Chapter 2
Problem Statement

In this chapter we present a description of the problem to be addressed focusing on its specific characteristics. In the sequence, we present the mathematical model formulated to represent the problem.

2.1 Problem Description

The addressed problem can be defined as the strategic planning of petroleum products distribution, where one seeks to select investments to be made in logistics infrastructure, taking into consideration decisions regarding the distribution of flows, inventory policies, and the level of the external commercialization of refined products. Such decisions arise in the context of strategic and tactical planning faced by petroleum companies operating over large geographical regions.

We consider this problem as an integrated distribution network design with discrete capacity expansion under a multi-product and multi-period setting. Typically, a logistics network for oil products is composed of a set

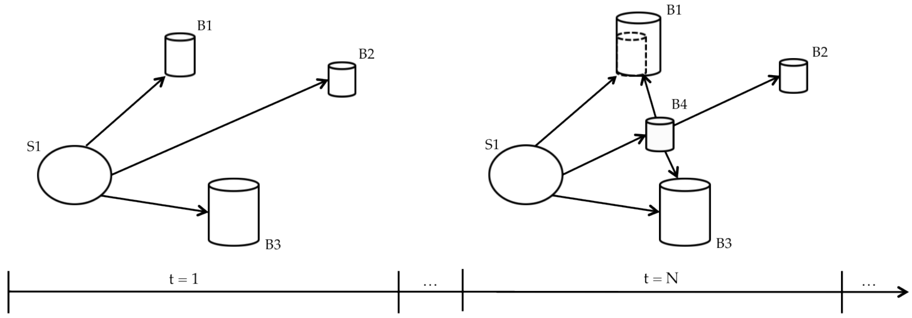

Figure 2.1: Example of a multi-period network investment planning

of nodes (international markets, refineries, terminals, and bases) that are connected by transportation arcs. A transportation arc is defined by its origin node, destination node, mode of transport, and product group. The product group defines which products are compatible with the operational specifications of the arc and, therefore, can flow through it. Some specific arc modes, such as pipelines, are reversible, which means that they can operate in both directions using the same structure. Figure 2.1 presents a small example of the network structure and its evolution during the time horizon. For the sake of illustration, we consider one supply node (S1) and three bases (B1, B2, B3) at the beginning, and four bases after N periods, assuming the representation of the decision of creating B4. One might notice that the structure changes along the time horizon, as is exemplified by the expansion of B1, the creation of B4, and the arcs that connect it to B1, B2, and B3.

After the refining process, petroleum products are stored in tanks before being shipped to distribution bases. These bases serve as aggregation points of demand for such products. They also might serve as intermediate points for other bases further away from the refineries. As the bases are capable of storing product when necessary, the problem is considered under a multi-period operation. The tanks of these bases are constantly being loaded and unloaded. This process is known as the tank rotation and is subject to the physical limitations that are inherent to the hardware associated with the tanks of the distribution base. The rotating capacity refers to the number of times a tank can be filled and emptied over a certain period of time.

Products may be transported by sea through marine terminals. Therefore, it is important to consider the issue of the time that the ships spend in port while waiting for permission to dock and unload, which increases the logistical costs due to the demurrage of the vessel at the site. This demurrage cost can be modeled by curves obtained from simulation models. The demurrage curves are drawn proportionally to the amounts that are handled at the port. These curves are typically nonlinear and depend mostly on the operational characteristics of each marine terminal. Normally, an increase in port workload leads to the augmented congestion of vessels in the harbor, which increases logistical costs.

The demurrage cost is approximated by a piecewise linear curve such that the linearity of the model is preserved. As can be seen in Figure 2.2, the precision level of such approximation is directly proportional to the number of linear segments used for the representation of the original curve (in the example, the nonlinear curve is approximated by four segments). However, there is a trade-off that must be considered because when one uses a greater number of these segments, the number of variables and constraints in the mathematical model increases.

Supplying internal demand depends on the characteristics of the network operations, refinery availability, and the sources of production. Although uncertain, meeting the internal demand is mandatory in the present context.

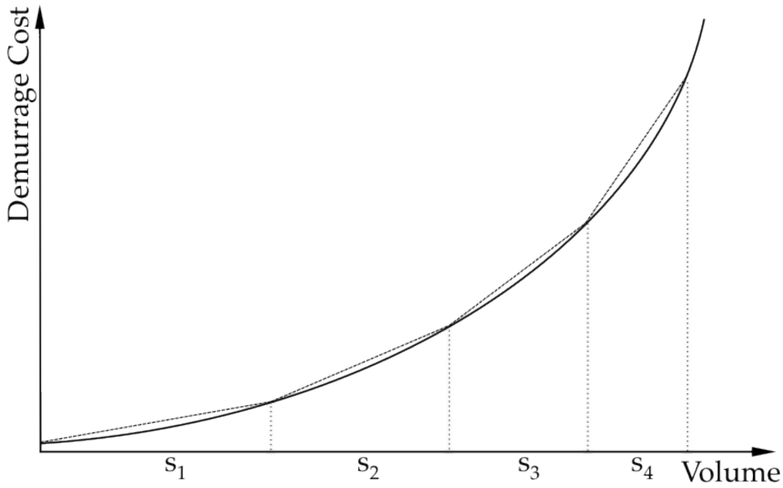

Figure 2.2: Piecewise linear representation of nonlinear demurrage cost

Therefore, it is imperative that the uncertain nature of the demand is considered into the modeling process so that the decisions are taken considering different possibilities for these uncertain outcomes.

To address this issue, we propose a two-stage stochastic mixed-integer linear programming model (Birge and Louveaux, 1997). The mathematical model's objective is to determine which of the possible investments should be implemented and when, together with the transportation and inventory decisions that will cope with the forecasted (although uncertain) growth of product demands while minimizing both investment and logistics present costs.

The first-stage decisions (i.e. the decisions that must be taken previously to the unveiling of the uncertainties) are the selection of which projects to implemented and when they should be available. These decisions are represented by binary variables. Typically, these investments are highly capital intensive and they all are built-to-order due to its technical complexity and particular specifications. For this reason, we assume that the same investment can be only implemented once along the time horizon. Also, we assume that investment decisions are available for use in the beginning of the chosen time period.

Given that the focus of the model is geared toward the commercialization, logistics, and allocation of petroleum products, aspects related to the petroleum refining process are not considered as objects of this decision model but as aspects that have already been decided in advance. Therefore, only the bases and the terminals of the model in question are locations subject to investment decisions. Each investment implies in improving the following characteristics individually or in combination:

- Changes in operational costs - justified by the introduction of more modern and / or more energy-efficient equipments;
- Changes in the rotation capacity of the tanks - investments in more powerful pumps that allow faster pumping (i.e., tanks are filled and emptied more quickly);
- Changes in demurrage cost curves - investing in ship management technology and/or adding new piers for berthing or modernizing existing berths;
- Changes in storage capacity - the installation of additional tanks for storage.

The most typical cases of investments in transportation arcs consist of improvements or expansions of the existing pipeline network, increasing pumping capacity, reducing operating costs and/or changing the types of products that can flow through that arc.

The second-stage decisions, to be taken after the unveiling of the uncertain parameters, are those related to the flows of products, inventory levels, supply provided to each demand site, and supply levels at sources.

2.2 Mathematical Model

In this subsection we present the mathematical model developed to deal with the aforementioned problem. The objective of this mathematical model is to minimize the costs related to investment, freight, inventory, operations, demurrage in marine terminals, commercialization, and penalties for non-fulfillment of demand (shortfall costs). The decisions are subject to constraints relating to network operational conditions, the demand for petroleum products and supply limits.

(a) Nomenclature

The nomenclature used in this model is as follows:

Indexes and sets	
$i, j, l \in \mathcal{L}$	Locations
$p, p_1, \ldots, p_k \in \mathcal{P}$	Products
$g = \{(p_1, \ldots, p_k) \in \mathcal{P}^k\} \in \mathcal{G}$	Product groups
$m \in \mathcal{M}$	Transportation modes
$a = \{(i, j, g, m) \in \mathcal{L} \times \mathcal{L} \times \mathcal{G} \times \mathcal{M} \in \mathcal{A}\}$	Arcs
$s \in \mathcal{S}$	Segments of the piecewise linearization of the demurrage cost function

$t \in \mathcal{T}$	Time periods
$\xi \in \Omega$	Scenarios
Subsets	
$A_I \subseteq \mathcal{A}$	Arcs that can operate in reverse
$A_K \subseteq \mathcal{A}$	Arcs subject to projects
$L_B \subseteq \mathcal{L}$	Bases
$L_E \subseteq \mathcal{L}$	International markets
$L_F \subseteq \mathcal{L}$	Production sources
$L_R \subseteq \mathcal{L}$	Refineries
$L_S \subseteq \mathcal{L}$	Locations with access via marine terminal
$L_K \subseteq \mathcal{L}$	Locations subject to projects
$M_M \subseteq \mathcal{M}$	Maritime modes

Table 2.1: Model Notation: Sets and subsets

Parameters		
CFD_{at}	$\forall a \in \mathcal{A}, \forall t \in \mathcal{T}$	Freight cost per unit of product transported by the arc a during period t
CFI_{at}	$\forall a \in \mathcal{A}_I, \forall t \in \mathcal{T}$	Reverse freight cost per unit of product transported by arc a in period t
CI_g	$\forall g \in \mathcal{G}$	Holding cost for product group g
CKA_{at}	$\forall a \in A_K, \forall t \in \mathcal{T}$	Cost of investment in the project for arc a in period t
CKL_{lt}	$\forall l \in L_K, \forall t \in \mathcal{T}$	Cost of investment in the project at location l for period t
CO_{lt}	$\forall l \in \mathcal{L}, \forall t \in \mathcal{T}$	Cost of site l operation during period t
COK_{lt}	$l \in L_K, \forall t \in \mathcal{T}$	Cost of site l operation after completion of project in period t
CS_{slt}	$\forall s \in \mathcal{S}, \forall l \in L_S,$ $\forall t \in \mathcal{T}$	Demurrage cost in segment s for period t in location l
CSK_{slt}	$\forall s \in \mathcal{S}, \forall l \in L_S \cap L_K,$ $\forall t \in \mathcal{T}$	Demurrage cost in segment s after completion of the project for location l in period t
D^{ξ}_{lpt}	$\forall l \in L_B, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}, \forall \xi \in \Omega$	Demand in location l for product p in period t for realization ξ
FS_{slt}	$\forall s \in \mathcal{S}, \forall l \in L_S,$	Maximum volume on segment s for

	$\forall t \in \mathcal{T}$	location l during period t
FSK_{slt}	$\forall s \in \mathcal{S}, \forall l \in L_S \cap L_K,$ $\forall t \in \mathcal{T}$	Maximum volume on segment s after the project conclusion for location l during period t
GR_{lpt}	$\forall l \in L_B \cup L_S, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Reference tank rotation at location l for product p during period t
GRK_{lpt}	$\forall l \in (L_B \cup L_S) \cap L_K,$ $\forall p \in \mathcal{P}, \forall t \in \mathcal{T}$	Reference tank rotation after project conclusion at location l for product p in period t
I_a	$a \in A_I$	Inversion factor for arc a
LE_{lpt}	$\forall l \in L_E, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Export limit for product p for location l in period t
LI_{lpt}	$\forall l \in L_E, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Import limit for product p for location l in period t
OI_{lpt}	$\forall l \in L_F \cap L_R, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Product p supply at location l in period t
PE_{lpt}	$\forall l \in L_E, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Sales price at location l of product p in period t
PI_{lpt}	$\forall l \in L_E, \forall p \in \mathcal{P},$ $\forall t \in \mathcal{T}$	Purchase price in location l of product p in period t
Q_a	$\forall a \in \mathcal{A}$	Arc a initial capacity
QK_a	$\forall a \in A_K$	Arc a capacity after project completion
R_{lp}	$l \in L_B \cup L_R \cup L_S,$ $\forall p \in \mathcal{P}$	Initial storage capacity of location l for product p
RK_{lp}	$l \in (L_B \cup L_R \cup L_S) \cap L_K,$ $\forall p \in \mathcal{P}$	Storage capacity after project conclusion in location l for product p
V_{ap}	$\forall a \in \mathcal{A}, \forall p \in \mathcal{P}$	Viscosity factor in arc a for product p
VI_{lp}	$\forall l \in L_B \cup L_R \cup L_S,$ $\forall p \in \mathcal{P}$	Initial inventory of product p in location l
θ		Penalty for not meeting the demand

Table 2.2: Model Notation: Parameters

Variables

$f_{slt}^{\xi} \in \mathbb{R}_+$	$\forall s \in \mathcal{S}, l \in L_S,$ $t \in \mathcal{T}, \forall \xi \in \Omega$	Amount handled in segment s subject to demurrage at location l in period t under realization ξ
$fk_{slt}^{\xi} \in \mathbb{R}_+$	$\forall s \in \mathcal{S}, l \in L_S \cap L_K,$ $t \in \mathcal{T}, \forall \xi \in \Omega$	Amount handled in segment s subject to demurrage after project completion at

		location l, in period t under realization ξ
$e^{\xi}_{lpt} \in \mathbb{R}_+$	$l \in L_E, p \in P,$ $t \in \mathcal{T}, \xi \in \Omega$	Amount of product p exported from location l in period t under realization ξ
$i^{\xi}_{lpt} \in \mathbb{R}_+$	$l \in L_E, p \in P,$ $t \in \mathcal{T}, \xi \in \Omega$	Amount of product p exported from location l in period t under realization ξ
$o^{\xi}_{lpt} \in \mathbb{R}_+$	$l \in \mathcal{L}, p \in P,$ $t \in \mathcal{T}, \xi \in \Omega$	Amount of product p supplied at location l in period t under realization ξ
$s^{\xi}_{lpt} \in \mathbb{R}_+$	$l \in L_B, p \in P,$ $t \in \mathcal{T}, \xi \in \Omega$	Shortage of product p at location l in period t under realization ξ
$v^{\xi}_{lpt} \in \mathcal{R}_+$	$l \in L_B \cup L_R \cup L_S,$ $p \in P, t \in \mathcal{T}, \xi \in \Omega$	Inventory level of product p at location l in period t under realization ξ
$xd^{\xi}_{apt} \in \mathbb{R}_+$	$a \in \mathcal{A}, p \in \mathcal{P}$ $t \in \mathcal{T}, \xi \in \Omega$	Flow in arc a of product p in period t under realization ξ
$xi^{\xi}_{apt} \in \mathbb{R}_+$	$a \in A_I, p \in \mathcal{P}$ $t \in \mathcal{T}, \xi \in \Omega$	Reverse flow in arc a of product p in period t under realization ξ
$y_{at} \in \{0, 1\}$	$a \in A_K, t \in \mathcal{T}$	Decision to implement a project for arc a in period t
$w_{lt} \in \{0, 1\}$	$l \in L_K, t \in \mathcal{T}$	Decision to implement a project for location l in period t
$z^{\xi}_{lpt} \in \mathcal{R}_+$	$l \in L_B \cup L_S, p \in P,$ $t \in \mathcal{T}, \xi \in \Omega$	Amount of product p handled at location l in period t under realization ξ
$zk^{\xi}_{lpt} \in \mathcal{R}_+$	$l \in (L_B \cup L_S) \cap L_K,$ $p \in P, t \in \mathcal{T}, \xi \in \Omega$	Amount of product p handled after project completion at location t in period t under realization ξ

Table 2.3: Model Notation: Variables

(b) Model Formulation

Bellow we state the equations that comprise the mathematical model.

First-stage problem

The first-stage model comprises the decisions regarding when and where the investments should be made prior to the realization of the uncertainty. The first-stage model can be written as follows:

$$\min_{w,y} \sum_{l,t} CKL_{lt} w_{lt} + \sum_{a,t} CKA_{at} y_{at} + \mathcal{Q}(w, y) \tag{2.1}$$

s.t.:

$$\sum_t w_{lt} \leq 1 \quad \forall l \in L_K \tag{2.2}$$

$$\sum_t y_{at} \leq 1 \quad \forall a \in A_K \tag{2.3}$$

Objective function 2.1 refers to the expenses related to investments in infrastructure (locations and arcs, respectively) that the model decides to deploy. The term $\mathcal{Q}(w, y) = \mathbb{E}_\Omega\{Q(w, y, \xi)\}$ represents the expected value second-stage problem considering all possible realizations $\xi \in \Omega$ of the uncertain parameters. Constraints 2.2 and 2.3 indicate that only a single investment can be made in each location or arc along the planning horizon.

Second-stage problems

The second-stage problems represent the costs of supplying the demand given an investment decision (w, y) for a certain realization ξ of the uncertain parameters. The second-stage problem $Q(w, y, \xi)$ is formulated as follows. The objective function is given equation 2.4:

$$\begin{aligned}\min_{xd,xi,v,z,zk,f,fk,i,e,s} & \sum_{a,p,t} CFD_{at} xd^{\xi}_{apt} + \sum_{a,p,t} CFI_{at} xi^{\xi}_{apt} + \sum_{l,g,p\in g,t} CI_g v^{\xi}_{lpt} + \sum_{l,p,t} CO_{lt} z^{\xi}_{lpt} \\ & + \sum_{l,p,t} COK_{lt} zk^{\xi}_{lpt} + \sum_{s,l,t} CS_{slt} f^{\xi}_{slt} + \sum_{s,l,t} CSK_{slt} fk^{\xi}_{slt} + \sum_{l,p,t} PI_{lpt} i^{\xi}_{lpt} \\ & - \sum_{l,p,t} PE_{lpt} e^{\xi}_{lpt} + \theta \sum_{l,p,t} s^{\xi}_{lpt}\end{aligned} \tag{2.4}$$

Terms $\sum_{a,p,t} CFD_{at} xd^{\xi}_{apt} + \sum_{a,p,t} CFI_{at} xi^{\xi}_{apt}$ compute the total freight cost, i.e., the cost of moving products through the arcs in direct and reverse directions, respectively. Term $\sum_{l,g,p\in g,t} CI_g v^{\xi}_{lpt}$ stands for the costs of maintaining the product inventories considering the level at the end of each period t. Terms $\sum_{l,p,t} CO_{lt} z^{\xi}_{lpt} + \sum_{l,p,t} COK_{lt} zk^{\xi}_{lpt}$ denote the operation cost of each site considering the current costs and the possible changes in operating costs due to investments. Terms $\sum_{slt} CS_{slt} f^{\xi}_{slt} + \sum_{slt} CSK_{slt} fk^{\xi}_{slt}$ calculate the demurrage cost to be incurred for locations that have port access also considering the current values and possible changes due to investments in infrastructure or operational improvements. Terms $\sum_{lpt} PI_{lpt} i^{\xi}_{lpt} - \sum_{l,p,t} PE_{lpt} e^{\xi}_{lpt}$ figure the net importation and exportation revenue. Finally, the last term $\theta \sum_{l,p,t} s^{\xi}_{lpt}$ represents the costs incurred by not meeting the demand.

$$\begin{aligned}& \sum_{a|j=l} xd^{\xi}_{apt} + \sum_{a|i=l} xi^{\xi}_{apt} + o^{\xi}_{lpt} + v^{\xi}_{lpt-1} = \sum_{a|i=l} xd^{\xi}_{apt} + \sum_{a|j=l} xi^{\xi}_{apt} + D^{\xi}_{lpt} + v^{\xi}_{lpt} - s^{\xi}_{lpt} \\ & \forall l \in L_B \cup L_R \cup L_S, \forall p \in \mathcal{P}, \forall t \in \mathcal{T}\end{aligned} \tag{2.5}$$

Equation 2.5 represents material balance conditions. All direct flows $\sum_{a|j=l} xd^{\xi}_{apt}$ and inverse flows $\sum_{a|i=l} xi^{\xi}_{apt}$ coming through arcs connected to location l plus the total amount o^{ξ}_{lpt} supplied by location l and the total remaining inventory from the previous period v^{ξ}_{lpt-1} should be equal to the sum of all direct and inverse flows ($\sum_{a|i=l} xd^{\xi}_{apt}$ and $\sum_{a|j=l} xi_{apt}$ respectively) going out from location l plus the local demand D^{ξ}_{lpt} and the total inventory v^{ξ}_{lpt} while discounting the eventual total of unmet demand s^{ξ}_{lpt} in the same period t. For $t = 1, v^{\xi}_{lpt-1} = VI_{lp}$. This constraint applies only to bases, refineries, and places with a marine terminal.

$$o^{\xi}_{lpt} \leq OI_{lpt} \quad \forall l \in \mathcal{L}, p \in \mathcal{P}, t \in \mathcal{T} \tag{2.6}$$

Constraint 2.6 states that the total supplied o^{ξ}_{lpt} must comply with the supply limit OI_{lpt}.

$$\sum_{p} V_{ap}\left(xd^{\xi}_{apt} + \frac{xi^{\xi}_{apt}}{I_a}\right) \leq Q_a\left(1 - \sum_{t' \leq t} y_{at}\right) + QK_a \sum_{t' \leq t} y_{at}$$
$$\forall a \in \mathcal{A}, t \in \mathcal{T} \tag{2.7}$$

Arc capacities are represented in constraint 2.7. The total capacity Q_a is shared by direct and inverse flows of all of products that pass along it. The flow volume must be adjusted by a viscosity factor V_{ap}, and a reversal factor I_a in case of inverse flows. In the event that the model choose an investment y^t_a to be made in arc a at a prior period $t' \leq t$ in the planning horizon, the arc operates based on its new capacity QK_a.

$$v^{\xi}_{lpt} \leq R_{lp}\left(1 - \sum_{t' \leq t} w_{lt}\right) + RK_{lp} \sum_{t' \leq t} w_{lt} \quad \forall l \in L_B \cup L_R \cup L_S,$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T} \tag{2.8}$$

Constraint 2.8 models storage capacities. The total tankage used for storing product p must not exceed the limit R_{lp}. If an investment w_{lt} is made at that location l at some previous period $t' \leq t$, the site operates at its new storage capacity RK_{lp}.

$$z^{\xi}_{lpt} \leq GR_{lpt} R_{lp}\left(1 - \sum_{t' \leq t} w_{lt}\right) \quad \forall l \in L_B \cup L_S, t \in \mathcal{T} \tag{2.9}$$

$$zk^{\xi}_{lpt} \leq GRK_{lpt} RK_{lp} \sum_{t' \leq t} w_{lt} \quad \forall l \in L_B \cup L_S, t \in \mathcal{T} \tag{2.10}$$

$$z^{\xi}_{lpt} + zk^{\xi}_{lpt} = \sum_{a|j=l} \left(xd^{\xi}_{apt} + xi^{\xi}_{apt} \right) + \sum_{a|i=l} \left(xd^{\xi}_{apt} + xi^{\xi}_{apt} \right)$$

$$\forall l \in L_B \cup L_S, t \in \mathcal{T} \qquad (2.11)$$

Constraints 2.9 to 2.11 model product movement through locations. Constraint 2.9 limits the total throughput z^{ξ}_{lpt} by the product of the tank reference rotation GR_{lpt} times the total available tankage R_{lp}. In the event that the model chooses an investment w_{lt} at period t, the throughput, which from the period t becomes represented by zk^{ξ}_{lpt}, is subject to the new reference rotation GRK_{lpt} times the new tankage RK_{lp} as represented in Constraint 2.10. Constraint 2.11 consolidates the flow of inputs and outputs (direct and reverse flows) into the respective variables. The binary characteristic of investment variables implies that $z^{\xi}_{lpt} = 0 \vee zk^{\xi}_{lpt} = 0, \forall l \in L_B \cup L_S, \forall p \in \mathcal{P}, \forall t \in \mathcal{T}, \forall \xi \in \Omega$.

$$\sum_{a|i=l} xd^{\xi}_{apt} + \sum_{a|j=l} xi^{\xi}_{apt} = e^{\xi}_{lpt} \quad \forall l \in L_E, \forall p \in \mathcal{P}, \forall t \in \mathcal{T} \qquad (2.12)$$

$$e^{\xi}_{lpt} \leq LE_{lpt} \quad \forall l \in L_E, \forall p \in \mathcal{P}, \forall t \in \mathcal{T} \qquad (2.13)$$

$$\sum_{a|j=l} xd^{\xi}_{apt} + \sum_{a|i=l} xi^{\xi}_{apt} = i^{\xi}_{lpt} \quad \forall l \in L_E, \forall p \in \mathcal{P}, \forall t \in \mathcal{T} \qquad (2.14)$$

$$i^{\xi}_{lpt} \leq LI_{lpt} \quad \forall l \in L_E, \forall p \in \mathcal{P}, \forall t \in \mathcal{T} \qquad (2.15)$$

For foreign trade partners, the amounts traded are represented by specific variables: e^{ξ}_{lpt} represents exports while i^{ξ}_{lpt} represents imports. Constraints 2.12 and 2.14 define the flows to and from foreign locations l as these variables. Constraints 2.13 and 2.15 bound the amounts of exports and imports, respectively.

$$f^{\xi}_{slt} \leq FS_{slt} \left(1 - \sum_{t' \leq t} w_{lt} \right) \quad \forall s \in \mathcal{S}, \forall l \in L_S, \forall t \in \mathcal{T} \qquad (2.16)$$

$$fk^{\xi}_{slt} \leq FSK_{slt} \sum_{t' \leq t} w_{lt} \quad \forall s \in \mathcal{S}, \forall l \in L_S, \forall, \forall t \in \mathcal{T} \qquad (2.17)$$

$$\sum_{s} f^{\xi}_{slt} + \sum_{s} fk^{\xi}_{slt} = \sum_{a|j=l \wedge m \in M_M, p} \left(xd^{\xi}_{apt} + xi^{\xi}_{apt} \right)$$

$$\forall l \in L_S, \forall t \in \mathcal{T} \qquad (2.18)$$

The amount of product handled in marine terminals, which is subject to demurrage costs, is modeled by Constraints 2.16 to 2.18. The demurrage costs increase nonlinearly as a function of the amount handled. Therefore, a piecewise linearization was performed along the demurrage curve dividing it into $|S|$ segments s, each with its respective linear cost. Constraints 2.16

and 2.17 bound the amount f^{ξ}_{slt} handled for each segment. Once again, given an investment w_{lt} at period t, the amount handled, which from the period t becomes represented by fk^{ξ}_{slt}, is subject to the new limit FSK_{slt}. Constraint 2.18 consolidates the terminal input flows undertaken by the maritime modal $m \in M_M \subseteq \mathcal{M}$ in the respective handling variables. The binary characteristic of investment variables implies that $f^{\xi}_{slt} = 0 \vee fk^{\xi}_{slt} = 0, \forall s \in \mathcal{S}, l \in L_S, \forall t \in \mathcal{T}, \forall \xi \in \Omega$.

In the following chapter we present in detail the techniques used to generate scenarios and obtain solutions for the stochastic problem for the present model. Furthermore, we will use the model proposed in this chapter to evaluate a real case study on the Brazilian petroleum product supply chain.