



Luiz Felipe Nobili França

Topological Properties of Partially Hyperbolic Attractors

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Matemática of the Departamento de Matemática da PUC-Rio, as partial fulfillment of the requirements for the degree of Doutor em Matemática.

Advisor : Prof. Lorenzo J. Díaz
Co-Advisor: Prof. Flavio Abdenur

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Abstract

Nobili, F; Díaz, L. J.; . **Topological Properties of Partially Hyperbolic Attractors**. Rio de Janeiro, 2012. 67p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

In this work we extend the results in (12) and (22) about the minimality of one of the strong foliations (stable or unstable), for the case of robustly transitive attractors that is partially hyperbolic with one dimensional center bundle. In our context the partial hyperbolicity is defined only in the attractor. Some consequences are obtained as the verification that these attractors are (robustly) homoclinic classes, have (robustly) empty interior and admit a spectral decomposition. Similar results still holds in the case of generically transitive attractors.

Keywords

attractor. foliation. minimality. transitivity. spectral decomposition. non-empty interior.

Resumo

Nobili, F; Díaz, L. J.; . **Propriedades Topológicas de Atratores Parcialmente Hiperbólicos**. Rio de Janeiro, 2012. 67p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Neste trabalho estendemos os resultados em (12) e (22), sobre a minimalidade de uma das folheações (estável ou instável forte), para o caso de atratores robustamente transitivos parcialmente hiperbólico e com direção central unidimensional. No nosso contexto a hiperbolicidade parcial esta definida somente no atrator. Algumas consequências são obtidas tais como a verificação de que estes atratores são (robustamente) classes homoclínicas, possuem (robustamente) interior vazio e admitem uma decomposição espectral. Resultados similares ainda valem no caso de atratores genericamente transitivos.

Palavras-chave

atrator. folheação. minimalidade. transitividade. decomposição espectral. interior não vazio.

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Whatever you can do or dream, begin it.

Goethe.