

Miguel Adriano Koiller Schnoor

**The non-existence of absolutely
continuous invariant probabilities is
 C^1 -generic for flows**

TESE DE DOUTORADO

DEPARTAMENTO DE MATEMÁTICA

Programa de Pós-Graduação em Matemática

Rio de Janeiro
August 2012

Miguel Adriano Koiller Schnoor

**The non-existence of absolutely continuous
invariant probabilities is C^1 -generic for flows**

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação em Matemática of the Departamento de Matemática, PUC-Rio, as partial fulfillment of the requirements for the degree of Doutor em Matemática.

Advisor: Prof. Jairo Bochi

Rio de Janeiro
August 2012



Miguel Adriano Koiller Schnoor

**The non-existence of absolutely continuous
invariant probabilities is C^1 -generic for flows**

Thesis presented to the Programa de Pós-Graduação em Matemática of the Departamento de Matemática do Centro Técnico Científico da PUC-Rio, as partial fulfillment of the requirements for the degree of Doutor em Matemática. Approved by the following commission:

Prof. Jairo Bochi

Advisor

Pontifícia Universidade Católica do Rio de Janeiro

Prof. Alexander Eduardo Arbieto Mendoza

Instituto de Matemática – UFRJ

Prof. Artur Avila Cordeiro de Melo

Instituto Nacional de Matemática Pura e Aplicada – IMPA

Prof. Maria José Pacífico

Instituto de Matemática – UFRJ

Prof. Rafael Oswaldo Ruggiero Rodriguez

Departamento de Matemática – PUC-Rio

Prof. Alex Lúcio Ribeiro Castro

Departamento de Matemática – PUC-Rio

Prof. Martin Andersson

Instituto de Matemática – UFF

Prof. José Eugenio Leal

Coordinator of the Centro Técnico Científico
Pontifícia Universidade Católica do Rio de Janeiro

Rio de Janeiro, 03/08/2012

All rights reserved.

Miguel Adriano Koiller Schnoor

B.A. and M.Sc. in Mathematics from Pontifícia Universidade Católica do Rio de Janeiro.

Bibliographic data

Koiller Schnoor, Miguel Adriano

The non-existence of absolutely continuous invariant probabilities is C^1 -generic for flows / Miguel Adriano Koiller Schnoor; advisor: Jairo Bochi . — 2012.

79 f. : il. ; 30 cm

1. Tese (Doutorado em Matemática) - Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2012.

Inclui bibliografia

1. Matemática – Teses. 2. Fluxo de frames ortonormais;. 3. probabilidade invariante absolutamente contínua;. 4. teoria ergódica;. 5. torre de Rokhlin não invariante.. I. Bochi, Jairo. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Matemática. III. Título.

CDD: 510

In memory of my father. His incredible wisdom and irresistible tenderness
continue to inspire me.

Acknowledgments

Foremost, I would like to express my gratitude to my advisor, Jairo Bochi, for his patience, motivation and for all knowledge that has been passed to me during the writing of this work. It was a huge privilege to work alongside him.

I would like to thank Capes and CNPq for the financial support that allowed me to devote myself exclusively to the Ph.D work.

To all my family, especially my mother and my sisters, for giving me support and love throughout my life; also to my uncle Jair and cousin José, who inspired me to follow this path.

My sincere thanks also goes to Flávio Abdenur, who enthusiastically encouraged me to study Dynamical Systems and to Lorenzo Díaz for introducing me to this field of Mathematics. To Enrique Pujals, for his guidance during IMPA seminars, to Alexander Arbieto and Martin Andersson, for the many mathematical conversations, and to Thomas Lewiner, for helping me with L^AT_EX issues.

I was lucky to work among several incredible friends in PUC and I am grateful not only for their suggestions and encouragement to my work, but also for the laughs during lunch and coffee time. In special, I thank Luiz Felipe Nobili, Rodrigo Pacheco, Yuri Ki, Débora Mondaini, Victor Goulart, Tiane Marcarini, Eduardo Telles, Américo Barbosa, André Zaccur, Guilherme Frederico Lima, João Paulo Romanelli, José Gondin, Márcio Telles, André Carneiro, Wilson Reis e Bernardo Pagnoncelli.

I owe my gratitude also to students from other institutions, like Rômulo Rosa, Jaqueline Rocha, Mariana Pinheiro, Tiago Catalan, Yuri Lima, Pablo Guarino, Pablo Barrientos, Arten Raibekas, Ivana Latosinski, Fernando Carneiro and Michel Cambrainha, with whom I had the opportunity to exchange great experiences at conferences and seminars I attended.

I must put on record my admiration and appreciation for the staff of the Mathematics Department of PUC. Without their daily help, this paper would be impossible to exist. Above all, I must especially thank my great heroine, Creuza.

I obviously need to thank my two loves, Sabrina and Valentina, for giving a new - and wonderful - direction to my life.

Abstract

Koiller Schnoor, Miguel Adriano; Bochi, Jairo. **The non-existence of absolutely continuous invariant probabilities is C^1 -generic for flows**. Rio de Janeiro, 2012. 79p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

We prove that C^1 -generic vector fields in a compact manifold do not have absolutely continuous invariant probabilities. This extends a result of Avila and Bochi to the continuous time case.

Keywords

Absolutely continuous invariant probability; ergodic theory; non-invariant Rokhlin tower; orthonormal frame flow.

Resumo

Koiller Schnoor, Miguel Adriano; Bochi, Jairo. **Fluxos C^1 -genéricos não possuem probabilidades invariantes absolutamente contínuas**. Rio de Janeiro, 2012. 79p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Provamos que campos de vetores C^1 -genéricos em uma variedade compacta não possuem probabilidades invariantes absolutamente contínuas em relação a uma medida de volume. Este trabalho estende ao caso de tempo contínuo um resultado de Avila e Bochi.

Palavras-chave

Fluxo de frames ortonormais; probabilidade invariante absolutamente contínua; teoria ergódica; torre de Rokhlin não invariante.

Contents

1	Introduction	10
1.1	Absolutely continuous invariant probabilities	10
1.2	Main Theorem	11
1.3	Remarks about the proof	11
1.4	Structure of the work	12
2	Preliminaries	15
2.1	Basic facts about vector fields and flows	15
2.2	Non-Conformality	21
2.3	Linear Cocycles	22
2.4	The orthonormal frame flow	27
2.5	Basic facts about volume crushing	28
2.6	Functions with bounded logarithmic derivative	30
2.7	Vitali Covering	32
3	Transverse Section	34
4	Tubular Chart	39
5	Local Crushing	52
5.1	Crushing-Time	54
5.2	Sliced Tube	57
5.3	Bump Function	64
5.4	Proof of the Fettuccine's Lemma	68
6	Global Crushing	74

List of Figures

2.1	Non-compatible cross-sections.	16
2.2	The image of an Euclidean ball by a linear invertible map L is inscribed in a sphere with radius $r_2 = \ L\ r$ and circumscribed on a sphere with radius $r_1 = \ L^{-1}\ ^{-1}r$.	22
2.3	A linear cocycle over the flow $\{\varphi^t\}$.	23
3.1	A saddle p with $\dim W_{loc}^s(p) = 2$ and $\dim W_{loc}^u(p) = 1$; the cross-sections Σ^u and Σ^s are respectively a cylinder and a union of two disks.	35
4.1	Tubular Chart	41
4.2	Choice of the initial orthonormal frame for $d = 3$.	43
4.3	The manifold \tilde{H} as a graph	45
4.4	The 1-codimensional submanifold $\tilde{H} = \{(x, w, y) : y = xw^2\}$ in Example 4.0.18 is graph of a function with unbounded second derivative.	50
5.1	Schematic illustration of V being crushed.	53
5.2	Crushing in the y -direction.	55
5.3	Distortion in the w -direction is not significant if the initial slice is thin enough.	60
5.4	$ \tau_s(p) - s < \Delta$ for all $s \in [a, T + \Delta]$.	62