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# Conclusions

This thesis is composed by four working papers with contributions to multistage risk averse stochastic programming models applied to financial problems. We develop different ways of modeling risk aversion which included risk measures, utility functions, direct and indirect penalties. Moreover, we develop contributions on modeling as well as a solution methodology for each one of the considered applications.

In the first paper, we developed a suitable economic interpretation for a particular set of risk-averse dynamic problems based on a recursive objective function. We prove that the objective function is the certainty equivalent with respect to the time consistent dynamic utility function defined as the composed form of one period preference functionals. We also prove that this objective is the composed form of certainty equivalents with respect to these one period preference functionals. This result gives us the intuition that, at stage $t$, the agent is maximizing the certainty equivalent of the portfolio value with respect to a given one-period preference functional. In addition, we developed a time consistent dynamic stochastic programming model for portfolio selection in which the objective function is a recursive setting of a convex combination between expectation and (negative of) CVaR applied to terminal wealth. We motivated our modeling choice using a numerical example to show some practical consequences of a time inconsistent policy and to compare the optimal solution to our time consistent alternative. We conclude that the first stage decisions might be suboptimal if an investor considers future planned decisions that are not actually going to be implemented. We also verify that time inconsistent policies may not take risk aversion into account at some intermediate states of the system. Finally, we illustrated our economic interpretation with a numerical example and we state that our model maximizes the certainty equivalent of the investor, given his / her one period preference functional.

In the second paper, we proposed an optimization model for portfolio and leverage selection considering the actual borrowing cost function faced by investors. Indeed, we considered a convex piecewise linear function whose slopes and segment sizes can be interpreted as the borrowing rates and credit limit of each available lender. We showed with a numerical example that the usual linear approximation for the borrowing cost function leads to suboptimal solution with a significant gap when compared to our proposed model. Moreover, we developed a multistage extension whose policy, for stage-wise independent returns and proportional credit limits, is myopic and can be easily obtained by solving a two-stage model.

In the third paper, we proposed a multistage linear stochastic programming model for optimal bond issuance of a firm considering fixed and floating interest rate bonds with different maturities and amortization patterns. We proposed an approximation for a numerically intractable long term multistage problem. We assume a hybrid model where the first $T^*$ stages are represented by a full event tree and the remainder described by subsamples approximated by independent paths. For the first period, we considered a full bond portfolio, while the simplified period considers only short term debt. Nonetheless suboptimal, we argue that our solution is a good approximation when most of the expenses are due over the first stages. Moreover, we proposed a objective function as the expected utility function that minimizes the cost of funding and penalizes negative values for the cash account at the terminal stage. In addition, we include intermediate excess of leverage penalties considering the market values of assets and debt. To do so, we develop valuation methods within the stochastic programming and considering a convex piecewise linear penalty with break points related to threshold levels of leverage. We examined the behavior of the model in an illustrative example that evidences the importance of this penalty function to introduce an appropriate risk aversion to the model.

In the forth paper, we developed a multistage stochastic program for optimal cash holding policy of a firm. As additional decisions variables, we considered production, investment and borrowing. As contributions to the stochastic programming literature, we developed a methodology to mitigate the end-effects and build a deterministic policy rule that approximates the optimal solution of the original stochastic problem. Indeed, we considered a infinite horizon problem whose truncated version gave unreliable solutions. Said so, extended the deterministic dual equilibrium methodology to a multistage stochastic model mitigating the end-effects of the truncated version, obtaining more reliable solutions to the original problem. Moreover, we developed and validated a policy rule by comparing the suboptimality gap on objective function. This policy rule was based on the regime switching model developed for the considered risk factors.