3.
The Truth Table as a notational means

> „...Die große Frage ist jetzt: wie muss ein Zeichensystem beschaffen sein, damit es jede Tautologie auf eine und dieselbe Weise als Tautologie erkennen lässt? Dies ist das Grundprobleme der Logik!"
> Wittgenstein, Brief an Russell, 1913, *Tagebücher* 14-16, p.128

My aim in this chapter is to analyze how Wittgenstein´s *Tractatus* can be reconstructed by the presuppositions and consequences of the assumption of the truth table as a notational system more suitable for the expression of propositions. This notation was inaugurated in the tractarian period as *WF Notation* (for example, in 4.442 for example Wittgentein call it *WF Schemata*). This special notation provides a means by which, in one movement, we could avoid philosophical confusion and linguistic nonsense and mirror the deep syntax of our language. The former outcome is actually a consequence of the latter. In doing so, my aim is to accommodate the tractarian project inside a Leibnizian intuition about the role of symbolic systems: Our symbols can be rather firm threads to the inner side or essence of the things than inevitable obstacles to it. This transparent language could then substitute the insistence of a very vague criterion for logic, making sense of the line of Wittgenstein´s criticism on Russell and Frege in *Tractatus*: "Das Einleuchten, von dem Russell so viel sprach, kann nur dadurch in der Logik entbehrlich werden, dass die Sprache selbst jeden logischen Fehler verhindert.—Dass die Logik *a priori* ist, besteht darin, dass nicht unlogisch gedacht werden *kann"* 5.4731. Following this intuition, a proper language should then prevent us from logical mistakes. The notation itself should eradicate logical misuses.

Ramsey, in his review of the *Tractatus* in 1923, had already recognized the truth table notation as an improvement in certain aspects of the expression of dependence between propositions and their operators in comparison with the notation of *Principia*, stating:

"It may, of course, be doubted whether it is possible to formulate this rule as it seems to presuppose the whole of symbolic logic; but in any perfect notation it might be possible; for example in Mr. Wittgenstein´s notation with T´s and F´s there would be no difficulty." (Ramsey, p.472)

Or also in this passage:

"But in a perfect language in which each thing had its own one name, that in the sense of a sentence a certain object occurred, would be also shown visibly by the occurrence in the sentence of the name of that object; and this might be expected to happen with regard to all internal properties of senses; that one sense, for example, is contained in another (i.e. one proposition follows from another) might always appear visibly in the sentences expressing them. (This is nearly achieved in Mr. Wittgenstein´s T notation). Thus in a perfect language all sentences or thoughts would be perfectly clear". (Ramsey, p.476 - 77)

3.1
A Perfect Language or Notation?: Searching for a Prospective Instrument

There is no problem, I believe, in speaking, as does Ramsey in the passages above, of a perfect language, if we think about a search for a perfect notation to express the already perfect logical syntax of current language. The search for a better expression of certain logical relations through a more perspicuous artificial language becomes clear when we think that a satisfactory notation to be developed could guide us in the way of contemplating and acknowledging logical truths. Thus, we would not need to postulate logical truths. We should just be able to build a perspicuous system of signs the rules for the construction of which could systematically show the syntactic rules of our language or of any possible language, as ingeniously projected by the young Wittgenstein. We would then have a kind of indirect approach to the logic of our language based on the construction of a prospective means that would reveal the syntactic complexity by its notational rules and expressive power, when its signal were to be applied. Such an approach is clearly outlined by Wittgenstein in the *Tractatus*:

„Daraus ergibt sich, dass wir auch ohne die logischen Sätze auskommen können, da wir ja in einer entsprechenden Notation (notação, notação, notation, notation)23 die formalen Eingenschaften der Sätze durch das bloße Ansehen dieser Sätze erkennen können. 6.122.

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23 Dos Santos (1993) in his translation of the *Tractatus* into Portuguese translates the different German terms Ausdrucksweise, Notation, Zeichensprache or even Sprache (3.325) as notação. Although I believe there is no loss to the text, it is difficult to justify why Zeichensystem is translated as sistema de sinais and not as notação as well. (5.475) In this way, in order to compare or build a perspicous vision of the possible tranlations for our disposal, from now on, besides using the more recent translation in Portuguese in brackets, I will also present Gianotti's translation (first Portuguese version in 1968), followed by the translation by Pears and McGuinness (1961), and the first translations into English by Ogden and Ramsey (1922), respectively. This survey is interesting to grasp the extent to which the translations diverge or converge in the context of a metaphysics of symbolism.

According to this symbolic ideal and just as Wittgenstein explicitly defends in 6.1223, we can have every logical law, if we can construct a proper notation: „Nun wird klar, warum man oft fühlte, als wären die „logischen Wahrheiten „von uns zu „fordern": Wir können sie nämlich insofern fordern, als wir eine genügende Notation [notation in all translations] fordern können." We would then need an ideal notation to silently reveal the deep syntactic rules of our language so that linguistic confusion could be systematically avoided, because a poor application or use of language could be automatically denounced. This should be clearly visible through the misuse of the notational system.

According to this sort of symbolic Leibnizian ideal, in the *Tagebücher* 14-16, the first entry about the notational elements of a language was made on 26/11/1914:

"Indem wir zwar in jeder möglichen Notation zwischen ~aRb und ~bRa unterscheiden, setzen wir in jeden eine bestimmte Zuordnung von Argument und Argumentstelle im negativen Satz voraus; die ja das Urbild des verneinten positiven Satzes ausmacht." p. 33.

This passage clearly shows the commitment with an essential feature of any language, which should be shown in a more appropriate notation. In this case, the necessary imbrication between positive and negative propositions. This relationship between key aspects of language and the expressive capacity of a notation is evidenced in annotations of 29/11/1914, now with respect to the lack of a need for the expression of identity sign in a notational system. An identity should be expressed exclusively by the equality of signs. As a result, to express the difference between two objects, it would then be enough to elegantly show them with different signs: "Ich glaube, man könnte das Gleicheitszeichen ganz aus unserer Notation entfernen und die Gleichheit immer nur durch die Gleichheit der Zeichen (u.u.) andeuten". On the same day, Wittgenstein still holds: "durch diese Notation verlören auch der Scheinsatz (x)x=a oder ähnliche allen Schein von Berechtigung." (p.34).

This shows us that the demand that the rules of a notation could prevent us from the formulation of pseudo-problems is an old idea in his philosophical journey. In response to Russell's letter of 30/10/1913, Wittgenstein makes it clear that the precursor to the truth table, the schemes of T and F, are a kind of

improvement of the already promising *AB Notation*: "(…) If you had only remembered the WF Scheme of ~p you would never have asked this question (I think). In fact all rules of the ab symbolism follow directly from the essence of the WF Scheme". This is the (insolent) response of Wittgenstein made to Russell´s subsequent question: "If apb is the symbol for p, is bpa the symbol for ~p? And if not, what is?" p.125. Wittgenstein was searching for a proper prospective means in the pre-*Tractatus* period and the development of a perspicuous notation follows the very development of his ideas.

The first mention in the *Tractatus* of a notational system occurrs in passage 3.325:

„Um diesen Irrtümern zu entgehen, müssen wir eine Zeichensprache [notação, linguagem simbólica24, sign-language, symbolism] verwenden, welche sie ausschließt, indem sie nicht das gleiche Zeichen in verschiedenen Symbolen, und Zeichen, welche auf verschiedene Art bezeichnen, nicht äußerlich auf die gleiche Art verwendet. Eine Zeichensprachen also die der logischen Grammatik __ der logischen Syntax __ gehorcht. (Die Begriffsschrift [ideografia, ideografia, conceptual notation, logical symbolism] Freges und Russells ist ein solche Sprache [notacao, linguagem, language, language], die allerdings noch nicht alle Fehler ausschließt.)".

In this passage, the general idea clearly becomes a project that connects the attempt to build an ideal notation with the possibility of avoiding the linguistic excesses, abuses and confusions of Philosophy. In this particular ideal notation we could then remedy our language from Philosophy, by making clear the syntactic rules of language: bringing to light the profound, hidden, logic of language. For example, the rule against using different symbols in the same way and against using the same symbol for different tasks. It shows how the young Wittgenstein rightly followed the logicist tradition represented by Russell´s idea that the grammar of everyday language does not necessarily reflect the logical syntax of language and that this can lead us to erroneous analysis (cf. 4.0031). This seems to be the first and only positive mention of Russell´s work in the *Tractatus*. In this context it seems natural to require a notation to be the safe thread and criterium for a more perspicuous analysis. Accordingly, Hintikka & Hintikka, articulate the notational ideal with the concept of logical form:

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24 Gianotti´s translation of *Zeichensprache* into "linguagem simbólica" is adequate in relation to the German, but redundant in relation to the *Tractatus*, because every language is symbolic there.

„Der Begriff der logischen Form ist, wie Frank Ramsey schon 1923 zutreffend hervorgehoben hat, eine der wichtigsten Ideen des *Tractatus*. Wichtig daran ist unter anderem, dass sie das Ziel jeglicher philosophischen Analysen spezifischer Begriffe bestimmt. Bei einer solchen Analyse muss man demnach so verfahren, dass für die betreffenden Begriffe eine Notation entwickelt wird derart, dass alle notwendigen Verbindungen zwischen ihnen zu logischen Wahrheiten und letzten Endes zu wahrheitsfunktionalen Tautologien werden, die sich ihrerseits an den rein notationsgebundenen Merkmalen unserer Sätze als solche erkennen lassen." (p.157)

Thus it is justifiable to bet that a notational system could be systematically improved to better expose the syntactic "innards" of language, thus excluding errors. As Wittgenstein strives to denounce throughout the *Tractatus*, the notation of Frege and Russell is not sufficient for this purpose. The project of developing a more suitable notation or a prospective instrument is pursued by Wittgenstein from the *Tagebücher* 14-16 with the AB notation, through the truth tables of the *Tractatus*, up until the early '30s, when he tries to think about alternatives that could follow the logical multiplicity of phenomena, such as the case of the octahedron in the case of colors system. Here we have a good criterion to decide the extent to which this middle period of his Philosophy is still somewhat tractarian. The search for a notation naturally accompanies the tractarian project of a prospective logical approach to language, as I defend here.

The idea of an essential core to all notations or of strictly necessary aspects for every notational system appears explicitly in 3.342:

„An unseren Notationen [all translation are the same] ist zwar etwas willkürlich, aber das ist nicht willkürlich: Dass, wenn wir etwas willkürlich bestimmt haben, dann etwas anderes der Fall sein muss. (Dies hängt von dem Wesen der Notation [all translation are the same] ab.)"

In this passage, the possibility of several different notations is also clear, i.e. the possibility of a plurality of possibly competing notational systems with distinct expressive power and accuracy that would accomplish the task of being a more or less suitable instruments for the purpose of a perspicuous analysis. However, I maintain that Wittgenstein continued in his search for the ideal notation throughout this time.

The possibility of multiple notational systems still appears explicitly in the middle phase of Wittgenstein's Philosophy when, unlike in the *Tractatus*, he tries to show that the representation of the deductive procedure directly depends on the response to the questions with which (or in which) notation we are operating, or in

which symbolic system we are pursuing our calculations. The tautology is shown then as something of collateral importance. In this phase, the emphasis on our logical system must then be in our notation, which would determine the limits of what can be legitimately expressed through its exclusions and implications. We could, in principle, have different notations for different kinds and forces of implications and exclusions in different systems. From this we could then have different forms of deduction directly dependent upon the kind of notation we use. Discussing this point in conversations with the Vienna Circle, Wittgenstein said:

"*Die Tautologie ist ja ganz nebensächlich.* Nur in einer bestimmten Notation stellt sich der Schluss als Tautologie dar. Wesentlich sind nur die Regeln der Syntax, die man ja immer angewendet hat, längst bevor man wusste, was eine Tautologie ist. (…) In meiner Notation [the truth table] zeigt sich nun die Richtigkeit des Schlusses daran, dass „p→q" eine Tautologie wird. Aber es ist absolut unnötig, die Richtigkeit des Schlusses gerade auf diese Art zu zeigen. Ebenso gut zeigt sich die Richtigkeit eines Schlusses an den üblichen Regeln des Schließens. *Es ist das nur eine von verschieden möglichen Notationen, die vielleicht nur den Vorteil hat, dass sie de Sache klare sehen lässt.* Aber an sich leisten die Russellschen Zeichen zusammen mit den Regeln ihrer syntaktischen Verwendung dasselbe. Dass der Schluss *a priori* ist, heißt nur, dass die Syntax darüber entscheidet, ob ein Schluss richtig ist oder nicht. Die Tautologie ist nur eine Art, um das Syntaktische zu zeigen." (91-92, WWK) my italics.

The assertion that tautologies are indeed irrelevant is blatantly at odds with the comparison to the Philosophy of Logic made in the *Tractatus*, which fully reduced logic to tautologies. In this passage, it is also interesting to note that the ideal of an ideal notation is replaced by the ideal of a notation that could be combined, directed and supplemented by the syntactical or grammatical grammar rules of a particular system. Thus the notation of *Principia* would not have to be replaced by a more perspicuous, or even the most perspicuous notation, the tractarian motto. We should only know the rules that underlie a system (color, length, volume, temperature...) and try to express them clearly in our notation, case by case. The movement here, unlike in the *Tractatus*, is clearly *ad hoc* and not normative. It then seems natural to think that some notations might be more appropriate for a system of propositions than others.

This demand for adequacy of the structural symbolic power of a symbolic system in the domain to be symbolized appears in the sequence of passages 3.33 - 3.334. The notational system does not need to address the meaning or semantics of their signals, but only assume that these must stand for something that through its descriptive power can capture the combinatorial possibilities of external

elements to the notation. Wittgenstein believed this to be one of Russell's (many) errors. Wittgenstein started to talk about the meaning of signs in order to establish his ideography. Subsequently, for Wittgenstein, not the whole language is ineffable. Semantic relations may remain ineffable, while its syntactic structure could be systematically revealed, showed, or exhibited by the rules of a perspicuous notational system. Accordingly, Hintikka & Hintikka write that:

"Die Syntax der Sprache dagegen lässt sich durchaus sprachlich zum Ausdruck bringen und erörtern. Ja im Grunde Laufen die im *Tractatus* vertretenen Ansichten auf einen kräftigen Ansporn hinaus, sich mit der "logischen Syntax der Sprache" zu befassen." (Hintikka & Hintikka, p.28).

Thus, we are forced to take a purely syntactic or formal approach to language, since the semantic sphere of the investigation is banned25. This partial possibility of meta-language, i.e., a notation representing the structural rules of language in the *Tractatus*, is also highlighted by Cuter in his article on the denial of names. The semantic sphere is banned from treating formal concepts, such as naming or conditions of truth of a proposition but, in principle, we could, indeed, tackle the rules of formation, and of syntactic articulations of language, particularly when we deal with the development of a notational system. Cuter subsequently comments on the important passage 3.33, writing:

"The first thing to notice in this passage is that Wittgenstein admits without hesitation the possibility of (i) establishing the logical syntax of a language and (ii) describing this syntax. If we want to call this kind of description "metalanguage" then we must recognize that the *Tractatus* has never excluded the possibility of a "metalinguistic" discourse. What is excluded is the possibility of referring to the relationship between name and object that it names. Language as combinatorial order to be observed in the use of certain signs is as describable as the game of chess or music theory. The *Tractatus* fits very exactly into what we call, in syntax, "rules of formation." The problem would arise only at the level of semantic rules. What we cannot definitively say is that "'snow' means snow," or that "'Snow is white' is true if (and only if) the snow is white". "Truth" and "meaning" are what the *Tractatus* calls "formal concepts". They are constitutive of the language, but cannot be an object of linguistic description." (Cuter, *Nomes*, p.72-73)

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25 Through his reaction to the *Color Exclusion Problem* in the middle period of his thought, Wittgenstein develops the concept of grammar that brings with itself semantic notions to the investigations of language. The negation is a syntactical device in the *Tractatus* that, at least, in some cases of propositions presupposes a semantic field, just like in the case of negated propositions about colors or measurements. Commenting on this direction, Hintikka and Hintikka claim that: „Nach dieser Konzeption stimmen die zulässigen Kombinationen der Symbole in einer vollständig analysierten Sprache mit den möglichen Konfigurationen der von ihnen dargestellten Entitäten überein. Kurz, in einer logisch analysierten Sprache entspricht die Grammatik der Ontologie. Und von dieser These ist es dann kein weiter Schritt mehr bis zu der Vorstellung, dass die Grammatik einer Sprache auch ihre Semantik reflektiert." p.30 We will come to this movement from a purely syntactic tractarian logical space to semantic and pragmatic notions of grammar as complete systems in the middle period in the last chapter of this investigation.

In passage 3.342 we clearly have a certain kind of tractarian motto that pervades the entire work: the idea that all possibilities are necessary. This feature is apparent in the ontology, and in *Bildkonzeption*. It is also apparent in the metaphysics of symbolism of the *Tractatus*26, as evidenced in the following passage:

„Eine besondere Bezeichnungsweise [modo de designacao, modo de designacao, mode of signifying, method of symbolising] mag unwichtig sein, aber wichtig ist es immer, dass diese eine *mögliche* Bezeichnungsweise ist. Und so verhält es sich in der Philosophie überhaupt: Das Einzelne erweist sich immer wieder als unwichtig, aber die Möglichkeit jedes Einzelnen gibt uns einen Aufschluss über das Wesen der Welt. (...)" (3.3421).

The mere possibility of a singular nods to an essential sphere of the world and language. The possibilities are always in the *Tractatus* as if they were immersed, inscribed, or fully planned in a system of exhaustive and eternal articulations. Thus, we learn something about the essence of a domain by knowing that one of its constituents is possible. And logic should somehow engage itself fundamentally with such possibilities. Logic would guide us from the contingent actualities to a necessary totality. In the *Tractatus*, logic always has a modal aception. It is always engaged with the necessary possibility of articulation of elements. The double modalization (a necessary possibility) here is important to *Tractatus*. The emphasis of philosophical investigation on logic would not be in the invention of notational systems themselves or expressions of syntactic relations between propositions, but with the possibility itself of this invention, which teaches us about the essence of language and the world:

"Es ist klar, wir haben vom Elementarsatz einen Begriff, abgesehen von seiner besonderen logischen Form. Wo man aber Symbole nach einem System bilden kann, dort ist dieses System das logisch wichtige und nicht die einzelnen Symbole. Und wie wäre es auch möglich, dass ich es in der Logik mit Formen zu tun hätte, die ich erfinden kann; sondern27 mit dem muss ich es zu tun haben, was es mir möglich macht, sie zu erfinden." 5.555

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26 That tractarian themes reappear in different parts of the book is not accidental nor an instance of an essential harmony between ontology and language. It is, rather, evidence or a product of a radical holism that will be outlined later in the last chapter. The understanding of a singular in reference to a whole (whose possibility is also necessary) is a mark of tractarian holism that is radicalized in the intermediate phase.

27 The Portuguese translation of *sondern* for "pelo contrário" makes sense by Dos Santos translation, but it is much stronger than the usual "mas sim".

This learning of general lessons from particular elements, or of necessities when we infer the mere possibility of something, also appears again in the context of debate about different notational systems at the end of 3.3441, with the conclusion in parentheses. „(Hiermit ist die Art und Weise gekennzeichnet, wie eine spezielle mögliche Notation [notação possível particular, notação especialmente possível, possibility of a specific notation, a special possible notation] uns allgemeine Aufschlüsse geben kann.)" In this passage, Wittgenstein deals with subsitutivity as an essential mark of all the notations that aim to express truth-functionality. They should always allow translations (or reductions in the case of more primitive notations) from one into another. What comes to mind here is the analogy of the case of one fragment of language which, although only possessing negation and disjunction, is able to generate, by truth-functional completeness, all the results of a language with all conventional logic operators. Wittgenstein outlined the case in the following way:

> „Man kann z.B. das Gemeinsame aller Notationen [the same for all translations] für die Wahrheitsfunktionen so ausdrücken: Es ist ihnen gemeinsam, dass sich alle __ z.B. __ durch die Notation von „~p" („nicht p") und „p∧q" („p oder q") ersetzen lassen. (Hiermit ist die Art und Weise gekennzeichnet, wie eine spezielle mögliche Notation uns allgemeine Aufschlüsse geben kann.)" 3.3441

Here we have a clear preview of the contemporary notion of truth-functional completeness. This example of a rule of subsitutivity as essential for the translation of a correct notation into another illustrates the possibility of translation being carried out systematically from definitions. In principle, we can dispose of a multitude of distinct notational systems, as far as they are interchangeable. As Wittgenstein states in 3.343: „Definitionen sind Regeln der Übersetzung von einer Sprache in eine andere. Jede richtige Zeichensprache [notação, linguagem simbólica, sign-language, symbolism] muss sich in jede andere nach solchen Regeln übersetzen lassen: Dies ist, was sie alle gemeinsam haben."

We can argue that the tractarian project fits well within the context of a kind of language optimism where the elements of a notational system can and should be adequately articulated to lead us to the interior of language so that our philosophical confusions dissolve. This notational system with its rules of formation can cover the possible articulations of language. As a tool, the prospective notation can, in principle, penetrate this layer of natural language to

reveal its essentialities, such as its very figurative essence. Indeed, in 4.011 Wittgenstein advocated a prospective application of notations in this way:

> „Auf den ersten Blick scheint der Satz __ wie er etwa auf dem Papier gedruckt steht __ kein Bild der Wirklichkeit zu sein, von der er handelt. Aber auch die Notenschrift scheint auf den ersten Blick der Musik zu sein, und unsere Lautzeichen- (Buchstaben-) Schrift kein Bild unserer Lautsprache. Un doch erweisen sich diese Zeichensprachen [notaçoes, linguagens simbólicas, sign-languages, symbolims] auch im gewöhnlichen Sinne als Bilder dessen, was sie darstellen28.“

We know that an important feature of the tractarian metaphysics of symbolism is that the language in principle can represent the whole reality or that facts can always be remitted to other facts, but it cannot represent what makes this representation possible, i.e. the community in the logical form between the represented and the representation. The tractarian distinction between saying and showing comes, among other things, to fulfill the role of clearly defining this impossibility of representation. Wittgenstein argues that what can be shown cannot be said. As the saying of something corresponds to the sphere of bipolarity, sense and contingency, it cannot cover the necessary sphere of language, for example, the sphere of the necessary notation. Something necessary to the language cannot be formulated meaningfully. Rather, it should be evidenced by its syntax. This is clear in this period. Syntax must be systematically exhibited by a notational system.

This requirement for transparency and perspicuity in the presentation of the syntactic rules of language functions as the test itself of legitimacy of a notational system, as we see in 4.1213: „Jetzt verstehen wir auch unser Gefühl: dass wir im Besitze einer richtigen logischen Auffassung seien, wenn nur einmal alles in unserer Zeichensprachen [notação, linguagem simbólica, sign-language, symbolism] stimmt.“ The role and importance of a suitable notation here is

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28 Here we notice a kind of incongruency in the Brazilian translation of Wittgenstein´s text. In 4.011, Zeichensprachen is translated into Portuguese as "notação", but in 4.1121 it is translated as "linguagem por sinais". "(...) Meu estudo da linguagem por sinais (Zeichensprache) não corresponderia ao estudo dos processos de pensar, estudo que os filósofos sustentaram ser tão essencial para a filosofia da lógica? No mais das vezes, eles só se emaranharam em investigações psicológicas irrelevantes, e um perigo análogo existe também no caso do meu método." Ambiguities of this kind between current language and artificial language appear in the Portuguese version that do not appear in the original text. Rather, there is the possibility of reading it as "an investigation of the language by means of or via signals", and that is not in the original German either. It would be important to have a unique word in Portuguese for the German word "Zeichensprachen", since the context in which it appears in the original is always about the same issue: "Zeichensprachen" are used for an artifical notational system, as a instrument of analysis. Despite of the previously alluded to problem of redundance, Gianotti always uses the same "linguagem simbólica" for Zeichensprachen.

obvious. If we have a perspicuous notation we have a logically correct concept, so that old misunderstandings are removed and new ones are avoided. The contrapositive here also appears revealing. If we do not have a logically correct concept, we will not be able to build a suitable notational system. One needs to have a correct view of the logic of our language. At first glance this seems to be the tractarian project, to establish a correct view of things, so we can then, through a notation built from this philosophical foundation, systematically avoid linguistic confusion. In passage 6.54, it is clear that the tractarian project is more ambitious than just bringing the correct view of the functioning of language to the reader. A correct view or posture towards the world (in all its vicissitudes) is also at stake:

„Meine Sätze erläutern dadurch, dass sie der, welcher mich versteht, am Ende als unsinnig erkennt, wenn er durch sie – auf ihnen – über sie hinausgestiegen ist. (Er muss sozusagen die Leiter wegwerfen, nachdem er auf ihr hinaufgestiegen ist.) Er muss diese Sätze überwinden, dann sieht er die Welt richtig.“

3.2.
Notation: Criterion or Result of a propositional analysys?

In the context, right after the recognition of the deadlocks of the *Color Exclusion Problem*s for the tractarian logic, the notation seems to be rather more a result of the complete analysis than a criterion for this. This is contrary, I believe, to the apologetic movement of Wittenstein´s pre-*Tractatus* writings, where it seems clear to me that the notation was much more of a criterion for the application of logic than its outcome. A notation would be the guide for the realization of the project of a complete analysis of the body of empirical propositions, leaving their logic anatomy transparent so that the notation would be an ultimate test of the meaningfulness of propositions. What cannot be expressed through the correct notation would be neither meaningful nor relevant. Here we may have a difference between the Philosophy of symbolism in the tractarian phase and the intermediate with regards to the problems of color: a symbolic system, which was previously a criterion, becomes the result of a perspicuous analysis. Although we continue with the general tractarian project of fully analysing language, the accent in the middle period should be on the search for a greater expressive sensitivity to capture the multiplicity of phenomena. The

normative cut in the tractarian phase proved to be too drastic. Wittgenstein expressed less than he should have in its normative radicalism. The tractarian logic under-generates the real sphere of what can be held as logical. It is as if, after the *Color Exclusion Problem*, Wittgenstein actually dropped out of the tractarian sight of purist logic. In other words, a notation would be led by the investigation of phenomena before guiding it. Now the notation must be accountable to conceptual arrangements of empirical systems, like the one of the colors.

By contrast, even before the *Tagebücher*, we could also see the appeal to notational systems, which at that stage Wittgenstein called ab Notation, as criteria and not the result for meaning and sense in linguistic constructions. In a letter to Russell from 1913, Wittgenstein highlighted the tentative nature of his notation, clarifying that it was a work in progress and an ongoing search. In this passage we identify a clear belief in the possibility of the typology of propositions made mechanically by a more perspicuous notation compared to the *Principia*, wherein we can determine without doubt when a proposition belongs to logic or would make sense or not. There we have the ultimate criterion for sense and for being logic, without surprises or miscalculations. This is a clear an announcement of the project of the truth table as notation:

"... I beg you notice that, although I shall make use in what follows of my ab notation, the meaning of this notation is not needed; that is to say, even if this notation should turn out not to be the final correct notation what I am going to say is valid if you only admit _as I believe you must do _ that it is a possible notation. Now listen! I will first talk about those logical propositions which are or might be contained in the first 8 chapters of *Principia Mathematica*. That they all follow from one proposition is clear because one symbolic rule is sufficient to recognize each of them as true or false. And this is the one symbolic rule: write the proposition down in the ab notation, trace all connections (of poles) from the outside to the inside poles: Then if the b-pole is connected to such *groups of inside poles only as contain opposite poles of one proposition*, the whole proposition is a true, logical proposition. If on the other hand this is the case with the a-pole the proposition is false and logical. If finally neither is the case the proposition may be true or false, but it is in no case logical. The same is the case – by the way – with your axiom of reducibility, *it is not a logical proposition at all* and the same applies to the axioms of infinity and the multiplicative axiom. *If these are true propositions they are what I shall call "accidentally" true and not "essentially" true."* (*Tagebücher*, p.126). My italics

In addition, we can take this revealing passage as a guide for Russell as to the usage of this notation, where Wittgenstein provides a test of logicality of some axioms of the *Principia*. Some axioms of *Principia* would not be able to pass this

test, and then could not be held as purely logical. At this point, it is important to emphasize that the critique goes through the establishment of a perspicuous notation as a criterion of meaningfulness. In the following passage taken from notes dictated to Moore in Norway in 1914, where Wittgenstein deals directly with a possibility of error of the ab notation, we see how the image of the notation (more as a criterion for and less as a result of the analysis of propositions) is steeped in the search for a suitable notation. If our notation gives us a somewhat strange result we have to rearrange our way of seeing things with respect to this strange result, and not rearrange our notation in order to deliver us more plausible results. This is clearly a normative movement:

"It seems at first sight as if the ab notation must be wrong, because it seems to treat true and false as on exactly the same level. It must be possible to see from the symbols themselves that there is some essential difference between the poles, if the notation is to be right; and it seems as if in fact this was impossible.(...)" (*Tagebücher*, p. 113.)

If there is a need for change or correction, this will be not undertaken in the notation, but with our "illusory" account of some language features. The notation seems to impose, as a thread, a correction of our picture of language. That T and F are on the same level in the language is not a result that should be followed by the notation, but it is a criterion of meaningfulness for the legitimate language that the notation makes clear and imposes.

This aspect, more a criterion for than rather an outcome of the application of logic, appears explicitly in an entry from 27/10/1914, when Wittgenstein adopts a notation as the best proof test or for the correction of a remark about the essence of language, again in the context of discussions on the necessity or meaningfulness of the sign of identity: "Es ist ja klar, dass "aRa" gleichbedeutend wäre mit "aRb.a=b". Man kann also den Scheinsatz "a=b" durch eine ganz analysierte Notation zum Verschwinden bringen. Bester Beweis für die Richtigkeit der obigen Bemerkung." (*Tagebücher*, p.19)

A phenomenological language should also mirror or follow the syntax of logic (or the deep grammar of the language) and logical problems should be avoided, or rather systematically excluded. In this context, a phenomenological language seems much more to be a notation than a language in strict sense, because it should show and not say the logical multiplicity of the phenomena. The idea of searching for a hidden opaque background for the current language

through a more perspicuous notational system is still tractarian, despite the way the tractarian notation proves to be insufficient (as is later to be the case with the phenomenological language itself). It is unclear what the priority would be here in this domain: the analysis of phenomena or the development of a perspicuous notational system. Thus, in this middle phase, still, as we defend, with a tractarian project, it is unclear whether a notation, now phenomenological, would be decisively a criterion or an outcome of the complete analysis. Somehow it stays open whether these two features should accompany and complement each other. This tension in the task of leadership is also highlighted by Kienzler:

> „Andererseits braucht man zur Bewältigung dieser Aufgabe schon eine Sprache, in der man seine Forschungen festhalten kann. Die Philosophen können ja nicht darauf warten, dass auf einmal eine solche fertig gefunden wird. Dieses Problem artikuliert der Aufsatz allerdings nicht. Es bleibt in Wittgenstein Beschreibung der Fragestellung unklar, ob die phänomenologische Sprache in erster Linie dem Philosophen die Analyse der Phänomene ermöglichen soll oder ob sie vor allem als Darstellungsmittel für bereits vorliegende Analyseergebnisse gedacht ist." (Kienzler, p.283).

3.3
Finding the Propective Means

In principle, we would not need evidence (*Selbständigkeit*) as a criterion for logic, as Frege or Russell thought, if we had a notation that prevented logic errors and philosophical confusions, as the passage 5.4731 points out. We would not need evidence, if we had a language through which we could not deceive ourselves, because it was simple, übersichtlich, transparent, regular, without surprises, without hierarchy between generalities and more specific things, wherein each expedient would be justified, and wherein each symbol would have an unequivocally meaning, and each meaning would have a single symbol. We would have the ultimate prevention against giving the wrong sense for signs of language. We would have a criterion for logic more severe than mere evidence. In 5.47321, Wittgenstein, using and justifying Occam's motto not only as a principle of economy, but as notational principle, therefore writes:

> „Occams Devise ist natürlich keine willkürliche, oder durch ihren praktischen Erfolg gerechtfertigte, Regel: Sie besagt, dass unnötige Zeicheneinheiten [unidades notacionais, unidades de signo, units in a sign-language, elements in a symbolism] nichts

bedeuten. Zeichen, die Einen Zweck erfüllen, sind logisch äquivalent, Zeichen, die keinen Zweck erfüllen, logisch bedeutungslos."

In the important passage 4.5 Wittgenstein finally announces the general descriptive form of propositions: "Es verhält sich und so so" ["as coisas estão assim", "Isto está do seguinte modo", "This is how the things stand", "Such and such is the case"]. With this, we have the role that a notational system can play in this context. Having the general form of the proposition means being able to describe every possible proposition of any notation. Here I believe we can also steer our thoughts in the opposite direction, i.e. if one has the exhaustive descriptions of the propositions in a notational system, one automatically would have a means for reaching the general form of the propositions, thereby showing the descriptive essence of language.

The bet here is that all possible propositions could, in principle, be anticipated, reduced, or built from this essential form. Put another way, having this elementary basis, characterized by the general form, we can generate and delimit all the possible linguistic spheres, the entire sphere of possible propositional senses. The idea of totality and exhaustion is important here. The elementary basis gives us the possibility of having every possible proposition, without fault or spare. Therefore, it is important to have a notation free of logical problems, such as the possibility of synonyms in the use of symbols, or the occurrence of symbols that designate differently thereby showing us how we can systematically generate complex propositions from elementary ones.

Each symbol should bring with it a sense. Every sense should be expressible through a symbol. Here we have the case of an ideal of a perfect coverage, without ambiguities and synonymies among the symbols of an ideal notation to be built and the senses to be transmitted through the language. The description of any notation must be able to cover the propositional general form, i.e., capable of expressing all that can be expressed from and by it. As Wittgenstein also writes in this passage:

„Nun scheint es möglich zu sein, die allgemeinste Satzform anzugeben: das heißt, eine Beschreibung der Sätze *irgend einer* Zeichensprachen [notação, linguagem simbólica, sign-language, sign language] zu geben, so dass jeder mögliche Sinn durch ein Symbol, auf welches die Beschreibung passt, ausgedrückt werden kann, und dass jedes Symbol, worauf die Beschreibung passt, einen Sinn ausdrücken kann, wenn die

Bedeutungen der Namen entsprechende gewählt werden. Es ist klar, dass bei der Beschreibung passt(…)." 4.5

It is interesting to note that with each new essential feature of language displayed in the *Tractatus*, be it figuration, or compositionality, we always have some consideration about a notation or notational systems punctuating or following them. This seems to show that the construction of a suitable notation can facilitate or even enable a proper understanding of the functioning of language. This fact is also clear in passage 5.21 and a comment of 5.2, where Wittgenstein states that among the propositional structures there are internal relations. For Wittgenstein, these logical relationships are essential to the understanding and construction of complex propositions, without which they could not convey meaning. We then have the presentation of an essential feature of language followed shortly after by a notational aspect that enables the understanding of this feature:

„Wir können diese internen Beziehungen dadurch in unserer Ausdrucksweise [notação, modo de expressão, mode of expression, manner of expression] hervorheben, dass wir einen Satz als Resultat einer Operation darstellen, die ihn aus anderen Sätzen (den Basen der Operation) hervorbringt" (5.21)

This operation (its base and its result) systematically displayed in a suitable notation express the internal relations between the logical structure of propositions. The notation should be able to show how one proposition is generated through another, showing the internal relation which orders the construction of molecular propositions from elementary ones, in the spirit that guides the compositionalist tractarian period. So, as Wittgenstein affirms in 5.474, the number of necessary basic operations would depend only on our notation. By understanding the notational system, we understand the rules for the construction of propositions. Here the number of instances for this rule does not matter. Moreover, the number of basic operators and fundamental concepts in a system is not relevant, but only its generality and perspicuity, or the generality and perspicuity of its rules29.

The important thing here is to build a system of signs (sign system, *Zeichensystem*) that has a certain number of logical dimensions - a certain

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29 We could clearly use "notação" in the more recent Portuguese translation. This would preserve the philosophical context of searching for a more perspicuous notation.

mathematical multiplicity - as Wittgenstein argues in 5.475. This idea of the same logical complexity or coverage between the representation and the represented dominates Wittgenstein´s Philosophy at least until the middle period, even though many other aspects of his metaphysics of symbolism, such as the idea of Vertretung or non-synonymous, are weakened or even abandoned. Bento Neto clarifies this when he writes:

"This same lack [of the concept of *Vertretung*] is to be found in the notes taken by D.Lee, in the classes given by Wittgenstein in the early 1930s. There too, the concept of "figuration" is focused. Wittgenstein aims to delimitate the concept of propositional figurativity contrasting it with the concept of figurativity by resemblance: the proposition need not to be "similar" to anything, what is needed is that it has "the correct multiplicity" etc... " (Bento Neto, p.103)

Contrary to Bento Neto, I believe that less than an abandonment of the notion of *Vertretung* in the intermediary period we have, rather, the consolidation of the growing importance of the concept of logical multiplicity. There is clearly a kind of development of the concept of logical multiplicity already presented in the *Tractatus* (cf. 4.04) in this interim period. There it seems to lead to the idea of the equinumerosity and coextensionality between the represented and the representation, with the "Am Satz muss gerade soviel unterscheiden zu sein, als ander Sachlage, die er darstellt." Then we note that this notion marks the logical accent of the search for an ideal notation which respects the combinatorial horizon of the represented complex. In other words, its logical multiplicity should respect the logical multiplicity of the complex which is depicted, this means not only having the same number of relevant elements, but also expressing possible concatenations of the depicted elements and interdicting their impossible concatenations in the notation articulation itself.

This is already outlined or hinted to in passage 5.511 when Wittgenstein responds to questions about how the logic of signal handling may correspond to the windings and tricks (*speziellen Haken*) of the logical syntax of language. The answer lies in the notion of a *großen Spiegel*, of a large mirror, or a kind of complete syntactic coverage. Taking advantage of this image and the difference between sign and symbol we can better understand another note about the importance of notation in this context:

„~p" ist wahr, wenn „p" falsch ist. Also in dem wahren Satz „~p" ist „~p" ein falscher Satz. Wie kann ihn nun der Strich „~" mit der Wirklichkeit zum Stimmen bringen? Das, was in „p" verneint, ist aber nicht das „~", sondern dasjenige, was allen Zeichen dieser Notation [all translations are the same], welche p verneinen, gemeinsam ist. (...)" 5.512

Here we have the, so to speak, notational version of the tractarian *Grundgedanke* of 4.0312. We can thus prevent the denotation of logical operators because, for example, nothing is added to a proposition when we articulate it with the negation signal. For even if p is false, not-p does not differently articulate the proposition with the state of things. The possibility of the symbol ~p must already be contained in the symbol p. The sign has no meaning outside of a notational system. The idea is that it carries with itself all that is common to other signs of notation that may deny the symbol p. What is common to "~p", "~~ p", "~p and ~p" etc. reflects what is common to the denial in a suitable notation. It should show the way we constitute the negation of p or the construction of conjunctions between p and q, from the "loose" propositions "p" and "q", and so on, so from each rule we have the meaning of the buildable symbols, as Wittgenstein outlined in 5.514:

„Ist eine Notation [all translations are the same] festgelegt, so gibt es in ihr eine Regel, nach der alle p verneinenden Sätze gebildet werden, eine Regel, nach der alle p oder q bejahenden Sätze gebildet werden, u.s.f. Diese Regeln sind den Symbolen äquivalent und in ihnen spiegelt sich ihr Sinn wieder".

These considerations about the importance of a notational system, or rather of its development, indicate the relationship of the construction of a suitable notation with the very attempt to show the functioning of our language. I bet here, as a possible reading of the *Tractatus*, on the prominent role played by a suitable notation in exhibition of the profound rules of language with the immediate expected consequence: the dissolution of linguistic confusion. After all, just as Wittgenstein writes in 6.124, if we know the logical syntax of any notation, symbolic language or *Zeichensprache*, all the propositions of logic are already given. Moreover, if the *Fragestellung der philosophischen Problemen auf dem Mißverständnis der Logik unserer Sprache beruht*, we can, by a secure and perspicuous notational thread, thus avoid that which silently and without surprises shows this *Logik unserer Sprache*.

3.4
Truth Tables as an Exegetical Key

From this point on, I intend to take the metaphysics of truth table as the theoretical basis or main conceptual marker of the *Tractatus*. This concerns the proposal for a reading strategy that articulates the main tractarian theses, assumptions and concepts, as compositionality and combinatorial neutrality of the logic, justifying the failure and abandonment of the project of Wittgenstein´s youth. I argue that this hypothesis for reading is certainly rich enough to accommodate some central theses of the *Tractatus* about the paradigms that would form the nature of language in general. Such hypotheses include:

1. the strong compositionality or truth-functionality (every proposition is either elementary or should be analyzed in terms of elementary propositions), for if we have a determined base, the complexity shall be determined from it. Or, in other words, any complexity can be reduced in terms of a base, with nothing missing or remaining.

2. the complete and unambiguous analysis of complex propositions in terms of elementary as well as intended logical independence,

3. the essential bipolarity of the propositions as a criterion of meaningfulness, of propositional sense (legitimate propositions must be able to be both true and false);

4. the role of elementary propositions in determining that the analysis has a logical end. We do not need further analysis if this limit is reached (here it is also interesting to note that writing a proposition, whatever it is, with a "p" entails, among other things, that the internal complexity of it, what and how much is inside it, and what is not relevant. Speaking in tractarian terms, if a proposition is composed solely of names, it is logically irrelevant to know what and how many they are);

5. the possibility of the full expression of reality with a potentially complete map of the architecture of the logical space.

And, more specifically, with regards to the status of logic, we still have the exegetical superiority of holding truth tables as notations, as shown in the following fields:

6. Propositions of logic are tautologies;

7. They must be recognized by examining the symbol itself (where a truth table would clearly be a decision algorithm, or more of a criterion of meaning);

8. They are complex, i.e., they arise from the special relationship of elementary propositions. We cannot, therefore, find any tautologies or contradictions at the basis of language. This means inter alia that there are no logical relations at the base of language;

9. propositions of logic systematically exhibit (*zeigen, und nicht sagen*) the structure of language;

10. logical operators denote nothing in reality. That is, they do not affect the logical multiplicity of propositions, nor the bars in the truth table, or the parentheses in conventional notations;

11. the irrelevance of the identity signal in a more appropriate notational system (5.533);

12. we also have a comprehensive view of all combinations of binary operators of the propositional calculus (5.101), and;

13. the important account of logical independence of elementary propositions.

This approach to accessing the *Tractatus* via the metaphysics embedded in the truth tables proposed here also indirectly shows us how the tractarian project

was committed to questions about an allegedly neutral status of logic and to the attempt to make logic categorically distinct of all the natural sciences: This distinction can be drawn "palpably, sensibly, diagrammatically" by the truth tables. Alongside this, I believe that, as a notation, the truth table, in addition to showing the positive theses of the *Tractatus*, also instructively shows where they fail. I can then read off the *Tractatus* from the truth table, so to speak, "for good and for bad". That is, what Wittgenstein tries to carry out, namely, is to determine a comprehensive horizon of all possible propositional sense and identify where these fail: and Wittgenstein finds the inability of the truth-functional analysis and the neutral paradigm of logic to express all empirical propositions, especially those that incorporate some kind of gradation. In particular, as the *Tractatus* does not satisfactorily account for generalities (e.g. quantification in infinite domains), subtleties of predicative denial and exclusions by contrarieties (e.g. the ascription of colors to visual points), the truth table fails to account for them as well. A fortiori, I believe that it is because of the truth table's expressive weaknesses in these cases that the *Tractatus* fails as well. The truth table incorporates the whole paradigm of compositionality and neutrality of logic. The abandonment of this notation involves the abandonment of the normative generality of compositionality and of a strictly neutral logic. In turn, this abandonment entails the abandonment of the very search for a notational system that would express these features.

I believe that much has been bet on a notational system that has reduced scope of expressive due to the abstraction level of its analysis. It is known that the greater the strength and handling of a calculation is, the lesser its expressive capability. It's like the power balance between computability and expressiveness on a system would never match each other. The *Tractatus* is another example of that. Truth tables are a powerful means, because they are complete, correct and decidable, but they are too abstract. A truth table is limited and not very expressive. This contributes to the isolation of propositional calculus in relation to other calculi, which are less potent but more expressive, since its principal algorithm shows itself to be strictly limited.

The truth table notation shows an appeal to logic as a exclusively neutral field, completely combinatorial or mechanical, where no possibility is excluded and no hierarchies are expected (cf. 5.556). These presuppositions are not enough,

for example, to account for the logic of exclusions of colors because some combinations should be conceptually and *a priori* excluded. As we have seen, since we would need another type of exclusion, more subtle, when the alternatives could be not true together, but false, we could not limit ourselves to the use of bipolarity, or of concatenating and non-concatenation of ultimate objects. If we really want to continue with the expectation of a complete analysis of propositions, we have to admit other kinds of less radical exclusions.

It is indeed by the privilege of this tractarian notation in the conceptual framework that we can understand in a single sight, its project and its downfall as well as explaining the strong metaphysical commitments of the truth table in its origin, something clearly not contemplated by the manuals of logic. The movement of this review is to compare the metaphysical commitment of the truth table as conceptualized in the *Tractatus* in its origin with its technical version (lay, secular, deflationary) that we find today in the manuals on propositional calculus.

All the technical characteristics and applications of truth tables - a form of tabular representation of truth conditions and truth value of propositions, which are conventionally advanced today in the manuals for propositional calculus – already appear in the *Tractatus* (be it directly or indirectly). Such truth table features include the definition of operators by their truth conditions, algorithmic power at the propositional level, completeness, possibility to test for consistency or for semantic equivalence of a group of sentences, i.e., when they have the same truth values in all possible interpretations, as well as tests of validity and invalidity of arguments, i.e., where the truth of the premises coerces the truth of the conclusion. I believe that these are technical sediments of the original concept of truth tables with generous contacts with metaphysics, now largely forgotten.

Under the paradigm of truth-functionality that clearly guides the truth tables of our logic manuals we have that every complex proposition articulated by connectives must have its truth value entirely determined by its parts, i.e., its propositional constituents. Consequently, only the truth conditions of these atomic propositions are relevant to determining the truth conditions of complex propositions. It is therefore assumed that the whole meaning of the propositions at their base is well determined, so that the sense of the complex depends exclusively on the sense of the propositions-base. In principle, any component of these atomic propositions – be they predicates, relationships, or "arities" - or even

the type of these components – be they space-temporal, abstract, merely linguistic or mental - are not important to the truth-functional composition of the complex proposition. This reveals why we write a proposition as p both in the *Tractatus* and in the truth tables: we do not have to care what's involved there in any allegedly logical point of view. Thus, the truth table can be taken as both a decision algorithm of propositions and the truth complex (a mechanical procedure which, while generally effective, in many cases is not effective enough for the analysis of exponentialized lines) and as a method for the systematic definition of truth-functional connectives as the articulators of logical propositions base.

However, if we restrict ourselves to the classical calculus, i.e., a system defined in terms of true and false without intermediate or complementary truth values and the rule of excluded middle and the absurd classic, we have that the truth tables. Besides not discriminating the fine differences in the denial of components of the proposition, truth tables are also clearly restricted regarding the treatment of generality in terms of logical sums and products, or for the gradation of contrary propositions, which can be false, but not true, together., Given its level of abstraction, the exclusion in truth tables can only be that of a strong type: the contradiction, in which two propositions are mutually exclusive if, and only if, they cannot be true together and false together. The truth table shows this phenomenon palpably through the exclusive occurrence of the truth value F (false) in its last column. Indeed, it lets in more complex cases for the calculation or ruled manipulation of truth foundations of the decomposed proposition, shown with each line generated in the table, so we can determine with certainty whether the proposition is contradictory, contingent or tautological. It is interesting to see that contingency is a criterion of meaningfulness in the *Tractatus*, a criterion that empirical propositions meet and logical propositions do not.

Following this strategy, the manuals teach us that truth tables allow an exhaustive typology of the propositions through a complex calculation, and clear, relatively simple criteria. Moreover, a truth table is the algorithm of the propositional calculus: it is the "best-behaved" and treatable formal system as it has the attractive meta-logical predicates of correctness, completeness, compactness and decidability.

Strictly speaking, we can see, what can and cannot be done technically with the algorithmic power of truth tables without making demands. However, thinking

about the positive and negative sphere of the truth table's expressive power in the *Tractatus* - its place of origin - can be revealing, both in dealing with truth table and with the *Tractatus* itself. Indeed, we can take the discussion on the limits to expressibility in truth tables to its real source, metaphysics. Its hidden, or even ignored, metaphysical origin, exhausted in manuals and in introductory classes in logic, is largely neglected. Thus, it can still be instructive to rescue this metaphysical sphere that has been gradually lost over the past decades.

We can note then that, since we can grasp much of the conceptual framework of metaphysics of the symbolism of the *Tractatus* through this special notation or method of decision and exhibition of the sense of every possible proposition, we can also identify its limitations, as well as identifying weaknesses in the treatment of some of the special cases in the *Tractatus*. Put simply, if the truth table promotes a positive understanding of tractarian theses, it can also be a revealing thread to understand the limitations of these same theses. Where the truth table fails, the *Tractatus* should also fail. What the truth table does not see as being too general and abstract, the *Tractatus* will not see either. Thus, a retrospective understanding of a more metaphysical original spirit in the truth table makes us see a symmetric way of clarifying things. We can approach the *Tractatus* from the metaphysics of the truth table and thereby see metaphysics in the truth table, its original interpretation from the *Tractatus*. The first clarification lends a more exegetical value to Wittgenstein's early work and the second, a more historical one, because if we stick to the manuals we will not know what the assumptions and issues (largely philosophical and metaphysical) were. These were to be answered when the truth table was actually conceived.

In general, the truth table is seen as the only positive contribution from the iconic (and wrong) tractarian logic to contemporary logic30. The book from Wittgenstein's youth was radical both in what it defended, solving im Wesentlichen all the problems of Philosophy, and in its abandonment by the

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30 „Es ist das besondere Merkmal der logischen Sätze, dass man am Symbol allein erkennen kann, dass sie wahr sind, und diese Tatsache schließt die ganze Philosophie der Logik in sich. Und so ist es auch eine der wichtigsten Tatsachen, dass sich die Wahrheit oder Falschheit der nichtlogischen Sätze nicht am Satz allein erkennen lässt" 6.113. It is worth thinking about the extent to which the Church´s Theorem can be a technical refutation of the tractarian thesis that every logical propositional could be by the symbol itself recognized as such. To what extent can a technical result refute a philosophical system? Maybe it can indeed refute only the part of a philosophical system which presents technical intentions. Nonetheless, I believe that, if confronted with this alleged refutation, Wittgenstein would say that nothing about his Philosophy has had been properly understood.

author himself, who devoted much of his later efforts to show his exemplary mistakes committed there. However, Wittgenstein did not see it only a means of defining the logical operators or an algorithmic possibility of an exhaustive typology of complex propositions in tautological, contradictory and contingent in the *Tractatus*, he also saw it as means of notation, of a more adequate expression of propositions.

The truth tables are metaphysical tools that give us a map of the articulations of a logical space thought as absolute, eternal, without competitors. Here we see another example of how something methodologically autonomous today was primarily conceived within a system or project that was clearly metaphysical. This is the same as with Modernity, and with Descartes´ coordinate system attached to his ontology of the extended thing, and Leibniz´ infinitesimal calculus, associated with his monadology (spiritual atoms). And this is the same with Fregean's more recent notational system and conceptual calculus built in order to reduce the mathematical building onto the logic. The phenomenon of a gradual and progressive obliteration or sublimation of metaphysical elements from practical and conceptual techniques reappears clearly in the case of truth tables. This shows us that metaphysical endeavors can be fertile and seminal even when their objectives fail or are abandoned. Working with tables and matrices of truth today involves working on the rich collections of a failure. The metaphysical can be seminal even when it is wrong. The truth table is designed primarily as a privileged notational means to meet and express metaphysical questions in Wittgenstein's *Tractatus*, as meaning, calculation and limits of expression based on notions of an essence of language. In the best Leibnizian spirit: we would not discuss, but calculate, if we had a language that captures all the relevant logical content of our discourse.

3.5
Reading the *Tractatus* through the Truth Table (for good and for bad!)

In the *Tractatus* Wittgenstein argues that his notation can be an alternative notation in relation to the one of the *Principia* to symbolize molecular propositions and to reveal the logic of our language. If we take this interpretation of truth tables seriously, we can see how some questions that usually arise in the context of debate on the *Tractatus*, especially in the context of the application of logic, can be best answered, or dissolved, by relying on poor understandings. For example, it is misleading to ask whether a conjunction can be analyzed in a simpler and more direct way than a disjunction, because a conjunction represents the simple junction of two states of affairs. There is no denotation of an operator. Indeed, there are no operators that can be more easily analyzed than others because, strictly speaking, there are no operators at the end of analysis. According to the tractarian period, a perspicuous notation must show that writing a conjunction or a disjunction must be an expedient as important to logic as the parenthesis in other notations. This type of question about the ease of analyzing some operators in relation to others is justified by the supremacy of the kind of notation in the *Principia*. In other words, people that are more used to the *Principia* notation may ask which operator is easier to analyzed, while in the notation used in *Tractatus*, this question would not make sense because we could live, in the end, without any representation of a logical operator. If, for example, we used truth tables, wherein only the combinatorial horizon of propositional possibilities need to be written and not operators, this sort of question would probably not appear. Here it is evident that our analytical method or notational representation can determine the outcome of the analysis, as well as our speculative horizon, i.e., questions and problems which are likely to appear and what questions it makes sense to formulate or pose.

As Wittgenstein saw things, and as revealed here, a truth table itself can be viewed as a propositional symbol even before being an algorithm. And it was by approaching the truth table as more perspicuous logical notation than the one from the *Principia*, for example, that Wittgenstein gained this strong power of decision, which constitutes the ultimate criterion of expressive legitimacy of complex propositions. The algorithmic power is the result of approaching the truth table as

a symbolic means, as a privileged notation. In *Tractatus*, the truth table is at once a more appropriate notation to mirror the logic of language, a criterion for meaning and an algorithm. At this stage, the young Wittgenstein subscribed to the thesis that claims that every proposition or sense could be expressed by elementary propositions, the basis of any language.

The counterpart of this ontological thesis is that the whole world can be expressed when we specify which elementary propositions are true and which are false. Even with the inconvenience of the exponentialization of the table rows by the number of propositions being considered, in the ratio of 2n, we have the table operating under an optimistic paradigm for the potentially complete expression of reality. Even with very long truth tables, potentially infinite, we would have, in principle, the possibility of completely mapping the facts of the world through truth tables. This was important for the *Tractatus*. If we had all elementary propositions we could generate models (*Bilder*) of all possible reality in a truth-functional way (cf. 4.26, 4023). Thus we could reveal the bipolar essence of propositions (i.e. they must be possibly true and possibly false): feature that had been damaged in the most part by tautologies and contradictions and the absurdities of Philosophy. Against to what Frege thought (cf. 6.1271), from this we would also hold that evidence would not be a good criterion for determining propositions of logic, and that their connective would not replace anything in reality as they would only express operations that were already contained in the elementary propositions (*sein Grundegedanke*! cf. 4.0312).

The completeness sought through the exhaustive typology of the propositions (tautological, contradictory, contingent) can be read as a strong tractarian thesis. According to Wittgenstein, the correct classification of syntactically well-formed propositions would be tautological, contradictory and *legitimate* because contingency was a criterion for determining the propositional legitimacy, i.e., if a proposition could not be true and false, or fail to present the two poles of truth, it could not be considered legitimate. Something in it had failed. The accent in the *Tractatus* was not on the contingency, but in the legitimacy, the main test for which was bipolarity. Nothing like a legitimate and necessary proposition (with sense) could exist. The *Tractatus* is a systematic attempt to make this ban visible. In fact, calling a proposition legitimate was

redundant for the young Wittgenstein. By definition, propositions should always be legitimate. This is clearly distinct to the Fregean view.

> „Frege sagt: Jeder rechtmäßig gebildete Satz muss einen Sinn haben; und ich sage: Jeder mögliche Satz ist rechtmäßig gebildet, und wenn er keinen Sinn hat, so kann das nur daran liegen, dass wir einigen seiner Bestandteile keine Bedeutung gegeben haben. (Wenn wir auch glauben, es getan zu haben)."

In the *Tractatus*, we could write a conjunction of any two propositions as p.q, or by showing the foundations of truth of p.q, by expressing when "p.q" were true, i.e., under which conditions p and q would be true together, as (p, q) TFFF , i.e. only when the two parts were true. So that we can see or exhibit p) q as (p, q) TTFT, or p) q is true, if p is F or q is true. The idea here is thus, following the intuition of correction of mathematical proof, to avoid that the antecedent´s truth go not lost in the consequent.

We can clearly see tractarian theses here. In a classical context, T and F are the only possible truth values attributable to propositions – there is no third alternative. In the *Tractatus*, every proposition must have the capacity to be true and the capacity to be false. There is no way, for example, that the combination could be confused with a disjunction here. In truth tables, when we bring a molecular proposition to its atomic basis, we assign the values T and F to each proposition. Not only true and not only false, and no other values. Even today, when we set up our truth tables in propositional calculus we oblige our atomic base to be bipolar. We don't insist they are just bivalent, but bipolar. They have to show the two poles. Today, this procedure is still entirely tractarian. To note the possibilities of truth values as T and F and not just T or F, in the basis of the construction of the truth table is the very graphic or diagrammatic representation of the tractarian bipolarit: the requirement for contingency at the elementary basis. That is, it is the mark of the requirement of the legitimacy of a proposition at the end of analysis. When the basis is legitimate, this legitimacy should echoes throughout the complex totality (what the *Color Exclusion Problem* comes to deny).

As we have seen, this bipolarity feature works as a proper criterion for the proposition, or propositional meaning. Claiming something is to put something in this expectation of truth or, falsity. And it means allowing the doubt. Thus, a consistent vision in metaphysics would lead us to wipe away the affirmations

from its field of discourse, or , rather, to wipe away the discourse from its field. Claiming something in this tractarian phase is a pretention: it is an expectation that cannot be *a priori* answered or, rather, it must be capable of not being answered *a priori*. Moreover, as we have seen, when we write a proposition only as p and ascribe the values T and F as the only possible values of it at the end of the analysis, we are indirectly saying that what happens inside the elementary proposition is irrelevant to the logical construction of molecular propositions. The combinatorial possibilities of molecular propositions are indifferent to the components that occur within the elementary propositions. This is important for the *Tractatus*, wherein elementary propositions are composed solely of names. This removes the opportunity to contain exclusions and implications from them, and excludes any other logical relationships between elementary propositions. The formation of complexity would then be entirely combinatorial, neutral and syntactic. This can be seen in the truth table as notation when we write an elementary proposition only as p and tabulate the combinatorial and exhaustive possibilities of articulation of truth values of p with truth-values, or of other elementary propositions. We do this without having to observe what is inside these propositions. Any logical relationship between elementary propositions is not actually expected.

The classical bivalence is weak for the *Tractatus*. In terms of legitimacy, the *Tractatus* makes no distinction between a tautology and a true proposition, or between a contradiction and a false proposition. The tautology is always true, and cannot be false. Now, if I demand that propositions must be able to be true and false, as the tautology is a proposition which, by definition, can only be true and a contradiction, a proposition that can only be false, we have, then, trivially, that they would not be propositions. That is, if more than bivalence, we require bipolarity, tautologies cannot be propositions for they cannot be false. Conversely, contradictions can't be propositions either because they cannot be true. Bipolarity is a much stricter criterion for propositional sense than bivalence.

Thus, in viewing the tractarian theses through the truth table and approaching bipolarity as a notational rule, it must always be expected that neither tautologies nor contradictions can form the atomic basis. That is, the basis on which the complex propositions are analyzed may not contain tautologies or contradictions. A first clear reason is that they are not atomic. The second reason

is because they are not bipolar but monopolar; they only exhibit one pole instead of two. In fact, in the *Tractatus*, tautologies and contradictions are complex propositions that articulate contingent propositions in a special way in which bipolarity disappears. They are, therefore, limited and radical cases of propositionality. Although syntactically well-built, they do not convey any sense.

„Dass die Sätze der Logik Tautologien sind, das zeigt die formalen _ logischen _ Eingenschaften der Sprache, der Welt. Dass ihre Bestandteile *so* verknüpft eine Tautologie ergeben, das charakterisiert die Logik ihrer Bestandteile. Damit Sätze, auf bestimmte Art und Weise verknüpft, eine Tautologie ergeben, dazu müssen sie bestimmte Eigenschaften der Struktur haben. Dass sie so verbunden eine Tautologie ergeben, zeigt also, dass sie diese Eigenschaften der Struktur besitzen. " 6.12

The truth table shows that systematically through the uniqueness of F or T in each proposition in the column-result. We then conduct a test to decide the contingency of propositions. In the *Tractatus*, we have a test for meaningfulness or sense because only contingent propositions can be meaningful31. We see this as a clear ban on the setting up of truth tables. If we set up a truth table in which the truth value holds true for all columns, we know we have done something wrong. The breakdown of the complex proposition must have been done incorrectly, and the analysis in the construction of the truth table must then be redone. In principle, because of this special possibility of exhibition of the propositional sense, all complex propositions can be analyzed systematically in a way that we can "see their truth-functional guts". I believe that in this context, the idea in the *Tractatus* was that the truth table as notation would point out, bring to light or *palpably* expose the internal relations that systematically generate every proposition from a base and operations. This special notation of the truth table reveals a kind of picture (or map of dependencies within) of a complex proposition in terms of elementary propositions that compose it.

„Die Strukturen der Sätze stehen in internen Beziehungen dadurch in unserer Ausdrucksweise [notacao, modo de expressão, mode of expression, manner of expression] hervorheben, dass wir einen Satz als Resultat einer Operation darstelesen, die

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31 As we have seen, this point is already clear in the *Tagebücher* 14-16, with the tentative ab notation oulined in the Notes on Logic (pp.93-6, 102-3) and in the Notes to Moore (pp. 113-5). It is interesting to note how (with the use of the same notational device) Wittgenstein already tried to show in one stroke that negation inverts the truth conditions of propositions, and that the notation of logical operators can be irrelevant as the parentheses or points, and that p and ~~p are the same proposition, with the same sense and the same truth conditions. We will come back to these topics in the next section.

ihn aus anderen Sätzen (den Basen der Operation) hervorbringt. Die Operation ist der Ausdruck einer Beziehung zwischen den Strukturen ihres Resultats und ihrer Basen. Die Operation ist das, was mit dem einen Satz geschehen muss, um aus ihm den anderen zumachen." (5.2 -5.23).

We can subsequently recognize the formal properties in our suitable notation through the type of simple examination indicated in paragraph 6.122. Having this *entsprechende Notation*, we can do without the formulation of logical propositions, since that might show through simple examination or inspection when propositions are tautological and what not. But if the sense of the proposition is the expression of its truth conditions, by explicitly capturing these conditions, the truth table captures explicitly what the proposition complex "says". Wittgenstein explains this point when he writes: „Der Ausdruck der Übereinstimmung und Nichtübereinstimmung mit den Wahrheitsmöglichkeiten der Elementarsätze drückt die Wahrheitsbedingungen es Satzes aus. Der Satz ist der Ausdruck seiner Wahrheitsbedingungen." (4.431).

The truth tables express molecular propositions (4.2-4.45), so that by systematically representing its truth conditions, they represent what is strictly relevant to their generation. Through the functioning or combination of the truth possibilities of the propositions that compose it, one can systematically show how the propositions analyzed are tautological, contradictory or contingent. And, in this way, reveal an exhaustive typology. Indeed, one can "see" if they make sense or not, in the case of very long propositions. And, as Wittgenstein outlines in 6.126, we can calculate if propositions have meaning or not:

„Ob ein Satz der Logik angehört, kann man berechnen, indem man die logischen Eingenschaften des Symbols berechnet. Und dies tun wir, wenn wir einen logischen Satz „beweisen". Denn, ohne uns um einen Sinn und eine Bedeutung zu kümmern, Bilden wir dein logischen Satz aus anderen nach bloßen Zeichenregeln [meras regras notacionais, meramente regras dos signos, rules that deal with signs, mere symbolic rules]. Der Beweis der logischen Sätzen besteht darin, dass wir sie aus anderen logischen Sätzen durch sukzessive Anwendung gewisser Operationen entstehen lassen, die aus den ersten immer wieder Tautologien erzeugen. (Und zwar folgen aus einer Tautologie nur Tautologien). Natürlich ist diese Art zu zeigen, dass ihre Sätze Tautologien sind, der Logik durchaus unwesentlich. Schon darum, weil die Sätze, von welchen der Beweis ausgeht, ja ohne Beweis zeigen müssen, dass sie Tautologien sind."

There is clearly a move to enable seeing all the tractarian "theses" in the Philosophy of Logic through the rules of handling a privileged notation.

Understanding the logic composed of tautologies, and bringing with this "gain", the possibility of calculating or of developing a mechanical method for the identification of a proposition as tautological or as a component of the logic itself is a notable step. And Ramsey emphasized this achievement in his review of 1923:

"So every proposition is a truth-function of elementary propositions, and many differently constructed propositional signs are the same proposition, because, expressing agreement and disagreement with the same truth-possibilities, they have the same sense and are the same truth function of elementary propositions. (…) There are two extreme cases of great importance; if we express disagreement with all the truth-possibilities we get a contradiction, if agreement with them all, a tautology, which says nothing. The propositions of logic are tautologies and to have made clear this, their essential characteristic, is a remarkable achievement". (Ramsey, p.471)

In an analogy with studies in biology, truth table gives us the physiology and anatomy of the propositions. Figuratively speaking, truth tables reveal the physiology, the functioning and interaction of the exhaustive truth conditions of elementary propositions, and the anatomy of the propositions, the exhibition of a map of possible combinations of truth values of the proposition. Taking this analogy further I could say, if I am not mistaken, that for Wittgenstein in the *Tractatus*, truth tables represent a method of dissection and analysis of (linguistic) parts (as well as, indirectly, their ontological counterparts). The molecular propositions could be dissected completely in terms of its constituent parts, without anything missing, or being left over, and without surprises32. The truth table notation would be more appropriate to express propositions, because by saying them we could already show its truth conditions, and bring its sense to public inspection or scrutiny. An ideal notation must be able to display or systematically show the sense of complex propositions just as Wittgenstein states: „Immer kann man die Logik so auffassen, dass jeder Satz sein eigener Beweis ist". (6.1265)

To reiterate, if there were no elementary propositions every sense of the complex propositions would be indeterminate and language would never leave itself. It would never become something non-linguistic. It would be terminally insularized within itself. Indeed, complex propositions only have sense and truth values because their constituent parts have them. The truth table notation shows

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32 Cf. „Darum kann es in der Logik auch nie Überraschungen geben". 6.1251. „In der Logik sind Prozess und Resultat äquivalent. (Darum keine Überraschung.) Der Beweis in der Logik ist nur ein mechanisches Hilfsmittel zum leichteren Erkennen der Tautologie, wo sie kompliziert ist." 6.1261-6.1262.

that complex propositions must be reduced into elementary propositions. These must be capable of being true and false. There is not the case in which a proposition has the possible values all true or all false already on in the first columns, where we distribute the possible values. This situation is enough to signal that either the decomposition or the distribution was not completely or correctly executed.

The sense of complex propositions is determined by the truth conditions of elementary propositions, so that it is enough to have a theory of meaning in these elementary propositions. This theory is the *Bildkonzeption*, which, once fulfilled, provides a sense of all possible complex propositions. (2.0211) According to the *Bildkonzeption*, if the components are the names of elementary propositions, we must have an eternal basis to serve as references for them. The ontology of the *Tractatus* fulfills this role sufficiently. It was needed to ensure that elementary propositions had sense, so that also had complex propositions. This was assured. The *Tractatus* made the truth functionality work indirectly, giving sense to this elementary basis by making the elementary propositions touch reality. If I understand the complex, I can analyze it in terms of elementary propositions and show its truth conditions. If the elementary propositions are true or not, nothing will change for the complete determination of its sense. If I assure the sense of the elementary propositions, I assure the determined sense of all complex propositions. I know how to generate it by strictly truth-functional means.

Assuming the propositions are in the left columns as elementary propositions, - the place where elementary propositions are usually displayed in a table - the right ones will show me a comprehensive inventory of their truth conditions, and, by showing me when the complex proposition is ultimately true and when it is false, the last column will show me their sense. This fully justifies why I decompose propositions in the first column on the left of a truth table whose truth value is all the same, either all true or all false. The possibility of a truth table showing only truths or falsehoods at the outset in the first column, where there are atomic propositions at the end of analysis, is already banned, i.e. it makes no sense. Reading this in the *Tractatus*, tautologies and contradictions are not elementary propositions. This also shows that there is no hierarchy among tautologies, so that one can be defined in terms of the other. A logic axiomatic would make no sense. It would always be arbitrary because all tautologies have

the same value. There would be no hierarchies in logic. All symbols or logical propositions will collapse into one. As shown in this criticism of Wittgenstein to Frege:

„Alle Sätze der Logik sind gleichberechtigt, es gibt unter ihnen nicht wesentlich Grundgesetze und abgeleitete Sätze. Jede Tautologie zeigt selbst, dass sie eine Tautologie ist. Es ist klar, dass die Anzahl der „logischen Grundgesetze" willkürlich ist, denn man könnte die Logik ja aus Einem Grundgesetz ableiten, indem man einfach z.B. aus Freges Grundgesetzen das logische Produkt bildet. (Frege würde vielleicht sagen, dass dieses Grundgesetz nun nicht mehr unmittelbar einleuchte. Aber ist merkwürdig, dass ein so exakter Denker wie Frege sich auf den Grad des Einleuchtens als Kriterium des logischen Satzes berufen hat). 6.127, 6.1271.

According to 3.326, tautologies could never be symbols because they don't have strict sense or significant use. A symbol is a signal with the set of possible syntactic connections that reflect the possible connections of the constituents of the represented complex. If we do not notice the important difference between sign and symbol, in this context, we could, for example, be led to believe that there is indeed some hierarchy in the case of tautologies, not of importance or value, but, perhaps, of complexity, as it would seem to be evident in the notation of tables truth. Wittgenstein clearly shows how, in a deductive system, it is arbitrary to approach some as more elementary than others, or to think of one as axiom and another as theorem.

Still, some tautologies appear to be more complex than others. In terms of the *Tractatus*, the truth table would represent the establishment of a criterion together with a method to take a proposition as a component of logic (cf. 6.1), differentiating it clearly from contingent propositions. The criterion for the membership in the logic could be: Tautologies are identified by the presence of T on all end lines, after the calculation. Thus, it would be impossible to determine what the difference is between tautologies of the same complexity as in the case of "p => p" (identity) and "~(p.~p)" (non-contradiction) and "~(p.q) => (~pv~q)" (De Morgan) and "(p=>q) => (~q => ~p)" (Contrapositive). If we take the truth table as a symbol and not as a sign, we may utilize a wrong criterion for negating that only a tautology would exist, even if all final values were T. We could then defend that they vary in number of Ts, for example. We might have, tautologies of different orders, so to speak. For example, there could be one of an order 4 (understood as having four Ts at the end), of order 8, order 16... growing

exponentially according to the number of elementary propositions they're composed of in a ratio of 2n. The number of different elementary propositions involved in the articulation could define the order or complexity which the tautology has. Those with the same order would be the same tautology. They would then be different when they were in different orders. But, although, we actually have different signs with differing complexity, they all mean the same: nothing33. They are all different representations of the same. They are all different signs that express the same symbol.

Different truth tables would show that there are different tautologies. But they are "the same symbol" through different signals. The natural question in this context would be: "But if we had a more suitable notation for more complex tautologies (i.e., a table with the last column exclusively for Ts, but with more lines) that would allow for variables, would it not show different tautologies?" If we do not differentiate between signs and symbols, we might be tempted to give an affirmative answer to that question. But, in fact, this is a false issue. If it were not, we would find ourselves at a deadlock: either we would adopt the opinion that truth table is not good notation because it leads to a error, by leading me to see that tautologies are distinguished by complexity, or we would assume that propositions of logic do have a hierarchy, exponentially organized, tautologies of order 2, of order 4, of order 16... The danger of this approach lies in bringing about the illusion that maybe we could put the more simple as primitive and the more complex as derived... Once again we would find an impasse: either the notation would be misleading, which contradicts our argument based on Wittgenstein´s writtings and on our Leibnizian intuition that guides this work, or tautologies allow some hierarchy, not of sense, but of complexity that goes against the tractarian "theses" as an undesirable byproduct of the truth table as a notational system. This is precisely created, among other things, to show that all the propositions of logic, in the end collapse, into a single tautology that says nothing.

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33 It is possible that when Wittgenstein wrote 6.124 he didn't note that the first sentences of this passage contradicts the distinction between to say and to show that he always employed coherently thereafter in the *Tractatus*. Indeed, in that passage, Wittgenstein wrote that the logical propositions describe (*beschreiben*)... To describe belongs to the sphere of the saying, while the logic should be circumscribed in the sphere of showing.

Another interesting point for reading off tractarian theses embedded in the truth table as a notation is the role of non-denotative logical operators. All logical operators can be defined by the truth table. Reading this thesis in the *Tractatus*, we can clarify two central theses. First, we can understand the functioning of denial. It should invert the sense of the denied proposition. As Wittgenstein categorically says in 5.2341: „Der Sinn einer Wahrheitsfunktion von p ist eine Funktion des Sinnes von p. Verneinung, logische Addition, logische Multiplikation, etc., etc. sind Operationen. (Die Verneinung verkehrt den Sinn des Satzes.)." The propositional negation reverses the sense, because it inverts the truth conditions of the denied proposition. This trvial fact about any propositional logic is clear in the truth table. It expresses the negation of a proposition pTF, as pFT. This shows what happens with p when one writes ~p. Negation should then reverse the direction of p, by reversing its truth conditions. What made p true before, turns out to make it false, what made it false starts to make it true.

As a result of this approach of denial, we clearly have a situation that negation can only be propositional, and not for instance precicative. In the *Tractatus*, it is clear that any predicative denial will have, by the end of analysis, to be able to be reduced to propositional negation, that is, a negation which ha the whole proposition as its scope. This treatment of negation announces what there is to come. According to Wittgenstein in his *Grundgedanke* (cf. 4.0312 and 5.4), logical operators do not have to refer to anything in the reality:

„Dass aber die Zeichen „p" und „~p" das gleiche sagen können, ist wichtig. Denn es zeigt, dass dem Zeichen „~" in der Wirklichkeit nichts entspricht. Dass in einem Satz die Verneinung vorkommt, ist noch kein Merkmal seines Sinnes (~~p=p). Die Sätze p und ~p haben entgegengesetzten Sinn, aber es entspricht ihnen eine und dieselbe Wirklichkeit." 4.0621

The second point in this approach to the logical operators through truth tables is that all operators and not only the denial should disappear in a perspicuous analysis. Operators should disappear at the end of analysis, when would have only independent propositions. If there is no logical complexity, there are no logical exclusions and implications. In fact, an examination of a suitable notation shows that the logical operator is a notational expedient only for abbreviation. It would be irrelevant to the essence of logic, in Wittgenstein's approach. We can do without operators, if we show systematically the truth

conditions of complex propositions. Because they do not actually exist, they denote nothing. They replace nothing in the reality. This is the standpoint of *Tractatus* in its attack against the Platonism.

We should fully understand the denial if we show how it operates the truth conditions of the denied proposition. If we deny a proposition twice, we exhibit what the iterated denial makes with the truth conditions. Once reversed by a denial, they return to the previously denied proposition. The second denial neutralizes the inversion of truth conditions of the first denial. In fact this becomes evident in the truth table. If we begin with pTF, denying it once we give us pFT, i.e., the inversion of the conditions. Denying it again, we reverse the order of the values and arrive at pTF, i.e., we get back to the first symbol, to the first proposition. What does this show us in the *Tractatus*? The sign or signal of denial, or the notation of negation (~) is irrelevant, as irrelevant as points or parentheses, because it is sufficient to have a more appropriate notation that shows how complex propositions can be manipulated in terms of possible values of the elementary, as passage 4.441 explicitly indicates (as cited above). Just showing the operation, but not the operator because the operator is a signal that summarizes, "wipes" the operation or shows how propositions should be handled. It may then be taken as a sign leaner, as an abbreviation to symbolize what is actually happening. And the truth table as notation effectively shows how this operation takes place. Denial is, in fact, the paradigmatic case of coverage of our syntactic rules by a logically perspicuous notation. This kind of deflationary treatment can be extended to the other logical operators. In a more perspicuous notation it would be clear that we do not need them to show the physiology of complex propositions, it is sufficient that we can show under which conditions they are true and under which conditions they are false. We have to make an instrumental distinction here between a truth table that is entirely set up (I will call this an extended truth table) and the notation of the last column of this table, which shows only the result of the algorithm itself. This appears clearly in 4.441-2, to illustrate the fact what is logically relevant in a notation, we need not write the entire distribution of Ts and Fs, or the logical operators, but the truth conditions of complex propositions. As Wittgenstein says:

„Es ist klar, dass dem Komplex der Zeichen „F" und „W" kein Gegenstand (oder Komplex von Gegenständen) entspricht; so wenig, wie den horizontalen und vertikalen Strichen oder den Klammern. _ „Logische Gegenstände" gibt es nicht. Analoges gilt natürliche für alle Zeichen, die dasselbe ausdrücken wie die Schemata der „W" und „F" (4.442)

In this passage-commentary 4.442, the combination of p to q becomes something like TFFF (p, q), the implication, something like TTFT (p, q), disjunction, something like TTTF (p, q), a bi-implication something like TFFT (p, q). However, there is a kind of notational superiority to the extended table: taking the biological metaphor proposed above, the extended table outlines the internal anatomy and physiology of complex propositions. For example, the functioning of the bi-implication would be clearer if by the extended truth table notation and not only in its last column. The extended shows that it is true when we have common truth values. With the extended truth table, one can clearly see what a binary operator is doing, and how it works from the manipulation of truth conditions. The conjunction is only true when both parts of the conjunction are true. The implication arises truly when the antecedent is false or the consequent true. The bi-implication is when the values are equal. The disjunction is when one of the disjuncts is true. And the extended truth table exhibits this fact. Moreover, with truth tables, I can define operators in terms of others, bi-implication in terms of two implications, implications in terms of disjunction and negation, or conjunction and negation and so forth. All these manipulation possibilities are made explicit in truth tables.

Taking this intuition seriously we can have only one operation, the jointly negation, and generates the same results. It is also important to note here that a fragment consisting only of negation and conjunction is a limitation of language and not of logic. Here is a fragment of the language and not of the logic. From this truth-functional completeness, a facet of logical holism34, is clear, and this must be presented by a more perspicuous notation. A complete understanding of a fragment, if complete, enables the generation of all the results with other operators. This fragment carries the logic completely. The truth table at once defines and exhibits the operations by showing the disappearance of operators for

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34 We will come back to this issue at the end of this thesis, where we will investigate other facets of this holism.

calculations of truth values, and thereby showing how some can be systematically reduced to others.

We can still apply the approach proposed in this chapter about perspicous notations to the paradigmatic case of identity. In the *Tractatus*, to say that two things are the same is absurd and to say that one thing is identical with itself is a triviality, thus, a perspicuous notation should suspend the use of identities. If the semantic content of names is exhausted in its connection with the named objects, a name that names another object is, in fact, another name, so that the most appropriate notation should also be sensitive to this aspect. As Wittgenstein states:

> „Ausdrücke von der Form „a=b" sind also nur Behelfe der Darstellung; sie sagen nichts über die Bedeutung der Zeichen „a", „b" aus. Können wir zwei Namen verstehen, ohne zu wissen, ob sie dasselbe Ding oder zwei verschiedene Dinge bezeichnen? __ Können wir einen Satz, worin zwei Namen vorkommen, verstehen, ohne zu wissen, ob sie Dasselbe oder Verschiedenes bedeuten? Kenne ich etwa die Bedeutung eines englischen und eines gleichbedeutenden deutschen Wortes, so ist es unmöglich, dass ich nicht weiß, dass die beiden gleichbedeutend sind, es ist unmöglich, dass ich sie nicht ineinander übersetzen kann. Ausdrücke wie „a=a", oder von diesen abgeleitete, sind weder Elementarsätze, noch sonst sinnvolle Zeichen. (Dies wird sich später zeigen). 4.242, 4.243.

Writing further on this point in passage 6.2322, Wittgenstein argues:

> „Die Identität der Bedeutung zweier Ausdrücke lässt sich nicht behaupten. Denn um etwas von ihrer Bedeutung behaupten zu können, muss ich ihre Bedeutung kennen: und indem ich ihre Bedeutung kenne, weiss ich, ob sie dasselbe oder verschiedenes bedeuten."

Under the notational ideal, each symbol, be it a constant, a variable, or a proposition, would show that for which it is an unambiguous signal, without synonyms. To avoid misunderstandings we should not have a sign symbolizing two different things or one thing being symbolized by two different signs. This notational requirement is fully met in the truth table. When I take p and q as my elementary propositions, there must be no mechanism to ensure that these propositions are different, as we have in the case of the notation from the *Principia*. It would be enough to display them as different, according to this tractarian standpoint. As Wittgenstein affirms, also as a notational ideal: „Gleichheit des Gegenstandes drücke ich durch Gleichheit des Zeichens aus, und nicht mit Hilfe eines Gleichheitszeichens. Verschiedenheit der Gegenstände durch Verschiedenheit der Zeichen." 5.53

This becomes even more evident in the truth table, because if p and q are different symbols, diferent propositions, they will have different truth conditions, then they will be different represented in the notation. If they are the same symbol they will have the same truth conditions, then having the same representation in the notation. This prohibition of ambiguity shows the role of notation for the improvement of the most fundamental confusions that philosophy is filled with (cf. 3.324). As Wittgenstein asserts then:

„Um diesen Irrtümern [of the traditional Philosophy] zu entgehen, müssen wir eine Zeichensprache verwenden, welche sie ausschließt, indem sie nicht das gleiche Zeichen in verschiedenen Symbolen, und Zeichen, welche auf verschiedenen Symbolen, und Zeichen, welche auf verschiedene Art bezeichnen, nicht äußerlich auf die gleiche Art verwendet. Eine Zeichensprache also, die der logischen Grammatik _ der logischen Syntax gehorcht. Die *Begriffsschrift* Freges und Russells ist eine solche Sprache, die allerdings noch nicht alle Fehler ausschließt.)   3.325

In one stroke, the truth table, taken as a notational means, shows the internal composition of complex propositions, the essential bipolarity of propositions, the irrelevance of identity and non-denotativity of logical operators.

Arriving at the last positive topic explored in the *Tractatus*, we have a strong thesis of the independence of elementary propositions. There, an elementary proposition could never exclude another. Exclusion appears only at a level of complexity where there is operation (or more precisely the denial). In this way, as there are no operators in the elementary basis of language, it cannot contain exclusions. According to 5.1241, every proposition that contradicts another denies that other, i.e. there are no propositions that contradict each other or exclude themselves, without entailing a denial. In this way, the logical product of p and q cannot generate a contradiction, if p and q are elementary. By the counter-positive, if the logical product generates contradictions, p and q are not elementary. The logical exclusion is a clear case of negative criterion for evaluating whether propositions are elementary or not. Following the Leibnizian intuition of this work, we can then "read off" this demand for independence from the construction of the first columns of the table of truth: is not the case of articulating these propositions free of operators and having final values of F. In the clear case of conjunction, we have as the last column TFFF (p, q). Nothing like FFFF (p, q) may be accepted in conjunction of elementary propositions. This

would be enough to signal that at some level (even, for example, if we do not exactly know whether or not, p is not-q or q is not-p) the propositions are not elementary, because they still comprise of some form of denial.

From the truth tables we could learn that if there is no propositional complexity, there is no possible incompatibility. This would be a clear tractarian tenet. And by the counterpositive, incompatibility means complexity in the truth table. There were only exclusion if there is a truth-functional operation. By the counter-positive again: there is no exclusion if there is no operation. At the atomic or elementary level there is no exclusion. The propositions there must therefore be independent (this idea is outlined in 5.134 and its ontological counterpart, 5.135)35. We can go further: I can only have some exclusion in a system of truth tables if there is denial of some element, and for that we need its repetition. As above, p and q can only be excluded truth-functionally if q can be analyzed in terms of a denial of p, or if p can be read in terms of the negation of q. That is, truth-functionality only allows one type of exclusion – contradiction in a form like (p.~p).

Reading this from the perspective of the *Tractatus*, exclusion is the mark of a restriction or of a disobedience of the bipolarity. Somehow a situation must be the case and also be not the case. Syntactically speaking, because the possibility of the negation of a proposition should be necessary, we would have always together a proposition and its negation. Semantically speaking, something must be true and (the same thing) must be false. So that is nonsense be the case and not be the case. This is always false, it is contradictory. I also have the need for repetition (I used "something" twice36) and denial ("it is not the case"). If there is exclusion between seemingly different propositions they are actually not entirely distinct from one another, but must have their sense, which is compared with other facts, somehow shared. In these case, we could say, that the sense is totally shared. In accordance to this reading, Bento Neto, following on from the interpretation of Dos Santos in the introduction to his Brazilian translation of the *Tractatus*, equates bipolarity, and determination of sense and complex propositional with natural ideas in this context of complete analysis. Bento Neto writes:

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35 This possibility of the passage of a linguist "result" to an ontological "result" in the *Tractatus* will be investigated in the last chapter of the present work as a interesting symptom of this tractarian holism.

36 Without repeated variables we do not have logic. Therefore the etimology of tautology is very correct in this context.

"The principle of bipolarity, together with the principle of complete determination of meaning, applied to propositions that hold logical relations of implication or exclusion among themselves, result in the termination of this false appearance that we have the case of propositions that are really and entirely different from each other. They put therefore the possibility of the movement from a apparent complete difference to the essential partial identification. These two principles, therefore, have a clear embryonic idea of analysis: the analysis is the reduction of the apparent distinction to the real, essential identity (albeit partial)." (Bento Neto, p.48)

One point missing from Bento Neto's interpretation is the investigation of how a perspicuous notation, in this case the truth table, should be able to help or even be a pre-requisite for a complete analysis. Yet he states:

"Adopting the determined form that the principle of complexity assumes in the *Tractatus*, the result of the analysis, the passage from appearance to essence, consist, then, in the constitution of a certain system of signs. The complete logical analysis shall be equivalent to the creation of a language that displays on the surface of its signals, the structure of its sense." (Bento Neto, p. 49)

I do not agree that the outcome of the analysis is a fully analyzed and transparent language but this language must be revealed. Or rather, I believe that a notational system helps the prospection of everyday language for the display of its deep logical form. A completely analyzed language does not need to be constituted, but prospected. It would be odd to say, for example, that the truth table notation _ before being a means to the truth-functional aid for the analysis of propositions _ is itself a completely analyzed language. It would be strange because this notation naturally finds its limit in the expression of the internality of elementary propositions. These are opaque to the notation that only requires that they be independent of each other. Everything that is logical should be complex. There would be nothing logically relevant inside the elementary propositions. In addition, a notational system is a system of signals and not of symbols (i.e., signs with the projective relations), therefore it cannot be a completely analyzed language, because it is not yet language in a strict sense. To reiterate: the exclusion in the *Tractatus* is of strong type, i.e., or a proposition is the case or it is not the case, a state of affairs is effective or not. Only in this order there is

exclusion. Exclusion in the *Tractatus* can only be established through denial and repetition37.

However, other kinds of exclusions, rather trivial and ordinary, are ubiquitous Exclusions such as finding that a table cannot be both 3 meters and 4 meters long, or a refrigerator cannot be set at both 15ºC and 16ºC, or that a bottle cannot hold exactly 2 liters and exactly 3 liters of liquid at once, or that a point in the sky cannot be both blue and red, are all common exclusions. These incompatibilities are not the fruits of a contradiction, because the two alternatives are not exhaustive, although they are exclusive ones. They cannot be true together, but can be false together. Here we have a classic case of contrariety and not of a contradiction, though we still have a case of exclusion or incompatibility between alternatives. What marks the contrariety is the idea of a degree or gradation, perhaps mappable through the linearity of numbers, as we see in the work of Cuter. Propositions of gradation are clearly mutually exclusive, but they are not contradictory because they can be false together. For example, it is possible that, in the case of a table, it is neither 3 meters nor 4 meters long.

The same holds for the other examples, not only length but temperatures, volume and color gradations allow themselves to be mapped by numerical indexes and some applications of arithmetic. Something cannot be green and yellow, even though green is created with yellow. And a meteorologist who says that the temperature tomorrow afternoon will be 30ºC and 31ºC will not be taken seriously, even if s/he says something more problematic like "it will rain and it will not rain." In each case, the members of the conjunction are mutually exclusive, but this is different from the exclusion involved in a contradiction. The exclusion in "it rains and it does not rain" seems to be more radical than the exclusion in "the temperature is 30º and 31º." The first is a contradiction as it involves exclusive and exhaustive alternatives that cannot be true together or false together, for tomorrow afternoon will either be rainy or not. However, it is possible that the temperature tomorrow will neither be 30º nor 31º. This is a

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37 But what would happen if there is an exclusion that does not use repetition, this could be expressed by the truth tables? How does function the exclusion in the numerical scale? Without implications or exclusions? To which extent this weak exclusion by contrariety carries the notion itself of functionality in a mathematical sense? For each element of the domain I can have only one element of the image. To which extent this image or metaphor presented in the definition of function, of a kind of radical disambiguation, is not connected to some empirical pictures of the kind of: a body cannot be in two different places at the same time? We will come back to these points in the end of this present work.

natural question that arises in this context: What is the role of repetition and denial in the case of the exclusion by contrariety? In the tractarian case, there is only exclusion if there is a well determined sharing of semantics in the repetition of a proposition. In the case of contrariety there is also a semantic sharing, but it is more refined. A proposition need not be repeated or denied, but the incompatibility comes from the co-belonging of singular values to a common system.

In the *Tractatus,* if p and q are elementary propositions they should be independent, because the concatenation of them does not generate contradictions, because there is no denial or repetition in elementary propositions. But if p and q belong to the same system, such as a length measuring system or the assignment of color to visual points, we can have exclusions without repetition or negation. We can pose here numerous questions, such as: In a system of color ascription, does something being green in some sense deny it from being red? Does an object being square prevent it from being round, in a classification system of geometric figures? In a biological taxonomy, does "being a lion" deny something from "being a leopard"? Does "to be a lion" deny "to be a domestic cat" more than "to be a leopard"? Does "being green" exclude something from "being red" more than it denies it from "being yellow"? Whatever their answer is, what these questions seem to show us is that there seems to be "negations" (or exclusions), or incompatibilities inour daily lives that appear not to be brought by repetition and denial and seem to be definitely distinguishable from the kind of exclusion by contradiction. Thus, these exclusions appear to be utterly beyond the power of analysis required by bipolarity and carried out by the prominence of the truth-functionality. If the base is meaningful, this meaningfulness does not guarantee the meaningfulness of the complex strictly generate from this basis.

Moreover, these exclusions might bear infinite degrees of strengh and proximities (*Nähe*). As Wittgesntein seems to speculate in the paragraph 218 of PB, regarding the color metric:

"Man kann nun unmittelbar Farben als Mischungen von rot, grün, blau, gelb, schwarz, und weiß erkennen. Dabei ist Farbe immer color, nie pigmentum, nie Licht, nie Vorgang auf oder in der Netzhaut etc. Man kann auch sehen, dass die eine Farbe rötlicher ist als die andere oder weißlicher etc. Aber kann ich eine Metrik der Farben etwa Bezug auf ihren Gehalt an Rot in der Mitte zwischen zwei anderen Farben steht? Es scheint

jedenfalls einen Sinn zu haben zu sagen, die eine Farbe steht einer andern in dieser Beziehung näher als einer dritten" (PB. 273) my italics

For example, it would be enough to take a point in the mosaic or in the continuous of colors and systematically compare this point to other points (perhaps spatially) closer and more distant in this system. If, in fact, colors form a dense system, i.e., for every two points in the range of colors we have a differentiable point between them, we will have, following this argument, infinite degrees of strength of exclusion. There is no homogeneity in the degree of exclusion within a system, just as there is no homogeneity in the degree of exclusion among systems. So further refinement of the analysis is required if we really expect that we will carry it out. This, in turn, is the demand for a kind of greater closeness to the empiria, less abstraction, and, unfortunately, less computability. As we shall see in the last chapter of this work, Ramsey[38] had already seen this problem with the exclusivity of thinking logic comprising tautologies and contradictions, but without linking it with measurement problems, or numbers, or exclusions by contrariety. As Ramsey pointed out, there would be other necessities that could be called logical, but that could not be reduced to tautologies, as the necessary properties of space and time, which, as discussed, seem to bring empirical or metaphysical aspects into the neutral and combinatorial tractarian logic. Ramsey pointed out that:

"It is a principle of Mr. Wittgenstein´s, and, if true, is a very important discovery that every genuine proposition asserts something possible, but not necessary. This follow from his account of a proposition as the expression of agreement and disagreement with truth-possibilities of independent elementary propositions, so that the only necessity is that of tautology, the only impossibility that of contradiction. There is great difficulty in holding this; for Mr. Wittgenstein admits that a point in the visual field cannot be both red and blue; and, indeed, otherwise, since he thinks induction has no logical basis, we should have no reason for thinking that we may not come upon a visual point which is both red and blue. Hence he says that "this is both red and blue" is a contradiction. This implies that the apparently simple concepts red, blue, (supposing us to mean by those words absolutely specific shades) are really complex and formally incompatible. He tries to show how this may be, by analyzing them in terms of vibrations. But even supposing that the physicist thus provides an analysis of what we mean by "red" Mr. Wittgenstein is only reducing the difficulty to that of the necessary properties of space, time and matter, or the ether. He explicitly makes it depend on the impossibility of a particle being in two places

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38 It is interesting to note that Kienzler also investigates this influence of the discussions with Ramsey in the turning point of Wittgenstein´s Philosophy, but without mentioning the criticism that would hit at the heart of the tractarian logic. Kienzler concentrates on Ramsey´s influence, even though it's negative regarding the status of mathematics in exclusively extentional formulations, and the infinitude and probability. Notably, these themes do not appear in his review form 1923. Bento Prado Neto doesn't investigate Ramsey's criticism at all in this context.

at the same time. These necessary properties of space and time are hardly capable of a further reduction of this kind. For example, considering A between points C and D and point C between B and D, then A must be between B and D; but it is hard to see how this can be a formal tautology." (Ramsey, p. 473) my italics

Accompanying this, there would be other exclusions that cannot be reduced to contradiction, as we have seen: the problem of measurements, colors, and taxonomic systems and of trichotomies or "politomies" (or situations wherein I have more than two alternatives in a unique system. Refer to the example of possible results for a football match). From Ramsey's visits to Wittgenstein in *Niederöstereich*, and the content of his review, as well as discussions on problems and obscure points in the *Tractatus*, we can speculate that Ramsey, at latest in his review of 1923, was the first to notice the so-called *Color Exclusion Problem*, i.e. the issue of the promissory note in passage 6.3751[39] that could not be paid in tractarian terms. To resolve this logical incompatibility, we should review the tractarian conceptual geography which was so certain for Wittgenstein at that time.

In accordance to this conceptual impossibility of the combinatorial tractarian logic to account for all the logical necessities, Stephen Read comments on the way the problem was postponed by analyzing it in terms of wavelengths or velocities of particles:

„Wittgenstein's belief expressed in 6.3751 was that further analysis, possibly physical analysis into wavelengths, could explain this incompatibility. (...) Following his return to Philosophy in the late 1920s, however, Wittgenstein realized that such reduction was impossible. For if the analysandum were compatible, the reduction would have failed; while if the reduction succeeded, it would simply repeat the incompatibility at the lower level. The problem is this: suppose p and q are incompatible ('This is red' and 'This is green', say). p and q are not just any two incompatible propositions-analysis is not always impossible. They ascribe incompatible determinates of a determinable. Then whatever analysis is applied to p, resulting in an analysans p', there will be a similar analysis q' of q. If p' and q' are compatible, something must have gone away with the analysis _ it won't explain why p and q are incompatible; while if p' and q' are incompatible, they will again ascribe incompatible determinates of a determinable (in Wittgenstein's case, different positions to a light-particle), and so p' and q' cannot be atomic. The original problem has simply been repeated. Not all necessity could be explained in the truth-functional, or combinatorialist manner he had hoped for in the *Tractatus*. Determinates of a determinable are inherently incompatible. (Read, *The unity of the fact*, p. 338-9)"

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39 With this quotation from Ramsey´s review it seems even more obvious for me that the central passage in 6.3751 points out something empirical or metaphysical, and nothing in the direction of numbers, quantification and formal series as Cuter´s work intends to prove. If we occupy, for example, an one seat-sofa it will be no place anymore for another person. As the metaphor from *Some Remarks*, p.169, seems to shows, this impossibility has something to do with the structure of the space.

The paradigm of reducing logic to tautologies and contradictions as limits of combinations of a completely independent atomic base finds its limit in the case of exclusions by contradiction. This led to a great fragmentation of the tractarian logical space in systems of propositions that also defined combinatorial horizons of singulars, but that dismissed the thesis of the logical independence. We highlight this change or development of Wittgenstein's Philosophy as a kind of Ramsey-effect, triggered by his critical insight. Ramsey was the first to point out, even incidentally, in his review already in the reception of the *Tractatus*, the nerve problem that led to the subsequent abandonment of the work. Although as Kienzler states:

"Von den Gesprächen Wittgensteins mit Ramsey gibt es keine unmittelbaren Aufzeichnungen, so dass die Frage, wie genau Ramsey durch seine Kritik die Gedankenentwicklung Wittgensteins beeinflusst hat, aus indirekten Quellen und verstreuten Bemerkungen erschlossen werden muss." (Kienzler, p.57)

Kienzler affirms that Ramsey visited Wittgenstein twice in Niederösterreich to discuss the *Tractatus*. Indeed, these discussions provided Ramsey with the origins for his 1923 review and his 1925 essay on the grounds of Mathematics. It does not seem inconsequential that Wittgenstein mentions Ramsey in PU as having influenced him through their discussions in the '20´s but not through his writings. Kienzler affirms that no central thought from Wittgenstein´s late Philosophy can be positively associated with Ramsey. Indeed, the majority of observations Wittgenstein makes about Ramsey are negative. And this clash shows in that many times Ramsey´s criticism were based on misunderstandings or on specific questions and circumstances, such as the issue of the colors, for example. Consequently, Kienzler asks:

„Worin ist dann die emphatische Widmung im Vorwort der Philosophischen Untersuchungen begründet? Ein erster und vielleicht der wichtigste Punkt ist ganz offensichtlich der intensive persönliche Umgang, den Ramsey trotz gelegentlicher Misstimmigkeiten, und der enormen Anstrengungen, der Wittgenstein seine Gesprächspartner aussetzte, nicht scheute, außerdem das Freundschaft und Vertrauensverhältnis zwischen beiden (…) Die Tatsache, dass er zum zweiten Mal nach Russell einen beständigen Partner für seine philosophischen Gespräche fand, hat sicher entscheidend dazu beigetragen, dass Wittgenstein seine im *Tractatus* niedergelegte Philosophie einer durchgreifenden Kritik und Revision hat unterziehen können". (Kienzler, p.72).

This possibility of critizing the *Tractatus* with a representative of it already justifies for Wittgenstein the mention of Ramsey in the preface of the book of his mature Philosophy. Actually, we could bet that, just like Wittgenstein at this time, Ramsey had not seen that the consequences of the *Color Exclusion Problem* would be so fatal to the whole project of the *Tractatus*.

What is evident here is the ineptitude of the truth table or of any scheme of truth-functionality to explain the exclusion of degrees. For example, the logical product and logical sum do not have sufficient sensitivity to explain the exclusion of non-exhaustive colors. If I take "this is white" as p and "this is black" as q, the logical product cannot be TFFF (p, q), precisely because the conjunction's parts cannot be true together. A row in the truth table should be completely eliminated (or mutilated as Von Wright often writes). Moreover, if p is the case, we have that q cannot be the case and vice versa. So there is a picture of exclusion and implications in the mosaic of colors. The result for the *Tractatus* seems to be trivial: if elements of a proposition are mutually exclusive, they are not elementary, so one must keep on analyzing to sublimate the operational complexity and display the elementary propositions at its base.

However, this problem is just postponed. All analysis of propositions of gradation will generate necessarily exclusions – while these are, of course, not exhaustive, they are still exclusions. According to our approach, as there is no truth functional treatment for exclusion of degrees in the truth table, there is none in the *Tractatus* either. Clearly referring to passage in 4.442 of the *Tractatus* on reduced forms of writing down the scheme of T and F, in § 79 of the PB Wittgenstein claims:

"Das würde aber heißen, dass ich zwei bestimmte Sätze zwar anschreiben darf, aber nicht ihr logisches Produkt. Die beiden Sätze kollidieren im Gegenstand. Der Satz f(g).f(r) ist nicht Unsinn, weil ja nicht alle Wahrheitsmöglichkeiten wegfallen, wenn sie auch alle abgewiesen werden. Man kann aber sagen, dass hier das "." eine andere Bedeutung hat, denn im allgemeinen bedeutet "x.y" (WFFF), dagegen hier (FFF). Und Analoges gilt für "xvy", etc." p.107

Here we clearly see how the limitation of the truth table shows a limitation in the conceptual framework of the *Tractatus*, and vice versa. The conjunction of propositions for the ascription of colors to the same point is not only false; it is a

nonsense that our notation could not prevent. And this incapability for prevention is a serious problem throughout the tractarian project. Marion agrees with this view, writing:

"In the "analysis" was conceived as decomposition from complex to simple; and, since complex propositions are concatenations of elementary propositions by means of truth-functional operators, to say that "statements of degree" are analyzable means therefore that they are logical products of even more elementary propositions". (Marion, p.120)

For example, I cannot analyze 3cm truth-functionally with the truth of their conjunction parts as equivalent to 1cm.1cm.1cm, which means trivially 1cm, and not 3 cm as intended. Nor can I analyze 3cm as 1cm.2cm, which would be absurd (by contrariety, not by contradiction). Similarly, if a table is 3 meters long it cannot be correctly analyzed as (1 meter.2 meters.3 meters), for that would mean that the analysed is in the analyzing. The statement "The table is exactly three meters" excludes the table from measuring any other length. But saying the table measures "at least three meters" implies other possibilities. Here we reveal a serious problem for the truth-functionality or the metaphysics of truth table. It's not just a momentary problem of incapacity. One just cannot analyze statements about the ascription of grades to empirical qualities by means of logic products. The characteristic of the addition, essential for these systems, is thereby lost. We have then necessities that are not grasped by the truth functional paradigm. As Wittgenstein affirms in PB §76:

"Und verschiedene Grade von Rot sind miteinander unverträglich. Das könnte man sich etwa so erklärt denken, dass irgendwelche kleine Quantitäten von Rot addiert, einen gewissen Grad von Rot ergeben. Was heißt es aber dann zu sagen, dass etwa fünf solcher Quantitäten von Rot vorhanden sind? Es kann natürlich nicht ein logisches Produkt sein, dass die Quantität No.1 vorhanden ist, und die Quantität No. 2 etc. bis 5; denn wie würden sich diese voneinander unterscheiden? Es kann also der Satz, dass der Grad 5 von Rot vorhanden ist, nicht so zerlegt werden. Und ich kann also keinen abschließenden Satz haben, dass das das ganze Rot ist, welches in dieser Farbe vorhanden ist; denn es hat keinen Sinn zu sagen, dass kein Rot mehr dazukommt, da ich nicht durch das logisch "und" Quantitäten von Rot addieren konnte. *Es heißt auch nichts, zu sagen, dass ein Stab, der 3 m lang ist, auch 2 m lang ist, weil er 2 +1 m lang ist, denn man kann nicht sagen, er ist 2 m lang und er ist 1 m lang. Die Länge von 3 m ist etwas Neues.*" p. 105. My italics.

As discussed, new alternative interpretations of the logical operators of limited validity and corresponding to some systems should be adopted, as pointed out by PB § 83, where there is clearly a demand for logical operators that would

not be truth functional as perhaps a special operator, let´s us say, "mix up". We propose it for the color system that would allow the junction of blue and red to create purple, but not the junction of blue and orange, for example. This limitation of the expressive power of the tractarian logic could not be supplied or supplemented by its metaphysics. Assuming then that the elementary propositions would be independent, we have would have then a situation wherein molecular propositions would indeed be formed by the uniform application of truth functions to totalities of propositions. In this way, we could always make a conjunction between any two given elementary propositions. The well-known *Color Exclusion Problem*, or even of any proposition of gradation, as of temperature, length, volume, etc.., or in the case of trichotomies or exclusions within a taxonomic system, are catastrophic counter-examples to the tractarian theses about exclusive paradigm of truth functionality in propositional analysis. As Wittgenstein claims later in PB § 78, all contradictions should be shown by symbolism, so absurdities or nonsense could also be avoided revisiting this Leibnizian intuition of the *Tractatus* which we advocate here:

"Der Widerspruch muss sich ganz im Symbolismus zeigen lassen, denn, wenn ich von einem Fleck sage, dass er grün und rot ist, so ist er ja eines dieser beiden sicher nicht, und der Widerspruch muss im Sinn der beiden Sätzen liegen. Dass zwei Farben nicht zu gleicher Zeit an den gleichen Ort gehen, muss in ihrer Form und der Form des Raumes liegen. Aber die Symbole enthalten ja die Form der Farbe und des Raumes, und wenn etwa ein Buchstabe eine Farbe, ein andermal einen Laut bezeichnet, so ist er jedes mal ein anderes Symbol, und das zeigt sich darin, dass andere Regeln der Syntax für ihn gelten. Das heißt natürlich nicht, das Folgern nun nicht nur formell, sondern auch materiell geschehen könnte. _ Sinn folgt aus Sinn und daher Form aus Form." (p.107).

Among other things, the presence of this Leibnizian intuition still seems to indirectly confirm the periodization proposed here. PB is still clearly marked by the tractarian project of exploring the language with an adequate symbolism to show systematically its essential features and to avoid nonsense. This common thread also marks the 1929 article and the conversations with the Vienna Circle.

3.6. Two attempts of solving the *Color Exclusion Problem* within the tractarian assumptions

There is also, however, the movement of authors trying to think over or solve the problem of incompatibility of colors in tractarian terms, i.e., in purely truth-functional terms, despite the fact that Wittgenstein himself abandoned this project, or dissolved the problem in 1929, showing that it arises from a misunderstanding of some tractarian themes. Hintikka & Hintikka, for example, accept this incompatibility by trying to reduce it to a wrong bet on the analysis of propositions in terms of subject and predicate. However, they do not bring the discussion to the field about the limits of the expressibility of the tractarian notation of the truth tables:

"Das durch das Phänomen der Fabeinkompatibilität aufgeworfene Problem ist dennoch lösbar. Argumente des gleichen allgemeinen Typs wie das von Anscombe beruhen – ganz gleichgültig, wie ihre Einzelheiten formuliert werden – auf einer wichtigen Voraussetzung, die von Wittgenstein nicht akzeptiert wird. Diese Argumente setzen voraus, dass wir, sofern Rot und Grün Gegenstände sind, wissen, welches ihr logischer Typ ist. Der Einwand geht im Grunde nicht nur davon aus, dass die in der natürlichen Sprache artikulierten Sätze "Dieser Fleck ist rot" und (auf denselben Fleck zeigend) "Dieser Fleck ist grün" inkompatibel sind, sondern auch, dass es sich dabei um Subjekt-Prädikat-Sätze handelt, die in der richtigen Schreibweise etwa als "R(a)" und "G(a)" wiederzugeben sind." (Hintikka & Hintikka, p. 163-164).

Rather, the young Wittgenstein would never accept that predicate analysis of sentences reveals the real, profound logical form of the propositions. However, one cannot also assume that analysis in terms of functions and arguments would even be sought. By the same argument we cannot anticipate the final logical form of elementary propositions, as Hintikka & Hintikka seem to try to do. In addition, they appear to falsely attribute this procedure to Wittgenstein:

„Denn wenn man andere Voraussetzungen macht, erscheint auch die Situation ganz anders. So können wir z.B. im Rahmen eines Gedankenexperiments annehmen, der allgemeine Begriff der Farbe sei in der Sprache nicht durch eine Funktion c wiederzugeben, die Punkte des Gesichtsraums in einen Farbenraum abbildet. Dann wären die jeweiligen logischen Formen der Sätze „Dieser Fleck ist rot" und „Dieser Fleck ist grün": c(a) = r und c(a) = g, wobei r und g die beiden separaten Gegenstände Rot bzw. Grün sind. Die logische Inkompatibilität, sind die beiden Farbzuschreibungen würde dann durch den Umstand gespiegelt, dass die Farben Rot und Grün durch verschiedene Namen wiedergegeben werden. Und wenn es sich so verhält, sind die beiden Sätze eben doch logisch inkompatibel. Dass sie inkompatibel sind, geht hervor aus ihrer logischen Darstellung in der üblichen Notation der Logik: Aufgrund ihrer „logischen Form" - d.h.

Aufgrund ihres logischen Typs – kann eine Funktion für dasselbe Argument nicht zwei verschiedene Werte annehmen." (Hintikka & Hintikka, p. 165).

We cannot assume that the logical analysis to be expected in the *Tractatus* is the kind of subject-predicate or the type of argument and function. Both analyses are "shots in the dark", because we cannot anticipate the forms of the elementary propositions. This should be a task of the appocation of logic. So, It seems they hold we could solve the *Color Exclusion Problem* by the very definition of mathematical function. I believe that in this way, Hintikka & Hintikka trivialize the problem by taking it out of their specificities, and by making it similar to any other predicative assignment analyzable in terms of function and argument. If a function can only generate a value for each argument, obviously there will not be the possibility to assign two colors to the same point. By definition, there is no possibility, to assign more than one value to any argument from a function. This interpretation would be more nominal than strictly logical.

Hintikka & Hintikka recognize the limitation of their interpretation from the notion of function to account for the expression of contradictions40 in the tractarian sense, i.e., based solely on truth-functionality, although believing, curiously, that the problem would be easily solved if we not assume the Philosophy of the *Tractatus*. However, if we do not take the *Tractatus* and its attractive neutral and combinatorial logic, we would not have the problem, or at least, the problem as it appears for us. Hintikka & Hintikka even claim that the *Color Exclusion Problem* is Wittgenstein's problems, not theirs (!), i.e., the difficulties are directed to Wittgenstein and not to their interpretation to the problem:

„Gegen die bisherigen Ausführungen lassen sich bestimmte Einwände erheben. (…) Insbesondere fällt es schwer einzusehen, wie diese Notation imstande sein soll, Farbeninkompatibilitäten in wahrheitsfunktionale Kontradiktionen in präzisen Sinne Wittgensteins zu verwandeln. Dies ist zwarein gescheiter Einwand, doch viel ist damit nicht zu erreichen. Es ist zugegenermaßen nicht leicht zu erkennen, wie Wittgenstein im einzelnen an den Versuch herangehen könnten, die Farbeinkompatibilitäten in die wahrheitsfunktionalen Kontradiktionen des *Tractatus* zu verwandeln. *Dies ist Wittgensteins Problem zu jener Zeit, doch unseres ist es nicht.*" (p. 171) my italics

We then ask: To what extent can we say that the restriction of no two values for each argument in a function is in fact a logical constraint? Would this be

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40 Following this problem we also have the difficulty or impossibility of expressing the logical truths and the logic of relations in tautologies that Wittgenstein intended to express.

necessarily logical in the tractarian sense? It would also be revealing to ask ourselves why precisely the colors, grades and ascriptions of the empirical qualities require this special functional treatment, when other assignments in our everyday language do not, assignments such as "the book is great," or as "the girl is sitting on the bench". Another problem: it seems rash to me to say that colors are objects in the *Tractatus*. For me, tractarian objects should be able to have color, but the idea that color is an object is a step beyond.

Regardless to this point, Hintikka & Hintikka really believe that if Wittgenstein had developed the idea of the logical form of the ascription of colors, he would have come closer to their proposal of a functional analysis of the colors, which will block any nonsensical propositional concatenation: „Aus diesem Grund nähert sich Wittgenstein der funktionalen Analyse als expliziter Theorie erst ganz am Schluss seiner Suche nach der logischen Form der Farbzuschreibungen, ohne dass er sie je ausdrücklich als These akzeptiert." (Hintikka & Hintikka, p. 173). However, it is clear to tractarian readers that Wittgenstein there was not trying to find out the logical form of determined groups of propositions but to excavate the logical form of all empirical propositions.

In one of his six essays on philosophical logic published in 1996 by *Philosophical Acta Fennica*, von Wright returns to *Tractatus* arguing that one can examine the ascription of colors in a manner consistent to its assumptions. In his article *On Colour,* with the suggestive subtitle, *a logical-philosophical Fantasy*, he tries to show that it is possible to analyze truth-functionally propositions about colors so that we can have color systems and even logically independent elementary propositions. The article deals with the legitimate fantasy of thinking about people from a tribe with different conceptual organizations of colors than ours; following a mental experiment Wittgenstein himself already thought up in *Über die Farben*. There Wittgenstein claims:

„Kann man sich nicht denken, dass Menschen eine andere Farbengeometrie hätten als wir? _ D.h. Doch: Kann man sich nicht Menschen mit anderen Farbbegriffen denken als den unsern; und das heißt wieder: Kann man sich nicht vorstellen, dass Menschen unsere Farbbegriffe nicht haben, und dass sie Begriffe haben, die mit unsern Farbbegriffen in solcher Weise verwandt sind, dass wir sie auch „Farbbegriffe" nennen möchten?" (Über die Farben, III, p. 154)

Von Wright believes that from this motivation we could think of a complete rebuilding of our experience with colors from independent units. Thus propositions on these units could be brought to an elementary basis, which, in turn, could also be independent. We know that propositions in the *Tractatus*, and in Article of 1929 are either elementary or molecular, which must be possible to be truth-functionally reduced to the elementary, bipolar and logically independent. Thus, the truth value of these propositions of the basis of the language would always be compatible with the distribution of truth values of others. We would have, then, tautologies and contradictions as extreme cases of this combinatory game. The ascription of colors, then, could not trivially be the case for displaying atomic logical complexity, i.e., as we have seen, we would still have implications and exclusions in this context. As von Wright claims:

> "This is red and blue (all over)", we feel is a contradiction. Its two component sentences are not logically independent. Hence "this is red" cannot be an atomic sentence. But since it is not atomic the sentence must be molecular "in disguise", i.e, it must be possible to analyze, exhibit it in the form of a truth-function of atomic sentences." (Von wright, p.9-10).

Here again is the insistence (or mistake) on calling this kind of relation between propostions as contradictory and not contrariety, just like Wittgenstein does in the passage 6.3751. Von Wright believes that Wittgenstein's digression into physics in this passage is just evasive, postponing the problem without solving it. Von Wright claims, therefore, that the problem of bringing color into the realm of numbers or frequency velocities does not solve the problem of incompatibility, because this correspondence would not be essential to the concepts of colors or to the color logic.

In any form, the output of the passage 6.3751 is unsatisfactory in the two lines of argumentation with numbers and with velocities of particles, because they imply, as Wittgenstein claims in 1929, the mutilation or restriction of the articulatory horizon of the truth tables, blocking *ad hoc* the free distribution of values of truth to propositions, as planned in the *Tractatus*.

We could try to think of a kind of operator of contrariety, expressed in truth-functional terms, as the negation of the conjunction of two propositions, something like (FTTT) (p, q). This corresponds to the second element in the exhaustive list of combination of binary operators that Wittgenstein draws in

5.101. However, this does not solve the problem either. Still, we should mutilate any row of the truth table for such an articulation. For example: assigning two colors to the same point is simply not authorized by the system of propositions in which we operate. There is a restriction in the distributions of truth-values in such molecular propositions. This notation does not capture the multiplicity of logic of the system in which it is used, i.e., it allows the articulation of symbols of things that cannot be articulated in reality. It is evident that the problem is less with the falsehood than with the absurdity. It is a nonsense that our notation does not prevent, even though being created to prevent it. Contradictions belong to symbolism, while nonsense or absurdities should not. This necessary mutilation of truth tables shows interesting cases of logical dependency between some propositions and their internal components. However, as von Wright claims:

"it lacks deeper interest unless we can give a satisfactory account why certain distributions of truth-values are ruled out as impossible. Wittgenstein´s solution in the paper [1929] is incompatible with the *Tractatus* idea of logically independent atomic sentences. In fact it signalizes the beginning of his later very extensive criticism of the *Tractatus*." (von Wright, p.10).

The first propositive movement of Von Wright is already perhaps the most problematic because it is anachronistic: trying to show that our notion of simplicity is contextually dependent, based on the assumption that the colors red, blue and yellow could no longer be simple primitive colors, but the bluish, the yellowish and the reddish. This freedom to change linguistic atoms seems to well accommodate notions of Wittgenstein's late Philosophy. However, this relativism of the simple in the *Tractatus* seems odd. Bluish is naturally taken not as something simple, but as something already containing a combination of blue with another color, or even a gradation of blue. However, according to von Wright, for the group of people for whom bluish is a single element, our blue would be taken as a compound, because it would be blue and not any other color (!). So blue would not be as simple in the language of this tribe, as it is for us. Von Wright then claims:

"The "logic" of the three simples of their colour language would differ from that of the corresponding three simples in our language. Their simples are logically independent. They can be present or absent in a given object in any of the eight commutations, beginning with the case when they are all present and ending with the one when they are all missing. That is to say: This is how we make it in our fantasy". (von Wright, p.11)

Indeed, to hold the three simple colors being bluish, reddish and yellowish seems to make all combinations possible, thus capturing all possible mixtures and also all the extreme cases such as blackish or whitish. We may also eventually make the language of this tribe richer if we add to it whitish or blackish. At the end of his article, Von Wright built truth tables without mutilation that capture the logical multiplicity of the conceptual articulation of the colors of this tribe, systematically showing that there is no problem of incompatibility, in these terms:

Bluish	Reddish	Yellowish			
+	+	+			dirty
+	+	-	violet		
+	-	+	green		
+	-	-		blue	
-	+	+	orange		
-	+	-		Red	
-	-	+		yellow	
-	-	-			colourless

Whitish	Swarthy	
+	+	Grey
+	-	White
-	+	Black
-	-	Colourless

According to von Wright, in this community, our blue would correspond to the interpretation that something is bluish and not reddish and not yellowish. In any way, von Wright takes us to the conclusion that there would be no pure blue. The colors are indeed always relative to the context in which the system is applied. A single blue which would be considered pure in a context where it were among red objects, for example, could be considered light blue in another, for example, when there are many shades of blue at our disposal. Which would the pure blue then be? There is a similar argument when we try to avoid using the notion of a unity of blue to allow some arithmetic notions of adding colors up to others. It is difficult to say (either estipulatively or arbitrarily) that a unity of blue added three times to a unity of red would generate a lighter purple.

We have one more problem in this mental experiment of von Wright. According to the discussions in the WWK under the heading *Die Welt ist rot*, a man who has lived all his life locked in a room that is completely red could not claim that "the world is red." To make sense of this assertion, the captive should be familiar with every colour i.e., our color systems would then be the same. The possible alternative is that he would have a different system, so that we could not even understand what his affirmation would actually mean. If the captive does not have our logical multiplicity or the same conceptual distinctions as we do, we could not understand him. Strictly speaking, he would not have the same experiences as we. He would not articulate things in the same way as us. The way we see colors are also determined by our conceptual system and the vocabulary we have available to describe and to differentiate objects of experience. So the question remains as to whether or not the members of this tribe conceived of by von Wright would actually experience the world of colors like we do. If we changed the atomic basis of colors, could we understand what these people identify as violet, or as green, or as dark? Either they would have the same system as ours, or we could not understand what they are talking about. If someone is to be familiar with a color, he or she must know all the colors, all the possibilities of the spectrum should already be foreseen41. If they have the same logic of colors, by the holism of propositional systems, they have to recognize all possibilities,

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41 *Die Welt ist rot.* If an animal distinguishes red from blue, does it means that it must distinguish all the other colors? Does it mean that it has a kind of "color logic"? Does this animal have to possess any level of language for this? Is it the case that, the richer our color spectrum, the richer our language is, and vice versa?

including black and white. Changing the atoms of the analysis can be disastrous here; it can lead to radical incommunicability between the "tribes". It can lead to a solitary and radical ethnographic confinement.

We have one more question here: If I say that an object is green, do I really mean that the object is yellow and blue? I think not. When I say that a point is white, do I use this statement as a kind of abbreviation for the assumption that all colors are there together at this point? The answer here also would be negative. Someone may well ignore the physics of the velocity of particles, or have little or no knowledge of a more refined mixing of colors for painting, and still competently use the vocabulary of our grammar of color without errors and in plain language. In fact, if we really take the idea that we should move in the grammatical sphere of language seriously, rather than trying to analyze it in terms of a first language, the need for the logical independence of elementary propositions about ascription of colors loses its relevance.

Even if we have actually freed the truth tables of their mutilation as in the case of the prohibition of the true conjunction of true propositions, by changing the atoms of analysis, we would still have a problem with color gradations, if we allow in this perspective variations of shades, lightness, or the saturation of these same colors. We will still have problems with any gradations of empirical qualities, such as temperature or length – problems Von Wright does not work through in his article. Von Wright recognizes the limitations of his approach:

"Can a colour-world of the same conceptual complexity as ours be built up in this way? I shall not investigate the question here. It is doubtful whether one can construe differences in degree of brightness and saturation of colors on a basis of logically independent "atoms". If one cannot, we would have to assume that people whose colour-worlds are constituted in the fashion we have here imagined either do not notice, pay attention to such differences or that they do not count them as differences in colors. (Do we?)" (von Wright, p. 14)

Even if one solves the problem with the colors with the adoption of logically independent color atoms, we would still have problems with all the gradations. Indeed, we have problems not only with colors, but also with tractarian numbers, which, as we have seen and Cuter´s works defend very well, serve, to count, but not to measure, because it is based on formal series. The problem is not restricted in any way to the colors, but, as we can see, there are problems with everything involving degrees. If we think that there is no empirical quality that cannot cope

with gradations in its constitution, that is, to have a gradation is fundamental for empirical qualities, we have the amplitude of the matter: for all qualities we have some gradation, because they are inserted in a logical space (or in many), therefore each empirical quality is embedded in a framework of logical implications and exclusions. Saying that a quality has essentially a gradation means that, negatively, there is no quality without the determination of gradation and, positively, thinking about a quality is considering the possibility of measuring it in some way. It is meaningless to speak of a quality that is not susceptible to empirical gradation and that doesn't compose a complex spectrum of combinatorial possibilities with other gradations. Just think of the interaction of two variables such as time and space to compose the notion of displacement, motion, speed, or the relationship of color and a surface temperature and length of a piece of metal. The *Color Exclusion Problem*, in fact, reveals itself as a problem pertaining to all empirical qualities and its determinations, implications and exclusions. "Man könnten sagen, die Farben haben zueinander eine elementare Verwandtschaft" (PB, 76). According to our reasoning, we could generalize this assertion: All empirical qualities have an elementary kinship in the same way that understanding a proposition is in a way to understand all the logical space (3.42). To this analogous conclusion also seems to lead Demos' article about particular negative propositions42.

It seems to make no sense either to think that these exclusions of qualities can be reduced to some extent to empirical exclusions, that is, to exclusions given in a multiplicity of data. Since these data are organized and belonging to categories or types of different qualities, we not only have an empirical complexity, but also a logical one. There is organization only where there are implications and exclusions. A multiplicity of data does not exclude itself, functionally or otherwise, without the addition of a logical space of gradations.

It seems naive to me to think of something as a strictly empirical exclusion. Indeed, something that is "purely empirical" is already an abstraction that is

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42 In the context of this problem, Bento Neto claims: "The fundamental is to note the modality of that notable exclusion: the other propositions, or the other "forms of concatenation", are not subject to a factual exclusion, but to a logical one (otherwise, indeed, this exclusion would not pose any problem for the Tractarian scheme)" (Bento Neto, p.126). Questions: What is a factual exclusion? To what extent may this indeed be differentiated from logical exclusion? Wouldn't talking about a exclusion that it is logical be redundant in the end? Doesn't speaking of exclusion imply the existence of a logical space of competing possibilities, of alternatives and abstract relations?

extremely sophisticated (if not just a word game). In our daily life we do not have any contact with anything that can even be considered strictly or purely empirical, totally apart of conceptual or logical relations. Something completely "clean" of gradations, of hypotheses, of an abstract sphere or of any conceptual or spatial relationships, cannot be found and is probably not even imaginable.

The case of colors seems to be an emblematic case: we cannot think of a color without thinking about its necessary articulation with another. I believe herein lies the key to understanding what Wittgenstein meant by an object in the *Tractatus*: a point or a place of articulations. The possibility of a color brings up the complete range of colors, and provides the logical space of the colors. The possibility of a contingent complex shows us a necessary totality. Maybe that's why all concepts of form in the *Tractatus* are always modal. When we think of the possibility of something, we will then introduce or discover an abstract network or combinatorial horizon of the things we deal with, its modal sphere - be they empirical qualities or mathematical concepts.

Furthermore, perhaps the best thing about the article by von Wright is that it shows us that the *Color Exclusion Problem*, despite Cuter, Ramsey and Wittgenstein himself in 1929, may indeed be truly independent of the issue or problem with measurements, (or of the expression of oppositions by contrariety as proposed here in this work). After all, making numbers, for example, to map people and drinks at a party in order to have the number and order of preference of drinking at different times does not make the nature of people or of cups the necessarily numerical. It is interesting to ask how philosophically interesting it would be to assume whether or not this correspondence with gradations and numbers is essential for the expression of the concepts of colors, or even for its organization. The color system can be held as a system by itself. The correspondence of colors and its organization to other elements and its organized, such as numbers and its linearity is not essential to them.

As a result to these arguments, the *Color Exclusion Problem* exists, even if Hintikka & Hintikka do not recognize it and work with the definition of function instead. And, contrary to von Wright's argument, I hold that the problem cannot be solved in tractarian terms.

3.7.
Recapitulating the tractarian negative points through the truth table

We have seen how the famous problem of the colors, which I take to be an umbrella concept for more general logical problems about expressibility of contrary propositions, appears in the truth tables. Or rather, according to the arguments I have outlined here, it appears only in the *Tractatus* if and only if it appears in the context of truth tables, as a privileged notation. After all, the *Tractatus* fails, where the truth table fails. I believe that it is not accidental that the *Color Exclusion Problem* is preferably presented by secondary literature using the truth tables. The intuition of these works, which I explore more systematically throughout mine, is how perspicuous the truth table can be in revealing problems in the *Tractatus*.

Thus, a second attack on the metaphysical tractarian flank of the truth table would be precisely to investigate what it means to take logical operators as non-denotative and without sensitivity to different contexts, particularly the denial, as our paradigmatic case. As we have seen, in order to identify the negation of a fact in an affirmative proposition, the negation should be thought of as an unary propositional operator and not as a predicative one. As we have seen in this way, the denial scope must be the whole proposition, and not just some part of the proposition. Under a strong truth-functional paradigm, as is the case of the *Tractatus*, the denial of a proposition actually seems to change the direction of the arrow pointed to the fact that will make it true. As we have seen, in the *Tractatus*, there is nothing internal to elementary propositions such as predicates, indexes, etc. and degrees or gradations that are logically relevant to determining the truth value of complex propositions, in this context.

It is important for the *Tractatus* that all operators, including the negation must be operators of propositions, so they do not interfere in the logical multiplicity of the proposition and are not denotative. The price paid here with this assumption is, among other things, the loss of the subtlety of predicative negation revealed, for example, by the type of analysis, for instance, recommended and defended by the work of Chateaubriand. I believe this is the price that the *Tractatus* has to pay for its conceptual mechanics works. Here the problem with a truth-functional notation is that it is rough and too abstract for

more refined analysis of operations conceptual within the proposition. In any way, within the conceptual geography of the *Tractatus*, accepting their assumptions, this problem is harmless. The tractarian denial is always propositional. It is so due to the pictorial demand that an affirmed or denied proposition represents the same fact, so that the negation functions as a simple inversion of sense, an inversion of truth conditions. The denial does not influence therefore in the logical multiplicity of the denied proposition. We would understand then nothing new about a fact represented by a proposition when we deny it. This Grundgedanke holds throughout the intervening period, as shown in conversations Wittgenstein had on positive and negative propositions at Schlick's home in January, 1930,:

> "Der negative Satz gibt der Wirklichkeit dieselbe Multiplizität wie der positive Satz. Wenn ich sage: "Ich habe keine Magenschmerzen", so habe ich der Wirklichkeit dieselbe Multiplizität gegeben, wie wenn ich sage: "Ich habe Magenschmerzen". Denn wenn ich sage: "Ich habe keine Magenshmerzen", so setze ich im Satz bereits die Existenz des positiven Satzes voraus, ich setze die Möglichkeit der Magensschmerzen voraus, und mein Satz bestimmt den Ort im Raum der Magenschmerzen. Es ist nicht etwa so, dass mein gegenwärtiger zustand nicht die geringste Verbindung mit den Magenschmerzen hätte. (…) Das letztere meine ich nun, wenn ich sage, der positive Satz hat nicht mehr Sinn als der negative. Beide geben der Wirklichkeit dieselbe Multiplizität". (p.86)

The notion of logical multiplicity becomes even more important for Wittgenstein in this intermediate phase, i.e., for representing things in reality we must always give our representations the logical multiplicity of the complex being represented. In analogy to this treatment of negation, we hold that nothing is added to propositions when we articulate it with an operator. This is important for the conceptual tractarian architecture (s*ein Grundgedanke*!), but leads to the problem of insufficient expressivity in the propositional negation, as noted above. Understanding Tp, Fp, F not-p, T not-P is to understand the very same fact p. The problem of denial could then be reduced to the problem of falsehood by the truth-functionality.

The unsatisfactory treatment for generalities is another problem that, beside the difficulty with the negation predicative, seems to lead our vision to the inner side of the elementary proposition, the need for a more refined analysis of its components. As alluded to in the beginning of the work, this hurts the truth-functional appeal, i.e., that places emphasis and relevance only on the truth value of elementary propositions. Again, in our daily life, in trivial contexts, we have to

the appeal to the use of propositions of a more general and abstract nature, but also descriptive, as with "there is only one table in my classroom", "I ate all the sandwiches in the fridge," "all the bottles of beer are cold ","no country has a sky more beautiful than mine"....What we have in the *Tractatus* is the suggestion of a treatment in terms of logic products and logic sums in the case of universal and existential propositions, respectively. So that the *Tractatus* could generate some treatment of generalities via a truth table using conjunctions and disjunctions. Already in *Tagebücher* Wittgenstein states categorically: "So könnte man statt (x)fx schreiben "fx.fy..."" (21.5.15, p. 49). In 5.1311 and 5.441-44, we see that Wittgenstein draws conclusions regarding the quantification through a stricly propositional approach. The problem here is that we should have ontology of eternal and simple elements available for this general work. We need something like a predication, that is, something inside the proposition should be used in order to have a better treatment for generality. In the same way one can assign a predicate to an individual, one can also predicate this predicate, by saying that it applies to all individuals or at least one in a given domain, as Frege and Russell do.

These three cases, the milder exclusion of degrees, the insensitivity of the propositional negation and the treatment of generality or quantifications are issues that lead beyond the expressive possibilities of the truth table. These are problems that demand a machinery - less abstract and more sophisticated, just as numerical indices or predicates of different orders and the attempts to see logical operators as negation and implication acting within the elementary propositions. To show the limits of the truth table, involves showing the limits to the kind of analysis advocated in the *Tractatus*. In other words, it is to show holes in the hull of the boat that was intended to carry us to all possible ports. Later, Wittgenstein sees that the proposed journey could not be made under those conditions, and that it should be aborted, and that the whole proposal or plan should also be abandoned.

Strictly speaking, in addition, leaving the paradigm of truth-functionality implies losing the notion of a decision algorithm, but also making gains in the expressiveness of meaning. We lose on one point but gain on another. From the metaphysics of truth tables we can finally get to the pragmatic of calculi of different expressive power, as expounded in textbooks today. We will return to this issue of the postponement of tasks in the last chapter of this work, but let's

complete the metaphysics of the tractarian sense by giving an account of the sub-propositional part of language: the *Bildkonzeption*, and its role in the *Tractatus* and in its collapse and abandonment.