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A

In this appendix, recalling notation in Section 3.2, our goal is prove equation (3.6):

$$\mathcal{Q}^{\hat\alpha,\hat\beta,i,j}_{0,\tilde k,\tilde t}(0^2) = x^c_Q.$$

More precisely, we want to prove that given a quotient family $(\mathcal{Q}^{\alpha,\beta,i,j}_{m,\ell,t})$ there are natural numbers $k, m, \ell, \tilde m, \tilde\ell$, with $(m,\ell) \neq (\tilde m, \tilde\ell)$, and numbers $\hat\alpha$ and $\hat\beta$ close to α and β, and small t such that x^c_Q is a common fixed point of $\mathcal{Q}^{\hat\alpha,\hat\beta,i,j}_{m,\ell,t}$ and $\mathcal{Q}^{\hat\alpha,\hat\beta,i,j}_{\tilde m,\tilde\ell,t}$, and $\mathcal{Q}^{\hat\alpha,\hat\beta,i,j}_{0,\tilde k,\tilde t}(0^2) = x^c_Q$.

Here we consider the case $(i,j) = (+,+)$, the other cases follows similarly. Without lost of generality, we can assume that in the local coordinates $x^c_P = (1,0)$ and $x^c_Q = (1,0)$. Recall that $\alpha = \rho\, e^{2\pi i \phi}$ and $\beta = \varrho\, e^{2\pi i\varphi}$, where $0 < \rho < 1 < \varrho$ and $\phi, \varphi \in [0,1)$.

Recall that for $t = (t_1, t_2)$ the bidimensional quotient map $\mathcal{Q}^{\alpha,\beta,+,+}_{m,\ell,t}(x,y)$ is of the form:

$$\varrho^\ell \begin{pmatrix} \cos\ell\, 2\pi\varphi & -\sin\ell\, 2\pi\varphi \\ \sin\ell\, 2\pi\varphi & \cos\ell\, 2\pi\varphi \end{pmatrix} \left[\rho^m \begin{pmatrix} \cos m\, 2\pi\phi & -\sin m\, 2\pi\phi \\ \sin m\, 2\pi\phi & \cos m\, 2\pi\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \right].$$

We choose $k > 0$ (we will explain this choice later) and consider

$$\tilde t = (\tilde t_1, \tilde t_2) = (\rho^{-k}\, \cos k\, 2\pi\phi, -\rho^{-k}\, \sin k\, 2\pi\phi).$$

We want to prove that

$$\mathcal{Q}^{\alpha,\beta,+,+}_{m,\ell,\tilde t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Using trigonometric formulae and considering $x^c_Q = (1,0)$ and $\tilde t$ above this equality can be read as follows

$$\varrho^\ell \begin{pmatrix} \cos\ell 2\pi\varphi & -\sin\ell 2\pi\varphi \\ \sin\ell 2\pi\varphi & \cos\ell 2\pi\varphi \end{pmatrix} \left[\rho^m \begin{pmatrix} \cos m2\pi\phi & -\sin m2\pi\phi \\ \sin m2\pi\phi & \cos m2\pi\phi \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \tilde t_1 \\ \tilde t_2 \end{pmatrix} \right] = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\varrho^{\ell}\begin{pmatrix}\cos\ell 2\pi\varphi & -\sin\ell 2\pi\varphi\\ \sin\ell 2\pi\varphi & \cos\ell 2\pi\varphi\end{pmatrix}\begin{pmatrix}\rho^{m}\cos m2\pi\phi+\varrho^{-k}\cos k2\pi\varphi\\ \rho^{m}\sin m2\pi\phi-\varrho^{-k}\sin k2\pi\varphi\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$$

$$\varrho^{\ell}\begin{pmatrix}\rho^{m}\cos(\ell 2\pi\varphi+m2\pi\phi)+\varrho^{-k}\cos(\ell 2\pi\varphi-k2\pi\varphi\\ \rho^{m}\sin(\ell 2\pi\varphi+m2\pi\phi)+\varrho^{-k}\sin(\ell 2\pi\varphi-k2\pi\varphi\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$$

Which is equivalent to the following system:

$$\begin{cases}\varrho^{\ell}\rho^{m}\cos(\ell 2\pi\varphi+m2\pi\phi)+\varrho^{\ell-k}\cos(\ell 2\pi\varphi-k2\pi\varphi)=1\\ \varrho^{\ell}\rho^{m}\sin(\ell 2\pi\varphi+m2\pi\phi)+\varrho^{\ell-k}\sin(\ell 2\pi\varphi-k2\pi\varphi)=0.\end{cases}$$

Multiplying second equation by i and adding first equation we get

$$\begin{aligned}&\rho^{m}\varrho^{\ell}\,e^{i\,(\ell\,2\pi\varphi+m\,2\pi\phi)}+\varrho^{\ell-k}\,e^{i\,(\ell\,2\pi\varphi-k\,2\pi\varphi)}=1\\ &\varrho^{\ell}\,e^{i\,\ell\,2\pi\varphi}\left(\rho^{m}\,e^{i\,m\,2\pi\phi}+\varrho^{-k}\,e^{-i\,k\,2\pi\varphi}\right)=1\\ &\left(\rho\,e^{i\,2\pi\phi}\right)^{m}+\left(\varrho\,e^{i\,2\pi\varphi}\right)^{-k}=\left(\varrho\,e^{i\,2\pi\varphi}\right)^{-\ell}\\ &\big(\underbrace{\rho\,e^{i\,2\pi\phi}}_{\alpha}\big)^{m}=\big(\underbrace{\varrho\,e^{i\,2\pi\varphi}}_{\beta}\big)^{-\ell}-\big(\underbrace{\varrho\,e^{i\,2\pi\varphi}}_{\beta}\big)^{-k}.\end{aligned}$$

Therefore we need to find a pair of natural numbers m and ℓ satisfying the previous equality. The rest of the proof follows exactly as in the proof of Proposition 3.5.

B

In the appendix we see that Lema 3.10 also holds for $(\mathbb{C},\mathbb{R})$-cycle, see Remark 3.13.

We want to prove that given a quotient family $(\mathcal{Q}_{m,\ell,t}^{\alpha,\beta_{s+1},\beta_{s+2},i,j})$ as Definition 4.5 for the case $(\mathbb{C},\mathbb{R})$-cycle, there are natural numbers $k, m, \ell, \tilde{m}, \tilde{\ell}$, with $(m,\ell) \neq (\tilde{m},\tilde{\ell})$, and numbers $\hat{\alpha}$ and $\hat{\beta}_{s+1}$ close to α and β_{s+1}, repectively, and small t such that x_Q^c is a common fixed point of $\mathcal{Q}_{m,\ell,t}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},i,j}$ and $\mathcal{Q}_{\tilde{m},\tilde{\ell},t}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},i,j}$, and $\mathcal{Q}_{0,\tilde{k},\tilde{t}}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},i,j}(0^2) = x_Q^c$.

The proof follows as Proposition 4.6. Let us remark the main differences.

Without lost of generality, we can assume that in local coordinates $x_P^c = (1,0)$ and $x_Q^c = (1,0)$. After an arbitrarily small perturbation we can assume that α has a rational argument ϕ. Fix $n > 0$ such that the map $C_\alpha^n = \rho^n R_\phi^n = \rho^n\,\mathrm{Id}$, where R_ϕ denotes the rotation of angle ϕ.

We consider the case $(i,j) = (+,+)$. Recalling Equations (4.5) and (4.4), for the case $(+,+)$ we have

$$\mathcal{Q}_{n,\ell,(t_1,t_2)}^{\alpha,\beta_{s+1},\beta_{s+2},+,+}(x,y) = \big((\beta_{s+1})^\ell[\rho^n\,x + t_1],\,(\beta_{s+2})^\ell(\rho^n\,M_1\,M_2\,y + t_2]\big).$$

Let $t = (t_1,t_2) = (t_1,0)$ and consider a point $(x,0)$. Then

$$\mathcal{Q}_{n,\ell,(t_1,0)}^{\alpha,\beta_{s+1},\beta_{s+2},+,+}(x,0) = \big((\beta_{s+1})^\ell[\rho^n\,x + t_1],\,0\big). \qquad \text{(B.1)}$$

We will choose pairs $(m,\ell\,j)$ and $(\tilde{m},\ell\,(j+1))$ and a parameter t_1 such that (after a small perturbation) the point $x_Q^c = (1,0)$ is a fixed point for these compositions.

After an arbitrarily small perturbation of β_{s+1} we can assume that there are arbitrarily large (even) ℓ, j and (multiple of n) m such that

$$\rho^m = (\beta_{s+1})^{-\ell\,j} - (\beta_{s+1})^{-\ell\,(j+1)}.$$

Consider a $\tilde{m} >> m$ such that $\tilde{\rho}^{\tilde{m}}$ is close to zero for all $\tilde{\rho}$ close to ρ. Take

$k > 0$ (close to $\ell\,(j+1)$), $\hat{\beta}_{s+1}$ close to β_{s+1} and $\hat{\rho}$ close to ρ such that

$$(\hat{\rho})^m - (\hat{\rho})^{\tilde{m}} = (\hat{\beta}_{s+1})^{-\ell\, j} - (\hat{\beta}_{s+1})^{-\ell\,(j+1)}, \tag{B.2}$$

$$(\hat{\beta}_{s+1})^{-k} = -(\hat{\rho})^m + (\hat{\beta}_{s+1})^{-\ell\, j}. \tag{B.3}$$

Let

$$t_1 = (\hat{\beta}_{s+1})^{-k} \quad \text{and} \quad t_2 = 0. \tag{B.4}$$

With these choices we have the following claims that prove our assertion:

Claim B.1. *The point* $(1,0)$ *is fixed for* $\mathcal{Q}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},+,+}_{m,\ell\, j,(t_1,0)}$ *and* $\mathcal{Q}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},+,+}_{\tilde{m},\ell\,(j+1),(t_1,0)}$.

Proof. Note that by (B.1), we have that

$$\mathcal{Q}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},+,+}_{m,\ell\, j,(t_1,0)}(1,0) = \big((\hat{\beta}_{s+1})^{\ell\, j}[(\hat{\rho})^m + t_1], 0\big)$$

and the choice of t_1 in (B.4) proves the first assertion of the claim.

Similarly, for the second assertion it is enough to see that

$$(\hat{\beta}_{s+1})^{\ell\,(j+1)}\,[(\hat{\rho})^{\tilde{m}} + t_1] = 1.$$

Indeed, by the definition of t_1 in (B.4), (B.3) and (B.2), respectively, we have

$$\begin{aligned}
(\hat{\beta}_{s+1})^{\ell\,(j+1)}\,\big[(\hat{\rho})^{\tilde{m}} + t_1\big] &= (\hat{\beta}_{s+1})^{\ell\,(j+1)}\,\big[(\hat{\rho})^{\tilde{m}} + (\hat{\beta}_{s+1})^{-k}\big] \\
&= (\hat{\beta}_{s+1})^{\ell\,(j+1)}\,\big[(\hat{\rho})^{\tilde{m}} - (\hat{\rho})^{m} + (\hat{\beta}_{s+1})^{-\ell\, j}\,\big] \\
&= (\hat{\beta}_{s+1})^{\ell\,(j+1)}\,\big[(\hat{\rho})^{\tilde{m}} - (\hat{\rho})^{\tilde{m}} + (\hat{\beta}_{s+1})^{-\ell\,(j+1)}\,\big] = 1,
\end{aligned}$$

proving the claim. □

Claim B.2. $\mathcal{Q}^{\hat{\alpha},\hat{\beta}_{s+1},\beta_{s+2},+,+}_{0,k,(t_1,0)}(0,0) = (1,0)$.

Proof. By the choice of t_1 and Equation (B.2) we have that

$$t_1 = -(\hat{\rho})^m + (\hat{\beta}_{s+1})^{-\ell\, j} = -(\hat{\rho})^{\tilde{m}} + (\hat{\beta}_{s+1})^{-\ell\,(j+1)}.$$

Using Equation (B.1), note that by (B.3) and the choice of t_1 above we get

$$(\hat{\beta}_{s+1})^k[\hat{\rho}^0 + t_1] = (\hat{\beta}_{s+1})^k[(\hat{\beta}_{s+1})^{-k}] = 1$$

ending the proof of the claim. □