## 2. Compositionality

"In meiner alten Auffassung der Elementarsätze gab es keine Bestimmung des Wertes einer Koordinate; obwohl meine Bemerkung, dass ein farbiger Körper in einem Farbenraum ist etc. mich direkt hätte dahin bringen können." Wittgenstein, PB, §83.

## 2.1. Towards the Logic, the Phenomenology, and the Phenomenological Language by the time of the *Some Remarks*

Before the rupture, Wittgenstein attempts to review and develop some tractarian positions. In his return to Philosophy with the article he wrote in 1929, we clearly see something like a continuation of the tractarian work through the abandonment of ideas and development of others. What this return means, in fact, is controversial. We have two main candidates here. Did he essentially solve the problems and come to apply or to extend what was begun in the *Tractatus*? Or did he come to modify what had previously been thought to be definitive and unassailable? In any case, one interpretation does not exclude the other. They can be made compatible, and can even complete each other to form a more diachronic view. For example, the new guidance and critical reading of the *Tractatus* in 1929 arises precisely from the consideration of its inadequacies when he tries to apply or to extend or to clarify and to complete what had been proposed for the essential, but had not been done in detail. As Kienzler claims:

"Das Jahr 1929, in dem Wittgenstein von Österreich wieder nach Cambridge übersiedelt, bedeutet ebenfalls noch keine Wende in seiner Philosophie, sondern in erster Linie die Wiederaneignung der bereits niedergelegten Auffassung, zusammen mit dem Versuch, sie auf neu hinzugekommene Probleme, wie Fragen der Wahrnehmung und des Unendlichen, die der *Tractatus* ausgeklammert hatte, anzuwenden. Dieser Prozess der Wiederaneignung führt dann allerdings Wittgenstein ziemlich rasch zu einer durchgreifenden Kritik am *Tractatus*, zu einer wesentlichen philosophischen Umorientierung, und damit zum eigentlichen Eintritt in die Spätphilosophie." (Kienzler, p. 13)

In 1929, what Wittgenstein wants to do or announces should be done, is still the need for a complete analysis of language, still within the tractarian project, but a more refined one. The failure of the central role of a completely truth-functional linguistic analysis represents more than the failure of the paradigm of complete

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analysis. It represents the failure of a project, which in turn leads to a negative or critical position in relation to a pictorial and normative image of language with clearly defined boundaries and senses. As a result, logic should not be so neutral and formal anymore. The problem with the truth-functionality, corresponding to the ideal of a completely neutral logic, seems to me to be the great paradigm in the Tractatus, in respect to which the Bildkonzeption should try to adjust itself. The logic should have, ultimately, taken care of it self. The collapse of the completeness of the compositional approach to language leads to problems with the bipolarity. This was no longer the only propositional form of exclusion. There must be not only one possible negation of a proposition as meant in 5.51310. There are in some cases of empirical propositions the explosions of possible alternatives. This mitigation of bipolarity led, in turn, to the need for reformulation or refinement of the Bildkonzeption. The truth-functionality fails as the only legitimate logical analysis of empirical propositions because this, at least in some special cases, seems to require a more contextually sensitive conceptual analysis. From this, the criteria for exclusion and implications seem to be more fluid. This demands a more refined or sensitive analysis for the conceptual arrangement of these propositions. These should then form a system of exclusions and implications. Otherwise, we could not have a more precise categorization of, at least, the continuum of colors, or of any measurement or possibly even of numbers.

The style of *Some Remarks*, this short article published in the *Proceedings* of the Aristotelian Society, is surprising in the case of Wittgenstein: it is nothing like the aphoristic structure of the *Tractatus* or the dialogical of the PU. Here we have a plain text, with traceable and organized arguments. Three more general points promptly call the reader's attention. Firstly, there is an appeal to a more contextual (rather than global) analysis of the language. This indicates a clear demand coming from the case of propositions that involve some kind of gradation of quality; ii) There is also a special motivation to consider ordinary knowledge, or rather, ordinary language as a criterion, as a intuitive and healthy point of reflection. For example, this happens when Wittgenstein identifies something like

<sup>10</sup> *Cf. Tagebücher* 14-16, p. 55. Here there is clearly the linkage of this vision to the account of bipolarity, because the passage that contains "Jeder Satz hat nur ein Negativ; ... Es gibt nur einen Satz der ganz außerhalb von "p" liegt" begins with questions about a theory of representation and language that would not yet encompass the necessity of a proposition having two poles. (2.6.1915)

"a point cannot have two colors" as a kind of tautology that "every one of us knows in ordinary life." (p. 167). Or when he justifies the interdiction on the ascription of two degrees to the same quality as saying that "to all of us in ordinary life is some sort of contradiction". (p.168); iii) There is a clear shift of emphasis from the realistic tractarian ontology of facts and states of affairs to a more phenomenological investigation11. In this way, affirms Wittgenstein: "I only wish to point out the direction in which, I believe, the analysis of visual phenomena is to be looked for." (p. 166). There is a demand on describing the logical form of actual (or even visual12) phenomena, and not anymore of state of affairs and of facts, which determines the truth values of propositions. If the use of "facts" could take advantage of the neutrality in the *Tractatus* in a kind of minimalist approach to ontology, now in 1929 there is a clear bet that the analysis should be directed to phenomena, that is, spatial and temporal objects of our experience. Kienzler seems to agree with this perspective:

"Ein wichtiges Ziel von Wittgensteins Studien im Jahr 1929 ist es, seiner Philosophie eine feste Grundlage zu geben. Neben den schon im *Tractatus* thematisierten Bereichen der Logik und der Naturwissenschaft hielt Wittgenstein in dieser Phase den

<sup>11</sup> Bento Prado Neto argues that there is no movement from a realistic sphere in the Tractatus to a phenomenological sphere in the middle period. Following the interpretation of Hintikka & Hintikka, Prado Neto already sees in the Tractatus this phenomenological sphere, identifying the pursuit of a phenomenological language of his middle period with the tractarian application of logic. He recognizes the project of a phenomenological language already in the tractarian passages of 5.6 's:"However, we will try to show later that, in a sense, the completely analyzed language of the Tractatus must be "phenomenological." If this adjective aims to characterize an essential dependence regarding a subject, this language will necessarily be "phenomenological" because fully analyzed the language will describe the world as it appears to "me"- that is, to the metaphysical subject. Thus, the phenomenological project reviewed in 1929-1930 is not the theoretical result of additions to the Tractatus. Instead of it, the project flows from the very heart of this work, and to tell the truth it is already there explicitly described: what would the "book" mentioned in the aphorism 5.631 be but a piece of "phenomenological prose"?" p.21 In pages 40-1, Bento Neto affirms even more strongly: "Then there is perfect continuity between the Tractatus and the project of a phenomenological language: the task to which Wittgenstein intend to devote himself, upon returning to work in 1929, is exactly the work that the Tractatus had left undone. The rupture of PB with the Tractatus, therefore, must be found essentially in the failure of this project." A missing point in the interpretation of Bento Neto is an investigation of the role, apparently very relevant, of the logical multiplicity for identifying the fully analyzed language of the Tractatus with the phenomenological middle period. It seems clear that in both cases a language or a perspicuous notational system must follow the logical multiplicity of what they represent. That is, they should allow the possible concatenation of signals that reflect possible concatenations of represented phenomena and prevent systematically unforeseen or impossible concatenations.

<sup>12 &</sup>quot;Wittgenstein verwendet den Ausdruck "Phänomenologie" also nicht, weil er sich damit auf andere Philosophen (wie z.B. Husserl) oder auf eine laufende Diskussion beziehen will, sondern als eine grobe Gebietsbezeichnung für die Untersuchungen, die er gerade anstellt. Das Gebiet, auf das Wittgenstein mit diesem Titel anspielt, ist das der Wahrnehmung im weitesten Sinne, insbesondere das der Gesichtswahrnehmung." (Kienzler, p. 110) Wittgenstein gives the character of phenomenological to the investigation of the present perception and sense. From this, science could represent the multiplicity of phenomena of perception, ordering them and qualifying them quantitatively. Physics would be responsible for the truth of the representation of the phenomenal world and phenomenology, for the sense of our representation. This follows the tractarian distinction between physical and logical space, i.e. the distinction between the modalities of actuality and of possibility. We will return to this theme in the last chapter of this work.

Rückgang auf die unmittelbar gegebenen Phänomene selbst für notwendig." (Kienzler, p.105)

Thus, it seems natural the requirement of thinking about a language or notation that would follow the multiplicity of phenomena, i.e., a language that would be strictly limited to the expression or representation of possible articulations of phenomena. The expected consequence would be the necessary exclusion, in its syntax, of some symbolic articulation which would represent a phenomenal impossibility. Thus it was founded on the need for a radically direct description of empirical reality the idea of a phenomenological language, transparent to the logical multiplicity of the phenomena. Indeed, the truth table would not be a good notation for representing phenomena because it allows, as is clear in the Color Exclusion Problem, symbols to articulate without mirroring possible combinations in reality. This is always a case in which elementary propositions are not logically independent. Some combinations of conjunctions or implications in the notation should be banned, even when this allows them. As Von Wright calls it, the truth table has to be multilate in some cases, what represents that some articulations are forbidden. The distribution of truth values cannot be neutral and purely combinatorial.

The description of the phenomena that appear directly in our perception should be able to then be captured by language or a notational system, which should be more appropriate or sensitive to this more refined expression of possible articulations of simple elements13. This language would not have the phenomenological unclearness or presuppositions about the physical nature of things that would appear due to an utterly imperfect or distorted characterisation of the current language, namely a physicalist one or the current language. This could not completely show transparently its sense, thus allowing ambiguities and indeterminacies. As Bento Neto says:

"The mechanics, therefore, in the strict sense (Newtonian mechanics, as opposed to the precise mode by which it describes the world), is not a series of propositions, it is not

<sup>13</sup> From the withdrawal of phenomenology is born a more global and comprehensive considerations of problems, through the investigation of grammar that compounds it. The sphere of the phenomenological shifts to the investigation of grammar. To abandon phenomenological language does not mean to abandon the complete analysis of language and of phenomena. Here's the classic place of the abandonment of the phenomenological language: "Ich halte sie jetzt nicht mehr für möglich" (*Nachlass*: 107,205)

a use of language. It is a language, and as such, there is no talk of "verification" or "falseness" (...) If succed in such a "translation" of the language of physics in the completely analyzed language, we will already have shown the unverifiable and unfalsifiable character of the mechanics. By showing that its "basic principles" are not be translated into "propositions" capable of truth and falsehood. Here, therefore, the language of physics contrasts itself with the completely analyzed language. The form of the language of mechanics (the metaphor of the squares or triangles) does not reflect the form of the world, which can only be clearly expressed in a completely analyzed language." (Bento Neto, p. 25.)

Also, while physics and mathematics seem to have found their suitable and sufficient notational systems for the analysis of phenomena and the perception we have only our current language. This, in addition to inaccuracies and ambiguities, is also permeated by abstract and hypothetical spheres that do not correspond directly to what is actually being seen. This becomes clear in the case of a trivial assertion: "Over there is a cinema." Here comes into play the probability of a façade and an entry corresponds to a cinema and that there are chairs set in front of a big screen, where a film will be watched maybe a few times a day by a curious or lazy public, by consumers of popcorn and so on. It is important to state: in such daily sentences, we are dealing with much more than mere perception or actual experience.

As affirms Bento Neto:

"Certainly the language of physics and the common language, both coincide in describing the data as a region of reality. The region to which we have immediate access. When I say "over there is a chair", or when the physicist says "over here is an electron", what is given is only a "part" or a "facet" of what "is there." Through these languages, we describe the data as a perspective on the reality. This perspective is determined by our location in the world, among other real prospects or mere possibilities. The "thing itself" would only be given by integrating all these perspectives. In this sense, these languages contrast a region of the "given" to a region of the hypothetical, of "the" inferred ". We do not see the chair itself, but one of its facets. We do not see the electron itself, but one of its macroscopic effects. The presence of the chair or the electron is therefore inferred from the "observational data". (Bento Neto, p.42)

This would sketch out in our everyday language our tendency to deal with objects and people instead of colors distribute in a Cartesian field, for example. Therefore, the task of a phenomenological language should be to clean or correct the current language from this hypothetical sphere and bring it to a more precise expression of current perceptions. It should establish the domain of the not hypothetical, of the direct, without intermediaries. And thereby describe only the verifiable. Avoiding assumptions is an attempt to stick to the realm of the radically given, of the explicit and transparent meaning. However the problem remains: is not this hypothetical sphere essential to the functioning of language? If not, it is reasonable to assume that we have to have a flawless memory and instant figurations strictly to avoid problems with space and time. Wittgenstein even assumes this in a thought experiment to investigate the consequences of something so radically different from our everyday lives:

"Angenommen, ich hätte ein so gutes Gedächtnis, dass ich mich meiner sämtlichen Sinneseindrücke erinnern könnte. Dann spricht nichts dagegen, dass ich sie beschriebe. Es wäre das eine Lebensbeschreibung fortlassen können? Ich könnte ja z.B. Die plastische Gesichtsbilder darstellen, etwa in verkleinertem Maßstab durch Gipsfiguren, die ich nur soweit ausführe, als ich sie wirklich gesehen habe, und den Rest etwa durch eine Färbung oder Ausführungsart als unwesentlich bezeichne." (PB, 97)

We have here the problem of space: how to deal with things that, strictly speaking, cannot be represented with individual figures, such as a landscape, or even the cinema. We are dealing with a whole composed of parts, where we in fact only see the parts. The parts themselves may also exhibit this same problem: for example, we do not strictly see the façade of the cinema, but perhaps an organized cluster of colors. To understand the description of these colors one may already know how to identify what these colors indicate. We must therefore be able to understand a previous interpretation. Even here, the description is not entirely clear or obvious, as intended by Wittgenstein. Indeed, one must have learned to master the language so that one can understand a description from it. We have to understand the language in which these *Gipsfiguren* would be relevant. To understand this radical phenomenological language we have to have already a language.

Moreover, when I reread a description of my current perception, would it not become hypothetical? Even here, the description should be instantaneous, generated simultaneously to the data, to prevent time bringing hypothetical elements to the phenomenological description. We have here the problem of time. In the case of the re-reading of a description, there is necessarily a delay. Simultaneity between the described and the description is no longer possible. A picture alone is unable to point to or indicate the subject to which it is bound. Its temporal indexing will not give us this specification. Perhaps only in the time of production of the picture, if this is truly instantaneous, of course. If not, the description should be as fluid as the data described without therefore a determined sense. As Bento Neto pointed out, this diagnosis leads to the impossibility of representing the temporal sphere of phenomena. We then come only to dead ends. The temporal determination cannot be properly figured, because there is no figuration without the temporal determination. In this sense, language is always physical and not phenomenological. Therefore, Kienzler affirms that, diagnosing the character more mechanical than linguistic of Wittgenstein's fantastic models:

"Es ist außerdem bemerkenswert, dass Wittgenstein einen konkreten Mechanismus skizzierte und nicht wirklich eine Sprache. Die Phänomenologische Sprache, wie Wittgenstein sie entwirft, ist gar keine Sprache, sondern ein technischer Apparat zur Erzeugung von Bildern. Die Merkwürdigkeit wird sich im weiteren noch öfter zeigen. Die Suche nach einer phänomenologische Sprache führt Wittgenstein regelmäßig aus der Sprache heraus; er bewegt sich mit seinen Annahmen ganz außerhalb der Sprache." (Kienzler, p. 119)

We find another problem here. Clearly, to avoid this problem of the remission of descriptions to their corresponding data, when we think of descriptions in terms of instantaneous and immediate production of empirical data whether in pictures, or in figures of plaster or whether in a phenomenological language, we are dealing with a paradigm of portrait, a denotative one, no more with a bipolar figuration of facts as in the *Tractatus*. The simultaneity or instantaneity in reproduction guarantees the univocity of what is being described, i.e., the correspondence of the frame of a picture to the actual landscape portrayed therein without ambiguity. But this solution brings us back to the classic problem of falsehood. This was "solved" in the *Tractatus* precisely by the refusal to hold propositions as points but as arrows. We will return later to this theme to consider the use of this analogy in the *Tractatus*.

Already Bento Neto argues that even if all these ideal conditions, such as an instantaneous and perfect memory in the production of models or descriptions were met, we would not be free of problems which time brings to this phenomenological investigation. Here there is a clear resemblance to the programmatic sphere of the *Tractatus* or to the postponement of the in facto logical analysis of language. And that is even with the announcement of the practical difficulties to be encountered, as Bento Neto defends:

"The reasoning has obviously as a whole the character of *a reductio ad absurdum*: it begins by assuming the most favorable conditions and shows that, even so, the aim is not reached. We should then understand the general meaning of these "initial conditions" (...) But it should be noted, first of all, that this is a language with a minimal syntax, almost a natural syntax, that is, a language in which a certain event is represented by a similar event: one explains how a determined action occurred by repeating it. Unless by the "coloring or any filling up", which indicates what I actually did not see, everything is an exact copy of the things I saw. My Lebensbeschreibung consists almost in a "revival" of my life. The role of quasi-natural syntax should be to put aside problems which are peripheral to the core issue that will be discovered. It is a well of practical and technical difficulties, but it is shallow of theoretical difficulties in the sense that the signs indicate what they represent by themselves. This empties the complicated network of relationships that normally binds signal to signalized. This "natural" character of syntax is exacerbated when Wittgenstein takes up the description of this language as a mechanism: this mechanism will produce the same visual sensations I've already had. And if a problem is detected in relation to this natural syntax, the same problem should be found in a different syntax, so perhaps just less obvious because of the symbolic network of relationships that it shuffles." (Bento Neto, p. 83)

The *Tractatus* is still clearly present in this period marked through the central role of the search for a phenomenological language, to guide and to be the result of a complete analysis of the current language. In *Some Remarks*, for example, we have been dealing with eminently tractarian themes: the form and content of propositions, a syntax restricting the combinatorial horizon of words, so that nonsense are excluded and prevented, the analysis of propositions in terms of sums, products or truth functions of propositions, the need for the existence of simple propositions and a last connection of terms (3.25) that have the same multiplicity of the represented complex (4.04) in the projection method. In *Some Remarks* we still have an atomic form that cannot be advanced by the analysis. However, we must learn the structure of the phenomenon with the phenomenon itself: "and [it] would be surprising if the actual phenomena had nothing more to teach us about their structure" (p. 163-4). In addition, we still have some mistrust towards ordinary language, as clearly Wittgenstein claims:

Then there is still a belief that a perspicuous language is sufficient to avoid the false problems of Philosophy. Throughout this work I try to defend that a

<sup>&</sup>quot;That is to say, where ordinary language disguises logical structure, where it allows the formation of pseudo propositions, where it uses one term in an infinity of different meanings, we must replace it by a symbolism which gives a clear picture of the logical structure, excludes pseudo propositions, and uses its terms unambiguously." (*Some Remarks*, p.163). Cf. 4.003.

tractarian motto, under the aegis of logical multiplicity, can still be summed up as follows: we need a notational system that reflects or show the rules of syntax, so that nonsense turns out to be automatically avoided. The task of finding this notational key seems more complex because the combinatorial and purely formal appeal of the tractarian logic, which is embedded in the truth tables, shows itself to be strictly limited to capture the multiplicity of the phenomena.

Even if the program for the foundation of direct description of the phenomenological language of phenomena were carried out (if it really could be done), it seems that we would deal with mechanisms for the reproduction or description of a fantastic, unattainable, non-intuitive reality, far from our everyday language. As already indicated, Wittgenstein's discussion in 1929 with members of the Vienna Circle, it was about the unpredictability of the logical form of elementary propositions, which himself in the *Tractatus* held to be composed just by names:

"Der logische Bau der Elementarsätze braucht nicht die geringste Ähnlichkeit zu haben mit dem logischen Bau der Sätze. Denken Sie einfach an die physikalischen Gleichungen: wie enorm komplex sind diese gebaut. Von dieser Komplexität werden auch die Elmentarsätze sein." (WWK, p.42)

In this "enormous complexity" of elementary propositions Kienzler also sees a priceless promissory:

"Es scheint also ein Weg zum Anfang der Philosophie gefunden zu sein: in der unmittelbaren Beschreibung der Wirklichkeit. Hier liegt der Anfang, hinter den nicht mehr weiter zurückgegangen werden kann, ohne in ein unartikuliertes Gestammel zu verfallen. Welchen Wert der so aufgefunden Anfang haben kann, wird sich allerdings erst zeigen müsse, *denn immerhin hat Wittgenstein in seiner Überlegung eine Mechanismus konstruiert, den es nicht gibt und den es in vollkommener Ausführung auch niemals geben wird.*" (Kienzler, p.121) my italics.

Later it seems for Wittgenstein that to the language is strictly forbidden this naive direct access to the phenomenon, because it can only move in the physicalist sphere. Language can only move within a certain time and space in relation to a context of utterance, with hypotheses and conceptual frameworks belonging to human activities, and not directly to the realm of pure and direct perception. The direct access of language to phenomena is an unattainable myth: to remove language from its hypothetical sphere is to strip the language of language. This makes it a reproduction, always late, of the immediate. Language has to gain at last its independence. The most direct imaginable description of the phenomena does not reach the phenomenon. It is still in the realm of language, a complex yet to correspond to another, a complex not yet sent to another complex. Reading a description is not the description itself:

"Wenn nämlich alle Beschreibung in der Sprache stattfindet, alle Sprache aber zur Welt der Physik, dann gibt es in der Welt der Phänomene gar keine Sprache und also auch keine Beschreibung." (Kienzler, p.121).

The description should be outside of the fact to be figured – an old tractarian theme. One cannot collapse into an interpretation the represented (phenomenon) and the representation. In the radicalism of the immediate "the here" and "the now" of the phenomena of the visual field we do not have language, even if we have an infinitely good memory and are capable of making instant reproductions of phenomena. Thus, we would no longer be dealing with them, but with reproductions of these that have yet to be read or understood, and then referred back to phenomena. The search for a phenomenological language appears to have radicalized the tractarian subject-matter of a notational system ideal to follow the logical multiplicity of the phenomenon:

So it really seems that what Wittgenstein searched for emphatically, but vaguely, in his return to Philosophy in 1929 no longer seemed to have a status of language, not in the wider sense of system of representation. Furthermore, he moved further and further away from our everyday language. The tractarian project to prospect the language to find its logical form has turned it in an unrecognizable und untangeble illusion. It has digged the current language too far, so that we could not recognize it any more.

As we have seen, forms of phenomena appear that cannot be measured by the tractarian means of expression, as coordinates in space and time, and especially colors, sounds, measurements and so on, "with their gradations,

<sup>&</sup>quot;Wittgenstein ist also in eine Sackgasse hineingeraten, und die Suche nach der phänomenologischen Sprache führt zu keinem greifbaren Ergebnis, auch nicht zu einer klaren negativen Schlussfolgerung, die etwa ihre Unmöglichkeit beweise. Etwas muss grundsätzlich an der Herangehensweise falsch sein, wenn dies das Ergebnis ist" (Kienzler, p. 123).

continuous transitions and combinations in various proportions" (*Some Remarks*, p. 165). The first substantial casualty to the *Tractatus* appears here in the article from 1929: "for their representation numbers (rational and irrational) must enter into the structure of the atomic propositions themselves". (*Some Remarks*, p. 165) This allows a coordinate and opposition system in the projection of propositions proper to gradations. "The occurrence of numbers in the forms of atomic propositions is, in my opinion, not merely a feature of a special symbolism, but an essential and, consequently, unavoidable feature of the representation." (*Some Remarks*, p. 166).

An additional problem here is that following passage 5.5562 "wissen wir aus rein logischen Gründen, dass es Elementarsätze geben muss, dann muss es jeder wissen, der die Sätze in ihrer unanalysierten Form versteht" and passage 4.002, which holds that: "Der Mensch besitzt die Fähigkeit Sprachen zu bauen, womit sich jeder Sinn ausdrücken lässt, ohne eine Ahnung davon zu haben, wie und was jedes Wort bedeutet. \_\_\_ Wie man auch spricht, ohen zu wissen, wie die einzelnen Laute hervorgebracht werden (...)". The philosophical analysis should make clear what we do trivially in everyday life, because the apparent logical form of proposition is not its real logical form, as pointed out by 4.0031. However, it is evident here that the phenomenological analysis provided in this article is increasingly complex and distant from our everyday language. The disguise of language seems more and more impenetrable, and we have problems to build up a notational means capable of removing this disguise. This is a reference to a direct tractarian theme:

"(...)Die Umgangssprache ist ein Teil des menschlichen Organismus und nicht weniger kompliziert als dieser. Es ist menschenunmöglich, die Sprachlogik aus ihr unmittelbar zu entnehmen. Die Sprache verkleidet den Gedanken. Und zwar so, dass man nach der äußeren Form des Kleides nicht auf die Form des bekleideten Gedankens schließen kann; weil die äußere Form des Kleides nach ganz anderen Zwecken gebildet ist, als danach, die Form des Körpers erkennen zu lassen. Die stillschweigenden Abmachungen zum Verständnis der Umgangssprache sind enorm kompliziert." 4.002

The second deviation is an amplification of the problem of the colors: "one might think \_ and I [Wittgenstein] thought so not long ago \_ that a statement expressing the degree of a quality could be analyzed into a logical product of single statements of quantity and a completing supplementary statement." (p. 168). So a sentence that assigns a grade to a quality cannot be analyzed in terms

of iterated conjunctions and also shows an internal relationship when it provides the difference between degrees: "The mutual exclusion of unanalyzable statements of degree contradicts an opinion which was published by me [Wittgenstein] several years ago and which necessitated that atomic propositions could not exclude on another." (p. 168). These are deviations of the passage 4.22: "Der Elementarsatz besteht aus Namen. Er ist ein Zusammenhang, eine Verkettung, von Namen." And of the passage 5.134: "Aus einem Elementarsatz lässt sich kein anderer folgern."

These two problems lead to the third great deviation that attacks the tractarian truth-functional heart: There is a failure of expression in the notation which was sketched there as a more perspicuous one: "Our symbolism, which allows us to form the sign of the logical product of RPT and BPT [the ascription of different degrees to a same point] gives here no correct picture of reality", because it allows or endorses the formation of absurdities, instead of preventing it. The truth table as a special notation does not prevent the formation of nonsense and fails to show the logic of our language. In this phase as is evident in § 83 of PB, Wittgenstein still thought he could reconcile this failure with the general theses of the *Tractatus*, as if the problem was to be solved because just localized: "Die Regeln über "und", "oder", "nicht", etc., die ich durch die W-F Notation dargestellt habe, sind ein Teil der Grammatik über diese Wörter, aber nicht die ganze" (PB, p.111). The problem in this phase would still be in the comprehensiveness and effectiveness of its notational means of logical analysis and not in the project itself.

The truth table is a powerful but insensitive notational system – it is insensitive precisely because it is powerful. The price to be paid to increase the sensitivity of a calculus is indeed the lack of decidability. This naturally reflects the logical isolation of the well-behaved propositional calculus in relation to the undecidable predicate calculus. There is in Wittgenstein's return, the recognition that there are logical constructs that do not recur to truth-functions. The parts of the complex are not sufficient to determine its sense or truth-value. Bipolarity in the form of non-concatenated and concatenated of objects does not capture the multiplicity of the exclusions and implications of contrary propositions, when the propositions exclude themselves, but can be false together. The contradiction, as an injury incurred against the bipolarity cannot be the only form of exclusion

expected. This is analogy to the tautologies, which cannot be the only form of logical consequence and necessity. These problems represent a fatal challenge to the picture of logic that Wittgenstein had by the time he wrote the *Tractatus*.

"Wie es nur eine logische Notwendigkeit gibt, so gibt es auch nur ein logische Unmöglichkeit". 6.375. With the passage 6.3751, we would have only an example for the thesis that there is only one kind of logical impossibility. The bet is on the tractarian picture of contradiction. The last sentence of this passage explains the choice of the example. At some point of the logical analysis the exclusion should show itself as a contradiction: "Es ist klar, dass das logische Produkt zweier Elementarsätze weder eine Tautologie noch eine Kontradiktion sein kann. Die Aussage, dass ein Punkt des Gesichtsfeldes zu gleicher Zeit verschiedene Farben hat, ist eine Kontradiktion." The argument is clear: propositions of color ascription cannot be elementary propositions, because they exclude each other. They still have a logical dependence, which must have been eradicated. Wittgenstein claims then:

"Das z.B. zwei Farben zugleich an einem Ort des Gesichtsfeldes sind, ist unmöglich, und zwar logisch unmöglich, denn es ist durch die logisch Struktur der Farbe ausgeschlossen. (…) Es ist klar, dass das logische Produkt zweier Elementarsätze weder eine Tautologie noch eine Kontradiktion sein kann. Die Aussage, dass ein Punkt des Gesichtsfeldes zu gleicher Zeit zwei verschiedene Farben hat, ist eine Kontradiktion."

The tautologies constitute the horizon of logic in the *Tractatus*. Given the bipolarity and in accordance with symmetry from the necessary syntactic possibility of denying any proposition, we have the contradictions. Tautologies and contradictions thus make up the logic of the *Tractatus*. Therefore we can say that it is not a matter of forgetting the reason for which Wittgenstein did not take the case of chromatic exclusion as an instance of exclusion by contrarieties, as has been done throughout the tradition. Obviously they can be false together, while in the case of contradiction, propositions cannot be true and false together. This is the meaning of the third excluded. There is no third possibility, or intermediaries between the only two possibilities. They are exhaustive. Because there are only contradictions in the *Tractatus*, the exclusion by contrariety must be reduced at some point to the contradiction. They are not elementary; they must be analyzed

for the sublimation of this exclusion. Here is clear Wittgenstein's bet on NOR14, the joint negation of elementary propositions to be the thread for this reduction. The problem of this bet on reducing all exclusions in terms of stronger exclusions, or even, in terms of one, already appears in 5.513 of the *Tractatus*: ,,(...) Und so kann man sagen: Zwei Sätze sind einander entgegengesetzt, wenn sie nichts miteinander gemein haben, und: Jeder Satz hat nur ein Negativ, weil es nur einen Satz gibt, der ganz außerhalb seiner liegt. (...)."

The sound interpretation of this passage is very relevant for this thesis. In fact in the case of colors, measurements and other contrary propositions we have this explosion of possible negations, all other values of the numerical scale, or a myriad of other unrealized possibilities. If you have "one meter", I do not have necessarily "two meters", "three meters", "four meters"... If I have blue, I do not have necessarily dark green or light yellow or dark magenta... Or even with individuals in an empirical taxonomic system: if in a jungle there is a feline and if this animal is not a lion, then it may be a leopard, a panther, a jaguar... The use of reticence here shows the indefinite iteration of alternatives, in some cases, clearly infinite.

Another analogy that seems to catch the reader's attention in this period is the one of the ruler affixed to reality. This analogy appears already in the *Tractatus* to comment on another analogy: propositions are *Bilder*. But *Bilder* do not have to have prolongations or a system of coordinate as the rulers. In this middle period, the analogy of propositions to rulers is naturally more predominant than the analogy to *Bilder*. As shown in 2.1512-2.15121, to measure an object one requires that the most extreme points of a ruler touches the object to be measured, but for the measurement of magnitudes we need the whole scale and the rules of measuring as well. One needs the whole paradigm of measurement. Thus, we can say that we cannot think of a singular, at least in these cases, without thinking of a whole natural system or logical region to which this singular is inserted and by which it is defined, organized and understood.

Here we have the measure of a radical semantic holism. I do not have "seven" without understanding the numbers and its linearity. I have no "lion"

<sup>14</sup> It is clear, for example, from the passage 5.101 that Wittgenstein is using the NOR and not the NAND in the *Tractatus*. In any way, the both are indeed interchangeable if we are talking about truth-functional completeness.

without a system for the taxonomy of animals (however rudimentary). I do not have "green" without all the shades of color. Here this kind of radical semantic holism can bring problems to the bipolarity requirement, making it turn into a requirement of "multipolarity", precisely due to the always expected possibility of negation, as a signal for propositional legitimacy. We cannot understand a proposition without understanding its negation. In order to understand a proposition, we must have an understanding of its opposite. There is not the possibility to say something truly without the notion of falsehood. The problem with the contrariety of sentences with gradations of qualities is precisely that the opposite is rebutted by a multitude of possibilities, and not a single alternative as in the case of all other, until then, well-behaved empirical propositions. To understand a proposition on colors and measurements means to understand this range of several alternatives, and to somehow keep all of them in mind.

The intuition here is that something must always be accompanied by its negatives, or all that which this something is not. Thus, differently from the tractarian paradigm, there is not only a negative for any proposition, but several, potentially endless alternatives. To identify the number seven, I have to be able to identify one, two, three... and their organization. To identify a lion, I should be able to differentiate it from other animals - not only from members of another family, but I should also be able to distinguish a lion from other members of in its own family. For example, a lion is different from human beings, but it is also different to a domestic cat and a leopard. To be able to identify a color such as red, I must be able to know that it cannot be circular or have a sound but also that red cannot be, in some sense, green, yellow, blue... Thus, there is the natural need for the reinterpretation of the metaphor of the ruler from the *Tractatus* to accommodate this explosion of alternatives and conceptual and categorical linkage, as is evident in this discussion of WWK:

<sup>&</sup>quot;Ich habe einmal geschrieben: "Der Satz ist wie ein Maßstab an die Wirklichkeit angelegt. Nur die äußersten Teilpunkte berühren den zu messenden Gegenstand." Ich möchte jetzt lieber sagen: Ein Satzsystem ist wie ein Maßstab an die Wirklichkeit angelegt. Ich meine damit folgendes: Wenn ich einen Maßstab an einen räumlichen Gegenstand anlege, so lege ich alle Teilstriche zu gleicher Zeit an. (...) Ich habe all das bei der Abfassung meiner Arbeit noch nicht gewusst und meinte damals, dass alles Schließen auf der Form der Tautologie beruhe. Ich hatte damals noch nicht gesehen, dass ein Schluss auch die Form haben kann: Ein Mensch ist 2 m groß, also ist er nicht 3 m groß. Das hängt damit zusammen, dass ich glaubte, die Elementarsätze müssten unabhängig; aus dem Bestehen eines Sachverhaltes könne man nicht auf das Nicht-

Bestehen eines andern schließen. Wenn aber meine jetzige Auffassung mit dem Satzsystem richtig ist, ist es sogar die Regel, dass man aus dem Bestehen eines Sachverhaltes auf das Nicht-Bestehen aller übrigen schließen kann, die durch das Satzsystem beschrieben werden." (WWK, p. 63-4)

It seems very interesting here also to highlight the "organic" role of metaphors. From the *Tractatus* to the intermediate period the metaphor of the ruler seems to grow, to develop and to determine more interesting consequences, and fix old ones, despite being the same and having basically the same role: trying to show how we can understand propositions. As we read in the entry from 9th, December 1931, in WWK regarding the dogmatism, we see that the metaphor of the ruler, which until then had only a subordinate importance illustrative of the famous and most comprehensive picture of the proposition as *Bild*, in fact gained in prominence in this middle period. This may also serve to weaken our current interpretations of the passage 2.1. From this perspective we can see that the passage 2.1 in the *Tractatus* is more a metaphor or bet itself than a necessary condition for everything that can be taken as representation, as the second literature often read it. In principle we could use a picture as a proposition or a proposition as a picture. We could introduce one in the reading of another. In fact the image of proposition as *Bild* is much mitigated during this transitional period. The sovereign metaphor, and expanded, then shows itself as the one about the ruler. There is clearly a displacement of importance of the comparison. In the Tractatus the image of propositions as Bilder is central and the ruler comes to illustrate this more specifically. In the intermediary period, when Wittgenstein thinks in retrospect, he sees that he could very well has inverted this kind of relationship of importance. The metaphor of the ruler appears now to communicate more than the metaphor of Bild. Moreover, it comes naturally from the theme of systems of propositions, from the need for extensions of the ruler to measure something, from the need of their markings or dashes, from the need of a system with coordinates and its possible determinations. As shown by Wittgenstein in WWK:

"(...) Als ich schrieb: "Der Satz ist ein logisches Bild der Tatsache", so meinte ich: ich kann in einen Satz ein Bild einfügen, und zwar eingezeichnetes Bild, und dann im Satz fortfahren. Ich kann also ein Bild wie einen Satz gebrauchen. Wie ist das möglich? Die Antwort lautet: Weil eben beide in einer gewissen Hinsicht übereinstimmen, und dieses Gemeinsame nenne ich Bild. Der Ausdruck "Bild" ist dabei schon in einem erweiterten Sinn genommen. (...) Das Wort "Bild" hat etwas Gutes: Es hat mir und vielen andern geholfen, etwas klar zu machen, indem es auf etwas Gemeinsames hinweist und zeigt: Also darauf kommt es an! Wir haben dann das Gefühl: Aha! Jetzt verstehe ich: Satz und Bild sind also von der gleichen Art. Ich könnte auch einen Maßstab als Symbol benutzen, d.h. Einen Maßstab in eine Beschreibung einfügen und so verwenden wie einen Satz. Ja, man kann sogar sagen: In vieler Hinsicht verhält sich ein Satz ganz so wie ein Maßstab, und ich hätte daher ebenso gut den Satz einen Maßstab nennen können. (Z.B. legen wir einer Farbaussage den ganzen Farbmaßstab an die Wirklichkeit an.). (WWK, p. 185)

It is interesting to think about the extent to which the *Color Exclusion Problem* would exist in the *Tractatus*, if the metaphor utilized therein had been the metaphor of the ruler, as that already carries the idea of an entire system of measurements. Using that metaphor, it would have been easier to notice that a proposition could exclude or necessarily imply another, even outside the standard of the tractarian logic consisting of tautologies and contradictions. The famous *Color Exclusion Problem* could have already been foreseen at the time of writing of the *Tractatus*.

## 2.2. Towards the hammer and the nails, or towards the notation and the complete analysis

The demand for the full and unambiguous analysis of the language is simple enough to be proposed, but impossible to be implemented without difficulties. The search for prospecting the real logical form of a proposition, hidden or disguised by its grammatical form, invariably generates embarrassment when attempting to make this analysis in a way perspicuous, unambiguous and comprehensive. "There is one and only one complete analysis of the proposition", states Wittgenstein in 3.25 of the *Tractatus*. There should be, therefore, a kind of final and complete logical dismemberment of the propositions of ordinary language. In this breakdown, the final parts would somehow designate logical simple objects in reality, i.e., no longer susceptible to description. All linguistic complexity should be reduced to this atomic base. Rightly or wrongly, this requirement is intuitively compatible with the demands of a determined representation of the facts that make up the world.

The explicit argument in the *Tractatus* is the demand that somewhere language has to find or touch (*berühren*) the world, otherwise the truth value of

propositions would always be determined by other propositions. This would lead to kind of insularization of language. Wittgenstein says in the *Tractatus*: "Hätte die Welt keine Substanz, so würde, ob ein Satz Sinn hat, davon abhängen, ob ein anderer Satz wahr ist. Es wäre dann unmöglich, ein Bild der Welt (wahr oder falsch) zu entwerfen". (2.021, 2.0211). Wittgenstein, in his return to the Philosophy in 1929, clearly remains with the drive for the full analysis of the language and the search for a notational system which, besides respecting the syntax of the language and mirroring the multiplicity of phenomena, would avoid nonsenses. However, he had to revise the assumption of the independence of elementary propositions - a conclusive signal of the end of the analysis on the ground of language, when it would touch, finally, the reality.

Indeed this mandatory review explains the reasoning and breadth behind why the great mottos of the *Tagebücher* 1914-16 and the *Tractatus*, "Logik muss für sich selbst sorgen" (c.f. 5.4711), should be revised. The limitations of the expressive power of the truth tables as a special notation, incorporates all the tractarian commitment to compositionality, for good and for bad. Or in other words, as we shall see in chapter II, this notation, besides systematically showing the entire conceptual framework underlying the tractarian logic, also shows its deficiencies. It passes judgment on the non-treatment of the exclusion of colors, which in turn implies a much larger problem, namely: the limitation of the paradigm of compositionality to treat all empirical propositions with trivial ascriptions of gradations to qualities, in which case two propositions cannot be true together, but false together.

In contrast to this, the trivial contradiction seems always to be capable of being captured by the paradigm of truth-functionality. This paradigm which accompanies the classical neutrality of connectors (truth-functionally could connect any primitive proposition without engendering contradictions) is not sensitive enough to address some conceptual links. These are not open to a strictly formal or combinatorial approach, for example, as the transitivity of some relations (such as "taller than") or as the analyticity of some concepts (such as "bachelors are unmarried"). In this case, we have to examine the conceptual systems to which these propositions belong. We cannot only focus on the structures of such propositions to grasp its validity. For example, in a universe where all individuals have the same height, the transitivity of the relation ,,taller than" does not hold or we could redefine *ad hoc* the meaning of being bachelor in order to avoid the above truism. All these logical relations of conceptual implications or exclusions should be sublimated until the end of a complete analysis of the propositions in which they occur.

In this way, for all possible counterexamples to compositionality the strict tractarian strategy would be the same: do not deny the meaning of these empirical propositions, but do indicate that their analysis is not over yet, because we still have logic complexity or logical dependency between their constituents requiring further analysis. We can then legitimately ask ourselves: Are there no logical constructs but truth-functions? The tractarian response: "Of course not! If anything points to this, the analysis must then continue to the end! Or, in the case that something has gone wrong with the analysis, keep on analyzing it!" This would be a promissory note that could never be truly paid. The curse of the tractarian project is always have to indefinitely to postpone its end.

The compositionality marked by truth-functionality does not capture all possible cases of empirical propositions. In this sense, we maintain that it postpones the problem of conceptual linkage, but does not solve it. In fact, the *Tractatus* seems to have bet too much on truth-functionality and its corresponding truth table notation, and seems to have missed the point that some required dependencies (such as the implication and the exclusion) can be seen in the conceptual relations *within* propositions, among its components. The "within" here is crucial: it is not enough to get primitive propositions, but we also have to analyze their components. We should look into the proposition, to its logical construction, to the logic complexity of its members. This directly challenges the passages of the complex 5.55 of the *Tractatus*.

Considered rigorously, the *Color Exclusion Problem* represents *prima facie* a challenge to the logic based on truth-functionality, and not to the tractarian account of Mathematics, of Ethics, or of Philosophy of Science. The problem there is ultimately with its image of logic and not with numbers or ethics and esthetics. Wittgenstein himself seems to acknowledge this interpretation in § 76 of the PB:

<sup>&</sup>quot;Man könnte sagen, die Farben haben zueinander eine elementare Verwandtschaft. Das lässt es erscheinen, als könne innerhalb des Elementarsatzes eine Konstruktion möglich sein. D.h., *als gäbe es eine logische Konstruktion, die nicht mit Hilfe der* 

*Wahrheitsfunktionen arbeitet.* Nun aber scheint es außerdem, dass diese Konstruktionen eine Wirkung auf das logische Folgen eines Satzes aus einem anderem haben. Denn wenn verschiedene Grade einander ausschließen, so folgt aus dem Vorhandensein des einen, dass der andere nicht vorhanden ist. Dann können zwei Elementarsätze einander widersprechen". (p.106) my italics.

These types of relationships or constructions of the components of elementary propositions would not be formal, if we think of formality collapsing with truth-functionality. These links are not expressible by the truth-functionality of logical operators (or, in the Tractatus, by the NOR operator (cf. 6), the combined denial of an elementary basis of propositions, by the truth-functional completeness). The sense of the complexity is not derived from its components' sense. We have to presuposse more. This leads us to believe that the tractarian logic of tautologies and contradictions is powerful, but rough. It may be rough, because it is too powerful! This lack of expression means that the tractarian logic would necessarily begin to incorporate some points usually taken as extra-logical. Logic collides with its application. Logic has to at last look the world to be executed properly. This quote taken form Some Remarks appears like a echo: "And it would be surprising if the actual phenomena had nothing more to teach us about their structure." (p.164). Logic has begun to appeal to intuitive empirical features. Where in the (arrogant) neutrality of the Tractatus could we expect an appeal to the ordinary or current language, like this in the following passage?

This is opposed to the revealing passage in parenthesis in the *Tractatus*: ,,(...)(Und wenn wir in die Lage kommen, ein solches Problem durch Ansehen der Welt beantworten zu müssen, so zeigt dies, dass wir auf grundfalscher Fährte sind)". In this way, Some Remarks teach us that, in contrast to his former vision, we have to take this "wrong" path. There is no proper language analysis, with or without a proper notation, without taking this wrong way: We have to look at the world!

<sup>&</sup>quot;Every one of us knows that in ordinary life. If someone asks us "What is the temperature outside?" and we said "Eighty degrees", and now he was to ask us again, "And is it ninety degrees?" "We should answer", "I told you it was eighty". We take the statement of a degree (of a temperature, for instance) to be a complete description which needs no supplementation." (*Some Remarks*, p.167)

"Wer nur einen Hammer hat, für den sieht jedes Problem wie ein Nagel aus"15. A proverb in German says that for those who only have a hammer, every problem looks like a nail. I believe that, condensed but explicit, we can see in this maxim the difficulties that the *Tractatus* faces when attempting to carry out a complete analysis of all legitimate propositions, or empirical ones, in terms of truth-functionality. Indeed, according to the *Tractatus* a proposition is complex and must be analyzed in terms of elementary propositions that define all the truth conditions of complex propositions or of any proposition with any degree of logical complexity. Or - in the best of the atomistic spirit nobility: if we have all elementary propositions, we would have the complete map of possible articulations of all complex propositions, i.e., a complete mapping of the comprehensive, absolute articulatory horizon of the things of world, without surprises.

The *Tractatus* requires complex propositions being truth-functions of elementary propositions, an articulated aggregate representative of facts, which by its names touches the reality. Following the suggestion from the exegetical maxim, the kind of compositional analysis is the hammer to carry out the nailing, ie, the complete analysis of all propositions. In this example, nothing is lacking and nothing is left, because there are only hammers and nails suited to the task of nailing16. However, when considering empirical propositions we note that they often contain generalities which are expressed traditionally in predicate calculus in terms of quantification, when a quantifier is analyzed, for example, as a predicate of a second order. The quantification of the *Tractatus* is made from logical products and sums, which lead to the largely metaphysical necessity that an elementary basis is fully defined and complete, always available and without empty references. Furthermore, we need there to always have a kind of implicit additional clause – a type of closing clause. A further proposition to complete the

<sup>15</sup> It seems that this *Sinnspruch* originally came from the Austrian psycotherapist and philosopher, Paul Watzlawick. It is a kind of popular sedimentation of his constructivist theories in psychology.

<sup>16</sup> Often the ridiculous elements of Philosophy is analogous to the ridiculous situation of a scientist who faces a problem by dismissing reality for an *ad hoc* model which could indeed be bold, lean, attractive, but non-operative on many levels. Or the case of a government that attempts to dissolve the people because there are many conflicts between them both. If your categorical system is poor, this should be discarded and not the reality. We have to keep ourselves aware. We know that the form of the analysis determines, interferes with and contaminates the product of the analysis.

construction of quantification in the *Tractatus* is always required, "and these are all disjunctions or conjunctions", "and that's all," "and nothing else" etc.

Yet, often when we deal with empirical propositions we need a more sensitive denial than the propositional one in order to differentiate what is really being denied within propositions. In a trivial example, we have: "It is not the case that there is a black cat on the big table". What is really being denied in this proposition? It is a predicate? It is a relationship? It is the instantiation of a predicate? (Is there no cat? Is there no table? Is the relationship "be on" which is false? Are the predicates "black" or "big" that are misapplied? Is the table that is black or the cat that is big? Are neither of them black or big?). In neither of these cases is the strictly truth-functional tractarian analysis fully satisfactory. We will return to these problems in the next chapter.

These are local or peripheral issues. The two "problems" can be harmless to the *Tractatus* if helped out by the metaphysical claims of the young Wittgenstein. That is, taking into account our image: the truth-functional hammer is sufficient for these shortcomings in quantification and in predicative denial, still coping with all the nails ... The tractarian metaphysics is generous enough to supplement these "flaws". However, the situation is dramatic if we take the problem of analysis of some common empirical propositions, such as assignments of color. For example, propositions like "this point is red" and "this point is blue" exclude each other, if combined. This naturally leads one to think of them as not yet fully analyzed because they still contain logical complexity. Being consistent with our principles, we then try to give more time to the work of the hammer, the spirit which is clear in the notes of 6.3751: the postponement of the task, as the secondary literature already shows. However, this kind of ascription leads to the collision of two central tractarian theses: the logical independence of elementary propositions and the complete analysis.

As we saw in the *Tractatus*, Wittgenstein indicates that this kind of exclusion should be considered in the ascription of different velocities to the same particle. However, we would still have an obvious exclusion. Hence we would have to continue the analysis. Nothing indicates that this exclusion may in fact be "sublimated" by a truth-functional analysis, especially because the conjunction does not work as an addition. I cannot consider 3 meters as "1 m and 1 m and 1 m," as this would simplistically be saying 1 m. Nor can I analyze 3 meters as "2 m

and 1 m" as that would be absurd. Logical products do not express gradations or degrees. This problem points out greater difficulties which indicate the need to consider different kind of conjunctions for different kinds of articulations in propositional systems. For example, there is a need for a combination which regards the number of occurrences of variables for the context of measurements or one which allows mixtures for some colors, but not for others, as in, respectively, of blue and red and of blue and orange.

This shows that neither of these problems with gradations (be they measurements or colors) cannot be a metaphorical nail and that the old "hammer" is not really enough to handle the analysis of all empirical propositions. We could perhaps have something like an embarrassing exit and deny the empirical status of the propositions that involve gradations in order to keep with the truthfunctionality of all propositions. Translating this exit to our guiding principles, we would have something like: "my hammer is still excellent, but your nails that are not actually nails." That is, this problem is indeed no problem. We could also adopt a more pragmatic exit reviewing our parameters or creating new ones from new perceived problems. In this vein, one could say: "I give up either the complete analysis, or the independence of elementary propositions, allowing, then, that they do exclude or imply each other in synthetically organized systems (Satzsystem)". This strategy is more ad hoc and irreversibly induces the revision of the Tractatus, since Wittgenstein assumes the need to start looking into the propositions. For it, the normative appeal has to be mitigated. The current language before being corrected by the impositive tractarian thread, now has shown to it a deficiency and waits a rearrangement of the project. In this case, we should also naturally improve our working instrument (here our notation). This is actually stated in the article of 1929, when Wittgenstein recognizes the limitation of its notation and bets, in the last paragraph of this text, on the project's improvement.

The return from the "philosophical holidays" in order to continue the work begun *im Wesentlichen* in the *Tractatus* means not a merely unfolding or detailing of what was done, but a revision and rearrangement of some conceptual foundations. The project continues, but the instruments to carry it out has to be rearranged and sharpened. This seems to be the opposite to what Bento Neto claims in the introduction to his book when he contrasts solving problems in the essential and in detail:

"(...)that same observation, saying that all philosophical problems were solved in essence, also says that they were not completely resolved, perhaps, "in detail". As we try to show, is precisely the project of phenomenological language where we can recognize such detailing (...)" (Bento Neto, p.12).

The requirement of a phenomenological language comes less from the demand on detailing of what was only done in the essential than from the demand on more sensitivity in relation to the diversity of the logical multiplicity of the phenomena that the tractarian logic could not, in essence, have done. During this period, in fact, some problems appear either demanding the detailing, or just a development, of the tractarian base, as with the Philosophy of Mathematics, its Epistemology and its Psychology. But the problem with the application of logic ending in names and the lack of sensitivity of the tractarian logic to express all the nuances of logical implication and exclusion are not problems that can be detailed. That's because they demand revision, reformulation or even the abandonment of assumptions. They are, in principle, impossible to be executable, with or without detail. There is in the tractarian project a unsolvable tension between this image of a neutral logic and the demand that this must be used to completely analyze the facts in the world. There are more logical connections than the tractarian logic, with its tautologies, contradictions and truth-functionality, can express.

The application of logic began then to collide irreversibly with the logic. The tractarian logic, representative of a purist vision of logic, should start, somehow, to look, at last, to the world. The elementary propositions, surviving through the permanence of the complete analysis are not only composed of simple names but must contain numbers, or even operators, to cope with a system of contraries, as, for example: "if x is 10 something, it is not 9 something, it is not 8 or 11 and so forth." The candidates of this kind of analysis are systems, wherein properties of length, intervals, intensity of sounds, hue or redness of a color and etc appear. For Wittgenstein, "It is characteristic of these properties that one degree of them excludes any other" (*Some Remarks*, p.167).

This problem leads to the revision of logic that would not take more care of itself, but would have to look at some empirical arrangements. There would not be only logical necessity, but a hybrid that would contain "synthetic necessities" too! "If a point is green, necessarily it cannot be red." Mathematics would also be revised, since the numbers do not disappear in a more complete analysis, possibly being in the end of the analysis of at least all the propositions of gradation. We need a drastically greater multiplicity in the propositions so they can represent and/or adequately reflect the multiplicity of facts in the world.

Thus, the hammer has shown itself to be too rough or even essentially inadequate to account for all problems. Not everything is a "nail". Thus, either the problem is with the complete analysis, or with this kind of proposition which would not be total empirical or total logical. Or, by following our analogy, the problem would be either with the hammer or with the other kinds of nails. Going a bit further, by the end of 1931 Wittgenstein reached the conclusion, as advocated by Kienzler and by myself here, that the problem was with both the hammer and the nails. The problem is largely with this way of trying to regulate language \_ trying to find an instrument or general method for prospecting and bringing to light a hidden essence. The problems arise with this kind of regulation of language through linear rules which produce more distortions than solutions. In 1931, the very idea of complete analysis was to be abandoned as, so to say, an idea of a hammer suitable for all nails. It was found that the problem was less about having a good hammer for all nails than to hold that everything could be taken as a nail. Neither the whole nor a part of our language must be exclusively pictorial. Not all linguistic contexts need linguistic or ontological atoms, or a harmonic and precise association between them. We do not need a complete analysis, or even any logical analysis, in all our language to guarantee the determination of the propositional sense. Indeed, Wittgenstein recognized that in some contexts sense would not need to be fully determined, and that at some times and, in some contexts and language activities, there are some essential indeterminacies.

## Rediscovering the *Color Exclusion Problem* (from colors to the space structure, passing by the numbers, degrees, the definition of function and exclusions by contrariety)

In his article Cores e Números Cuter provides an interesting interpretation of the problematic passage 6.3751. He does this with the same spirit that guides this work, internally to the Tractatus. That is, he assumes its concepts and assumptions, exploring their positive and negative consequences, without bringing Wittgenstein's later Philosophy into the discussion. In his article, Cuter takes the chromatic exclusion as an indication that this exclusion could be considered in the context of measurement, i.e. where numbers must appear, largely following the spirit of Some Remarks. Cuter argues that the analysis of colors would be correlated to the tractarian process of analysis of numbers through quantification. The project is pretty much tractarian: the names of colours and numbers do not appear in the elementary propositions because they show logical complexity, and should therefore be analyzed. Although not declared, the idea here is to try to reduce the contrariety pattern that appears in ascriptions of colors to the contradiction, as well as a philosophical paradigm to reduce synthetic truths to analytic truths, as we see for example in Leibniz. According to Cuter, this project would be in total contextual harmony with the tractarian picture of logic:

Here the passage from ontology to language and vice versa is clear - an issue that will be investigated later in the last chapter of this work. Ramsey makes a similar heuristic and exegetical proposal in his review of the *Tractatus*, published in Mind in 1923. But he didn't develop it. In that article, Ramsey analyzes the *Color Exclusion Problem* accentuating the Wittgensteinian idea that each and every necessity or impossibility would be logical, i.e., exposed in terms

<sup>&</sup>quot;A logical impossibility must always correspond to a contradiction \_ it must have importance in the affirmation and negation of the simultaneous occurrence of a same fact in the world. On the linguistic level, this means that an elementary proposition could never contradict another elementary proposition, at the ontological level, we would say that the occurrence of a state of affairs could never exclude the possibility that another state of affairs occurs" (Cuter, p.183-4).

of tautologies or contradictions. With this idea, he anticipates Cuter's proposal of analyzing "colors" in terms of numbers.

"(...) Just as the explanation of some apparently necessary truths as tautologies met with difficulty in the field of color, so does the explanation of the remainder as pseudopropositions. "This blue color and that" says Mr. Wittgenstein, "stand in the internal relation brighter and darker *eo ipso*. It is unthinkable that these two objects should not stand in this relation" (4.123). Accordingly a sentence apparently asserting that one named color is brighter than another named color must be a pseudo-proposition; but it is hard to see how this can be reconciled with the indubitable significance of a sentence asserting that a described color is brighter than another, such as "my cushion at home is brighter than my carpet". But in this case the difficulty could be completely removed by the supposition that the physicist is really analyzing the meaning of "red"; for his analysis of a color comes eventually to a number, such as the length of a wave or what not, and the difficulty is reduced to that of reconciling the non-significance of inequality between two given numbers with the significance of an inequality between two described numbers (...)" (Ramsey, p.476)

In fact, the color assignment is an unfortunate example of contradiction. For in the passage 6.3751, the *Color Exclusion Problem*'s *locus classicus*, Wittgenstein writes: "Die Aussage, dass ein Punkt des Gesichtsfeldes zu gleicher Zeit zwei verschiedenen Farben hat, ist eine Kontradiktion". This unfortunate example (if we do not want to call it a mistake) is hidden in the low numbers of the *Tractatus*, far away from its main themes. And it seems to point out to an exception, which I would call trivial, for its paradigm of logical analysis. This example seems innocent but is fatal because it clearly shows a limit of expressibility in the tractarian logical analysis and project, and, therefore, in its notational system as well.

Every contradiction displays a contrariety, an opposition, but not every contrariety presents a contradiction. The exclusion in "It rains and it does not rain" is intuitively stronger than the exclusion "every Brazilian knows how to dance samba and no Brazilian knows how to dance samba", despite the fact they're both logical exclusions. Interestingly, in a previous section of the above cited article Cuter deals with the case of logical exclusions. He seems to touch on the problem of the difference between exclusions by contrariety and by contradiction, but doesn't develop it. Cuter distinguishes facts from states of affairs, in the tractarian ontology because the latter do not allow exclusions, while the former does: "Consider, for example, the assertion that there are five people in this room and the assertion that there are three people in this room. These statements are clearly incompatible." (Cuter, p.182) It is no coincidence that the author seems to have chosen this example. With that he already prepared the ground for his suggestion to reconstruct chromatic exclusions in terms of exclusions of numbers.

Chromatic ascriptions should be eliminated or analyzed because still embedding some logical complexity. But how? Cuter's strategy is to recognize measurement and numbers in this famous passage of Wittgenstein which already points out an analogy to the physics, to justify a kind of major program to be fulfilled. Here it would involve the task of analysis to be outsourced, as is typical of Wittgenstein in the Tractatus: he resolves problems in essence, by pointing out the steps for their future dissolution (that would often be impossible). He postpones fine and sophisticated activities in the belief that they can be fulfilled according to the gross or essential insights made in the Tractatus. So says Cuter: "I think Wittgenstein is simply remembering, in this passage [6.3751] that physics chromatic incompatibilities the form numbers represents the in of incompatibilities" (Cuter, p.183)

When operating this reduction we have the tractarian arsenal of analysis of numbers until then developed to be systematically applied to the case of colors, doing justice to the tractarian theory of numbers and trying to make all the exclusions becoming logical contradictions. We would have then a potentially uniform and elegant way of interpreting two problems in the *Tractatus*: the numbers and the colors. According to Cuter:

In fact, this strategy fits well with the tractarian spirit, especially with the bastion of bipolarity that states that everything that has meaning is contingent; that all that is necessary does not make sense; that elementary propositions are independent; and that any one state of affairs does not imply or exclude another. Here we have the very paradigm of realist theories of truth, where the sense of a

<sup>&</sup>quot;With the mechanisms of quantification, we have the numbers and with the numbers, we hope to reproduce in the phenomenal plane, the same kind of analysis that allows us to show the contradictory nature of incompatible chromatic assignments in terms of theories of physics." (Cuter, p.191)

proposition must be compared with the reality. This corresponds to its essence, i.e. its conditions of truth and falsehood. As summarized by Cuter:

"Ultimately, all the necessity associated to tautologies and contradictions emanate from bipolarity and from what in the world corresponds to it: the necessarily contingent character of each of the immediate concatenations of objects of which the world is composed." (Cuter, p.191).

This is anticipated in the passage 5.513 about the univocity of the negation of a proposition explicitly preparing the ground for the *Color Exclusion Problem* to come. The colors just do not respect this picture of strict form of exclusion. The consequences of our decisions or bets always appear in the future somehow. There are indeed some necessary exclusions of empirical propositions but, although one might say they're logically necessary. They are by no means always truth-functional. In his article "Unity of the facts - an attack on the philosophical idea of possible worlds", Stephen Read uses the *Color Exclusion Problem* to discuss the limits of truth-functionality and bipolarity as a model for analysis of language:

"(...)Wittgenstein's notion of bipolarity is a false model. Patches are not just black or white. Propositions do not have just two poles. Determinates are not bipolar: any ascription of a determinable corresponds to a single determinate fact. That the patch is crimson, say, makes it false that it is scarlet, vermilion, ultramarine, cyan and so on, just as the fact that a man is 2 m tall makes it false that he is 1 m, 3 m, 2.5 m tall-that he is any other height." (Read, *The Unity of the Facts*, p. 341)

The complexity of the architecture and organization of colors imposes sophistication on the tractarian logic that it cannot provide. In other words, the color ascription shows that there are, at least, some empirical propositions that have many (potentially infinite) corresponding negatives, as in the case of the ascription of degrees to empirical qualities. At least in these cases, this happens because we have several other propositions that are somewhat, but not "totally", outside of them, as argued in 5.513. We have two problems here: There are some propositions which are fully meaningful as they admit many possible and meaningful negations and these negations are not completely outside of them. In these cases, they all belong together to the same system or conceptual background, which builds up this logical affinity or familiarity. If I say that a table is green, this necessarily excludes it from being red or yellow. In the same way, if I say that the table is 3 meters long that means that it isn't (nor can it be) 4 meters nor 2 meters long... I believe we can also think here this kind of map of exclusions in all component elements of any system of classification as well, i.e., wherever we have a multiplicity categorically organized. For, if I say that a certain animal in front of me is a lion, this excludes this animal from being a turtle, or a human being, and it is also excluded from being a domestic cat or a leopard. And this is the same with trichotomies: as is the case when, for example, we are informed that a football match did not end in a draw and we can say that one of the two teams won or lost. Here for the proposition "the game ended up in a draw" there is not only one possible negative but two, for each team either won or lost. And these two remaining alternatives are not totally outside the first proposition as they share the same conceptual background or system of familiarites, in this example a league table of soccer results.

The elements of these conceptual systems seem to be grouped in categories that exclude elements of other categories. But within these categories, components also differ from each other in a way that the presence or the identification of one automatically excludes the other. In this way it may make sense to develop the idea that there is not just one kind of exclusion, but infinite kinds of exclusions, with different degrees or forces. In an organized group or system17 that consists of, say, (green, yellow, blue, circular) we can easily identify which element is the "outsider", the "alien". However, we also know that if to an object in the visual field is assigned one of these qualities that are left, the other ones will automatically or necessarily be excluded. Logically excluded, I say. In a group or system that consists of (4 meters, 3 meters, 5 meters, 4 grads Celsius) again we

<sup>17</sup> Here I am adopting an intuitive sense of categorical organization, commonly adopted by linguists to deal with paradoxes of exclusions - a sort of micro-system. A micro-system that can give us lessons on complex systems in fact, as the one of numbers or colors. My objective with this is to show the naturalness of contrariety exclusions in everyday life. I am not interested here in more sophisticated or abstract kinds of groups organized by logical characteristics as (rot, blau, gelb, blauer), where blauer is the element that does not belong to the group because it is a binary relation and not a simple predicate. Correlatively, we could build a group with the following terms (blauer, grösser, länger, schneller, rot), where rot is detached because it is not a binary relation. We could represent a group of first-order predicates, without representing exclusions by contrariety, as (rot, bunt, gross). Such a group would allow the three elements here to be simultaneously assigned to an object in the field of discourse. What is interesting here is that we place "blue" into this group. We would then have again automatically the phenomenon of exclusion by contrariety between "rot" and "blau". This is because an object, regardless of what it is, in a domain of discourse, regardless of what it is, cannot have these two predicates simultaneously. This becomes clearer if we think about the truth-functional conjunction. We could think of a "color" operator to simulate the impossibility of conjuncting blue and red, but allowing the mixture of both in the form of violet. This operator "color" does not necessarily need to match the additive conjunction of arithmetic. After all, white does not mean every color, in the same way that three means the junction of three units. We will return to this issue at the end of this study.

can identify, or exclude the alien component, and again we have to exclude elements of the same class, if one is ascribed to an object in the visual field. In a group or system that consists of animals, for example, (a lion, a leopard, a domestic cat, a turtle) we can identify, isolate, or exclude one as the "categorical alien" (in this example the non-feline turtle). And if we identify something as being in fact one of the leftover components, others will be necessarily excluded. And logically, I say.

All the components grouped into a system (or group) are obviously cognates, excluding components of other groups, but they also are excluded within their own group. This is the fact and the problem. There is a paradox here with groups which is investigated by linguists, as is clear in the work of Dan Jaspers. Even when elements are organized into a group based on affinities or familiarities, and excluding other elements of other groups, they also exclude elements within their own group. We can easily think of levels or degrees of strength in the exclusion of external components of other categories and in the exclusion of internal elements. These exclusions, the internal and the external, must not be uniform. All exclusions here are not by contradiction and they do not admit being reduced in terms of contradictions. For example, a system or group consisting of a (lion, leopard, domestic cat, seven) logically seems to exclude the "7" more intensely than the way the turtle is excluded from a group consisting only of animals, for example, a group with (lion, leopard, domestic cat, turtle). This is because the "number 7" is even more intensely foreign to the group of felines than a turtle would be. But the turtle would be more strongly excluded from its group, than a dog would be if added amongst the felines. As these examples show, the exclusion of foreign elements clearly accepts variations of degree and strength.

In the case of exclusion within the system, we have this phenomenon, but more subtly, when ascribing a group component to an object in the world, other components are necessarily, automatically, logically excluded. Apparently we always have the same strength of exclusion, regardless of whichever system we are operating in. Here, however, there is also a variation in the strength of internal exclusion, as when attributing a degree of temperature, a volume or length to some empirical element, or color to a visual point. This is even the case when we work on taxonomic systems, which in no way appear to belong to be logic but also involve exclusions, as in the case of a group of felines. If something is a lion, it cannot be a domestic cat, a leopard, a jaguar, or a panther... "Lion" excludes "domestic cat" more strongly than "leopard" as lions and leopards are both big cats found in the wild while domestic cats are not. Similarly, if a point is blue, is not green, nor purple, nor orange... Here blue excludes orange more strongly than green and purple because blue enters in its composition. All exclusions presented here seem to be automatic, necessary, logical, but they are not truth-functional. The sense and the truth of the complex do not depend strictly only on its present parts or elements. These logical exclusions cannot be represented in these terms, because they belong to some propositions that explode in many (perhaps infinite) alternatives when negated, always in relation to other propositions within the same system.

We could then also think of the interaction between systems or organizations and systems that include or exclude others or involve others. One can conceive of relations of a second order, i.e., exclusions and implications operating, not between elements within a system but between systems themselves. This would also seem to enable us to organize systems within systems, so that we can exclude the alien category or system. For example, in the case of a system consisting of (countries, cities, continents, numbers), one would exclude numbers. We could even try to find structural equivalences or isomorphisms in a system of systems, which at first glance seem not to belong to the same second order system, nor to even have anything in common with one another. For example: (colors, numbers, musical notes, first order logic). Such research can reveal surprising structural equivalence between categorical systems completely different, perhaps forcing us to rearrange our knowledge. This makes Dany Jaspers's approach fascinating, although quite ambitious. Jaspers considers the question about the classical square of oppositions in the article Logic of colors in historical perspective where he tries to argue that:

<sup>&</sup>quot;The patterning of these color oppositions and of semantic oppositions in logic [as well in the case of music and numbers] turns out to be so similar that it would be extremely surprising if there was no cognitive algorithm common to both domains. A important question that arises in view of this isomorphism of physical/physiological patterns of color opposition and natural logical patterns of opposition more traditionally mapped onto triangles, squares and stars is the issue of whether the opposition pattern represents a separate cognitive module that feeds two different cognitive domains or

whether it originated in one or the other faculty first and was later utilized by the other." (Jaspers, p.33).

Despite promises to be historic inherent in its title and the truly diachronic approach to the subject, this article proposes argumentative support to the innatist vision of human cognitive faculties *a la* Chomsky. Japers argues:

"Clearly, the present proposal leads to the conclusion that logic can be generalized. But if that is correct, there is more. Everybody acknowledges that colour perception is determined by innate structures in the eyes and the human mind/brain. If its logical architecture is indeed the same as that of natural language, that is strong evidence for the viewpoint that the latter is an innate biological endowment of the species too and hence strong confirmation of the correctness of a central tenet of the Chomskyan conception of natural language. (...) That the transfer occurs or at least that the isomorphism is there, is undeniable; but that it might be the result of learning (rather than explanation or something along those lines), makes no sense." (Jaspers, p. 34).

It is not relevant for this thesis to discuss the extent to which this Chomyskian approach to the development of our language skills is true or not. The interesting point for us here is to understand the extent to which logic can be generalized in many different systems, as Jaspers defends. This would mean that this generalization may be something to be discovered or established structurally among different complexes in different fields! The generalization of logic in this article means finding the pattern of oppositions represented diagrammatically by the Aristotelian square of oppositions in other categorical systems, in principle, independent from the first-order logic, as the case of color system. Without even realizing it, I believe Jaspers moves in his work from the opposition of colors to the important problem in the middle Wittgenstein: the necessity of propositional systems to express some kinds of exclusions. What, in the intermediate phase, Wittgenstein would call a propositional system, Jaspers calls a "closed set of lexical items with interrelated meanings". An example of this would be the system of logical operators of predicate logic which can be found in many natural languages, if not in all of them.

<sup>&</sup>quot;It was clear from the start that the interrelatedness of the operators is oppositional in nature, a fact that came to be represented by means of opposition lines in the diagrammatic representation called the Square of Oppositions" (JASPERS, p.1).

Jaspers's point of investigation is to show that the pattern of the four corners of the square of opposition and its different types of exclusions will appear in other domains and this appearance is indicative of essential, non-causal, features of these systems. Thus we classically would have the four forms of propositions of predicate logic18, as with the following examples:

a. **Universal affirmative**, which have the form "All S is P". E.g. "Every Brazilian is happy."

b. Universal negative: "No S is P". E.g. "No Brazilian is happy."

c. Particular affirmative: "Some S is P". E.g. "Some Brazilians are happy."

d. **Particular negative**: "Some S is not P". E.g. "Some Brazilians are not happy."

or "Not every Brazilian is happy."

For any of the two propositions above there are the following oppositions between them. There is obviously the case of sub-alternation but \_ as this is not an exclusion (be it weak or strong) \_ it is not relevant for this work:

i. **Contradiction**: when the two propositions cannot be true nor false together. As with the propositions "Every Brazilian is happy" and "not every Brazilian is happy." Or "some Brazilians are happy" and "no Brazilian is happy".

ii. **Contrariety:** when the two propositions cannot be true together, but can be false together. Eg "Every Brazilian is happy" and "no Brazilian is happy."

iii. **Subcontrariety**: when both can be true together, but not false. Eg. "Some Brazilians are happy" and "Not every Brazilian is happy."



<sup>18</sup> Here is important to note that we do not have to deal with predicates for generating the logical relationships of the Aristotelian square. Instead, we can use quantification, modalization or just the propositions themselves. For example, we can use p and q as propositions, and conjunctions, disjunctions and negations for the oppositions and subalternation between them according to the De Morgan rules.

Jaspers argues that the configuration and nature of the oppositions or even the layout of the diagram appear in other areas. What makes this research interesting is the fact that these other areas seem to be purely empirical, not prima facie immersed in logical linkage and oppositions, just as in the case of colors.

"One of the key points of the present article will be that the pattern of oppositions represented by the lines in this diagram (subaltern entailments, contradictories, contraries and subcontraries) transcends the conceptual filed of quantifier words." (Jaspers, p.2)

This is a kind of vision analogous to the development of the article by Cuter. However, I believe the problem in the *Tractatus* is not with the colors or the numbers or how colors can be reduced in terms of numbers. The problem is in how the logic of the *Tractatus* is unable to express the milder kind of exclusion found in contrary propositions that appear in color systems, numbers (measurements) and perhaps in any organized system of propositions: where two propositions can be false together, but not true together. We can then, in the interpretation of the *Tractatus* proposed in this thesis, go "from the *Color Exclusion Problem* to the contrary propositions" as well as going in the direction Jaspers' proposes, i.e. "from contrary propositions to the system (or logic) of colors". What Aristotle's square of oppositions seems to show us is that there are at least two kinds of exclusions: the strong contradiction-like one, and a weak contrary-like one, involving sub-contraries (and of course the necessary implication in the case of subaltern propositions completing this system of exclusions and logical implications).

With his work, Jaspers shows us that seeing a system of oppositions in the logic is not new, and that it was indeed studied (not casually) by Aristotle. Jaspers investigates colors as well as logical operators forming a system or set of oppositions lexical or closed fields (color field). In it, patterns of exclusion also emerge, and these are also diagrammatically representable. As Jaspers says: "Yet if we look at the relevant lexical expressions, they can once again be characterized as a set of cognate lexical items arranged in a closed lexical field (red, green, blue, yellow, black, white...)" p.3. It is not just a question of finding patterns and interrelationships between operators and the mixture of colors, but also to

investigate a kind of naturalness restriction for some logical possible components of such systems. That is, this involves the search for the reason why some elements of these groups - although logically possible - should be artificially created because they do not appear in nature. And this feature could be another hallmark of the structural identity of these systems. Jaspers claims:

"Natural repertories of this kind are extendable, but only by conscious effort and learning, at times by calculated breach of certain natural constraints. It is these claims I wish to defend and substantiate in this contribution and whose consequences will be explored." (Jaspers, p.4.)

The idea is to find or to discover this logical or mathematical pattern of oppositions through their diagrammatic representation in what he calls different (yet logically equivalent) cognitive modalities - just as in the case of color and language structures. The possibility of comparing prima facie independent systems, through the comparison of diagrammatic notations or models used to represent these systems is interesting to us here. At this point we clearly have another tractarian theme: We can indirectly study a domain through a model (or notation) that expresses it. The more appropriate or perspicuous is our representational system (be it diagrammatic or not), the better is our understanding of the domain to be investigated. The use of the same kind of graphical representation in different contexts would then be evidence of the persistence or pervasiveness of relevant connections and interrelations between independent system components. As Japers maintains:

"There may be a single logical or mathematical pattern at work in more than one cognitive modality \_ in color cognition and the musical faculty, to be specific, is as old as logic itself and has been a central concern throughout the ages in many of the same circles where logic and/or colors were studied intensively" (JASPERS, p.4.)

Jaspers' representation of colors system assumes the expansion of Aristotle's Square of Oppositions to a hexagon of oppositions, where the vertices A, E, I, O, U, Y would stand, respectively, for the primitive colors: red, yellow, blue, cyan, green, and magenta. The reason for choosing these primitive colors and not others is not relevant here.



Interestingly, in the paragraph 221 of the PB, Wittgenstein presents a diagram of an octahedron, which represents the logical relationships between colors, and this is very similar to, if not the same, the one of the Austrian Philosopher and Psychologist Alois Höfler (1853-1923) presented in his Psychologie (1899). This has also focused on a diagrammatic structure of a double pyramid on a rectangular base. Jaspers presents this without mentioning the octahedron from Wittgenstein's PB, as Wittgenstein presents his without mentioning his compatriot and contemporary.



Jaspers bets that: as in the case of quantifiers, the concepts and perceptions of color are much more attractive from the speculative point of view of both science and Philosophy. Thus, attempts to represent these patterns of relationship and opposition can lead to many different diagrams, which can be developed and expanded in many ways conforming to the field of knowledge and its aims. In the passage quoted above, Wittgenstein makes claims to the figurative superiority of the octahedron with respect to a line with the continuum of colors when trying to represent diagrammatically the logic of colors:

"Man kann freilich auch alle Farbtöne in einer geraden Linie anordnen, etwa mit den Grenzen Schwarz und Weiß, wie das geschehen ist, aber dann muss man eben durch Regeln gewisse Übergänge ausschließen und endlich muss das Bild auf der Geraden die gleiche Art des tautologischen Zusammenhangs bekommen wie auf dem Oktaeder. Es ist dies ganz analog, wie das Verhältnis der gewöhnlichen Sprache zu "einer logisch geklärten" Ausdrucksweise. Beide sind einander vollkommen äquivalent, nur drückt die eine Regeln der Grammatik schon durch die äußere Erscheinung aus." (PB, § 278, p. 277)

This movement of comparison between *Ausdruckweise* and notations certainly follows two themes of the *Tractatus* and of this intermediate phase. I find these to be:

a) The development of a perspicuous notation to reveal the hidden basis of a syntactic system and thus prevent absurd formulations;

b) Propositional systems (as with the color system) are fragmented "logical spaces", which use internal relations between their components to define the horizon of possibilities for the articulation of expressions.

Thinking of this definition of boundaries as arising from an empirical whole or as being determined by experiments is to misunderstand the nature of grammar, of a logical system and its internal relationships19. And this is what Wittgenstein seems to mean when he afirms:

"Wenn ich mit meiner Auffassung recht habe, so ist es kein Satz: "Rot ist eine reine Farbe", und was damit angezeigt werden soll, keiner experimentellen Entscheidung fähig. Es ist dann nicht denkbar, dass uns einmal Rot, ein andermal Blaurot rein erscheinen sollte . (...) Nun meine ich aber nicht, dass es durch ein Experiment der Mischung festgestellt wird, dass gewisse Farben so aus anderen entstehen. Ich könnte das Experiment etwa mit einer rotierenden Farbenscheibe anstellen. *Es kann dann gelingen oder nicht gelingen, aber das zeigt nur, ob der betreffende visuelle Vorgang auf diese physikalische Weise hervorzurufen ist oder nicht; es zeigt aber nicht, ob er möglich ist.*"(PB 279-80, my italics).

<sup>19</sup> We will come back to this topic in the last chapter of this thesis.

We see that a philosophical theory about the logic of color is ultimately revealed as a conceptual inquiry about which propositions about colors or about the combination or mixture of colors are legitimate and which not. This analysis of the logic of colors is not about the research of the effect of colors on eyes or how people perceive different shades, but should turn to the logical relationship (internal amongst the colors) so that we can somehow make possible the conceptual organization of the appearance of colors. Importantly, this phenomenological analysis or grammar tries to deal with colors as such and not with the cognitive or sensorial apparatus of the people who see the colors. Or, as Kienzler argued:

"Die Fragen nach der phänomenologischen Sprache, nach dem Farboktaeder, der Farbigkeit des Raumes oder nach der Harmonielehre, sind alles eigentlich Fragen nach der Grammatik der darin vorkommenden Ausdrücke und keine Aufgaben zur Erforschung des Unmittelbaren \_ so wie Machs Gedankenexperiment kein Experiment, sondern eine grammatische Betrachtung ist." (Kienzler, p. 137)

Another issue which might be called typically tractarian in Jaspers's approach is his emphasis on a kind of atomistic paradigm in analyzing systems to show their formal similarities in a more perspicuous way. And Jaspers maintains that, in order to investigate the logical relations of a system we have to bring it to its ultimate elements:

"That the molecules built from them are nonetheless different and hence weave a pattern of oppositions within the confines of the field is because each of them is a configurationally different end product of compositional activity on the basis of the small set of atoms. In sum, a closed field (whether lexical-conceptual or perceptual) is arguably created by means of a very small – arguably triadic, set of cognate atoms plus a few rules of composition, which generate the whole paradigm of interrelated different items." (Jaspers, p.9).

Dany Jaspers does speak specifically about unification between logic and colors as an objective for future research. And this theme certainly resembles the intermediate phase of Wittgenstein's Philosophy:

<sup>&</sup>quot;It will be clear that what is attempted here is very much in the same spirit: the occupants of the vertices in the diagrams are different (color percepts versus predicate logical propositions/operators), but all the relations obtaining between the vertices are arguably identical." (Jaspers, p.7)

The first step would then be to find similarities in diagrammatic presentations of both systems. The idea is that from equivalent diagrammatic representations we could show that the internal relations of predicate logic and the perception of colors are the same. Jaspers says that, when analyzed together, the representations converge to geometric identities. This is particularly the case with fundamental exclusions: contradiction and contrary in the case of logic and different types of incompatibility in the case of colors. However, the combination of colors to the vertices of the square of oppositions of Aristotle often appears mysterious, unjustifiable and arbitrarily in many parts of his article. Jaspers wants to defend the possibility of extending the paradigm of logical opposition to the perceptual field of color. This recalls Wittgenstein's youthful project of trying to express the pattern of logical exclusion present in colors through a system of propositions about colors accompanied by a corresponding notation, diagrammatic or not.

Despite the programmatic similarities in the article by Jaspers, nothing is said about the difficulty or intractability of the *Tractatus* with the colors. Nothing is said about the interesting logic of the color system of his intermediate phase, especially in paragraphs 218-224 of the PB. The book *Über die Farben* (published late in 1950) is mentioned *en passant* to basically show that the theme on colors may have philosophical relevance. As we have seen, the notion of logical exclusion between colors in the visual field is a subject eminently Wittgensteinian in the subsequent period of reformulating the *Tractatus* based on its mistakes and shortcomings. The movement in Jaspers' article in seeing colors in the framework of oppositions is inverse (and complementary) to our problem in this work as we are looking at opposition and classical problems in the *Color Exclusion Problem* in the *Tractatus*. This complementarity becomes clear when Jaspers claims:

<sup>&</sup>quot;Note that the idea that there are further closed conceptual (lexical) fields beyond predicate logical operators that can be mapped onto the square is in itself not very controversial. Thus the square has been used successfully to represent relations between propositions containing propositional operators (and, or, nor), modal operators (must, can, can't) and comparative operators (more, equally as much/many, less), amongst others. Yet, the idea that the same pattern surfaces at the perceptual level of cognition too, where representations arise wholly reflexively and without agent intervention or coinage, is new, surprising and potentially rich in implications for research into the relationship between semantic meaning and other forms of mental knowledge." (Jaspers, p.8)

It is clear that Jaspers' ambitious project is to extend this pattern of oppositions to the perceptual level of human cognition. This pattern would also organize our perception of colors, music and numbers. Jaspers' article is an important resource for my argument, even if he does not mention the "Color Exclusion Problem" of Wittgenstein. Jaspers' interpretation was born in the way logic extends itself to colors, as he tries to explain problems with asymmetries in lexicalizations and degrees of naturalness of colors for the evolution of human cognitive apparatus. I believe that the opposite should be seen in Cuter's article. Moving from the colors and numbers and their deadlocks in the *Tractatus*, Cuter helps us see the very intractability of more subtle kinds of exclusions than the contradiction and not expressible in terms of truth tables, i.e. in truth-functional terms. As every ascription of a degree to a quality is an empirical case of contrariety (as with the assignment of number values) it cannot be represented in terms of truth-functional, as shown by the Article from 1929. Therefore, it is surprising that Cuter only covered the Color Exclusion Problem in his article (he focused on colors and numbers, more specifically in arithmetic) rather than, for example, also investigating the problem of the expressibility of the tractarian logic itself, and its alleged and defended neutrality as well. For, as Marion indicates:

That numbers come into propositions about gradations is a consequence and not the cause of the failure of the truth-functional conjunction to express logical relationships in these articulations. Logical problems are the protagonists here, not mathematical problems. The mathematics should be changed only in function of logical impossibilities. I believe that Cuter's vision is sometimes so internal to the tractarian presuppositions and concepts that it doesn't allow itself to see the ways the problem is connected to classic discussions on exclusions by contrariety. This is a logical problem and not a mathematical one. All these kinds of propositions of gradation, not only of color ascriptions, require some form of multiplicity greater than, for example, simple thruth functional conjunctions, in order to assign values

<sup>&</sup>quot;That numbers must enter in elementary propositions is, however, a fairly obvious consequence of the fact that so-called "statements of degree" are not analyzable further: since such statements cannot be broken into further more elementary propositions, the multiplicity of the phenomena will not be captured by the use of the conjunction" (Marion, p. 122)

in a coordinate system that accompany the possible degrees of these empirical qualities. However, when proposing the reduction of colors to the numbers, it makes us naturally think of a reduction of all ascriptions of degrees in terms of numbers, i.e. the problem becomes more abstract and general, and therefore more interesting. In PB § 81 Wittgenstein returns to this matter of logical (but not truth-functional) exclusions which can be mapped by numbers:

"Es verhält sich übrigens mit Farben nicht anders als mit Tönen oder elektrischen Ladungen. Es handelt sich immer um die vollständige Beschreibung eines gewissen Zustandes in einem Punkt oder zur selben Zeit. (...) Wie verhält es sich aber mit allen scheinbar ähnlichen Aussagen, wie: Ein materieller Punkt kann nur eine Geschwindigkeit auf einmal haben, in einem Punkt einer geladenen Oberfläche kann nur eine Spannung sein, in einem Punkt eines Dampfkessels nur ein Druck etc.? Niemand kann dran zweifeln, dass das alles Selbstverständlichkeiten sind und die gegenteiligen Aussagen Widersprüche" (PB, p. 108)

Indeed, we can ask ourselves how to reduce contrariety to numbers. And we can ask if this expedient is really possible or even necessary, if we adopt a system of propositions. It would not be difficult to do reduce contrariety to numbers by using the notion of mapping. From this mapping task, we gain the notion of linear organization. Numbers lend linearity to the complex that they map by determining, one and only one value for the coordinates of a system. However, the ascription of numbers is also exclusive in the sense of contrariety. Contrariety is a general characteristic that also marks numerical organization. So, from this perspective, numerical organization is a rather particular case of this kind of logical exclusion. This makes us understand that the problem lies not in numbers but in the truth-functional paradigm itself, which leads us to think contradiction as the only case of exclusions that are not truth-functional:

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<sup>&</sup>quot;The *Tractatus* tells us nothing about, but one can speculate, depending on what has been said that the only available alternative would be to seek a kind of "color metric", in which each color was associated with a number (or double or triplet of numbers), and the exclusion of colors could be viewed as a particular case of the exclusion of incompatible numerical ascriptions. The proposition "This is blue and this is red" should have, after all, the same logical form that the proposition "This table is 2 feet long, and this table is 3 feet long", or that the proposition "There are exactly two people in this room." (Cuter, p.191)

The problem is with the image itself of a neutral logic, based exclusively on the bipolarity. One cannot reduce contrariety to contradiction. The problem is not exactly only with numbers, but with the very truth-functional paradigm for the analysis of any empirical proposition. It is not enough to have just the Vertretung of a logical object by a logical name. We need a whole scale, a whole system with its multiplicity of contrarieties, to be referred to reality in order to determine a degree and so also to determine the remaining values that are not there. As with the earlier example of a ruler, to measure things we do not need only the extremes of the ruler, but the entire ruler and all its possible extensions. We need the entire metric system for being able to measure. Similarly, we need the whole system of colors or of numbers or of measurements in order to understand propositions, i.e., we need systems containing such oppositions by contrarieties.

It matters little to reduce color systems to number systems. We need a representation that accounts for, maps and organizes the multiplicity of this dense frame of exclusions in measuring empirical qualities. Using numbers to map colors tells us as little about the nature of colors, just as distributing numbers in a bank to determine the order of arrival of customers tells us about the nature of people. Using numbers to map the multiplicity of colors does not make the nature of the colors numerical, in the same way that numbers distributed in a physician's office to prioritize service does not make the patients numerical. That a complex map another does not necessarily give us its nature. Similarly, a triangle that is mapped out by an equation does not become essentially algebraic nor does the equation necessarily become geometric. Here this strategy of mapping has a pragmatic value rather than a metaphysical one. We can even consider the inverse. There may be another reason (any reason, perhaps an aesthetic one) for which one may want to map out numbers in terms of colors. Nonetheless, this would not make the nature of the numbers chromatic.

This kind of *non sequitur* via the concept of mapping appears in the previously cited passage when Cuter says, "if each color was assigned a number...," thus the exclusion of colors would result from the exclusion of numbers, and also the arrangement of colors would also result from the organization of numbers. Real numbers (with their metric ability, density and linearity) are good candidates to be used as a guiding thread in this mapping. I believe that numbers do not necessarily need to be found within the elementary

propositions. Elementary propositions *must* not be composed of numbers but numbers *must be able* to map such propositions, expressing exclusions through contrarieties, and not by contradictions. Moreover, we do not need to express the framework of implications and exclusions within the system of colors with numbers, for we have seen it is enough to show them diagrammatically (perhaps with octahedral or hexagonal shapes) depending on the perspicuity of each representation or notation required for our purposes. However, regardless of form, being discreet, natural numbers, although linear, do not have enough multiplicity for such an undertaking. As Cuter clarifies, numbers serve to count but not to measure. They serve to organize but they cannot comprehensively map the dense wide range of phenomena.

However, if we go back to the problematic passage 6.3751, we note that Cuter's analysis conveniently stops at the notion of speed of particles proposed by Wittgenstein, without mentioning his next intriguing point:

We begin with a kind of application of bipolarity to the ontology. We then have a situation whereby a certain state of affairs must be able to happen and to not happen, thereby founding a principle of contingency in the world, from where we can derive that there is only logical impossibility, for there is only necessity in the logic. Then we have the example of the colors. It should show us that because they exclude, they should still be analyzed. Whether contrary or contradictory, these propositions necessarily exclude each other, they are necessarily incompatible. Cuter applies the analogy to the speed of particles20 as introduced

<sup>(...)</sup> Ungefähr so, dass ein Teilchen nicht zu gleicher zeit Geschwindigkeiten haben kann; das heißt, dass es nicht zu gleicher Zeit an zwei Orten sein kann; das heißt, dass Teilchen an verschiedenen Orten zu Einer Zeit nicht identisch sein können. (...)" my italics

<sup>20</sup> Even here there is the problem of time. Speed measurement is not restricted to a measurement of space. It takes, so to say, more than one ruler to measure speed. With time, the multiplicity of the representation must be much greater. Additionally, we have to ask ourselves how (in terms of tractarian quantification) the two scales could interact. Is there a way to show displacement and movement without a notion of time and space? Bento Prado Neto sees the problem of time in relation to the visual field and the postponement of this investigation as the major conceptual weakness of *Some Remarks*. These would be the main ideas responsible for changing the theme of the Aristotelian Society conference in 1929, where Wittgenstein presented reflections on infinity instead of something about the time. According to Bento Neto: "We think therefore that during the time when Wittgenstein wrote observations about mathematics, he was looking for a solution to the problem of time, or alternatively, that he had postponed the examination of the idea of a phenomenological language until this solution could be found . And when the time came to the conference, in which he should speak publicly about the phenomenological, this issue has become

verbatim by Wittgenstein to justify his reading of the chromatic exclusions by means of numbers. However, we still have the rest of the passage as shown in italics. We begin with purely logical principles, and we come (via the notion of speed) to a kind of trivial, physical exclusion exhorted as evident and primitive in any classic metaphysical system: One thing cannot be in different places at the same time. If particles are in distinct places at the same time then they have to be different. They cannot be the same particle. Put another way, there cannot be two things in any one point. The question that we can pose is: To what extent is the contrariety such an "empirical exclusion" as the one of "something cannot be in two different places" or "one place cannot be occupied by two things"? Why is contrariety always thought of in spatial terms? Two different grades cannot be attributed to the same coordinate, because here there is no room for more than a degree. Is this a mere metaphor? Two colors cannot be attributed to the same point. At the same point there cannot be two different colors. Two bodies cannot occupy the same place. Or just, as a mathematical function where there is only place for a single argument. As Wittgenstein asserts in paragraph 84 of PB:

Hintikka & Hintikka indicate this conceptual similarity in the treatment of the field of color and space in the *Tractatus*, highlighting the role of a character essentially functional in its figurative conception. However, they do not investigate the kind of incompatibility in terms of gradations of quality that Wittgenstein points out in *Some Remarks*. Nor do they look at contrary propositions, as pointed out in this work.

"Auch die Ähnlichkeit zwischen Raum- und Farbbegriffen, auf die sich Wittgenstein im *Tractatus* stützt, deutet auf etwas von der Art unserer Abbildungsidee

<sup>&</sup>quot;Es stellt die Sache falsch dar, wenn man sagt, man dürfe einem Gegenstand nicht zwei Attribute beilegen, die miteinander unvereinbar sind. Denn so scheint es, als müsse man in jedem Falle erst untersuchen, ob zwei Bestimmungen miteinander vereinbar seien oder nicht. Die Wahrheit ist, dass zwei Bestimmungen Derselben Art [Koordinate] unmöglich sind. Unsere Erkenntnis ist eben, dass wir es mit Maßstäben und nicht quasi mit isolierten Teilstrichen zu tun haben. Jeden Aussage bestünde dann gleichsam im Einstellen einer Anzahl an Maßstäben, und das Einstellen eines Maßstabes auf zwei Teilstriche zugleich ist unmöglich."

inescapable, and the result became disastrous for the phenomenological language and for *Some Remarks*" (BENTO NETO, p.81)

hin. Diese Ähnlichkeit spielt in Wittgenstein Erörterung der Farbbegriffe im *Tractatus* eine wichtige Rolle. Ein Grund für ihre Bedeutung ist der, dass das gleiche Inkompatibilitätsproblem, wie es die Farbbegriffe aufweisen, auch von den Raum aufgeworfen wird. So ist es z.B. ausgeschlossen, dass sich ein und derselbe Farbfleck an zwei verschiedenen Stellen des Raums befindet." (p.167)

Hintikka & Hintikka correctly believe that this kind of incompatibility was taken as harmless by Wittgenstein because there was the idea of a notation that could dissolve such an exclusion, or analysis that could reduce this exclusion in terms of *NOR*. In a way, in opposition to Cuter's ideas on reducing colors into numbers, Hintikka & Hintikka believe that Wittgenstein hoped to reduce colors analogously to spatial concepts. They read then the passage 6.3751 until its end, with an addendum: the reduction should be made via a suitable notation. This analogy between the analysis of spatial structure and of the structure of color actually already appears in notes in the *Tagebücher* 14-16 from 16th August, 1916:

"Dass ein Punkt nicht zugleich rot und grün sein kann, muss dem ersten Anschein nach keine Logische Unmöglichkeit sein. Aber schon die physikalische Ausdrucksweise reduziert sie zu einer kinetischen Unmöglichkeit. Man sieht, zwischen Rot und Grün besteht eine Verschiedenheit der Struktur. Und nun ordnet sie die Physik gar noch in eine Reihe. *Und nun sieht man, wie hier die wahre Struktur der Gegenstände ans Licht gebracht wird. Dass ein Teilchen nicht zu gleicher Zeit an zwei Orten sein kann, das sieht schon vielmehr aus wie eine logische Unmöglichkeit.* Fragen wir z.B. warum, so taucht sofort der Gedanke auf: Nun, wir würden eben Teilchen, die sich an zwei Orten befänden, verschieden nennen, und *das scheint alles wieder aus der Struktur des Raumes und der Teilchen zu folgen.*" (my italics)

As is evident in this passage from *Tagebücher*, the *Color Exclusion Problem* was primarily explicitly thought of in the context of the structure of space and not of the numbers. I believe that the mentioning of velocities of particles (6.3751) is more tentative or heuristic for an analysis of this kind of exclusion rather than being properly metaphysical, that is, it does not have to do with the nature of colors. I feel it's mentioned with the aim of giving the nature of colors – appearing in this way to defend Cuter's work. Moreover, it seems difficult to think that Wittgenstein (or Hintikka & Hintikka) really believed that the truth table, in a purely syntactic or combinatorial meaning, might actually sometimes cope with this kind of exclusion of "space". I believe that here there is a kind of anachronism

in the way Hintikka & Hintikka already see in the *Tractatus* the demand for a special notation for the cases of color and spatial concepts. Rather, I see there is the demand (both in the tractarian time as well as in the intermediaries in the texts to PB) for a suitable notation that could be used systematically to express the logical multiplicity of states of affairs. However there is a fundamental difference between these two searches: the generality or the scope of the notation. In the *Tractatus* there was the idea of a notational system that could capture the truth-functionality based on the complexity of all empirical propositions.

By the time of his return in 1929 - triggered by the *Color Exclusion Problem* - Wittgenstein begins working with the idea of a multiplicity of notational systems that could each be applied to a different system of propositions according to their logical and conceptual peculiarities. This is important: the fragmentation of the logical space led to the idea of propositional systems, parallel to the fragmentation of the idea of a "super notational system" that leads to the idea of different notation systems appropriate to certain system of propositions.

The purely syntactic approach of the truth table follows the logical multiplicity based on truth-functionality. Since this paradigm of analysis is not exhaustive any more it is natural to think of the demand for other notational systems (other systems for analysis) which are not truth-functional. So what Hintikka & Hintikka claim about the *Tractatus* holds only for the period of Wittgenstein's period to Philosophy:

Indeed, in this context of exclusions, we have non-truth-functional images that accompany our intuitions about mathematical functions: Given one imput, I have one and only one output. Given one argument to a function, I only have one final value. Here we see the same structure as with the colors exclusion. Given a point in the visual field we can only ascribe a color to it. In turn, this functional image is again full of spatial images. Somehow it seems that we do not actually have room for anything else, that the vacant place was already occupied. The function can only give me one and only one value for the argument in question.

<sup>&</sup>quot;Er hält es für möglich, eine Notation zu entwerfen, die die notwendigen Beziehungen zwischen Farbbegriffen spiegelt, und dadurch zu zeigen, wie die passenden Strukturen (Formen) in den Gegenständen angelegt sind, mit denen wir es bei Farbzuschreibungen zu tun haben." (Hintikka & Hintikka, p. 168)

These are all images and metaphors loaded - at least so it appears - with empirical elements. Another empirical metaphor appears already in this context of demarcation of logic in the *Tractatus*:

"Die Wahrheitsbedingungen bestimmen den Spielraum, der den Tatsachen durch den Satz gelassen wird. (Der Satz, das Bild, das Modell, sind im negativen Sinne wie ein fester Körper, der die Bewegungsfreiheit der anderen beschränkt; im positiven Sinne, wie der von fester Substanz begrenzte Raum, worin ein Körper Platz hat.) Die Tautologie lässt der Wirklichkeit den ganzen \_\_\_\_\_ unendlichen \_\_\_\_\_ logischen Raum; die Kontradiktion erfüllt den ganzen logischen Raum und lässt der Wirklichkeit keinen Punkt. Keine von beiden kann daher die Wirklichkeit irgendwie bestimmen." 4.463.

Making use of this metaphor and keeping in mind that contrary propositions are not contradictions but exclusions, one might ask the question: What connection do the contrary propositions have with the reality? Do these propositions determine reality to some extent? If we take these propositions, (such as the ones related to the *Color Exclusion Problem*) as really empirical, their negation must then be possible. Can two objects occupy the same space? Can an object be in two different places at the same time? Can two colors occupy the same point in the visual field? Can something that is a lion be a leopard too? Can a table that's three meters long also be four meters long? Can a circle be a square? Can a soccer game strictly end with a draw and have a winning team? In discussion with the Vienna Circle on the possibility of identifying this kind of system that organize propositions as synthetic *a priori* propositions, Wittgenstein attacks this kind of interpretation a la Husserl. This attack occurs via the role of negation for the understanding of a proposition, reaffirming the new logical or grammatical nature of such propositions:

"Wenn ich sage: "Ich habe keine Magenschmerzen", so setzt das bereits die Möglichkeit eines Zustandes der Magenschmerzen voraus. Mein jetziger Zustand und der Zustand der Magenschmerzen liegen gleichsam im selben logischen Raum. (So wie wenn ich sage: Ich habe kein Geld. Diese Aussage setzt bereits die Möglichkeit voraus, dass ich ja Geld habe. Sie zeigt auf den Nullpunkt des Geldraumes.) *Der negative Satz setzt den positiven voraus und umgekehrt.* Nehmen wir nun die Aussage: "Ein Gegenstand ist nicht rot und grün zugleich." Will ich damit bloß sagen, ich habe bisher einen solchen Gegenstand nicht gesehen? Offenbar nicht. Ich meine: "Ich kann einen solchen Gegenstand nicht sehen.", "Rot und grün können nicht im selben Ort sein" Hier würde ich nun fragen: Was bedeutet hier das Wort "kann"? Das Wort "kann" ist offenbar ein grammatischer logischer Begriff, nicht ein sachlicher. Gesetzt nun, die Aussage: "Ein Gegenstand kann nicht rot und grün sein" wäre ein synthetisches Urteil und die Worte "kann nicht" bedeuten die logische Unmöglichkeit. Da nun ein Satz die Negation seiner Negation ist, muss es auch den Satz geben: "Ein Gegenstand kann rot und grün sein".

Dieser Satz wäre ebenfalls synthetisch. Als synthetischer Satz hat er Sinn, und das bedeutet, die von ihm dargestellte Sachlage kann bestehen. Bedeutet also kann nicht die logische Unmöglichkeit, so kommen wir zu der Konsequenz, dass das Unmögliche doch möglich ist. Hier bleib Husserl nur der Ausweg, dass er erklärt, es gäbe noch eine dritte Möglichkeit. Darauf würde ich erwidern: Worte kann man ja erfinden; aber ich kann mir darunter nichts denken." (WWK.67-68.) my italics

We do not then need to consider a third kind of proposition: empirical, logical and, say, synthetic *a priori*. We should only amplify or stretch the tractarian logic which consists of only tautologies and their negations provided by the syntax (the contradictions) to bring the kind of implication and exclusion found in a more refined system such as of the color, into the logical field. Before bringing empirical features to logic we can shed light on logical features of empirical complexities. And I believe that Wittgenstein thought this way at that time.

However, he seems to stop writing about colors in 1931, at the end of the short period of phenomenological Philosophy. In 1950, he writes about color with a new conceptual horizon opened up by the consolidation of his mature Philosophy. But even so, examples from *Über die Farben*, II, passages 2 and 3, reveal a Wittgenstein who is still forced to protect himself from the temptation of thinking about color judgments as belonging to a third category of judgments, synthetic but *a priori*. I would even say that we can see a Wittgenstein who is insecure about the attractive idea of thinking about colors propositions as examples of a sort of middle way between logic and science - a hybrid that he has always tried to avoid throughout his life:

"Die Beimischung des Weißes nimmt er Farbe das Farbige; dagegen nicht die Beimischung von Gelb. \_ Ist das am Grunde des Satzes, dass es kein klar durchsichtiges Weiß geben kann? Was aber ist das für ein Satz: dass die Beimischung des Weißen der Farbe das Farbige nimmt? Wie ich es meine, kann es kein physikalischer Satz sein. *Hier ist die Versuchung groß, an eine Phänomenologie, ein Mittelding zwischen Wissenschaft und Logik, zu glauben.*" my italics

Or even in passage 4, of the third part, the problem surrounding the extent to which the logic has to differentiable or can be separable from the *Empirie* is explicit:

"Aber auch das reine Gelb ist heller als das reine, satte Rot oder blau. Und ist dies ein Satz der Erfahrung? \_ ich weiß z.B. Nicht , ob Rot (d.h. Das reine) heller oder dunkler ist als Blau; ich müsste sie sehen, um es sagen zu können. Und doch, wenn ich es gesehen hätte, so wüsste ich es nun ein für allemal, wie das Resultat einer Rechnung. Wo trennen sich hier Logik und Erfahrung (Empirie)?" p.41

Where can we separate logic from experience (or experience from logic) in this field of propositions that allow themselves to be arranged in a system or grammar? Anyway, it seems that we are dealing with a kind of exclusion more empirical - or less logical - than the contradiction. The exclusion by contrariety seems to bring logic to the empirical world. Inevitably, it seems to be throwing alien elements, empirical or metaphorical ones, into the allegedly neutral ground of logic. It seems to challenge the limits of logic itself, when this has to touch the world, when it has to take a look at and examine things and its laws of organization in closed systems. These features challenge the set 5.55 of passages in the *Tractatus*. And Wittgenstein affirms this in a talk with the Vienna Circle towards the question about a colors system, although he had yet to develop the idea:

"(...) Die Aussagen, welche mir die Länge eines Gegenstandes beschreiben, Bilden ein System, ein Satzsystem. Ein solches ganzes Satzsystem nun wird mit der Wirklichkeit verglichen, nicht ein einzelner Satz. Wenn ich z.B. Sage: Der und der Punkt im Gesichtsfeld ist blau, so weiß ich nicht nur das, sondern auch, dass der Punkt nicht, grün, nicht rot, nicht gelb usw. Ist. Ich habe die ganze Farbenskala auf einmal angelegt. Das ist auch der Grund dafür, warum ein Punkt zu gleicher Zeit nicht verschiedene Farben haben kann. Denn wenn ich ein Satzsystem an die Wirklichkeit anlege, so ist damit \_ genau wie beim räumlichen \_ schon gesagt, dass immer nur ein Sachverhalt bestehen kann, nie mehrere." (WWK, p.64.) my italics

Such statements always make it tempting to say that at this phenomenological period (in this period that Wittgenstein is dedicated to the study of possibilities of meaning of empirical propositions in systems) we have an investigation of something midway between logic and natural science, a hybrid being. Wittgenstein continues to work in this context of exclusions by contrariety with empirical metaphors or spatial images of exclusion, even in PB. I then highlight some passages:

<sup>&</sup>quot;Wie ich es möglich, dass f(a) und f(b) einander widersprechen, wie es doch der Fall zu sein scheint? Z.B., wenn ich sage "hier ist jetzt rot" und "hier ist jetzt grün"? Es hängt das mit der Idee der vollständigen Beschreibung zusammen: "Der Fleck ist grün", beschreibt den Fleck vollständig, *und es ist für eine andere Farbe kein Platz mehr*. Es hilft auch nichts, dass rot und grün in der Zeitdimension gleichsam aneinander vorbei

können; denn wie, wenn ich sage, dass während eines gewissen Zeitraums ein Fleck rot und dass er grün ist?" (PB, § 77, p. 106) (my italics)

"(...) Das zwei Farben nicht zu gleicher Zeit an den gleichen Ort gehen, muss in ihrer Form und der *Form des Raumes* liegen." (PB, § 78, p.107) (my italics)

"(...) Das würde aber heißen, dass ich zwei bestimmte Sätze zwar anschriben darf, aber nicht ihr logisches Produkt. *Die beiden Sätzen kolliedieren im Gegenstand*." (PB, § 79, p.107) (my italics)

"Ich kann gelb und rot nicht eigentlich mischen, d.h., nicht wirklich zugelich sehen, denn wenn ich hier gelb sehen will, so muss das Rot von diesem Platz weg und umgekehrt." (PB, § 80, p.108) (my italics)

As Cuter says, it does seem that, in this context, via the problem with the exclusion of colors, we are getting closer:

"to one of the most exciting problems in the history of Philosophy: the problem of the relationship between logic and the world. (...) To which extent can logic be seen as a product of human conventions that create a language? And, to which extent, despite being linked to human conventions, logic is to be concerned with the world that our languages try to represent?"(Cuter, p.182)

The advantage of Cuter's proposal is that we can see a logical identity of the kind of exclusions among colors and numbers, showing how the two problems represent dead ends for the kind of analysis proposed by Wittgenstein in the Tractatus. We have two examples of expressive inadequacy, whether they are reducible to each other or not. I believe this is the reason for the early problems with the Tractatus: expressive incapacity. These problems at this time are not yet with its presuppositions, but with its expressive limitations - sometimes expressing less (colors), sometimes expressing erroneously (numbers). Again, it is not relevant here to know for sure how much one can be reduced to the other. We can, in principle, maintain the independence of the Color Exclusion Problem in relation to the problem of numbers by showing that the truth-functional paradigm does not express contrarieties of colors, but natural numbers through quantification and iteration of operations. In the case of numbers, we also have an expressive failure, but in relation to the tractarian quantification, as Cuter shows well:

<sup>&</sup>quot;The *Tractatus* project comes to an end at the exact moment that Wittgenstein realizes that the analysis of numbers there proposed is inconsistent with any mensurative context.

The numbers of the *Tractatus* are great for counting, but are terrible for measuring. (...) Measurements cannot be displayed in the form of quantificational structures. Quantifiers are used to count. They do not serve to measure (...)." (Cuter, p.192)

The only form of opposition by contrariety that seems to be expected in the *Tractatus* seems to be in using generalizations, as in "All the students speak German" and "No student speaks German", form of the example given by Cuter in his article. Somehow it seems that in the *Tractatus* Wittgenstein hopes to sublimate this kind of exclusion as well as numbers, quantifications, operators, the colors at some stage in an analysis exclusively truth-functional. Perhaps this is why Wittgenstein did not realize at the time of writing the *Tractatus* that this kind of exclusion by contrariety could cause irreversible damage to his project of complete analysis of empirical propositions in truth-functional terms.

The truth table as the proposed notation in this context is particularly revealing. One might think to define an operator of contrariety that would be displayed exactly as *NOR*, thereby making false the junction of two propositions only when both are true together. However the problem is not in the last column of the truth table, which gives us the final values of all combinations, but in the line that allows the articulation of propositions not provided, or absurd ones. The deficiency is in the construction itself of the truth table. It is in its indiscriminate distribution of truth values. Our problem here is not with the falsehood of some combinations but with the possibility itself of combining some propositions, where sometimes the parts make sense but the combination of the parts does not. This is the challenge for the truth-functional paradigm. We will return to this issue in more detail in the next chapter.

In fact this kind of exclusion that appeared only in terms of quantification could perhaps be diluted or sublimated by the end of truth-functional analysis. But this attempt has no future, as *Some Remarks* has shown. The ways of generating contrary propositions with numbers and colors seem to provide precisely exclusions which still remain logical, although they must not be analyzed in terms of quantification and cannot be analyzed in terms of truth-functional means. As Wittgenstein states later in the passage 46, in *Über die Farben*: "In den Farben: Verwandtschaft, und Gegensatz. (Und das ist Logik)". (p. 50)

In his interesting article, despite of being almost forgotten by the secondary literature, *The discussion of a certain type of negative proposition*, Raphael

Demos suggests an interpretation for the role of the propositional negation which in many ways resembles the tractarian interpretation, including its difficulties in expressing exclusions by contrariety. I believe that the more general reason for this resemblance is departing, as well as the *Tractatus*, from a kind of Russelian mistrust regarding the common grammar. This could never deliver us the alleged real logical form of the propositions. Here it is also easy to realize the need for using a perspicuous analysis in order to reveal the true logical foundation of a proposition hidden by its surface or grammatical disguise.

Demos helps us see that by attempting to understand the mechanism of the propositional negation, we can departure from colors linkage to contrary propositions. We can also do the opposite: we can departure from the problem of oppositions by contrarieties in empirical propositions and reach oppositions of colors, as Jaspers does in his article. The movement that I argue is happening in the *Tractatus*, essentially shows its inability to address contrary propositions through the paradigmatic example of the intractability of colors. This problem seems to be independent of the conceptual geography of the *Tractatus*. That is it may arise without any mention of the collapse of the project tractarian, as we saw in Jaspers' article. The problem for Jaspers as a linguist does not begin in the analysis of particular negative propositions, such as it does for Demos, but in the difficulty of lexicalization of some points of opposition in Aristotle's square. However, both articles point to a kind of radical semantic holism: to understand a proposition is to understand the system of oppositions, exclusions and implications in which it is necessarily inserted.

Unfortunately Demos' work is not complete because it does not contemplate generality. He did not mention or even indicate how to treat the negative generality following on from his results on negative particulars. Moreover, Demos does not develop the idea of a negative proposition being related to all other propositions, when he shows a certain semantic holism a la the tractarian phase of Wittgenstein's thought. In reading his article we are clear that he finds that there is no possibility of saying something true without the notion of falsehood. This resembles the famous tractarian bipolarity. It only makes sense to assert something when we can suppose its falsehood. As Wittgenstein asks rhetorically at 5.5151:

"Muss das Zeichen des negativen Satz mit dem Zeichen des positiven geBildet werden? Warum sollte man den negativen Satz nicht durch eine negative Tatsache ausdrücken können. (Etwa: Wenn "a" nicht in einer bestimmten Beziehung zu "b" steht, könnte das ausdrücke, dass aRb nicht der Fall ist.) Aber auch hier ist ja der negative Satz indirekt durch den positiven gebildet. Der positive Satz muss die Existenz des negativen Satzes voraussetzen und umgekehrt."

What makes Demos' article interesting in this context of discussion towards the tractarian negation and its commitment to a vision of a formal, neutral, combinatorial, truth-functional logic, and, therefore, with its inability to expressing assignment of degrees to empirical qualities, is the very announcement of his goals. About his goals, Demos writes: "my aim will be to discover the definition of the negative propositions of the sort in question [particular negation], that is to say, their general form and their relation to the total field of propositions." (Demos, p.188). Here we highlight the idea of a general negation form to be pursued or discovered for simply not being visible in non-analyzed everyday sentences. Furthermore, there is also the relation of this general form with a propositional whole, an exhaustive linguistic whole of legitimate possibilities. These themes are markedly tractarian.

Interestingly, assuming tractarian principles and objectives, Demos arrives, I argue here, to the same *aporias* of Wittgenstein's youth Philosophy. In positive considerations concerning negation, Demos concludes that negative propositions are objective entities, i.e. they are independent of a knowing subject. This interpretation is against the view that propositions become negative only if a subject denies it. From this perspective, the interpretation is directly related to a cognitive attitude. "Any work on symbolic logic contains many propositions as members of deductive systems. There, no attitude seems to be involved, and yet the propositions are not devoid of their peculiar character as positive or negative." (Demos, p.188).

Even more tractarian is the conclusion and the argument about the need to explore ordinary sentences of ordinary language to discover its real hidden form. As previously suggested, this is certainly a consequence of the affiliation with Russell. cf. 4.004. As Cuter claims in his article on the negation of names in the *Tractatus*:

"Russell's theory of descriptions made in the presence of certain assumptions, that the notion of "propositional part" ceased to have a criterion of identification merely visual. Since the article about the denotation (1905), the propositional "parts" will be object of a discovery and not of a connotation. They will be the result of a long process of analysis. The idea, actually, was not new. In a sense, it is inseparable from the logicist project as a whole. Long before 1905, Frege's *Begriffschrift* had already exhibited logical complexities in certain propositions which do not find expression in everyday language (...) The logical "parts" of the analyzed proposition have nothing to do with the words we use in ordinary expression." (Cuter, p. 34).

With Demos we revisit the Russellian theme of distrust of the ability of the grammatical analysis to reveal the logical form of a sentence. We can clearly see his work as a continuation of Russell's:

"The parallelism in the further treatment on the one hand of simple phrases by Mr. Russell, and on the other of negative propositions by myself, such that the former are supplemented by an assertion of existence, and the latter by an assertion of truth, is obvious" (Demos, p.195).

Demos accepts and uses the Russelian theory of descriptions: "The exclamation "rain" is really of the form, "It is raining" or "there is rain", and the proposition, "I saw the servant of Y" is "there is one who is the servant of Y and I saw him". (Demos, p. 193.) The negative propositions should not be taken at their face value, but must be interpreted in order to exhibit their logical form and their inherent reference to positive propositions and these, in turn, to the positive facts. In the excerpt below Demos refuse the idea of negative facts:

"Hence a view which adopted appearances would have to add to the world of positive propositions a new class of propositions which are negative, and to the world of positive facts, a new class of negative facts. Now, the reason why such a view must not be entertained is the empirical consideration that strictly negative facts are nowhere to be met with in experience, and that any knowledge of a negative nature seems to be a derived form perception of a positive kind (...) Granting that there are no negative facts, then, in so far as a negative proposition is asserted of fact at all, the term of reference must be the world of positive facts. Hence, appearances must be discarded and *a special interpretation* given to the negative proposition, the term of reference must be the world of positive facts21." (Demos, p.189). My italics

This analysis or special interpretation (or even "symbolic prospective search" respecting the Leibnizian intuition of the present work) in order to show

<sup>21</sup> We will come back to this issue about negative facts when we cover the tractarian *Bildkonzeption* and the falsehood problem in the section 3.3 of this work.

the logical form of a legitimate proposition should be guided to the particle "not". But Demos argues that we should not consider it as qualifying a predicate within the proposition. As in the *Tractatus*, Demos takes "not" as a propositional operator, and then attempts to make the vision of a predicative "not" unfeasible.

"I have in mind the general view which makes the peculiarity of the negative proposition appear to be a peculiarity of the predicate and is thus enabled to define the class of negative propositions as simply a subdivision in the class of positive propositions that contain a "not"-predicate." (DEMOS, p.190)

Demos does not assume a negative proposition relates to any individual element present within it, as a predicate or the grammatical subject, but with all its propositional content. Thus, the contents of a negative proposition must be positive, insofar as the negative proposition is a kind of negative function of some positive particular proposition. To Demos the "not" would then be a modification relation of a affirmative proposition, i.e. a positive function on a particular proposition, whatever it is. The "not" works as a relational modification of a proposition "p".

In other words, "not" means subsequently "the opposite of p" or "the contrary of p". Demos argues that the use of "not" attests the truth of a proposition "q" which is in opposition to the denied "p". This seems to be easy to propose in trivial cases of empirical negative propositions as when "Peter is not father of John" is analyzed as "not (Peter father of John)" or "the opposite of (Peter father of John)". In another example, "The bananas were not bought at the market" can be understood as "not (Bananas bought at the market)" or "the opposite of (bananas bought at the market)." I want to make clear with this tentative notation that the "not" should reach the entire content of the proposition to be denied, as Demos holds:

<sup>&</sup>quot;Now, there is a certain relation among propositions which, in accordance with traditional usage, I shall call the relation of opposition or of contrariety or of inconsistency, and which gives rise to the qualifying "opposite", or "contrary", or "inconsistent with", the word "not" is precisely a symbol for this qualifying predicate, and "not-p" means "opposite, or contrary, of p". The relation of opposition is such that if p opposes q, p and q are not both true (at least one of them is false)." (Demos, p. 190)

However, here we see some of our already known problems: it is not always easy - or even possible - to find a particular candidate to be the opposite or contrary to an affirmative proposition. Even if a negative proposition is in fact a description of some positive proposition which is in opposition to it, there are clearly instances when we cannot determine this proposition because there is not only one opposite proposition but several, and, indeed, in some cases there are endless alternatives. According to Demos, we refer with the negation to a true proposition that is opposite to the negative proposition. However, in some cases we cannot have this uniqueness in the reference, even without mentioning it. This is something Wittgenstein doesn't cover in the *Tractatus* when for example, he claims in 5.513 – an important passage in this thesis - that every proposition would only have one negative, because there would only be one negative that would be entirely outside of it. This strong interpretation of the uniqueness of a denial in relation to a proposition does not seem to be very problematic in a disjunctive horizon of only two possibilities, namely in the case of a dichotomy: if we do not have one, we have the other. In such cases it is easy to determine the opposite, or the negation of a proposition. However, we can bring events to the discussion that allow a trichotomy (as in the prior example of a football match). What would happen to the uniqueness or exclusivity of alternatives, where the opposite of winning can be losing as much as drawing. The disjunction can be even more complex if we take the case of our traditional problem with the colors.

However here we can already pose a simple question-challenge: What is the opposite or the contrary of green? When we say "this shirt is not green," although we understand this proposition, say, determinately, nothing is determined if we analyze this sentence, following the paraphrase proposed by Demos, as "not (green shirt)" or "opposite of or contrary of (green shirt)." We have infinite possible candidates for non-green – indeed; we have the whole spectrum of colors which, if we allow for shade variations, for example, is potentially infinite. Or: What would be the opposite of "It's 24° today"? With both "Not (24 degrees today)" and "opposite of (24 degrees today)," we again have an explosion of possible candidates for the contrary of a temperature degree. Demos expressly states: "As such, a negative proposition constitutes a description of some true positive proposition in terms of the relation of opposition which the latter sustains to some other positive proposition." (Demos p. 194). But what if only one positive

proposition that it is contrary to this negative proposition does not exist? The contrary of some propositions, namely, at least, all that contain gradations of empirical qualities, are always essentially ambiguous. There is an essential feature here in the tractarian jargon because this ambiguity of the negation of these propositions would characterize their logical form. It is not enough to say that negation means the opposite in the case of this kind of empirical propositions. In these cases, if we take the negation of a proposition as a description of a description we will draw the inevitable conclusion that it is an ambiguous description, essentially ambiguous. All negation would make to some extent a proposition ambiguous. In other words, we can have the unpleasant consequence that all negation is ambiguous. Demos' negation seems to throw the denied proposition (regardless of whether or not it's about degrees), into a context of tacit assumptions or into, say, a system of propositions.

According to Demos, if we know, for example, that John is not at the market, we would then have the opposite of (John in the market). But to do so we must assume that John should be somewhere else, i.e. at home, at the corner, on the street... In the case of the trivial proposition "This is not mine" (and so that its opposite makes sense) must be analysed as "not (this mine)," or "the opposite of (this mine)", which is not determinable, because, in principle, it can belong to anybody else. But can we really understand the opposition. Why? In this paraphrase, affirming that "this is not mine" necessarily involves assuming that "this belongs to another person" – someone, perhaps, I may not actually know. Affirming that "John is not in the market" is to assume that John is in another place, or somewhere else, somewher which I do not really know? Should any such denial, therefore, be essentially ambiguous? It does not seem to be problematic for Demos to make the propositional negation always ambiguous. This essential ambiguity seems to be accepted by him and foreshadowed by what he calls the reference without mention:

<sup>&</sup>quot;It is as a descriptive phrase in the above sense that the reader is invited to regard the negative proposition and more particularly as an ambiguous description of some positive proposition in terms of its opposition to some other positive proposition (...) it must be pointed out next that reference to a positive proposition in terms of the negative proposition describing it is achieved without having the former as a datum to the subject

referring, or, in general, without having it as a constituent in the complex of reference; in a word, it is reference without mention." (Demos, p. 192).

Wittgenstein's objective in the *Tractatus* seems to surround trying to make all the exclusions capable of being reduced to contradictions. Demos' objective, on the other hand, is to try to surreptitiously make all the negations capable of being reduced to contrarieties, to ambiguities and indeterminacies. This means negative propositions are never complete or determined. To be complete, clear, and/or explicit, they must be completed by the truth of the proposition they describe, despite us not knowing exactly what that is.

A particular proposition must be interpreted as negative description of a true proposition in opposition to the corresponding positive proposition denied. To affirm "John is not at home" means "not (John at home)," according to the interpretation of negations as propositional operator, which, in turn, would mean the contrary or opposite of "John at home", which in turn refers to a proposition "q" – also a bet or speculation - which is in opposition to the "John is at home". Such alternatives could be that "John is at the market," "John is in the cinema", "John is in the library" ... This means that from "John is not at home" we come to "John is somewhere else," – clearly supposing that John must be somewhere. This seems to be to some extent a trivial supposition in a conversation, but as Demos introduce it, it would certainly be an inhospitable and artificial conversation:

Another relevant question in this context could be: Who, other than perhaps a philosopher, would answer a question "where is John?" with "John is not at home" knowing that John is actually in the store? Indeed, Demos' interpretation

<sup>&</sup>quot;Suppose you ask me where John is and suppose I reply "John is not at home", what is it that I convey in my reply? In asking me where John is, you are asking for the truth about John, i.e., for a true proposition as to John's whereabouts. Now, I know that John is at the store, i.e., I know that the true proposition or again I may refer to it indirectly, that is, I may describe the truth. Actually, I choose the latter alternative and reply by describing the true proposition. The true proposition "John is at the store" is in fact a contrary of "John is at home", and hence may be described as a proposition which is a contrary of the latter. Thus, in reply to your question as to the truth about John, I furnish the statement "The true proposition, or the truth as to John's whereabouts, is a contrary of the proposition, "John is at home". However, as it is understood that I am referring to the true proposition, I make no mention of that, and in my reply I give its description only, i.e., I state "a contrary of "John is at home". (Demos, p. 194).

ensures that something that "ii is not mine" must, in last instance, belong to someone else. Demos seem to recognize this explosion of alternatives and assumptions when he writes: "Inasmuch as there may be several propositions contrary to a given proposition, a negative proposition interpreted to mean "an opposite, or, a contrary, of p is to be regarded as an ambiguous description." (Demos, p. 192).

It is revealing to note here that there is a clear movement towards the softening or mitigation of the relation of opposition. We can subsequently include a multiplicity of possible contrarities, such as color, or any case of ascription of degrees to qualities as we have seen. This also happens in the radicalization of the consequences of Demos' interpretation in the case of denial of any proposition and the explicitation of many assumptions of trivial propositions. We do not only have one proposition contrary to the "my shirt is white." Even at the time of writing his article, Demos undertook a remarkable shift in his lexicon. When before Demos spoke only of "not-p" being interpreted as "the opposite, or contrary of p", we now have "not-p" as "an opposite, or contrary of p". The indefinite article "a/an" is important here to accommodate this explosion of contrarities on a system of propositions.

Demos' interpretation only makes sense if it includes every proposition p in a system of propositions, to account for the numerous contrary propositions in relation to p. Not all propositions are opposite to a negated proposition p, but just the ones that belong to the same system, where p also is. That is, the thesis that negative propositions should be analyzed in terms of an opposition forces us to accept a certain radical semantic holism. If every proposition must admit its negation and if every denied proposition brings its opposition to the discussion, and if we accept that the possible or legitimate opposition of a proposition bring us the idea of a system of propositions where these oppositions are embedded, we find that all propositions. This step seems to be a consequence of Demos argumentation, even if the author does not seem to want it or even see it coming.

Interestingly, it is precisely the possibility of the denial of any legitimate proposition that makes Wittgenstein think of systems of propositions such as that of colors. As is clear from the discussions *Die Welt ist rot* and *Anti-Husserl* of the WWK, if I understand that "*a* is green" I must be able to understand also "*a* is not

green", and this implies or brings the whole system of colors with the exclusion by contrariety, "it is not green, so it may be red, yellow, blue ... ". If *a* is a point of the visual field, it must have a color, even if I do not know which color that is. This combination of the possibility of negation with the need to make all propositions belong to systems becomes clear in the following argument organized in a section entitled *Liegt jeder in einem Satz System?* in the WWK. This passage appears immediately after a commentary by Waissman which states that the possibility of the negation presupposes, or brings with it, a logical space of possibilities. In this period, Wittgenstein suggests that this collapses and fragments itself into the notion of various propositional systems:

"Es kommt darauf hinaus, ob das Zeichen "a" ein notwendiges Zeichen ist. Wenn es bloß den Satz " $\varphi$ a" gäbe, aber nicht " $\varphi$ b", so wäre die Erwähnung von "a" überflüssig. Es würde genügen, " $\varphi$ " allein zu schreiben. Der Satz wäre also nicht zusammengesetzt. Das Wesentliche am Satz ist aber, dass er ein Bild ist und Zusammensetzung hat. Soll also " $\varphi$ a" ein Satz sein, so muss es auch eine Satz " $\varphi$ b" geben, d.h. Die Argumente von " $\varphi$ ()" Bilden ein System. (...) Setzt aber " $\varphi$ a" auch " $\psi$ a" voraus? Jawohl. Denn dieselbe Überlegung lehrt: Gäbe es zu "a" nur eine einzige Funktion " $\varphi$ ", so wäre sie überflüssige; man könnte sie weglassen. Das Satzzeichen wäre also einfach und nicht zusammengesetzt. Er Bildet nicht ab. Zeichen, die entbehrlich sind, haben keine Bedeutung. Überflüssige Zeichen bezeichnen nichts." Ergebnis: So viele Konstanten in einem Satz vorkommen, in so viele Dimensionen ist ein Satz variierbar. So viele Dimensionen hat der Raum, in dem der Satz liegt. Der Satz durchgreift den ganzen logischen Raum. Sonst wäre die Negation nicht verständlich." (WWK, p.90-91)

The arguments used here to a certain extent pressupose the metaphysic of symbolism in the *Tractatus* and its peculiar reading of Occam's lema. Cf. 3.328 e 5.47321. Waissman points this out when summarising Wittgenstein's theses at the end of WWK:

"Und so verhält es sich mit jedem sinnvoll gebrauchten Zeichen. Kommt das Zeichen "a" in dem Satz "fa" vor, so setzt das schon andere Sätze dieser Art, z.B. Den Satz "fb" voraus. Denn wenn es bloß den Sachverhalt fa gäbe, aber nicht den Sachverhalt fb, so wäre die Erwähnung von "a" überflüssige und überflüssige Zeichen bedeuten nichts. Dies zeigt, das jeder Satz in einem System von Sätzen liegt." (WWK, p.261).22

Only by including p in a system of propositions can we cope with this explosion of contrarieties by understanding the negation of a proposition about

<sup>22</sup> We will come back to these discussions about what I call here holism in understanding the tractarian period, including the intermediate phase in the last chapter of this study. There the exceptical protagonism will be given to the passage 3.42 and not to the Occam motto; because I believe that the holism advocated is found more naturally in 3.42. But, although it is possible to take a holistic approach to the Occam motto, we need more interpretive steps because it is not straightforward.

colors or length, for example. The price to pay is that every negative proposition will generate an indefinite explosion of contrarieties - not only in the case of ascriptions of degrees but in all propositions somehow. As we have seen, we have to think a kind of system of propositions about "ownership" as comprising of all the possible individuals of a domain in order to go from "This is not mine" to "this belongs to Mary or John or Louis..." This premise comes automatically if we are in a propositional system a la intermediary Wittgenstein, which seems to be anticipated by Demos, even if he was apparently unaware of the consequences of his interpretation. In the example given above about John, we have to think about a kind of system of propositions covering John's actions. We need, for example, to assume that John is always doing something, to make "John does not go jogging" refer indirectly to "John swims", "John reads" etc., i.e., all such propositions are contrary to "John goes jogging." As in the classic case of colors, these propositions cannot be true together, but they are false together. And while the elementary propositions are meaningful, the junction of them is not. While the statements "John reads" and "John swims" are by themselves meaningful, the conjunction "John reads and swims" is not. Similarly, while both statements "point a is blue" and "point a is red" are by themselves meaningful, the conjunction of both is not.

Instead of making the paradigm of colors enter into the paradigm of truthfunctional analysis of propositions in the *Tractatus* as Wittgenstein tried in 6.3751, in his article, Demos makes every proposition to fit into the paradigm of exclusions by contrariety similar to the interpretation of 1929 for the proposition of color ascription. And he does this without perhaps even being aware of this consequence. Recapitulating, Demos does not seem to notice that he makes every proposition belonging to a propositional system as a result of his approach to negative propositions. If, indeed, we accept the intuitive tractarian principle that proposition should always indirectly refer to another one, whether through exclusions or implications, even if we do not know for sure how many or indeed what these other propositions are. This seems to be consistent with Demos' theses when he states: "In this respect, we may characterize negative assertion as always positive in reference but never positive in content." (Demos, p. 193) Without doubt, what makes Demos' interpretation attractive, despite (or precisely because of) this explosion of candidates through the opposition to the content denied by a negative proposition is that we need not to compose our ontology with negative facts or objects. This is because negative propositions would ultimately refer to positive propositions which, in turn, would refer to facts: in this light negative propositions are revealed to always be positive. The negative proposition could finally be taken as a kind of second-order proposition, i.e., for describing a proposition, which is itself a description. The negative proposition does not describe negative facts, but it indirectly describes positive facts, via opposition to positive propositions. A negative proposition can indeed have cognitive value. It presents an opposite to the truth content. The problem here is that this opposite content is always ambiguous and potentially infinite depending on the reference system in which we are operating.

Besides the demand for a perspicuous analysis that finally reveals the hidden deep logical form of negative propositions, the interesting corollary of Demos' approach, which resembles Wittgenstein's problems by also demanding a more perspicuous analysis of the discourse, may be the understanding that every proposition would be immersed in a system with other propositions that exclude or imply it. While perhaps unintentional, Demos' denial shows a facet of this system: every empirical proposition can have multiple oppositions. As pointed out in 3.42 of the *Tractatus*, it is also the possibility of negation, among other things, that brings up the "grammar" or "logical space" altogether. If we think about this in connection with an investigation of a general form of propositions, we could modalize the modality already present here and again affirm that every empirical proposition must be able to have multiple oppositions, capable of being in a system wherein propositions would be logically dependent on each other.

These problems lead us to analyze the actual role of the truth table in the *Tractatus*, often considered peripheral. I argue that the *Tractatus*'s notation has more to tell us about tractarian successes, and especially about tractarian failure than we expect. I hold that the truth table is underestimated as an exegetical key. My hypothesis is that the *Tractatus* fails where the truth table fails. Besides proving to be a notation that does not prevent nonsense, it shows us that the very idea of logic being neutral to what is happening in the world, neutral to the

conceptual connections or to systems of exclusions and affinities of perception, entirely syntactic and combinatorial, should be revised. Rather, logic should look at the world and at its (empirical?) systems.