5
Corporate Cash Holding Policy with an Application in the Agribusiness Sector

Classical economic theory suggests that high amounts of cash in a corporation can be detrimental to shareholder value. Cash savings can be better invested in high return assets, hence corporate cash holding is costly to the shareholders. Indeed, excess cash is usually invested in a risk-free asset whose return to the shareholders is often lower than the risk-free interest rate due to the level of corporate taxes. The agency conflict proposed by Jensen & Meckling (JENSEN; MECKLING, 1976) highlights another disadvantage of excess corporate cash holding. Managers of the firm may not have the same value-maximizing goal as the shareholders of the firm. A high level of excess cash allows, and may even motivate, managers to invest in negative net present value projects due to the manager's objective to achieve a certain asset growth rate. Jensen & Meckling (JENSEN; MECKLING, 1976) proposes that to minimize this agency problem, it is better for corporations to engage in the capital markets as funding needs arise instead of hoarding internal cash in anticipation of future funding demands.

On the other hand, Fazzari et al. (FAZZARI et al., 1988) and Froot et al. (FROOT; SCHARFSTEIN; STEIN, 1993) argue that external funding costs may be too large to bear in the face of capital market imperfections. Hence, it may be in the firm's best interest to ensure that it has sufficient internal funding for all investment demands. Otherwise, the firm may have to reduce its investments or incur in expensive financing costs. Corporations hold cash to maintain the ability to finance investments even when current cash flow is insufficient to meet the investment demands. Thus, the availability of cash or corporate liquidity may impact firm value through its investment policy.

In spite of arguments against firms keeping cash reserves, Bates (BATES; KAHLE; STULZ, 2006) reports that the average cash-to-assets ratio of U.S. public companies has risen from 10% in 1980 to 24% in 2004. More recent studies on corporate cash holding seem to suggest that there are reasonable cause and benefit to holding cash. Companies that have a large cash reserve, like IBM, have been able to snap up smaller companies at the cheap during market downturns. Using a sample of publicly-traded manufacturing firms over the 1981-2000 period, Almeida et al. (ALMEIDA; CAMPELLO; WEISBACH, 2002) found that financially constrained firms tend to save a positive fraction of their cash inflow. Opler (OPLER et al., 1999) examine the determinants of corporate cash holding and finds a link between growth and excess cash, arguing that cash-rich firms have more growth opportunities. Moreover, Simutin (SIMUTIN, 2010) and Palazzo (PALAZZO, 2008) find that firms with a high excess cash level generate significantly higher stock returns compared to those with low excess cash. On the other hand, Harford (HARFORD, 1999) and Lie (LIE, 2002) report that cash-rich firms tend to make more value-decreasing investments, nonetheless Mikkelson (MIKKELSON; PARTCH, 2003) find that a persistent large cash holding does not hinder the firm's operating performance.

Even though there are many empirical evidence on the importance of corporate cash saving, there are few papers that provide a model to explain the cash holding behavior of firms. We would like to understand further the benefits to cash holding in the face of a stochastic cost of investment and cost of external financing under a regime-switching framework. To do so, we present a multistage model of a firm's cash holding, production, investment, financing, and dividend payout activities. Our model differs from the two mentioned above in its inclusion of production decision and regime-dependent risk factors. We argue that there is a benefit to cash hoarding during good economic times in order to facilitate investment spending during bad economic times. Stochastic programming is our chosen approach due to its modeling flexibility.

5.1 The Model

We consider a firm seeking to maximize the expected utility of its payout to shareholders over the planning horizon. The firm achieves its objective by controlling its cash holding, dividend payout, investment, production and financing policies at time $t \in \{0, \dots, T\}$. Also, assume that the firm has two sources of funding: internal cash savings and a single period debt. Let us now discuss the model assumptions in more detail.

5.1.1 Model Notation

Sets

- $\mathcal{H} = \{0, \dots, T\}$: Set of stages.
- $\mathcal{S} = \{1, \dots, S\}$: Set of scenarios.

Deterministic Parameters

- C_0: Initial amount in cash account at the beginning of the planning horizon, $t = 0$.
- K_0: Initial cash amount of capital at the beginning of the planning horizon, $t = 0$.
- ϕ: Fixed cost as a proportion of capital.
- δ: Depreciation rate of capital.
- λ: Penalty for terminal debt.
- ν: Multiplier to account for the carrying cost of cash.

Risk Factors

- $\tilde{r}_t\,(s)$: Short term nominal risk-free interest rate between $t-1$ and t under scenario s, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $\tilde{r}_{d,t}(s)$: Borrowing rate for a single-period debt issued at $t-1$, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $\tilde{c}_t(s)$: Unit cost of production at time t, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $\tilde{p}_t(s)$: Unit sale price of product at time t, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $\tilde{w}_t(s)$: Investment cost multiplier at time t, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $\tilde{\gamma}_t(s)$: Borrowing limit as a percentage of asset value at time $t\,\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.

Decision Variables

- $C_t(s)$: Amount of cash holding at t under scenario s, $\forall\, t \in \mathcal{H}$.
- $K_t(s)$: Value of capital at time t under scenario s, $\forall i \in \mathcal{N}$, $\forall t \in \mathcal{H}$.
- $D_t(s)$: Amount of debt borrowed at time t under scenario s, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $Q_t(s)$: Production quantity at time t under scenario s, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $I_t(s)$: Additional capital investments at time t under scenario s, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.
- $E_t(s)$: Dividend payout to shareholders at time t under scenario s, $\forall\, t \in \mathcal{H}$, $s \in \mathcal{S}$.

Other Definitions

- $f(K_t(s))$: Production capacity function.

Production

The production quantity during period t, Q_t, is decided at the beginning of period t and the revenue from selling those products is realized at the end of the period. We also assume that the firm can always sell every unit of production at the prevailing market sales price $\tilde{p}_t$. Moreover, the production quantity during period t is constrained by the production capacity function $f(K_t(s))$, which depends on the level of existing capital at time t, $K_t(s)$.

$$Q_t(s) \leq f(K_t(s)),$$

where $f(K_t(s))$ is an increasing and concave function.

Financing

The model considers only single-period debt, that must be paid at the end of every period. The firm borrows at the prevailing market rate of borrowing. The borrowing rate $\tilde{r}_{d,t}$ over period t is stochastic and depends on the economic regime during the period. Negative borrowing or lending is not allowed, $D_t \geq 0$. Our historical data shows that the borrowing rate is highest during market busts and lowest during market booms. This will motivate the firm to save cash and borrow more during market booms than during market busts.

Investments

Investing activity increases the firm's level of capital expressed by $K_t = (1-\delta)K_{t-1} + I_t$. An increase in the level of capital in turn increases the firm's capacity of production. The unit cost of investment is stochastic and equal to $\tilde{w}_t I_t$. We can consider the cost of investment as the cost of buying new machinery or building a new plant and even acquiring another firm that has an identical line of business as the firm in focus. Then, we expect the cost of investment to be highest during market booms and lowest during market busts. Therefore, the investment cost $\tilde{w}_t$ should also be dependent on the economic regime during period t.

Dividend Payout

The firm can also choose to payout dividends to its shareholders at the end of period t. The amount of dividend, E_t, must be non-negative because we are not considering equity issuance.

Assuming that the managers' objective is aligned with the shareholders', then firm's objective is to maximize the utility of its shareholders. Let

$\mathbf{C}, \mathbf{Q}, \mathbf{I}, \mathbf{K}, \mathbf{D}, \mathbf{E}$ denote the vectors of cash, production quantity, investments, capital, borrowing, and dividends, respectively, over time. Denoting the scenario in our scenario tree with s, we write the stochastic program of the firm's problem as the following.

$$
\begin{aligned}
\underset{\mathbf{C},\mathbf{Q},\mathbf{I},\mathbf{K},\mathbf{D},\mathbf{E}}{\text{Maximize}} \quad & \frac{1}{S}\sum_{s\in\mathcal{S}} U(\mathbf{E}(\mathbf{s}), \mathbf{D}(\mathbf{s}), \tilde{\mathbf{r}}(\mathbf{s})) \\
\text{subject to} \quad & C_1(s) + E_1(s) = C_0 - Q_1(s)c_1(s) - \phi K_0 && \text{(5-1)} \\
& \qquad - w_1(s)I_1(s) + D_1(s), \quad \forall s \in \mathcal{S} \\
& C_t(s) + E_t(s) = (1 + \tilde{r}_t(s))\, C_{t-1}(s) + Q_{t-1}(s)\tilde{p}_t(s) && \text{(5-2)} \\
& \qquad - Q_t(s)\tilde{c}_t(s) - \phi(1-\delta)K_{t-1} - \tilde{w}_t(s)I_t(s) + D_t(s) \\
& \qquad - (1 + \tilde{r}_{d,t}(s))\, D_{t-1}(s), \quad \forall t \in \mathcal{H}/\{1\}, s \in \mathcal{S} \\
& K_1(s) = K_0 + I_1(s), \quad \forall s \in \mathcal{S} && \text{(5-3)} \\
& K_t(s) = (1-\delta)K_{t-1}(s) + I_t(s), \quad \forall t \in \mathcal{H}/\{1\}, s \in \mathcal{S} && \text{(5-4)} \\
& Q_t(s) \leq f(K_t(s)), \quad \forall t \in \mathcal{H}, s \in \mathcal{S} && \text{(5-5)} \\
& I_t(s), Q_t(s), D_t(s), C_t(s), E_t(s) \geq 0, \quad t \in \mathcal{H}, s \in \mathcal{S} && \text{(5-6)} \\
& I_t(s) = I_t(s'), \quad Q_t(s) = Q_t(s'), \quad D_t(s) = D_t(s'), && \text{(5-7)} \\
& \qquad C_t(s) = C_t(s'), \quad E_t(s) = E_t(s'), \\
& \qquad \forall \text{ scenario } s \text{ that has the same past as } s' \text{ up to time } t.
\end{aligned}
$$

The first two equality constraints (5-1) and (5-2) are the budget or cash flow constraint to match the cash in-flow with the cash out-flow at each decision stage. The next set of constraints (5-3), (5-4), and (5-5) are the investment and production constraints. Finally, due to the tree structure of our scenarios, we need to include a set of non-anticipativity constraints (5-7). These constraints are in place to ensure that scenarios that have the same past have the same decision or solution.

5.1.2
On the End-Effect of Cash Holding Policy

Using the multistage stochastic programming technique to solve our cash holding policy problem presents us with the issue of choosing the appropriate planning horizon parameter. Essentially, we are taking an infinite horizon problem and solving only the finite horizon approximation of it. Hence, we have to be able to account for this end-effect properly as the solution to our program may change drastically depending on the treatment of the model past the planning horizon. In this section, we consider the impact of accounting for end-effects using the method presented by Grinold (GRINOLD, 1983).

The original multistage problem described above falls under the so-called truncation method by Grinold (GRINOLD, 1983). The truncation technique essentially assumes that the company ceases to exist past the planning horizon T and we would like to value the firm as a going-concern. Therefore, we choose to model all the activities of the firm during periods t where $t > T$ by aggregating the constraints for those periods, we call this the dual equilibrium method.

Cash Constraint

Assuming that the return on cash is equal to the risk-free rate, or $\nu = 1$, our cash constraint for the periods after the planning horizon T is summarized by the following equation.

$$\begin{aligned} E_{T+i} + C_{T+i} &= (1+\bar{r})C_{T+i-1} + Q_{T+i-1}\bar{p} - Q_{T+i}\bar{c} + D_{T+i} \\ &\quad -(1+\bar{r_d})D_{T+i-1} - I_{T+i}\bar{w} - \phi(1-\delta)K_{T+i-1} \qquad ,\forall i \geq 1 \end{aligned} \tag{5-8}$$

In our notation, $\bar{x}$, when x is a risk factor, denotes the long-run level of that risk factor. For example, $\bar{p}$ is the long-run unit sale price of the firm's product given the chosen price process. Let us now define $\alpha := \dfrac{1}{1+\bar{r}}$ and the following notation:

$$\begin{aligned} \bar{Q} &= \textstyle\sum_{i=1}^{\infty} \alpha^i Q_{T+i} \\ \bar{E} &= \textstyle\sum_{i=1}^{\infty} \alpha^i E_{T+i} \\ \bar{D} &= \textstyle\sum_{i=1}^{\infty} \alpha^i D_{T+i} \\ \bar{K} &= \textstyle\sum_{i=1}^{\infty} \alpha^i K_{T+i} \\ \bar{I} &= \textstyle\sum_{i=1}^{\infty} \alpha^i I_{T+i} \end{aligned} \tag{5-9}$$

We can rearrange the equation 5-8 and multiply through with α to obtain

$$\begin{aligned} C_{T+i-1} &= \alpha[E_{T+i} + C_{T+i} - Q_{T+i-1}\bar{p} + Q_{T+i}\bar{c} - D_{T+i} \\ &\quad +(1+\bar{r_d})D_{T+i-1} + I_{T+i}\bar{w} + \phi(1-\delta)K_{T+i-1}] \qquad ,\forall i \geq 1. \end{aligned} \tag{5-10}$$

Next, we can multiply through with α^{i-1} and add up all the constraints.

$$\begin{aligned} \textstyle\sum_{i=1}^{\infty} \alpha^{i-1} C_{T+i-1} &= \textstyle\sum_{i=1}^{\infty} \alpha^i [E_{T+i} + C_{T+i} - Q_{T+i-1}\bar{p} + Q_{T+i}\bar{c} - D_{T+i} \\ &\quad +(1+\bar{r_d})D_{T+i-1} + I_{T+i}\bar{w} + \phi(1-\delta)K_{T+i-1}] \\ C_T &= \bar{E} - \alpha(Q_T + \bar{Q})\bar{p} + \bar{Q}\bar{c} - \bar{D} + \alpha(1+\bar{r_d})(D_T + \bar{D}) \\ &\quad +\bar{I}\bar{w} + \alpha\phi(1-\delta)(K_T + \bar{K}) \end{aligned} \tag{5-11}$$

Production Constraint

The capital balance for $t > T$ is

$$K_{T+i} = (1-\delta)K_T + I_{T+i}, \quad \forall \text{i} \geq 1.$$

Multiplying the capital balance equations with α^i and then adding them up, we arrive at the following equation:

$$\begin{aligned}
\sum_{i=1}^{\infty} \alpha^i K_{T+i} &= \alpha(1-\delta)\sum_{i=1}^{\infty} \alpha^{i-1} K_{T+i-1} + \sum_{i=1}^{\infty} \alpha^i K_{T+i} \\
\bar{K} &= \alpha(1-\delta)(K_T + \bar{K}) + \bar{I} \\
K_T &= \frac{1}{(1-\delta)}(\bar{r}+\delta)\bar{K} - (1+\bar{r})\bar{I}
\end{aligned} \tag{5-12}$$

Let us now consider the production constraints. Assume that the production capacity function $f(K_t)$ can be approximated by a piecewise linear function. Define $f_j(K_t) = A_j K_t + B_j$, where A_j and B_j are constants. We can write

$$f_(K_t) = \min_j f_j(K_t).$$

Summing up the above constraints $Q_t \leq f(K_t)$ for $t \geq T$, we obtain

$$\begin{aligned}
\sum_{i=1}^{\infty} \alpha^i Q_{T+i} &\leq \sum_{i=1}^{\infty} \alpha^i f(K_{T+i}) \\
\bar{Q} &\leq \sum_{i=1}^{\infty} \alpha^i \min_j f_j(K_{T+i}) \\
\bar{Q} &\leq \min_j \sum_{i=1}^{\infty} \alpha^i f_j(K_{T+i}) \\
\bar{Q} &\leq \min_j \sum_{i=1}^{\infty} \alpha^i A_j K_{T+i} + B_j \\
\bar{Q} &\leq \min_j A_j \sum_{i=1}^{\infty} \alpha^i K_{T+i} + \frac{\alpha B_j}{1-\alpha^i} \\
\bar{Q} &\leq \min_j A_j \bar{K} + \frac{\alpha B_j}{1-\alpha^i}
\end{aligned} \tag{5-13}$$

Equivalently, the constraint can be written as

$$\bar{Q} \leq A_j \bar{K} + \frac{B_j}{\bar{r}}, \quad \forall j. \tag{5-14}$$

5.2
Scenario Generation: An Application in the Agribusiness Sector

Stochastic programming usually utilizes a tree structure to represent the set of future uncertainties. Each path that can be traced from the beginning node to the end node is considered as one scenario or realization of future uncertainties. An example of a two-period scenario tree with a branch structure of 3-2 is illustrated in figure 5.1. The sample tree has 3x2 scenarios and the red-colored path marks scenario 2.

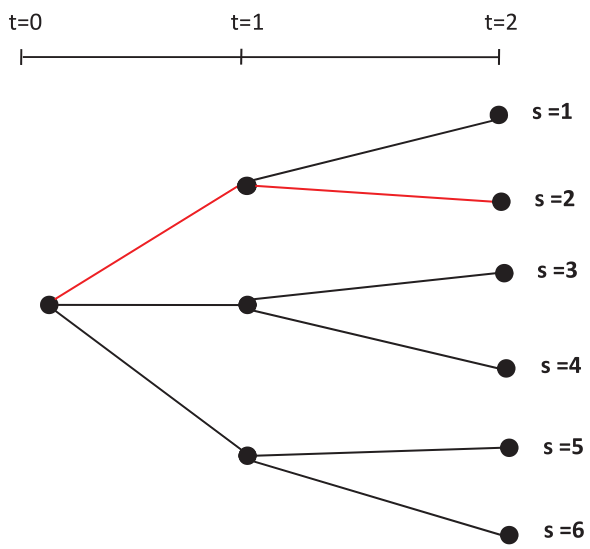


Figura 5.1: A scenario tree with a branch structure of 3-2, which means that there are 3 branches coming out of each node at $t = 0$ and 2 branches coming out of each node at $t = 1$.

Modeling future uncertainties is by no means an easy task. There are many mathematical models available that can be chosen to simulate different types of processes. Sometimes, however, only a limited data set is available. In this case, bootstrapping from the historical sample data may be the best approach. We use both approaches, calibrating a parametric model and bootstrapping from our sample data, in simulating future realizations of the risk factors.

In our model, the uncertainties arise from our risk factors: unit cost of production, unit sale price of production, unit cost of investment, and the borrowing spread. Because we believe that the processes of these factors are driven by the business cycle or economic regime, we also have one more source of uncertainty, which is the economic regime itself. In summary, we first simulate scenarios of the economic regime and then the rest of the risk factors are simulated from probability distributions that are dependent upon the given regime.

The agribusiness sector data is chosen as the case study for our model. Using a Hidden Markov Model, we are able to identify the sector's business cycles over our sample data. We then "cluster" the rest of our risk factors based on the identified regimes and calibrate each of the risk factor model the using the clustered data.

Hidden markov models (HMM) are used in modeling unobservable states that follow a markov process. The main assumption of a HMM is that, instead of observing the sequence of (hidden) states, one can only observe the output that has a probability distribution dependent on the state. A graphical illustration of the standard HMM is provided in figure 5.2.

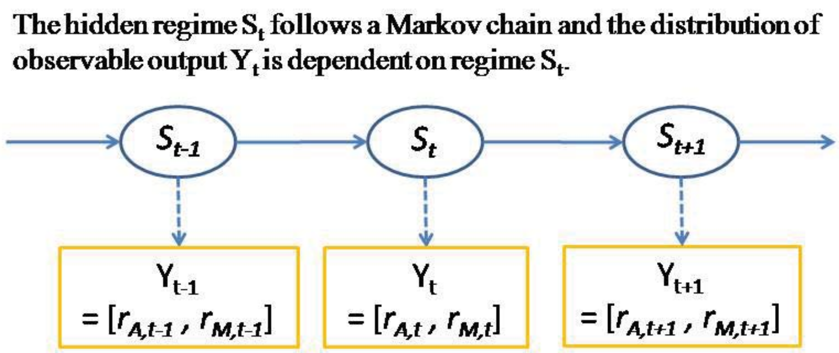


Figura 5.2: A graphical illustration of a Hidden Markov Model with hidden states S and output Y.

The model was first popularized by Rabiner (RABINER, 1989), who applied the HMM to a problem of speech recognition. In finance or economics, the hidden markov model is an important tool through its use in modeling business cycles. The earliest application of HMM to predicting business cycles using U.S. real GNP data was proposed by Hamilton (HAMILTON, 1989). A later work by Gregoir & Lenglart (GREGOIR; LENGLART, 2000) uses the HMM to measure the business cycle turning points of the French industry. Further discussions and improvements on this methodology can be found in Lahiri & Moore (LAHIRI; MOORE, 1992) and Koskinen & Oller (2003).

5.2.1

A Hidden Markov Model for the Agribusiness Sector Business Cycle

Suppose that the monthly returns of the S&P 500 and the cap-weighted Agribusiness Index (”Agri Index”) have a multivariate normal distribution that depends on the business cycle. We cannot observe the business cycle regimes directly, but we can observe the monthly returns of the equity market and the agribusiness sector in each month t, $Y(t) = (r_{A,t}, r_{M,t})'$. As such, we can model the returns following a Hidden Markov Model (”HMM”), where the business cycle regime is the latent variable and the monthly stock returns are the observed variables. Assume that there are S regimes and the regime in month t is denoted by the variable $S(t)$, $t \in \{1, ..., T\}$. We assume that the distribution of a particular month's returns depend upon that month's regime as follows:

$$Y(t)|S(t) = s \sim \mathcal{N}(\mu_s, C_s), \quad \forall s \in \{1, ..., K\}, \tag{5-15}$$

where μ_s denotes the monthly returns of the S&P 500 and the Agri Index at the end of month t, given $S(t) = s$, and C_s denotes the covariance matrix of the returns under state s.

The parameters of the assumed HMM model are calibrated to the historical returns of the S&P 500 and the Agri Index from January 1, 1990 to March 31, 2010. The number of states can be chosen using the BIC. A Hidden Markov Model with 3 regimes, $S = 3$, has the lowest BIC and provides an intuitive interpretation. The estimated HMM parameters are summarized in table 5.1. The top part of table 5.1 displays the transition probability matrix of the HMM. The element contained in row i and column j of this matrix shows us the probability of moving from state i to state j. The bottom part of table 5.1 displays the probability distributions of the returns, given the regime. The first column contains the estimate for $(\mu_{s,A}, \mu_{s,M})^T$, the second column displays the covariance matrix, and the third column shows the correlation coefficient estimate.

Interestingly there is zero probability of moving from regime 1 to regime 3 and of moving from regime 2 to regime 1. The transition matrix shows that in order to move from regime 1 to regime 3, one must first go through regime 2. Moreover, the emission distributions has positive expected returns and positive covariance in regime 1, pointing to a ”good”state of the world. The expected returns in regime 3, however, are both negative with also positive covariance, suggesting a ”bad”state of the world. Finally, regime 2 is the most unusual because of the negative covariance estimate between the two returns. Combining these observations together, we can interpret Regime 1 as an expansionary period, Regime 2 as a transition period, and Regime 3 as a recessionary period in the agribusiness sector.

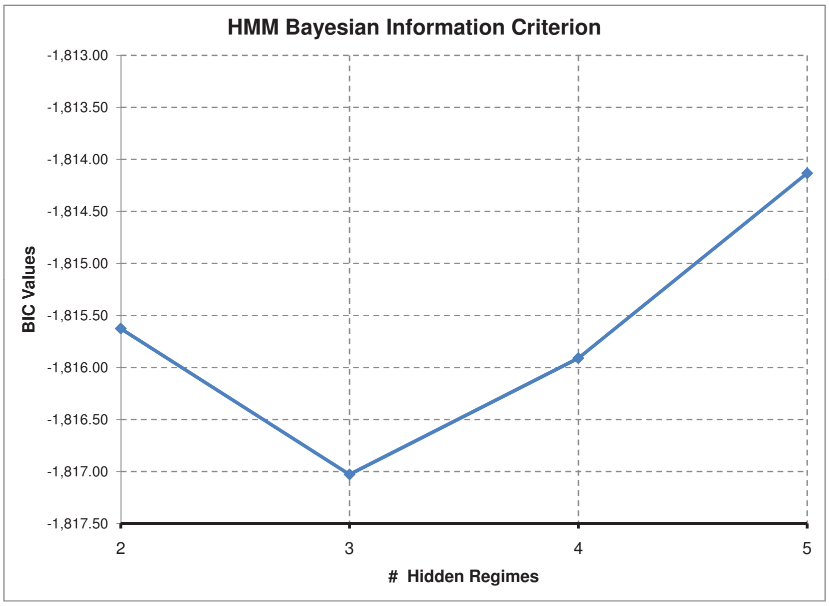


Figura 5.3: Bayesian Information Criterion used in determining the number of hidden states in the Hidden Markov Model.

The most probable sequence of regimes over the historical period, assuming our HMM parameter estimates are true, is then estimated using the Viterbi algorithm. Figure 5.4 plots the smoothing probabilities, $P_{S(t)|Y_1^T}(s|y_1^T)$, and figure 5.5 plots the color-coded total return of the agribusiness index to signify each regime. The Agri Index total returns has an increasing trend in those periods that are identified as expansionary and a decreasing trend in those months estimated as recessionary. It would not be desirable to have our regimes jump around too much and so, it is also important to note the persistence of the regimes. In other words, there is a relatively high probability of staying in one regime which is representative of the reality.

Transition Probability Matrix, $P(S(t)|S(t-1))$

	Regime 1	Regime 2	Regime 3
Regime 1	0.979	0.021	0
Regime 2	0	0.676	0.324
Regime 3	0.076	0.078	0.846

Emission distribution parameters

	Mean, μ_s	Covariance, C_s	Correlation
Regime 1	$\begin{bmatrix} 1.53\% \\ 1.50\% \end{bmatrix}$	$\begin{bmatrix} 0.11\% & 0.06\% \\ 0.06\% & 0.08\% \end{bmatrix}$	0.647
Regime 2	$\begin{bmatrix} 2.48\% \\ -1.59\% \end{bmatrix}$	$\begin{bmatrix} 0.25\% & -0.02\% \\ -0.02\% & 0.13\% \end{bmatrix}$	-0.139
Regime 3	$\begin{bmatrix} -1.22\% \\ -0.32\% \end{bmatrix}$	$\begin{bmatrix} 0.30\% & 0.25\% \\ 0.25\% & 0.51\% \end{bmatrix}$	0.629

Tabela 5.1: Regime Analysis: Model parameters estimated using S&P 500 and cap-weighted agribusiness index monthly returns from 1/1/1990 to 3/31/2010.

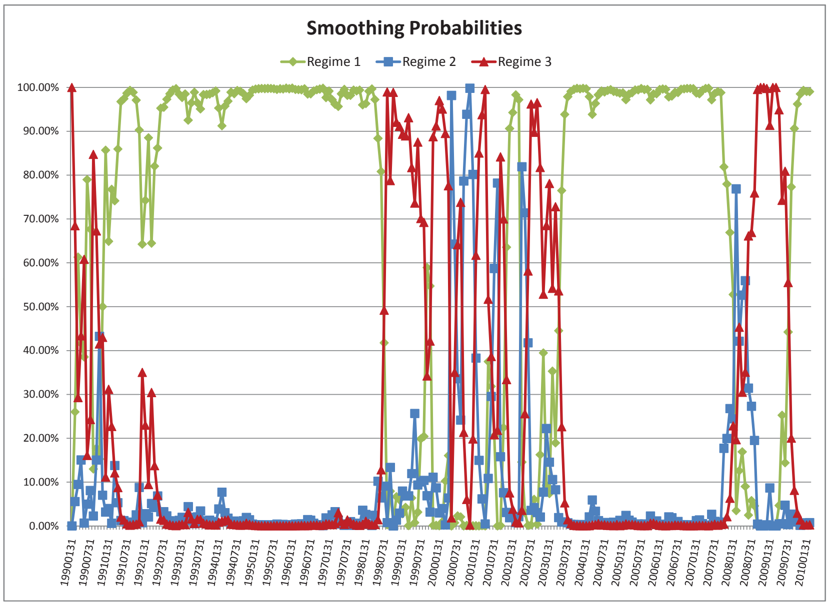


Figura 5.4: Smoothing probabilities.

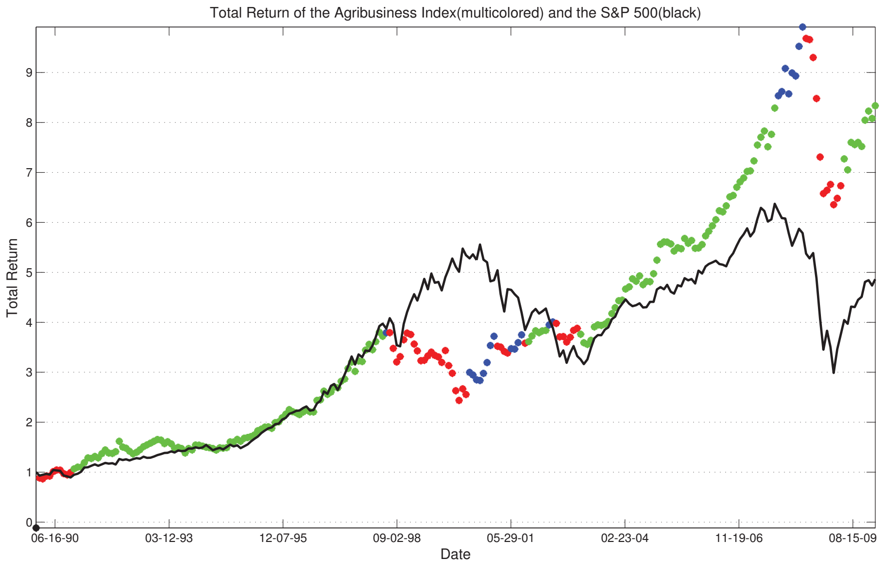


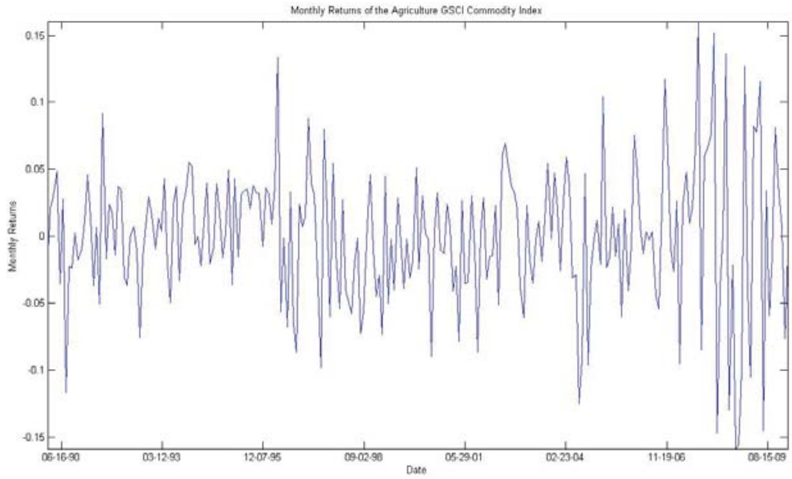
Figura 5.5: Total return of the S&P 500 and Agri Index with color-coded regimes.

5.2.2 Agriculture Commodity Index

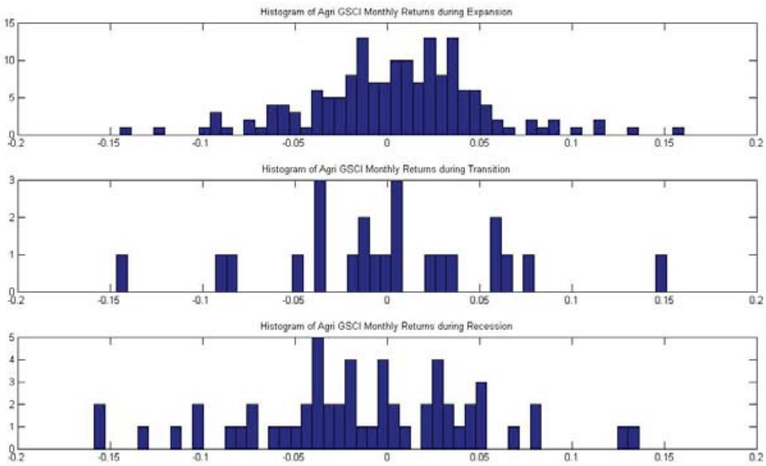
The typical company in the agribusiness sector produces a multitude of products using a wide variety of raw commodities. This fact adds a layer of complexity to the modeling of the agribusiness companies that we would like to avoid at this time. We believe that the simplified model would still provide the same qualitative results as the more complex one and choose the agriculture commodity index as a proxy to model the aggregate cost of raw materials used in production. This section details an analysis on the behavior of the chosen agriculture commodity index, namely the Goldman Sachs Commodity Index ("GSCI").

Some volatility clustering behavior are observed in the Agriculture GSCI's monthly returns from 1 January 1990 to 31 March 2010, as shown in figure 5.6. An Autoregressive Conditional Heteroskedasticity ("ARCH") test performed on the monthly return series rejects the null hypothesis that the coefficients to the ARCH components in an ARCH model is equal to 0.

As such, the agriculture GSCI return during month t, $\tilde{p}_t$, is modeled as a GARCH(1,1) process. The model is calibrated using the maximum likelihood



5.6(a): Volatility clustering behavior observed in the monthly returns.



5.6(b): Histogram of the monthly returns under each regime

Figura 5.6: The monthly returns of the Agriculture GSCI from 1/1/1990 to 3/31/2010 exhibit some volatility clustering behavior.

method and the coefficient estimated are summarized in table 5.2.

$$\tilde{p}_t = a_0 + \tilde{\epsilon}_t \quad (5\text{-}16)$$
$$\text{where } \tilde{\epsilon}_t = \sigma_t \tilde{z}_t,$$
$$\tilde{\sigma}_t = \alpha_0 + \alpha_1 \tilde{\epsilon}^2_{t-1} + \beta_1 \tilde{\sigma}^2_{t-1}, \text{ and}$$
$$\tilde{z}_t \sim \mathcal{N}(0,1).$$

5.2.3 The Cost of Borrowing

We choose the spread between the Barclays Aggregate AAA US Corporate borrowing rate and the 3-month US T-Bill rate on the secondary market

	Regime 1 Expansion	Regime 2 Transition	Regime 3 Recession	All Data
a_0	0.0050	-0.0091	-0.0119	8.1070e-05
α_0	2.9207e-04	0.0013	1.3253e-04	1.2957e-04
α_1	0.1252	0.6428	0.1984	0.0943
β_1	0.7438	0	0.7990	0.8563

Tabela 5.2: GARCH(1,1) model calibrated using the sample data from 1/1/1990 to 3/31/2010 for price process of the agriculture commodity index.

as a measure of the cost of borrowing over the different regimes. Intuitively, the credit spread of borrowing is expected to increase when the market is experiencing a negative shock and risk free interest rate is decreasing. On the other hand, the credit spread of borrowing should decrease during times of economic expansion or when the market is experiencing a positive shock. The sample statistics for our chosen spread over the historical period, summarized in table 5.3, support the hypothesis.

	Regime 1 Expansion	Regime 2 Transition	Regime 3 Recession	All
Mean	2.39%	2.06%	2.54%	2.39%
Std Dev	1.05%	0.99%	1.44%	1.15%
Minimum	0.13%	0.62%	0.98%	0.13%
Maximum	4.32%	3.65%	6.63%	6.63%

Tabela 5.3: Sample statistics of the Barclays Aggregate US Corporate AAA spread over the 3-month US T-Bill rate under each regime from 1/1/1990 to 3/31/2010.

5.2.4 Gross Margin

The sales price per unit of product is simulated indirectly by first simulating the gross profit margin of the agribusiness company. This somewhat convoluted modeling is necessary because we do not have any reliable data on the unit sales price of the products, but we can extract the gross profit margin data from each company's income statement. Hence, the unit sales price follows the following formula:

$$\tilde{p}_t(s) = \tilde{c}_t(s)(1 + \frac{\tilde{m}_t(s)}{(1 - \tilde{m}_t(s))})$$

where $m_t(s)$ is the gross profit margin during period t.

The gross profit margin is calculated as the difference between revenue and cost of goods sold, divided by the revenue. Because gross margin is a balance sheet data that is reported quarterly at the most, we used a linear interpolation of the quarterly numbers to estimate the monthly gross profit for each company. The cross-section monthly weighted-average of the linearly-interpolated gross margin is then assumed to represent the gross margin of the agribusiness sector as a whole. Figure 5.7 plots the weighted-average profit margin for the agribusiness sector over the period from January 1, 1990 to March 31, 2010.

Even though profit margin values can take both positive or negative values, our sample data is very concentrated in the range between 19% to 36%. Therefore, bootstrapping historical data will be a more appropriate method in simulating gross profit rather than calibrating a parametric model to the historical data.

5.2.5 Investment Unit Cost Multiple

The probability distribution of the unit cost of investing also depends on the economic regime. If we consider the cost of investment as the market value one would have to pay to acquire the assets of a company, then investment cost intuitively should be higher during expansionary periods than during recessionary periods. As such, we can model the total cost of investment as the dollar amount of the capital acquired multiplied by a multiplier. In other words, an I_t amount of capital costs $\tilde{w}_t * I_t$, where $\tilde{w}_t$ is distributed as the following:

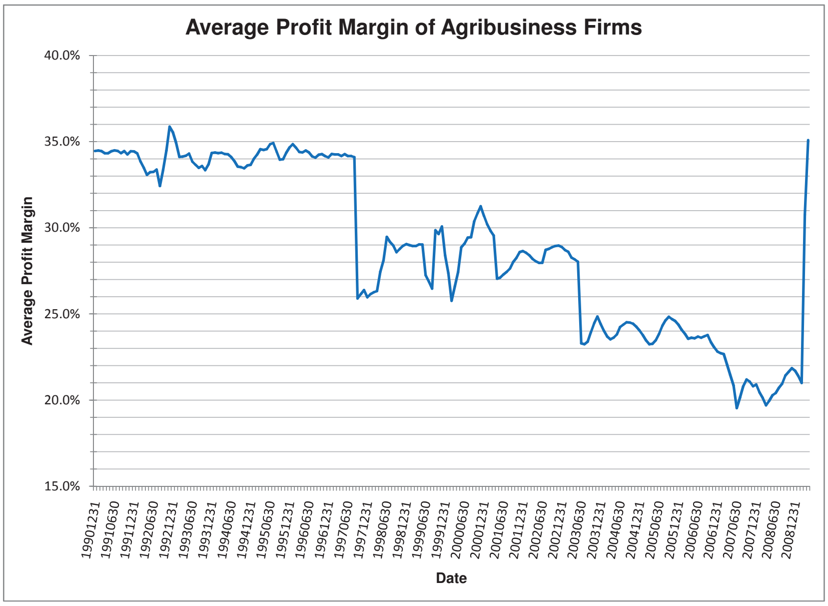
$$\tilde{w}_t = \begin{cases} w_1, & \text{if } S_t = 1, \\ w_2, & \text{if } S_t = 2, \\ w_3, & \text{if } S_t = 3, \end{cases} \tag{5-17}$$

and S_t is the economic regime during period t.

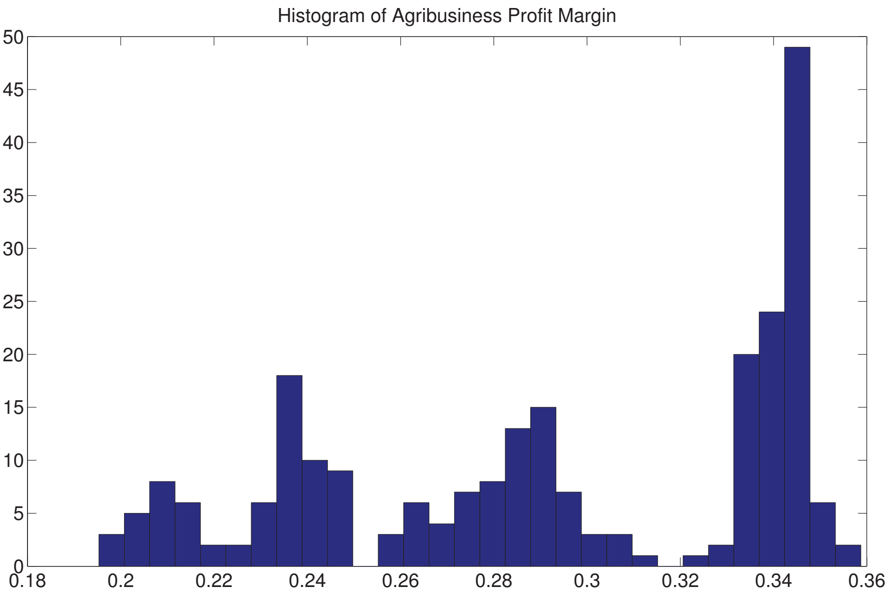
We can consider the market-to-book ratios or the price-to-earnings ratio as a measure of the cost multiple to acquiring a new investment, $\tilde{w}_t$. In particular, the three measure we considered are the market-to-book value of equity, market-to-book value of assets, and the price-to-earnings ratio of the agribusiness companies. Table 5.4 summarizes the sample statistics of those chosen variables.

Even though the sample means are not lower during recession than during expansion, it is more important to note that the standard deviation of these measures increases during recession and thus, there are more cheap investment opportunities available during a recession.

5.2.6



5.7(a): Cross-sectional average of profit margins for agribusiness firms, weighted by the book value of assets.



5.7(b): Histogram of cross-sectional average of profit margins for agribusiness firms, weighted by the book value of assets.

Figura 5.7: Weighted-average profit margin in the agribusiness sector from 1/1/1990 to 3/31/2010.

Production Capacity Function

The production capacity function chosen is of the Cobb-Douglas functional form. The amount of quantity that can be produced given K_t amount of

Variable	Statistic	Regime 1 Expansion	Regime 2 Transition	Regime 3 Recession	All
MB Equity	Sample Mean	5.0	22.8	10.2	7.8
	Sample Std Dev	3.0	57.3	21.5	20.5
	Minimum	0.8	4.4	0.1	0.1
	Maximum	30.5	268.4	156.1	268.4
MB Asset	Sample Mean	1.96	1.83	1.93	1.94
	Sample Std Dev	0.20	0.16	0.24	0.21
	Minimum	1.57	1.69	1.51	1.51
	Maximum	2.47	2.41	2.46	2.47
P/E ratio	Sample Mean	113.0	131.7	119.4	116.3
	Sample Std Dev	38.8	61.6	56.7	46.2
	Minimum	53.6	65.0	6.9	6.9
	Maximum	260.8	283.1	246.5	283.1

Tabela 5.4: Sample statistics of agribusiness companies from 1/31/1990 to 3/31/2010.

capital is

$$f(K_t) = \beta_0 K_t^{\beta_1}.$$

If $\beta_1 < 0$, then the production function is said to have a decreasing returns to scale. If $\beta_1 > 0$, then the production function is said to have an increasing returns to scale. Otherwise, $\beta_1 = 0$ means that the production function has a constant returns to scale. We assume that there will be no significant changes in the production technology over time and across regimes, allowing β_0 and β_1 to remain constant. A graphical illustration of the capacity function with various β_0 and β_1 values are depicted in figure 5.8.

We fit the parameters of the production function by performing a least-squares regression using the quarterly balance sheet items reported by our sample of agribusiness companies over the 20-year period from January 1, 1990 to March 31, 2010. But first we need to transform the equation into a linear one by applying the natural logarithm function to obtain the following equation.

$$\log(f(K_t)) = \log(\beta_0) + \beta_1 \log(K_t) + \epsilon_t.$$

K_t is measured by the natural logarithm of book-value of assets less cash and $f(K_t)$ is measured by the cost of goods sold (COGS) divided by the market price of the agriculture GSCI. The simple least-squares regression gives us the following parameter estimates of $\hat{\beta}_0 = 5$ and $\hat{\beta}_1 = 0.38$.

5.2.7

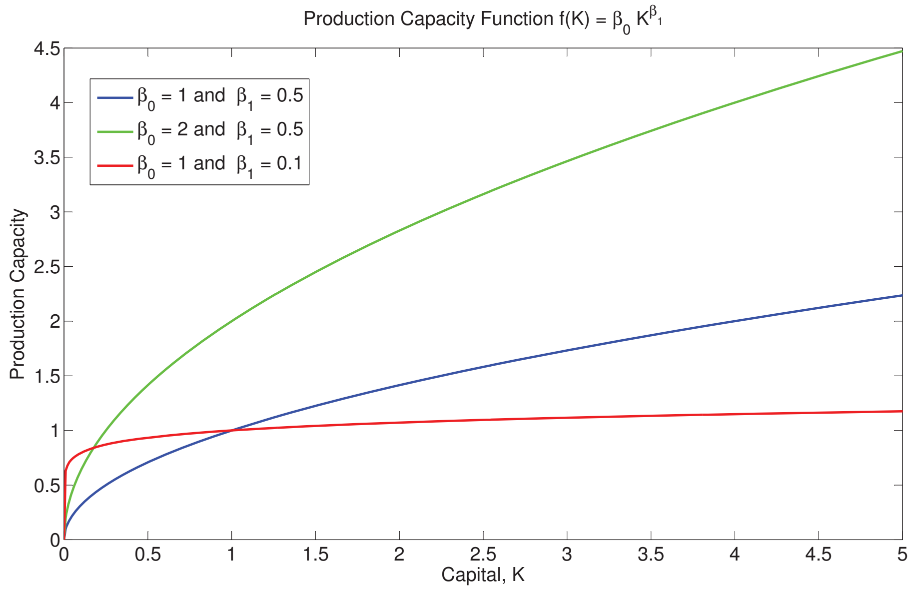


Figura 5.8: Production capacity function with various parameter values for β_0 and β_1.

Stochastic Program Numerical Results

The model was solved with 2,560 scenarios and a branch structure of 20-4-4-2-2-2, which means that there are 20 branches coming out of the first node at $t = 0$ and 4 branches coming out of each node at $t = 1$ and so on. The stochastic program solved has 143,360 variables with 1,518,957 constraints. A set of base case parameter values were chosen for this exercise. The parameter values are summarized in table 5.5

The free GLPK (GNU Linear Programming Kit) is utilized to solve our stochastic program. The interior point method takes 358.9 seconds to solve the problem on a MacBook Pro with 2.66 GHz Intel Core i7 and 8 GB memory. The solution trees using base case parameter values are depicted in figure 5.9

Tabela 5.5: Base case parameter values.

C_{ini}	Initial Cash Amount	\$1.00
K_{ini}	Initial Capital Amount	\$20.00
ϕ	Fixed Cost Ratio	0%
δ	Depreciation Rate	1%
T	Planning Horizon (months)	6
r_t	Risk-Free Rate	1%
β_0	Production Capacity Function Scaling Parameter	5
β_1	Production Capacity Function Concavity Parameter	0.38
c_0	Initial cost per unit	\$1.00
m_0	Initial profit margin	0.35

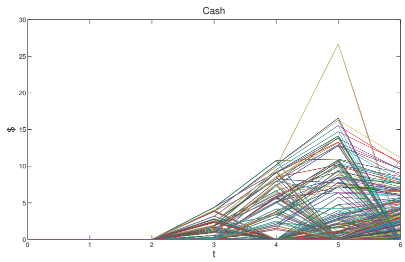
Branch structure	20-4-4-2-2-2

Investment cost multiplier

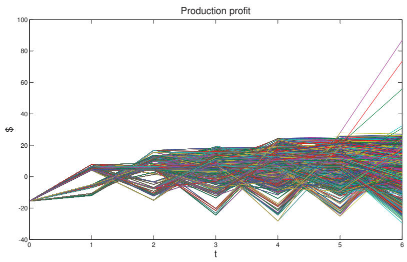
Regime 1	1.6
Regime 2	1.7
Regime 3	1.5

Borrowing spread

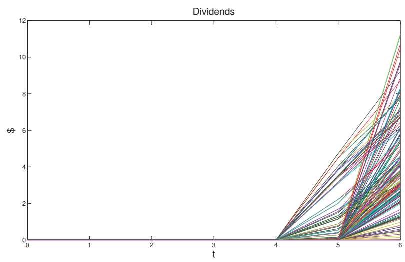
Regime 1	4.32%
Regime 2	3.65%
Regime 3	6.65%



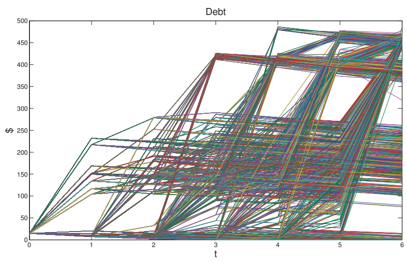
5.9(a): Cash savings tree



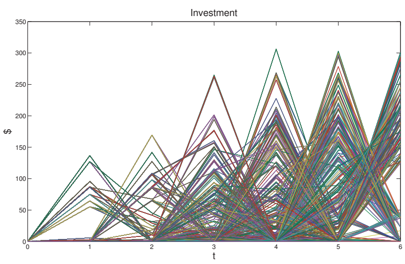
5.9(b): Gross profit tree



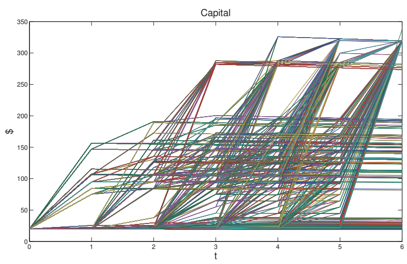
5.9(c): Dividend payout tree



5.9(d): Borrowing tree



5.9(e): Investment cost tree



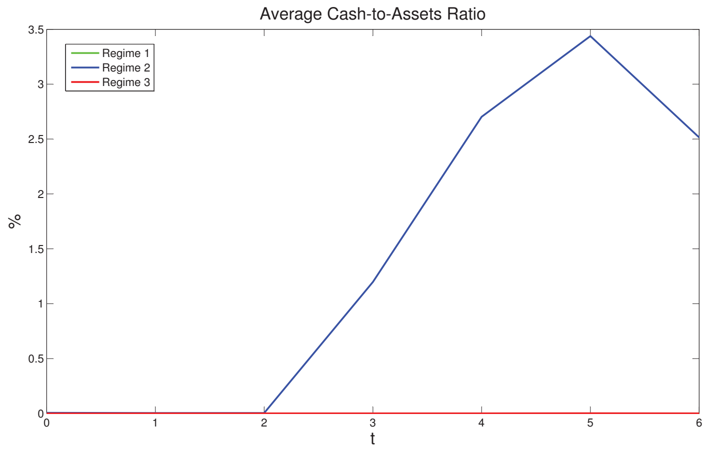
5.9(f): Production capital tree

Figura 5.9: The solution tree of the stochastic program under one set of scenarios.

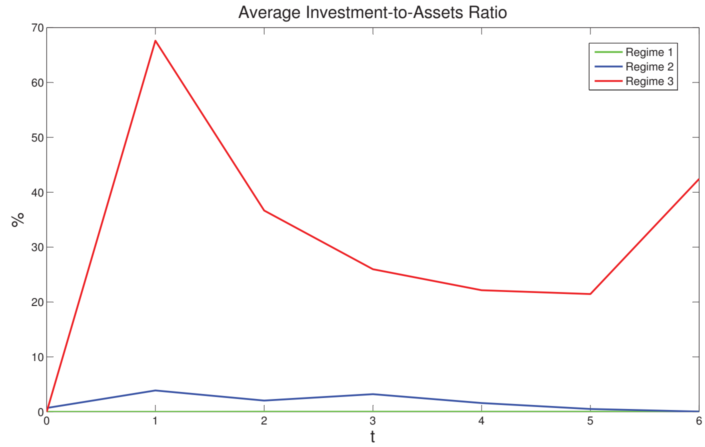
In order to make more sense of the stochastic program solution, we first computed ratios of the solution with respect to the asset value of the firm at each node. We define the asset value at time t and scenario s as $A_t(s) := K_t(s) - I_t(s) + (1 + \tilde{r}_t)C_{t-1}(s) + \tilde{p}_t(s)Q_{t-1}(s)$. Then, the cash-to-asset ratio, $\text{cratio}_t(s) = \dfrac{C_t(s)}{A_t(s)}$, debt-to-asset ratio, $\text{dratio}_t(s) = \dfrac{D_t(s)}{A_t(s)}$, and investment-to-asset ratio, $\text{iratio}_t(s) = \dfrac{\tilde{w}_t(S)I_t(s)}{A_t(s)}$, are computed for each node. These three ratios will generally be referred to as the solution ratios throughout the rest of this study.

We make the following observations from the plots of ratio averages in figure 5.10:

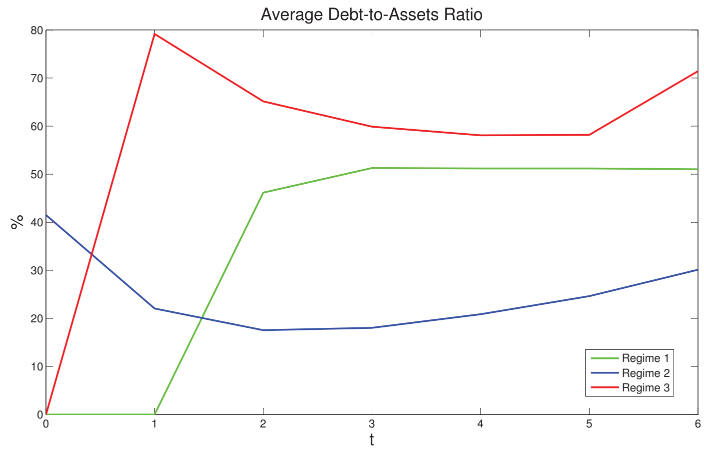
- The model shows that it is optimal for firms to save cash during the planning horizon. On average, the firm saves more cash, as a proportion to asset, in regime 2(transition) than in any other regime.
- It is optimal for the firm to invest most in regime 3 than in any other regimes. This behavior is motivated largely by the cheap cost of investments in regime 3.
- Debt is mostly issued to fund production in regime 1 and 2, but debt is used largely to finance investments in regime 3.



5.10(a): Average cash-to-asset ratios



5.10(b): Average investment-to-asset ratios



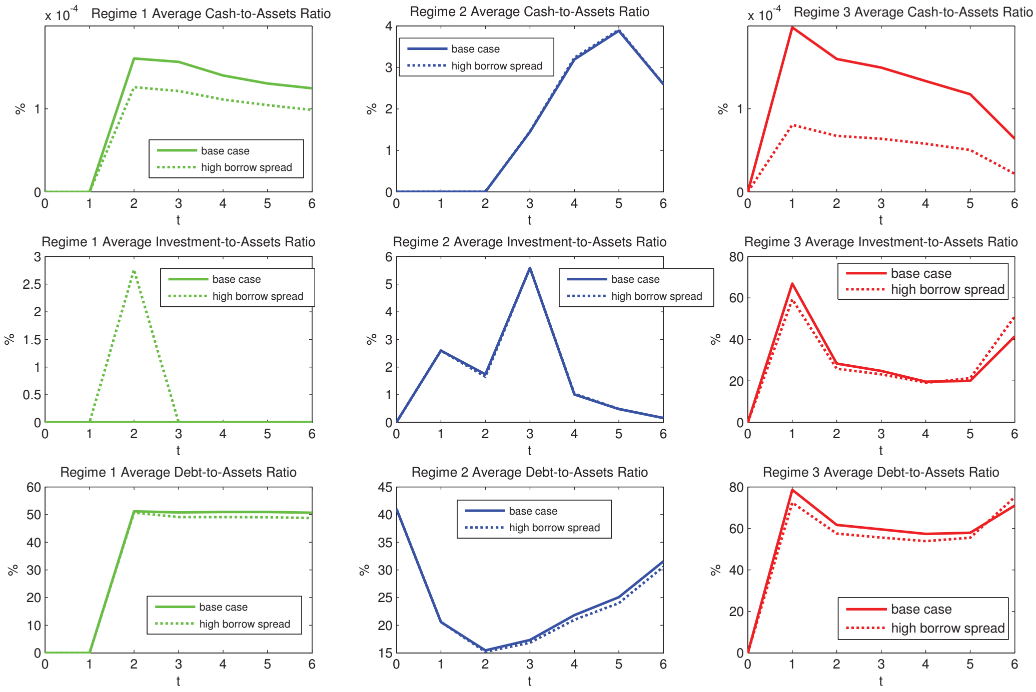
5.10(c): Average debt-to-asset ratios

Figura 5.10: The average solution ratios under each regime and each stage of the stochastic program.

Sensitivity to the Borrowing Rate

In order to test the sensitivity of the solutions to a change in the borrowing rate, the model is solved using a higher rate of borrowing in regime

3. In particular, the regime 3 borrowing spread is doubled from 6.65% to 13.3% and all other parameter values are kept the same as the base case. Figure 5.11 compares the average ratios under each regime using this new rate of borrowing against the base case solution.



Note: All other parameters are assumed to have the base case values. On average, the optimal debt ratios decrease across all regimes when we increase the borrowing spread. Moreover, the investment ratio increases in regime 1, possibly to compensate for the decrease in investments during regime 3.

Figura 5.11: Sensitivity plot comparing the solution using regime 3 borrowing spread = 6.65% vs regime 3 borrowing spread = 13.3%.

The optimal cash-to-asset ratios do not change significantly for any of the regimes. Even though the optimal cash-to-asset ratios in regime 1 and 3 are slightly higher than the optimal ratios given by the base case, both sets of ratios are very close to zero. Therefore, we feel that the increase is insignificant. A higher borrowing rate in regime 3 has lowered the firm's ability to borrow, and thus lowers the amount of investment spending, in regime 3. It seems like the firm compensates its decrease in investments in regime 3 (recessionary period) by increasing investments in regime 1 (expansionary period). However, the average investment spending in regime 1 is still very close to zero.

Sensitivity to the Investment Cost

We also solved the stochastic program with higher investment cost multipliers. The lines denoted as high investment cost in figure 5.12 are the solution ratios of the model using 3.2, 3.4, and 3 as the investment cost multipliers for regime 1, 2, and 3, respectively.

As expected, the firm saves a higher proportion of its assets in cash across all regimes. The increase in savings is most significant during regime 2 (the transition period). The firm also does not invest in regime 2. Investment spending is also lower in regime 3 up until stage $t = 4$. Interestingly, the firm's investment spending increases in regime 1, although the ratio is extremely close to zero. The firm's optimal leverage ratio also decreases accordingly in regime 2 and 3 because the firm does not spend as much on investments in these two regimes.

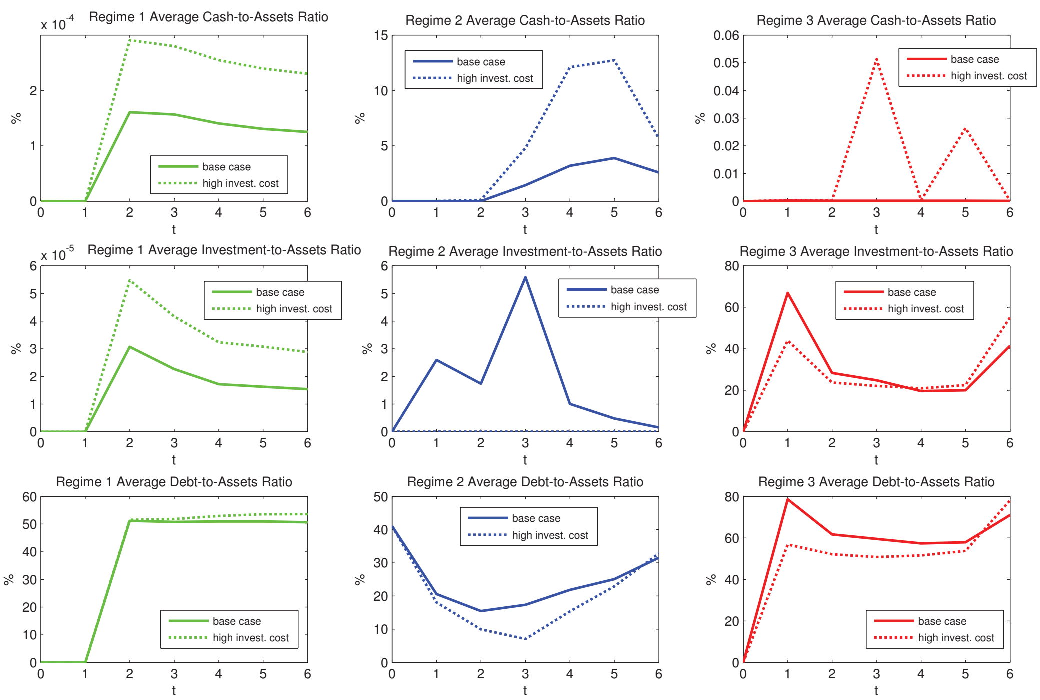


Figura 5.12: Sensitivity plot comparing the solution using base case investment cost multiplier vs high investment cost. All other parameters are assumed to have the base case values.

Sensitivity to Fixed Cost

The stochastic program using a higher fixed cost of production (defined as a percentage of existing capital), $\phi = 5\%$, is solved and the average solution ratios compared to the base case solution ratios. These are plotted in figure 5.13.

It is intuitive that with an increase in fixed cost decreases the firm's optimal investment spending in regime 2 and 3 as investments will not only increase the capacity of production but also increase the fixed cost of production. Interestingly, the investment spending in regime 1 is higher than that of the base case solution. It is also intuitive that, on average, cash savings is higher across the regimes because the firm needs to make sure that it has enough cash to produce.

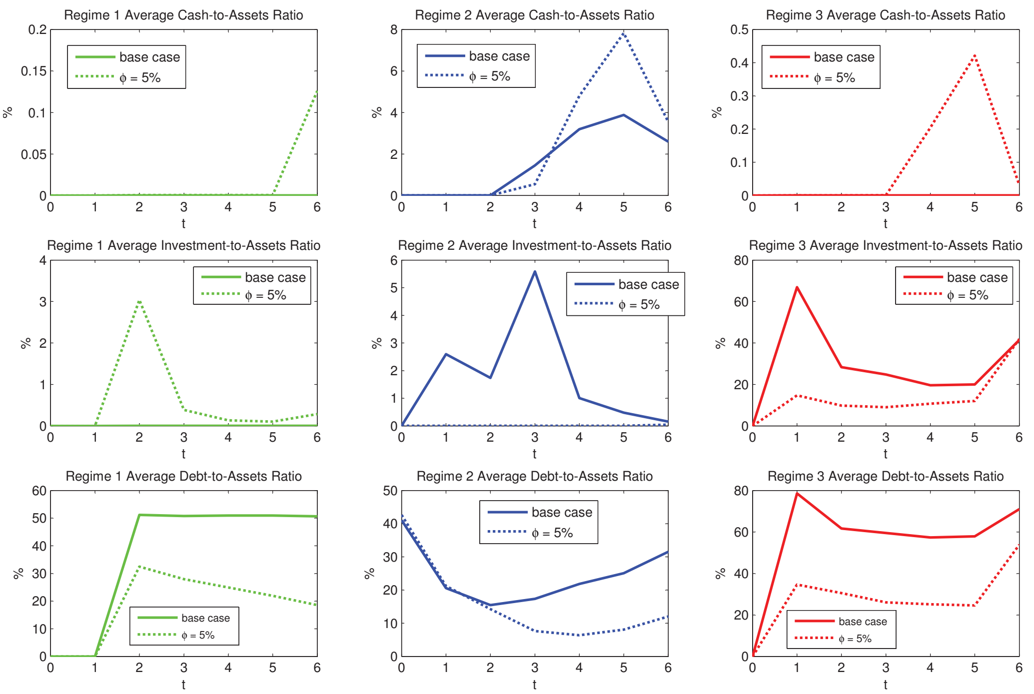


Figura 5.13: Sensitivity plot comparing the solution using $\phi = 5\%$ vs $\phi = 0\%$. All other parameters are assumed to have the base case values.

5.3
A Fixed Policy Rule Approximation for Corporate Cash Holding

A significant drawback of stochastic programming is that its solutions are often times hard to interpret. Most practitioners are reluctant to bet all their money on directly implementing the solution of a stochastic program that lacks an intuitive explanation. A stochastic program can be useful, however, in identifying patterns in the solution. Those patterns may be a good starting point to discovering policy rules that can be implemented more easily and provide sufficiently robust results.

In this chapter, we investigate a fixed policy rule derived from the stochastic programming solution described in the preceding sections. In particular, the firm's cash and investment holdings at each node must be at least as great as the pre-determined cash-to-asset and investment-to-asset ratio targets. These target ratios are given by the average cash-to-asset and investment-to-asset ratios of the stochastic programming solution. We compare the objective value given by this fixed policy rule against the optimal objective value given by the stochastic program.

5.3.1 Implementation of the Fixed Policy

Let us denote γ_c and γ_i as the target cash-to-asset and the target investments-to-asset ratio, respectively. Under the fixed policy rule, the firm is required to invest and save a proportion of its assets at the level of the target ratios. Moreover, the firm must satisfy the cash flow balance summarized in equation (5-19) for time $t = 0$ and equation (5-24) for $t > 0$.

The firm's optimal decision at each node was determined by maximizing the current payout to shareholders while minimizing debt as follows:

– At $t = 0$ and scenario s, we solve

$$\underset{C_0(s),D_0(s),I_0(s),Q_0(s),E_0(s)}{\text{maximize}} \quad E_0(s) - \frac{(1+r_{d,0}(s))}{(1+r_0(s))}D_0(s)$$

$$\text{subject to} \quad C_0(s) = C_{ini}(s) - Q_0(s)c_0(s) + D_0(s) - w_0(s)I_t(s) - \phi K_{ini}(s) - E_0(s), \tag{5-18}$$

$$C_0(s) \geq \gamma_0^c(s)\left(C_{ini}(s) + D_0(s) + K_{ini}(s)\right), \tag{5-19}$$

$$w_0(s)I_0(s) \geq \gamma_0^i(s)\left(C_{ini}(s) + D_0(s) + K_{ini}(s)\right), \tag{5-20}$$

$$Q_0(s) \leq f(K_{ini}(s) + I_0(s)), \tag{5-21}$$

$$0 \leq I_0(s), Q_0(s), D_0(s), C_0(s), E_0(s). \tag{5-22}$$

– At time $t \in \mathcal{H}/\{0\}$ and scenario s, we solve

$$\underset{C_t(s),D_t(s),I_t(s),Q_t(s),E_t(s)}{\text{maximize}} \quad E_t(s) - \frac{(1+r_{d,t}(s))}{(1+r_t(s))} D_t(s) \tag{5-23}$$

$$\text{subject to} \quad C_t(s) = (1+r_{t-1}(s))C_{t-1}(s) + Q_{t-1}(s)p_t(s) \tag{5-24}$$

$$- Q_t(s)c_t(s) + D_t(s) - (1+rd_{t-1}(s))D_{t-1}(s)$$

$$- w_t(s)I_t(s) - \phi(1-\delta)K_{t-1}(s) - E_t(s),$$

$$C_t(s) \geq \gamma_t^c(s)\left((1+r_{t-1}(s))C_{t-1}(s) + Q_{t-1}(s)p_t(s) + D_t(s) + (1-\delta)K_{t-1}(s)\right), \tag{5-25}$$

$$w_t(s)I_t(s) \geq \gamma_t^i(s)\left((1+r_{t-1}(s))C_{t-1}(s) + Q_{t-1}(s)p_t(s) + D_t(s) + (1-\delta)K_{t-1}(s)\right), \tag{5-26}$$

$$Q_t(s) \leq f(K_t(s)), \text{and} \tag{5-27}$$

$$0 \leq I_t(s), Q_t(s), D_t(s), C_t(s), E_t(s). \tag{5-28}$$

Optimization for the End-Effect

Under the fixed policy rule, the model takes on the same end-effect treatment as in Chapter 6. The reader can refer to the previous chapter for the derivation of the end-effect constraints. Also following the notation in Chapter 6, we define the same end-effect decision variables $\bar{E}, \bar{D}, \bar{I}, \bar{K}$, and $\bar{Q}$. Recall that if x is a risk factor, then $\bar{x}$ denotes the long-run average level of that risk factor. In summary, the firm optimizes the following problem at time T:

$$\underset{\bar{D}(s),\bar{K}(s),\bar{I}(s)}{\text{maximize}} \quad \bar{E}(s) \tag{5-29}$$

$$\text{subject to} \quad \bar{E}(s) = C_T(s) + \alpha(Q_T + f(\bar{K}(s)))\bar{p} + f(\bar{K}(s))\bar{c} + \bar{D}(s) \tag{5-30}$$

$$- \alpha(1+\bar{r_d})(D_T(s) + \bar{D}(s)) - \bar{I}(s)\bar{w}$$

$$- \alpha\phi(1-\delta)(K_T(s) + \bar{K}(s))$$

$$\bar{K}(s) = \alpha(1-\delta)(K_T(s) - \bar{K}) + \bar{I}(s) \tag{5-31}$$

$$\bar{D}(s) \geq 0 \tag{5-32}$$

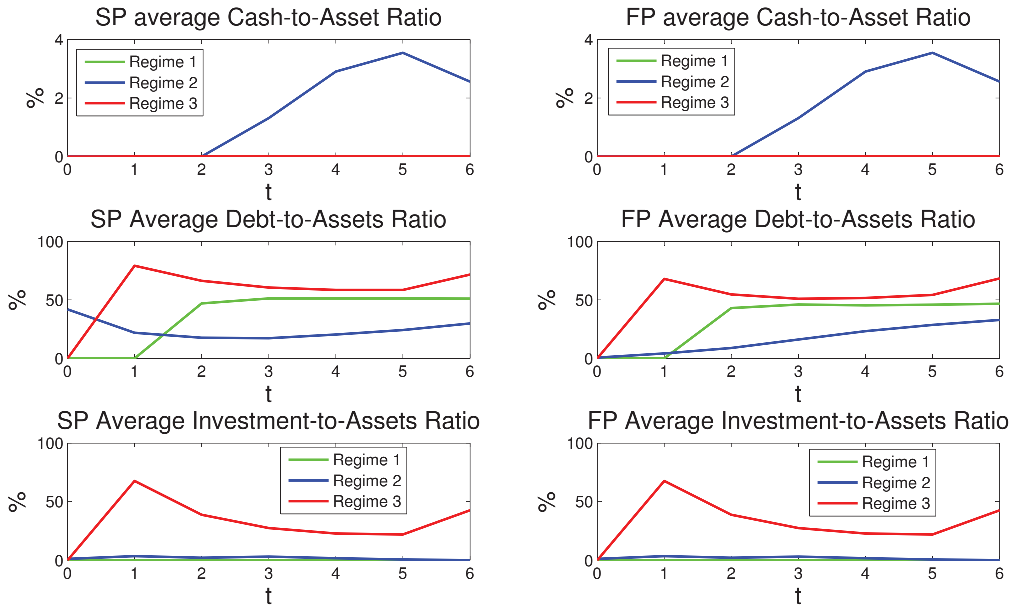
$$\bar{K}(s) \geq 0 \tag{5-33}$$

Finally, the firm's objective value for each scenario path s is the same as that of the stochastic program's, and is computed as the average of the total present value of all dividend payments made by the firm to its shareholders.

5.3.2 Numerical Results and Discussion

To test the robustness of the fixed policy rule, both the stochastic program and its corresponding fixed policy model were solved 10 times; each iteration is solved using a different scenario tree[1]. Let us first look at the solution ratios. Figure 5.14 compares the average solution ratios given by the fixed policy (FP) rule against the optimal ratios given by the stochastic program (SP) in Chapter 6. We can easily observe the fixed policy cash-to-asset and investment-to-asset ratios meet their target values. However, the average debt-to-asset ratio given by the fixed policy is much lower than that of the stochastic program's because of the firm's objective at each node, which is to maximize dividend payout while minimizing debt issuance. The difference between the two sets of debt-to-asset ratios is largest under regime 2 at the beginning of the planning horizon as shown in figure 5.15. The fixed policy average debt-to-asset ratio is 41% lower than the stochastic program's at $t = 0$ given regime 2.

[1]Each scenario tree has 2560 sample paths that are constructed using a branch structure of 20-4-4-2-2-2 as in the base case of Chapter 6.



Note: The average cash-to-asset and investment-to-asset ratios given by the fixed policy rule are very close to the target ratio values given by the stochastic program. The debt-to-asset ratios given by the fixed policy rule, however, is on average lower than that of the stochastic program's due to the firm's fixed policy objective, which aims to maximize current dividend with the least amount of debt possible.

Figura 5.14: Plots of average solution ratios given by the original stochastic program vs. fixed policy rule.

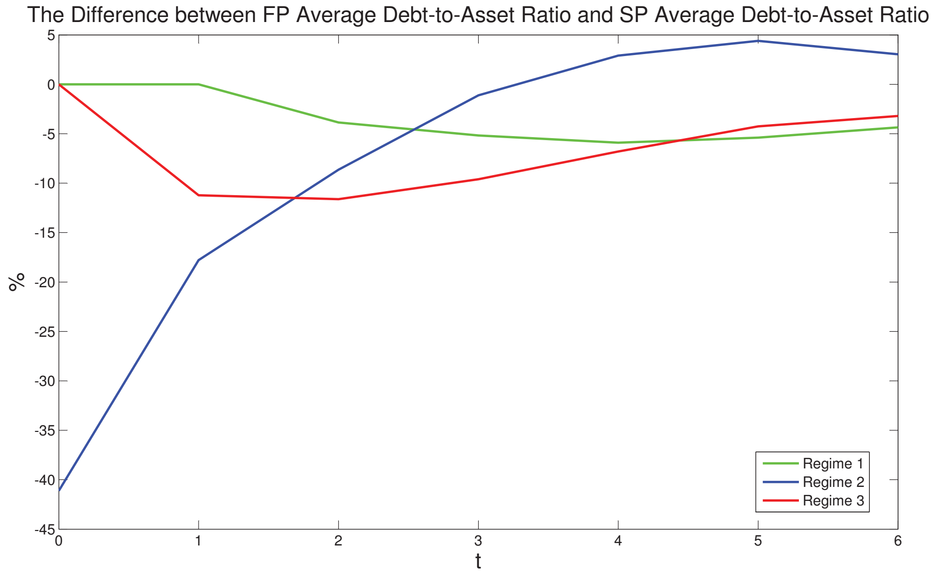


Figura 5.15: The average difference in debt-to-asset ratios given by the stochastic program and the fixed policy rule.

Parameter	Objective Gap	Std Dev
base case	9.85%	0.56%
High borrowing spread	9.04%	0.36%
High investment cost	9.41%	0.12%
$C_{ini} = \$10$	10.77%	0.85%
$\delta = 10\%$	16.73%	0.09%
$\phi = 5\%$	9.88%	0.07%

Tabela 5.6: Objective value gap between the objective value given by the optimal solution and the fixed policy rule approximation.

Our fixed policy gives us a suboptimal solution to the original problem, therefore our next task is to examine how well the policy rule approximates the "true" optimal solution given by the stochastic program. Using the base case parameter set detailed in Chapter 6, we first compare the difference in the fixed policy (FP) and stochastic program (SP) objective values by computing the following ratio:

$$\text{Objective Gap} = \frac{\text{SP Objective} - \text{FP Objective}}{\text{SP Objective}}.$$

We computed the sample average and standard deviation of the objective gaps given by the different scenario trees. This procedure is performed with the base case parameter values and repeated for various changes in the base case parameters, such as in the borrowing spread or the investment cost multiple. The results are reported in table 5.6.

The optimal objective values achieved by the FP rule were on average 9.04% to 16.73% lower than the SP objective value, depending on the parameters. This range of difference is reasonable as a trade-off for a more implementable solution to our cash holding problem. By modeling the underlying economic regimes, we are able to customize the firm's fixed policy rule appropriately according to the state of the business cycle. The optimal solution can be approximated quite well simply by matching the average optimal cash-to-asset and investment-to-asset ratios conditioned on the regimes.