Pontifícia Universidade Católica
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A Regularized Benders Decomposition with Multiple Master Problems to Solve the Hydrothermal Generation Expansion Problem

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Engenharia Elétrica, do Departamento de Engenharia Elétrica da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

Advisor: Prof. Alexandre Street

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Abstract

Soares, Alessandro; Street, Alexandre (Advisor). **A Regularized Benders Decomposition with Multiple Master Problems to Solve the Hydrothermal Generation Expansion Problem**. Rio de Janeiro, 2021. 42p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

This paper exploits the decomposition structure of the hydrothermal generation expansion planning problem with an integrated modified Benders Decomposition and Progressive Hedging approach. We consider a detailed representation of hourly chronological short-term constraints based on typical days per month and year. Also, we represent the multistage stochastic nature of the hydrothermal operational policy through an optimized linear decision rule, thereby ensuring investment decisions compatible with a nonanticipative implementable operational policy. To solve the resulting large-scale optimization problem, we propose an improved Benders Decomposition method with multiple instances of the master problem, each of which strengthened by primal cuts and new Benders cuts generated by each master's trial solution. Additionally, our new approach allows using Progressive Hedging penalization terms for regularization purposes. We show that our method is 60% faster than the traditional ones and also that the consideration of a nonanticipative operational policy can save, on average, 8.27% of the total cost in out-of-sample tests.

Keywords

Generation expansion planning; Benders Decomposition; Hydrothermal power system; Progressive hedging; Linear decision rules.

Resumo

Soares, Alessandro; Street, Alexandre. **Uma Decomposição de Benders com Múltiplos Problemas Masters Regularizada para Resolver o Problema da Expansão da Geração Hidrotérmica**. Rio de Janeiro, 2021. 42p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Este trabalho explora a estrutura de decomposição de um problema de planejamento da expansão da geração hidrotérmica, utilizando uma integração entre uma Decomposição de Benders modificada e um Progressive Hedging. Consideramos uma representação detalhada das restrições cronológicas de curto prazo, com resolução horária, baseando-se em dias típicos para cada etapa. Além disso, representamos a natureza estocástica de uma política operacional hidrotérmica multiestágio por meio de uma Regra de Decisão Linear otimizada, garantindo decisões de investimento compatíveis com uma política operacional não antecipativa. Para resolver este problema de otimização em grande escala, propomos um método de decomposição de Benders aprimorado com várias instâncias do problema mestre, onde cada uma delas é reforçada por cortes primários além dos cortes de Benders gerados a cada candidato a solução do mestre. Nossa nova abordagem permite o uso de termos de penalização de Progressive Hedging para fins de regularização. Mostramos que o algoritmo proposto é 60 % mais rápido que os tradicionais e que a consideração de uma política operacional não antecipativa pode economizar, em média, 8.27 % do custo total em testes fora da amostra.

Palavras-chave

Planejamento da expansão da geração; Decomposição de Benders; Sistema de potência hidrotérmicos; Progressive hedging; Regras de decisão linear.

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BD – Benders decomposition

BDMM – Benders decomposition with multiple masters

DD – Dual decomposition

DE – Deterministic equivalent

GEP – Generation expansion planning

INV – Investment

LDR – Linear decision rule

MILP – Mixed-integer linear programming NLP – Non-linear programming

PH – Progressive hedging

r-BDMM – Regularized Benders Decomposition with Multiple Masters

TBD – Traditional benders decomposition

Nomenclature

Constants

$\bar{x}^k$ Average of the first-stage decision from previous iteration.

$\hat{x}_s^k$ Candidate solution at scenario s in the iteration k.

$\mathcal{I}$ Cost of first-stage variables.

π_s^k Dual variable associated to the constraint that couples the investment and operation problem, at iteration j.

ρ Progressive hedging static parameter.

A Matrix allocating generators to buses.

$a_t(\omega)$ Vector of hydro inflows at stage t and scenario ω.

B DC power-flow matrix.

c Cost for all operational variables.

c_v Vector units' operational costs.

$D_{t,d,h}(\omega)$ Vector of demand per bus at stage t, typical day d, hour h and scenario ω.

G Matrix representing the maximum capacity of the plants.

$H_{t,d}$ Number of hours in typical day d, stage (month) t.

I Vector of investment costs of generation units.

L Matrix selecting hydro units.

P Coefficient matrix modeling the productivity of hydro units.

p_ω Probability of scenario ω.

R Incidence matrix allocating hydros outflow to reservoirs according to rivers topology.

W, T Matrices modeling first- and second-stage coupling constraints.

w_s^k Progressive hedging dynamic parameter.

Index

ω Index representing the operative scenario ω of the operation problem (subproblem).

s Index representing an operative scenario selected to build a primal cut and identifying a master problem realization, its trial solution, and optimal value.

Sets and Indices

$\mathcal{X}$ Set of feasible investment decisions.

$\aleph_{t,d}$ Set of feasible hourly operating points modeling ramping constraints and transmission lines maximum flow capacity at stage t and typical day d.

Λ_ω Feasibility set representing operational constraints.

Ω Set containing the scenarios ω.

$\tilde{V}_t$ Set of hydro operative constraints at stage t.

Decision Variables

α_ω Approximation of the real operational cost at scenario ω.

π_ω Vector of dual variables associated with operation-investment coupling constraints used to build the Benders cuts.

$\theta_{t,d,h}(\omega)$ Vector of buses' phase angle at stage t, typical day d, hour h and scenario ω.

$g_{t,d,h}(\omega)$ Generation vector at stage t, typical day d, hour h, and scenario ω.

$Q(x)$ Real operational cost function.

$s_t(\omega)$ Vector of water spilled during stage t and scenario ω.

$u_t(\omega)$ Vector of amount of water used to generate electricity during stage t and scenario ω.

$v_0(\omega)$ Vector of initial storage condition at stage t and scenario ω.

$v_t(\omega)$ Vector of reservoirs storage at stage t and scenario ω.

x Vector of first-stage decision variables comprising the binary investment variables and the affine policy coefficients.

x^{INV} Binary vector of investment decisions (existing generators are modeled as fixed entries equal to 1).

$x_{t,0}^{LDR}$ Linear decision rule vector of linear coefficients per hydro unit for stage t.

x_t^{LDR} Linear decision rule vector of angular coefficients for stage t.

$y(\omega)$ Vector of decision variable representing all operative variables at scenario ω.

To become good at anything you have to know how to apply basic principles. To become great at it, you have to know when to violate those principles

Gary Kasparov, *Deep Thinking: Where Machine Intelligence Ends and Human Creativity Begins.*

1
Introduction

Generation expansion planning (GEP) models aim to minimize the total cost of investment and operation through a long-term horizon. They bring relevant insights for market agents and also provide planners and regulators with valuable information about long-term equilibrium generation portfolios under the absence of market power abuse [1, 2]. This is especially important in hydrothermal systems, where relevant metrics needed to induce efficient generation expansion rely on the assessment of the opportunity cost of water under a long-term investment and operational equilibrium [3]. Because of that, GEP models should take into account the main long- and short-term characteristics of the system and uncertainties that should enhance the description of opportunity costs of first-stage decisions [4, 5]. Furthermore, this class of models is usually non-convex due to the necessity of representing integer investment and operative decisions. Conventional approaches to solve these large-scale non-convex stochastic programs are: (i) non-linear programming (NLP) and mixed-integer linear programming (MILP) to solve the extensive form of the problem; and (ii) decomposition techniques (such as Benders decomposition). All of these approaches may be used together with approximations and assumptions to make the problem computationally tractable [6, 7].

A wide range of applications on GEP are found in the literature [8–20], each of which considering different aspects and system characteristics. In this work, we focus on hydrothermal power systems [13–16, 19, 20]. In this setting, the main challenge is to consider, within the expansion problem, an integrated and computationally efficient nonanticipative water-value assessments. Therefore, hydrothermal-based GEP largely relies on the co-optimization of investment decisions and long-term multistage stochastic dispatch policies. Also, there are several other applications with a similar structure, such as the maintenance optimization problem [21], where the algorithms and idea exploited in this work is also valid.

1.1
Energy resource planning models

In energy resource planning literature, [8] solves the investment and operation problems simultaneously. To reduce the computational burden and avoid decomposition, the authors adopt a linear relaxation of investment decisions and propose a clever time-clustering scheme to reduce the number of hours yet keeping a chronological representation of externalities. A clustering algorithm is also proposed in [22], where the author shows the strategy's effectiveness comparing clustered problems with the unclustered version. In [23] the authors proposes a novel optimization problem to minimize the approximation errors of the typical days. Time clustering schemes are commonly used in the literature as typical days or weeks. For instance, [9] uses typical days per month and year, and considers a deterministic model for long-term planning with a detailed representation of short-term constraints. In this setting, the problem can be solved without a decomposition approach. [13] proposes a hydrothermal-based GEP, using typical days to represent hourly constraints and scenarios to represent hydro inflows uncertainties. The authors use the progressive hedging (PH) technique to decompose the problem scenario-wise. Several other recent planning models in literature adopt simplifications to make the investment and operational problems computationally tractable [24, 25].

Notwithstanding, decomposition methods are largely applied in this subject. For instance, [10] proposes a decomposition approach where the master consists of a deterministic investment problem and the subproblem is a detailed short-term operational problem that produces feasibility cuts. [11] addresses uncertainties in the net load by a robust optimization approach, considering uncertainty in the hourly ramping, but without other detailed short-term constraints. [12] proposes a multiscale multistage stochastic model, addressing short-term constraints and uncertainties, and decomposes the problem with PH, where the problem is convex (with linear investment and commitment decisions), which guarantees optimal solutions but may not be as fast as BD. Table 1.1 summarizes the comparison between the proposed approach and the energy resource planning literature. In this table, symbols "✓" and "-" indicate whether a particular aspect is considered or not.

Table 1.1: Proposed approach compared to literature

Approach	**Representation of Uncertainties**	**Hourly constraints**	**Operational policy**	**Binary investment decision**	**Co-optimization of energy and reserves**	**Decomposition Technique**
Koltsaklis, N. E. et al. (2015) [9]	-	✓	-	✓	✓	-
Pina, A. et al. (2013) [10]	-	✓	-	✓	-	Feasibility cuts
Li, J. et al. (2018) [11]	✓^{A}	✓	-	✓	✓	Column-and-Constraint
Liu, Y. et al. (2018) [12]	✓	✓	-	-	-	Progressive Hedging
Thome, F. et al. (2019) [26]	✓	-	✓	✓	✓	Benders Decomposition
Proposed approach	✓	✓	✓^{B}	✓	✓	BDPH

^{A} The uncertainties are represented through robust optimization methods

^{B} The hydro operation policy is represented through linear decision rules

1.2 Decomposition structures

Deterministic planning models can be solved in a reasonable amount of time, even considering the co-optimization of investment and operational decisions. Uncertainty representation drastically increases the size of the problem, leading to intractability issues, especially when the number of scenarios is large. This is the case for most real problems. Notwithstanding, these issues can be especially worsened in the presence of time-coupling constraints requiring a multistage model to characterize the opportunity costs of operational resources such as water. Hence, decomposition approaches such as PH [27] and Benders Decomposition (BD) [28] are frequently used. PH algorithms guarantee convergence to the optimal solution when the problem is convex. However, since real expansion problems have binary investment variables, PH is usually used as a heuristic to obtain solutions [12, 29, 30]. BD techniques guarantee optimal solutions when the problem is convex. The BD approach was first proposed in the context of GEP by Campodonico et al. [31] and is used to solve lots of real problems [19, 20, 26, 32–35] since the optimal solution is guaranteed in a reasonable amount of time.

Guo et al. [36] used the PH to speed up other algorithms that converge even in the presence of binary variables, i.e., Dual Decomposition (DD) [37]. The authors used PH to generate initial Lagrangian dual variables to the Lagrangian relaxation subproblems in DD. Van Roy [38] proposed the cross decomposition, a primal-dual decomposition that integrates Lagrangian Relaxation and BD. Barnett et al. [39] applied PH in a branch and bound scheme to obtain convergence in nonconvex problems.

Crainic et al. [40] proposed the partial BD, where they add a subset of constraints and variables to the master problem. The algorithm we present in this work improves this idea considering multiple copies of the master problem, each of which accounting for primal cuts generated based on scenario information and new Benders' cuts obtained from each master's trial solution. Due to the multiplicity of trial solutions generated by the information of different scenarios, we use Progressive Hedging penalization terms to regularize and accelerate our method.

1.3 Nonanticipative hydrothermal dispatch

The solution of a long-term GEP problem applied to a hydrothermal power system requires the consideration of a multistage reservoirs' operational policy. The main reason for that is to avoid the threat of optimistically biasing the water opportunity-cost assessments based on an anticipative operational model [41, 42]. In other words, we need to consider in our GEP a decision rule (see [43]) that is as closer to an implementable (nonanticipative) policy as possible to avoid under investments due to artificially reduced operational costs (based on optimistic anticipative policies) that will not be implementable in practice. In this context, the customary two-stage approximation, in which given the investment decisions the system operation follows with perfect (anticipative) information of the uncertainty realizations (i.e., per scenario), is not valid as we demonstrate in our case study.

A nonanticipative operational policy is a rule defining decision variables of a given period t based on previously reveled information, i.e., without assuming access to the information of uncertainty factors after t. The linear decision rule (LDR) methodology defines an implementable nonanticipative policy based on an optimized linear combination of functions applied to the previously revealed uncertainty scenario. [44] first proposed the LDR for reservoir management, and this approach is gaining more and more attention in the literature (see [45–48] and [49]) and will be used in this work to propose a new stochastic hydrothermal GEP with multistage (nonanticipative) dispatch policy based on linear decision rules.

1.4 Contributions and work organization

The main contributions of this work are threefold:

- A new stochastic hydrothermal GEP model considering multistage (nonanticipative) dispatch policies based on linear decision rules. Out-of-sample tests based on real data demonstrate that the consideration of nonanticipativity has significant impacts on first-stage investment decisions and subsequent operation costs.
- An improved Benders Decomposition with multiple master problems (BDMM) method, each of which strengthened by primal cuts based on scenario information and new Benders' cuts generated by each master's trial solution.
- Leveraging the diversity of trial solutions our BDMM method provides, we combine the proposed BDMM with Progressive Hedging penalization terms for regularization purposes, thereby generating a novel regularized-Benders Decomposition with Multiple Masters (r-BDMM) method.

The remainder of this paper is organized as follows. In Section 2, the GEP problem is introduced and formulated. The decomposition algorithm is presented as the solution strategy to solve the problem in Section 3. Section 4 provides numerical results illustrating the performance of the proposed algorithm and an analysis of the anticipative policy in the GEP formulation. Finally, in Section 5 final remarks are drawn.

2
The generation expansion planning model

This section introduces the GEP problem formulation as a MILP optimization problem, which will be referred to as the Deterministic Equivalent (DE) problem. We assume a discrete and finite sample space $\Omega = \{1, .., \omega, ...\}$, in which each scenario ω is assumed to have a known conditional probability p_ω. For the operational variables, we use a LDR to consider a monthly nonanticipative multistage operational policy under uncertainty of inflows [49,50] for the reservoirs.

The system operation constraints and costs are computed within an hourly resolution based on monthly hydro generation targets dictated by the LDR. Thus, based on monthly inflow scenarios and subsequent hydro generation amounts given by the LDR, the uncertainties of intermittent renewable sources are used to define the operation of typical (representative) days within an hourly granularity. In this sense, we approximate the daily operation within each stage (months) by weekdays, weekends, and holidays multiplied by their number of hours per month. Hourly scenarios for renewables are conditionally generated based on the inflows scenarios to present correlations.

Mathematically, for each month (stage) $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ we have: i) strategic stagewise decisions, such as $u_t(\omega), u_t(\omega), v_t(\omega)$, defining the total amount of water used to generate electricity and spilled during the stage, and the storage level at the end of the stage; ii) short-term operational decisions, such as $g_{t,d,h}(\omega), \theta_{t,d,h}(\omega)$, defining the hourly generation and the bus angles for each typical day $d \in \mathcal{D}$ and hour $h \in \mathcal{H}$ of the stage t; iii) strategic to short-term linking constraints, $Pu_t(\omega) - L\sum_{d,h} H_{t,d} g_{t,d,h}(\omega) = 0$; and iv) x^{INV} and $(x_{t,0}^{LDR}, x_t^{LDR})$ representing the investment decisions and LDR coefficients (first-stage decision vectors).

Figure 2.1 summarizes the proposed scheme. The idea is that we will have a first-stage solution (which is the investment and hydro operating policy decisions), represented as the first white circle, and then evaluate the operation of this solution through all of the scenarios. For each scenario, we have T months (represented as the solid black circles), and for each month, we will have hydro balance constraints. Inside each of the months, we solve the hourly operational problems for each of the typical days (a representative day of the month with 24 consecutive hours). The relation between the monthly operation and the hourly operation is made by the coupling constraint (highlighted in the figure), which will restrict the model to use, in the hourly operation, the hydro energy that was decided in the monthly hydro balance.

The proposed model is detailed as follows in expressions (2-1)-(2-10):

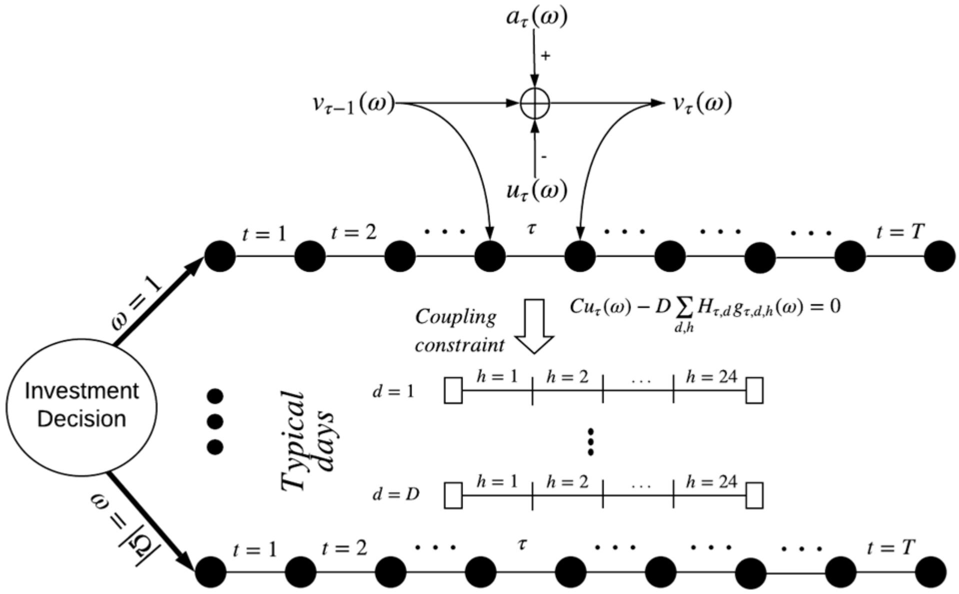

Figure 2.1: Typical days within each stage

$$\min \quad I^T x^{INV} + \sum_{t,d,h,\omega} p_\omega c_v^T H_{t,d} g_{t,d,h}(\omega) \tag{2-1}$$

$$s.t \;\; x^{INV} \in \mathcal{X} \tag{2-2}$$

$$v_t(\omega) + R(u_t(\omega) + s_t(\omega)) - v_{t-1}(\omega) = a_t(\omega) \;\; \forall t, \omega \tag{2-3}$$

$$v_T(\omega) - v_0(\omega) \geq 0 \;\; \forall t, \omega \tag{2-4}$$

$$Pu_t(\omega) - L\sum_{d,h} H_{t,d} g_{t,d,h}(\omega) = 0 \;\; \forall t, \omega \tag{2-5}$$

$$g_{t,d,h}(\omega) - Gx^{INV} \leq 0 \;\; \forall t, d, h, \omega \tag{2-6}$$

$$Ag_{t,d,h}(\omega) + B\theta_{t,d,h}(\omega) = D_{t,d,h}(\omega) \;\; \forall t, d, h, \omega \tag{2-7}$$

$$u_t(\omega) - (x_{t,0}^{LDR} + x_t^{LDR} a_t(\omega)) = 0 \;\; \forall t, d, h, \omega \tag{2-8}$$

$$\{g_{t,d,h}(\omega), \theta_{t,d,h}(\omega)\}_h \in \aleph_{t,d} \;\; \forall t, d, \omega \tag{2-9}$$

$$\{v_t(\omega), u_t(\omega), s_t(\omega)\} \in \tilde{V}_t \;\; \forall t, \omega. \tag{2-10}$$

We omit the sets in which indexes range for the sake of conciseness. The objective function has two parts, the investment cost and the present value of the expected operational costs. Constraint (2-2) refers to investment constraints, such as capacity target, energy policies, and limits for the LDR coefficients. Constraint (2-7) refers to the load balance. Constraint (2-3) refers to the hydro balance equation. Constraint (2-5) refers to the hourly hydro generation modeling, where we restrict the model to use in the hourly operation, the energy that was decided in the monthly decision. Similar approaches are widely adopted in long-term studies (see relevant publications in the last five years [5, 42, 51, 52]). Also, the matrix P is representing an average production factor for the hydro plant, so we are simplifying the hydro operation assuming that the production factor does not vary with the storage. Constraint (2-6) represents the relation between the investment variable and the operation maximum capacity constraint. Constraint (2-4) represents the hydro operational strategy used to prevents the end-of-horizon effect. This constraint aims to obligate the model to use only the water that arrives along the years comprised in the study horizon. The constraint (2-8) represents the linear decision rule. In this expression, $x_{t,0}^{LDR}$ and x_t^{LDR} are decision variables representing the vector of linear and matrix of angular coefficients defining the LDR, respectively. This strategy could be "improved" increasing the number of coefficients defining the policy, for example, we could add a constraint such as $u_t(\omega) - (x_{t,0}^{LDR} + x_{t,1}^{LDR} a_t(\omega) + x_{t,2}^{LDR} a_{t-1}(\omega))$, but the improvement in this LDR is not being discussed in this work. Finally, constraint (2-9) represents other operational constraints, such as ramping constraints and limits on angles modeling transmission lines maximum flow capacity. Constraint (2-10) represents hydros constraints, such as minimum and maximum values for water storage, maximum generation and spillage amounts.

3
Solution Strategy

Problem (2-1)-(2-10) provides optimal investment decisions considering a multistage stochastic operational policy. However, because we are using a LDR parametrization, the problem can be formulated as a large-scale multiperiod two-stage stochastic optimization problem, where the LDR is estimated as part of first-stage variables. To efficiently solve this problem, in the next sections, we present a variant of the Benders decomposition approach. For the sake of simplicity and didactic purposes, we re-write problem (2-1)-(2-10) in a compact formulation as follows:

$$\min \quad \mathcal{I}^T x + \sum_{\omega \in \Omega} p_\omega c^T y_\omega \tag{3-1}$$

$$s.t \quad W y_\omega - T x \leq h_\omega \quad \forall \omega \in \Omega \tag{3-2}$$

$$x \in \mathcal{X}, \; y_\omega \in \Lambda_\omega \quad \forall \omega \in \Omega, \tag{3-3}$$

where, y_ω is the vector comprising the operational variables ($g_{t,d,h}(\omega)$, $\theta_{t,d,h}(\omega)$ and $u_{t,d,h}(\omega)$); Λ_w is a set containing the feasibility constraints (2-2)-(2-10); x is the vector comprising the first-stage variables (x^{INV}, $x^{LDR}_{t,0}$, and x^{LDR}_t); and the constraint (3-2) couples the first- and second-stage variables.

3.1
Traditional Benders decomposition

Now we present the traditional benders decomposition (TBD) applied to solve problem (3-1)-(3-3). We decompose the problem into master and subproblem problems, where the master will be solved as a MILP and the subproblems as a LP. The master selects the first-stage variables (vector x comprising investment and LDR coefficients), while the subproblem evaluates the recourse function, $Q(x)$, by solving the operative problem given x.

So, we start defining the recourse function (the expected cost of the second-stage) for a given point x as follows:

$$Q(x) = \sum_{\omega \in \Omega} p_\omega q_\omega(x). \tag{3-4}$$

The evaluation of $Q(x)$ can be decomposed per scenario ω and solved in parallel. For each scenario, $q_\omega(x)$ represents the second-stage cost and can be calculated as follows:

$$q_\omega(x) = \min \; c^T y_\omega \tag{3-5}$$

$$s.t \; W y_\omega \leq T x + h_\omega \; :\xrightarrow{\text{dual}} \; \pi_\omega \tag{3-6}$$

$$y_\omega \in \Lambda_\omega. \tag{3-7}$$

Where, $\pi_\omega = (\text{dual})^T T$ is the subgradient of q_ω with respect to x. Then, for a given iteration k of the algorithm, we run the master problem to obtain a new trial solution, $\hat{x}^k$, and a lower bound, LB_{TBD}. The master is a relaxation of problem (2-1)-(2-10) because the recourse function is approximated from below by supporting planes. These planes are also called Benders' cuts and are obtained in previous iterations of the method. Using the multi-cut method [53], the master problem of a TBD returns a new trial solution and a lower bound as follows:

$$z^k, \hat{x}^k \leftarrow \min \quad \mathcal{I}^T x + \sum_{\omega \in \Omega} p_\omega \alpha_\omega \tag{3-8}$$

$$s.t \ \alpha_\omega \geq q_\omega(\hat{x}^j) + \pi_\omega^j (x - \hat{x}^j) \ \forall \omega \in \Omega, j \in [k-1] \tag{3-9}$$

$$\alpha_\omega \geq 0 \ \forall \omega \in \Omega, \ x \in \mathcal{X}, \tag{3-10}$$

where, α_ω represents the best approximation of the epigraph of q_ω until iteration k. Furthermore, hereinafter, we adopt the notation in which $[k-1] = \{1, ..., k-1\}$ and $[0] = \emptyset$. Then, by solving (3-5)–(3-7) for the newly obtained trial solution $\hat{x}^k$, a new Benders' cut can be generated to feed the next iteration master problem. Additionally, a lower and upper bound can be assessed to check the optimality GAP of the current solution as follows:

$$LB_{\text{TBD}} = z^k \tag{3-11}$$

$$UB_{\text{TBD}} = \mathcal{I}^T \hat{x}^k + Q(\hat{x}^k). \tag{3-12}$$

If the $GAP = UB_{\text{TBD}} - LB_{\text{TBD}} \leq \epsilon$, then the algorithm stops and x^k is returned as the optimal solution. Otherwise, the k is incremented, and the master problem is called once again.

3.2 The Benders Decomposition with multiple master problems

In this section, we present our proposed BDMM method. First, we make $S = |\mathcal{S}|$ copies of the master problem (3-8)-(3-10), i.e., in each iteration of our BDMM method we define one master problem for each $s \in \mathcal{S}$. Second, each master problem, now indexed by s, instead of considering only Benders' cuts, also considers the second-stage primal constraints associated with a given scenario s (this strategy may significantly increase the size of the problem, but as we show in Section 4, this is worth it). It is worth highlighting that $\mathcal{S}$ could be generated based on different clustering strategies. Hence, we will develop our method for a general set $\mathcal{S}$. However, in this paper, we will use $\mathcal{S} = \Omega$. Then, we end up with $S = |\Omega|$ master problems, each of which differing from each other by a (stochastic) primal cut related to a given scenario. Furthermore, due to the multiplicity of master problems, multiple (S) trial solutions are also generated. Therefore, for each one of the S newly generated points $\{x_s^k\}_{s \in \mathcal{S}}$, the multi-cut approach generates $|\Omega|$ new cuts, each of which approximating one function in $\{q_\omega(\cdot)\}_{\omega \in \Omega}$. Consequently, in each master problem s of a given iteration k, a total of $S \cdot |\Omega|$ Benders' cuts are considered for each previous iterations. Thus, the s–master problem and the associated lower bound and trial solution are defined as follows:

$$z_s^k, \hat{x}_s^k \leftarrow \min \ \mathcal{I}^T x + \sum_{\omega} p_\omega \alpha_\omega \tag{3-13}$$

$s.t$

$$\alpha_\omega \geq q_\omega(\hat{x}_{s'}^j) + (\pi_{\omega,s'}^j)^T(x - \hat{x}_{s'}^j)$$
$$\forall \omega \in \Omega, s' \in S, j \in [k-1] \tag{3-14}$$
$$\alpha_s \geq c^T y_s \tag{3-15}$$
$$W y_s - T x \leq h_s \tag{3-16}$$
$$y_s \in \Lambda_s \tag{3-17}$$
$$x \in \mathcal{X}. \tag{3-18}$$

Note that traditional Benders' cuts are built based on local-dual information of the recourse problem, thereby providing the master problem with loose linear approximations of the recourse function. The primal cut defined by (3-15)–(3-17), on the other hand, provides a much richer polyhedral information about the second stage to the master problem. This improvement proposed in this work is inspired by the success of column-and-constraint-generation algorithms applied to robust optimization, where few primal cuts are actually needed to support the optimal decisions (see [54]). Furthermore, the number of Benders' cuts in (3-14) is S times greater than in the TBD method, which significantly improves the description of the recourse function.

After solving the S instances of the master problem, we have $\{z_s^k, \hat{x}_s^k\}_{s\in\mathcal{S}}$. Because all values in $\{z_s^k\}_{s\in\mathcal{S}}$ are valid lower bounds for the problem, an improved Benders' lower bound can be calculated based on the maximum among all values, i.e.,

$$LB_{\text{BDMM}} = \max_{s\in\mathcal{S}}\{z_s^k\}. \tag{3-19}$$

A similar approach can be used to improve the upper bound. By evaluating the recourse function on each $\hat{x}_s^k$, we get S new candidates for upper bounds, $\{\mathcal{I}^T\hat{x}_s^k + Q(\hat{x}_s^k)\}_{s\in\mathcal{S}}$. Thus, we can select the lowest upper bound, i.e., we define

$$UB_{\text{BDMM}} = \min_{s\in\mathcal{S}}\{\mathcal{I}^T\hat{x}_s^k + Q(\hat{x}_s^k)\}, \tag{3-20}$$

and store the solution associated with the best upper bound, $\hat{x}^k(s^*)$, as the best trial solution at iteration k. A comparison with the best solution found so far is also advisable to keep the global best solution at hand.

Finally, note that although a lower and upper bound comparison between the two methods, TBD and the proposed BDMM, is not directly possible, because the methods should follow different paths. However, it is clear that the BDMM provides tighter approximations in every master problem since the first iteration. The improvement in lower bounds comes at the cost of a higher computational effort. The tradeoff between improving the lower bound assessment and the additional computational effort will be depicted in our Case Study Section. Leveraging the diversity of trial solutions our BDMM method provides, in the next section, we propose a new regularization scheme based on PH penalization terms.

3.3 Accelerating convergence of the Benders Decomposition with multiple masters

Now we present a new regularization scheme based on the PH method to accelerate our BDMM. To do that, we add the PH penalty terms in the master problem formulation (3-13)-(3-18). So, we rewrite the master problem (3-13)-(3-18) as problem (3-21)-(3-22), adding the terms related to the augmented Lagrangian relaxation following the PH approach (see [55]).

$$\hat{x}_s^k \leftarrow$$

$$\min \quad \mathcal{I}^T x + \sum_{\omega} p_\omega \alpha_\omega + \frac{\rho}{2}\|x - \bar{x}^k\|^2 + w_s^k(x - \bar{x}_k) \tag{3-21}$$

$$s.t \quad \text{Constraints (3-14)–(3-18).} \tag{3-22}$$

Then, after solving the S master problems, following the BDMM approach, w_s^k is updated following the sub-gradient method, i.e.,

$$w_s^{k+1} = w_s^k + \rho(\hat{x}_s^k - \bar{x}^k) \tag{3-23}$$

where $\hat{x}_s^k$ is the solution of the problem (3-21)-(3-22) from the iteration k and scenario s, and $\bar{x}^k$ is the average of all the S trial solutions obtained with (3-21)-(3-22) at iteration k.

To obtain a lower bound however, we have to solve the following modified version of the problem, which considers only the simple Lagrangian relaxation (without the quadratic terms):

$$\zeta_s^k \leftarrow \min \quad \mathcal{I}^T x + \sum_{\omega} p_\omega \alpha_\omega + w_s^k(x - \bar{x}_k) \tag{3-24}$$

$$s.t \quad \text{Constraints (3-14)–(3-18).} \tag{3-25}$$

Thus, the lower bound can be calculated according to the following Theorem:

Theorem 1. *Problem* (3-1)-(3-3) *admits the following lower bound:*

$$LB_{\text{r-BDMM}} = \sum_{s=1}^{S} p_s \zeta_s^k. \tag{3-26}$$

Although slightly different, the proof to **Theorem 1** goes very much like that provided in [55]. Therefore, for the sake of conciseness, we omit the proof here. Finally, the upper bound remains unchanged and follows expression (3-20).

We summarize the proposed r-BDMM in the following algorithm:

Algorithm 1 The proposed r-BDMM method

Initialization ($\rho \leftarrow$ **input**)
  $k \leftarrow 0, GAP^k \leftarrow +\infty, w_s^k \leftarrow 0 \;\; \forall s \in \mathcal{S}$
while $GAP^k > \epsilon$ **do**
  $k \leftarrow k+1$
  for each $s \in \mathcal{S}$ (computed in parallel) **do**
    Solve master problem (3-21)–(3-22) and store x_s^k
    for each $\omega \in \Omega$: compute $q_w(x_s^k)$ and store $\pi_{\omega,s}^k$
    Solve problem (3-24)–(3-25) and store ζ_s^k
  end for
  Compute:
    $\bar{x}^k \leftarrow \sum_{s\in\mathcal{S}} p_s \hat{x}_s^k$
    $LB^k_{\text{r-BDMM}} \leftarrow \sum_{s=1}^{S} p_s \zeta_s^k$
    $UB^k_{\text{r-BDMM}} \leftarrow \min_{s\in\mathcal{S}} \{\mathcal{I}^T \hat{x}_s^k + Q(\hat{x}_s^k)\}$
    $GAP^k \leftarrow UB^k_{\text{r-BDMM}} - LB^k_{\text{r-BDMM}}$
    $w_s^{k+1} \leftarrow w_s^k + \rho(\hat{x}_s^k - \bar{x}^k)$
end while
Return solution with the lowest UB so far

The figure 3.1 illustrates the proposed scheme, in which, for each iteration, the master problems and the real operating problems are solved in parallel (we are hiding the problem (3-24)-(3-25) for the sake of simplicity since it does not affect the analysis of the parallelization). The parallelization scheme is synchronized after computing the Benders cuts for each master problem.

Let us assume that we are solving the problem with S scenarios. So, without any parallelization, we would have to solve S master problems. For each of them, the real operating problem is calculated by solving S deterministic operating problems (3-4), resulting in S^2 problems.

Considering the parallelization scheme, let us assume that we also have S processors available. In this case, we would solve, for each processor, one master problem and S deterministic operating problems. Moreover, if S^2 processor are available we can solve all the operation problems in parallel too.

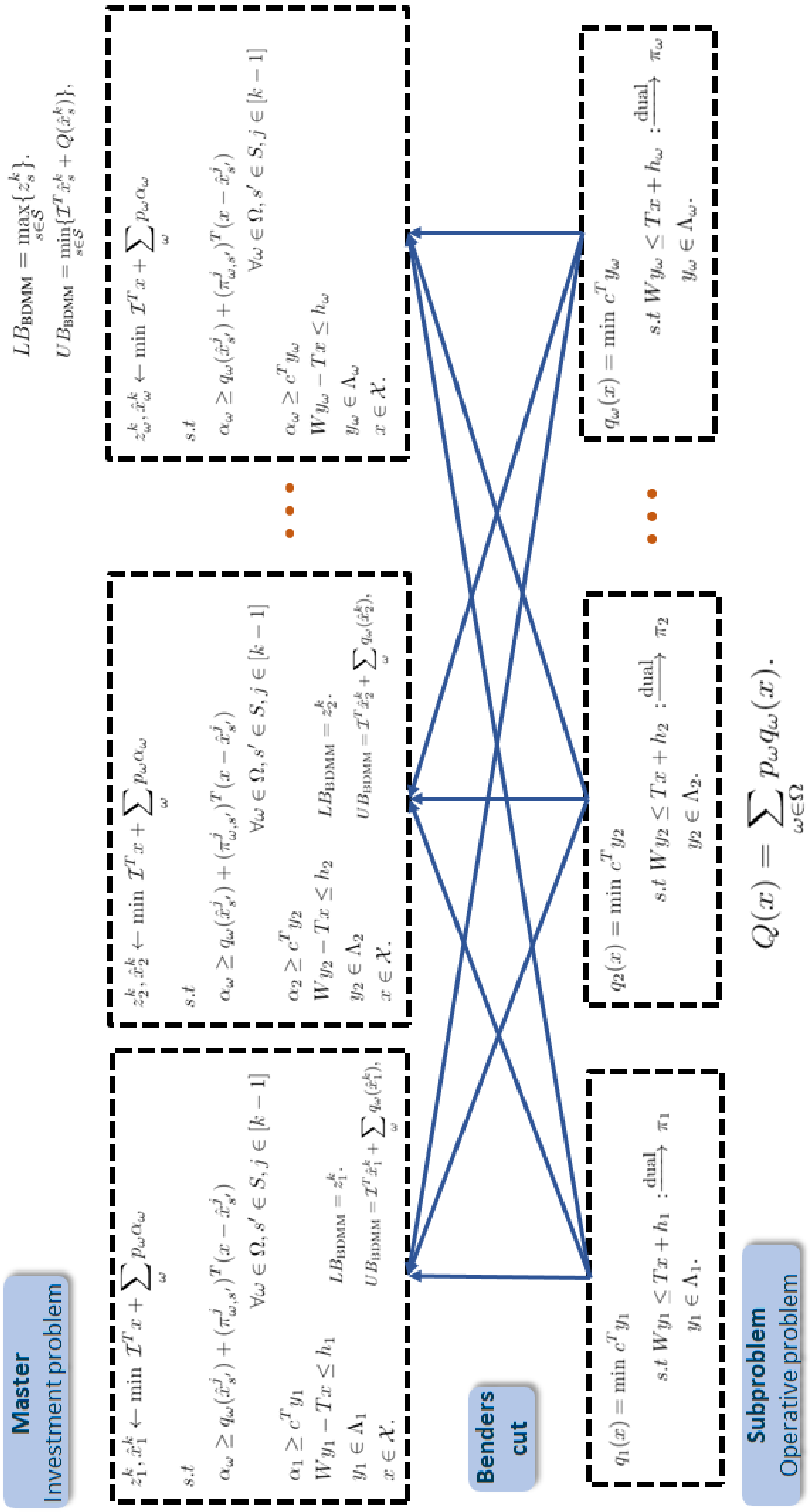

Figure 3.1: Summary of the algorithm and the parallelization scheme

4
Case study - The Brazilian power system

This section will study the proposed r-BDMM method to solve the GEP for the Brazilian power system. The Brazilian power system is interconnected by a transmission network comprising 50 transmission lines connecting subsystems. The system has 550 thermal plants, 150 renewable plants, 200 hydro plants, 10 batteries, 35 buses, and a total of 100 projects. We consider monthly stages (time steps), with three typical days per stage (week, weekend, and critical days). For each hydro unit, an affine policy with 24 coefficients is considered - thus, we have 4,900 first-stage variables per year to solve. The database configuration is based on [56], where the main assumptions are:

- Starting point: PDE 2026 final system configuration (see in [57])
- We analyze a target year where the demand is considered to be twice the demand of 2017, which amounts to 166 GW/1200 TWh (peak/annual energy)
- The investment cost of the projects is annualized to be comparable with the operational cost
- No hydropower project candidates other than those included in the PDE 2026 expansion plan

We compare the proposed method r-BDMM against the TBD, the unregularized BDMM method, and the deterministic equivalent (DE) formulation (3-1)-(3-3) solved directly through a MILP algorithm. We compare the algorithm in terms of number of iterations and computational time. The analyses include 10 instances considering 5, 10, ..., 50 scenarios in Ω representing uncertainties in renewable generation, hydro inflows, and demand. Furthermore, we also analyze the effects of the nonanticipativity constraints in the expansion planning.

We used the following relevant parameters for the algorithm: GAP tolerance of 0.1%; the maximum number of Benders' iterations equal to 200; 12 stages representing months; and three typical days (weekdays, weekend days, and a critical day) for each stage. Because the proposed method significantly benefits from parallelism, we used the same computational resources to compare all methods. We used the Xpress solver (FICO, optimizer version 34.01) and an Amazon EC2 c5.12xlarge computer (48 processors and 96 GB of RAM), for the LP problems we used the barrier algorithm.

It is relevant to highlight that vector x comprises very different components, with different images and different weights in the original objective function (3-1). While investment decisions are binary and already appear in the original objective function weighted by investment costs, LDR coefficients are real numbers and do not participate in the original objective function. Therefore, in our implementation, we modified the penalization term $\frac{\rho}{2}\|x - \bar{x}^k\|^2$ from expression (3-21) to consider different penalty weights for components in x^{INV} and $(x_{t,0}^{LDR}, x_t^{LDR})$. Therefore, we used the following quadratic penalty term:

$$\|x^{INV} - \overline{x}^{INV}\|_{\frac{\mathcal{I}}{2}}^2 + \\ \frac{1}{2}\|x_t^{LDR} - \overline{x}_t^{LDR}\|^2 + \frac{1}{2}\|x_{t,0}^{LDR} - \overline{x}_{t,0}^{LDR}\|^2, \quad (4\text{-}1)$$

where, $\|x\|_P := x^T P x$. In this context, the quadratic deviation of investment decisions is penalized with half of their original weight (investment costs), whereas deviations of LDR coefficients are penalized with $\rho = 1$. This selection strategy constitutes a selection rule that improved the algorithm's efficiency (in terms of iterations) for all tested instances.

In the following sections, we analyze the performance, in terms of iterations and computational time, of the proposed method for different instance sizes, and the impact of nonanticipativity in the investment decisions, total cost and spot prices.

4.1 Analysis of the decomposition algorithm

Table 4.1 summarizes some macro results of the proposed r-BDMM method for some selected instances. The number of constraints and variables are referring to the size of the DE version of the problem (instance (3-1)-(3-3)).

Table 4.1: Results of the proposed algorithm for some instances (identified by the number of scenarios considered)

$\|\Omega\|$	**15**	**25**	**30**	**50**
Constraints (10^7)	1.75	2.92	3.50	5.84
Variables (10^7)	2.45	4.08	4.92	8.16
Execution time (min)	28	65	120	206
Number of iterations	36	33	37	31
Upper bound (M$)	71,423	57,087	42,600	30,036
Lower bound (M$)	71,359	57,053	42,562	30,012
Optimality GAP (%)	0.09	0.06	0.09	0.08

Table 4.2 shows, for each instance and method, the number of iterations required to achieve a GAP of 0.1%. Note that the number of iterations required by the proposed BDMM method is always smaller than that required by the TBD. Furthermore, the regularized version of the algorithm, the r-BDMM method, further reduced this number, showing that the PH regularization scheme is effective in reducing the number of Benders' loops needed to achieve the required GAP. Indeed, the r-BDMM required, on average (over all instances), 53% fewer iterations than the unregularized version of the algorithm (BDMM) and 68% fewer iterations than the TBD. So, results show that the proposed r-BDMM method outperforms the benchmarks in terms of Benders' iterations needed to solve the GEP for the Brazilian power system.

Table 4.2 also shows that the DE converges faster than the TBD and the proposed unregularized BDMM as long as the computer's memory is enough to address the problem. However, for larger instances in which MILP solvers fail to address the DE, Benders' approaches are still capable of providing high-quality solutions. Notwithstanding, it is important to highlight that the proposed r-BDMM breaks this pattern, achieving the required GAP faster than all methods for the larger eight out of ten tested instances. After considering 25 scenarios, the DE method fails to load the problem. Additionally, the same pattern observed for the number of iterations was observed for the computational time. The BDMM outperformed the TBD, and the r-BDMM outperformed the BDMM. Indeed, the r-BDMM is, on average (over all instances), 46% faster than the BDMM and 60% faster than the TBD.

Table 4.2: Number of iterations and computational time for different instance sizes and methods

Number of Benders loops (iterations)				**Execution time (in minutes)**			
$\|\Omega\|$	**TBD**	**BDMM**	**r-BDMM**	**TBD**	**BDMM**	**r-BDMM**	**DE**
5	176	78	34	29	17	11	7
10	132	82	33	48	33	17	11
15	136	89	36	88	59	28	31
20	125	83	36	132	88	46	59
25	113	83	33	193	142	65	-
30	101	72	37	276	203	120	-
35	87	64	34	287	226	135	-
40	73	61	32	252	237	131	-
45	88	70	33	356	320	163	-
50	78	63	31	443	407	206	-

For comparison purposes, the algorithm r-BDMM for the instance with 50 scenarios, the total execution time was 206 minutes, where 21% (44 minutes) was used to solve the master problems and 79% (162 minutes) to solve the subproblems.

Finally, Figure 4.1 compares the convergence over time of the r-BDMM, BDMM, and TBD, for the instance with 50 scenarios. It's clear that the convergence of the r-BDMM outperforms the BDMM and TBD. Also, after 1 hour of running time, the GAP values are: 9.76% for the r-BDMM; 53.15% for the BDMM; and 490% for the TBD. The GAP after 2 hours decreases to: 1.59% for the r-BDMM; 11.75% for the BDMM; and 68.3% for the TBD. These results corroborate the superiority of our proposed method to solve the GEP for the Brazilian system.

4.2 Benefits of a nonanticipative operational policy

This section analyzes the benefits of considering a multistage (nonanticipative) operational policy when deciding the investment plans for the Brazilian power system. To do that, we consider two cases:

Case 1 (multistage nonanticipative policy) – We solve problem (3-1)-(3-3) with the r-BDMM algorithm and $|\Omega_{50}|= 50$ scenarios (the same case study analyzed in Section 4.1). The operational results obtained in the optimization will be referred to as "in-sample". Then, we fix the optimal value

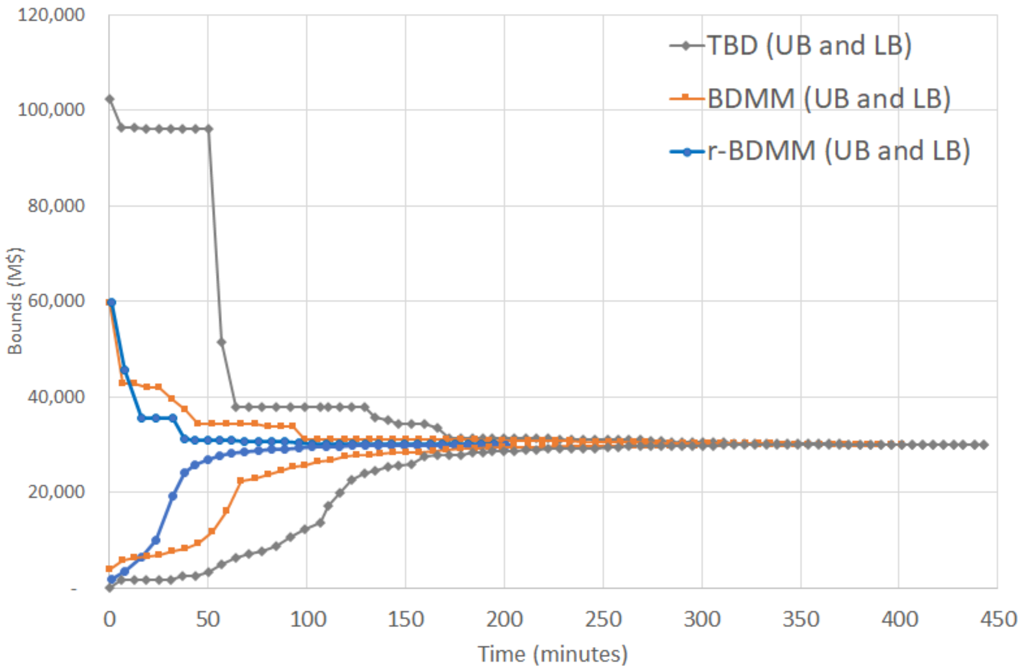

Figure 4.1: Lower bound and Upper bound for each algorithm

obtained for x^* and evaluate $Q(x^*)$ with $|\Omega_{1000}|= 1000$ scenarios (i.e., we evaluate the operational part of the problem, (3-4), with one thousand unseen scenarios). These results will be referred to as "out-of-sample".

Case 2 (Anticipative policy) – We solve the same problem (3-1)-(3-3), disregarding the nonanticipative constraints (2-8) for $|\Omega|= 50$. In this context, we are considering an anticipative approximation for the hydrothermal dispatch costs when deciding the generation investment plans. Then, as in the previous case, we will provide results for both in-sample and out-of-sample cases. However, in order to assess the benefit of a multistage nonanticipative operational policy when making investment decisions in hydrothermal power systems, the out-of-sample analysis must be carried out based on an implementable (nonanticipative) policy. To do that, we fix the optimal investment part of the solution found with the anticipative approximation, i.e., x_A^{INV*}, load the nonanticipative constraints to the in-sample problem, and solve it again to define $x^{LDR*}(x_A^{INV*})$. Then, with the complete first-stage vector, $x_A^* = [x_A^{INV*} \; x^{LDR*}(x_A^{INV*})]$, we evaluate the out-of-sample operational cost. To do that, we use the same 1000 scenarios used in the out-of-sample evaluation of Case 1.

Notwithstanding, it is clear that the anticipative case 2 is motivated by its lower computational burden. In this case study, the in-sample optimization of case 1 took 206 min, whereas the anticipative case 2 took 68 min. Therefore, this work aims to highlight the benefits of considering a nonanticipative operational policy to justify its higher computational times. In our comparison, we used the following metrics:

- *Expected total cost* – assessed with out-of-sample nonanticipative operational results.

- *Regret* – measured as the difference between the in-sample and out-of-sample expected total cost.
- *The average spot price* – assessed with out-of-sample operational results.
- *The 95th-percentile of the spot price* – assessed with out-of-sample operational results.
- *The average uncertainty level of spot price* – assessed with out-of-sample operational results. This metric is defined as (see [58]): $\frac{1}{T}\sum_{t=1}^{T}\left(\mathbf{Q}_t^{95\%} - \mathbf{Q}_t^{5\%}\right)$, where $\mathbf{Q}_t^{\alpha\%}$ represents the $\alpha\%$ quantile of a given variable at stage t.
- *The time variability of the spot price* – assessed with out-of-sample operational results. This metric is defined as (see [58]): $\frac{1}{T-1}\sum_{\omega\in\Omega_{1000}} p_\omega \sum_{t=2}^{T}\left|\frac{\pi_{t,\omega} - \pi_{t-1,\omega}}{\pi_{t-1,\omega}}\right|$.
- *Value of the nonanticipative policy* – the difference between the expected total cost, assessed with out-of-sample operational results, of Case 1 (nonanticipative) and Case 2 (anticipative), i.e., $VNAP = \mathcal{I}^T x_A^* + Q_{\Omega_{1000}}(x_A^*) - (\mathcal{I}^T x^* + Q_{\Omega_{1000}}(x^*))$.

Table 4.3 shows the in-sample and out-of-sample costs for Cases 1 and 2. Table 4.3 shows that the anticipative case 2, albeit 15.84% cheaper than the nonanticipative case 1, when analyzed with in-sample results, is actually 6,579M$ (or 8.27%) more expensive when analyzed with out-of-sample results. This difference defines the benefit or value of considering a nonanticipative policy when making the investment decisions: $VNAP = 86,116 - 79,537 = 6,579M\$$ in absolute terms, or 8.27% of the total cost and 16.18% of the investment cost obtained with the anticipative operational policy. The difference between what was expected when optimizing and what we get when actually implementing the solutions defines the regret metric, which values 20,679 M$ (or 24% of the total cost and 50.85% of the investment cost obtained with the anticipative operational policy).

Table 4.3: Impact of the stochastic policy in terms of total cost

In-sample results			
Policy	**Investment cost (M$)**	**operational cost (M$)**	**Total cost (M$)**
Nonanticipative	42,646	33,155	75,801
Anticipative	40,662	24,775	65,437
Difference	4.88%	33.82%	15.84%
Out-of-sample results			
Policy	**Investment cost (M$)**	**operational cost (M$)**	**Total cost (M$)**
Nonanticipative	42,646	36,891	79,537
Anticipative	40,662	45,454	86,116
Difference	4.88%	-23.21%	-8.27%

Table 4.4: Out-of-sample metrics for the temporal inconsistency

Policy	Total costs (M$)	Regret (M$)	Average spot price (R$/MWh)	95th-percentile of the spot price (R$/MWh)	Average uncertainty level of the spot price (R$/MWh)	Time variability of the spot price (%)
Anticipative	86,116	20,679	335.4	900.9	1276.7	105%
Non Anticipative	79,537	3,736	156.2	542.9	478.0	65%
Difference	-8.27%	-453.51%	-114.72%	-65.94%	-167.09%	-61.54%

Figure 4.2 shows the 90% confidence interval for the spot price in both cases. Since both cases considered building interconnections between areas, the spot prices for each of the regions are exactly the same. This figure shows that the investments made under an anticipative policy, when actually operating the system, produces much higher and uncertain spot prices. Furthermore, Table 4.4 shows the metrics presented at the beginning of this section. We can see that, besides being 8.27% more expensive, the non-implementable policy brings higher spot price on average and for high quantiles, higher volatility (uncertainty level) and higher temporal variability. Additionally, the deficit risk for the nonanticipative case 1 achieved a 0% probability in the out-of-sample, while the anticipative case 2 exhibited several scenarios with deficit as shown in Figure 4.3.

These results are consistent with the results obtained in previously reported works where simplifications were used in the opportunity cost assessment [5, 42, 47]. In this work, however, we extend this idea to the expansion planning level.

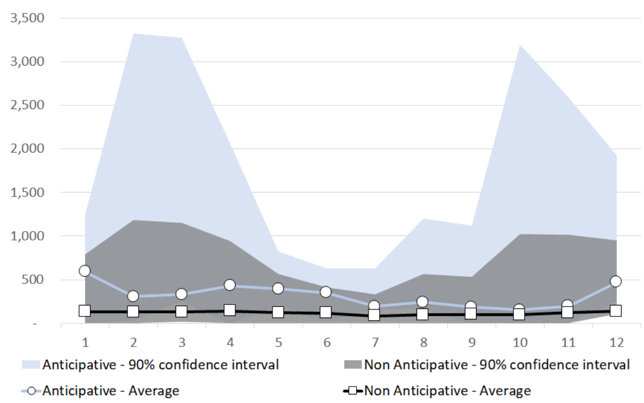

Figure 4.2: 90% confidence interval of the energy price for both cases

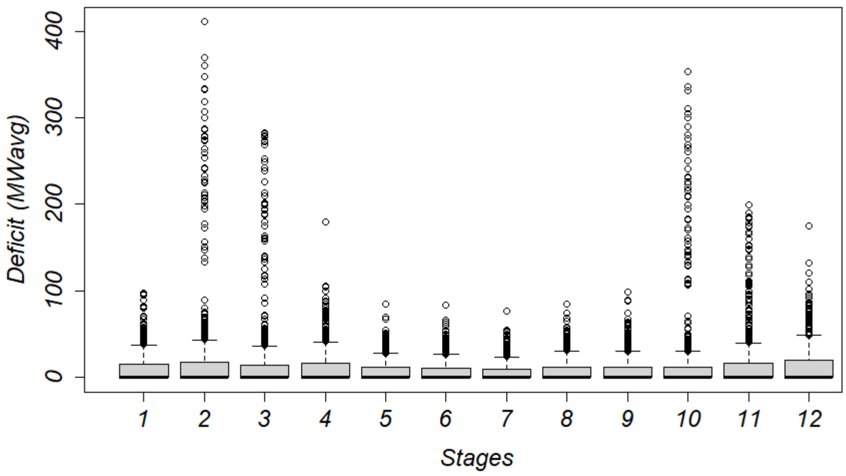

Figure 4.3: Variability and amount of deficit for the anticipative policy case

5
Conclusions

This work presented a novel regularized Benders Decomposition with multiple master problems. We show that the proposed method significantly improves the performance of the traditional Bender decomposition by using different parallel master problems, each of which considering a primal cut associated with a given scenario. Furthermore, we also showed that the consideration of a regularization scheme based on the Progressive Hedging is capable of significantly improve the proposed method performance when applied to the generation expansion planning problem.

In this work, we studied the Brazilian hydrothermal power system, which highly relies on the assessment of the opportunity cost of water through multistage nonanticipative operational policies. We show that the consideration of a multistage nonanticipative policy, rather than the less computationally intensive anticipative approximation, brings relevant benefits to the optimal investment decisions.

The case studies presented in this work allows us to convey the following concluding remarks:

- The unregularized version of our method outperforms the traditional Bender decomposition benchmark in both the number of iterations (30% on average) and computational time (23% on average) under the same computational resources.
- The proposed regularized version of our method outperforms the unregularized version in both number of iterations (53% on average) and computational time (46% on average). So, the proposed regularized version of our method outperforms the traditional Benders decomposition benchmark in terms of the number of iterations (68% on average) and in terms of computational time (60% on average).
- The proposed algorithm can solve huge instances of the expansion planning problem, such as the Brazilian power system.
- The value of considering the nonanticipative operational policy when deciding the investment plans is 8.27%, investment decisions disregarding anticipativity constraints can be reduced by this amount on average.
- The consideration of a nonanticipative operational policy also brings other benefits, such as (i) a reduction of 115% on average energy prices; (ii) a reduction of 66% on the 95th-percentile of the spot prices; (iii) a reduction of 167% in the spot prices uncertainty metric; and (iv) a reduction of 61% on the temporal variability of the spot prices.
- The regret of considering an anticipative (non-implementable) operational policy in the expansion problem is 20,679 M$, which is 24% of the total cost and 51% of the investment cost.

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