2 The Model

2.1. Basic Setup and Equilibrium

A buyer with an inelastic unitary demand wishes to procure a single, indivisible good while paying up to the maximum price $\bar{p}$. A total of $N \geq 2$ firms compete to supply the good. Firms observe privately their cost of executing the project, c_i. All bid packages consist of a pair (p_i, q_i) of price and quality, respectively, submitted simultaneously. The buyer utilizes a two-stage decision rule to decide the winning bid. Any bid package with quality greater than or equal to some minimum level $\tilde{q}$ is qualified to the price competition phase, where the quality of the package does not matter. The lowest price, conditional on qualifying, wins the procurement and must execute the project as specified on the bid package while earning profits equal to $p_i - c_i$.

We assume quality and costs are randomly distributed, with each firm observing the independently distributed random variables q_i and c_i, with distributions $G(.)$ and $F(.)$, respectively. Furthermore, we assume for simplicity that q_i and c_i are also independent of each other and defined on the closed support $[0, \bar{q}]$ and $[\underline{c}, \bar{c}]$. Realizations of the random variables are private information of the firms. q_i is assumed to be observable ex-post by the agency and contractible, so no informational rents need to be given by the agency to enforce truth-telling on the quality dimension.

We define the minimum threshold of quality as $\tilde{q}$. We also assume that firm 1 will be the beneficiary of the agency's favoritism, who will decide on the level of $\tilde{q}$ as a function of firm 1's quality level, so that $\tilde{q} = \tilde{q}(q_1)$. For simplicity, we assume that the agency will choose $\tilde{q}(q_1)$ by eliminating the greatest possible number of competing firms in the procurement, which happens by setting $\tilde{q}(q_1) = q_1$. Since all quality levels are observable ex-post, an additional simplifying assumption is that any qualifying rule set by the agency should be monotonic. That is, for all $q_i \geq \tilde{q}$, the project will necessarily be qualified to the price competition phase.

While costs and quality levels are private information, the threshold as defined above is revealed *before* bid packages are submitted. This can be thought of as part of the general procurement guidelines that are published by government agencies and available to all firms before bids are submitted. In that sense, the set of public information in the model includes the distributions $F(.)$ and $G(.)$, as well as the minimum quality threshold $\tilde{q}$. Finally, we also assume for simplicity that all participating firms are aware of the favoritism of the agency for firm 1.

In short, the line of events is as follows:

1) Firm 1 observes q_1, and the agency sets $\tilde{q}(q_1) = q_1$
2) Procurement rules and the quality cutoff are revealed to all firms
3) Firms simultaneously submit bid packages
4) The agency awards the contract to the lowest bid that satisfies the qualification rule and that is lower than or equal to a publicly known maximum price $\bar{p}$.

We look for the Bayesian Nash equilibrium, as characterized by the system of differential equations derived from the procurement game. The equilibrium is defined as a set of functions $\beta_i(c_i, q_i, \tilde{q})\colon [\underline{c}, \bar{c}]\, X[0, \bar{q}]^2 \rightarrow [\underline{p}, \bar{p}]$ that associate cost and quality realizations and the quality threshold written in the procurement rules to pricing decisions. Due to $\tilde{q}$ being determined exogenously by the agency, we derive equilibrium bid functions conditional on $\tilde{q}$.

Since $N-1$ firms will behave in a completely symmetric fashion throughout the model in equilibrium, we need only to consider the two bid functions β_1 and β_i in order to characterize the equilibrium, and investigate the relationship between favoritism and price distortions in the model.

Lemma 1: $\boldsymbol{\beta_i(c_i, q_i, \tilde{q}) = \beta_i(c_i, \tilde{q}),\ \forall\, i \neq 1}$

Proof: See Appendix.

Following directly from the fact that the probability of qualifying is determined by firm 1's quality level, the maximization problem of the other firms is not affected by their own observed quality.

The lemma allows us to characterize the equilibrium bid functions β_1 and β_i conditional on any previously determined qualification threshold $\tilde{q}$ and also to define the inverse bid functions as follows, where p is used as the argument of the domain of the inverse bid function:

$\lambda_1(p|\tilde{q}) \equiv \beta_1^{-1}(p|\tilde{q}): \left[\underline{p}, \bar{p}\right] \rightarrow \left[\underline{c}, \bar{c}\right]$ and $\lambda_i(p|\tilde{q}) \equiv \beta_i^{-1}(p|\tilde{q}): \left[\underline{p}, \bar{p}\right] \rightarrow \left[\underline{c}, \bar{c}\right]$

Since all bid functions are derived conditional on $\tilde{q}$, we omit it from the notation from here on out. It should be noted that the results that follow do not require that the quality threshold be set exactly as q_1. While we assume that this is the case, all results hold for any agency function $\tilde{q}(q_1)$ where firm 1 always qualifies. It may be the case that the agency has limited discretionary power over how much it can distort the minimum quality threshold or that it may not wish to set $\tilde{q}$ exactly equal to q_1 in order to maximize firm 1's profits. As long as $q_1 > \tilde{q}$ and the favorite firm always qualifies, all of the results that follow hold.

2.2. Beliefs

The qualifying rules determine a distribution of qualified participants in the price competition that is unknown to bidders ex-ante. More importantly, however, firm 1 and all others will have different Bayesian-consistent beliefs regarding the distribution of potential qualified competitors. To see this, note that firm 1 will always qualify to the price competition with probability 1, while all other firms will do so with probability $1 - G(\tilde{q})$ ex-ante.

This asymmetry in participation governs the probability that each firm attributes to its number of rivals. Given the total number of potential firms N, the probabilities are determined by q_1, and we can write the asymmetric distribution of beliefs for firms 1 and i, respectively, as

$$\Pr(n_1 = k|\tilde{q}) = \binom{N-1}{k}[1 - G(\tilde{q})]^k * [G(\tilde{q})]^{N-k-1}, \;\; for\; 0 \leq k \leq (N-1)$$

$$\Pr(n_i = k|\tilde{q}) = \binom{N-2}{k-1}[1 - G(\tilde{q})]^{k-1} * [G(\tilde{q})]^{N-k-1}, \;\; for\; 1 \leq k \leq (N-1)$$

Since firm 1 always qualifies, no other firm attributes positive probability to facing zero competitors. Note that asymmetries in the two distributions will be particularly relevant if the quality cutoff is high, and if the number of potential participants is low.

Defining $A(k) \equiv \Pr\,(\#\, Rivals\; of\; firm\; 1 = k)$, $B(k) \equiv \Pr\,(\#\, Rivals\; of\; firm\; i = k)$, we can write ex-ante expected profits as weighted functions of profits and the probability of making the lowest qualifying bid where p_j is the price set by the firm:

$$\Pi_1^e = \sum_{k=0}^{N-1} A(k) * \Pr(p_1 \leq \beta_i^*)\,[p_1 - c_1]$$

$$\Pi_i^e = \sum_{k=0}^{N-1} B(k) * \Pr(p_i \leq \beta_i^*, p_i \leq \beta_1^*)\,[p_i - c_i]$$

Profits are functions of these probabilities, and characterize equilibrium behavior through the first order conditions for any interior solution, with the exception of the atom at $\bar{p}$ on firm 1's bid function. The model differs from the McAfee & McMillan (1987) setup in two fundamental ways: First, the distribution of participants is endogenously determined by the agency's decision to distort qualifying levels in order to minimize the beneficiary's competition. Furthermore, this distortion not only potentially increases ex-ante profits of firm 1 by guaranteeing qualification, it also results in asymmetric distributions on the unknown number of qualified bidders.

2.3. Existence and Equilibrium Characterization

Due to the asymmetric nature of the game, both existence and unicity of equilibrium are not directly guaranteed by the first order conditions. Like in Lebrun (1999) and Lebrun(2006), existence will require a binding reserve price $\bar{p}$, so that the support of the bid distributions is equal for all firms and there are no incentives to deviate towards the maximum price for high-cost firms. As discussed on Lebrun (1996), Athey (2001), and Jackson & Swinkels (2005), the problem of existence may arise in auction models when player's payoffs are discontinuous at points of tied bids. By assuming that the reserve price $\bar{p}$ will be binding, we guarantee that the non-benefitted players' payoffs will necessarily be zero at the point where a tie may occur with positive probability due to firm 1's atom, which avoids discontinuities in the payoffs.

We assume throughout that the reserve price $\bar{p}$ will be binding at the top for firm 1[3]. That is, for a maximum price $\bar{p}$, there will be no incentive for high cost firms to deviate from the bid functions induced by the system of differential equations, since the potential deviation of high cost, non-benefitted firms will necessarily result in negative profits. By restricting the set of actions to a subset of $[\underline{c}, \bar{c}]$ we eliminate the possibility of positive profits for high cost firms.

The Bayesian Nash Equilibrium of the model is then defined as the pair of functions β_1 and β_i determined by the solution to the system of differential equations resulting from the

[3] We show on the appendix as Proposition 2 that if the reserve price is greater than the superior limit of the support of the cost distribution, there exists no Pure Strategy Nash Equilibrium to the game, due to the discontinuity of the payoff functions of all other firms on the strategy space of firm 1.

maximization problems of both sets of firms for an interior solution and by an atom at $\bar{p}$ due to firm 1's decision to bid the maximum price for sufficiently high realizations of the cost random variable. Deriving the expected profit and defining λ_1 and λ_i as the equivalent inverse bid functions, the following equations characterize the equilibrium:

$$\lambda_i' = \frac{(\sum_{k=0}^{N-1} A(k)[1 - F(\lambda_i)]^k)}{(\sum_{k=0}^{N-1} A(k)k[1 - F(\lambda_i)]^{k-1} f(\lambda_i)[p_1 - \lambda_1])}$$

$$\lambda_1' = \frac{(\sum_{k}^{N-1} B(k)[1 - F(\lambda_1)][1 - F(\lambda_i)]^{k-1} - \sum_{k=0}^{N-1} B(k)(k-1)\lambda_i'[1 - F(\lambda_1)][1 - F(\lambda_i)]^{k-2}[p - \lambda_i])}{\left(\sum_{k=0}^{N-1} B(k) f(\lambda_1)\left[1 - F\left(\lambda_i(p)\right)\right]^{k-1}[p_i - \lambda_i]\right)}$$

There is no closed form solution for the general case, but we can nevertheless prove the existence of a pure strategy Bayesian equilibrium and characterize the relationship between the bidding functions of both types of firms under the assumptions previously mentioned.

Lemma 2: The inverse bid functions $\boldsymbol{\lambda_1}, \dots, \boldsymbol{\lambda_N}$ are strictly increasing

Proof: See Appendix.

We prove existence by following closely the first theorem of Lebrun (1999) for generic asymmetric auctions. The main assumptions required by the theorem (common support of the cost distributions, existence of density functions, differentiability of F) are also made here. Along with Lemma 2, the second proposition proves that there exists a pure strategy Bayesian equilibrium to the game proposed, characterized by the solution to the system of differential equations.

Proposition 1: Under the assumptions of the model, an n-tuple of strategies $(\boldsymbol{\beta_1}, \dots, \boldsymbol{\beta_N})$ is a pure strategy Bayesian equilibrium if $\boldsymbol{\beta_1}, \dots, \boldsymbol{\beta_N}$ are equal to pure strategies over $[\underline{\boldsymbol{p}}, \bar{\boldsymbol{c}})$, and there exists $\underline{\boldsymbol{c}} < \bar{\boldsymbol{p}} \leq \bar{\boldsymbol{c}}$, such that the inverses $\boldsymbol{\lambda_1} = \boldsymbol{\beta_1^{-1}}, \dots, \boldsymbol{\lambda_N} = \boldsymbol{\beta_N^{-1}}$ exist,, and form a solution over $[\underline{\boldsymbol{p}}, \bar{\boldsymbol{c}})$ of the system of differential equations, satisfying the boundary conditions $\boldsymbol{\lambda_1}(\underline{\boldsymbol{c}}) = \cdots = \boldsymbol{\lambda_N}(\underline{\boldsymbol{c}}) = \underline{\boldsymbol{p}}$ and $\boldsymbol{\lambda_1}(\bar{\boldsymbol{c}}) = \cdots = \boldsymbol{\lambda_N}(\bar{\boldsymbol{c}}) = \bar{\boldsymbol{c}}$.

Proof: See Appendix.

The proof is based on two central assumptions: First, it is necessary that the expected profit for firms that observe $\bar{c}$ be equal to zero. The assumption that the maximum price $\bar{p}$ is lower than the maximum quality value $\bar{c}$ guarantees this. Second, we also require that the bid support is equal for all firms. This is guaranteed by assuming that there exists a common lower bid for all participating firms, so that $\beta_i(\underline{c}) = \underline{p}, \forall\, i$.

Proposition 2: For any distributions of cost and quality $F(.)$ and $G(.)$, the two-stage procurement auction with favoritism is such that $\boldsymbol{\beta_1(c)} \geq \boldsymbol{\beta_i(c)}, \forall \boldsymbol{c}$ where $\boldsymbol{\beta_1(c)}$ follows the system of differential equations. The inequality holds strictly, for all $\boldsymbol{c} > \underline{\boldsymbol{c}}$, if $\tilde{\boldsymbol{q}} > 0$.

Proof: See Appendix.

The proposition shows that despite complete ex-ante symmetry, independence between quality and cost variables, and separation of the decision rule, the agency's bias towards firm 1 result in prices that are point-by-point greater for firm 1 compared to all others. Regardless of having no advantages in cost or through favoritism in the price competition, the beneficiary of the agency's bias is nonetheless less aggressive than its competitors due to the advantage given in the qualifying phase.

If the reserve price $\bar{p}$ is binding, since we have firm 1 bidding less aggressively than all others for the solution of the system of differential equations, there will exist an interval $[\tilde{c}, \bar{c}]$ where firm 1 will be binded by the reserve price restriction and the firm will bid $\bar{p}$. The economic intuition is that firm 1 faces a smaller expected number of competitors and a strictly positive probability of having no competition at all for $\tilde{q} > 0$. Thus, at some level cost $\tilde{c}$ where the bid function reaches the reserve price, this firm defaults to choosing the highest price, only winning when no other firms manages to qualify to the price competition phase.

The inefficiency generated by equilibrium bidding behavior happens because, since the favorite firm bids less aggressively then all others, the probability of it not being awarded the contract when it is more efficient than all others is strictly positive for any $\tilde{q} > 0$. This means that the favorite firm purposefully shades its bidding further than all others, due to the fact that it expects to face less competition when compared to non-favorite firms.

To illustrate the equilibrium behavior, we assume that costs and quality are independently and uniformly distributed on the compact support $[0,1]$, with maximum price $\bar{p} = 1$, and simulate[4] equilibrium inverse bid functions for different numbers of potential firms and different minimum quality thresholds. The simulations show that the asymmetries observed on the bidding functions are decreasing on the number of potential firms and increasing on the quality threshold chosen.

Figure 1 **– Equilibrium Inverse Bid Functions (N=3)**

$G(\tilde{q}) = 0.5$ $\qquad\qquad$ $G(\tilde{q}) = 0.75$

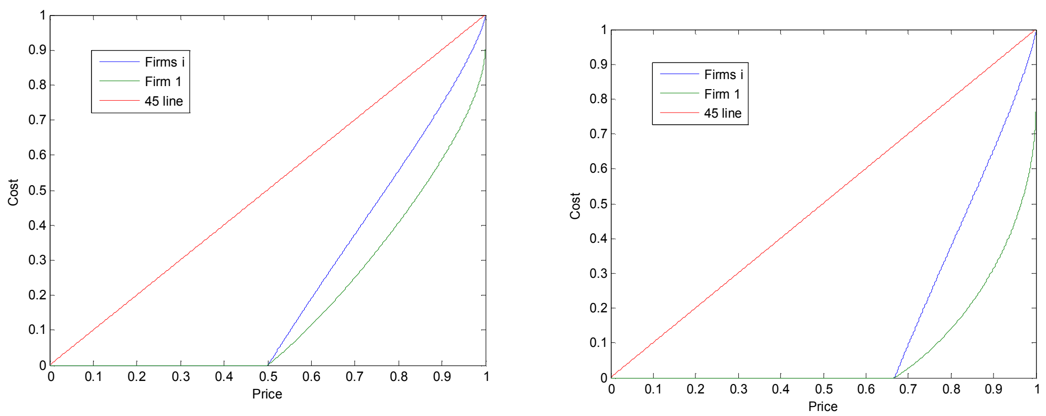
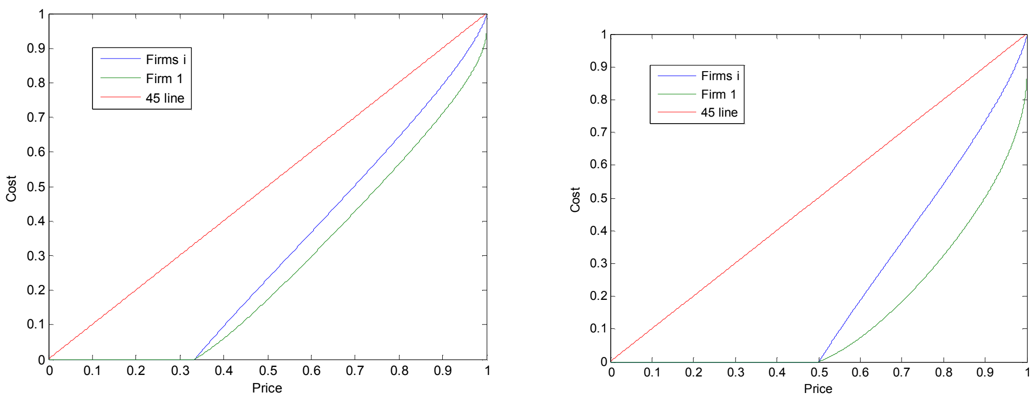


Figure 2**– Equilibrium Inverse Bid Functions (N=5)**

$G(\tilde{q}) = 0.5$ $\qquad\qquad$ $G(\tilde{q}) = 0.75$



Firm 1's Ex-Ante Profit Maximization. As a side note, the simulations suggest one more possible result from the model: Profit functions for firm 1 are strictly increasing on the quality threshold $\tilde{q}$ for the uniform case. If this result holds in general, the agency's choice not only

[4] See Appendix B for a detailed discussion on the simulation methods utilized.

eliminates the largest possible number of competitors, but also maximizes the favorite firms' ex-ante profits.

Choosing a higher quality threshold (up to q_1) has two effects: First, the intended direct effect of reducing the probability of any one firm qualifying ex-ante. Second, raising the quality threshold also results in changes in the strategic decisions on the bid functions. The former is straightforward and unequivocally raises expected profits due to the higher probability of having a reduced number of participants. The second, strategic effect involves the firm's reaction to less expected competition, which is to be less aggressive and therefore bid higher, but also how the asymmetric bid functions interact in equilibrium. It may be the case that a higher quality threshold results in firm 1 choosing higher prices in relation to the remaining firms, which would result in a decrease in expected profits. If that is the case, which can be thought of as a usual commitment problem in mechanism design, the favorite firm would be better off with a *lower* quality threshold, $\tilde{q} < q_1$. The simulations show, however, that the positive impact in expected profits dominates the potentially offsetting second effect – for any quality that firm 1 may observe on its project, setting $\tilde{q} = q_1$ also maximizes firm 1's expected ex-ante profits.

2.4. The Mechanism Design Problem

This section approaches the favoritism problem from a mechanism design perspective. Consider that the government will take the favoritism displayed by the agency as given, in the sense that it does not attempt to design a corruption-proof mechanism. Therefore, the objective is to design a price competition mechanism that minimizes how much it expects to pay for the project taking as given that the agency will be responsible for the decision of the qualifying phase cutoff. Formally, this assumption means that quality and qualifications are taken as exogenous.

We follow the Myerson approach closely, defining the government's maximization problem as a minimization of expected payments for the project conditional on the quality threshold. $a_i(\hat{c}_i, c_{-i})$ is defined as the allocation rule and$a_i(\hat{c}_i, c_{-i})$ as the payment schedule.

Firms maximize $U_i = E_{c_{-i}}\,[t_i(c_i, c_{-i}) - a_i(c_i, c_{-i}) * c_i]$ and the designer wishes to maximize

$$Max_{(a_i(.),t_i(.))_{i\in N,a_i\in A}} E_c\left[\sum_{i=1}^{N} -t_i\right]$$

subject to incentive compatibility and participation constraints.

Let A be the set of allocation rules the mechanism designer can utilize. In our setting, $A = \{a \in [0,1]^N | \sum a_i \leq 1; a_i = 0 \; if \; q_i < \tilde{q}\}$. Since $\tilde{q}$ has been exogenously set as q_1 for the designer, allocation restrictions are fully determined exogenously and cannot be distorted by the designer. The consideration of the favoring rule as exogenous is crucial for the results that follow.

From a price competition mechanism design perspective, the only novelty to the maximization problem is determined by the restriction on the set of allocations he may utilize. Utilizing the first order approach, it follows directly from the envelope theorem that we can write$U_i(c_i) = U_i(\bar{c}) + \int_{c_i}^{\bar{c}} E_{c_{-i}}\,[a_i(\tau, c_{-i})]d\tau$, along with a monotonicity rule on $E_{c_{-i}}\,[a_i(\tau, c_{-i})]$. The participation constraint is binding at the top, so that $U_i(\bar{c}) = 0$.

Substituting into the maximization problem, we have

$$Max_{a_i\in A} \sum_{i=1}^{N} E_c\left[-c_i * a_i(c_i, c_{-i}) - \int_{c_i}^{\bar{c}} E_{c_{-i}}\,[a_i(\tau, c_{-i})]d\tau\right]$$

Integrating by parts, the maximization problem reduces to

$$Max_{a_i\in A} \sum_{i=1}^{N} E_c\left[a_i(c) * \left[\left\{\frac{F(c_i)}{f(c_i)} - c_i\right\}\right]\right]$$

The solution to the problem is completely equivalent to a direct mechanism subject to a regularity condition on the distribution F, where the good is allocated according to the ranking of virtual valuations. While the allocation rule will be further restricted by the set A, the fact that the set is determined exogenously by another agency does not change the fact that the optimal allocation rule will be based on the virtual valuation, and that payments need only to be made by the firm with the lowest cost restricted to the set. The mechanism will therefore allocate the contract to the firm with the lowest cost, conditional on qualifying. It follows naturally, then, that the direct mechanism derived is equivalent to a second-price auction. Firms truthfully reveal their types, are qualified by their realizations of q_i, and the qualified firm with the lowest cost is awarded the right to execute the project. The general result of the optimality in regards to price competition of this mechanism holds under the basic assumptions of the literature and with no additional frictions.

2.5. Alternative Mechanisms

We now discuss possible solutions to the asymmetry in the procurement setup. Our goal is to once again investigate how procurement rules may be modified in a two-stage setting to mitigate the price distortions derived, *given* the agency's favoritism towards one of the participants. We assume that the agency will still set the minimum quality threshold equal to firm 1's quality level as before, and propose two alternatives based on the results of the mechanism design problem: Second-price procurement and revealing the number of qualified firms before prices are submitted.

Proposition 3: The second-price, two-stage procurement auction with favoritism is such that $\boldsymbol{\beta = \beta_1 = \beta_i}$***, and*** $\boldsymbol{\beta(c_i) = c_i}$

Proof: See Appendix

The general result of Vickrey auctions holds in the model; for all types, the weakly dominant strategy is to truthfully reveal costs. Given this strategy, it is straightforward that bidding will be symmetric for all participants. This symmetry guarantees that expected costs under a second-price rule will be lower, since the inefficiency introduced in the original model was precisely due to firm 1's less aggressive bidding resulting in it losing the procurement with positive probability even when it has the lowest cost.

All of the usual caveats need to be made regarding second price auctions. Risk aversion is a natural concern when adopting second price auctions. In the symmetric, independent private value case, revenue equivalence does not hold and the auctioneer is strictly better off with a first-price mechanism. In our case, which effect would dominate with risk-averse buyers would depend on the intensity of the risk aversion observed. Furthermore, Che & Gale (1998) highlight some of the implications of financially constrained bidders when faced with first and second price auctions, concluding that a second price mechanism results in less favorable returns for the auctioneer and in a decrease in efficiency, even when bidders have access to credit. Finally, collusion is another major concern in public procurement that may be aggravated by adopting mechanisms other than the usual first-price auction. The collusion concern is particularly relevant in this case, since the favoritism modeled may be interpreted as collusion between the agency and one of the firms.

Alternatively, consider the following game:

1) Firms observe c_i and q_i, and truthfully reveal q_i
2) Given $\tilde{q} = q_1$, firms are qualified if $q_i \geq q_1$
3) The number of qualifying firms n is revealed to all participants
4) Firms compete in a first-price procurement auction with n participants

Note that, by construction, there is no asymmetry in the price competition phase. Since quality values are independent of cost, all informational advantages previously awarded to firm 1 are eliminated by revealing, *before* prices are set, the number of qualified participants. Expected project execution costs are once again reduced when compared to the model with an unknown number of qualified bidders.

Moreover, the elimination of the informational effects has consequences on the expected level of quality in both cases. This happens for two reasons: In this setup, firm 1's quality level q_1 represents the lower bound of all qualifying firms. Furthermore, when the asymmetry is eliminated and firm 1 bids on the same fashion as the remaining firms, its ex-ante probability of winning the price competition increases. Since the probability of the winning project having the lower bound of quality increases, the expected quality level on this alternative mechanism will necessarily decrease when compared to the baseline model.

In fact, any mechanism we may choose to adopt that keeps the biased qualification rule but results in symmetric price competition will exhibit these two properties – increased cost efficiency and a decrease in the expected quality of the project. The intuition to the effects discussed previously depends only on the resolution of the asymmetry in equilibrium bid functions, and not on how the price competition phase will actually occur.

Expected Prices and Cost. This section investigates the relationship between the asymmetric equilibrium characterized previously and potential concerns regarding efficiency and government expenses under our proposed alternative. Despite the straightforward result that revealing the number of participants increases efficiency, it is not clear what will happen with the price that the government expects to pay for the good. To see this, we consider the original model with an unknown number of bidders, and the alternative mechanism proposed where the number of qualified firms is revealed.

For the second case, we can write ex-ante expected prices as

$$E[p] = \sum_{k=1}^{N} \Pr(n = k) * k * \int_{\underline{p}(k)}^{p_i(k) \le p_i^*(k)} p(k)\, dF'(p)$$

Where $F'(p)$ is the distribution of bids. The expression is simply the weighted average of the expected winning price in a symmetric, first-price procurement auction and has the usual closed form solution given by the order statistics of the distribution of costs.

Under the original rules where the number of qualified firms is not revealed:

$$E[p] = \int_{\underline{p}}^{p_1 \le p_{-1}^* | q_{-1} \ge q_1} p\, dF_1'(p) \;+\; (N-1) * \int_{\underline{p}}^{p_i \le p_{-i}^* | q_{-i} \ge q_1} p\, dF_i'(p)$$

Where $F_1{}'$ and $F_i{}'$ are the distributions of bids for firm 1 and all other firms, respectively. We would like to rewrite both expressions as functions of cost and the common cost distribution $F(c)$ and compare expected prices.

Given the intractability of the expressions for the model with an unknown number of qualified bidders, we will focus on the uniformly distributed case and simulate expected prices and cost for the full range of quality thresholds that the agency may choose. Throughout the simulations, we assume once again that costs and quality are independently and uniformly distributed on the compact support $[0,1]$. Equilibrium bid functions are simulated from the system of differential equations for any given level of the quality threshold $\tilde{q}$ .Finally, we simulate expected prices and costs with and without revealing the number of participants. Since both mechanisms converge when the minimum quality threshold $\tilde{q}$ is close to zero, prices and expected costs also converge. For higher values of $\tilde{q}$, the mechanism that reveals the number of qualified participants exhibits improvements in both expected costs and price.

Figure 3 Expected Prices and Cost (N=3)

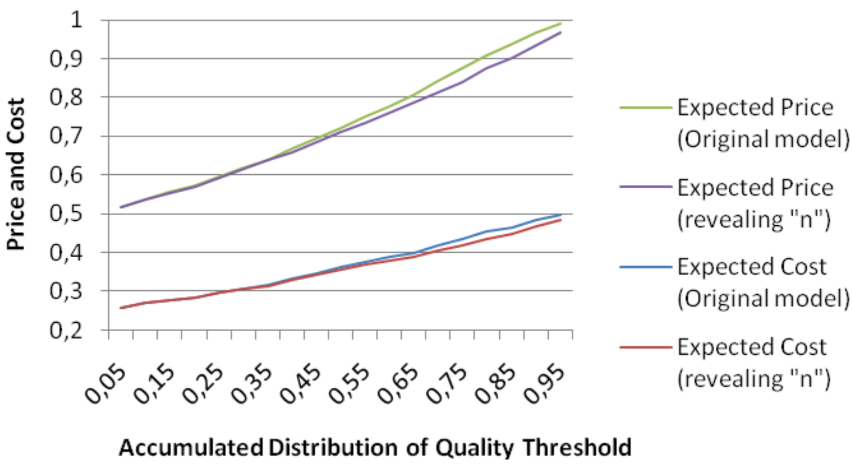
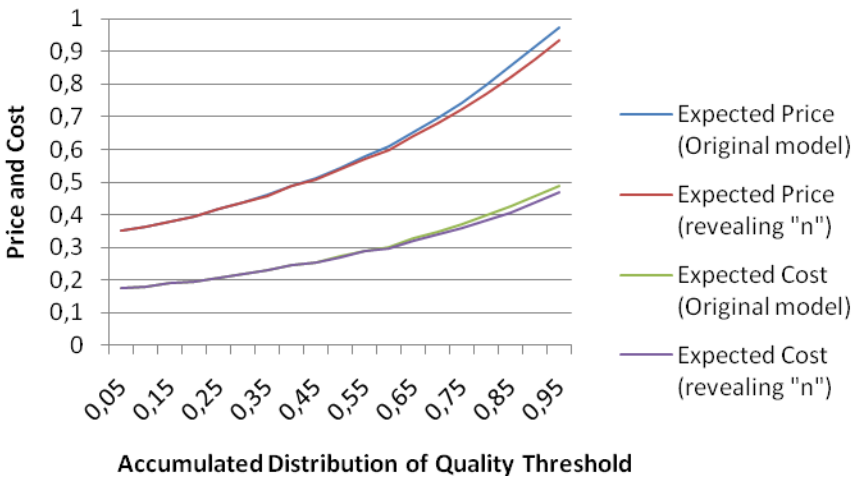


Figure 4 - Expected Prices and Cost (N=5)



The expected cost decreases when revealing the number of qualified participants due to the induced symmetry on the bidding behavior of the benefitted firms and all others that have qualified. We also find, however, that expected prices are lower when revealing the number of participants. Under a uniform distribution, resolving the asymmetry in bidding has the effect of at least part of the efficiency rents being accrued by the auctioneer in the form of a lower expected price.

Expected Profits. It is not immediately clear what may happen to the benefited firm's profits once this information is revealed. To see this, we can write firm 1's expected profits for both models as functions of the same distribution of the cost random variable. Let $c^* \equiv$

$Min_i(c_i|q_i \geq \tilde{q})$. For firm 1, the relevant cost is the lowest one, conditional on qualification. This is true for either model; whether or not the number of qualified participants is revealed, the relevant competition to firm 1 is the most efficient of the symmetric competitors. Defining F_{c^*} as the distribution of c^*, we can write conditional profit functions under both models as:

$$\Pi_1(c_1|\tilde{q})_{M1} = (\beta_1(c_1) - c_1) * \left[1 - F_{c^*}\left(\beta_i^{-1}(\beta_1(c_1))\right)\right]$$

$$\Pi_1(c_1|\tilde{q})_{M2} = (\beta(c_1) - c_1) * [1 - F_{c^*}(c_1)]$$

From proposition 1, we know that $\beta_1(c_1) \geq \beta_i(c_1)$, and therefore $\beta_i^{-1}(\beta_1(c_1)) \geq c_1$. As discussed earlier, the probability of firm 1 winning the contract increases due to symmetry.On the other hand, we should expect firm 1 to bid more aggressively under the new rule, so the equilibrium effect on profits is not obvious. For the uniform case, the result of firm 1 increasing its profits when the number of participants is revealed or a second-price procurement rule is applied holds. The same is not true for all other firms: Revealing the number of participants results in a decrease in ex-ante expected profits for non-favorite firms.

Figure 5 – **Expected Profits conditional on $\tilde{q}$ (N=3)**

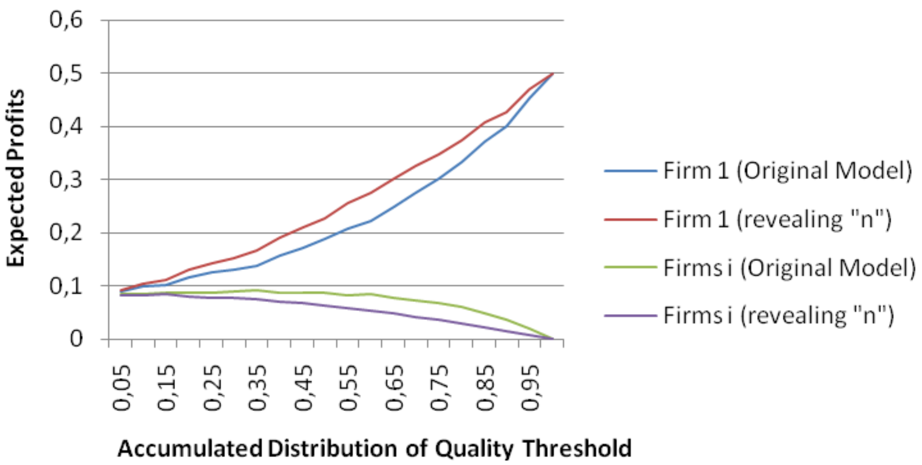
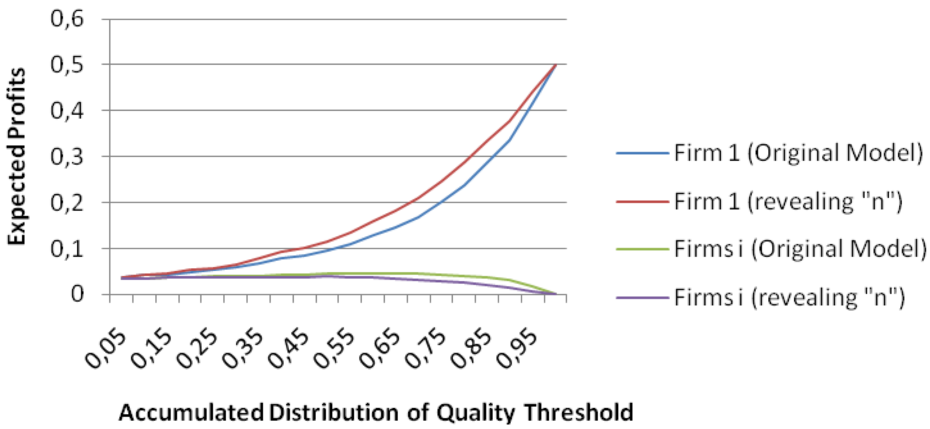


Figure 6 Expected Profits conditional on $\tilde{q}$ (N=5)



This result shows that we should expect an increase in expected gains from collusion by both parties, which suggests a higher probability of maintaining a collusive agreement. This particular solution to the price asymmetry may have an additional unintended effect: Since both firm 1's and the agency's expected payoffs increase under the alternative rule, the problem of favoritism may be *strengthened* when adopting a second-price rule or its revenue-equivalent mechanism of revealing the number of participants. Our results are further evidence that, despite second-price auctions being very robust in terms of price efficiency, they are nonetheless particularly susceptible to a series of distortions, including collusion.

Additionally, while the model presented is silent on how the competition for the agency's bias works, the simulations nonetheless allows us to gain some insight into its process. Ex-ante profits are larger for benefitted firms than for those that are not, and this clearly presents an incentive for firms to compete for the opportunity to be favored.

2.6. Extension - Correlated quality and cost

One assumption made throughout the paper is that the quality and cost random variables are independently distributed. In this section, we show that the model is extendable to consider the case where q_i is positively correlated with c_i. Our main result does not depend on any additional assumptions other than the correlation between the two random variables, though some additional structure may be required in order to fully characterize equilibrium behavior.

Consider the same line of events as before. Note, however, that two potentially useful new sets of information are available to each firm in their pricing decisions: Their own and the benefitted firm's quality. The former adds no information to price competition, since it is an ancillary signal to already-observed costs. The observation of $\tilde{q}$, however, presents additional informational effects in two ways: First, firms will form expectations on c_1 conditional on $\tilde{q}$. Second, all firms will know that despite not having exact signs for all other quality levels, it is a necessary condition to participation that $q_i \geq \tilde{q}$. Therefore, all relevant expectations to non-favorite firms will be taken conditional on $q_i \geq \tilde{q}$.

To see this we can rewrite expected profits as functions of new conditional distributions. Belief distribution asymmetries remain the same, and we observe an additional asymmetry on conditional cost distributions. Much like in Maskin & Riley (2000), equilibrium relationships between the bid functions will depend on assumptions made on the conditional distributions.

Proposition 4: **Let $\boldsymbol{F_1(c) \equiv \Pr(c_1 \leq c|\tilde{q})}$ and $\boldsymbol{F_i(c) \equiv \Pr(c_i \leq c|q_i \geq \tilde{q})}$. Expected profit functions can be rewritten as:**

$$\boldsymbol{\Pi_1 = \sum_{k=0}^{N-1} A(k)\left[1 - F_i\left(\beta_i^{-1}(p_1)\right)\right]^k * [p_1 - c_1]}$$

$$\boldsymbol{\Pi_i = \sum_{k=0}^{N-1} B(k)\left[1 - F_1\left(\beta_1^{-1}(p_i)\right)\right] * \left[1 - F_i\left(\beta_i^{-1}(p_i)\right)\right]^{k-1} * [p_i - c_i]}$$

Proof: See Appendix

Proposition 4 can be used to obtain properties of equilibrium bids based on assumptions on the cost distribution. For example, first-order stochastic dominance in conditional distributions, where for all $\tilde{q}' > \tilde{q}$ we assume that $F(c_i|\tilde{q}')$ FOSD $F(c_i|\tilde{q})$ is equivalent to considering q_i "bad news" about c_i as in Milgrom (1981) and has been thoroughly characterized on Maskin & Riley (2000) without uncertainty in the number of bidders.