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A Proofs

Proof of Proposition 2

We divide the proof in three parts. First, we determine the equilibrium mortgage payments and which condition imply that a household is able to get a new loan in $t = 2$. Next, we describe the payment for each type on a mortgage got in $t = 0$. Once we have the mortgage payments, we finally prove the proposition.

Part A: It is useful to establish the equilibrium payment of a mortgage given in $t = 2$. In the event of a bubble burst in $t = 1$, $B_1 = 0$ and $S_1 = f$. If the household defaults on their mortgage, they have $\bar{y}_i$ to use as down payment. Because the representative bank is risk neutral, the payment X_i' is such that

$$(f - \bar{y}_i) = (1 - p_i) \cdot \gamma f + p_i \cdot X_i'$$

Rearranging terms, we get the payment:

$$X_i' = \frac{(f - \bar{y}_i) - (1 - p_i) \cdot \gamma f}{p_i} \quad \text{(A-1)}$$

In order to this level of payment be an equilibrium, it must be affordable and the household must be willing to make the payment. The second restriction is implied by the first.

$$X_i' \leq \bar{y}_i \Rightarrow X_i' - r \leq \bar{y}_i - r \leq \bar{U}$$

where the second inequality holds because of Assumption 1.

In order to be affordable, the household income must exceed the mortgage payment. That is,

$$X_i' \leq \bar{y}_i \iff \bar{y}_i \geq y^{*,i} \equiv \frac{[1 - (1 - p_i)\gamma] \cdot f}{1 + p_i}$$

If $\bar{y}_i < y^{*,i}$ the household cannot afford the mortgage in any state in $t = 3$. Hence, the household is better off by not getting the new mortgage and saving the money that would be given as down payment.

Part B: It is also useful to determine $X_i(w_0, S_0)$. Once we have the value of the mortgage payment, we will show that it is optimal for families to behave like stated in Proposition 2.

On the one hand, if the household defaults only when the mortgage payment is not affordable, the payment in a competitive credit market is such that

$$(f - w_0) = (1 - p_i) \cdot \gamma\big(f + q\xi B_0\big) + p_i \cdot X_i(w_0, S_0)$$

Rearranging terms, we get the payment:

$$X_i(w_0, S_0) = \frac{[1 - (1 - p_i)\gamma]f + [1 - \gamma q\xi(1 - p_i)]B_0 - w_0}{p_i} \tag{A-2}$$

On the other hand, if the household defaults strategically when the bubble bursts, the payment in a competitive credit market is such that

$$(f - w_0) = (1 - p_i) \cdot \gamma\big(f + q\xi B_0\big) + p_i(1 - q) \cdot \gamma f + p_i q \cdot X_i(w_0, S_0)$$

Rearranging terms, we get the payment:

$$X_i(w_0, S_0) = \frac{[1 - (1 - qp_i)\gamma]f + [1 - \gamma q\xi(1 - p_i)]B_0 - w_0}{qp_i} \tag{A-3}$$

Part C:

1. First let us analyze the behavior of families when the bubble bursts in which case $B_1 = B_3 = 0$. Consider Low-income families that are shut from the mortgage market if they walk away from their mortgage, so for them $P_1(Ref) = 0$. Assume that families of type L default strategically, then condition 2-5 must hold.

   $$X_i(w_0, S_0) > \bar{U} + r$$

   Because $X_i(w_0, S_0)$ must be affordable to the family, it is true that $X_i(w_0, S_0) \leq \bar{y}_L$. Therefore, the above condition implies

   $$\bar{y}_L - r > \bar{U}$$

   which contradicts Assumption 1. Hence, Low-income families do not default strategically when bubble bursts.

2. Now consider families of type H that are able to return to the mortgage market if they default on the mortgage. In order to default strategically, condition 2-5 must hold when $P_1(Ref) = 1$.

   $$X_i(w_0, S_0) - (p_i X_i' + \bar{y}_H) > \Big(1 - p_i\Big) \cdot \bar{U}$$

Substituting the expressions A-1 and A-3, it is possible to show that the above inequality is equivalent to

$$w_0 < w'_H \equiv \underline{w}^*_H + qp_H \cdot \left[\bar{y}_H - f + (1-p_H)\cdot\left(\gamma f - \bar{U}\right)\right] \quad \text{(A-4)}$$

where $\underline{w}^*_H \equiv [1-(1-qp_i)\gamma]f + [1-\gamma q\xi(1-p_H)]B_0 - qp_H \cdot \bar{y}_H$.

Rearranging equation A-4

$$w_0 < w'_H \equiv \underline{w}^*_H + qp_H \cdot \left[\bar{y}_H - \left((1+p_H)y^{*,H} + (1-p_H)\bar{U}\right)\right] \quad \text{(A-5)}$$

Families of type H with $w_0 < w'_H$ are better by defaulting on their mortgage when the bubble bursts.

3. Now turn to the case in which the bubble continues: $B_1 > 0$. If the household is not offered a mortgage after a default, a similar argument of part 1 proves that paying the mortgage is the best action.

   To analyze the case in which a new mortgage is offered, it is necessary to evaluate the new contract's payment. Households of both types pay the mortgage $t = 3$ independent of the bubble realization due to Assumption 1. The equilibrium payment is defined by:

$$(f - y_i) = (1-p_i)\cdot\gamma\left(f + q\xi B_1\right) + p_i \cdot X'_i$$

   Rearranging terms, we get the payment:

$$X'_i = \frac{[1-(1-p_i)\gamma]f + [1-\gamma q\xi(1-p_i)]B_1 - \bar{y}_i}{p_i} \quad \text{(A-6)}$$

   In order to this level of payment be an equilibrium, X'_i must be affordable and the household must be willing to make the payment. The second restriction is implied by the first.

$$X'_i \leq \bar{y}_i \Rightarrow X'_i - r \leq \bar{y}_i - r \leq \bar{U}$$

   where the second inequality holds because of Assumption 1.

   If the household is offered a new mortgage, it entails the payment describes in equation A-6. It is possible to show that if the bubble rises fast enough or if $\bar{y}_i < w_0$, then $X'_i > X_i(w_0, S_0)$ for both types. Hence, condition 2-5 does not hold and it is not optimal to default strategically.

   □

Proof of Proposition 3

A family get a mortgage in $t = 0$ if $X(w_0, S_0)$ that satisfies equation 2-6 and if $\bar{y}_i \geq X_i(w_0, S_0)$.

Because a family of type L does not default strategically when the bubble bursts, their mortgage payment is given by expression A-2. Substituting equation A-2, it is possible to show that $\bar{y}_L \geq X_L(w_0, S_0)$ is equivalent to

$$w_0 \geq \underline{w}_L^* \equiv [1 - (1 - p_L)\gamma]f + [1 - \gamma q\xi(1 - p_L)]B_0 - p_L \cdot \bar{y}_L \qquad \text{(A-7)}$$

Therefore, a Low-income family get a mortgage in $t = 0$ if $w_0 \geq \underline{w}_L^*$.

If a family of type H defaults strategically when the bubble burst, their mortgage payment is given by expression A-3. Substituting equation A-3, it is possible to show that $\bar{y}_H \geq X_H(w_0, S_0)$ is equivalent to

$$w_0 \geq \underline{w}_H^* \equiv [1 - (1 - qp_H)\gamma]f + [1 - \gamma q\xi(1 - p_H)]B_0 - qp_H \cdot \bar{y}_H \qquad \text{(A-8)}$$

Therefore, a High-income family that defaults strategically in $t = 1$ gets a mortgage if $w_0 \geq \underline{w}_H^*$.

A High-income household that does not default strategically has a minimum initial wealth threshold analogous to the one in expression A-7 that is given by

$$w_0 \geq \underline{w}_H^{**} \equiv [1 - (1 - p_H)\gamma]f + [1 - \gamma q\xi(1 - p_H)]B_0 - p_H \cdot \bar{y}_H \qquad \text{(A-9)}$$

The minimum wealth requirement for High-income families that do not default strategically is not important if $\underline{w}_H^{**} < w'_H$ because these families already have $w_0 > w'_H$. From equations A-5 and A-9, it is possible to show that

$$w'_H - \underline{w}_H^{**} = qp_H \cdot \left[\bar{y}_H - \left((1 + p_H)y^{*,H} + (1 - p_H)\bar{U}\right)\right] + p_H(1 - q)(\bar{y}_H - \gamma f)$$

The second term in the expression above is positive if $\gamma f < \bar{y}_H$. The first term determines the existence of strategic default in the economy and it is positive if $\bar{y}_H > \big((1+p_H)y^{*,H} + (1-p_H)\bar{U}$. Therefore, High-income households that do not default strategically in $t = 1$ also get a mortgage if condition (A-8) is satisfied for values of $\bar{y}_H$ sufficiently high. In this case, High-income families get a mortgage in $t = 0$ if $w_0 \geq \underline{w}_H^*$.

It is easy to see from the expressions above that $\underline{w}_i^*$ is decreasing in $\bar{y}_i$, q and ξ. $\square$

Proof of Proposition 4

From the definition of $\bar{w}'$ in equation A-4, it is possible to show that the mass of families defaulting strategically in this economy is given by:

$$\alpha(\bar{w}' - \underline{w}_H^*) = \alpha q p_H \cdot \left[\bar{y}_H - \left((1+p_H)y^{*,H} + (1-p_H)\bar{U}\right) \right]$$

The expression inside the brackets is positive if $\bar{y}_H > \left((1+p_H)y^{*,H} + (1-p_H)\bar{U}\right.$. That is, for a value of $\bar{y}_H$ sufficiently high, there is strategic default in the economy. In such a case, it is easy to see that the incidence of strategic default in the economy is increasing in q and $\bar{y}_H$ and that it is decreasing in $\bar{u}$. $\square$

Lemma 8

Lemma 8 *Consider a mortgage contract such that delinquent households are not allowed to apply for a new mortgage in the future.*

1. *The gain from walking away from this mortgage contract at time t is given by the following expression:*

$$U_t(tenant_t) = U_t^{i,D} + \bar{u} + \max\{\gamma S_t - D_t; 0\} \tag{A-10}$$

2. *The gain from paying this mortgage contract at time t is given by the following expression:*

$$U_t(owner_t; i) = U_t^{i,D} + \bar{u} + A_t(x, w_t, S_t, r_t; i) \tag{A-11}$$

where A_t is the expected gain from paying the mortgage at t and $w_t = Rw_{t-1} + y_t - x$.

3. *The expected gain from paying the mortgage is given by*

$$A_T(x, w_T, S_T, r_T; i) = \beta\frac{\bar{u}}{1-\beta} - x + \beta E_T[S_{T+1}]$$

$$\begin{aligned} A_t(x, w_t, S_t, r_t; i) &= \beta r_t + \beta\bar{u} - x + \\ &+ \beta Prob_t\big(Rw_t + y_{t+1} \geq x\big) \cdot E_t\Big[\max\left\{A_{t+1}; \max\{S_{t+1} - D_{t+1}, 0\}\right\}\Big| Rw_t + y_{t+1} \geq x\Big] + \\ &+ \beta Prob_t\big(Rw_t + y_{t+1} < x\big) \cdot E_t\Big[\max\{S_{t+1} - D_{t+1}, 0\}\Big| Rw_t + y_{t+1} < x\Big] \end{aligned} \tag{A-12}$$

4. *The expected gain from paying the mortgage is decreasing in the periodic payment, x, and increasing in the borrower's wealth, w_t, and in the rent value, r_t.*

Proof of Lemma 8:

Part 1: When households decide to walk away from this mortgage contract, we assume that they are not allowed to apply for a new mortgage, so they have to become tenants from period $t+1$ onwards. Hance, households gain the homeownership value at t, $\bar{u}$, and the utility of permanently renting a home $U_t^{i,D}$ in equation 4-1 which corresponds to the family's income expected present value minus the expected rent expenses.

Furthermore, as discussed in the paper, the household may choose to default or prepay the mortgage depending on the selling house value to debt ratio. If the selling house value, γS_t, exceeds the debt balance, D_t, a household that walks away form the mortgage is better off by selling the house, prepaying the mortgage and keeping the difference $\gamma S_t - D_t$. Hence, families that walk away also gain $\max\{\gamma S_t - D_t; 0\}$.

Part 2:

We prove Part 2 by retroactive induction. Consider first the problem faced by the borrower at period T, the final payment due date. If the borrower decides to pay the mortgage at the final payment due date, he solves the following problem:

$$\max_{(c_T,w_T)} c_T+\bar{u}+\beta E_T\left[Rw_T+y_{T+1}+\beta\frac{E[y_i]}{1-\beta}+\left(\frac{\bar{u}}{1-\beta}+S_{T+1}-\frac{r_{T+1}}{1-\beta}\right)\right] \quad \text{(A-13)}$$

subject to

$$c_T + w_T = Rw_{T-1} + y_T - x$$

In Problem A-13, we assume that at $T+1$ the household's utility is their wealth, Rw_T, plus the present value of their life-time expected income, $y_{T+1} + \beta\frac{E[y_i]}{1-\beta}$, and the final value of homeownership. We assume that property of the house yields the private benefit $\bar{u}$ at each period from time $T+1$ onwards, so the homeowner gains the present value of these private benefits $\frac{\bar{u}}{1-\beta}$. Furthermore, homeowners gain, as in the simple model of Section 2, the house value, S_{T+1}, discounted of the implicit rent paid for living in the house from time $T+1$ onwards, $\frac{r_{T+1}}{1-\beta}$. Note that we assume that $r_T = E_T[R_{T+1}]$.

Because $\beta \cdot R = 1$, the borrower is indifferent between any level of w_T. In particular, he chooses $w_T = Rw_{T-1} + y_T - x$ and we can write his utility as

$$U_T(owner_T) = Rw_{T-1} + y_T - x + \bar{u} + \beta\frac{E[y_i] - r_T}{1-\beta} + \beta\frac{\bar{u}}{1-\beta} + \beta E_T[S_{T+1}]$$

$$U_T(owner_T) = \Big(Rw_{T-1} + y_T + \beta\frac{E[y_i] - r_T}{1-\beta}\Big) + \bar{u} + \Big(\beta\frac{\bar{u}}{1-\beta} - x + \beta E_T[S_{T+1}]\Big)$$

Using the definitions of $U_T^{D,i}$ in equation 4-1 and of $A_T(x, w_T, S_T, r_T)$ in equation A-12, we can write the expected utility of paying the house at time T.

$$U_T(owner_T) = U_T^{D,i} + \bar{u} + A_T(x, w_T, S_T, r_T)$$

As can be seen, the expression of $A_T(x, w_T, S_T, r_T)$ is decreasing in x and increasing in w_t and in r_T. To finish the proof, we assume that equation A-11 and the properties of A_{t+1} hold for period $t+1$. In period t, the problem faced by the borrower that decides to pay the is given by:

$$\max_{(c_t,w_t)} c_t + \bar{u} + \beta Prob_t(Rw_t + y_{t+1} < x)E_t\big[U_{t+1}(tenant_{t+1})\big|Rw_t + y_{t+1} < x\big] +$$
$$+ \beta Prob_t(Rw_t + y_{t+1} \geq x)E_t\big[\max\{U_{t+1}(tenant_{t+1}); U_{t+1}(owner_{t+1})\}\big|Rw_t + y_{t+1} \geq x\big] \tag{A-14}$$

subject to

$$c_t + w_t = Rw_{t-1} + y_t - x$$

In problem A-14, we use the fact if the borrower cannot afford the mortgage at $t+1$, $Rw_t + y_{t+1} < x$, then he has to become a renter in which case his gain is $U_{t+1}(tenant_{t+1})$. Moreover, if the borrower can afford the mortgage at $t+1$, $Rw_t + y_{t+1} \geq x$, then he maximizes his utility by choosing whether to pay the mortgage or walk away from it. Substituting equations A-11 and A-10 into problem A-14 and using the fact that $c_t = Rw_{t-1} + y_t - x - w_t$,

$$\max_{w_t} \quad \big(Rw_{t-1} + y_t - x - w_t\big) + \bar{u} + \beta Prob_t(Rw_t + y_{t+1} < x)\cdot$$
$$\cdot E_t\big[U_{t+1}^{D,i} + \bar{u} + \max\{\gamma S_{t+1} - D_{t+1}; 0\}\big|Rw_t + y_{t+1} < x\big] + \beta Prob_t(Rw_t + y_{t+1} \geq x)\cdot$$
$$\cdot E_t\big[\max\{U_{t+1}^{D,i} + \bar{u} + \max\{\gamma S_{t+1} - D_{t+1}; 0\}; U_{t+1}^{D,i} + \bar{u} + A_{t+1}\}\big|Rw_t + y_{t+1} \geq x\big]$$

Rearranging items,

$$\max_{w_t} \quad \big(Rw_{t-1} + y_t - w_t + \beta E_t[U_{t+1}^{D}]\big) + \bar{u} + \beta\bar{u} - x +$$
$$+ \beta Prob_t\big[y_{t+1} + Rw_t < x\big]E_t\big[\max\{\gamma S_{t+1} - D_{t+1}; 0\}\big|y_{t+1} + Rw_t < x\big] +$$
$$+ \beta Prob_t\big[y_{t+1} + Rw_t \geq x\big]E_t\big[\max\{A_{t+1}; \max\{\gamma S_{t+1} - D_{t+1}; 0\}\}\big|y_{t+1} + Rw_t \geq x\big]$$

The first term in parenthesis equals $U_t^{D,i} + \beta r_t$. Therefore, the above expression can be rewritten as

$$\max_{w_t} \quad U_t^{D,i} + \bar{u} + \beta\bar{u} + \beta r_t - x +$$
$$+ \beta Prob_t\big[y_{t+1} + Rw_t < x\big] E_t\big[\max\{\gamma S_{t+1} - D_{t+1}; 0\}\big| y_{t+1} + Rw_t < x\big] +$$
$$+ \beta Prob_t\big[y_{t+1} + Rw_t \geq x\big] E_t\big[\max\left\{A_{t+1}; \max\{\gamma S_{t+1} - D_{t+1}; 0\}\right\}\big| y_{t+1} + Rw_t \geq x\big]$$

Finally, using the definition of $A_t(x, w_t, S_t, r_t)$ in equation (A-12),

$$\max_{w_t \in [0, Rw_{t-1} + y_t - x]} \quad U_t^{D,i} + \bar{u} + A_t(x, w_t, S_t, r_t) \tag{A-15}$$

If $A_t(x, w_t, S_t, r_t)$ is increasing in w_t, then $w_t = Rw_{t-1} + y_t - x$ and the utility of paying the mortgage at t is given by the equation A-12. Hence, to finish the proof we still need to prove Part 4 of Lemma 8.

Part 3: It follows directly from the proof of Part 2.

Part 4: To show that $A_t(x, w_t, S_t, r_t)$ is increasing in w_t, assume that $A_{t+1}(x, w_{t+1}, S_{t+1}, r_{t+1})$ is increasing in w_{t+1} which implies that $w_{t+1} = Rw_t + y_{t+1} - x$. Consider two wealth values such that $w'_t > w_t$, then $\forall y_{t+1}\ w'_{t+1} > w_{t+1}$ and $A_{t+1}(w'_{t+1}) \geq A_t(w_{t+1})$.

It is possible to show that $A_t(x, w'_t, S_t, r_t) - A_t(x, w_t, S_t, r_t)$ is bounded from below by the expression:

$$0 < \beta \underbrace{\left(F(x - Rw'_t) - F(x - Rw_t) \right)}_{<0} \cdot$$
$$\cdot \underbrace{\left(E_t\Big[\max\{\gamma S_{t+1} - D_{t+1}; 0\}\Big] - E_t\Big[\max\left\{A_{t+1}; \max\{\gamma S_{t+1} - D_{t+1}\}\right\}\Big| y_{t+1} + Rw_{t+1} \geq x\Big] \right)}_{<0}$$

With the same steps, it is possible to prove that $A_t(x, w_t, S_t, r_t)$ is increasing in r_t and decreasing in x. $\square$

Proof of Proposition 5

Consider a household that walks away from their mortgage at time t. To prove Proposition 5 we first have to compute the expected utility of this delinquent household in the case they finance a new home purchase with a loan like the one analyzed in Lemma 8. With such contract, the household is permanently shut from the credit market if they default on the new mortgage.

Consider a delinquent household of type i that decides to finance a new home purchase at time $t+1$ with a mortgage that entails the periodic payment x_i^{**}. At $t+1$, the household receives income, y_{t+1}, and uses its savings, Rw_t, so the expected utility is given by:

$$U_{t+1}(owner_{t+1};i) = \Big[(1-d^D)\cdot(Rw_t+y_{y+1})+\beta\frac{E[y_i]-r_{t+1}}{1-\beta}\Big]+\bar{u}+x_i^{**}+$$
$$+A_0(x_i^{**},1-d^D)\cdot(Rw_t+y_{y+1}),S_{t+1},r_{t+1};i) \quad \text{(A-16)}$$

In equation A-16, the first term is the household's outside option of becoming a tenant from period $t+2$ onwards in which case the household has a wealth of $(1-d^D)\cdot(Rw_t+y_{t+1})$ after making the new mortgage down payment at t+1. The household also evaluates the present value of the difference between their life-time income minus rent expenses. The other terms correspond to the gains from the mortgage as described by Part 1 of Lemma 8.

From Lemma 8, we know that $U_{t+1}(owner_{t+1};i)$ is decreasing in the mortgage periodic payment x, therefore the equilibrium payment of the new mortgage contract, x_i^{**}, is such that

$$x_i^{**} = \min\{x/ \quad V_{t+1}(x,w_{t+1},S_{t+1};i) \geq S_{t+1}-d^D\cdot w_{t+1}\} \quad \text{(A-17)}$$

In equation A-17, the representative bank accepts the new mortgage request from the household at time $t+1$ if the present value of the mortgages's cash flow, $V_{t+1}(x,w_{t+1},S_{t+1};i)$, exceeds the amount lent $S_{t+1}-d^D\cdot w_{t+1}$. Among the available contracts, the household chooses the mortgage with the smallest periodic payment.

Alternatively, a delinquent household also may decide to remain a tenant at $t+1$ in which case its expected utility is given by:

$$U_{t+1}(tenant_{t+1};i) = Rw_t+y_{t+1}+\beta\frac{E[y_i]-r_{t+1}}{1-\beta}]-r_{t+1} \quad \text{(A-18)}$$

In equation A-18, a delinquent household that does not buy a house at $t+1$ has to rent a home permanently. In this case the family keeps its wealth, Rw_t+y_{t+1}, and evaluates the expected present value of its future income discounted of rent costs. Furthermore, the household pays r_{t+1} for renting a home at $t+1$.

Now we turn to the problem faced by a household that decide to walk away from the mortgage at time t. The problem is given by:

$$\max_{(c_t,w_t)} c_t+\bar{u}+\beta Prob_t[\overline{Ref_t^{i,*}}]E_t\big[U_{t+1}(tenant_{t+1})\big|\overline{Ref_t^{i,*}}\big]+$$
$$+\beta Prob_t[Ref_t^{i,*}]E_t\big[\max\{U_{t+1}(tenant_{t+1});U_{t+1}(owner_{t+1})\}\big|Ref_t^{i,*}\big] \quad \text{(A-19)}$$

subject to

$$c_t + w_t = Rw_{t-1} + y_t + \max\{\gamma S_t - D_t; 0\}$$

In problem A-19, the household that walks away from the mortgage still gains the current private benefit, $\bar{u}$, at time t. At time $t+1$, the household is eligible for a new mortgage contract with probability $Prob_t[Ref_t^{i,*}]$. In this case, the family chooses the best action between remaining to be a tenant or becoming a homeowner again. At $t+1$, the household also may be excluded from the credit market with probability $Prob_t[\overline{Ref_t^{i,*}}]$ in which case the family has to rent a home. The problem's constraint states that besides savings and income, a delinquent household also counts with the money from possibly selling the house and prepaying the mortgage, $\max\{\gamma S_t - D_t; 0\}$.

Substituting $U_{t+1}(tenant_{t+1})$ in equation A-18 and $U_{t+1}(owner_{t+1})$ in equation A-16 into problem A-19 and using the fact that $c_t = Rw_{t-1} + y_t + \max\{\gamma S_t - D_t; 0\} - w_t$,

$$\begin{aligned}\max_{w_t} \quad & \Big(Rw_{t-1} + y_t + \max\{\gamma S_t - D_t; 0\} - w_t\Big) + \bar{u} + \\ & + \beta Prob_t[\overline{Ref_t^{i,*}}] \cdot E_t\Big[Rw_t + y_{t+1} + \beta\frac{E[y_i] - r_{t+1}}{1-\beta} - r_{t+1}\Big|\overline{Ref_t^{i,*}}\Big] + \\ & + \beta Prob_t[Ref_t^{i,*}] \cdot E_t\Big[\max\Big\{Rw_t + y_{t+1} + \beta\frac{E[y_i] - r_{t+1}}{1-\beta} - r_{t+1}; \\ & (1-d^D)(Rw_t + y_{y+1}) + \beta\frac{E[y_i] - r_{t+1}}{1-\beta} + \bar{u} + x_i^{**} + A_0\Big\}\Big|Ref_t^{i,*}\Big]\end{aligned}$$

Using the facts that $\beta R = 1$ and $r_t = E_t[r_{t+1}]$, we can rewrite the problem as

$$\begin{aligned}\max_{w_t} \quad & \Big(Rw_{t-1} + y_t + \beta\frac{E[y_i] - r_t}{1-\beta} + \max\{\gamma S_t - D_t; 0\}\Big) + \bar{u} + \\ & + \beta Prob_t[\overline{Ref_t^{i,*}}] \cdot E_t\Big[- r_{t+1}\Big|\overline{Ref_t^{i,*}}\Big] + \beta Prob_t[Ref_t^{i,*}] \cdot \\ & \cdot E_t\Big[\max\Big\{- r_{t+1}; -d^D \cdot (Rw_t + y_{y+1}) + \bar{u} + x_i^{**} + A_0\Big\}\Big|Ref_t^{i,*}\Big]\end{aligned}$$

Again, using that $r_t = E_t[r_{t+1}]$ and the definition of $U_t^{D,i}$ in equation 4-1,

$$\begin{aligned}\max_{w_t} \quad & U_t^{D,i} + \bar{u} + \max\{\gamma S_t - D_t; 0\} + \\ & + \beta Prob_t[Ref_t^{i,*}] E_t\Big[\max\Big\{A_0 + r_{t+1} + \bar{u} + x_i^{**} - d^D \cdot (Rw_t + y_{y+1}); 0\Big\}\Big|Ref_t^{i,*}\Big]\end{aligned}$$

From Lemma 8, we know that $A_0(w)$ is increasing in the household's wealth, so the problem is solved with $w_t = Rw_{t-1} + y_t + \max\{\gamma S_t - D_t; 0\}$. Therefore, the expected utility of walking away at time t is given by:

$$U_t(tenant_t;i) = U_t^{D,i} + \bar{u} + \max\{\gamma S_t - D_t; 0\} + \\ + \underbrace{\beta Prob_t[Ref_t^{i,*}] E_t\Big[\max\left\{A_0 + r_{t+1} + \bar{u} + x_i^{**} - d^D \cdot (Rw_t + y_{y+1}); 0\right\}\Big| Ref_t^{i,*}\Big]}_{=M_t^i}$$

This proves also proves Part 2. □

Proof of Proposition 6

The proof of Proposition 6 is analogous to the proof of Lemma 8. The only difference is that the gain from walking away that now includes the expected utility of returning to the credit market.

Part 1: We prove Part 1 by retroactive induction. To prove that equation 4-4 holds at the final payment due date, T^*, we follow the same steps as in the proof of Lemma 8. If the borrower decides to pay the mortgage at the final payment due date, he gains property of the house and solves the following problem:

$$\max_{(c_{T^*}, w_{T^*})} c_{T^*} + \bar{u} + \beta E_{T^*}\left[Rw_v + y_{T^*+1} + \beta\frac{E[y_i]}{1-\beta} + \left(\frac{\bar{u}}{1-\beta} + S_{T^*+1} - \frac{r_{T^*+1}}{1-\beta}\right)\right] \tag{A-20}$$

subject to

$$c_{T^*} + w_{T^*} = Rw_{-1} + y_{T^*} - x$$

In Problem A-20, we assume that at T^*+1 the household's utility is their wealth, Rw_{T^*}, plus the present value of their life-time expected income, $y_{T^*+1} + \beta\frac{E[y_i]}{1-\beta}$, and the final value of homeownership. We assume that property of the house yields the private benefit $\bar{u}$ at each period from time T^*+1 onwards, so the homeowner gains the present value of these private benefits $\frac{\bar{u}}{1-\beta}$. Furthermore, homeowners gain, as in the simple model of Section 2, the house value, S_{T^*+1}, minus the implicit rent paid for living in the house from time T^*+1 onwards, $\frac{r_{T^*+1}}{1-\beta}$. Note that we assume that $r_{T^*} = E_{T^*}[R_{T^*+1}]$.

Because $\beta \cdot R = 1$, the borrower is indifferent between any level of w_{T^*}. In particular, he chooses $w_{T^*} = Rw_{T^*-1} + y_{T^*} - x$ and we can write his utility as

$$U_{T^*}(owner_{T^*}) = Rw_{T^*-1} + y_{T^*} - x + \bar{u} + \beta\frac{E[y_i] - r_{T^*}}{1-\beta} + \beta\frac{\bar{u}}{1-\beta} + \beta E_{T^*}[S_{T^*+1}]$$

$$U_{T^*}(owner_{T^*}) = \left(Rw_{T^*-1} + y_{T^*} + \beta\frac{E[y_i] - r_{T^*}}{1-\beta}\right) + \bar{u} + \left(\beta\frac{\bar{u}}{1-\beta} - x + \beta E_{T^*}[S_{T^*+1}]\right)$$

Using the definitions of $U_{T^*}^{D,i}$ in equation 4-1 and of $A_{T^*}(x, w_{T^*}, S_{T^*}, r_{T^*})$ in equation 4-5, we can write the expected utility of paying the house at time T^*.

$$U_{T^*}(owner_{T^*}) = U_{T^*}^{D,i} + \bar{u} + A_{T^*}(x, w_{T^*}, S_{T^*}, r_{T^*})$$

As can be seen, the expression of $A_{T^*}(x, w_{T^*}, S_{T^*}, r_{T^*})$ is decreasing in x and increasing in w_{T^*} and in r_{T^*}. To finish the proof, we assume that $U_{t+1}(owner_{t+1})$ in equation 4-4 holds and the properties of A^*_{t+1} are valid at $t+1$. At time t, the problem faced by the borrower that decides to pay the is given by:

$$\max_{(c_t, w_t)} c_t + \bar{u} + \beta Prob_t(Rw_t + y_{t+1} < x) E_t\left[U_{t+1}(tenant_{t+1}) \middle| Rw_t + y_{t+1} < x\right] + \\ + \beta Prob_t(Rw_t + y_{t+1} \geq x) E_t\left[\max\{U_{t+1}(tenant_{t+1}); U_{t+1}(owner_{t+1})\} \middle| Rw_t + y_{t+1} \geq x\right] \tag{A-21}$$

subject to

$$c_t + w_t = Rw_{t-1} + y_t - x$$

In problem A-21, we use the fact if the borrower cannot afford the mortgage at $t+1$, $Rw_t + y_{t+1} < x$, then he has to become a tenant in which case his gain is $U_{t+1}(tenant_{t+1})$ in equation 4-2. Moreover, if the borrower can afford the mortgage at $t+1$, $Rw_t + y_{t+1} \geq x$, then he maximizes his utility by choosing whether to pay the mortgage or walk away from it. Substituting equations 4-4 and 4-2 into problem A-21 and using the fact that $c_t = Rw_{t-1} + y_t - x - w_t$,

$$\max_{w_t} \quad \left(Rw_{t-1} + y_t - x - w_t\right) + \bar{u} + \beta Prob_t(Rw_t + y_{t+1} < x) \cdot \\ \cdot E_t\left[U_{t+1}^{D,i} + \bar{u} + \max\{\gamma S_{t+1} - D_{t+1} + M_t^i; 0\} \middle| Rw_t + y_{t+1} < x\right] + \beta Prob_t(Rw_t + y_{t+1} \geq x) \cdot \\ \cdot E_t\left[\max\{U_{t+1}^{D,i} + \bar{u} + \max\{\gamma S_{t+1} - D_{t+1} + M_t^i; 0\}; U_{t+1}^{D,i} + \bar{u} + A^*_{t+1}\} \middle| Rw_t + y_{t+1} \geq x\right]$$

Rearranging items,

$$\max_{w_t} \quad \left(Rw_{t-1} + y_t - w_t + \beta E_t[U_{t+1}^D]\right) + \bar{u} + \beta\bar{u} - x + \\ + \beta Prob_t\left[y_{t+1} + Rw_t < x\right] E_t\left[\max\{\gamma S_{t+1} - D_{t+1}; 0\} + M_t^i \middle| y_{t+1} + Rw_t < x\right] + \\ + \beta Prob_t\left[y_{t+1} + Rw_t \geq x\right] E_t\left[\max\{A^*_{t+1}; \max\{\gamma S_{t+1} - D_{t+1}; 0\} + M_t^i\} \middle| y_{t+1} + Rw_t \geq x\right]$$

The first term in parenthesis equals $U_t^{D,i} + \beta r_t$. Therefore, the above expression can be rewritten as

$$\max_{w_t} \quad U_t^{D,i} + \bar{u} + \beta\bar{u} + \beta r_t - x + \\ + \beta Prob_t\left[y_{t+1} + Rw_t < x\right] E_t\left[\max\{\gamma S_{t+1} - D_{t+1}; 0\} + M_t^i \middle| y_{t+1} + Rw_t < x\right] + \\ + \beta Prob_t\left[y_{t+1} + Rw_t \geq x\right] E_t\left[\max\left\{A^*_{t+1}; \max\{\gamma S_{t+1} - D_{t+1}; 0\} + M_t^i\right\} \middle| y_{t+1} + Rw_t \geq x\right]$$

Finally, using the definition of $A^*_t(x, w_t, S_t, r_t)$ in equation (4-6),

$$\max_{w_t \in [0, Rw_{t-1} + y_t - x]} \quad U_t^{D,i} + \bar{u} + A_t(x, w_t, S_t, r_t) \tag{A-22}$$

The household with a higher wealth at $t+1$ may choose whether to pay or not the mortgage rather than being forced to default due to lack of resources. Since households are indifferent between consuming at t or at $t+1$ because $\beta \cdot R = 1$, it is optimal for them to set $w_t = Rw_{t-1} + y_t - x$. Hence,

$$U_t(owner_t; i) = U_t^{D,i} + \bar{u} + A_t(x, w_t, S_t, r_t) \tag{A-23}$$

where $w_t = Rw_{t-1} + y_t - x$.

Part 2: It follows directly from the proof of Part 1. □

Proof of Proposition 7

First, consider the value of the contract's cash flow after the last mortgage payment due date. Because there are no other payments, the value of the contract is zero and $V_{T^*} = 0$.

Consider now the cash flow of a mortgage contract that the representative bank holds at $t+1$. If the borrower decides to pay the mortgage, the lender receives the mortgage periodic payment, x, plus the value of the mortgage contract at $t+1$, V_{t+1}.

If the borrower decides to walk away from the mortgage, the lender receives the mortgage's debt balance, D_{t+1}, whenever $\gamma S_{t+1} \geq D_{t+1}$ because the household chooses to sell the house and prepay the mortgage in this case. Alternatively, the lender receives γS_{t+1} whenever $\gamma S_{t+1} < D_{t+1}$ because in this case the household prefers to default on the mortgage. Hence, the mortgage's cash flow is $\min\{\gamma S_{t+1}; D_{t+1}\}$ if the borrower walks away from the mortgage.

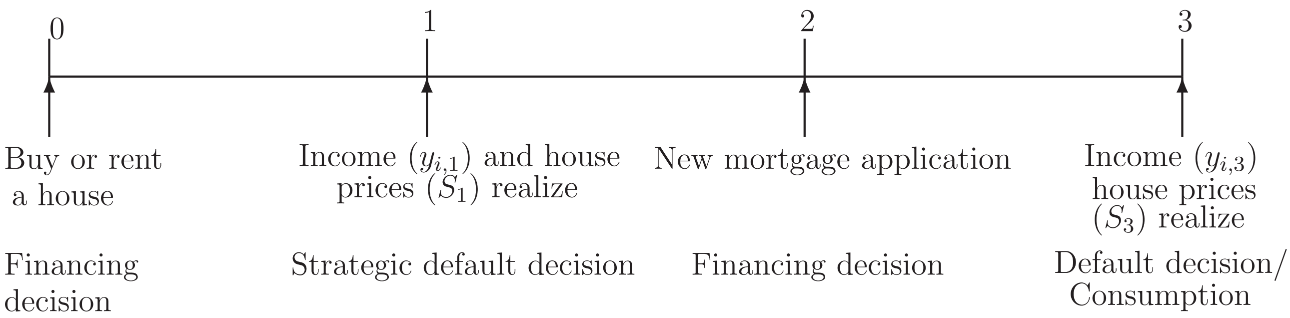
The lender's cash flow at $t+1$ can be summarized by the function $h_t(.)$ bellow.

$$h_{t+1}(x, w_t, S_{t+1}, y_{t+1}; i) = \begin{cases} x + V_{t+1}(x, S_{t+1}, w_{t+1}; i) & \text{if the borrower pays at t+1} \\ \min\{\gamma S_{t+1}; D_{t+1}\} & \text{if the borrower walks away at t+1} \end{cases} \tag{A-24}$$

At any period t, the contract's value is the expected present value of its cash flow. Therefore, the contract's value is $V_t(x, S_t, w_t; i) = E_t[h_t(x, w_t, S_{t+1}, y_{t+1})]$. □

B Tables and Figures

Figure B.1: Timing of events



	PRIME: Median Income of 80 thousand and unemployment chance of 8% per period				SUBPRIME: Median Income of 40 thousand and unemployment chance of 20% per period			
Initial wealth (thousands of dollars)	Interest Rate	Periodic payment (thousands of dollars)	Default probability Strategic	Default probability Non-strategic	Interest Rate	Periodic payment (thousands of dollars)	Default probability Strategic	Default probability Non-strategic
0.0	7.0%	0.54	3.1%	8.2%	-	-	-	-
10	6.0%	50	0.9%	8.2%	-	-	-	-
20	6.0%	47	0.0%	8.3%	-	-	-	-
30	6.0%	45	0.0%	8.3%	-	-	-	-
40	6.0%	43	0.0%	8.3%	-	-	-	-
50	6.0%	40	0.0%	4.1%	-	-	-	-
60	6.0%	38	0.0%	0.2%	6.0%	38	1.9%	1.1%
70	6.0%	36	0.0%	0.2%	6.0%	36	1.9%	0.9%
80	6.0%	33	0.0%	0.2%	6.0%	33	1.1%	1.1%
90	6.0%	31	0.0%	0.2%	6.0%	31	0.8%	0.9%

Table B.1: Model Simulation - Mortgage on 220 thousand of dollars house

Mortgage characteristics on 220 thousand of dollars house for several initial wealth values. Bubble parameters: $B_0 = 0.8$ (80 thousand dollars); $\xi = 1.4$; $q = 0.02$. Rent parameters: $r_0 = 0.07$ (7 thousand dollars); $u = 1.05$; $d = 0.95$; $p = 0.5$. Income Parameters of Prime families: $\bar{y} = 0.8$; $\pi = 0.08$. Income Parameters of Subprime families: $\bar{y} = 0.4$; $\pi = 0.2$. Mortgage contract: $T^* = 6$; $\gamma = 0.7$; $d = 1$; $d^D = 80\%$. Private benefit: $\bar{u} = 0.3$.

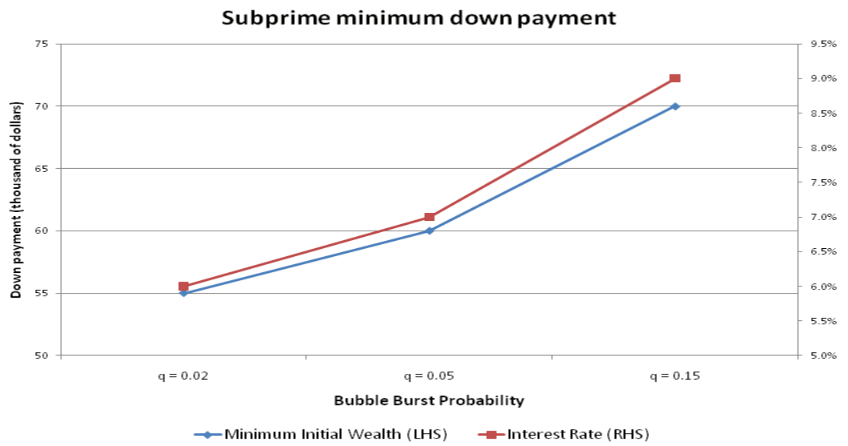


Figure B.2: Mortgage characteristics on 220 thousand of dollars house for several bubble burst probabilities and initial wealth values.

Bubble parameters: $B_0 = 0.8$ (80 thousand dollars); $\xi = 1.4$. Rent parameters: $r_0 = 0.07$ (7 thousand dollars); $u = 1.05$; $d = 0.95$; $p = 0.5$. Income Parameters of Prime families: $\bar{y} = 0.8$; $\pi = 0.08$. Income Parameters of Subprime families: $\bar{y} = 0.4$; $\pi = 0.2$. Mortgage contract: $T^* = 6$; $\gamma = 0.7$; $d = 1$; $d^D = 80\%$. Private benefit: $\bar{u} = 0.3$.

Table B.2: Summary Statistics: Variables Means per Year.

Year	Mortgage Delinquency	House Prices	House Prices Growth (2 years)	House Prices Growth (4 years)	Per Capita Personal Income (thousands of dollars)	Personal Income Growth (2 years)	Unemployment
1999	0.99%	127.4	10.72%	16.48%	29.16	12.53%	4.49%
2000	0.94%	136.2	13.48%	21.55%	31.12	12.86%	4.19%
2001	1.25%	146.5	15.39%	27.91%	31.74	11.12%	4.93%
2002	1.24%	158.3	16.81%	32.82%	32.05	5.42%	6.08%
2003	1.36%	172.9	18.56%	37.25%	32.89	5.87%	6.18%
2004	1.18%	194.6	23.25%	44.73%	34.55	9.84%	5.62%
2005	0.99%	222.3	28.24%	53.16%	36.24	12.03%	5.16%
2006	1.16%	236.4	20.77%	49.96%	38.48	12.97%	4.75%
2007	2.33%	230.5	4.08%	33.39%	40.21	12.31%	4.82%
2008	5.19%	203.5	-12.01%	5.53%	41.19	8.46%	6.23%
2009	9.04%	185.2	-17.64%	-14.04%	39.92	0.79%	10.03%
2010	9.68%	178.4	-11.74%	-21.44%	40.35	0.02%	10.58%
Full sample mean	2.95%	182.7	9.16%	23.94%	35.66	8.69%	6.09%
Min	0.03%	100.2	-45.00%	-52.70%	23.87	-7.60%	3.30%
Max	20.89%	319.4	61.70%	103.40%	51.85	24.90%	14.90%
Overall SD	3.78%	44.9	18.52%	30.02%	5.74	5.66%	2.26%
Between-state SD	1.01%	16.9	4.28%	9.90%	4.02	2.26%	0.60%
Within-state SD	3.66%	41.9	18.07%	28.49%	4.27	5.23%	2.19%

Table B.3: Testing the wealth effect on strategic default

	Dependent Variable: Mortgage Delinquency						
	Full Sample			2006 - 2010		1999 - 2005	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Unemployment	0.970***	0.975***	0.803***	1.109***	0.688***	0.109**	0.0457
	(0.0877)	(0.0876)	(0.0916)	(0.118)	(0.123)	(0.0430)	(0.0447)
Personal Income Growth (2 years)	-0.154***	-0.160***	-0.215***	-0.125**	-0.148***	0.00843	-0.00904
	(0.0306)	(0.0307)	(0.0321)	(0.0586)	(0.0534)	(0.0101)	(0.0108)
Log(Per capita Personal Income)	0.199***	0.201***	0.227***	-0.042	0.0118	-0.016	-0.0097
	(0.0246)	(0.0246)	(0.0246)	(0.0436)	(0.0404)	(0.0110)	(0.0107)
House Prices Growth (2 years)	-0.0737***	-0.0102	-0.0302***	0.0548	-0.00613	0.0232	-0.00219
	(0.00718)	(0.0362)	(0.011)	(0.0532)	(0.0154)	(0.0159)	(0.0050)
House Prices Growth (2 years) x Per capita Personal Income		-0.00168*		-0.00360**		-0.00128***	
		(0.000936)		(0.0014)		(0.000445)	
House Prices Growth (4 years) x Per capita Personal Income			-0.000856***		-0.00162***		-0.000376***
			(0.000167)		(0.000231)		(0.0000801)
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Constant	-2.087***	-2.144***	-2.406***	0.410	-0.093	0.167	0.111
	(0.258)	(0.264)	(0.265)	(0.466)	(0.425)	(0.110)	(0.109)
Observations	528	528	528	220	220	308	308
R-squared	0.89	0.89	0.89	0.90	0.92	0.52	0.54
Number of States	11	11	11	11	11	11	11

Standard errors in parentheses *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$