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**Analysis of pneumatic structures considering
nonlinear material models and pressure–volume
coupling**

TESE DE DOUTORADO

DEPARTAMENTO DE ENGENHARIA CIVIL
Programa de Pós-Graduação em
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July 2012



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Thesis presented to the Programa de Pós-Graduação em Engenharia Civil of the Departamento de Engenharia Civil, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Engenharia Civil.

Advisor: Prof. Deane de Mesquita Roehl
Co–Advisor: Prof. Kai-Uwe Bletzinger

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Abstract

Coelho, Marianna Ansiliero de Oliveira; Roehl, Deane de Mesquita; Bletzinger, Kai-Uwe. **Analysis of pneumatic structures considering nonlinear material models and pressure–volume coupling**. Tese de Doutorado — Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro.

In this work a study of pneumatic structures considering pressure–volume coupling under plastic and viscoplastic material behavior is developed. Pneumatic structures are membrane structures acted on by air or gases stabilized by tension. These structures are lighter than conventional structures resulting in economic structural solutions. They present also some characteristics that contribute to the sustainable development, such as the utilization of natural lighting and ventilation and its possibility of reuse. When pneumatic structures are subjected to external loads these structures present both internal pressure and volume variation. This coupling is one of the objects of the present work. Analytical solutions are developed to describe this coupling. In conventional finite element systems this coupling is not considered. A formulation for pressure–volume coupling by closed chambers is included in the framework of a finite element large strain model. The variety of material models implemented has the purpose to cover the behavior of the many kinds of membrane materials used in pneumatic structures. In the literature the study of the membrane materials for pneumatic structures focuses on experimental analysis. Membrane material models are incorporated in the finite element model for small and large strains. The constitutive material models considered in this work are hyperelastic, elastoplastic and elastoviscoplastic. The onset of large strains is enclosed. A new material model based on NURBS surfaces is proposed and validated on hand of experimental results and classic material models. In this work emphasis is given to the material ETFE (Ethylene tetrafluoroethylene), which is widely used in pneumatic structures. The models developed here, such as the pressure–volume coupling and the material models, are implemented in finite elements on the program used in the Static Chair at TUM (Technische Universität München), which is called CARAT++ (Computer Aided Research Analysis Tool).

Keywords

Pneumatic structures; Material models; Finite element method; Pressure–volume coupling; Large strains; NURBS material

Resumo

Coelho, Marianna Ansiliero de Oliveira; Roehl, Deane de Mesquita; Bletzinger, Kai-Uwe. **Análise de estruturas pneumáticas considerando modelos não lineares do material e o acoplamento pressão–volume.** Tese de Doutorado — Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro.

Neste trabalho um estudo de estruturas pneumáticas considerando acoplamento pressão–volume e modelos constitutivos plásticos e viscoplásticos são desenvolvidos. Estruturas pneumáticas são estruturas de membrana sobre as quais atuam pressão de gases estabilizadas por tensões de tração. Essas estruturas são mais leves que estruturas convencionais resultando em soluções mais econômicas. Elas possuem ainda algumas características que contribuem para um desenvolvimento sustentável, como a utilização de luz natural e ventilação e a possibilidade de reutilização. Quando as estruturas pneumáticas são submetidas a cargas externas, essas estruturas apresentam variação da pressão interna e do volume. Este acoplamento é um dos objetos de estudo do presente trabalho. Soluções analíticas são desenvolvidas para descrever este acoplamento. Em programas convencionais de elementos finitos esse acoplamento não é considerado. Uma formulação para o acoplamento pressão–volume para câmaras fechadas é incluído no modelo de elementos finitos com grandes deformações. A variedade de modelos de material implementados tem a finalidade de abranger o comportamento de muitos tipos de materiais de membrana usados em estruturas pneumáticas. Na literatura o estudo dos materiais de membrana para estruturas pneumáticas tem foco na análise experimental. Modelos para material de membrana são incorporados no modelo de elementos finitos para pequenas e grandes deformações. Os modelos constitutivos considerados neste trabalho são hiperelástico, elastoplástico e elastoviscoplastico. A ocorrência de grandes deformações é incluída. Um novo material baseado em superfícies NURBS é proposto e validado com base em resultados experimentais e modelos clássicos de materiais. Neste trabalho ênfase é dada ao material ETFE (Etileno tetrafluoretileno), o qual é amplamente usado em estruturas pneumáticas. Os modelos desenvolvidos aqui, como o acoplamento pressão–volume e os modelos de materiais são implementados em elementos finitos no programa usado na cadeira de estática das construções da TUM (Technische Universität München), chamado CARAT++ (Computer Aided Research Analysis Tool).

Palavras–chave

Estruturas pneumáticas; Modelo de material; Método dos elementos finitos; Acoplamento pressão-volume; Grandes deformações; Material NURBS

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