2 A Simple Model

2.1 The model

We can convey the main ideas of our paper through a simple four-period model in which a continuum of risk-neutral households decide at the initial period whether to seek financing to buy a home. If households choose not to buy a house, then they go to the rental market to satisfy their demand for hosing services.

In choosing whether to buy or rent a house, households take into account that the banking sector - summarized by a representative bank - faces a moral hazard problem when financing home acquisitions. We model this moral hazard problem by assuming that households cannot commit to pay principal plus interest whenever they can afford the debt obligation. Banks are aware that, rather than paying the mortgage, households may default strategically if the balance of their loans is above the market value of their homes. Accordingly, competitive interest rates account for this moral hazard problem.

The main novelty of our paper is to combine this well known moral hazard problem with the banks response to a loan request from a household that borrowed in the past and did not honor its debt obligation. In deciding to default strategically, households compare the short-term gain of strategic default with the long-term costs of losing access to the credit market. We shall show that low-income families are more likely to refrain from strategic default because they expect the bank to deny credit to past delinquent low-income borrowers.

To make our point, we assume that households either have High-income (type H) or Low-income (type L) and that the household's type is common knowledge at the initial period. For simplicity, households consume only at the final period and are able to transfer wealth to future periods at the risk free interest rate. Moreover, the price of a house is the sum of a stochastic bubble component and a fundamental constant component. We interpret the fundamental house price as the value of housing services provided after the terminal period. Figure B.1 below summarizes the sequence of actions.

At $t = 0$, households decide whether to buy or rent a home. If they choose to buy a house, they need to obtain a loan to finance the difference between the house price, S_0, and their initial wealth, w_0. In turn, the representative bank decides to finance a mortgage request if there is a contract whose expected return is at least equal to the risk free interest rate which we assume to be zero. The mortgage contract offered by the bank specifies a fixed payment for the following period that we denote by $X_i(w_0, S_0)$ where i is the household type. The mortgage contract is thus supposed to end at $t = 1$.

At $t = 1$, the household's income, $y_{i,1}$, realizes and so does the house price, S_1. Nonetheless, the mortgage contract is risky, for two reasons. For any type i household, income is either $\bar{y}_i$ with probability p_i or zero with probability $(1 - p_i)$. If the household receives a zero income, it has no choice but to walk away from the mortgage contract. Lack of income is not the only reason for default, though. Even if the borrower receives a positive income, he may choose to breach the contract for strategic reasons if the debt outstanding is larger than the price of the house. We assume that a High-income household has both a higher positive income, $\bar{y}_H > \bar{y}_L$, and a higher probability of receiving this positive income, $p_H > p_L$. Moreover, families excluded from the mortgage market pay a rent of r at $t = 1$.

At $t = 2$, households that defaulted on their mortgages in the previous period may become tenants or get a new mortgage to buy a house. Delinquent households use their first period income as down payment, so the required loan amount for house purchase is $S_1 - y_{i,1}$. Again the representative bank decides to lend if there is a contract with expected return at least equal to the risk free rate. The mortgage determines a fixed payment for the next period that is denoted by $X_i'(S_1)$.

At $t = 3$, both household's income, $y_{i,3}$, and house price, S_3, realize once more. In the final period, households consume and gain the bequest value of their home. Only homeowners enjoy this bequest value that is assumed to be sum of a private benefit for ownership, $\bar{U}$, and the house value discounted the implicit housing services. Also at period $t = 3$, delinquent households that accessed the credit market again at $t = 2$ decide whether or not to pay their new mortgages. Furthermore, tenants pay r for living in their homes.

The preferences of households at time t are represented by the following expected utility function:

$$U_t(h_t, C_3) = E_t\Big[C_3 + 1_{owner_3} \cdot (\bar{U} + S_3 - f)\Big] \tag{2-1}$$

where 1_{owner_3} is the indicator function that assumes one if the family owns a house $t = 3$.

Equation (2-1) states that households care about their consumption level at the final date, C_3, and about their bequest value. Homeowners at $t = 3$ ($h_3 = owner_3$) gain the a private benefit, $\bar{U}$, plus the difference between the house value, S_3, and the implicit home services paid for living in the house that is defined as f.

Likewise, the representative bank evaluates the debt contract's cash flow taking into account the default probability of a household of type i. This value can be expressed by the following expression:

$$V_t(X, S_t, D_t; i) = P_t(owner_{t+1}) \cdot X + \big(1 - P_t(owner_{t+1})\big) \cdot \gamma \cdot E_t[S_{t+1}] \quad (2\text{-}2)$$

where X is the mortgage payment and D_t is the debt balance at time t.

In equation (2-2), we assume that the bank's recovery rate in case of mortgage foreclosure is γ. In equilibrium, the competitive bank closes the contract on its break even payment level. That is, the equilibrium mortgage payment at time t is such that $V_t(X, S_t, D_t; i) = D_t$.

Having described the basic setup, we can turn to the three main insights of this simple model. First, we analyze the impact of the household's income profile on their ability to return to the mortgage market and, therefore, on their decision to default strategically. Second, we analyze the composition of mortgage holders in the economy and the distortions created by the bubble in house prices on this composition. Third, we assess the economy's mortgage delinquency in the event of a bubble burst.

2.2 Strategic default decision

To characterize the strategic default decision, our starting point is the household utility at $t = 1$ under the two possible actions. The gain of a household that decides to pay the mortgage in $t = 1$ is given by:

$$U_1(owner_1, X_i; i) = \bar{U} + E_1[S_3 - f] + \big(\bar{y}_i - X_i(w_0, S_0) + p_i\bar{y}_i\big) \quad (2\text{-}3)$$

When the household pays the mortgage, it receives the terminal value of owning a house in exchange for making the mortgage payment, $X(w_0, S_0)$. In equation (2-3), the first two terms, $\bar{U} + E_1[S_3 - f]$, correspond to the payoff from owning the home, that is, the bequest $\bar{U}$ plus the difference between the expected price of the house and the rental value, f. The term in parenthesis is the expected consumption of the type i household in the terminal date, which, from the budget constraint, is equal to what is left from date-1 income after paying the mortgage, $\bar{y}_i - X(w_0, S_0)$, plus the date-3 expected income, $p_i\bar{y}_i$.

Alternatively, households that received a positive income at $t = 1$ may choose to default strategically in which case their gain is given by the following expression:

$$U_1(tenant_1; i) = P_1(Ref) \cdot p_i \Big(\bar{y}_i - X_i'(S_1) + \bar{U} + E_1[S_3 - f]\Big) + (1 - P_1(Ref)) \cdot (\bar{y}_i + p_i \bar{y}_i - r) \quad (2\text{-}4)$$

where $\{Ref\}$ is defined as the event of getting a new mortgage in $t = 2$.

In equation (2-4), the first term is the expected utility of a delinquent household that accesses the mortgage market at $t = 2$ with probability $P_1(Ref)$. In such case, the household gives their first period income as down payment on the new mortgage. We assume that the household pays the mortgage whenever they receive a positive income in $t = 3$. Therefore, with probability p_i the family gains the bequest home ownership value and consumes the difference between the third period positive income and the new mortgage payment, $X_i'(S_1)$. Later, we provide a condition that implies that the household pays the mortgage at the final date when it is possible.

The second term is the expected utility of a delinquent household that becomes a tenant at $t = 1$ with probability $1 - P_1(Ref)$. In this case, the family keeps the income received at $t = 1$ and pays the rent for their home at $t = 3$. Tenants do not receive the bequest value of ownership, so their utility corresponds to the expected consumption which is $\bar{y}_i + p_i \bar{y}_i - r$.

At this point, it is convenient to introduce some structure on the bubble in house prices. We assume that the house price is expressed by $S_t = f + B_t$ where B_t is the bubble component. The bubble can either explode, $B_t = 0$, with probability $1 - q$ or grow, $B_t = \xi \cdot B_{t-1}$, with probability q. We assume that $q \cdot \xi \leq 1$. If $q \cdot \xi = 1$, then the bubble is rational. Otherwise, the bubble is said to be irrational and the hypothesis implies that the representative bank prefers lending to families over buying all the house supply.

The decision to default strategically is optimal if, and only if, the gain from walking away in (2-4) is higher than the gain from paying the mortgage in (2-3). Taking into account that $S_3 = f + B_3$ and the bubble structure, it is possible to show that a household defaults strategically if, and only if,

$$X_i(S_0, w_0) - P_1(Ref) \cdot \big(p_i X_i'(S_1) + \bar{y}_i\big) > \big(1 - P_1(Ref) \cdot p_i\big) \cdot \big(q\xi B_1 + \bar{U}\big) + (1 - P_1(Ref)) \cdot r \quad (2\text{-}5)$$

The left hand side of inequality (2-5) is the difference between the current mortgage payment and the expected cost of the new mortgage. This expected cost is the sum of the first period income given as down payment and the mortgage payment that is made only when a positive income occurs at the final period. When the bubble bursts, the new mortgage payment can be quite small because the amount financed, $f - \bar{y}_i$, is small. The mortgage payment reduction due to a decrease in the house value is the short-term benefit of strategic default.

The right hand side of inequality (2-5) is the cost of defaulting strategically. If the delinquent household gets a new mortgage at $t = 2$, then $P_1(Ref) = 1$ and the cost is the bequest value loss due to liquidity driven default at $t = 3$. In this case, condition (2-5) states that in order to default strategically the household's potential gain in mortgage payment reduction must exceed the cost of losing the bequest value with probability $(1 - p_i)$.

If credit is denied for a delinquent household, then $P_1(Ref) = 0$ and the strategic default cost is the rent paid in $t = 3$ plus the bequest value loss. In this situation, condition (2-5) states that households default if the gain of leaving the payment $X_i(w_0)$ over paying rent exceeds the homeowners bequest value. We now assume that it is never optimal for households without access to the credit market to default strategically on their mortgage.

Assumption 1 *For households excluded from the credit market, homeowner's bequest value exceeds the current benefit of strategic default.*

$$\bar{y}_L - r < \bar{y}_H - r \leq \bar{U}$$

We want to highlight the role of the fear of losing access the mortgage market on the strategic default decision, so we assume that the private benefit from owning a house exceeds the current benefit of walking away from a mortgage. Assumption 1 states that the private benefit of homeownership exceeds the maximum income minus the rent value for both types. Hence, a household without access to the credit market prefers to pay the mortgage if the mortgage payment is lower than the family's positive income.

Even with this assumption, some households choose to default strategically because they are able to return to the mortgage market in better terms after a large price drop. In other words, in the event of a bubble burst, the new mortgage payment, $X_i'(S_1)$, can be so low that the household is willing to take the risk of losing the homeownership bequest value. It is possible to establish conditions under which families of different types behave differently when facing a large house price drop.

Proposition 2 *Under Assumption 1:*

1. *If $\bar{y}_L$ is sufficiently low, then $P_1(Ref) = 0$ for Low-income families. Moreover, if the bubble bursts at $t = 1$, then Low-income families choose to keep paying the mortgage.*

2. *If $\bar{y}_H$ is sufficiently high, then $P_1(Ref) = 1$ for High-income families. Moreover, if the bubble bursts at $t = 1$, then there is a wealth level $\bar{w}'$ such that High-income families default strategically if, and only if, $w_0 < \bar{w}'$.*

3. *If the bubble increases fast enough (that is, ξ big enough) and does not burst at $t = 1$, then both types pay the mortgage.*

To get some intuition on Proposition 2 consider first Low-income families with a bad income profile such that credit is denied for them even at the reduced house price after the bubble burst. As stated by inequality (2-5), a household without access to the mortgage market defaults strategically if the difference between the current mortgage payment, $X(w_0, S_0)$, and rent is higher than the bequest value of owning a house. However, Assumption 1 implies that the private benefit of home ownership overcomes the current benefit of default for any affordable mortgage. For Low-income families, the certain loss of the private benefit of home ownership avoids strategic default.

Alternatively, consider a High-income family that has access to the mortgage market after defaulting in $t = 1$. Although Assumption 1 holds for High-income families as well, they may choose to default strategically because their new mortgage entails a lower payment due to the lower house price after the bubble burst. A household that gave a low initial wealth as down payment have an expensive mortgage, so their new mortgage payment is smaller enough than the original mortgage payment so that the household decides to default strategically.

High-income families with a high initial wealth made a large down payment at time $t = 0$ which implied a small mortgage payment. For these families the incentive to default, the right hand side of inequality (2-5), is low because the payment reduction with the new mortgage is not high enough to compensate for the risk of losing the bequest value of home ownership due to liquidity problems at the final period.

The third part of Proposition 2 states that both types prefer to pay the mortgage when the bubble continues. If the bubble rises by a great amount, the necessary payment of a new mortgage is higher than the current payment. In this case, it is optimal for households to pay the mortgage at $t = 1$ and guarantee their homeownership value at the lower current payment.

2.3 Who gets access to the credit market?

Households may default on their mortgages either because they experience liquidity problems or because they cannot commit to honor their debt obligations when prices fall. The risk of the costly foreclosure state is incorporated into the competitive interest rate which implies a more expensive mortgage. To compensate for this risk the representative bank require a minimum down payment to accept a mortgage request. Only households that can afford to make down payment higher than the minimum requirement have access to the credit market at the initial period.

A bubble in house prices changes the composition of borrowers in the economy by lowering the minimum down payment requirement of a mortgage at the initial period. Therefore, it allows the entrance of more financially fragile borrowers into the credit market. While the bubble is alive, home prices are expected to increase, and so are their salvage values. Accordingly, the collateral value of mortgage contracts increase with the bubble, thereby lowering their down payment and the interest rates.

Once the behavior of households at $t = 1$ is characterized, it is possible to determine which families are able to get a mortgage in the initial period. In equilibrium, the mortgage payment is such that the lender's value expressed in equation (2-2) equals the amount lent at $t = 0$:

$$V_0(X_i(w_0, S_0), S_0, w_0) = S_0 - w_0 \tag{2-6}$$

In turn, the household decides to buy a house in the initial period if the mortgage payment $X_i(w_0, S_0)$ is smaller than the family's positive income, $\bar{y}_i$. In the case that $X_i(w_0, S_0) > \bar{y}_i$, the household defaults in $t = 1$ with probability one and is better off by keeping the initial wealth that would be used as down payment. Therefore, given the household's income distribution, there is a minimum initial wealth used as down payment on the mortgage that implies $X_i(w_0, S_0) \leq \bar{y}_i$. Households with a initial wealth superior to this threshold choose to finance their home purchase with a mortgage.

Proposition 3 *Assume that $\bar{y}_H$ is sufficiently high.*

1. *There is a minimum initial wealth level, $\underline{w}_i^*$, such that households of type i get a mortgage in $t = 0$ if, and only if, $w_0 \geq \underline{w}_i^*$.*
2. *Moreover, $\underline{w}_i^*$ is decreasing in the positive income, $\bar{y}_i$, and in the bubble continuation probability, q, and in the bubble increase factor, ξ.*

Proposition 3 states that each type family has a minimum wealth level to access the credit market at time $t = 0$. The representative bank lowers the minimum required mortgage down payment if any change in parameters makes the loan safer. Consider first an increase in the expected bubble value (higher q or ξ). If the expected bubble value increases, then the expected salvage value also increases which allows the access the mortgage market of families with a lower wealth level.

Alternatively, a greater borrower's capacity to repay the mortgage also leads to a reduction in the required down payment. A higher positive income allows the household to pay more expensive mortgages which reduces the down payment necessity.

The presence of a bubble in the price of houses changes the composition of families that are able to get a mortgage. The expectation of large price gains lead to a increase in the proportion of homeowners in the economy. These new homeowners gave a lower down payment and, as suggested by Proposition 2, are more likely to default strategically when the bubble bursts.

2.4 Economy's Mortgage Delinquency

Now we turn to the incidence of strategic default in this simplified economy. When the bubble bursts, part of the mortgage holders decide to default strategically in order to enjoy a smaller mortgage payment. The extent to which strategic default occurs is related to the proportion of High-income with low initial wealth in the mortgage market.

To compute the economy's delinquency assume that a fraction α of households is of type H. We have already identified which households have the required wealth level to get a mortgage at $t = 0$ and their default behavior at $t = 1$. Once we know the mortgage holders behavior, it is straight forward to establish the mortgage delinquency in the economy.

Proposition 4 *Assume that each household's initial wealth, w_0, is uniformly distributed in the interval $[\underline{w}, \bar{w}]$. The delinquency in the economy can be evaluated in each possible realization of the bubble in $t = 1$:*

1. *If the bubble continues, the proportion of families defaulting at $t = 1$ is given by:*

$$\alpha(1 - p_H) \cdot (\bar{w} - \underline{w}_H) + (1 - \alpha)(1 - p_L) \cdot (\bar{w} - \underline{w}_L)$$

2. *For $\bar{y}_H$ sufficiently high, there is strategic default in case of a bubble burst. The fraction of families defaulting strategically is given by $\alpha(\bar{w}' - \underline{w}_H)$. This fraction is increasing in positive income, $\bar{y}_H$, and in the bubble continuation probability q. Moreover, it is decreasing in home bequest value, $\bar{U}$.*

3. *If the bubble bursts, the proportion of families defaulting at $t = 1$ is given by:*

$$\alpha(1-p_H)\cdot(\bar{w}-\bar{w}')+\alpha(\bar{w}'-\underline{w}_H)+(1-\alpha)(1-p_L)\cdot(\bar{w}-\underline{w}_L)$$

The results in Proposition 4 follow direct from Propositions 2 and 3 and the Law of the Large Numbers. When the bubble continues only families with liquidity problems default on the mortgage. By the Law of Large Numbers, a fraction $(1-p_i)$ of mortgage holders of type i default at time $t=1$.

When the bubble bursts, families of type H such that $w_0 \in [\underline{w}_H, \bar{w}']$ decide to default strategically, so there is a mass $(\bar{w}'-\underline{w}_H)$ of High-income households that default strategically. Strategic default in the economy increases with the probability of a bubble continuing (q) and with type H's high income $(\bar{y}_H)$. When these parameters increase, default risk falls and the representative bank reduce the down payment requirement. The entrance of High-income families with a more fragile financial position increases the incidence of strategic default in the economy. Moreover, if $\bar{u}$ increases, then strategic default in the economy falls due to a higher cost of losing the home ownership bequest value.

Now that we have characterized the difference in strategic default behavior among families with different income levels, we can generalize the model to replicate mortgage delinquency among different US regions. we extend the model to an infinite horizon setting that let us link the cross-sectional variation of delinquency in the U.S. to differences in income across different state.