Miguel Adriano Koiller Schnoor

The non-existence of absolutely continuous invariant probabilities is C^1-generic for flows

TESE DE DOUTORADO

DEPARTAMENTO DE MATEMÁTICA

Programa de Pós-Graduação em Matemática

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Pontifícia Universidade Católica
do Rio de Janeiro

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Thesis presented to the Programa de Pós-Graduação em Matemática of the Departamento de Matemática, PUC–Rio, as partial fulfillment of the requirements for the degree of Doutor em Matemática.

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In memory of my father. His incredible wisdom and irresistible tenderness continue to inspire me.

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Abstract

Koiller Schnoor, Miguel Adriano; Bochi, Jairo. **The non-existence of absolutely continuous invariant probabilities is C^1-generic for flows**. Rio de Janeiro, 2012. 79p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

We prove that C^1-generic vector fields in a compact manifold do not have absolutely continuous invariant probabilities. This extends a result of Avila and Bochi to the continuous time case.

Keywords

Absolutely continuous invariant probability; ergodic theory; non-invariant Rokhlin tower; orthonormal frame flow.

Resumo

Koiller Schnoor, Miguel Adriano; Bochi, Jairo. **Fluxos C^1-genéricos não possuem probabilidades invariantes absolutamente contínuas**. Rio de Janeiro, 2012. 79p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Provamos que campos de vetores C^1-genéricos em uma variedade compacta não possuem probabilidades invariantes absolutamente contínuas em relação a uma medida de volume. Este trabalho estende ao caso de tempo contínuo um resultado de Avila e Bochi.

Palavras–chave

Fluxo de frames ortonormais; probabilidade invariante absolutamente contínua; teoria ergódica; torre de Rokhlin não invariante.

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