2 An Empirical Comparison of Two Portfolio Selection Approaches

2.1 Introduction

Portfolio selection are tools of foremost importance in the real world, where investment decisions directly affect the life of people. The traditional approach of Markowitz requires the estimation or modelling of all variances and covariances, leading to unstable results when applied to a large set of assets. The evolution of financial markets increases the number of asset groups and the number of distinct assets in each group, leading the traditional approach to be less suitable to be used by practitioners. Therefore, we consider portfolio selection approaches which can deal with large set of assets a relevant field of study.

This paper aims to compare and combine two approaches which supposedly work well on large set of assets (in the case, stocks). The first is a modified version of the mean-variance optimization approach of Markowitz (1952) proposed in Chapter 1, while the second is the parametric approach to portfolio selection proposed by Brandt et al. (2009). The objective of comparing and attempting to combine the two approaches lies in the fact they assume distinct paradigms. On one hand, the Markowitz approach attempts to obtain better risk-adjusted returns by exploiting the covariance structure of stocks. On the other hand, the parametric approach overweights stocks according to some characteristics, leading to greater returns but with no success at reducing volatility below the level of a value-weighted portfolio. Therefore, the comparison between the two techniques can be seen as a comparison between the economic values of the covariance structure of stocks and deviations from the benchmark according to stocks' characteristics and its relations to past returns. In order to reinforce this idea, it is important to discuss two points. First, why is the covariance structure more important to explain the Markowitz's results? Second, why isn't the parametric approach able to reduce volatility by deviating from the benchmark according to some characteristic? The first point can be justified by the fact the optimized portfolios have lower volatility than the value-weighted, so its increase in Sharpe Ratio comes mainly from volatility reduction. Besides, the expected return models over monthly horizons do not show forecasting power greater than a naive model which assumes there is no cross-sectional variation, as seen in Chapter 1. The second point is harder to justify. In theory, nothing prevents the reduction of volatility by overweighting stocks according to some characteristic. However, it can be seen that no configuration of the three parameters lead to a volatility, say, 5% lesser than the value-weighted.

The combination of techniques can be easily achieved by setting the initial weights of the parametric optimization as the optimal weights obtained from the modified Markowitz approach. The comparison uses data from stocks listed in NYSE, between 1974 and 2008, defines a comparison environment without look-ahead bias, imposing limits on leverage, and including transaction costs. Finally, both approaches attempt to use the same stock characteristics to pursue better results and optimize the same expected utility. The modified Markowitz approach uses a multi-factor model to explain expected returns and covariances and includes adjustments to restrict leverage and deal with transaction costs. The parametric approach defines the weight of each stock as deviations from the benchmark (at first, the value-weighted portfolio) according to the characteristics (market value, momentum and book-to-market) and one parameter to each characteristic, estimated to maximize ex-post utility over the sample. Our work is related to papers which attempt to compare portfolios obtained from different models or approaches, like Fama & Macbeth (1973), Frost & Savarino (1988), Michaud (1989),Chan et al. (1999), and Ledoit & Wolf (2008).

We reach three main results. First, the portfolios generated by using our modified Markowitz approach have better risk-adjusted returns than naive portfolios like the value-weighted and equal-weighted portfolios, as well as the parametric portfolios, even in the presence of transaction costs and absence of look-ahead bias in the estimation of coefficients. Second, the parametric portfolios perform barely better than value-weighted and worse than equal-weighted or portfolios generated by the modified Markowitz approach. This contradicts the results presented in Brandt et al. (2009). Given that the characteristics, the parameterization and policy to minimize turnover (due to transaction costs) are all the same, we can attribute the difference in performance to the use of a restricted set of stocks (we do not include stocks listed in AMEX and NASDAQ) and another sample period. Third, the combination of techniques is unable to obtain better results than the ones obtained by using only the modified Markowitz approach. These results suggest the covariance structure of stocks can not be ignored when pursuing better return-risk ratios.

The paper is organized as follows. Section 2.2 describes briefly both approaches and the comparison environment. Section 2.3 describes the data and show the results. Section 2.4 concludes.

2.2 Comparison Methodology

In order to obtain a fair and realistic comparison between the parametric approach and the adjusted Markowitz approach, we establish the following criteria:

1. The investable set is the same, at all periods;
2. Both techniques use the same stock characteristics searching for a better portfolio;
3. The leverage is limited to a short position of 30% of the investor's wealth;
4. The estimation of parameters, volatilities, coefficients and risk premia for each factor to be used at period $t+1$ uses only information available at period t;
5. Neither of the approaches attempt to capture risk premia variation over time;
6. The investor has the same preferences;
7. The performance covers returns net of transaction costs.

As the second criterion suggests, both approaches use the same stock characteristics: book-to-market, stock size and momentum. The parametric approach directly includes the three characteristics in the parameterization: book-to-market, stock price times shares outstanding and the accumulated return from the last 12 months are the characteristics . The use of these characteristics by the traditional approach is indirect: in addition of the market excess return, three portfolios' returns are used as factors. The two first additional factors are the small-minus-big and high-minus-low portfolios described by Fama & French (1996). These factors attempt to exploit size and value anomalies. The third additional factor is the winner-minus-loser portfolio suggested by Carhart (1997). This factor attempt to exploit the 1-year momentum anomaly.

The fourth criterion removes look-ahead bias from the estimation. However, the entire approach still suffer from some degree of look-ahead-bias. This occurs because the characteristics used are known to be correlated to the risk-return ratio given our knowledge from the entire sample.

We include the fifth criterion because the estimation procedure in the parametric approach assumes the optimal parameters are constant over time. As a first attempt to evaluate the impact of this restriction in the relative performance, we estimate both techniques considering two regimes. This procedure is described in 2.2.3.

The inclusion of transaction costs aims to penalize turnover. Commonly, an active technique which supposedly performs better than a naive approach such as value-weighted portfolio without considering transaction costs depends on excessive turnover and the absence of transaction costs. Considering transaction costs favors lower turnover approaches. In order to deal with transaction costs, we use the same policy described in Brandt et al. (2009). Briefly, this policy estimates a hypersphere centered at the current weights. If the optimal weight lies inside this regions, it is optimal not to trade. If the optimal weight lies outside, the weights are changed up to the border of the hypersphere centered at the optimal weights.

2.2.1 Utility function

To compare the performance of the two approaches for an investor with the same preferences, we represent his preferences by an expected utility function. The utility function the investor maximizes is the ex-ante Sharpe Ratio. We choose the Sharpe Ratio because the investable set is restricted to ordinary stocks. Given the fact the investor can not include other asset classes due to the parameterization used by the parametric approach, the Sharpe Ratio is a good measure in the search for a portfolio superior to the value-weighted portfolio. In a second step, the optimal portfolio could be treated as an asset in an allocation optimization including all asset classes.

The maximization of the Sharpe Ratio in the parametric approach is straightforward. The estimated parameters are the ones which maximize the ex-post Sharpe Ratio. In the Markowitz approach, the portfolio depends upon the expected returns and covariances and the target expected return. For each target expected return, there is an optimal portfolio with expected return and volatility. Therefore, we choose the target expected return which maximizes the Sharpe Ratio.

2.2.2 Out-of-sample procedure

In order to prevent look-ahead bias in the estimation of parameters, volatilities, coefficients and risk premia associated to each factor, we employ a growing estimation window.

For the parametric approach, we estimate the optimal parameters from the first 104 months of the sample. This parameters are used to define the optimal portfolio for the next 12 months. Then, the estimation sample is increased to include these months, the parameters are re-estimated and used for the next 12 months. The procedure continues until we reach the end of the sample.

In the traditional approach, we re-estimate factor loadings, risk premia, residual volatility and factor covariances for each period t, by using all information available at period $t-1$. Besides, the factor portfolios are created by following the procedure described by Fama & French (1993a) and Carhart (1997). Therefore, the definition of the set of stocks in each group (small, high, winner, etc) does not require any information unavailable by the moment the portfolio return is used.

2.2.3 Comparison assuming two regimes

The assumption of constant parameters through time is used to estimate the optimal parameters in the parametric approach. This restriction motivates one of the fairness criteria: that neither of the approaches attempt to capture risk premia variation over time. As a first attempt to evaluate how the removal of this assumption impacts both portfolio selection techniques, we divide the sample in two. The first part contains periods in which the yield curve is positively sloped, while the second part contains periods in which the yield curve is negatively sloped. All parameters, factor loadings, risk premia and covariances, and residual volatilities, are estimated separately for each part. This division is analogous to the procedure described in Brandt et al. (2009). The same division is used in Chapter 1 and the portfolios perform worse than without 2 regimes.

From the business cycle view, periods in which the yield curve is negatively sloped tend to be associated to recessions. Hence, correlations among stock characteristics and returns should vary if the parameters are estimated separately for each part. Likewise, the risk premia associated to each factor should be different across the two sample parts.

From the econometric view, it is important that each part contains periods across the entire sample. In particular, the sample used to estimate initial parameters should have periods from both parts.

From the portfolio manager view, a smoother transition would be a better approach. If the estimated values are different, the transition between regimes should generate great losses due to transaction costs. Besides, the transitions can be associated to lesser market liquidity. Hence, the transaction costs may be underestimated.

2.2.4 Parametric approach

We describe the parametric approach proposed by Brandt et al. (2009).

We parameterize the optimal portfolio weights as a linear function of stock characteristics:

$$\omega_{i,t} = \bar{\omega}_{i,t} + \frac{1}{N_t}\theta^T \hat{x}_{i,t} \tag{2-1}$$

where N_t be the number of stocks in the investable set at each date t, $\bar{\omega}_{i,t}$ is the base weight of stock i, θ is a vector of coefficients to be estimated, and $\hat{x}_{i,t}$ are the characteristics of stock i, standardized cross-sectionally to have zero mean and unit standard deviation across all stocks in the investable set at date t. The characteristics we use to generate three factors are related to equity value (me), book-to-market (btm) and accumulated return (mom).

The accumulated return is the accumulated return for the last twelve months. The equity value is the the log of 1 plus book equity (with a lag of at least 6 months) divided by market equity computed using shares outstanding and closing prices from the previous month. The book-to-market is the market value of the stock, computed using data from the previous month. The three factors are related to the coefficients θ_{mom}, θ_{btm}, and θ_{me}, respectively. We use lagged values in order to enable all data required to define the characteristics to be already known at period t .

The assumption of constant θ across stocks and over time is the key to estimate its optimal values. Since the coefficients are constant through time, the coefficients that maximize the investor's conditional expected utility at a given date are the same for all dates. Therefore, these coefficients maximize the investor's conditional expected utility.

$$\hat{\theta} = \arg\max_{\theta} E[u(r_{p,t+1})] = \arg\max_{\theta} E\Big[u\Big(\sum_{i=1}^{N_t} \omega_{i,t} r_{i,t+1}\Big)\Big] \tag{2-2}$$

Thus, we can estimate the optimal coefficients $\hat{\theta}$ by maximizing the corresponding sample analogue.

2.2.5 Markowitz approach

We use the same portfolios described in Chapter 1.

The optimal weights ω^* which solve the Markowitz problem can be viewed as a function of the target expected return μ_{target}, the conditional expected return μ and the conditional covariance matrix Σ_t. For each period t, we choose the target expected return which maximizes the investor's expected utility function. This 2-step optimization does not require additional computational power: it can be shown that the weights of optimal Markowitz portfolios are a linear function of the target expected return.

Given the large number of stocks, we use a multifactor model for returns (and covariances). Each stock return is given by the risk-free rate plus a constant coefficient related to the stock plus the factor set multiplied by stock sensitivity to each factor plus a residual.

The multi-factor model we use is an extension to the Fama-French 3-factor model (Fama & French (1996)) proposed in Carhart (1997) to include the portfolio winner-minus-loser as a factor. The 4-factor model considers the market return and return anomalies over distinct classes of size, momentum and book-to-market - the same characteristics used to define the active deviation from the benchmark portfolio in the parametric case. We evaluate the performance when imposing $A = 0$. Given these factors, the conditional expected return is given by:

$$\begin{aligned} E_t[r_{i,t+1} - r_{f,t+1}] = \alpha_i + \beta_i E_t[RM_{t+1} - r_{f,t+1}] + s_i E_t[RSMB_{t+1}] \\ + h_i E_t[RHML_{t+1}] + p_i E_t[RWML_{t+1}] \end{aligned} \tag{2-3}$$

The estimation follows the Fama-Macbeth procedure. Given the good results obtained in Chapter 1 by the alternative technique to estimate factor loading for each stock, we also check the performance using factor loadings estimated by the aggregated method discussed in the paper. Finally, Σ_e is assumed to be diagonal and the i-th element of its diagonal is the variance of the time series comprising differences between realized and fitted returns.

2.3 Empirical results

2.3.1 Data

Our sample consists of stocks listed in NYSE. We use monthly holding period returns, shares outstanding and closing prices from CRSP monthly database and quarterly data from Compustat to calculate the book equity. The sample period is from June 1970 to December 2008. Before this period, there was no quarterly data available for more than 20 companies with stocks listed in NYSE. After exclusions, the number of valid stocks varies through the sample, ranging from 520 to 714. We exclude from the sample:

1. Stocks with an asset code different from 10 or 11, according to CRSP database[1];

[1]This excludes certificates, ADRs, SBIs, Units, companies incorporated outside the U.S.,

2. Stocks from a company not listed in Compustat;
3. Stocks from companies with negative book-to-market at any moment during the sample period.

Removing stocks from companies with negative book-to-market generate a quality bias in the sample. Therefore, a naive value-weighted strategy over this sample outperform an index like the S&P500.

Figure 2.3.1 shows the accumulated returns for value-weighted and equal-weighted portfolios using the stocks in the sample. Furthermore, the mean, standard deviation and skewness of the value-weighted portfolio are 0.87%, 0.041, and -0.855, respectively. Finally, the mean, standard deviation and skewness of the equal-weighted portfolio are 1.10%, 0.046, and -1.275, respectively.

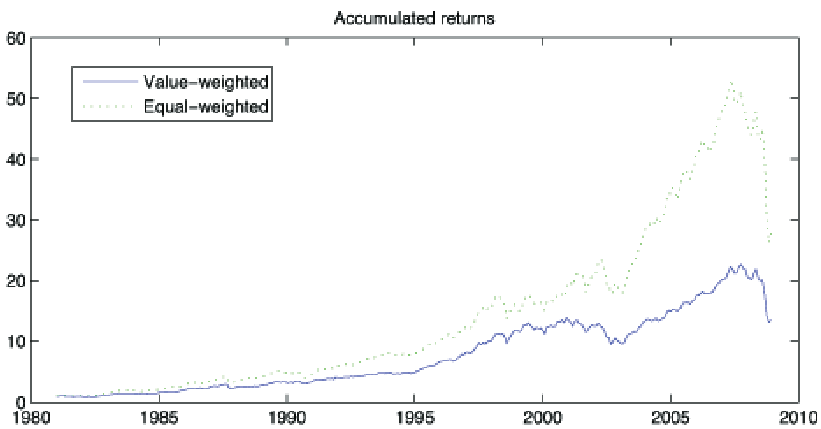

Figure 2.1: Accumulated returns for value-weighted and equal-weighted portfolios

For each month, the book equity of a company is (in parenthesis, the field code in Compustat): total assets (ATQ) minus liabilities (LTQ) plus balance-sheet deferred taxes and investment tax credits (TXDITCQ), minus preferred stock value (PSTKQ); the market value of a stock is shares outstanding times closing price; the market equity of a company is the market value of all stocks in CRSP from the same company. Finally, the book-to-market of a company is the log of 1 plus book equity divided by market equity.

As risk-free rate, we use the 3-month Treasury bill secondary market rate from FRED database. From the same source, we compute the yield slope as the difference between market yield on U.S. Treasury securities at 10 and 1 year.

closed-end funds and REIT's.

2.3.2 Transaction costs

Our modelling of the transaction costs $c_{i,t}$ for stock i at period t follow the approach used by Brandt et al. (2009), which attempts to capture two empirical facts. First, that transaction costs vary across distinct stocks, being larger for small caps than for large caps. Second, the decrease in transaction costs over time. Among others, these results can be found in Domowitz et al. (2001).

The transaction costs $c_{i,t}$ are modelled as:

$$c_{i,t} = 0.006 - 0.0025 \times ME_{i,t} * T_t \tag{2-4}$$

where $ME_{i,t}$ is the log of the market value of the stock normalized to the interval $[0, 1]$ and T_t captures declining costs over time. In the first month of the sample, $T_t = 2^2$ and it decreases linearly over each month, until $T_t = 1$ at the last month of the sample. As an example, the stock of lesser market value at January, 74, has transaction costs of 1.2% at the same month. Likewise, the stock of greater value at December, 2008, has transaction costs of 0.35% at this month.

Our costs are lesser than the costs used in Brandt et al. (2009) for two reasons. First, our sample starts 10 years later. Besides, since we use only NYSE stocks, transaction costs are supposed to be lesser than in a sample also containing stocks from AMEX and NASDAQ.

2.3.3 Results

In this Section, we present the performance of 12 distinct portfolios:

- Value-weighted and equal-weighted portfolios;
- Parametric portfolio, parameters optimized from the entire sample;
- Parametric portfolio, parameters optimized from the growing windows scheme described in Section 2.2.2;
- 4 portfolios from the Markowitz optimization approach (M1, M2, M3 and M4, as described in Chapter 1);
- Parametric portfolio, deviating from alternate portfolios as initial weights:
  - Active weights deviating from the equal-weighted portfolio;
  - Active weights deviating from M2;
  - Active weights deviating from M4;
  - Active weights deviating from M4, parameters divided by 2.

[2] We also test $T_1 = 3$ and achieve similar results.

By evaluating these portfolios, we can compare both techniques to the naive portfolios, compare the performance of one technique relative to the other and check the possibility to combine the approaches. The division of parameters by 2 can be seen as a shrinkage towards the initial weights. Several empirical articles present performance improvement after shrinking estimated parameters towards a prior. An example is Ledoit & Wolf (2008).

All returns are net of transaction costs. We use the procedure to handle transaction costs described in Chapter 1 in all portfolios except the value-weighted and equal-weighted. The leverage is limited to a short position of 30% of the investor's wealth.

Table 2.1 presents statistics for the value-weighted and equal-weighted portfolios, as well as the in-sample and out-of-sample parametric portfolios. The value-weighted portfolio has an annualized average return of 10.4% and a 14.3% volatility. The Sharpe ratio is 0.362, and alpha, beta and residual volatility are -0.006, 1.000 and 0.011, respectively. Alpha, beta and residual volatility are measured against the returns from the value weighted portfolio without imposing transaction costs. The largest average position on a stock is 4.6% and the average monthly turnover is 0.2% of the investor's wealth. The equal-weight portfolio has greater returns 13.2% and volatility 15.8%. Its Sharpe ratio is 0.506 and average monthly turnover is 7% of the investor's wealth.

The parametric portfolio optimized from the entire sample has 12.1% average returns and 18.1% volatility, causing the Sharpe ratio to be 0.384. The average turnover is 1.8% and average proportion of stocks shorted is 38%. The portfolio's alpha, beta and residual volatility are 0.016, 0.914 and 12.5%, respectively. The average short position is 30%. This means the restriction on leverage was active at virtually all periods. Finally, the parameters θ_{mom}, θ_{me}, and θ_{btm} are 0.965, -1.906, and 1.957, respectively. Therefore, the portfolios overweights stocks that performed above average in the last year, have market value below average and a greater book value in relation to its market value. The portfolio optimized following the out-of-sample procedure performs even better. The volatility falls to 17.6% and the Sharpe ratio is 0.396. The better performance of the out-of-sample over the in-sample suggests the freedom to adjust the parameters throughout the sample compensates for the absence of periods in which the parameters are used. However, the parametric approach does not perform better than the equal-weighted case.

It is worth comparing the results with those of Brandt et al. (2009). There, the sample also includes stocks from AMEX and NASDAQ, there is no limit on leverage and the evolution of transaction costs through time

Table 2.1: Naive and parametric portfolios performance

This table presents statistics from four portfolios, considering transaction. All statistics are from January, 81 to December, 2008. The columns labeled "VW" and "EW" displays the value-weighted and equal-weighted portfolios, respectively. The last 2 columns present statistics for the parametric approach. The column labeled "IS" displays the performance of the parametric technique optimizing θ over the entire sample. The column labeled "OOS" displays the performance of the portfolio estimated by following the Out-of-sample procedure. The first set of rows shows the estimated average coefficients for each parameter followed by the standard deviation. The second set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The third set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The fourth set of rows presents average characteristics of the portfolio over time. The rows labeled "me", "mom" and "btm" show measures of market value, momentum and book-to-market, respectively. The returns are net of transaction costs.

Variable	VW	EW	IS	OOS
$\bar{\theta}_{mom}$	-	-	0.965	1.987 (1.268)
$\bar{\theta}_{me}$	-	-	-1.906	-0.924 (0.747)
$\bar{\theta}_{btm}$	-	-	1.957	5.247 (2.684)
$\lvert\omega_i\rvert \times 100$	0.105	0.157	0.268	0.257
$\max \omega_i \times 100$	4.689	0.187	3.054	2.594
$\min \omega_i \times 100$	0.001	0.142	0	0.004
$\sum \omega_i I(\omega_i < 0)$	0	0	-0.300	-0.299
$\sum I(\omega_i \leq 0)/N$	-	-	0.384	0.433
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.002	0.070	0.018	0.049
$\bar{r}$	0.104	0.132	0.121	0.122
$\sigma(r)$	0.143	0.158	0.181	0.176
SR	0.362	0.506	0.380	0.396
α	-0.006	0.020	0.016	0.016
β	1.000	1.024	0.914	0.924
$\sigma(\epsilon)$	0.011	0.056	0.125	0.115
me	1.297	0	-0.995	-0.576
mom	0.107	0	0.140	0.045
btm	-0.437	0	1.499	1.787

is different. Their out-of-sample portfolio including the same approach to handle transaction costs has average returns of 28.4% and volatility of 21.2%. The average short position is 156% of the investor's wealth. The optimal parameters for θ_{mom}, θ_{me}, and θ_{btm} are 3.154, -0.845 and 4.021, respectively. The parameters are of the same sign, but they imply a greater deviation towards past winner and value stocks and a lesser deviation towards small stocks. The huge gap in performance is probably caused by two factors: our limit on leverage and the absence of stocks from AMEX and NASDAQ.

Table 2.2 presents the portfolios generated by the Markowitz optimization, using 4 distinct versions of the 4-factor model. The average returns range from 10.9% to 12.6%, the volatility range from 13.1% to 13.6%. Excluding the M1 model,which performs worse than the others, the Sharpe ratio is close to 0.525. The average monthly turnover is approximately 10% of the investor's

Table 2.2: Markowitz portfolios performance

This table presents statistics from four portfolios, considering transaction costs. All statistics are from January, 81 to December, 2008. The columns correspond to Markowitz optimized portfolios using return models M1, M2, M3 and M4, respectively. The first set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The second set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs. The returns are net of transaction costs. The third set of rows presents average characteristics of the portfolio over time. The rows labeled "me", "mom" and "btm" show measures of market value, momentum and book-to-market, respectively.

Variable	M1	M2	M3	M4
$\lvert\omega_i\rvert \times 100$	0.235	0.160	0.171	0.142
$\max \omega_i \times 100$	4.113	1.747	3.792	1.971
$\min \omega_i \times 100$	0.016	0.006	0.004	0.002
$\sum \omega_i I(\omega_i < 0)$	-0.180	-0.121	-0.111	-0.044
$\sum I(\omega_i \leq 0)/N$	0.197	0.182	0.133	0.094
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.116	0.099	0.094	0.091
$\bar{r}$	0.109	0.125	0.121	0.126
$\sigma(r)$	0.133	0.136	0.131	0.139
SR	0.426	0.531	0.527	0.527
α	0.006	0.023	0.020	0.020
β	0.873	0.862	0.850	0.904
$\sigma(\epsilon)$	0.043	0.055	0.047	0.049
me	0.507	0.115	0.175	0.045
mom	0.369	0.030	0.017	0.001
btm	-0.396	0.223	0.197	0.152

wealth. All portfolios have positive alpha, the beta is close to 0.9 and the residual volatility ranges from 0.013 to 0.056. All portfolios perform better than the value-weighted and parametric portfolios. Excluding the M1 model, the Markowitz portfolios also perform better than the equal-weighted portfolio. Still comparing with the parametric portfolios, the portfolios generated by the traditional approach present a larger turnover, but require lesser leverage.

The imposition of $A = 0$ improves the performance when the factor loadings are estimated directly from the stocks returns. This suggests the abnormal returns do not persist and reduce return forecasting power. There is no impact when the factor loadings are estimated from portfolios. In this case, the intercepts are already very close to 0 and the adjustment does not affect performance.

Finally, it is worth to evaluate whether it is possible to combine the portfolio approaches. Therefore, we change the benchmark portfolio $\bar{\omega}_t$. Originally, $\bar{\omega}_t$ corresponds to the value-weighted portfolio. In our attempt to combine the approaches, we change $\bar{\omega}_t$ to the equal-weighted portfolio, as well as Markowitz portfolios with return models M1 and M3. Unfortunately, the portfolios generated do not perform better than the original portfolios. The statistics for

Table 2.3: Parametric technique deviating from alternate portfolios
This table presents statistics from four portfolios, considering transaction costs. The portfolios obtained by applying the parametric optimization to initial weights distinct from the value-weighted benchmark. The first column presents the portfolio obtained by using the equal-weighted portfolio as initial weights. The following two columns display statistics from the portfolio by using the Markowitz portfolios from return models M2 and M4, respectively. The last column presents the result by applying the parametric approach to the Markowitz portfolio using return model M4, with optimal parameters divided by 2. All statistics are from January, 81 to December, 2008. The first set of rows shows the estimated average coefficients for each parameter followed by the standard deviation. The second set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The third set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The fourth set of rows presents average characteristics of the portfolio over time. The returns are net of transaction costs.

Variable	EW	M2	M4	M4 + shrinkage
$\bar{\theta}_{mom}$	1.845 (1.144)	1.691 (1.159)	1.777 (1.137)	0.889 (0.568)
$\bar{\theta}_{me}$	-0.169(0.571)	-0.426 (0.606)	-0.294 (0.591)	-0.147 (0.295)
$\bar{\theta}_{btm}$	4.939 (2.426)	4.370 (2.488)	4.660 (2.417)	2.330 (1.209)
$\lvert\omega_i\rvert \times 100$	0.257	0.255	0.256	0.245
$\max \omega_i \times 100$	2.390	2.012	2.159	1.623
$\min \omega_i \times 100$	0.004	0.003	0.004	0.002
$\sum \omega_i I(\omega_i < 0)$	-0.297	-0.299	-0.299	-0.267
$\sum I(\omega_i \leq 0)/N$	0.416	0.425	0.418	0.340
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.050	0.029	0.036	0.017
$\bar{r}$	0.133	0.131	0.132	0.130
$\sigma(r)$	0.185	0.161	0.167	0.149
SR	0.438	0.484	0.478	0.514
α	0.026	0.029	0.029	0.028
β	0.935	0.858	0.870	0.846
$\sigma(\epsilon)$	0.127	0.104	0.110	0.086
me	-0.866	-0.754	-0.783	-0.559
mom	0.042	0.039	0.033	0.047
btm	1.910	1.719	1.798	1.447

the three portfolios are in Table 2.3. However, the portfolios perform better than the parametric portfolio obtained using value-weighted portfolio as initial weights. This suggests that the initial weights matter and can influence the performance

In addition, we shrink the parameters estimated with an active deviation from the Markowitz portfolio with return model towards 0: all parameters are divided by 2. This portfolio performs better than the combined portfolio (the Sharpe Ratio goes from 0.479 to 0.514), but still performs worse than the M3 portfolio. However, this experience can not be seen as an appropriate test of the performance of portfolios with shrunk parameters. The limit on leverage can also be seen as a form of shrinkage, so it is not possible to separate both effects.

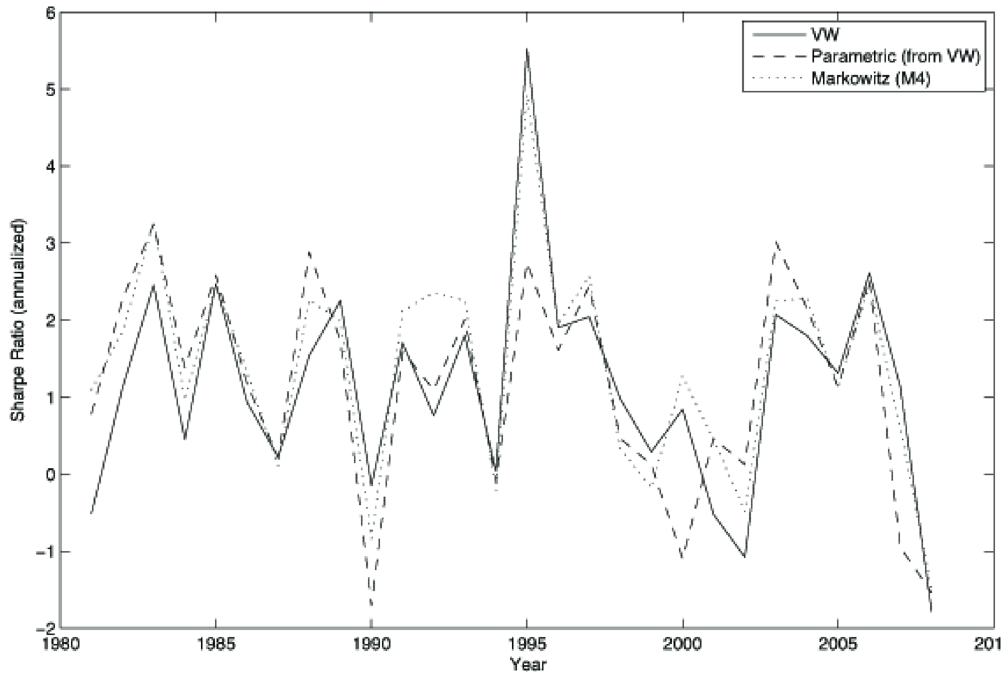

Figure 2.2: *Sharpe Ratio for each year*

Figure 2.2 presents the Sharpe Ratio for each year. The value-weighted portfolio performs equal or worse than the Markowitz portfolio over almost all years, except 1988 to 1990, 1995 and 1998 to 1999. The better performance of the Markowitz than the parametric portfolios seems to come from the years 1990 to 1995 and 2000. Finally, the parametric portfolio performs better than the value-weighted portfolios over most of sample and its poor overall performance comes from 1990, 1995 and 2000.

2.3.4 Results: two regimes

Table 2.4 presents statistics for portfolios obtained from both traditional and parametric approaches, following the 2-regimes strategy described in 2.2.3. The two portfolios' performance is far below the obtained without 2 regimes. It seems the several discrete switches between regimes generate excessive transaction costs.

Tables 2.5 and 2.6 present the returns statistics during the positive slope and inverted regime, respectively. The parametric portfolio performs better in the positive slope regime, but Markowitz portfolio's performance on inverted slope regime compensates this loss.

Table 2.4: Two regimes

This table presents statistics from two portfolios, considering transaction costs and following the procedure to allow 2 regimes, as described in Section 2.2.3. The columns labeled “Markowitz” and “Parametric” show statistics for the Markowitz (M3 return model) and the parametric approach, respectively. All statistics are from January, 81 to December, 2008. The first set of rows shows the estimated average coefficients for each parameter followed by the standard deviation. The second set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The third set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The fourth set of rows presents average characteristics of the portfolio over time. The returns are net of transaction costs.

Variable	Markowitz	Parametric
$\bar{\theta}_{mom,slope>0}$	-	1.865
$\bar{\theta}_{me,slope>0}$	-	-1.309
$\bar{\theta}_{btm,slope>0}$	-	5.216
$\bar{\theta}_{mom,slope<0}$	-	2.388
$\bar{\theta}_{me,slope<0}$	-	-0.145
$\bar{\theta}_{btm,slope<0}$	-	5.708
$\lvert\omega_i\rvert \times 100$	0.210	0.257
$\max \omega_i \times 100$	6.617	2.573
$\min \omega_i \times 100$	0.006	0.005
$\sum \omega_i I(\omega_i < 0)$	-0.169	-0.299
$\sum I(\omega_i \leq 0)/N$	0.201	0.434
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.202	0.052
$\bar{r}$	0.112	0.121
$\sigma(r)$	0.146	0.177
SR	0.403	0.388
α	0.001	0.016
β	0.948	0.928
$\sigma(\epsilon)$	0.053	0.117
me	0.104	-0.626
mom	0.014	0.012
btm	0.119	1.801

Table 2.5: Two regimes - positive slope regime

This table presents statistics from two portfolios over the positive slope regime, considering transaction costs and following the procedure to allow 2 regimes, as described in Section 2.2.3. The columns labeled “Markowitz” and “Parametric” show statistics for the Markowitz (M3 return model) and the parametric approach, respectively. All statistics are from January, 81 to December, 2008.The rows display annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs.

Variable	Markowitz	Parametric
$\bar{r}$	0.104	0.122
$\sigma(r)$	0.153	0.186
SR	0.367	0.396
α	-0.003	0.016
β	0.977	0.950
$\sigma(\epsilon)$	0.051	0.122

Table 2.6: Two regimes - inverted slope regime

This table presents statistics from two portfolios over the inverted slope regime, considering transaction costs and following the procedure to allow 2 regimes, as described in Section 2.2.3. The columns labeled “Markowitz” and “Parametric” show statistics for the Markowitz (M3 return model) and the parametric approach, respectively. All statistics are from January, 81 to December, 2008.The rows display annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs.

Variable	Markowitz	Parametric
$\bar{r}$	0.158	0.118
$\sigma(r)$	0.086	0.105
SR	0.905	0.353
α	0.053	0.010
β	0.613	0.667
$\sigma(\epsilon)$	0.055	0.551

2.4 Conclusions

We presented an empirical comparison of two portfolio selection techniques, with distinct paradigms. On one hand, the modified Markowitz approach attempts to obtain better risk-adjusted returns by exploiting the covariance structure of stocks. On the other hand, the parametric approach overweights stocks according to some characteristics, leading to greater returns but with no success at reducing volatility below the level of a value-weighted portfolio. Therefore, the comparison between the two techniques can be seen as a comparison between the economic values of the covariance structure of stocks and deviations from the benchmark according to stocks' characteristics and its relations to past returns. We also attempted to combine both approaches and a simple 2-regimes approach.

We reach three main results. First, the portfolios generated by using the modified Markowitz approach have better risk-adjusted returns than naive portfolios like the value-weighted and equal-weighted portfolios, as well as the parametric portfolios, even in the presence of transaction costs and absence of look-ahead bias in the estimation of coefficients. Second, the parametric portfolios perform barely better than value-weighted and worse than equal-weighted or portfolios generated by the modified Markowitz approach. This contradicts the results presented in Brandt et al. (2009). Given that the characteristics, the parameterization and policy to minimize turnover (due to transaction costs) are all the same, we can attribute the difference in performance to the use of a restricted set of stocks (we do not include stocks listed in AMEX and NASDAQ) and another sample period. Third, the combination of techniques is unable to obtain better results than the ones obtained by using only the modified Markowitz approach. These results suggest the covariance structure of stocks can not be ignored when pursuing better return-risk ratios. The experiment using 2 regimes fails for both approaches, in the sense the performance is worse than in the 1-regime case.

Further improvements may include: consider others expected utility functions, attempt to dissociate the effects of modeling expected returns and expected covariances. Finally, an approach to exploit successfully time-varying investment opportunities could be another source of improvement.