1 Evaluating an Adjusted Markowitz Approach on a Large Set of Stocks

1.1 Introduction

Portfolio selection is of foremost importance in the real world, where investment decisions directly affect the life of people. The traditional mean-variance approach of Markowitz requires the estimation or modeling of all variances and covariances, leading to unstable results when applied to a large set of assets. The evolution of financial markets increases the number of asset groups and the number of distinct assets in each group, leading the original Markowitz approach to be less suitable to be used in practice. Besides, there are transaction costs and leverage limits in the real world. These aspects must be taken into account, even if the resulting solution lacks some proof of theoretical optimality. Therefore, we propose a modified version of the Markowitz approach which can deal with large number of assets as well as leverage limits and transaction costs. Our methodology combines techniques previously proposed in the literature to control for the possible "curse of dimensionality" when estimating the covariance matrix of the assets with the imposition of leverage limits and transaction costs. Furthermore, we modify the utility function in order to maximize the Shape ratio of the final portfolio. Our empirical tests use only stocks, but other asset classes can be easily included.

This paper evaluates a modified version of the mean-variance optimization approach of Markowitz (1952). We set an evaluation environment which prevents look-ahead bias in the estimation of coefficients, includes transaction costs and maximizes the expected Sharpe Ratio (SR) of a portfolio containing stocks listed in the NYSE, between 1981 and 2008. The adjustments we include are:

1. Use of a multi-factor approach to model expected returns and covariances;

2. Definition of a maximum leverage, setting the maximum weights on short positions;

3. Shrinkage of the estimation of factor coefficients by estimating portfolios containing similar stocks, given the factors;

4. Use of a policy to reduce transaction costs.

The first adjustment was proposed by Sharpe (1964) using the single factor model (CAPM). Chan et al. (1999) study the performance of different factor models specifications. We use a four-factor model as in Carhart (1997) which was derived from the three-factor model of Fama & French (1993a). The fourth factor is important in order to include momentum.

We estimate the factor loadings for each stock by creating a portfolio containing the stock itself and others with similar characteristics. This can be seen as a shrinkage of the factor loading towards a portfolio containing related stocks. By assuming that all covariance between different stocks are explained by factor covariances and factor loadings, we greatly reduce the amount of coefficients to be estimated, mitigating multi-dimensionality. The second and third adjustments are related to the shrinkage of coefficients, which have already been used to obtain better results in Ledoit & Wolf (2008). In order to deal with transaction costs, we follow a "no-trade" region policy proposed in Brandt et al. (2009) which greatly reduces turnover and performance loss. We consider the setup of the evaluation environment, together with the shrinkage of the estimation of factor loadings and the results we achieve, to be an original contribution to the literature.

Our results show the portfolios generated by using our modified Markowitz approach have better risk-adjusted returns than naive portfolios like the value-weighted and equal-weighted portfolios, even in the presence of transaction costs and absence of look-ahead bias in the estimation of coefficients. These good results do not depend on the fact the sample has more bull years than bear years. The optimized portfolios tend to perform better than the benchmark in both periods. This suggests there is a positive economic value in the covariance structure of stocks.

The paper is organized as follows. Section 1.2 describes the traditional Markowitz approach, the multi-factor model of conditional expected returns and covariances, the estimation of coefficients, and the adjustment we propose to simplify the otimization problem. Section 1.3 describes the approach to deal with transaction costs, the out-of-sample estimation procedure and the utility function we choose. Section 1.4 presents the data and the results. Section 1.5 concludes.

1.2 Markowitz optimization and extensions

1.2.1 General Formulation

Let r_{t+1}, $\mu_t \equiv \mathbb{E}_t[r_{t+1}]$, and $\Sigma_t \equiv \mathbb{E}_t[(r_{r+1} - \mu_t)(r_{r+1} - \mu_t)']$ denote, respectively, an $N_t \times 1$ vector of stock returns, the expected returns and covariance matrix conditional based on information up to period t. Both μ_t and Σ_t are not observable, so we use estimated values $\hat{\mu}_t$ and $\hat{\Sigma}_t$, respectively.

The investor's problem at t is to minimize portfolio volatility, subject to a target expected return and weight constraints:

$$\begin{aligned} \omega_t^* &= \arg\min_{\omega_t} \omega_t' \hat{\Sigma}_t \omega_t \\ \text{s.t.} \quad & \omega_t' \hat{\mu} = \mu_{target} \\ & \omega_t' \mathbf{1} = 1 \end{aligned} \tag{1-1}$$

where ω_t is an $N_t \times 1$ vector of portfolio weights on stocks, μ_{target} is the target expected rate of return from t to $t+1$ and $\mathbf{1}$ is a $N_t \times 1$ vector of ones.

This formulation has a correspondent formulation as quadratic expected utility maximization, each value of μ_{target} yields weights equivalent to the optimal weights for a given risk aversion in the quadratic utility function (see Brandt (2004)).

The optimal weights ω^* which solve the optimization problem described in (1-1) can be viewed as a function of the target expected return μ_{target}, the conditional expected return μ_t and the conditional covariance matrix Σ_t.

1.2.2 Adjusting other expected utility functions

The general formulation given by Equation 1-1 requires three components: the vector of expected returns μ, the covariance matrix Σ, and the target expected return μ_{target}. In this section, we take μ and Σ as given. The target expected return at each period t is chosen in order to maximize the investor's expected utility. For each target return there is a portfolio $\omega_t^*(\mu_{target})$ (we add μ_{target} to emphasize the fact the optimal weights depend on the target expected return). Given $\omega_t^*(\mu_{target})$, we can obtain the portfolio's expected return and variance. Hence, any expected utility function related to these characteristics has a value for each μ_{target}.

The first impression is that the optimization of the expected utility by this reasoning is a 2-step optimization: while choosing the target expected return, we must compute the optimal weights for each μ_{target}, which is also an optimization problem. However, this 2-step optimization does not require excessive computational power and the resulting problem is simple. This occurs because the weights of optimal Markowitz portfolios are a linear function of the target expected return, as shown in Campbell et al. (1997). Besides, the resulting problem is a optimization problem over a single variable.

In order to obtain the relation between ω_t and μ_{target}, let L_t be the Lagrangian function of Equation 1-1:

$$L \equiv \omega_t' \Sigma_t \omega_t + \delta_1(\mu_{target} - \omega_t' \mu) + \delta_2(1 - \omega_t' \mathbf{1}) \tag{1-2}$$

Differentiating L with respect to ω_t and setting to zero generates:

$$2\Sigma_t \omega_t - \delta_1 \mu - \delta_2 1 = 0 \tag{1-3}$$

By combining (1-3) to the equality constraints from (1-1), we find:

$$\begin{aligned}
\omega_t^* &= \mathbf{g}_t + \mathbf{h}_t \mu_{target} \\
\mathbf{g}_t &= \frac{1}{D}[B_t(\Sigma_t^{-1}\mathbf{1}) - A_t(\Sigma_t^{-1}\mu)] \\
\mathbf{h}_t &= \frac{1}{D}[C_t(\Sigma_t^{-1}\mu_t) - A_t(\Sigma_t^{-1}\mathbf{1})] \\
A_t &= \mathbf{1}'\Sigma_t^{-1}\mu_t \\
B_t &= \mu_t'\Sigma_t^{-1}\mu_t \\
C_t &= \mathbf{1}'\Sigma_t^{-1}\mathbf{1} \\
D_t &= B_t C_t - A_t^2
\end{aligned} \tag{1-4}$$

We further restrict the set of possible portfolios by removing all portfolios which require a leverage level greater than 30%. Hence, we search over the restricted set of optimal portfolios for the one which maximizes a given expected utility function.

These results and restrictions turn the investor problem to:

$$\begin{aligned}
\mu_{target}^* = \arg \max_{\mu_{target}} \; & \mathbb{E}_t[U(r(\omega_t(\mu_{target})))] \\
\text{s.t.} \quad & \omega_t = \mathbf{g}_t + \mathbf{h}_t \mu_{target} \\
& I(\omega_t < 0)'\mathbf{1} <= 0.3 \\
& \omega_t'\mathbf{1} = 1
\end{aligned} \tag{1-5}$$

where $I(\omega_t < 0)'\mathbf{1}$ represents the sum of negative weights and $\mathbb{E}_t[U(r(\omega_t(\mu_{target})))]$ represents the investor's expected utility.

There are two issues related to this approach. First, the selected portfolio is not necessarily optimal, given the selected expected utility function. Second, the set of mean-variance optimal portfolios with leverage lower than 30% may be empty, given expected returns μ_t and covariances Σ_t.

The use of others expected utility functions adds flexibility to the approach and may allow a better performance comparison to other portfolio selection approach.

1.2.3 Conditional expected returns and covariances

Since our investable set is a large set of stocks, we use multi-factor models for expected returns and covariances. The use of these models facilitates the assurance that the covariance matrix is positive definite and reduces the number of coefficients to be estimated, leading to results less noisy and unstable. Otherwise, the number of estimated coefficients would be too great, given the sample size. For example, a set containing 1000 stocks would require the estimation of 495,500 parameters. Michaud (1989) suggests the unconstrained mean-variance optimization has a tendency to maximize the effects of errors, and can yield results that are inferior to those of simple equal-weighted or value-weighted schemes. The factor model for each stock return is given as:

$$r_{i,t+1} = r_f.\mathbf{1} + A + B.f_{t+1} + \epsilon_{t+1} \tag{1-6}$$

where r_f, $\mathbf{1}$, A, B, f_{t+1}, and Σ_e, denote, respectively, the risk-free rate, a $N_t \times 1$ vector of ones, a $N_t \times 1$ vector containing the constant coefficient for each stock, a $N_t \times k$ matrix containing the factor sensitivity for each stock and factor, the realized values for each factor at $t+1$, and a $N_t \times 1$ vector containing unexpected chocks to returns.

By assuming $\mathbb{E}_t[\epsilon_{t+1}] = 0$, this return model gives expected conditional returns and covariance as follows:

$$\begin{aligned} \mathbb{E}_t[r_{t+1}] &= r_f.\mathbf{1} + A + B.\mathbb{E}_t[f_{t+1}] \\ \Sigma_t &= B.\Sigma_f.B' + \Sigma_e \end{aligned} \tag{1-7}$$

where Σ_f is the expected conditional covariance matrix for the factors and Σ_e is the covariance matrix not explained by $B.\Sigma_f.B'$. We assume the residuals are uncorrelated across stocks. Hence, Σ_e is diagonal and the main diagonal equals $\mathbb{E}_t[\epsilon^2_{t+1}]$.

Multi-factor models like this may find theoretical support on the Arbitrage Pricing Model, introduced in Ross (1976), or the Intertemporal Capital Asset Pricing Model, introduced in Merton (1973). The first is based on absence of arbitrage arguments, while the second relies on equilibrium arguments. The multi-factor model we use is an extension to the Fama-French 3-factor model (Fama & French (1996)) discussed in Carhart (1997). It includes the winner-minus-loser portfolio[1] as a factor. The 4-factor model considers the

[1]The winner-minus-loser portfolios is a zero investment portfolio which takes long position in stocks which performed above average in the previous 11 months and takes short positions in stocks which performed below average in the same period.

market return and return anomalies over distinct classes of size, momentum and book-to-market. Following the literature which proposes the absence of persistent abnormal returns, we also evaluate the performance when imposing $A = 0$. Given these factors, the conditional expected return is given by:

$$\begin{aligned} \mathbb{E}_t[r_{i,t+1} - r_{f,t+1}] = \alpha_i + \beta_i \mathbb{E}_t[RM_{t+1} - r_{f,t+1}] + s_i \mathbb{E}_t[RSMB_{t+1}] \\ + h_i \mathbb{E}_t[RHML_{t+1}] + p_i \mathbb{E}_t[RWML_{t+1}] \end{aligned} \quad (1\text{-}8)$$

where RM_{t+1}, $RSMB_{t+1}$, and $RHML_{t+1}$ are the returns from the three factor-mimicking portfolios proposed by Fama & French (1996), and $RWML_{t+1}$ is the return from the winners-minus-losers factor-mimicking portfolio discussed in Carhart (1997). All factors are created by following the procedure described in each paper over the sample of stocks we use. Section 1.4.1 show the factor correlations.

1.2.4 Estimation

For each period t, we need to estimate A,B,Σ_f,Σ_e and $\mathbb{E}_t[f_{t+1}]$ from the subsample containing information available at period t.

Estimation of $\mathbb{E}_t[f_{t+1}]$

Given that the factors are aggregated excess returns, there are several works which find some degree of previsibility. For example, Cochrane (2008) and Santa-Clara & Ferreira (ming) defend previsibility for stock market returns, exactly the factors which explains most of single stock returns. However, we assume historical means as predictors for $\mathbb{E}_t[f_{t+1}]$, as suggested in Campbell et al. (1997). We use this predictor for 2 reasons. First, to simplify the model structure (which already contains too many variables to estimate). Besides, the articles defending return prediction present only a small coefficient of determination for monthly returns.

Estimation of A and B

In order to estimate A and B, we adopt 2 distinct procedures. The first procedure uses the returns of the stock to estimate its coefficients a_i and b_i. In the second procedure, we generate for each stock i a portfolio i^* containing stocks equivalent to stock i. Then, the return of this portfolio is used to estimate the coefficients a_i and b_i. The second procedure removes the biases caused by abnormal returns in the estimation of a_i and b_i. Furthermore, the coefficients estimated for each portfolio tend to deviate lesser from the market aggregated return. Hence, the aggregated estimation can be seen as a shrinkage towards a model which suggests no cross-sectional variation of expected returns. This technique is consistent to the adjustment we propose to check and or restrict the expected return model.

In the first procedure, the coefficients a_i and b_i which form the matrixes A and B are estimated by an OLS regression of the following equation:

$$r_{i,t}-r_{f,t} = \alpha_i+\beta_i(RM_t-r_{f,t})+s_iRSMB_t+h_iRHML_t+p_iRWML_t+u_{i,t} \quad (1\text{-}9)$$

where RM_t,$RSMB_t$,$RHML_t$,$RWML_t$ denote returns of the aggregated market in excess of the risk-free rate, and returns for SMB, HML and WML factor portfolios, respectively.

As stated before, the second procedure generates for each stock i a portfolio i^* containing stocks equivalent to stock i, in the sense the stocks have similar characteristics. Let $\hat{x}_{i,t}$ denote the $M \times 1$ vector containing the M normalized characteristics of stock i at period t[2] . Define $d_{i,j,t} \equiv (\hat{x}_{i,t} - \hat{x}_{j,t})'.(\hat{x}_{i,t} - \hat{x}_{j,t})$ as a measure of distance between two stocks i and j. For each stock i, we define at each period t an portfolio containing the value-weighted combination of the 30 stocks closer to stock i (including the stock i itself). That is, we generate a portfolio containing the 30 stocks with lesser values of $d_{i,j,t}$, for each stock i and period t. Let $r^*_{i,t}$ denote the return of this portfolio. The coefficients a_i and b_i which form the matrices A and B are estimated by an OLS regression of the following equation:

$$r^*_{i,t}-r_{f,t} = \alpha_i+\beta_iRM_t-r_{f,t}+s_iRSMB_t+h_iRHML_t+p_iRWML_t+u_{i,t} \quad (1\text{-}10)$$

where RM_t,$RSMB_t$,$RHML_t$,$RWML_t$ denote returns of the aggregated market in excess of the risk-free rate, and returns for SMB, HML and WML factor-mimicking portfolios, respectively.

Estimation of Σ_f and Σ_e

For each period t, we estimate Σ_f as the sample covariance matrix of the factor portfolio returns. Hence, every element $\sigma_{i,j}$ of Σ_f is estimated by:

$$\hat{\sigma}_{i,j} = \frac{1}{t-1}\sum_{q=1}^{t}(f_{i,q} - \bar{f}_i)(f_{j,q} - \bar{f}_j) \quad (1\text{-}11)$$

where $f_{i,q}$ is the i-th factor portfolio return at period q and $\bar{f}_i$ is the average factor portfolio return between periods 1 and t.

In order to estimate Σ_e, we assume it to be diagonal. This is equivalent

[2]The M characteristics we include are: (i) accumulated return over the last 12 months, (ii) stock price times shares outstanding, and (iii) book-to-market. All characteristics are normalized at each period so the cross-section average and variance are 0 and 1, respectively.

to assume all covariance among distinct stocks is explained by the factors. The elements of the main diagonal are the sample variance of the part of returns not explained by the model (difference between realized and forecast returns).

1.3 Evaluation Environment

In order to obtain a fair and realistic comparison between the Markowitz approach and the naive value-weighted and equal-weighted portfolios, we establish the following criteria:

1. The investable set is the same, at all periods;

2. The estimation of parameters, volatilities, coefficients and risk premia for each factor to be used at period $t+1$ uses only information available at period t;

3. The performance covers returns net of transaction costs.

The first criterion is useful because using only stocks the choice of portfolios to be used as benchmark is straightforward. The second criterion removes look-ahead bias from the estimation. However, the entire approach still suffer from some degree of look-ahead-bias. This occurs because the characteristics used are known to be correlated to the risk-return ratio given our knowledge from the entire sample. The use of these characteristics is indirect: in addition of the market excess return, three portfolios' returns are used as factors. The two first additional factors are the small-minus-big and high-minus-low portfolios described by Fama & French (1996). These factors attempt to exploit size and value anomalies. The third additional factor is the winner-minus-loser portfolio suggested by Carhart (1997). This factor attempt to exploit the 1-year momentum anomaly.

The inclusion of transaction costs aims to penalize turnover. Commonly, an active technique which supposedly performs better than a naive approach such as value-weighted portfolio without considering transaction costs depends on excessive turnover and the absence of transaction costs. Considering transaction costs favors lower turnover approaches. In order to deal with transaction costs, we use the approach described in Section 1.3.3.

We also test the performance of the technique by assuming 2 regimes, with distinct factor loadings and risk premia for each regime. The procedure is described in Section 1.3.4

1.3.1 Utility function

The utility function the investor maximizes is the ex-ante Sharpe Ratio. We choose the Sharpe Ratio because the investable set is restricted to ordinary stocks and excludes bonds, commodities and other asset classes. Therefore, the investor cannot allocate his wealth optimally given his risk aversion. The Sharpe Ratio utility function is useful to enable the search of the better risk-return ratio without considering risk aversion. In a second step, the optimal stocks portfolio could be mixed with a risk-free bond in order to define the optimal risk level. Besides, the Markowitz optimized portfolio with greater Sharpe Ratio is the portfolio which generates the mean-variance efficient frontier in the presence of a risk-free asset. Finally, the technique we propose can be easily extended to include other asset types.

The maximization of the Sharpe Ratio in the Markowitz approach is straightforward. The portfolio depends upon the expected returns and covariances and the target expected return. For each target expected return, there is an optimal portfolio with expected return and volatility. Therefore, we choose the target expected return which maximizes the Sharpe Ratio, following the procedure described in 1.2.2.

1.3.2 Out-of-sample procedure

In order to prevent look-ahead bias in the estimation of parameters, volatilities, coefficients and risk premia associated to each factor, we employ a growing estimation window.

Factor loadings, risk premia, residual volatility and factor covariances are re-estimated for each period t using all information available at period $t-1$. Besides, the factor portfolios are created by following the procedure described by Fama & French (1993a) and Carhart (1997). Therefore, the definition of the set of stocks in each group (small, high, winner, etc) does not require any information unavailable by the moment the portfolio return is used.

1.3.3 Transaction costs

We describe here a technique to take trading costs into account while estimating optimal portfolios, from Brandt et al. (2009). The technique allows transaction costs to vary across stocks and through time. Transaction costs may impact differently the approaches to portfolio selection, even with equivalent average turnover.

Let the "hold portfolio" at period $t+1$ be defined as the portfolio resulted from keeping the stocks from period t. Let $\omega_{i,t-1}$, $r_{p,t}$, and $r_{i,t}$ be the portfolio weight on stock i at period $t-1$, the portfolio return from $t-1$ to t and the return from stock i from $t-1$ to t, respectively. The "hold portfolio" weight on stock i at period t is given by:

$$\omega_{i,t}^{h} = \omega_{i,t-1}\frac{1+r_{i,t}}{1+r_{p,t}} \tag{1-12}$$

Let $c_{i,t}$ be the estimated transaction costs of trading stock i at t. The portfolio return net of trading costs is the portfolio return less the absolute change on weight from the "hold portfolio" multiplied by its transaction costs:

$$r_{p,t+1} = \sum_{i=1}^{N} \omega_{i,t} r_{i,t+1} - c_{i,t}|\omega_{i,t} - \omega_{i,t}^{h}| \tag{1-13}$$

Several theoretical studies suggest the optimal strategy with transaction costs should consider a no-trade region, given current position (e.g. Leland (2003)). If the desired portfolio weights (hereafter called "target portfolio") is inside the no-trade region, it is optimal not to trade. The intuition of this result lies in the fact there is a first-order loss when trading inside the no-trade region and only a second-order gain. Motivated by these theoretical results, Brandt et al. (2009) propose a technique which model the no-trade region as an hypersphere and shrinks the target portfolio to the hold portfolio.

Let ω^{target} be the optimal portfolio obtained by the Markowitz approach, ω^{h} the weights from the "hold portfolio" and κ^2 the radius of the no-trade region. If $\sum_{i=1}^{N_t}(\omega_{i,t}^{target} - \omega_{i,t}^{h})^2/N_t <= \kappa^2$, the target portfolio is inside the no-trade region at t. Therefore, the optimal policy is keeping the current portfolio.

However, if $\sum_{i=1}^{N_t}(\omega_{i,t}^{target} - \omega_{i,t}^{h})^2/N_t > \kappa^2$, the optimal policy is change the weights towards the target portfolio, up to the no-trade region centered in the target weight:

$$\begin{aligned} \omega_{i,t} &= \alpha_t \omega_{i,t}^{h} + (1-\alpha_t)\omega_{i,t}^{target} \\ \alpha_t &= \frac{\kappa\sqrt{N_t}}{\sqrt{\sum_{i=1}^{N_t}(\omega_{i,t}^{target} - \omega_{i,t}^{h})^2/N_t}} \end{aligned} \tag{1-14}$$

This weighted average can be seen as shrinkage of the optimized portfolio to the hold portfolio.

Section 1.4.2 presents and justifies the values of $c_{i,t}$ and κ used.

1.3.4 Experiment assuming two regimes

As a first attempt to evaluate the effect of relaxing the assumption of constant factor loadings and risk premia over time, we divide the sample in two. The first part contains periods in which the yield curve is positively sloped, while the second part contains periods in which the yield curve is negatively sloped. All parameters, factor loadings, risk premia and covariances, and residual volatilities, are estimated separately for each part. This division is analogous to the procedure described in Brandt et al. (2009).

From the business cycle view, periods in which the yield curve is negatively sloped tend to be associated to recessions. Hence, correlations among stock characteristics and returns should vary if the parameters are estimated separately for each part. Likewise, the risk premia associated to each factor should be different across the two sample parts.

From the econometric view, it is important that each part contains periods across the entire sample. In particular, the sample used to estimate initial parameters should have periods from both parts.

From the portfolio manager view, a smoother transition would be a better approach. If the estimated values are different, the transition between regimes should generate great losses due to transaction costs. Besides, the transitions can be associated to lesser market liquidity. Hence, the transaction costs may be underestimated.

1.4 Empirical results

1.4.1 Data

Our sample consists of stocks listed in NYSE. We use monthly holding period returns, shares outstanding and closing prices from CRSP monthly database and quarterly data from Compustat to calculate the book equity. The sample period is from June 1970 to December 2008. Before this period, there was no quarterly data available for more than 20 companies with stocks listed in NYSE. After exclusions, the number of valid stocks varies through the sample, ranging from 520 to 714. We exclude from the sample:

1. Stocks with an asset code different from 10 or 11, according to CRSP database[3];

2. Stocks from a company not listed in Compustat;

3. Stocks from companies with negative book-to-market at any moment during the sample period.

Removing stocks from companies with negative book-to-market generates a quality bias in the sample. Therefore, a naive value-weighted strategy over this sample should outperform an index like the S&P500. However, the

[3]This excludes certificates, ADRs, SBIs, Units, companies incorporated outside the U.S., closed-end funds and REIT's.

S&P500 performs better than the value-weighted portfolio from January 81 to December 2008: the annualized values for average return, volatility and Sharpe ratio are 0.109, 0.151, and 0, 365, respectively, surpassing the Sharpe ratio from the value-weighted portfolio, which is 0.357.

Figure 1.4.1 shows the accumulated return for value-weighted and equal-weighted portfolios using the stocks in the sample. Furthermore, the mean, standard deviation and skewness of the value-weighted portfolio are 0.87%, 0.041, and −0.855, respectively. Finally, the mean, standard deviation and skewness of the equal-weighted portfolio are 1.10%, 0.046, and −1.275, respectively.

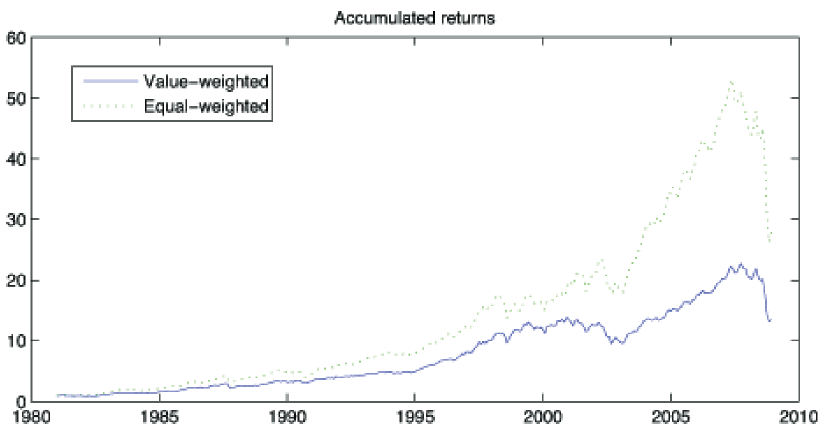

Figure 1.1: *Accumulated return for value-weighted and equal-weighted portfolios*

For each month, the book equity of a company is (in parenthesis, the field code in Compustat): total assets (ATQ) minus liabilities (LTQ) plus balance-sheet deferred taxes and investment tax credits (TXDITCQ), minus preferred stock value (PSTKQ); the market value of a stock is shares outstanding times closing price; the market equity of a company is the market value of all stocks in CRSP from the same company. Finally, the book-to-market of a company is the log of 1 plus book equity divided by market equity.

As risk-free rate, we use the 3-month Treasury bill secondary market rate from FRED database. From the same source, we compute the yield slope as the difference between market yield on U.S. Treasury securities at 10 and 1 year.

Finally, Table 1.1 presents the correlations among monthly returns from each factor-mimicking portfolio. The largest correlation in absolute value is between HML and WML portfolios: −0.28.

Table 1.1: Correlation matrix

This table presents correlations among the factor portfolios. The correlation is over the entire sample.

	M	SMB	HML
SMB	0.09		
HML	-0.24	-0.01	
WML	-0.17	-0.18	-0.28

1.4.2 Transaction costs

Our modelling of the transaction costs $c_{i,t}$ for stock i at period t follow the approach used by Brandt et al. (2009), which attempts to capture two empirical facts. First, that transaction costs vary across distinct stocks, being larger for small caps than for large caps. Second, the decrease in transaction costs over time. Among others, these results can be found in Domowitz et al. (2001).

The transaction costs $c_{i,t}$ are modelled as:

$$c_{i,t} = 0.006 - 0.0025 \times ME_{i,t} \times T_t \tag{1-15}$$

where $ME_{i,t}$ is the log of the market value of the stock normalized to the interval $[0, 1]$ and T_t captures declining costs over time. In the first month of the sample, $T_t = 2$and it decreases linearly over each month, until $T_t = 1$ at the last month of the sample[4]. As an example, the stock of lesser market value at January, 74, has transaction costs of 1.2% at the same month. Likewise, the stock of greater value at December, 2008, has transaction costs of 0.35% at this month.

Our costs are lesser than the costs used in Brandt et al. (2009) for two reasons. First, our sample starts 10 years later. Second, since we use only NYSE stocks, transaction costs are supposed to be lesser than in a sample also containing stocks from AMEX and NASDAQ.

1.4.3 Results

In this Section, we present the performance of value-weighted and equal-weighted portfolios, as well as four portfolios from the Markowitz optimization approach with the following expected return models:

1. Factor loadings estimated from Equation (1-9) (M1);
2. Factor loadings estimated from Equation (1-9), imposing $A = 0$ (M2);
3. Factor loadings estimated from Equation (1-10) (M3);

[4]We also test $T_1 = 3$ and achieve similar results.

4. Factor loadings estimated from Equation (1-10), imposing $A = 0$ (M4);

All returns are net of transaction costs. We use the procedure to handle transaction costs described in Section 1.3.3 in all portfolios except the value-weighted and equal-weighted. The leverage is limited to a short position of 30% of the investor's wealth.

Table 1.2 presents statistics for the value-weighted and equal-weighted portfolios. The value-weighted portfolio has an annualized average return of 10.4% and a 14.3% volatility. The Sharpe Ratio is 0.362, and alpha, beta and residual volatility are −0.006, 1.000 and 0.011, respectively. Alpha, beta and residual volatility are measured against the returns from the value weighted portfolio without imposing transaction costs. The largest average position on a stock is 4.6% and the average monthly turnover is 0.2% of the investor's wealth. The equal-weight portfolio has greater returns 13.2% and volatility 15.8%. The Sharpe Ratio is 0.506 and the average monthly turnover is 7% of the investor's wealth.

Table 1.2 also presents the portfolios generated by the Markowitz optimization, using 4 distinct versions of the 4-factor model. The average returns range from 10.9% to 12.6%, the volatility range from 13.1% to 13.6%. Excluding the M1 model,which performs worse than the others, the Sharpe Ratio is close to 0.525. The average monthly turnover is approximately 10% of the investor's wealth. All portfolios have positive alpha, the beta is close to 0.9 and the residual volatility ranges from 0.013 to 0.056. All portfolios perform better than the value-weighted portfolio[5]. Excluding the M1 model, the Markowitz portfolios also perform better than the equal-weighted portfolio.

The imposition of $A = 0$ improves the performance when the factor loadings are estimated directly from the stocks returns. This suggests the abnormal returns do not persist and reduce return forecasting power. There is no impact when the factor loadings are estimated from portfolios. In this case, the intercepts are already very close to 0 and the adjustment does not affect performance.

1.4.4 Economic value

By following a procedure similar to Fleming et al. (2001), we estimate the economic value of the Markowitz optimized portfolios for a Sharpe Ratio maximizer investor.

For a given portfolio p, we take the average and standard deviation of monthly returns over each year i: $\bar{r}_i^p$ and std_i^p. The economic value Δ_p of

[5] If we reduce the limit on leverage to 100% of the investor's wealth, all portfolios perform worse and the average short position goes from 58% to 87%.

Table 1.2: Naive and Markowitz portfolios performance

This table presents statistics from six portfolios, considering transaction costs. All statistics are from January, 81 to December, 2008. The columns correspond to value-weighted and equal-weighted portfolios, followed by Markowitz optimized portfolios using return models M1, M2, M3 and M4, respectively. The first set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The second set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs. The returns are net of transaction costs. The third set of rows presents average characteristics of the portfolio over time. The rows labelled "me", "mom" and "btm" show measures of market value, momentum and book-to-market, respectively.

Variable	VW	EW	M1	M2	M3	M4
$\lvert\omega_i\rvert \times 100$	0.107	0.158	0.235	0.160	0.171	0.142
$\max \omega_i \times 100$	4.665	0.187	4.113	1.747	3.792	1.971
$\min \omega_i \times 100$	0.001	0.142	0.016	0.006	0.004	0.002
$\sum \omega_i I(\omega_i < 0)$	0	0	-0.180	-0.121	-0.111	-0.044
$\sum I(\omega_i \leq 0)/N$	-	-	0.197	0.182	0.133	0.094
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.011	0.070	0.116	0.099	0.094	0.091
$\bar{r}$	0.104	0.132	0.109	0.125	0.121	0.126
$\sigma(r)$	0.143	0.158	0.133	0.136	0.131	0.139
SR	0.357	0.506	0.426	0.531	0.527	0.527
α	-0.007	0.020	0.006	0.023	0.020	0.020
β	1.000	1.024	0.873	0.862	0.850	0.904
$\sigma(\epsilon)$	0.002	0.056	0.043	0.055	0.047	0.049
me	1.297	0	0.507	0.115	0.175	0.045
mom	0.107	0	0.369	0.030	0.017	0.001
btm	-0.437	0	-0.396	0.223	0.197	0.152

portfolio p is given by:

$$\sum_{i=1}^{n} \frac{\bar{r}_i^p - \Delta_p}{std_i^p} = \sum_{i=1}^{n} \frac{\bar{r}_i^b}{std_i^b} \qquad (1\text{-}16)$$

where n, $\bar{r}_i^b$, and std_i^b are the number of years and the average and standard deviation of the benchmark monthly returns, respectively. The benchmark is the value-weighted portfolio in the presence of transaction costs. The value of Δ_p represents the amount a Sharpe Ratio maximizer investor is willing to pay to be indifferent between portfolio p and the benchmark.

Table 1.3 shows in the first column the economic value for several portfolios optimized using the Markowitz approach. The relative performance equals the relative performance according to the Sharpe Ratio. A Sharpe Ratio maximizer investor would be willing to pay more than 200 basis points to hold Markowitz optimized portfolios using return models M2, M3 or M4. These results suggest the multi-factor model of stocks covariance has relevant economic value.

We also present the values obtained by applying the mean-variance utility used in Fleming et al. (2001). In this case, the economic value Δ_p of portfolio

Table 1.3: The economic value of Markowitz optimized portfolios

This table presents the economic value (in basis points) according to three criteria of four distinct portfolios. The first column is the economic value for a investor with expected utility function given by the Sharpe ratio. The next two columns consider an investor with mean-variance utility and distinct values of risk aversion parameter λ.

Portfolio	Sharpe-ratio (basis points)	$\lambda = 1$ (basis points)	$\lambda = 10$ (basis points)
Equal-weighted	160	264	54
Markowitz (M1 model)	24	68	218
Markowitz (M2 model)	259	220	328
Markowitz (M3 model)	257	197	365
Markowitz (M4 model)	265	227	292

p is given by:

$$\sum_{t=1}^{T}(1+r_{p,t}-\Delta_p)-\frac{\lambda}{2\times(1+\lambda)}(1+r_{p,t}-\Delta_p)^2=\sum_{t=1}^{T}(1+r_{b,t})-\frac{\lambda}{2\times(1+\lambda)}(1+r_{b,t})^2 \quad (1\text{-}17)$$

where $r_{p,t}$ is the monthly return of portfolio p at t,$r_{b,t}$ is the monthly return of the benchmark at t and λ is a measure of risk aversion.

As the last two columns of Table 1.3 show, all portfolios have a positive economic value. Furthermore, investors with greater risk aversion prefer the Markowitz optimized portfolios (200 to 370 basis points of economic value) over the equal-weighted portfolio.

1.4.5 Results: segmentation by bull and bear years

Table 1.4 shows the segmented performance of value-weighted, equal-weighted and Markowitz optimized portfolios using models M2 and M4. In order to segment the performance, we divide the sample in two: bull and bear years. We classify bull years as years in which the benchmark return is greater than the risk-free rate. Conversely, bear years are years in which the benchmark return is lesser than the risk-free rate. The sample contains nine bear years.

In general, the statistics related to weight distribution are the same across the two sub-samples. Besides, bear years tend to be associated to a greater volatility. The results show that the portfolios perform better than the benchmark both in bull and bear years. M2 has a negative alpha (-0.004), but it is greater than the value-weighted's alpha in presence of transaction costs (-0.009). The main point of this segmentation is to show that the superior performance of the Markowitz portfolios do not depend on the fact the sample contains much more bull years. Even if we consider a random sub-sample containing the same number of bull and bear years, these portfolios would perform better than the market.

Table 1.4: Naive and Markowitz portfolios performance segmented by bull and bear years

This table presents statistics from six portfolios, considering transaction costs. All statistics are from January, 81 to December, 2008. The columns correspond to value-weighted and equal-weighted portfolios, followed by Markowitz optimized portfolios using return models M2 and M4, respectively. The first set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The second set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs. The returns are net of transaction costs. We classify bull years as years in which the benchmark return is greater than the risk-free rate. Conversely, bear years are years in which the benchmark return is lesser than the risk-free rate. The sample contains nine bear years.

Variable	VW	EW	M2	M4
	Bull years			
$\lvert\omega_i\rvert \times 100$	0.109	0.158	0.158	0.141
$\max \omega_i \times 100$	4.482	0.184	2.151	2.300
$\min \omega_i \times 100$	0.001	0.141	0.005	0.002
$\sum \omega_i I(\omega_i < 0)$	0	0	-0.107	-0.042
$\sum I(\omega_i \leq 0)/N$	-	-	0.171	0.070
$\bar{r}$	0.187	0.209	0.209	0.202
$\sigma(r)$	0.116	0.123	0.109	0.110
SR	1.171	1.276	1.437	1.375
α	-0.006	0.017	0.037	0.026
β	1.000	0.989	0.840	0.877
$\sigma(\epsilon)$	0.001	0.045	0.048	0.041
	Bear years			
$\lvert\omega_i\rvert \times 100$	0.112	0.157	0.167	0.142
$\max \omega_i \times 100$	5.053	0.187	1.526	1.890
$\min \omega_i \times 100$	0.002	0.141	0.003	0.003
$\sum \omega_i I(\omega_i < 0)$	0	0	-0.150	-0.049
$\sum I(\omega_i \leq 0)/N$	-	-	0.206	0.084
$\bar{r}$	-0.072	-0.029	-0.052	-0.035
$\sigma(r)$	0.179	0.205	0.169	0.178
SR	-0.718	-0.416	-0.637	-0.512
α	-0.009	0.043	-0.004	0.020
β	0.998	1.068	0.863	0.927
$\sigma(\epsilon)$	0.004	0.074	0.065	0.063

Table 1.5: Two regimes

This table presents statistics from two portfolios, considering transaction costs and following the procedure to allow 2 regimes, as described in Section 1.3.4. The column labelled "M3" shows statistics for the Markowitz (M3 return model). All statistics are from January, 81 to December, 2008. The first set of rows shows statistics of the portfolio weights, averaged across time. These statistics are: average absolute weight, maximum and minimum portfolio weight, the average sum of negative weights in the portfolio, average fraction of non-positive weights in the portfolio, and average turnover. The second set of rows displays annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs. The third set of rows presents average characteristics of the portfolio over time. The rows labelled "me", "mom" and "btm" show measures of market value, momentum and book-to-market, respectively.

Variable	M3
$\lvert\omega_i\rvert \times 100$	0.210
$\max \omega_i \times 100$	6.617
$\min \omega_i \times 100$	0.006
$\sum \omega_i I(\omega_i < 0)$	-0.169
$\sum I(\omega_i \leq 0)/N$	0.201
$\sum \lvert\omega_{i,t} - \omega_{i,t}^h\rvert$	0.202
$\bar{r}$	0.112
$\sigma(r)$	0.146
SR	0.403
α	0.004
β	0.948
$\sigma(\epsilon)$	0.053
me	0.104
mom	0.136
btm	0.119

1.4.6 Results: two regimes

Table 1.5 presents statistics for portfolios obtained from the Markowitz optimization, using the "M3" model and following the 2-regimes strategy described in 1.3.4. The portfolio's performance is far below the obtained without 2 regimes: the Sharpe Ratio falls from 0.527 to 0.403 when applying 2 regimes to the M3 model. The performance loss comes from both average returns and volatility. The results suggest the several discrete switches between regimes generate excessive transaction costs.

Table 1.6 presents the returns statistics during the positive slope and inverted regimes.

1.5 Conclusions

This paper reaches three main results. First, the covariance structure of stocks has positive economic value, that is, an investor with expected utility given by the Sharpe Ratio (we also test risk-averse quadratic utility) would be willing to spend basis points in order to exchange value-weighted portfolio by the portfolios optimized by the adjusted Markowitz approach we propose,

Table 1.6: Two regimes - results segmented by regime

This table presents statistics from two portfolios over the positive slope and inverted slope regimes, considering transaction costs and following the procedure to allow 2 regimes, as described in Section 1.3.4. All statistics are from January, 81 to December, 2008.The rows display annualized portfolio return statistics: average return, standard deviation and Sharpe Ratio of returns, and the alpha, beta, and volatility of idiosyncratic shocks of a market model regression. The returns are net of transaction costs.

Variable	Positive Slope	Inverted Slope
$\bar{r}$	0.104	0.158
$\sigma(r)$	0.153	0.086
SR	0.367	0.905
α	-0.003	0.053
β	0.977	0.613
$\sigma(\epsilon)$	0.051	0.055

even with no look-ahead bias and in the presence of transaction costs. The adjustments we include are fundamental to improve the performance and beat the benchmark. Second, the "smooth" estimation approach we propose to estimate factor loadings seems to generate better results than the traditional estimation. In particular, there is no difference between results allowing or not abnormal returns (returns not explained by the factors). Third, the "no-trade" adjustment to handle transaction costs is successful to reduce the monthly turnover to approximately 9%, providing a good balance between minimizing losses due to transaction costs and adjusting the portfolio towards weights with better risk-adjusted expected returns. These good results do not depend on the fact the sample has more bull years than bear years. The optimized portfolios tend to perform better than the benchmark in both periods. The 2-regimes experiment does not yield proper results and another approach should be used to attempt to exploit time-varying investment opportunities.

Further improvements and insights could be achieved by an experiment attempting to separate the effects of cross-sectional variation in returns and the covariance structure. As mentioned previously, an approach to exploit successfully time-varying investment opportunities could be another source of improvement. Finally, the approach could be extended to include additional asset classes.