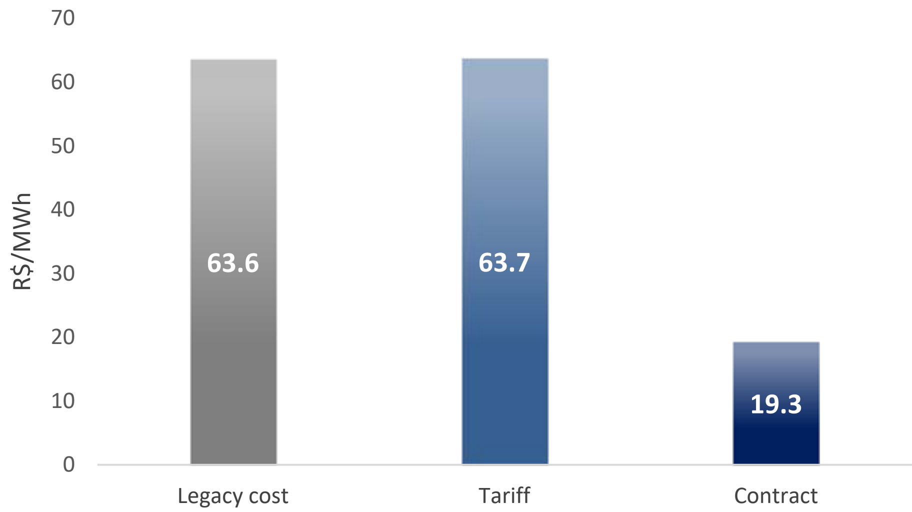

Figure 5-15 – Risk premium with maintenance of legacy contracts (Base Case)

Analyzing the results found, we notice that not only the distributor's tariff is higher than the price of contracts in the free market, but it even slightly exceeds the initial cost of legacy contracts. This is because the migration of consumers to the free market generates a small over-contracting in the regulated one and, thus, legacy contracts whose prices were R$ 187.1/MWh (premium of R$ 63.6/MWh) end up generating a final tariff of R$ 187.2/MWh (premium of R$ 63.7/MWh), precisely because the number of legacy contracts (numerator) remains unchanged, but is now distributed over a slightly smaller number of regulated consumers (denominator).

Still, the reader may wonder why migration was not even greater then, given that the free market is more attractive (and regulated even less) than in the previous case. This is simply because the simulated system has a physical supply limitation in the free market, with a well-adjusted supply-demand balance. This only allows for a small over-contracting of the distributor and does not allow for the complete migration of type II captive consumers, as there would be no physical guarantee for the execution of more contracts in the free market (as the Disco is holding the legacy contracts). In turn, this total contracting of the free market supply makes the risk premium in this environment assume the value of R$ 19.3/MWh, while in the case with replacement of legacies in this market, the premium was R$ 17.3/MWh.

However, as in practice it is to be expected that these price signals would generate greater availability of supply in the free market, we show below other cases in which we consider more supply in this market for the absorption of consumers who would like to migrate.

5.2.2.2
Sensitivity 1: addition of physical guarantee to allow total migration

As seen in the previous case, the migration of consumers to the free market without replacement of physical guarantee in the free market generates a greater difference between markets and tends to leave the regulated market over-contracted. However, in the planned system, there is not enough supply in the free market to absorb this demand. In this Sensitivity 1, we assume that there is enough physical guarantee in the free market to sell contracts to all consumers who want to migrate. For this, an additional 4,857 MW-average of supply in the system was considered, that is, 12,573 MW-average (total demand of Captive Consumers type II) minus 7,716 MW-average (how much it was already possible to migrate in the Base Case), distributed proportionally to the initial physical guarantee generators, thus maintaining the participation of each one in the market, which would allow the total migration of type II captive consumers, should they decide to migrate to the free market. It should be noted that this exercise did not change the physical supply nor the prices in the short-term market, that is, the impact of increasing the supply of physical guarantee was purely commercial.

The migration result was, as expected, the total migration of 12,573 MW-average. This results in 54% for the free market and 46% for the regulated, while the Base Case presented the proportion of 48% free vs. 52% regulated.

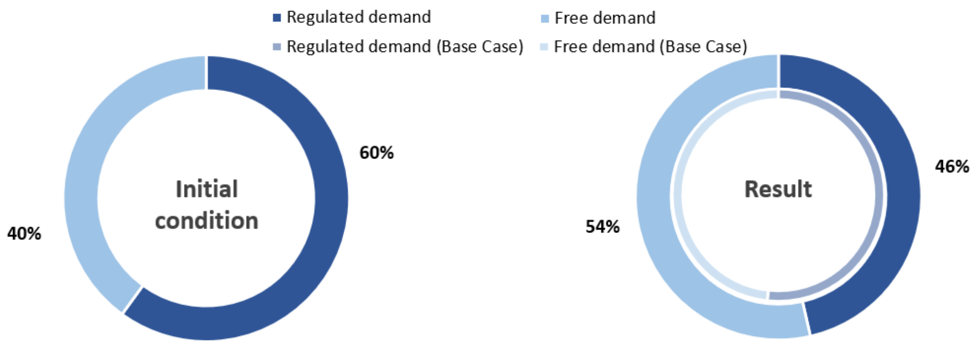

Figure 5-16 – – Shares of regulated and free markets in total demand with maintenance of legacy contracts (Sensitivity 1)

On the other hand, although this proportion of 54% for the free market and 46% for the regulated was the same found in the case with where the legacies were broke and the supply was resent to the generators for sales in the free market (section 5.2.1), there was a change in prices, particularly in the regulated tariff, which increased substantially, even with respect to the Base Case with maintenance of legacies, which had presented only a slight tariff increase.

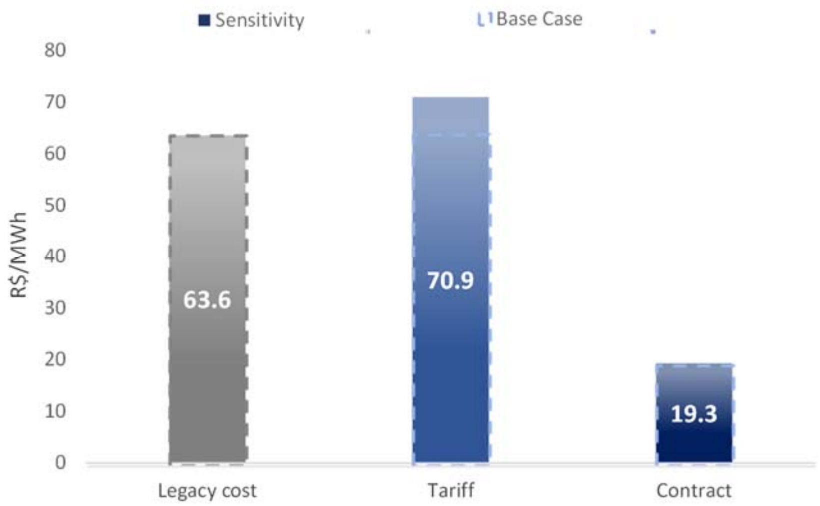

Figure 5-17 – Risk premium with maintenance of legacy contracts (Sensitivity 1)

The reason for this result, with a tariff of R$ 194.4MWh (premium of R$ 70.9/MWh), much higher than the free market price and than the original price of the legacy contracts, is an exacerbation of the result already found in the Base Case. That is, the remaining regulated consumers (in this case, only captives type I) have to absorb the entire cost of the legacy contracts. With this lower denominator to pay for the legacies, the tariff cost goes up – and in this case, even more, as there are now no more Type II captive consumers to help share this burden. In the free market, on the other hand, prices remain equivalent to those found previously, as the supply needed for migration was added, but without generating surpluses in this market, which continues with a tight supply-demand balance and a premium of around R$ 19.3 /MWh. In other words, the physical guarantee limit constraint for the sale of energy from generators remains active, resulting in the same risk premium.

5.2.2.3
Sensitivity 2: unlimited physical guarantee

In the previous sensitivity, it was seen that an additional supply in the free market could allow a total migration of type II captive consumers, which increases the regulated tariff and increasingly dissociates from the prices practiced in the free market. Even so, the amount of physical guarantee added in the free market was just enough to allow this total migration, but it remained with an adjusted supply-demand balance.

In this second sensitivity, we emulate a situation where it is possible for the generator to sell energy above its physical guarantee in the free market (in other words, a contracting environment not limited to the physical guarantee). The objective is to show how the free market price can be indefinitely dissociated from the regulated market in a regulatory environment where there is no mechanism for the replacement of the physical guarantee of market legacies or any sharing of this liability among agents. Still, it is noteworthy that this case does not have additional generation (the system dispatch remains the original, as well as in the previous sensitivity). However, it is possible to give an indication of results that may occur when the system has more supply available to enter into contracts.

The migration result found was, as expected, the same 12,573 MW-average found previously, which corresponds to the total demand of type II captive consumers. This generates, as seen above, a proportion of 54% of consumers in the free market and 46% in the regulated one.

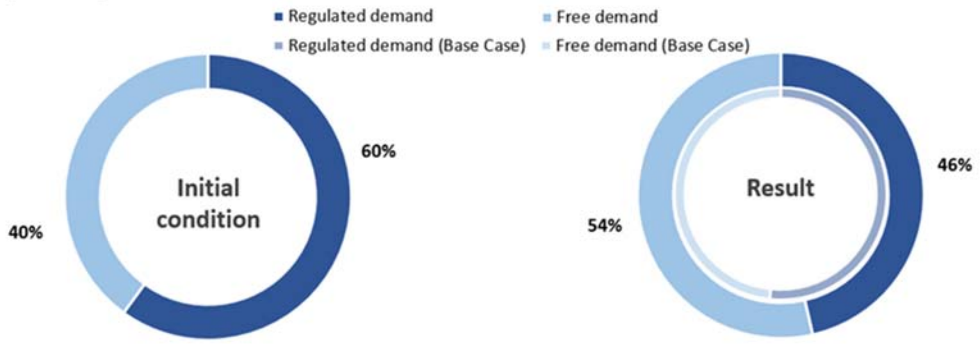

Figure 5-18 – Shares of regulated and free markets in total demand with maintenance of legacy contracts (Sensitivity 2)

With respect to prices, tariffs do not show changes with respect to the previous sensitivity, in which a total migration of type II captive consumers had already been possible, making the tariff basically the distribution of legacy costs to captive consumers type I, in an over-contracting situation. Free market prices, on the other hand, had a significant drop, as shown below.

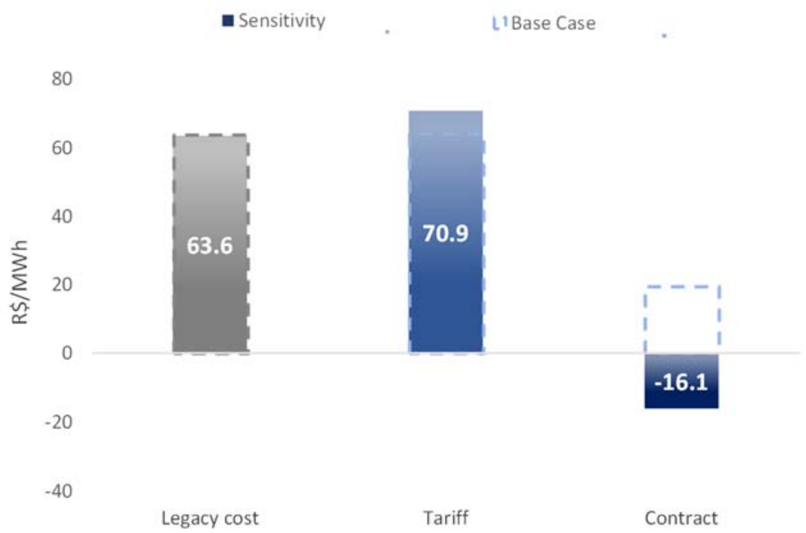

Figure 5-19 – Risk premium with maintenance of legacy contracts (Sensitivity 2)

This huge price drop in the free market, which reaches a level of R$107.4/MWh, is due to the non-limitation of contracting to the physical guarantee by the generators. With this, the system has more (unlimited) physical guarantee to enter into contracts, which leaves the supply-demand balance of this product looser (although physical supply has not been added in the dispatch model). Although this was a theoretical result, given that physical back-up is required for the celebrating contracts in Brazil, the main message of this case is the extreme dissociation that the free and regulated markets can assume, with free consumers benefiting from competitive prices while the captives carry the burden of high tariffs, potentiated by the distributor's involuntary over-contracting.

5.2.3 Alternatives for allocating legacy costs

In the exercises presented so far, all the losses associated with the migration of consumers from the regulated to the free market are allocated to a single type of agent. Firstly, in section 5.2.1, as already mentioned, the risk lies entirely with the generator, due to the existence of an adjustment mechanism in the legacy contracts, causing the contract to be terminated and forcing it to seek to resell this energy at lower values in the free market. In the simulations of section 5.2.2, migration becomes a burden for the Disco (and consequently for captive consumers who cannot migrate), which retains the legacy contracts already signed, but now with a lower demand to pay it, increasing the “slice” (the tariff) to each remaining consumer.

In this section, different ways of allocating this liability among market agents are modeled, in order to: (i) dilute these costs, reducing the losses individually faced by each agent and (ii) make the consumer who opted for migrating to be held responsible, at least partially for the financial consequences of their decision.

In this way, the liabilities of legacy contracts corresponding to the energy contracted by the Distributor to satisfy the demand of consumers who were originally in the regulated environment and migrated to the free market will be redistributed, through a regulatory charge, in different ways, which are detailed and modeled in the next subsections.

5.2.3.1 Case 1: costs allocated to the entire demand

One possibility is to carry out a complete socialization of the costs associated with migration, allocating a charge to be paid by both the free and the regulated markets. The increase in the amount paid individually is small, given the greater number of payers.

In addition, as costs increase equally in both markets, the addition of this charge does not change the relationship between prices charged in the free market and the Disco's tariff, with the former remaining considerably cheaper. Thus, as expected, the result is a total migration of eligible captive consumers to the free market and the same price equilibrium obtained in Section 5.2.1 – however, producing a charge of R$ 6.3/MWh to be paid by all consumers (instead of allocating the cost to the generators), as shown in the figure next.

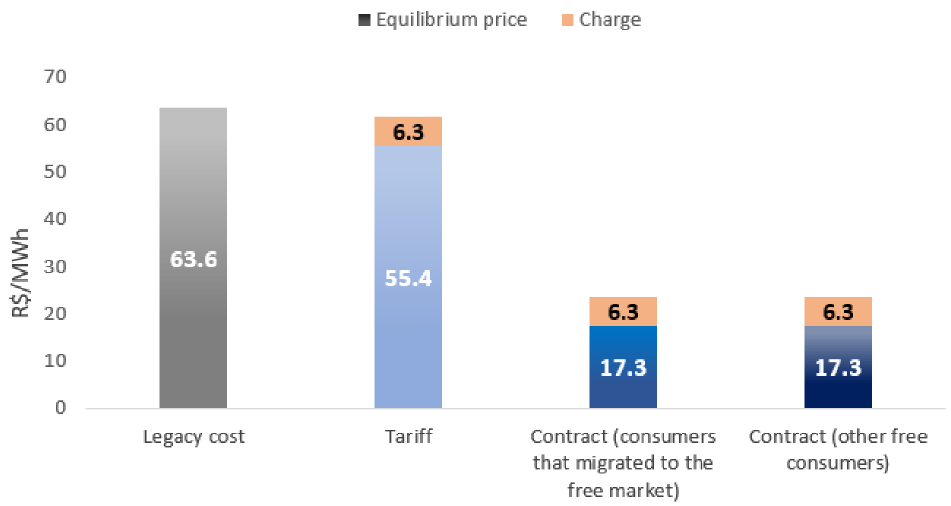

Figure 5-20 – Risk premium and charges assigned (Case1)

5.2.3.2
Case 2: costs allocated to the regulated market and to the consumers that migrate

A second approach involves redistributing the migration costs only between the Disco (regulated market) and consumers who migrated to the free market, exempting agents originally from the free market from paying. The argument for this design is that this last portion of the demand should not be held responsible for issues involving solely the Distributor (which signed the legacy contracts and is responsible for honoring these commitments) and the newly migrated consumer (which harms the Distributor when seeking more attractive prices in the free market, leaving the remaining regulated users to bear the portion of legacy contracts corresponding to its demand).

Again, similarly to the previous case, the price of free market contracts paid by the consumer who opts for the migration, including the charge, remains substantially lower than the Distributor's tariff, even before adding this additional component. With this, there is a total migration of the same 12,573 MW-average captive consumers type II to the free market, who pay, together with captive consumers type I, a charge in the amount of R$ 10.5/MWh, as illustrated below.

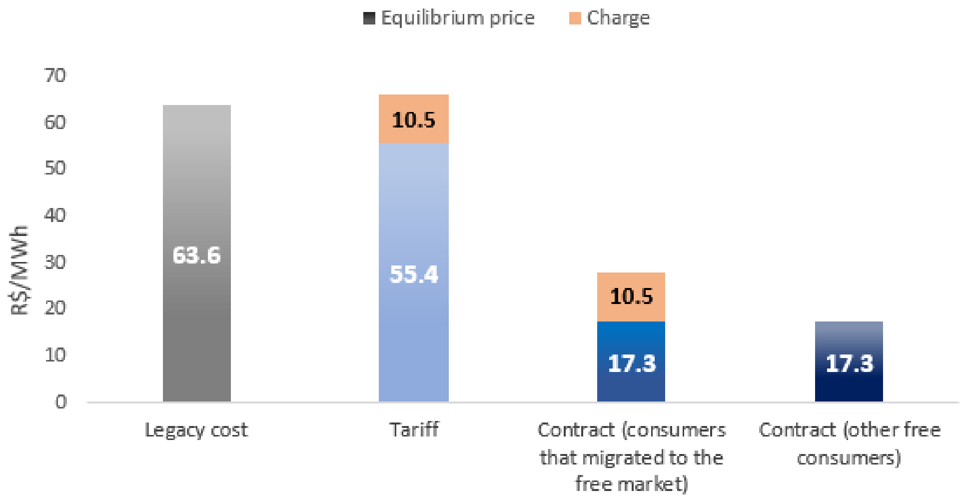

Figure 5-21 – Risk premium and charges assigned (Case 2)

5.2.3.3
Case 3: costs allocated to the free market

Another possibility is to transfer the burden produced by the migration of captive consumers to the free market to the entire free market, including the newly migrated agents themselves. This case would represent a sort of "historic repair", through which free consumers would be "rewarding" the Distributor for enabling the system expansion through long-term contracts, contributing to the supply adequacy, which has been benefitting the whole system, without the free consumers assuming any commitments or risks.

However, as shown in the Figure, the increase in the charge of R$ 11.7/MWh in the price of free market contracts is still insufficient to balance the amounts paid in both markets. Thus, once again, this difference represents incentives for the eligible regulated consumers to migrate completely to the free market.

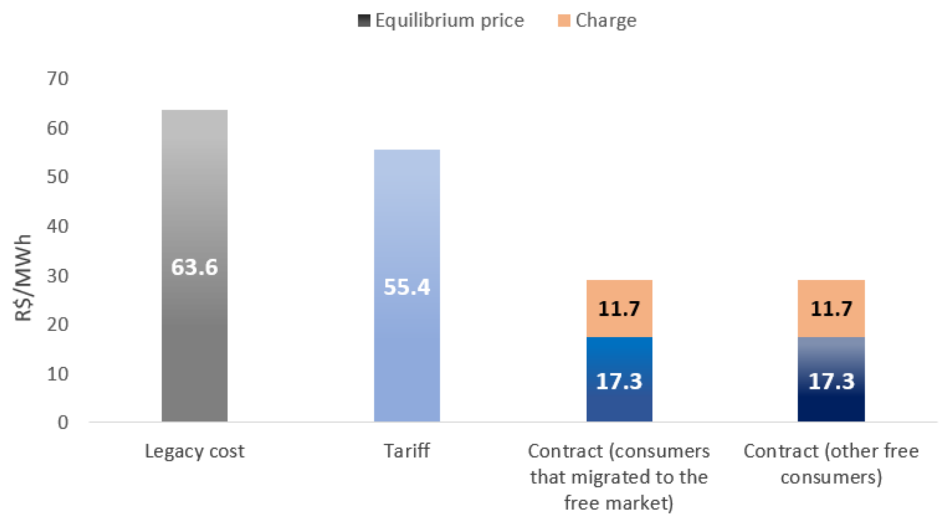

Figure 5-22 – Risk premium and charges assigned (Case 3)

5.2.3.4
Case 4: costs allocated to the consumers that migrate

Finally, in a more extreme case, the liabilities of legacy contracts associated with the migration could be entirely allocated to the consumer who decides to migrate. In that case, the agent responsible for producing this cost would also carry the duty to compensate the Distributor for this loss.

This type of allocation produces results that are quite intuitive, but not necessarily interesting: by being fully responsible for bearing the costs, the benefits of migration are automatically nullified, completely eliminating its attractiveness. In this way, the result of this regulatory alternative was a configuration identical to the initial one, since there are no migration incentives.

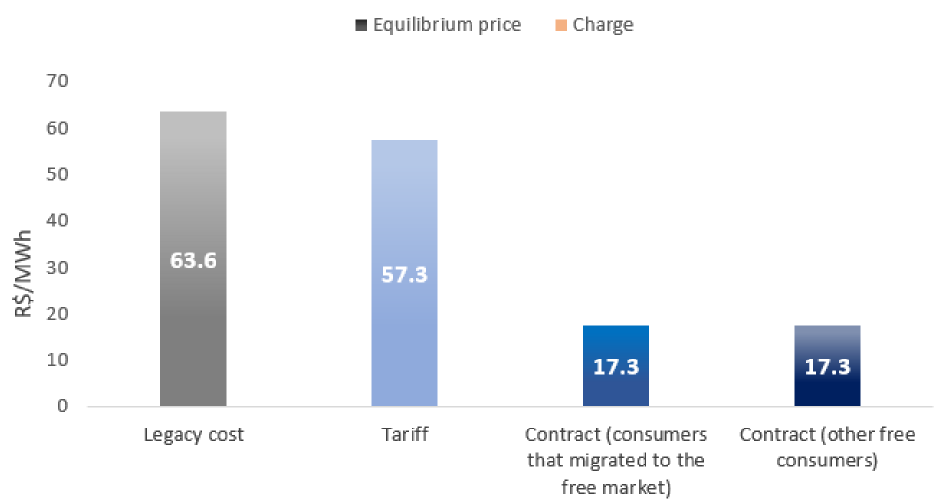

Figure 5-23 – Risk premium and charges assigned (Case 4)

5.2.3.5
Case 5: costs allocated to the free market in a descending price/longevity order

After analyzing cases 3 and 4, it is possible to reach a direct conclusion. As the supply-demand balance in the free market is maintained (given that, upon migration, the physical guarantee of legacy contracts will be resold by the distributor to consumers in this environment), in both cases, the price in the free market (excluding the charge) remained the same. And then, the difference between the price of the legacy contract and the price of contracts in the free market, which will be divided into the charge, will be prorated by the consumers in some stipulated way (we tested some of them in the previous subsections). Case 4 shows the intuitive result that, if the payer of the charge is only the consumer who migrates, this consumer fully bears this difference, which would leave them indifferent between migrating or not. In case the entire free market shares this charge with along with this consumer (Case 3), the denominator that will pay this cost increases, and the charge decreases. Thus, and given again that the price of the contracts in the free market (without the charge) remains unchanged, the free market remains more cheaper than the regulated, even if the charge is allocated to free consumers, as they will pay less than the difference between the price of the legacy and the equilibrium price of this market. Thus, in all cases, the solution was a complete migration, as the free market always remained more competitive.

In order to investigate ways of actually achieving an equilibrium between the two markets in the model, that is, balancing costs among all system consumers, equaling their payment obligations, such that migration would occur up to a certain point and then would stop being attractive, we propose this additional Case 5. The proposal is that, similarly to Case 3, the charge (difference between cost of legacies caused by over-contracting and the equilibrium price of the free market) is prorated by all free consumers. However, in this case, the calculation of the cost of over-contracting does not consider that all legacy contracts have the same price: a price distribution of legacy contracts is adopted, and the allocation of legacy costs is based on the descending order of their costs (which has a strong correlation with the longevity of contracts, in the sense older contracts tend to be more expensive). The rationale for this approach is to consider a merit order for legacy contracts in order to increase the cost signal of legacy costs to the free market and thereby reduce the cost imbalance between contracting environments, avoiding the "death spiral" (that is, a full migration that does not lead to a balance). Additionally, this approach is a proxy for allocating to the free consumers primarily the energy costs that were contracted in the past to serve these consumers, when several of them they had not yet migrated. For example, in the first Brazilian new energy auction, held in 2005, energy was contracted for delivery in 2008 for 15 years. Such legacy costs would be prioritized in the calculation of the charge, since this energy was contracted with the objective of also supplying consumers who migrated to the free market environment after 2008.

To model this case, unlike the previous ones, we do not use one legacy contract per generator with an average price of R$187.11/MWh. Instead, based on a database of individual contracts, price ranges were identified. In more detail, prices of legacy contracts were divided into four groups, and four contracts per generator were assigned, instead of one. It should be noted that, just as using only the average price was a simplification, this is also a simplified way of dealing with the dispersion of contract prices (as the approach is not individualized in contracts, nor the allocation customized for each generator), but which allows for a differentiated treatment of legacy contracts by segments. The figure below illustrates the price ranges found (uniform segments of R$ 65/MWh):

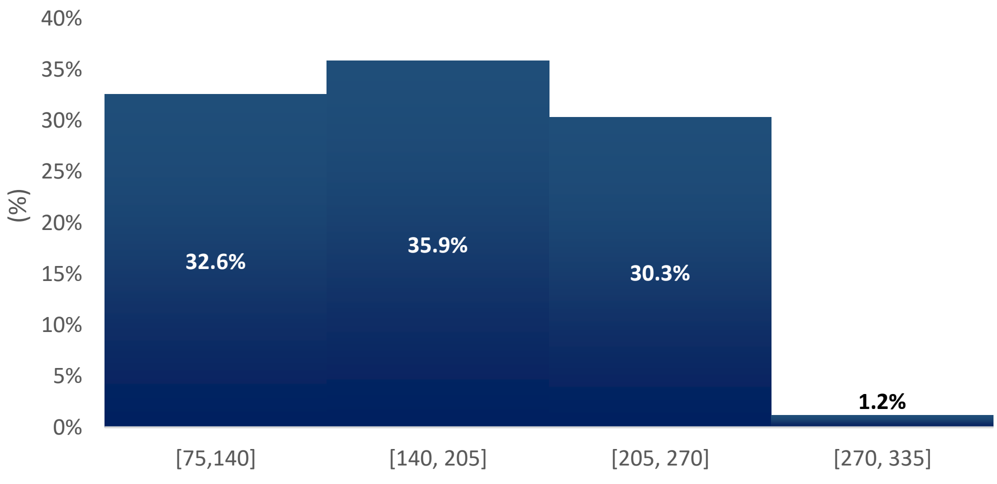

Figure 5-24 – Segmentation of legacy contracts proposed in Case 5

The next step was to define a representative price per range. Taking an average value per interval and performing a slight normalization, we arrive at the following representative values for each interval: R$ 116.3/MWh, R$ 187.0/MWh. R$257.7/MWh and R$328.9/MWh. The average of these values, weighted by the frequencies shown in the histogram above, result in the average price of legacy contracts used in the previous cases (R$ 187.11/MWh). In this way, for each generator, we replace its individual contract at the mentioned average price by four contracts whose shares in the total are shown by the histogram, and with the representative prices mentioned above.

With the database updated, we ran the model. In this case, then, the migration of the captive consumers type II to the free market allows the Disco to resell the most expensive contracts, in a descending order, to the free market. The results, starting with the final amounts of regulated and free markets, are shown below.

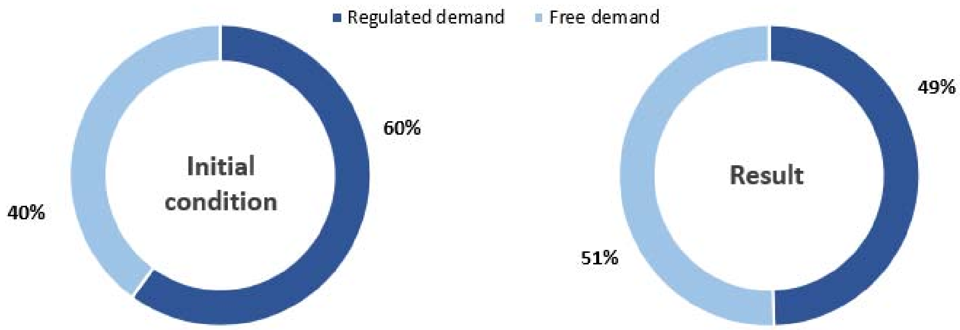

Figure 5-25 – Shares of regulated and free markets in total demand in Case 5

Unlike most of the previous cases, in which there was a total migration of the eligible captive consumers to the free market, in this one there is a migration of 9,980 MW-average (of the total 12,753 MW-average of consumers of this type), resulting in a final condition of 49% regulated and 51% free (instead of 46% and 54% of the previous cases).

This is because, when there is migration, the charge added to the free market comes from the difference between more expensive contracts and the average price in this market, which simultaneously makes the free market more expensive and the regulated one cheaper, until a price balance is reached between them, as shown below.

This result is more remarkable than what a 3% difference of participation in the free market may indicate (51% instead of 54% in the previous cases). The result of the previous cases was 54% because the eligible portion of the captive market is limited (captive consumers type II); otherwise, they would result in 100% migration to the free market. Thus, the present case shows the possibility of price equilibrium, instead of a full migration of whatever eligible demand, which was the result of the previous cases analyzed.

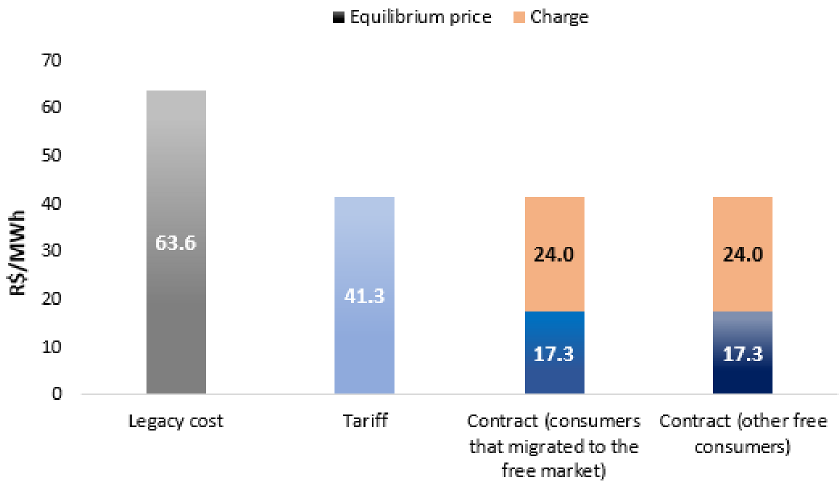

Figure 5-26 – Risk premium and charges assigned (Case 5)

As expected, the amount paid by consumers who migrated and by the initially free consumers is the same, given that the legacy cost is assigned to the entire free market (original free consumers and the newcomers). In this case, however, they pay a total price of R$ 164.8/MWh (the premium of R$ 17.3/MWh plus a charge of R$ 24.0/MWh), which equals the distributor's tariff (whose total premium drops to R$41.3/MWh). The premium of R$ 17.3/MWh was maintained because the supply-demand balance in the free market remains the same (migration is compensated by equivalent extra supply made available in the free market). The moment the free market, including the charge, is no longer more attractive than the tariff, migration ceases and market equilibrium is reached.

It is also important to highlight that, in order to achieve balance (without a simpler solution of total migration), it was necessary for the model to seek to reduce the gap of the iterative process until the convergence between the free and the regulated market values (see Chapter 4 for the details about the process). Therefore, it is interesting to analyze the process of model convergence to the optimal solution. Below, it is shown the evolution of market share and contract and tariff prices (including the charge) through the model's iterations.

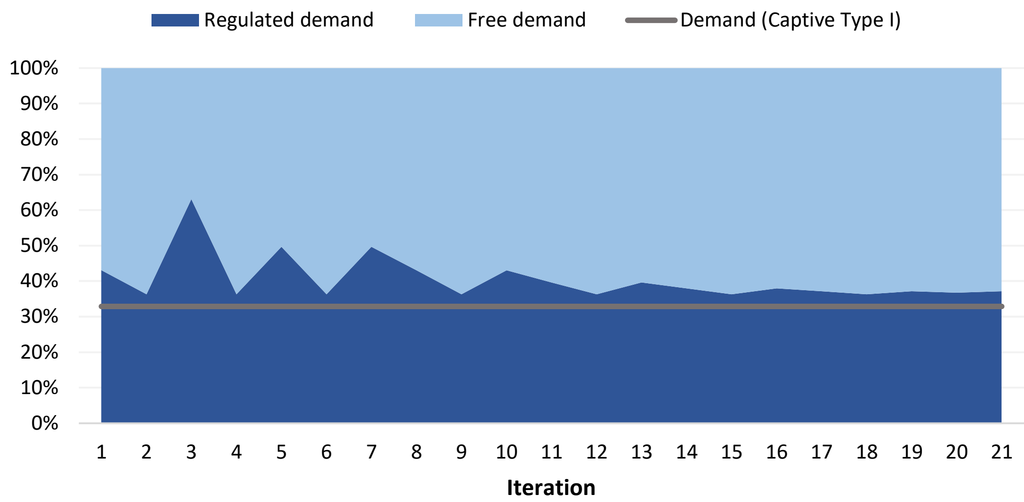

Figure 5-27 – Share of each market environment throughout iterations (Case 5)

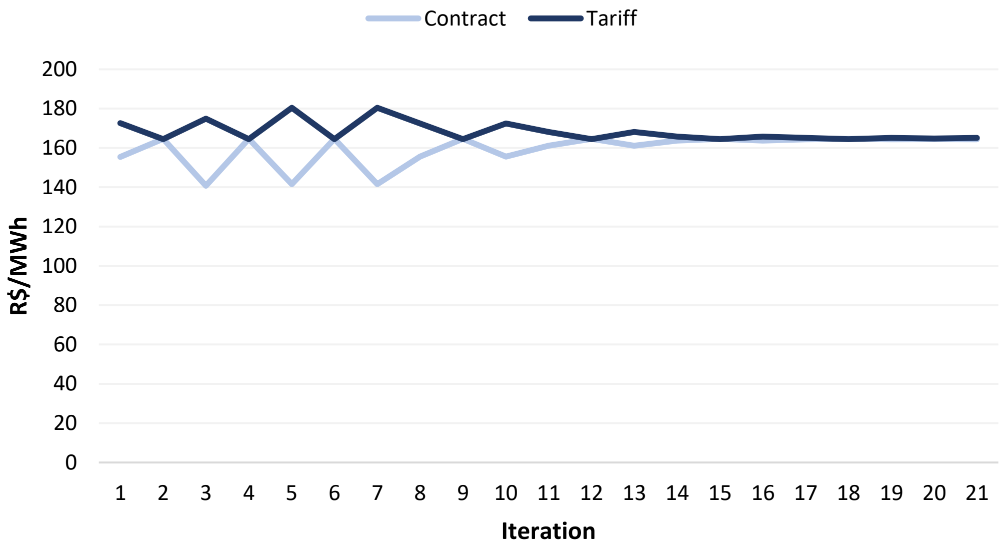

Figure 5-28 – Contract and tariff prices throughout iterations (Case 5)

The graphs above show the convergence process towards the optimal solution. Note that it is exactly the behavior of prices that influences the resulting quantities. In iterations where the tariff is more expensive than the contracts in the free market (including the charge), consumers seek to migrate to the free market. However, after the migration of all those who could migrate, the free market contract price (slightly) surpasses the resulting tariff, which makes part of the consumers actually remain in the regulated market. However, they do not all return (which would result in a non-convergent process), due to the gradual reduction of the smoothing factor δ (see Chapter 4). Finally, when the final migration value change and market prices converge (δ is lower than a very small threshold), the model reaches equilibrium, with a final migration of 9,980 MW-average and energy price in both markets of R$164.8/MWh (energy price plus charges in the case of the free market).

5.2.4 Final notes

In the case studies presented, we analyze the market equilibrium in terms of quantities and prices in the free and regulated environments, under different regulatory assumptions, in particular for the treatment of legacy contracts and liabilities associated with migration. The table below summarizes the main results in a comparative fashion.

Table 5.1 – Summary of results of the Brazilian case study

Case	Migration (MW-average)	Legacy contracts premium	Tariff premium + charge (R$/MWh)	Free market contract premium (migrating consumer) + charge (R$/MWh)	Free market contract premium (other consumers) + charge (R$/MWh)
Adjustment in legacy contracts	12.573	63.6	55.4	17.3	17.3
WIthout adjistment (Base Case)	7.716	63.6	63.7	19.3	19.3
WIthout adjistment (Sens. 1)	12.573	63.6	70.9	19.3	19.3
WIthout adjistment (Sens. 2)	12.573	63.6	70.9	-16.1	-16.1
Socialization of costs (total demand)	12.573	63.6	61.7	23.6	23.6
Socialization of costs (regulated + migration)	12.573	63.6	65.9	27.8	17.3
Socialization of costs (free + migration)	12.573	63.6	55.4	29.0	29.0
Socialization of costs (migration only)	0	63.6	57.3	-	17.3
Descending socialization of costs (free + migration)	9.980	63.6	41.3	41.3	41.3

As noticeable, there is a trend of total migration of captive consumers type II to the free market, whenever there is available supply for such. This is because these consumers can take advantage of the more competitive prices of the free market, compared to the Disco's tariff. In cases where there is a socialization of the liabilities of legacy contracts associated with migration (that is, reallocating these costs among market agents in different ways than only at the regulated market alone), migration to the free market continues to be advantageous, in general, despite the charge added to the contract price paid by these agents. One of the exceptions is Case 4, in which the legacy costs associated with the migration are entirely allocated to the migrating consumer. In this case, there is no incentive to migrate, as any benefits are fully offset by the charge. In Case 5, in which the legacy costs resulting from migration are sorted in decreasing order of cost/longevity, a market equilibrium is reached with partial migration of captive consumers type II (9,980 MW-average from 12,573 MW-average). This occurs because this order sharply increases the charge and reduces the tariff, until both environments reach an equivalent equilibrium price including the charge (R$164.8/MWh, that is, the same premium of R$41.3/MWh). In this equilibrium, the contract premium of contracts in the free market was 17.3 R$/MWh and the charge for free consumers was 24 R$/MWh.

6 Incorporating the equilibrium model in the expansion problem

The implementation of the equilibrium model presented in the previous chapters applies to a given, fixed configuration of a power system (i.e., one expansion case). However, the results of the equilibrium model (contract prices, tariffs, contracting levels, etc.) interfere in the agents' investment decisions and, consequently, in the system expansion.

In this context, this chapter proposes an iterative methodology for defining an optimal market equilibrium, taking into account that the agents' decisions and results at each iteration have an impact on the system expansion. The expected result of this methodology is the determination of an optimal expansion that takes into account the dynamics of the equilibrium of contracting environments in the recovery of the agents' investment and operation costs.

The chapter is divided into two main sections. The first one discusses theoretically the incorporation of market conditions and the investor's perspectives in the generation expansion problem. The second presents in detail the methodology proposed to carry out such incorporation, with the market equilibrium model being part of the iterative process.

6.1 Central planning *versus* decentralized investment decisions

This section presents a brief conceptual discussion on the opposition between the investment decisions resulting from centralized planning and the individual decisions of investors. A more detailed discussion is presented in [7].

6.1.1 Central planning

In the centralized planning, one agent is responsible for determining the optimal supply that should meet the electricity demand maximizing the social welfare (minimizing total cost for society), and respecting certain reliability criteria. This approach relies on a pillar that the central planning framework is that the planner has accurate knowledge of investment costs, of operating costs (supply curve) and of the load profile (demand curve), and so can confection an expansion plan that leads to an optimal system to electricity customers.

If that was the case, this would be an appealing approach, as this decision maker would have the tools and information to build an economic system to society. Nonetheless, those assumptions are seldom true: the planner does not have perfect knowledge about the supply chain, about the demand, let alone from inherently uncertain variables, such as the availability of natural resources for power generation.

The lack of perfect information makes the plan deviate from optimality and jeopardizes the planner's original goal of designing an economic system, with costs overruns ultimately paid by society. In this context, since the 1980s, competition in the generation segment began to be sought as a path for economic systems, as it presented two main advantages: (i) the investor has better knowledge of its individual production costs; and (ii) the investor is accountable for costs and risks incurred in its investments.

It is for this reason that one of the main goals of auctions practiced in Brazil and several other countries is to enable competition for entering the sector. This allows the revealing of the true investment and operation costs by the investors, accountability of risks on the investors, and the selection of the most economic projects to society. It also encourages innovation and the development and improvement of new technologies and projects.

6.1.2 Decentralized investments

In a decentralized environment, the expectation on returns of each individual agent is the driver for their participation in the market. Since the supply-demand conditions dictate energy prices, the agents' decisions would, in theory, lead to an equilibrium. It occurs at the point where the market price is such that each agent does not obtain economic profits (additional to a competitive required rate of return): if agents obtain additional profits, others would enter this market until the equilibrium is achieved. This is achieved at the point where the price is equal to the average cost of production, which is the division of the agent's total costs by the quantity produced. In energy markets, the average cost is often referred to as the levelized cost of energy.

In this context, a simplified interpretation of the generation expansion problem is that the marginal cost of electricity should be equal to the levelized cost of the most expensive unit selected – in this way, this agent can recover its costs with no extra profits. In practice, the conciliation of costs and revenues is less direct, since both generation and demand are not constant (nor deterministic), and so the *captured* prices from a generation project are not equal to the average system price. This is why a detailed representation of the system's and projects' characteristics is desired to identify efficient investment decisions.

6.1.3 Frictions between ideal planning and investors' perspectives

As previously discussed, investments in a decentralized environment are recovered through the market conditions faced by developers. Ideally, these conditions should be determined by the supply-demand balance obtained in this competitive environment. However, in practice, there are usually distortions in energy markets that prevent reaching the purely economic equilibrium.

Firstly, there are uncertainties, both in the short term (e.g., the availability of natural resources and demand profile) and in the long term (e.g., fuel prices, investment costs, political landscape etc.). Such uncertainties can incur supply and demand unbalances, so that investments, which are irreversible, and were attractive at the time of decision-making may no longer be so and vice-versa.

Moreover, the planners' concern with of energy shortage makes them seek to maintain a constant excess supply of generation in the system. This surplus, if participating in the wholesale market, contributes to lower short-term prices, jeopardizing the recovery of investments.

Furthermore, it is a common practice in wholesale electricity markets the adoption of caps for the short-term prices (to protect agents from exorbitantly high prices), which limits the recovery of investments.

For these reasons, short-term prices are often not enough to remunerate generation assets. Thus, the optimal expansion plan according to a criterion of maximizing social welfare may not lead to sufficient remuneration to the selected projects.

6.1.4 Conciliating central planning and private investment decisions

The question, therefore, is how to reconcile the view of the centralized planner and a purely market environment, where energy prices are given by short-term prices or result from bilateral negotiations. In addition to the distortions related to private knowledge of the investment and operating costs, the goals and risks of investors and planners are different.

Agents seek to invest in economically attractive assets, according to market prices. In turn, the planner seeks to guarantee the operation and expansion of the system at the least-cost possible to the society, respecting the supply reliability criteria and meeting the guidelines of the energy policy.

Regarding the risks, agents are usually averse to the risk of not recovering their investment, thus they seek higher energy prices. On the other hand, the planner is usually neutral to this risk (or cannot emulate accurately the agents' aversion and the market conditions). On the other hand, the generators are individually indifferent to security of supply issues, whereas the planner is sensitive to the system's reliability levels and will seek a certain oversupply, which puts downward pressure on short-term prices.

In the case of the free market (for instance, in Brazil), energy contracting is currently based in the economic logic: consumers seek projects with the lowest energy costs; investors seek to make their projects viable through the remuneration of these contracts and revenues from the short-term market.

Then, concerns arise with regards to the transition to a fully or highly liberalized market (without centralized auctions to coordinate the contracting of new capacity). They include identifying the optimal expansion in this environment, the resulting short-term and contract prices, and how to ensure security of supply. In this sense, one of the goals of the proposed separation between firm energy ("physical guarantee") and energy itself in Brazil is to guarantee the coordination of generation expansion in a liberalized environment.

To address the abovementioned concerns, it is necessary to incorporate the individual investment decisions of each agent into the expansion optimization dynamics. The following section proposes a methodology for doing so.

6.2 Methodology to incorporate the investor's view in the expansion problem

This section proposes the methodology that aims to incorporate, to the result of the optimal system expansion, information about the attractiveness and economic feasibility of the different candidate projects in the expansion plan, according to the individual perspective of investors. For simplicity, the approach considers that all developers have the same information regarding the investment and operation costs of market agents, with the risk aversion profiles being what differentiate them. The purpose is to reach a reasonable relationship between the optimality of the expansion plan, from the point of view of a central planner, and the practical feasibility of the different proposed projects, by evaluating the economic attractiveness of each one of them. In practice, this is done through an iterative process that considers three fundamental modules: (i) the generation system expansion; (ii) determination of market equilibrium prices; and (iii) the evaluation of the necessary investments.

Figure 6-1 – Optimal expansion plan overview incorporating individual investment decisions

The generation expansion module, a function performed by the OptGen software [50,51], aims to build an optimal expansion schedule from a systemic point of view, which would be the result of centralized planning. This is done by minimizing, under the uncertainty of inflows and renewable generation scenarios, the sum of investment costs in expansion – to effectively increase generation capacity – and system operation over a given horizon.

The result is a set of projects that, combined and from a systemic point of view, result in the optimal expansion schedule. However, such a schedule is not necessarily attractive to different investors. Note that if some unattractive projects are not effectively implemented, the generation stack will be different from that planned, and there is a deviation from the optimal solution – possibly incurring in situations of lack of installed capacity.

The investment evaluation module, a function performed by the OptFolio software [54], aims to maximize the return on each project. It is also used to assess the economic and financial attractiveness of different projects, from the perspective of individual investors. For this purpose, information regarding financing, potential future contracts, settlement in the short-term market, risk aversion profile of each investor etc., is used.

The result is an optimal contracting strategy, from the investor's point of view, for each asset, and its net present value, adjusted to the risk profile – this is the result actually used for the proposed iterative process. By evaluating the individual viability of different projects, it is possible to indicate to the expansion module which projects are economically viable and, for those that are not, calculate the risk premium necessary to achieve viability.

In order to optimize the contracting strategy for each asset, it is desirable to have good information regarding the prices of contracts in the free market. For that, the energy price equilibrium model, presented in the first chapters of this work, and from this point onwards referred to as OptContract, is used. This model receives as inputs a system's supply and demand configuration and the generation probability distributions and prices in the short-term market, resulting from an optimal expansion plan (obtained by the OptGen model) and the simulation of its operation.

The model results in optimal equilibrium prices and contracting levels in free and regulated markets. However, as discussed previously, the contracting decisions and price levels resulting from the model are, in principle, unrelated to the system expansion decisions. In this way, it is possible that the equilibrium model results in prices and contracts that exceed (or do not cover) the investment costs of new plants, which would not be sustainable, as it would generate incentives for a greater (or lesser) expansion to an efficient physical system configuration balance. In particular, and as will be seen later, information about the attractiveness of prices, especially in the free market (as we aim to emulate a liberalized environment), feeds the expansion optimization process, allowing the entry of new supply in the system, which shifts prices in the free market until convergence.

6.2.1 Iterative algorithm

This subsection presents in more detail the iterative process proposed for the application of the methodology. The following figure displays each step, which are then explained in more detail.

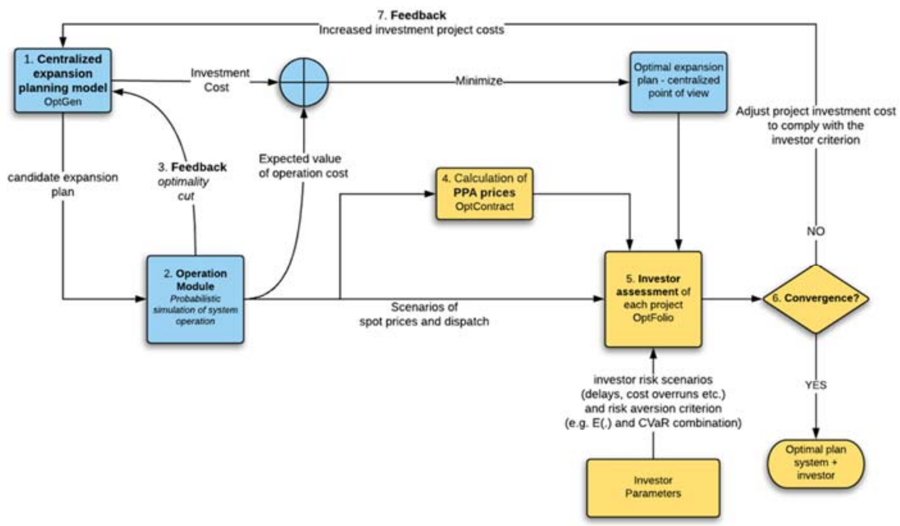

Figure 6-2 – Iterative process for defining the optimal expansion plan with considering market conditions and the individual investment decisions

Steps 1, 2 and 3: investment decisions and simulation of the system operation. These steps aim to build an optimal expansion schedule, from a systemic point of view, which would be the result of centralized planning. This is done by minimizing, under the uncertainty of flow and renewable generation scenarios, the sum of investment costs in expansion – to effectively increase the generation capacity – and system operation over a given horizon.

In step 1, an expansion schedule is decided, considering the investment costs of different candidate projects. In step 2, the operation of this configuration is simulated, considering different operating characteristics of the system under study and inherent uncertainties in the decision-making process for the system's dispatch. In step 3, there is feedback between steps 1 and 2: an optimality constraint is created and inserted to the problem solved in step 1. The purpose is to have an expansion schedule such that it pursues equilibrium between the marginal costs of short and long term – that is, the solution of this equilibrium problem, where the expectation of the marginal costs of the operation approaches the marginal cost of the expansion. This is done by the OptGen software.

Step 4: Definition of the free-market contract prices. In possession of an expansion schedule, as well as generation scenarios and short-term prices resulting from the simulation of the system operation for this schedule, the prices of contracts in the free market are calculated. At this stage, the equilibrium model developed within the scope of this work (OptContract) is used.

Step 5: evaluation of individual investment decisions. Using, among others, the information about the contractual prices obtained in step 4, the agent's risk profile, generation scenarios and short-term prices and project financing conditions, this module calculates the investor's cash flow and defines the present value risk-adjusted for each project. As a result, which projects are economically viable (positive present value) or not (negative present value) are indicated. For projects that are not viable, an additional step is responsible for communicating to the planning stage what premium would be necessary to be added in the costs perceived by the expansion model for the feasibility of this project to be achieved – step 7.

An additional (possible) feature of this step is the determination of the optimal contracting strategy, using the contract prices in the free market obtained by the price equilibrium model – for the different assets. In this case, in addition to the risk-adjusted present value, the energy amounts that should be committed in contracts to maximize the investor's return would be obtained as a result. In this case, the agents are considered price-takers (i.e., each individual contracting decision does not affect the equilibrium price already determined by the OptContract).

Step 6: convergence check. If all selected projects have a non-negative risk-adjusted present value, the process has reached its convergence – all projects selected by the expansion model are economically viable.

Otherwise, for each of the unfeasible projects, a risk premium is calculated equal to the minimum amount that the investor should additionally receive to assume the risks of that specific project. This amount is the risk-adjusted present value itself, but now with the positive sign.

Step 7: Feedback. For a given project, the calculated premium can be interpreted as a change in the rate of return expected by the investor for that project: projects that, given the market prices obtained from the expansion plan simulation, are identified as unattractive, will require rates higher returns.

Specifically, the calculated premiums are added to the costs of each project used in the iteration in question, so that in the next iteration the expansion module seeks a new configuration for the expansion schedule. It is important to note that, in the iterative process, risk premiums are considered for the elaboration of expansion plans, effectively impacting the resulting short-term (and contract) prices. However, they are not considered in the calculation of the risk-adjusted present value of subsequent iterations. The motivation for this is precisely to incorporate the decision logic of an individual investor: they may want a level of short-term prices considerably higher than their level of energy cost to ensure the recovery of their investment.

6.2.2 Mathematical formulation

This section presents the mathematical formulation of the optimization modules involved in the iterative process presented previously.

6.2.2.1 Module of generation expansion

In a simplified fashion, the problem solved by the expansion module is described by the Equations (6.1) to (6.5) below. Equation (6.1) is the problem's objective function, denoting the minimization of the expected value of the operation costs plus the investment cost I_p, including a risk premium PR_p; Equation (6.2) represents the investment constraints (e.g., associated with energy policies, budget limitations etc.), Equation (6.3) represents the operative constraints (such as supply-demand balance, plants' operative limits, hydrological balance etc.); Equation (6.4) represents the limit of generation for the projects selected (note that the limits are non-null only for the selected projects, otherwise, the binary variable x_p is null); and Equation (6.5) represents the supply-demand balance. This module is denoted by $OPTSDDP(PR_p)$.

$$OPTSDDP(PR_p) = \underset{g_{p,t,s}\ \epsilon_{s,t}\ x_p}{\text{Minimize}} \frac{1}{S}\sum_{p,\mathrm{t,s}} p_s\left(C_p g_{p,t,s} + C_\epsilon\, \epsilon_{p,t,s}\right) + \left(I_p + PR_p\right)x_p \tag{6.1}$$

s.t.:

$$x_p \in \aleph \tag{6.2}$$

$$g_{p,t,s} \in \tilde{G} \tag{6.3}$$

$$g_{p,\mathrm{t,s}} \leq \overline{g_p} x_p \tag{6.4}$$

$$A g_{p,\mathrm{t,s}} + \epsilon_{s,t} = L_{t,s} :\rightarrow \pi_{s,t} \tag{6.5}$$

This is a simplified representation of the expansion module carried out by the OptGen software.

6.2.2.2
Module of market equilibrium

As presented in detail in Chapter 2 and discussed in all subsequent ones, the optimization problem solved by the equilibrium model is the maximization of the welfare of all market agents. With all the mathematical treatment and assumptions considered in more detail in that chapter, this is a linear problem, whose objective function and equilibrium constraints are presented in a simplified way below. It is denoted as $OptContract(g_{p,t,s}, \pi_{t,s})$, and its objective function and equilibrium constraints are presented again below.

$$
\begin{aligned}
&OptContract(g_{p,s,t}, \pi_{s,t}) \\
&\quad = \text{ Maximize } \lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] \\
&\quad + (1-\lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] \\
&\quad + \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] \\
&\quad + (1-\lambda^{CII}) \times \left[a^{CII} + \frac{\sum_s y_s^{CII}}{S \times (1-\alpha)}\right] \\
&\quad + \lambda^L \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] \\
&\quad + (1-\lambda^L) \times \left[a^L + \frac{\sum_s y_s^L}{S \times (1-\alpha)}\right] \\
&\quad + \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(g_{t,s} - q_t^G\right)}{(1+r)^t}\right] \\
&\quad + (1-\lambda^G) \times \left[a^G + \frac{\sum_s y_s^G}{S \times (1-\alpha)}\right]
\end{aligned}
\tag{6.6}
$$

s.t.:

$$d_t^D + m_t^{CII} - m_t^L = d_t^{CI} + d_t^{CII} \tag{6.7}$$

$$q_t^G - q_t^D - q_t^{CII} - q_t^L = 0 \tag{6.8}$$

It should be highlighted that the formulation above is the “simplified” one, developed in Chapter 2. However, for the implementation itself, it was used the extended one (considering legacy contracts) presented in Chapter 4, whose process is suppressed here for the sake of conciseness.

6.2.2.3
Module of evaluation of investments

As discussed, an important step of the iterative process is to check whether the market conditions faced by the agents are enough for them to enter the market, given their risk aversion. For doing so, we emulate the decisions of investors seeking to maximize their risk-adjusted revenues. In case such revenues are not enough, a risk premium is calculated and informed in the feedback of the iterative process. Below, we show the optimization problem of the agent seeking to maximize its revenues given the market decisions (they are considered price-takers in this step), and next, the calculation of the risk premium.

$$OptFolio(g_{p,t,s}, \pi_{t,s}, \tau_{km}) = \underset{x}{\text{Max}}\ (1-\lambda)\ CVaR_{\alpha}[NPV_s] + \lambda \frac{1}{S}\sum_{t} NPV_s \quad (6.9)$$

s.t.:

$$x \in \chi \quad (6.10)$$

$$R_{s,t} = \left(g_{p,t,s} - x\, q_t\right)\pi_{t,s} + q_t \tau_{km} - g_{p,t,s} \cdot c_p - I_{p,t} \quad (6.11)$$

$$NPV_s = \sum_{t} \frac{R_{s,t}}{(1-r)^t} \quad (6.12)$$

The calculation of the risk premium is carried out after the execution of the expansion investment evaluation. Specifically, the risk premium is equal to the risk-adjusted present value if it is negative, or zero otherwise (as no extra revenues would be required in the latter case). This is represented by Equation (6.13):

$$OptFolio\left(g_{p,t,s}, \pi_{t,s}, p^c\right) = \begin{cases} PR_p = 0\ , f\left(g_{p,t,s}, \pi_{t,s}\right) > 0 \\ PR_p = -f\left(g_{p,t,s}, \pi_{t,s}\right), f\left(g_{p,t,s}, \pi_{t,s}\right) \leq 0 \end{cases} \quad (6.13)$$

Considering the optimization problems formulated by the equations above, the iterative algorithm can be described as follows.

Integration algorithm of expansion, market equilibrium and evaluation of investments

```
PR_p ← 0, p^C ← 0
For k ← 1 to K Do
    g_{p,t,s}, π_{t,s}, x_p ← argmin OPTSDDP(PR_p)
    p^C ← OptContract(g_{p,t,s}, π_{t,s})
    PR_p ← OptFolio(g_{p,t,s}, π_{t,s}, p^C)
    If PR_p == 0
        Stop
    End if
End For
Return x_p
```

Figure 6-3 – Integration algorithm of the optimization modules

7
Case study of the equilibrium model in the Brazilian system expansion

This chapter presents an application of the methodology described in the previous one to the Brazilian market. In Chapter 5, the equilibrium of the Brazilian power market was presented for a fixed configuration of the power system. In this one, the methodology proposed in Chapter 6 is used to obtain the equilibrium in the optimal system expansion, which takes into account the investors' perspectives, including their risk profiles and the market conditions faced (including the prices set by the equilibrium model itself).

Moreover, in addition to the modelling of the market equilibrium under the current configuration of the Brazilian regulation, in which energy contracts are backed-up by physical guarantee (bundling both products), this chapter also explores the impacts in expansion of explicitly adding the firm energy (physical guarantee) and firm capacity products, separately to the energy contract, as well as their remunerations under the assumptions that are described throughout the chapter.

7.1 System and market assumptions

This section describes the assumptions used in the modeling of this case study. In terms of system configuration, the starting point (current configuration of the Brazilian power system in 2021) is the same as the one used in Chapter 5. This encompasses not the physical system configuration, but also the entire commercial structure presented in that chapter, including the representation of the generators, consumers, distributor, as well as their risk aversion.

The year studied is also 2030, and the demand evolution until this target year is also the same one applied in that case, i.e., demand is projected to grow from 67.7 GW in 2020 to 92.7 GW in 2030.

However, with respect to supply evolution, instead of considering the customized expansion used in that case, this one considers that it is the result of an optimization procedure, according to the methodology described in Chapter 6. In order to apply the methodology, it is necessary to establish the costs of the candidate technologies, which are informed to the expansion model OptGen. These are shown below.

Table 7.1 – Investment costs by technologies used in the case study

	Natural gas (CC)	Natural gas (OC)	Wind	Solar	Biomass
CAPEX (R$/kW)	3,640	2,550	4,330	2,665	5,500
Useful life (years)	25	25	20	20	25
Discount rate (%)	8%	8%	8%	8%	8%
Fixed O&M (R$/kW-year)	35	35	85	30	85
Annualized fixed costs (R$/kW-year)	**1,135**	**807**	**708**	**530**	**772**
Operation cost (R$/MWh)	**260**	**500**	**0**	**0**	**0**

The expansion model considers the annualized fixed costs (investment and fixed O&M) as the cost to add a certain project into the system, and also its operation costs, in order to evaluate its entry in operation. In the optimal solution, the model selects the project (or the combination of projects) that minimizes the sum of fixed and variable costs. Moreover, as previously mentioned, during the iterative process, the fixed costs may be adjusted between iterations in order to portray the investors' perspectives.

7.2 Case 1: Current regulation

7.2.1 Expansion results

In this case, the iterative expansion process is carried out, considering the contract prices calculated by the equilibrium model and contractual rules similar to the current ones applied in the Brazilian market, that is, the contracts celebrated between consumers and generators (new and existing) are energy contracts backed by firm energy (physical guarantee) – and there are no firm energy/capacity products explicitly separated from energy. The expansion result is shown below.

Case 1 (iteration 1): prices	
Marginal cost (R$/MWh)	125
Spot price (R$/MWh)	114
Contract price (R$/MWh)	119

Case 1 (iteration 1): expansion	
Plant	Wind (NE)
Capacity (MW)	10,371
Physical guarantee (% of installed capacity)	56%
Levelized cost of energy (R$/MWh)	134
Contracting level (% of physical guarantee)	100%
Risk premium required (R$/MWh)	18

Case 1 (iteration 2): prices	
Marginal cost (R$/MWh)	180
Spot price (R$/MWh)	154
Contract price (R$/MWh)	160

Case 1 (iteration 2): expansion	
Plant	Wind (NE)
Capacity (MW)	8,251
Physical guarantee (% of installed capacity)	56%
Levelized cost of energy (R$/MWh)	134
Contracting level (% of physical guarantee)	100%
Risk premium required (R$/MWh)	-

Figure 7-1 – Results of the iterative expansion process for the Case 1

The case converges in two iterations. The plant responsible for the expansion in both of them is a wind farm located in the Northeast region, with a capacity factor of 56% (one of the highest among the candidates). In the first iteration, the expansion model selects 10,371 MW from this candidate plant. This leads to a marginal cost of R$ 125/MWh, a spot price[3] of R$ 114/MWh and a contract price of R$ 119/MWh, that is, with a small risk premium with respect to the spot price. However, when analyzing such result from the investor's point of view, these market conditions are not satisfactory to enable that expansion – both because they perceive spot prices (and not the marginal cost, as in the expansion model), and also as even the contract that it can sell is cheaper than the average marginal cost. In addition, the investor is represented as risk-averse, and thus would require even more remuneration to enter the system (than the average one to cover its costs). Thus, at the end of the first iteration, the investor requires an additional premium of R$18/MWh to enter the system with the amount initially chosen by the model.

The previous fact leads to an additional iteration, in which the model selects a smaller quantity from the same plant (8,251 MW), which leads to an increase in prices: an average marginal cost of R$ 180/MWh, a spot price of R$ 154 /MWh and contract price of R$ 160/MWh. In this iteration, the requirements of this plant to enter the system are met, including its risk aversion – note that the levelized cost of this plant is R$ 134/MWh. Therefore, market conditions remunerate it, even

[3] The spot price is calculated as the marginal cost capped by the floor (39.68 R$/MWh) and ceiling (559.75 R$/MWh) at every scenario and load block.

weighting the (average of) more negative scenarios in which it has exposures to honor its contractual obligations.

7.2.2 Results of the equilibrium model

In this subsection, we analyze the results of OptContract – specifically for iteration 2 (final one). It is noteworthy that, with respect to the treatment of liabilities associated with legacy contracts, it was considered that this would be divided equally among the entire demand. That is, the distributor would resell contracts that would make it over-contracted and the difference between the cost of that contract and the resale price (price of contracts in the free market) is passed on equally to all consumers (see Chapter 5, Section 5.2.3 – Case 1). Evidently, other options, extensively discussed in Chapter 6, could be adopted – however, to focus the analyses, we opted for a "standard" choice in this regard.

The results of contract price migration in free and regulated markets for the expanded system configuration according to iteration 2 of the expansion procedure are shown below.

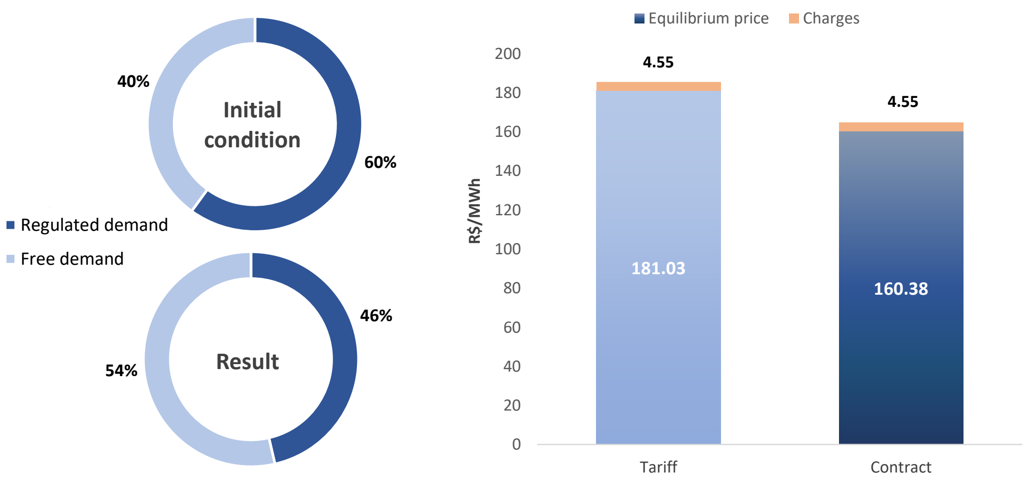

Figure 7-2 – Results of the equilibrium model for the Case 1

As expected (and also obtained in Chapter 5), there was a complete migration of captive consumers type II to the free market. This is because the prices in this market environment are more attractive than the regulated tariff, which carries the costs of legacy contracts. Regarding the price of contracts in the free market for the year 2030, this assumed the value of R$ 160/MWh, already anticipated in the previous subsection - which represents a spread of R$ 6/MWh with respect to the average spot price (R$ 154/MWh). It is precisely the fact that this price is more attractive (lower) than the tariffs that generates the migration of consumers to this market environment. In addition, as the expansion process is now iterative and incorporates market dynamics, new agents enter providing sufficient physical guarantee to enable the migration of consumers to the free market (in this case, wind power in the Northeast). Finally, it can be highlighted the charge applied on an equal basis in both markets (at the amount of R$ 4.55/MWh): however, it is necessary to emphasize that the free market contract price for new generators is R$ 160/MWh only (without the charge) – that is, the charge is paid by demand specifically to reimburse the distributor, not being part of the economic signals for the expansion of new projects in the iterative process.

7.3 Case 2: Separation of energy and firm energy (physical guarantee)

In this case, it is considered the explicit payment of the firm energy product (physical guarantee), in addition to payments for the energy itself (through energy contracts or short-term prices). It is assumed that the optimal supply selected by the expansion model should receive a payment that would complement its energy revenues, if these were not sufficient for its economic viability, considering its risk aversion. To do so, we calculate the difference between the revenue required by the generators selected by the expansion model, and the expected value of their risk-adjusted remuneration obtained from the execution of the model. Then, we consider that the firm energy price is precisely the amount that would cover this "missing money" of the incoming generator – instead of, for example, informing a risk premium to the expansion model, leading to a smaller expansion and higher energy prices, as observed in Case 1. In this way, the firm energy assumes an economic sense, which is the additional revenue needed to make the agents of optimal expansion viable (which is the concept of the missing money).

Finally, should be highlighted that the analysis of payment for firm energy will be restricted to new expansion candidates – one way to interpret this assumption is that the existing system is widely contracted supplying energy and firm energy in current contracts.

7.3.1 Expansion results

The expansion results obtained for this case are displayed below.

Case 2 (single interation): prices	
Marginal cost (R$/MWh)	125
Spot price (R$/MWh)	114
Contract price (R$/MWh)	119
Firm energy price (R$/MWh)	18

Case 2 (single interation): expansion	
Plant	Wind (NE)
Capacity (MW)	10,371
Physical guarantee (% of installed capacity)	56%
Levelized cost of energy (R$/MWh)	134
Contracting level (% of physical guarantee)	100%
Risk premium required (R$/MWh)	-
Bid in a firm energy auction (R$/MWh)	18

Figure 7-3 – Results of the iterative expansion process for the Case 2

Note that the results are the same as those found in the first iteration of the previous case. This is because, once the optimal expansion is obtained by the generation expansion model, the missing money of the marginal agent that entered the system (in this case a single agent, wind generation located in the Northeast region), calculated in the individual feasibility analysis of the project, instead of being incorporated in a next iteration of the expansion model, becomes the remuneration for firm energy. And so, this generator receives an additional payment of R$ 18 per megawatt-hour of physical guarantee, which complements its revenue and makes it economically viable, considering its risk aversion.

7.3.2 Results of the equilibrium model

Analogously to the previous case, the results of the equilibrium model are shown below, considering once again the charge assigned to the entire demand to reimburse the distributor's over-contracting.

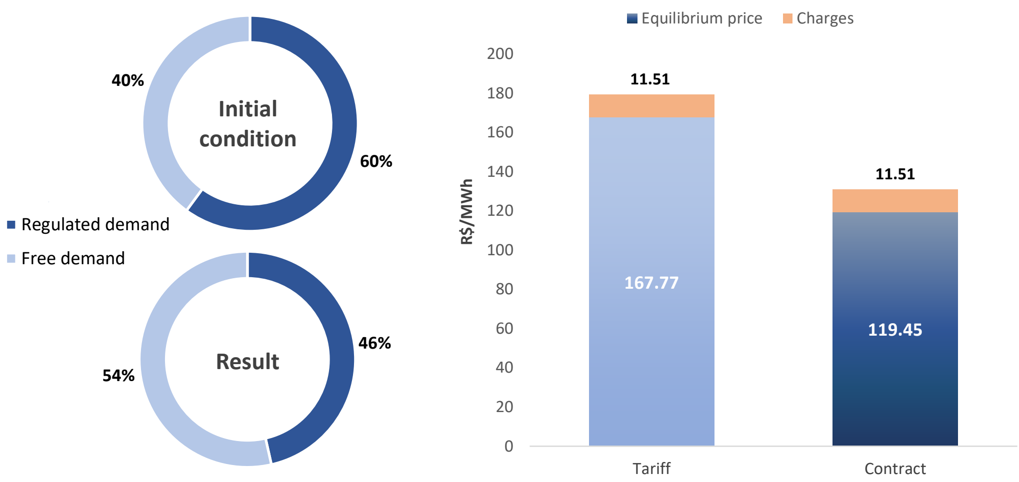

Figure 7-4 – Results of the equilibrium model for the Case 2

Again, a total migration of captive consumers type II can be noticed, given that prices in the free market are more attractive (even more attractive in this case, despite the fact that total migration is achieved in both cases). These lower prices in the free market also generate a comparatively cheaper regulated tariff than in Case 1, given that the distributor makes up part of its portfolio with contracts at the projected price. However, evidently, because the price of the free market is even lower, this latter remains the most attractive market for consumers.

Finally, it should be noted that, in this case, demand would still be paying R$ 18/MWh of firm energy for the wind farm that joined the system.

7.3.3 Cost comparison between Cases 1 and 2

Next, we show the total cost results of Case 1, in which the contracts include energy and firm energy products in an integrated manner (current regulation), with Case 2, in which there is an explicit separate payment for the firm energy product. It is noteworthy that such costs are composed of the fixed annualized cost of incoming projects (referred to as “investment cost”) and the cost of operating the system in the simulated year (2030).

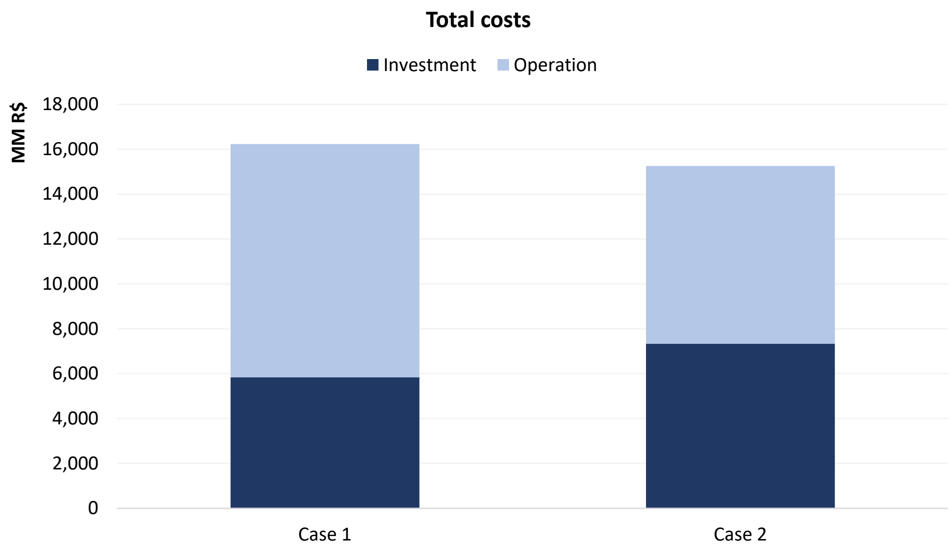

Figure 7-5 – Investment and operation costs in Cases 1 and 2

As expected, Case 1 has lower investment costs, given that in its second iteration there was a reduction in the amount of wind entering the system (from 10.37 GW to 8.25 GW). On the other hand, this allowed their remuneration through higher energy prices, with average marginal cost of R$ 180/MWh, spot price of R$ 154/MWh and contract price of R$ 160/MWh, which relates to the higher operating costs shown. In Case 2, the investment cost is higher, as the original expansion proposed by the 10.37 GW expansion model is maintained – since, despite lower prices (average marginal cost of R$ 125/MWh, spot price of R$ 114/MWh and contract price of R$ 119/MWh), the generator receives payment for firm energy (R$ 18/MWh) to supplement its revenue.

Still, it is interesting to highlight that, in terms of total costs (investment + operation), Case 2 is more economical. This is expected: note that this case maintains the original optimal expansion proposed by the generation expansion model, while in Case 1 a risk premium was included in the candidates' investment costs to expansion. Therefore, Case 2 has a total cost of R$ 15.25 billion, while Case 1 reached a total cost of R$ 16.23 billion – an increase of virtually R$ 1 billion (6.4%) resulting from informing the expansion model the costs, from the investor's point of view, for making the new projects viable – while in Case 2 it was possible to count on this expansion through the payment of firm energy to cover for such requirements from the agents. This is a remarkable result: adding a separate firm energy payment to cover for the new agents' missing money allowed the maintenance of a more economical system expansion.

7.4 Case 3: Separation of energy, firm energy and firm capacity

Although the Brazilian system has historically been an energy-constrained one (i.e., whose challenge has been to meet the load throughout the whole year, dealing with droughts that can take several months), an ongoing discussion in Brazil over the last few years is the growing challenge of meeting the system's peak demand (capacity constrain), as well as how to remunerate agents who provide the service to do so. Historically, Brazil has had a large hydroelectric share in its energy mix, which manages to follow the load without major difficulties, complemented by thermoelectric plants, which are usually also capable of doing so due to their dispatchability. With the insertion of intermittent sources in the Brazilian electrical system over the last few years, the system's net demand has become increasingly erratic and less predictable, increasing the need for equipment that is available in hours of need – providing power to the system.

One way to guarantee the presence of equipment that can provide energy during peak hours is to contract and pay for a specific product. In 2021, an auction was held in Brazil for the contracting of equipment to provide capacity services [55]. In other countries, especially those historically restricted in power, it is also common to have a "capacity" or "power" product for similar purposes.

7.4.1 Assumptions and proposed methodology

Given the above, we propose a new case in which an explicit capacity constraint should be met. For this, we consider a series of assumptions. First, we assume that the capacity constraint is 105% of the system's maximum demand in a given year. Some assumptions about the existing system's contributions to firm capacity by technology were adopted, in order to have a starting point. From there, it was then possible to calculate the need for additional power in the system until the year 2030, as shown below.

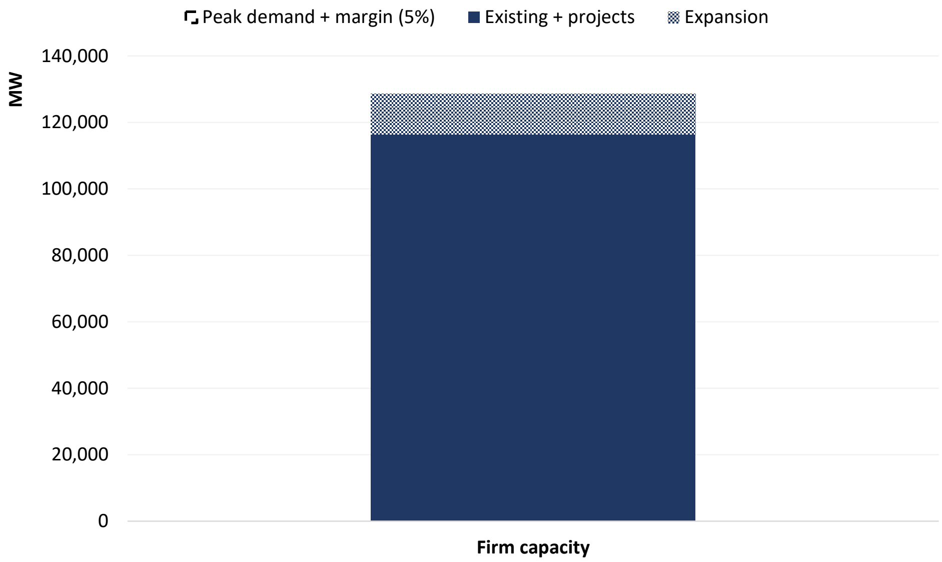

Figure 7-6 – Firm capacity balance in 2030 (assumption)

The chart above shows that, for the assumptions adopted, it would be necessary to add 12.2 GW of firm power to the system by 2030. Evidently, these are approximate estimates, and are not intended to determine in detail the security requirements of supply of the Brazilian electricity system – but rather portray a situation that can illustrate simulations with the explicit representation of this product.

Another important assumption is how much capacity each candidate technology can provide during peak hours. Usually, this determination is related to the "dispatchability" of the equipment or the historical contribution in these periods. In the case of thermoelectric plants, usually in the international experience, their nominal capacity is used, discounted by unavailability forecasts, leading to high values of firm capacity. Hydroelectric power plants, especially the ones with great modularization capacity, also tend to have relevant values of firm power, portrayed by their capacity to supply peak demand in possible situations of water scarcity. In the case of non-conventional renewable plants, the international experience is quite diverse: from cases in which renewables do not have firm assigned to them them to cases in which some statistics associated with their historical contributions during the peak hours. Given the above, the following assumptions of firm capacity of the candidate technologies were adopted.

Table 7.2 – Assumptions of firm capacity of the expansion candidates

	Natural gas (CC)	Natural gas (OC)	Wind	Solar	Biomass
Firm capacity (% of capacity)	95%	95%	Variable	Variable	90%

As displayed, the firm capacity provided by renewable plants is variable, because each one has a different seasonal and hourly behavior. Added to this is the fact that it is not defined in the regulation whether these plants would be selectable to provide this product – nor how much could they provide. In this sense, variations in the value considered for the contribution of renewable plants will be adopted (from 0% to 50% in the case of wind farms), as will be shown in detail in the results section.

In terms of modeling and product pricing procedure, the same iterative expansion procedure was used, which includes the expansion optimization model (OptGen), detailed system simulation (SDDP), contract price calculation and equilibrium of markets (OptContract) and assessment of the feasibility of the proposed expansion from the investor's point of view (OptFolio). In this case, however, the firm capacity product remuneration is added. A summary of each product's pricing is shown below:

- The price of the firm capacity product is calculated as the value of the dual variable of the firm capacity constraint added in the expansion model (OptGen), which is represented explicitly. That is, if the minimum cost expansion of the system does not comply with the constraint of firm capacity (105% of peak demand), the model will "impose" the entry of more capacity so that the constraint is met. This generates a positive value for the dual variable of the capacity restriction (in R$/kW), which is interpreted as the firm capacity value, that is, how much demand would be willing to pay for the marginal unit that would be providing the end service (in addition to payments via the energy market).
- The value of the short-term energy price is still being calculated by the dispatch model (SDDP)
- The price value of energy contracts is still determined by the equilibrium model (OptContract)

- The firm energy price keeps being given by the missing money of the marginal agent that entered the system. That is, agents selected by OptGen (including those receiving firm capacity) may require additional revenues to enter the system, when considering dispatch conditions, contract prices and risk aversion not captured by the expansion model.

7.4.2 Results

This section presents the results of Case 3, comparing them to the ones obtained in the other two cases.

7.4.2.1 System expansion

As previously mentioned, a relevant discussion for determining the expansion of this case is how much firm capacity will be provided to renewable plants. In this sense, the expansion procedure was performed eleven times, varying the firm power of the wind farms from 0% to 50%, in steps of 5%.[4] The expansion results for each case are shown in the graph below, comparing them not only with each other, but also with the expansion obtained in cases 1 and 2.

[4] The wind technology was selected for this variation as it was found empirically that it was the renewable technology being selected by the model due to its competitiveness, i.e., varying the solar contributions did not produce differences in the results.

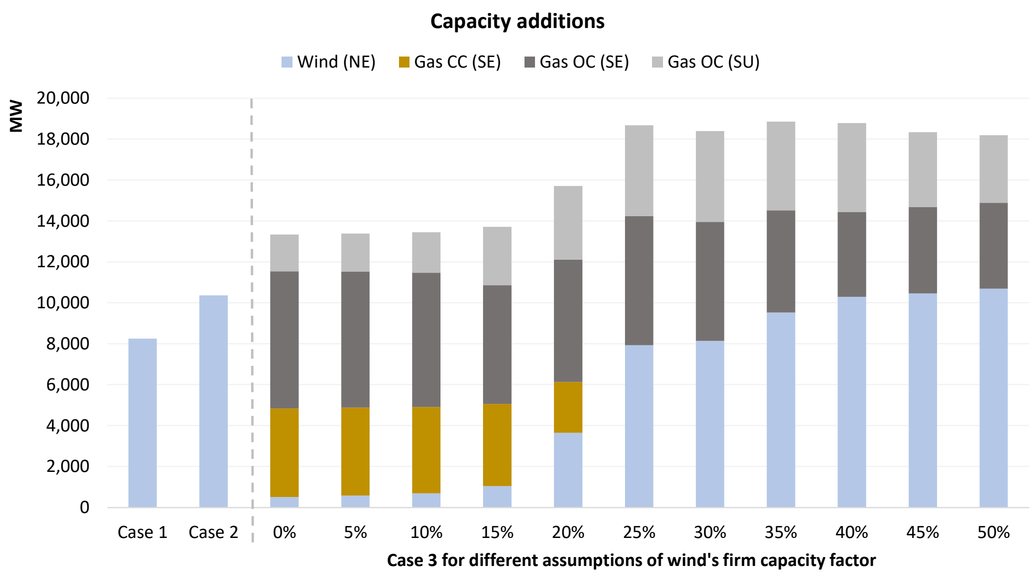

Figure 7-7 – Capacity additions in Cases 1, 2 and 3

The graph shows that the addition of the firm capacity constraint generates a significantly greater expansion in Case 3 than those found in Cases 1 and 2. In addition, the expansion counts with thermoelectric plants, which can provide more of this product (95% of their installed capacity). It is also shown that, for cases where wind contributes with little or no firm power, the system has combined cycle (CC) and open cycle (AC) expansion, the former being mainly responsible for supplying energy to the system, especially in dry scenarios. As the firm capacity assigned to wind farms increases, they start to replace the combined cycle, as a technology that can supply the system with energy and capacity.

Another relevant comment is that, in all sub-cases of Case 3, the added firm capacity was exactly what the amount needed to meet the requirement presented in the assumptions section (12.2 GW). The total capacities in each case differ precisely because each technology provides a different percentage of its installed capacity for peak service (in addition to the fact that in the case of wind, there is variation between cases).

7.4.2.2
Energy prices

Next, we show the energy prices obtained in the spot market (yearly averages, given by the dispatch model) and in contracts (given by the equilibrium model), in each case. A downward trend in energy prices can be noted as wind farms have assigned more firm power and consequently start to represent a greater portion of the system's expansion. This result is intuitive, since such plants have low operating costs, while thermoelectric plants, which dominate expansion when wind has little firm power, have higher variable costs, contributing to set prices at higher levels.

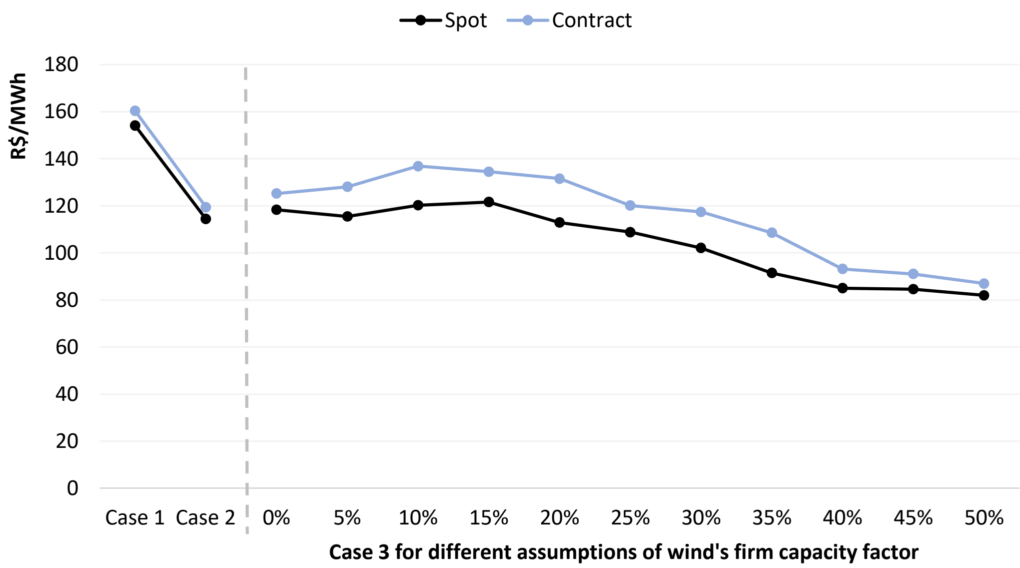

Figure 7-8 – Average spot and contract prices in Cases 1, 2 and 3

7.4.2.3
Prices of firm energy and firm capacity

Next, the firm energy and firm capacity prices obtained are analyzed. Note that the firm capacity price, determined by the expansion model, assumes lower values when the expansion is more thermoelectric (when wind has lower values of firm power). This is because the thermoelectric plants themselves contribute more to the system's energy supply, especially in drier scenarios, and earn more infra-marginal revenues in the energy market than when they provide almost exclusively the capacity service. Consequently, the additional remuneration required (paid through the firm capacity product) is reduced.

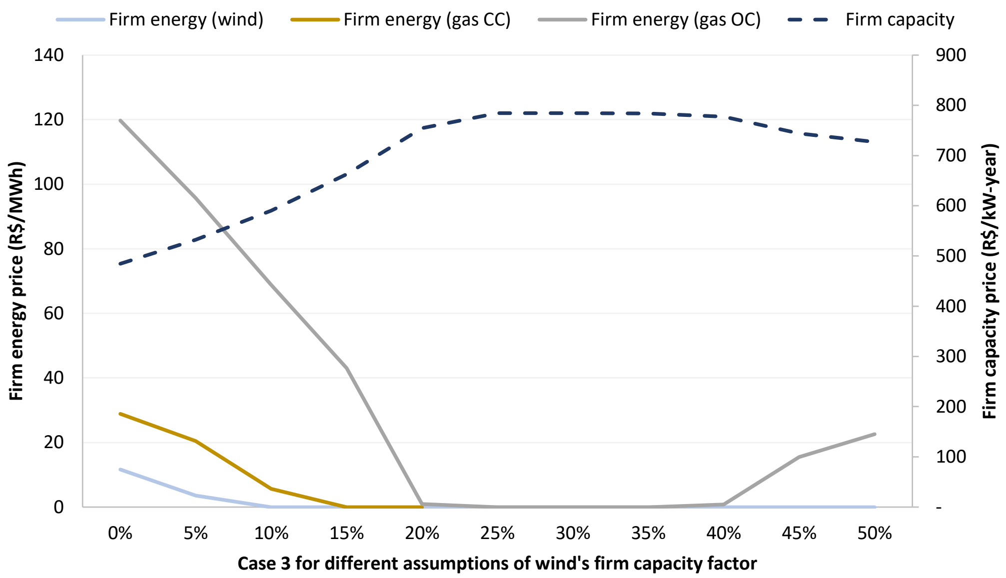

Figure 7-9 – Prices of firm energy and firm capacity in Case 3

As wind farms gain more firm capacity, they start to supply the system with energy and the thermoelectric plants that enter the system operate almost exclusively as peak-shavers, with little infra-marginal revenues. Thus, the entry of thermal plants is largely dependent on the payment of the firm capacity, which then assumes a higher value, comparable to the investment cost of the plant itself.

In the final part of the graph, the effect is partially reversed: as the amount of wind capacity that enters is quite expressive, reducing thermal input, the probability distribution of prices becomes more disperse (despite a lower average, there is a longer tail). So, there are some scarcity scenarios (of water and wind resources) where open-cycle thermoelectric plants generate and receive a certain amount of infra-marginal revenues in the expansion and dispatch models.

With respect to firm energy prices, it is possible to highlight the complementary behavior to that of firm capacity. This behavior is somewhat expected, since this product is defined as the missing money of the generators that entered the system. When the firm capacity, which is a guaranteed revenue for the generators that entered the system, is larger, the firm energy payment tends to be smaller, and vice versa. In particular, generators' missing money increases more significantly in cases where they rely on more inframarginal revenues perceived in the expansion model. However, from the investor's point of view, such revenues are substantially limited, not only by considering their risk aversion, but mainly by the remuneration through the spot price, and not the marginal cost, as considered in the expansion exercise. The price cap between the marginal cost and the spot price generates a greater missing money in these cases and, consequently, a higher price for the firm energy product.

Finally, the graph above represents individually the firm energy prices for each of the technologies that entered the system (obtained from their values of missing money). This could be seen as, for example, the result of a technology-specific firm energy auction; or even the result of a multi-technology auction, in which each one offered its missing money and was paid accordingly (a “pay-as-bid” auction). It would also be possible to consider a competition between technologies and payment according to the marginal offer (auction in the “marginal pricing” modality) – in this case, all the technologies would receive what was offered by the open cycle plant. It is not part of the scope of this work to design the auctions for contracting such product(s), but this topic can certainly be explored in future work.

7.4.2.4
Total costs

A relevant result of all expansion cases under Case 3, compared to Cases 1 and 2, is the increase in their total costs. This is because these scenarios have the firm power requirement, that is, a more stringent supply security criterion, which leads to additional costs. Among the exercises Case 3, it is noted that there is a reduction in total costs as more firm power is assigned to wind farms. This is due to the greater ease in complying with the established criterion – it would be analogous to imagining a less rigorous criterion, since less resources are needed to meet it.

It should also be noted that, with the increase in the entry of wind farms, there is a trend towards a reduction in operating costs, since these plants have null operating costs and contribute to the reduction of energy prices.

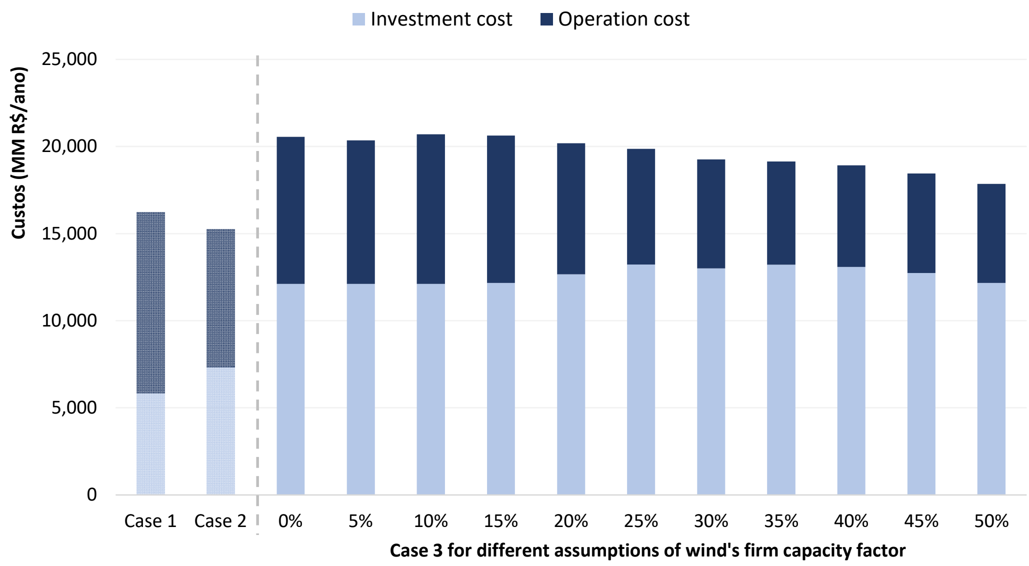

Figure 7-10 – Investment and operation costs in Cases 1, 2 and 3

7.4.3 Final notes

This case study assessed the expansion of the Brazilian electricity system with a 2030 horizon taking into account the feasibility from the investor's point of view and the dynamics of free and regulated contracting environments, both under current regulation and with the addition of new products (firm energy an capacity).

In the case with the current regulation, the iterative expansion process needs a second iteration to converge, since the optimal expansion initially proposed from the planner’s point of view does not remunerate the investor, considering contractual dynamics, energy price differences between the expansion and operation model (including regulatory floor and ceilings and other details of representation of the operation) and the risk aversion of the agent. Thus, a risk premium of R$ 18/MWh is informed to the expansion model, which is added to the investment cost perceived by the planner, making an adjustment in the expansion in the second iteration. In this case, the expansion reduces from 10.37 GW to 8.25 GW, and the increase in energy prices (spot and contracts) are enough to remunerate the expansion from the investor's point of view.

Considering the existence of a firm energy product, which precisely remunerates the missing money of the expansion agents (R$ 18/MWh), the initial optimal decision of the expansion model is maintained (10.37 GW of wind farms in the Northeast). This case then had a greater expansion than Case 1 and a lower total cost, as it is the optimal expansion from the systemic point of view (feasible from the investor's point of view thanks to the payment for the firm energy).

Finally, the representation of a firm capacity product in the Brazilian electrical system is explored. This representation leads to costlier expansions and more capacity additions, as there is a new, more stringent requirement on expansion. In addition, natural gas-fired thermoelectric power plants also start to configure the expansion, as they provide significant contributions to this product. Also, as there is no definition of how an eventual firm power of renewable plants would be defined (a topic without consensus, even internationally), the contribution of wind farms, natural candidates for expansion, was varied in this product. This definition was preponderant in determining the system expansion, which indicates the relevance of this issue for the Brazilian electricity sector.

8
Conclusions and future work

This dissertation has proposed a methodology for the determination of an equilibrium between free and regulated environments in an electricity market where most consumers can opt for their source of supply. Through Multiple Optimization Problems with Equilibrium constraints (MOPEC), it was possible to emulate the agents' (distributor, consumers, generators) interdependent decisions in such a way to maximize the risk-adjusted expected revenues of each one of them (thus, also maximizing Social Welfare).

This type of approach is especially relevant given the ongoing transition of energy markets, characterized by an increasing liberalization, in which consumers are more participative and seek for their best options either in the regulated or free market. Thus, the methodology challenges the traditional approach of a price-taker regulated demand that will assume the costs of whatever decisions taken by other agents in the sector (especially the distribution companies). Also, it is able to reveal, given the assumptions applied, what are the optimal levels of migration between markets and the prices of electricity contracts that should apply and be accepted by the agents given certain system/market conditions. These are obtained as results of the equilibrium model, either by primal variables (migration) or dual variables in the model (tariff and contract prices).

A particularly advantageous feature of the proposed methodology is that, through algebraic manipulations (shown in Chapter 2 of this work), the equilibrium problem becomes linear, simplifying the obtention of its solution with limited computational effort. Furthermore, the work presents (in Chapter 4) an extension of the methodology, which allows the incorporation of legacy contracts of the distribution companies (a relevant feature of the real problem and a motivation for the migration of regulated consumers in the first place), which turns the problem into an iterative process (of linear problems with recalculation of variables) – still with limited computational demand, carried out using open, linear solvers.

When treating the abovementioned realistic problem, considering legacy contracts, developments were also made to allow different allocations of their costs whenever the migration of regulated consumers to the free market generate over-contracting to the Disco. This emulates the various regulatory treatments that can be implemented in a situation of massive migration to the free market, which intends, among other goals, to avoid a disequilibrium caused by an increasing burden assigned to the regulated market (known informally as the “death spiral”).

Then, acknowledging that the proposed methodology works for a certain configuration of an electricity system (supply-demand balance) and that, on the other hand, the equilibrium prices obtained through it would impact the signals for new investments in the sector, this work proposed the incorporation of this methodology in the generation expansion problem. This is done through the integration of the equilibrium model in an iterative process for the determination of the optimal generation fleet, which counts with sequential adjustments in the economic parameters informed to a least-cost expansion tool, given the market conditions perceived by the agents obtained by the equilibrium model at each iteration. When no adjustments in the parameters are needed (i.e., when the proposed expansion is viable, given the market conditions, in the investor’s perspective), the problem converges, and the optimal expansion is found.

All the abovementioned developments, carried out in this dissertation, were implemented and presented in the form of case studies. Firstly, the intuitions of the model were assessed through a case with simplified systems and no legacy contracts. It was shown the interdependence of the agents’ decisions and the variations of results depending on parameters such as the supply configuration, supply-demand balance and agents’ risk aversion (both in the supply and demand sides). Also, features such as demand’s contracting obligation and supply’s contracting limitation based on physical back-up were explored, in order to assess the impacts of such “real-life” conditions in electricity markets (both of them applicable in Brazil, for instance). It was found that limiting the generators’ contractual sales increases the risk premium of the ones with lower risk aversion, and the contracting obligation increases the premium of more risk-averse generators. On the consumer side, it was seen that, if consumers are forced to contract and this constraint is active, the equilibrium price depends on the supply-demand balance and on the competitiveness of generating agents, but not on the consumer's preferences.

A case study of the extended methodology considering legacy contracts was presented for the Brazilian power market, portraying realistic configurations of both the physical and commercial spheres. The case has shown a full migration of eligible regulated consumers to the free market for most of the assessed regulatory frameworks regarding the allocation of legacy costs. Since contracts in the free market are cheaper than the regulated tariffs, the regulated consumers have such incentive to migration, either when the Disco retains the legacy contracts in their portfolio or when they are able to resell them in the free market (which keeps enough supply in the free market at attractive equilibrium prices). It was shown that the market equilibrium (partial migration of eligible consumers and equivalent prices of tariffs and contracts) was found only when a significative portion of the Disco's over-contracting costs were allocated to the free consumers (an assignation through a descending price order of legacy contracts was emulated for this purpose). On the other hand, it may be difficult to justify and implement such costs to the free market in practice. Thus, the results show the challenges arising from the market liberalization, as it can overload captive consumers with costs, in case its implementation is not carefully designed.

Finally, a case study incorporating the equilibrium model in the generation expansion problem applied to the Brazilian system was carried out. It was shown that, under more realistic market conditions and considering the generators' risk aversion, they require a market premium with respect to the costs initially parametrized in the expansion model. Firstly, this premium was obtained through a new iteration of the model, which informed such requirement to the expansion model and resulted in a smaller expansion, with lower investment costs and higher operation (and marginal) costs to recover the investments. In a second case, it was assumed that this premium, labeled as the missing money, could be recovered through the payment of a new product, the firm energy. Such fixed revenue prevents the need of reducing the system expansion and enables the convergence at the first iteration (optimal in terms of total costs). This result shows that a fixed, separate payment for a product alongside the energy may be desirable and decrease the system's total costs, which corroborates with the current regulatory discussions in the Brazilian power market of separating the energy's and firm energy's (physical guarantee's) contracts. At last, it was assessed a case in which a new requirement was added: to meet the system's peak demand plus a margin with a new product, the firm capacity. It was shown that such requirement, as expected, increases the system's total costs, as a new, active constraint is added in the model. It was also found that other technologies, particularly natural gas plants, become more valuable to the system, as they can provide high availability at peak times, and so take part in the system expansion alongside the renewable plants. Finally, given the current undefinitions in this regard, this case varied the assumed firm capacity of renewable (wind) plants: the material impacts that such variations had in the system expansion evidenced the importance of determining the contributions of each plant/technology to the system's firm capacity, in order to obtain an optimal and reliable system configuration.

For future work, regarding the core of this dissertation (the equilibrium model), new elements could be added in order to sophisticate its representation. In particular, the addition of further options to consumers, such as distributed generation and demand response are desirable as they would interplay in the consumers' decisions and in the market equilibrium. Also, these are elements gaining pace in electricity markets worldwide, and its inclusion in the model would enable it to incorporate increasingly relevant issues. Moreover, the model would also benefit from the incorporation of a wider variety of contractual options. Currently, the proposed model can have multi-stage contracts, however they are defined with a flat profile only. Other options of seasonality, modulation, among other features, could be added in order to increase realism in future applications.

Regarding the expansion planning process, as already anticipated, studies are desired to explore and evaluate the contributions of the various technologies to the system's reliability requirements (such as firm capacity), as this has direct impacts in the system's optimal expansion and security of supply. Also, the assignation of firm energy and/or capacity to candidate agents is a complex process, whose methodology can have relevant impacts in the expansion. In principle, auctions should be a desirable route to assign these products in a competitive fashion – still, designing them incurs own complexities and tradeoffs that could be explored in future work.

Finally, although this dissertation has presented realistic case studies, with the representation of the Brazilian system and its main agents, more elements can be added in the future to enhance the case applications, such as multi-year horizons, individualized representation of all generators and distributors and of their risk profile, among others. Evidently, seeking and adding new data and elements is a laborious and gradual process, which can be carried out in parallel with enhancements in the model.

9
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10
Annex A – Proof of the equivalence of the MOPEC and the Social Welfare maximization problems

In this annex it is demonstrated that the solution of the MOPEC presented in Chapter 2 can be obtained through the maximization of social welfare. For this, the optimality conditions of the MOPEC, for each agent, and of the social welfare problem will be presented. From these sets of equations, it will be possible to show the equivalence between the solution of the two problems.

10.1 Optimality conditions of the problems associated to the MOPEC

This section presents the optimality conditions of the MOPEC associated to each agent. For doing so, the optimization problem of each agent, already in their treated version comprising the separation of the deterministic and stochastic components, as presented in Chapter 3, is adopted as a starting point.

10.1.1 Optimization problem of the Distributor

The optimization problem of the Distributor, including its optimality conditions is denoted as follows.

$$\text{Max } \sum_t \frac{p^R \times d_t^D - p^C \times q_t^D}{(1+r)^t} + \lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] + (1-\lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] \tag{10.1}$$

s.t.

dual variables

$$d_t^D \geq 0 \qquad \theta_t^D \tag{10.2}$$

$$q_t^D \geq 0 \qquad \beta_t^D \tag{10.3}$$

$y_s^D \leq 0$ $\quad \gamma_s^D$ (10.4)

$y_s^D - \sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} + a^D \leq 0$ $\quad \eta_s^D$ (10.5)

Optimality conditions:

Primal feasibility:

$$d_t^D \geq 0 \tag{10.6}$$

$$q_t^D \geq 0 \tag{10.7}$$

$$y_s^D \leq 0 \tag{10.8}$$

$$y_s^D - \sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} + a^D \leq 0 \tag{10.9}$$

Dual feasibility:

Variable d_t^D:

$$\frac{\left(p^R - \lambda^D \times \frac{1}{S} \sum_s \pi_{t,s}\right)}{(1+r)^t} - \theta_t^D - \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.10}$$

Variable q_t^D

$$\frac{\left(-p^C + \lambda^D \times \frac{1}{S} \sum_s \pi_{t,s}\right)}{(1+r)^t} - \beta_t^D + \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.11}$$

Variable y_s^D:

$$(1 - \lambda^D) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^D - \eta_s^D = 0 \tag{10.12}$$

Variable a^D:

$$(1 - \lambda^D) - \sum_s \eta_s^D = 0 \tag{10.13}$$

Primal-dual equality of linear programming:

$$\sum_t \frac{p^R \times d_t^D - p^C \times q_t^D}{(1+r)^t} + \lambda^D \times \frac{1}{S} \sum_s \left[\sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] + (1 - \lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] = 0 \tag{10.14}$$

$$\theta_t^D \leq 0 \tag{10.15}$$

$$\beta_t^D \leq 0 \tag{10.16}$$

$$\gamma_s^D \geq 0 \tag{10.17}$$

$$\eta_s^D \geq 0 \tag{10.18}$$

10.1.2 Optimization problem of the Captive Consumer Type II

The optimization problem of the Captive Consumer Type II, including its optimality conditions is denoted as follows.

$$\text{Max} \sum_t \frac{-p^R \times (d_t^{CII} - m_t^{CII}) - p^C \times q_t^{CII}}{(1+r)^t} + \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] + (1 - \lambda^{CII}) \times \left[a^{CII} + \frac{\sum_s y_s^{CII}}{S \times (1-\alpha)}\right] \quad (10.19)$$

s.t.

	dual variables	
$m_t^{CII} \geq 0$	θ_t^{CII}	(10.20)
$m_t^{CII} \leq d_t^{CII}$	σ_t^{CII}	(10.21)
$q_t^{CII} \geq 0$	β_t^{CII}	(10.22)
$y_s^{CII} \leq 0$	γ_s^{CII}	(10.23)
$y_s^{CII} - \sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} + a^{CII} \leq 0$	η_s^{CII}	(10.24)

Optimality conditions:

Primal feasibility:

$$m_t^{CII} \geq 0 \quad (10.25)$$

$$m_t^{CII} \leq d_t^{CII} \quad (10.26)$$

$$q_t^{CII} \geq 0 \quad (10.27)$$

$$y_s^{CII} \leq 0 \quad (10.28)$$

$$y_s^{CII} - \sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} + a^{CII} \leq 0 \quad (10.29)$$

Dual feasibility:

Variable m_t^{CII}:

$$\frac{\left(p^R - \lambda^{CII} \times \frac{1}{S} \sum_s \pi_{t,s}\right)}{(1+r)^t} - \theta_t^D - \sigma_t^{CII} - \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \quad (10.30)$$

Variableq_t^{CII}:

$$\frac{-p^C + \lambda^{CII} \times \frac{1}{S} \sum_s \pi_{t,s}}{(1+r)^t} - \beta_t^{CII} + \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \quad (10.31)$$

Variable y_s^{CII}:

$$(1-\lambda^{CII})\times\frac{1}{S\times(1-\alpha)}-\gamma_s^{CII}-\eta_s^{CII}=0 \quad (10.32)$$

Variable a^{CII}:

$$(1-\lambda^{CII})-\sum_s \eta_s^{CII}=0 \quad (10.33)$$

Primal-dual equality of linear programming::

$$\sum_t \frac{-p^R\times(d_t^{CII}-m_t^{CII})-p^C\times q_t^{CII}}{(1+r)^t}+\lambda^{CII}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times(q_t^{CII}-m_t^{CII})}{(1+r)^t}\right]+(1-\lambda^{CII})\times \left[a^{CII}+\frac{\sum_s y_s^{CII}}{S\times(1-\alpha)}\right]=\sum_t \sigma_t^{CII}\times d_t^{CII} \quad (10.34)$$

$$\theta_t^{CII}\le 0 \quad (10.35)$$

$$\theta_t^{CII}\le 0$$

$$\sigma_t^{CII}\ge 0 \quad (10.36)$$

$$\beta_t^{CII}\le 0 \quad (10.37)$$

$$\gamma_s^{CII}\ge 0 \quad (10.38)$$

$$\eta_s^{CII}\ge 0 \quad (10.39)$$

10.1.3 Optimization problem of the Free Consumer

The optimization problem of the Free consumer, including its optimality conditions is denoted as follows.

$$\text{Max}\sum_t \frac{-p^R\times m_t^L-p^C\times q_t^L}{(1+r)^t}+\lambda^L\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times\left(q_t^L-(d_t^L-m_t^L)\right)}{(1+r)^t}\right]+ (1-\lambda^L)\times\left[a^L+\frac{\sum_s y_s^L}{S\times(1-\alpha)}\right] \quad (10.40)$$

s.t.

	dual variables	
$m_t^L\ge 0$	θ_t^L	(10.41)
$m_t^L\le d_t^L$	σ_t^L	(10.42)
$q_t^L\ge 0$	β_t^L	(10.43)
$y_s^L\le 0$	γ_s^L	(10.44)
$y_s^L-\sum_t\frac{\pi_{t,s}\times(q_t^L-m_t^L)}{(1+r)^t}+a^L\le 0$	η_s^L	(10.45)

Optimality conditions:

Primal feasibility:

$$m_t^L \geq 0 \tag{10.46}$$

$$m_t^L \leq d_t^L \tag{10.47}$$

$$q_t^L \geq 0 \tag{10.48}$$

$$y_s^L \leq 0 \tag{10.49}$$

$$y_s^L - \sum_t \frac{\pi_{t,s} \times (q_t^L - m_t^L)}{(1+r)^t} + a^L \leq 0 \tag{10.50}$$

Dual feasibility:

Variable m_t^L:

$$\frac{-p^R + \lambda^L \times \frac{1}{S} \sum_s \frac{\pi_{t,s}}{(1+r)^t}}{(1+r)^t} - \theta_t^L - \sigma_t^L - \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.51}$$

Variable q_t^L:

$$\frac{-p^C + \lambda^L \times \frac{1}{S} \sum_s \frac{\pi_{t,s}}{(1+r)^t}}{(1+r)^t} - \beta_t^L + \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.52}$$

Variable y_s^L:

$$(1 - \lambda^L) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^L - \eta_s^L = 0 \tag{10.53}$$

Variable a^L:

$$(1 - \lambda^L) - \sum_s \eta_s^L = 0 \tag{10.54}$$

Primal-dual equality of linear programming:

$$\sum_t \frac{-p^R \times m_t^L - p^C \times q_t^L}{(1+r)^t} + \lambda^L \times E\left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] + (1 - \lambda^L) \times CVaR_\alpha\left[\sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] = \sigma_t^L \times d_t^L \tag{10.55}$$

$$\theta_t^L \leq 0 \tag{10.56}$$

$$\sigma_t^L \geq 0 \tag{10.57}$$

$$\beta_t^L \leq 0 \tag{10.58}$$

$$\gamma_s^L \geq 0 \tag{10.59}$$

$$\eta_s^L \geq 0 \tag{10.60}$$

10.1.4 Optimization problem of the Generator

The optimization problem of the Free consumer, including its optimality conditions is denoted as follows.

$$\max \sum_t \frac{p^C \times q_t^G}{(1+r)^t} + \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t}\right] + (1-\lambda^G) \times \left[a^G + \frac{\sum_s y_s^G}{S \times (1-\alpha)}\right] \quad (10.61)$$

s.t.

dual variables

$$q_t^G \geq 0 \qquad \beta_t^G \quad (10.62)$$

$$y_s^G \leq 0 \qquad \gamma_s^G \quad (10.63)$$

$$y_s^G - \sum_t \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t} + a^G \leq 0 \qquad \eta_s^G \quad (10.64)$$

Optimality conditions:

Primal feasibility:

$$q_t^G \geq 0 \quad (10.65)$$

$$y_s^G \leq 0 \quad (10.66)$$

$$y_s^G - \sum_t \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t} + a^G \leq 0 \quad (10.67)$$

Dual feasibility:

Variable q_t^G:

$$\frac{p^C - \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s}}{(1+r)^t}\right]}{(1+r)^t} - \beta_t^G - \sum_s \eta_s^G \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \quad (10.68)$$

Variable y_s^G:

$$(1-\lambda^G) \times \frac{1}{S \times (1-\alpha)} - \eta_s^G = 0 \quad (10.69)$$

Variable a^G:

$$(1-\lambda^G) - \sum_s \eta_s^G = 0 \quad (10.70)$$

Primal-dual equality of linear programming:

$$\sum_t \frac{p^C \times q_t^G}{(1+r)^t} + \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t}\right] + (1-\lambda^G) \times \left[a^G + \frac{\sum_s y_s^G}{S \times (1-\alpha)}\right] = \sum_{t,s} \eta_s^G \times \frac{\pi_{t,s} \times g_{t,s}}{(1+r)^t} \quad (10.71)$$

$$\beta_t^G \leq 0 \tag{10.72}$$

$$\gamma_s^G \geq 0 \tag{10.73}$$

$$\eta_s^G \geq 0 \tag{10.74}$$

10.2 Optimality conditions of the Social Welfare maximization problem

Using the social Welfare maximization problem developed in Chapter 2, which already comprises the separation of the deterministic and stochastic components of each agent's revenues, as well as the cancellation of the terms with opposing signals, and adding its optimality conditions, we have:

$$\begin{aligned}
\text{Max } \lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] + (1-\lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] \\
+ \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] \\
+ (1-\lambda^{CII}) \times \left[a^{CII} + \frac{\sum_s y_s^{CII}}{S \times (1-\alpha)}\right] \\
+ \lambda^L \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] \\
+ (1-\lambda^L) \times \left[a^L + \frac{\sum_s y_s^L}{S \times (1-\alpha)}\right] \\
+ \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t}\right] \\
+ (1-\lambda^G) \times \left[a^G + \frac{\sum_s y_s^G}{S \times (1-\alpha)}\right]
\end{aligned} \tag{10.75}$$

s.t.

dual variables

$d_t^D \geq 0$ $\qquad \theta_t^D$

$q_t^D \geq 0$ $\qquad \beta_t^D$ (10.76)

$y_s^D \leq 0$ $\qquad \gamma_s^D$ (10.77)

$y_s^D - \sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} + a^D \leq 0$ η_s^D (10.78)

$m_t^{CII} \geq 0$ θ_t^{CII} (10.79)

$m_t^{CII} \leq d_t^{CII}$ σ_t^{CII} (10.80)

$q_t^{CII} \geq 0$ β_t^{CII} (10.80)

$y_s^{CII} \leq 0$ γ_s^{CII} (10.81)

$y_s^{CII} - \sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} + a^{CII} \leq 0$ η_s^{CII} (10.82)

$m_t^L \geq 0$ θ_t^L (10.83)

$m_t^L \leq d_t^L$ σ_t^L (10.84)

$q_t^L \geq 0$ β_t^L (10.85)

$y_s^L \leq 0$ γ_s^L (10.86)

$y_s^L - \sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t} + a^L \leq 0$ η_s^L (10.87)

$q_t^G \geq 0$ β_t^G (10.88)

$y_s^G \leq 0$ γ_s^G (10.89)

$y_s^G - \sum_t \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t} + a^G \leq 0$ η_s^G (10.90)

$d_t^D + m_t^{CII} - m_t^L = d_t^{CI} + d_t^{CII}$ ξ_t^R (10.91)

$q_t^G - q_t^D - q_t^{CII} - q_t^L = 0$ τ_t^C (10.92)

Optimality conditions:

Primal feasibility:

$d_t^D \geq 0$ (10.93)

$q_t^D \geq 0$ (10.94)

$y_s^D \leq 0$ (10.95)

$y_{s,D} - \sum_t \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t} + a_D \leq 0$ (10.96)

Dual feasibility:

Variable $d_t^D \geq 0$:

$-\xi_t^R - \frac{\lambda^D \times \frac{1}{S} \sum_s \pi_{s,t}}{(1+r)^t} - \theta_t^D - \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0$ (10.97)

Variable $q_t^D \geq 0$:

$$\tau_t^C + \frac{\lambda^D \times \frac{1}{S} \sum_s \pi_{s,t}}{(1+r)^t} - \beta_t^D + \sum_s \eta_s^D \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.98}$$

Variable y_s^D:

$$(1 - \lambda_D) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^D - \eta_s^D = 0 \tag{10.99}$$

Variable a^D

$$(1 - \lambda_D) - \sum_s \eta_s^D = 0 \tag{10.100}$$

Primal feasibility:

$$m_t^{CII} \geq 0 \tag{10.101}$$

$$m_t^{CII} \leq d_t^{CII} \tag{10.102}$$

$$q_t^{CII} \geq 0 \tag{10.103}$$

$$y_s^{CII} \leq 0 \tag{10.104}$$

$$y_s^{CII} - \sum_t \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t} + a^{CII} \leq 0 \tag{10.105}$$

Dual feasibility:

Variable m_t^{CII}:

$$-\xi_t^R - \lambda^{CII} \times \frac{1}{S}\left[\sum_s \frac{\pi_{t,s}}{(1+r)^t}\right] - \theta_t^{CII} - \sigma_t^{CII} - \sum_s \eta_s^{CII} \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.106}$$

Variable q_t^{CII}:

$$\tau_t^C + \lambda^{CII} \times \frac{1}{S}\left[\sum_s \frac{\pi_{t,s}}{(1+r)^t}\right] - \beta_t^L + \sum_s \eta_s^{CII} \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.107}$$

Variable y_s^{CII}:

$$(1 - \lambda^{CII}) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^{CII} - \eta_s^{CII} = 0 \tag{10.108}$$

Variable a^{CII}:

$$(1 - \lambda^{CII}) - \sum_s \eta_s^{CII} = 0 \tag{10.109}$$

Primal feasibility:

$$m_t^L \geq 0 \tag{10.110}$$

$$m_t^L \leq d_t^L \tag{10.111}$$

$$q_t^L \geq 0 \tag{10.112}$$

$$y_s^L \leq 0 \tag{10.113}$$

$$y_s^L - \sum_t \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t} + a^L \leq 0 \tag{10.114}$$

Dual feasibility:

Variable $m_t^L \geq 0$:

$$\xi_t^R + \lambda^L \times \frac{1}{S}\left[\sum_s \frac{\pi_{t,s}}{(1+r)^t}\right] - \theta_t^L - \sigma_t^L + \sum_s \eta_s^L \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.115}$$

Variable $q_t^L \geq 0$:

$$\tau_t^C + \lambda^L \times \frac{1}{S}\left[\sum_s \frac{\pi_{t,s}}{(1+r)^t}\right] - \beta_t^L + + \sum_s \eta_s^L \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.116}$$

Variable y_s^L:

$$(1 - \lambda^L) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^L - \eta_s^L = 0 \tag{10.117}$$

Variable a^L:

$$(1 - \lambda^L) - \sum_s \eta_s^L = 0 \tag{10.118}$$

Primal feasibility:

$$q_t^G \geq 0 \tag{10.119}$$

$$y_s^G \leq 0 \tag{10.120}$$

$$y_s^G - \sum_t \frac{\pi_{t,s} \times (g_{t,s} - q_t^G)}{(1+r)^t} + a^G \leq 0 \tag{10.121}$$

Dual feasibility:

Variable q_t^G:

$$-\tau_t^C - \lambda^G \times \frac{1}{S}\left[\sum_s \frac{\pi_{t,s}}{(1+r)^t}\right] - \beta_t^G - \sum_s \eta_s^G \times \frac{\pi_{t,s}}{(1+r)^t} = 0 \tag{10.122}$$

Variable y_{s,G,k_G}:

$$(1 - \lambda^G) \times \frac{1}{S \times (1-\alpha)} - \gamma_s^G - \eta_s^G = 0 \tag{10.123}$$

Variable a^G:

$$(1 - \lambda^G) - \sum_s \eta_s^G = 0 \tag{10.124}$$

$$d_t^D + m_t^{CII} - m_t^L = d_t^{CI} + d_t^{CII} \tag{10.152}$$

$$q_t^G - q_t^D - q_t^{CII} - q_t^L = 0 \tag{10.125}$$

Primal-dual equality of linear programming:

$$\lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s}\times(q_t^D - d_t^D)}{(1+r)^t}\right] + (1-\lambda^D)\times\left[a^D + \frac{\sum_s y_s^D}{S\times(1-\alpha)}\right] + \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s}\times(q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] + (1-\lambda^{CII})\times\left[a^{CII} + \frac{\sum_s y_s^{CII}}{S\times(1-\alpha)}\right] + \lambda^L \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s}\times\left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] + (1-\lambda^L)\times\left[a^L + \frac{\sum_s y_s^L}{S\times(1-\alpha)}\right] + \lambda^G \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s}\times(g_{t,s} - q_t^G)}{(1+r)^t}\right] + (1-\lambda^G)\times\left[a^G + \frac{\sum_s y_s^G}{S\times(1-\alpha)}\right] = \sigma_t^{CII}\times d_t^{CII} + \sigma_t^L \times d_t^L + \sum_{t,s}\eta_s^G \times \frac{\pi_{t,s}\times g_{t,s}}{(1+r)^t} + \sum_t (d_t^{CI} + d_t^{CII})\times \xi_t^R \tag{10.126}$$

$$\theta_t^D \le 0 \tag{10.127}$$

$$\beta_t^D \le 0 \tag{10.128}$$

$$\gamma_s^D \ge 0 \tag{10.129}$$

$$\eta_s^D \ge 0 \tag{10.130}$$

$$\theta_t^{CII} \le 0 \tag{10.131}$$

$$\sigma_t^{CII} \ge 0 \tag{10.132}$$

$$\beta_t^{CII} \le 0 \tag{10.133}$$

$$\gamma_s^{CII} \ge 0 \tag{10.134}$$

$$\eta_s^{CII} \ge 0 \tag{10.135}$$

$$\theta_t^L \le 0 \tag{10.136}$$

$$\sigma_t^L \ge 0 \tag{10.137}$$

$$\beta_t^L \le 0 \tag{10.138}$$

$$\gamma_s^L \ge 0 \tag{10.139}$$

$$\eta_s^L \ge 0 \tag{10.140}$$

$$\beta_t^G \le 0 \tag{10.141}$$

$$\gamma_s^G \ge 0 \tag{10.142}$$

$$\eta_s^G \ge 0 \tag{10.143}$$

10.3 Equivalence of the solutions of the two problems

Firstly, the optimality conditions associated with the MOPEC problems coincide with the ones of the Social Welfare when the following correspondences are made:

$$\xi_t^R = -\frac{p^R}{(1+r)^t} \tag{10.144}$$

$$\tau_t^C = -\frac{p^C}{(1+r)^t} \tag{10.145}$$

Regarding the verification of the primal-dual equality, note that the conditions of primal and dual feasibility of each MOPEC problem imply, for each agent, the following.

For the Distributor:

$$\sum_t \frac{p^R \times d_t^D - p^C \times q_t^D}{(1+r)^t} + \lambda^D \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^D - d_t^D)}{(1+r)^t}\right] + (1-\lambda^D) \times \left[a^D + \frac{\sum_s y_s^D}{S \times (1-\alpha)}\right] \leq 0 \tag{10.146}$$

For the Captive Consumer Type II:

$$\text{Max} \sum_t \frac{-p^R \times (d_t^{CII} - m_t^{CII})}{(1+r)^t} + \lambda^{CII} \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times (q_t^{CII} - m_t^{CII})}{(1+r)^t}\right] + (1-\lambda^{CII}) \times \left[a^{CII} + \frac{\sum_s y_s^{CII}}{S \times (1-\alpha)}\right] \leq \sigma_t^{CII} \times d_t^{CII} \tag{10.147}$$

For the Free Consumer:

$$\text{Max} \sum_t \frac{-p^R \times m_t^L - p^C \times q_t^L}{(1+r)^t} + \lambda^L \times \frac{1}{S}\left[\sum_{t,s} \frac{\pi_{t,s} \times \left(q_t^L - (d_t^L - m_t^L)\right)}{(1+r)^t}\right] + (1-\lambda^L) \times \left[a^L + \frac{\sum_s y_s^L}{S \times (1-\alpha)}\right] \leq \sigma_t^L \times d_t^L \tag{10.148}$$

For the Generator:

$$\sum_{t}\frac{p^{C}\times q_{t}^{G}}{(1+r)^{t}}+\lambda^{G}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times\left(g_{t,s}-q_{t}^{G}\right)}{(1+r)^{t}}\right] \\ +(1-\lambda^{G})\times\left[a^{G}+\frac{\sum_{s}y_{s}^{G}}{S\times(1-\alpha)}\right]\leq\sum_{t,s}\eta_{s}^{G}\times\frac{\pi_{t,s}\times g_{t,s}}{(1+r)^{t}} \quad (10.149)$$

Summing these conditions and using the equilibrium constraints from the MOPEC, the following is obtained.

$$\lambda^{D}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times(q_{t}^{D}-d_{t}^{D})}{(1+r)^{t}}\right]+(1-\lambda^{D})\times\left[a^{D}+\frac{\sum_{s}y_{s}^{D}}{S\times(1-\alpha)}\right] \\ +\lambda^{CII}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times(q_{t}^{CII}-m_{t}^{CII})}{(1+r)^{t}}\right] \\ +(1-\lambda^{CII})\times\left[a^{CII}+\frac{\sum_{s}y_{s}^{CII}}{S\times(1-\alpha)}\right] \\ +\lambda^{L}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times\left(q_{t}^{L}-(d_{t}^{L}-m_{t}^{L})\right)}{(1+r)^{t}}\right] \\ +(1-\lambda^{L})\times\left[a^{L}+\frac{\sum_{s}y_{s}^{L}}{S\times(1-\alpha)}\right] \\ +\lambda^{G}\times\frac{1}{S}\left[\sum_{t,s}\frac{\pi_{t,s}\times\left(g_{t,s}-q_{t}^{G}\right)}{(1+r)^{t}}\right] \\ +(1-\lambda^{G})\times\left[a^{G}+\frac{\sum_{s}y_{s}^{G}}{S\times(1-\alpha)}\right] \\ \leq\sigma_{t}^{CII}\times d_{t}^{CII}+\sigma_{t}^{L}\times d_{t}^{L}+\sum_{t,s}\eta_{s}^{G}\times\frac{\pi_{t,s}\times g_{t,s}}{(1+r)^{t}} \quad (10.150)$$

However, the primal-dual condition of the Welfare Maximization problem implies that the inequality above is satisfied with equality, using the correspondences denoted by Equations (10.144) and (10.145).

Therefore, each inequality is satisfied using the equalities denoted in Equations (10.144) and (10.145) and the conditions of each problem of the MOPEC, directly implying that both problems are optimized.