Pontifícia Universidade Católica
do Rio de Janeiro

Camillo Vianna Cantini

Portfolio Selection Incorporating Macroeconomic Views Using Black-Litterman Model

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Engenharia de Produção of PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Engenharia de Produção.

Advisor : Prof. Davi Michel Valladão
Co-advisor: Prof. Betina Dodsworth Martins Froment Fernandes

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Prof. Davi Michel Valladão
Advisor
Departamento de Engenharia Industrial – PUC-Rio

Prof. Betina Dodsworth Martins Froment Fernandes
Co-advisor
Departamento de Engenharia Elétrica – PUC-RIO

Prof. Luiz Eduardo Teixeira Brandão
Departamento de Engenharia Industrial – PUC-Rio

Prof. Frances Fischberg Blank
Departamento de Engenharia Industrial – PUC-Rio

Rio de Janeiro, August the 22nd, 2019

Camillo Vianna Cantini

Graduated in chemical engineering by Instituto Militar de Engenharia. Working at Petrobras since 2012, first at Exploration and Production, then at Business Risks and now manages Valuation and Value Base Management on Business Performance.

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Abstract

Cantini, Camillo Vianna; Valladão, Davi Michel (Advisor); Fer-nandes, Betina Dodsworth Martins Froment (Co-Advisor). **Portfolio Selection Incorporating Macroeconomic Views Using Black-Litterman Model**. Rio de Janeiro, 2019. 39p. Dissertação de mestrado – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

Black and Litterman proposed a portfolio selection model that blends investor's views on asset returns with market equilibrium concepts to construct optimal portfolios. However, the model efficiency relies on the performance of investors' views regarding tradable assets, which is challenging in practice. Focusing on improving Black-Litterman practical application, this work consists in providing new allocations based upon views on macroeconomic factors, which are largely available but not directly tradable. The main advantage is that predictions on these factors are usually provided by market players. A case study based on the information disclosed by the Brazilian Central Bank is presented to test the proposed framework. The out-of-sample risk-adjusted returns obtained incorporating the players' macroeconomic expectations through the use of the proposed framework outperformed the traditional mean-variance model as well as the local benchmark.

Keywords

Portfolio Selection; Black-Litterman Model; Macroeconomic Views; Finance.

Resumo

Cantini, Camillo Vianna; Valladão, Davi Michel; Fernandes, Betina Dodsworth Martins Froment. **Seleção de Portfólio Incorporando Visões Macroeconômicas Utilizando o Modelo Black-Litterman**. Rio de Janeiro, 2019. 39p. Dissertação de Mestrado – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

Black e Litterman propuseram um modelo de seleção de portfólio que combina a visão dos investidores acerca de ativos com conceitos de equilíbrio de mercado para construir portfólios ótimos. Entretanto, a eficiência do modelo depende da qualidade da visão futura acerca do retorno dos ativos, o que é desafiador na prática. Com o objetivo de melhorar a aplicação prática do modelo Black-Litterman, o foco desse trabalho é viabilizar novas alocações com base em visões de fatores macroeconômicos que estão fora do universo de alocação. A principal vantagem é que a previsão desses fatores é amplamente fornecida por agentes de mercado. Um estudo de caso baseado nas informações disponibilizadas pelo Banco Central do Brasil é apresentado para validar a estrutura proposta. Os retornos obtidos fora da amostra e ajustados ao risco incorporando a visão de fatores macroeconômicos com a estrutura proposta superaram o modelo de média-variância tradicional e o *benchmark* local.

Palavras-chave

Seleção de Portfólio; Modelo Black-Litterman; Visões Macroeconômicas; Finanças.

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Symbols

$\hat{}$ - is an operator that denote estimates

$\tilde{}$ - is an operator that denote random variables

t - is a general timestep

n - is a scalar for a general dimension of vectors and matrices

$\boldsymbol{w}$ - is the portfolio allocation vector

$\boldsymbol{\Sigma}$ - is the covariance matrix of asset point returns estimate

$\boldsymbol{\mu}$ - is the vector of expected returns

δ - is the risk aversion parameter

β - is the CAPM market risk coefficient

r_f - is the risk free asset return

r_{mkt} - is the average market return

σ_{mkt} - is the market standard deviation

w_{mkt} - is the market weights of the assets

$\boldsymbol{r}$ - is the vector of the return of the portfolio

$\boldsymbol{\pi}$ - is the vector of the estimates of the expected returns of the securities

τ - is the constant of proportionality for $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}_\pi$

$\boldsymbol{\Omega}$ - is the matrix of covariance of the errors on the views estimates

$\boldsymbol{V}$ - is the posterior return covariance matrix using the Black-Litterman model

$\boldsymbol{m}$ - is the posterior return estimate vector using the Black-Litterman model

C - is the Idzorek confidence parameter, defined between 0 and 1

$\boldsymbol{\Sigma}_\pi$ - is the asset expected returns covariance matrix estimate

$\boldsymbol{P}$ - is the views structure matrix

$\boldsymbol{y}$ - is the investors' updated views vector

$\boldsymbol{\epsilon}$ - is the view estimation's errors vector

$\boldsymbol{q}$ - is the investor best estimates vector for $\boldsymbol{y}$

1 - is an elements 1 vector or a matrix

0 - is an elements 0 vector or a matrix

σ - is the portfolio standard deviation

ss - is the sample size

n - is the number of securities

k - is the number of views

$_s$ - is a subscript for securities' parameters

$_f$ - is a subscript for macroeconomic factor's parameters

l_r - is the securities' expected return estimation window estimation length

l_c - is the securities' covariance estimation window estimation length

"It had long since come to my attention that people of accomplishment rarely sat back and let things happen to them. They went out and happened to things."

Leonardo da Vinci, .

1
Introduction

Goldfarb and Iyengar [2003] define the portfolio selection process as a problem where the investor wants to allocate his capital in order to maximize his returns and minimize his risks. The Markowitz model (Markowitz [1952]) was the first mathematical model to achieve relevance on this subject. In that work the investor's return was represented by the expected return of the portfolio and it's risk represented by the overall portfolio variance. By solving the risk minimization problem, given a minimum target return, the investor would obtain his optimal allocation. Using this mean-variance framework, every investment decision could be taken considering the assets expected returns and covariances.

Despite presenting an intuitive framework for handling risk-return relationship, the Markowitz model is avoided in practical applications. Some authors attribute this to intrinsic estimation errors (Michaud [1989], Michaud et al. [2013] and Idzorek [2002]). In order to address the estimation issue it is possible to use a robust approach developed in Soyster [1973], Ben-Tal and Nemirovski [1998], Ben-Tal and Nemirovski [1999], Ben-Tal and Nemirovski [2000], Bertsimas and Sim [2004] and Fernandes et al. [2016] or a re-sampling one developed in Michaud [1989] and Michaud et al. [2013] to minimize those errors.

Black and Litterman [1992] indicate that the Markowitz model is also avoided because it's outcomes many times are hard-to-explain corner solutions, and also because investor's perceptions play no role in it. To address these issues Black and Litterman [1992] propose a simple approach that enables investors to combine their beliefs regarding securities' performance with market equilibrium allocation.

The Black-Litterman framework (Black and Litterman [1992], Satchell and Scowcroft [2000], Walters [2011] and Cheung [2009]) enables investors to blend their subjective views on securities returns with market equilibrium returns using a simple formula to achieve optimal balanced portfolios. We see extensions of the model in the literature, and we highlight: Fusai and Meucci [2003] which removes the τ parameter from the model, Meucci [2006] whose work continues the previous extension, Idzorek [2002] that proposes the use of an intuitive investor confidence parameter C instead of the covariance matrix for the views $\mathbf{\Omega}$ and Krishnan and Mains [2005] which develop the prior returns estimation including factors other than the market risk premium. But the Black-Litterman model, praised for its simplicity, is often criticized for two issues: (i) being insensitive to historical data; and (ii) lacking of an organized framework for setting the investors' views. Both Zhou [2009], who proposes mixing the historical returns with the Black-Litterman returns, and Michaud et al. [2013], who criticizes the model for reaching a previously wanted portfolio and proposes his alternative model for handling estimation errors, address (i). Fernandes et al. [2018] who proposed a mixed use of historical returns and fundamentalist data to set securities views, and Cheung [2013] who proposed views based on linear factors which explain securities returns, are good empirical applications that address (ii).

1.1 Contributions

We propose a further extension of the Black-Litterman model, motivated by the same idea of the factor-based approach presented by Cheung [2013]. The model proposed by Cheung [2013] has the drawback of associating views to factors only after establishing a linear factor model for the returns.

The proposed Macrofactor Black-Litterman model (MBL) does not require a return modelling step, avoids the use of Black-Litterman parameters τ and δ[1], and enables the use of views on non-tradable factors (which may be easier to obtain) to update tradable securities expected returns and covariance matrix, relying on their relationship with tradable securities (measured by their correlations) to improve the results. In the proposed model an investor does not have to be concerned about modelling returns, and is able to blend his views on macroeconomic factors to obtain an updated allocation, which is an advantage when compared to the approach presented in Cheung [2013].

With the MBL model our objective is to present an alternative framework for investors to incorporate expectations on macroeconomic factors to their portfolio allocation. We summarize the main contributions of this thesis as follows:

1. This novel framework allows views on factors to be incorporated even when there is no explicit linear factor model linking the factors to the securities returns. Among these factors, we highlight those related to

[1] τ is the constant of proportionality between assets returns covariance matrix and assets expected returns covariance matrix and δ is the risk aversion parameter defined in the CAPM.

macroeconomics, which have many predictors available on the marketplace.

2. We develop a framework for setting views based on the public data of macroeconomic indicators available on the Brazilian Central Bank (BCB) database. This framework based on historical information does not make use of market or equilibrium information for the priori returns and avoids parameters τ and δ.

3. Our proposed MBL model was tested in a case study based on Brazilian financial data provided by BCB to set the views. The results show that the MBL model generates greater out-of-sample risk-adjusted returns compared to some chosen benchmarks.

1.2 Organization

This thesis is organized as follows. In Chapter 2 we show a brief review of the Black-Litterman model, starting with the modelling of the returns, then how views are incorporated and finally the optimization framework.

In Chapter 3 we describe our proposed model. In Chapter 4 we present a case study using Brazilian financial market data where views are set with BCB information. The conclusion are finally presented in Chapter 5.

1.3 Assumptions and Notation

Throughout this thesis, we use bold-faced capital letters to indicate matrices $(\mathbf{\Sigma}, \mathbf{\Omega}, ...)$, bold-faced lowercase letters to indicate vectors $(\boldsymbol{\mu}, \boldsymbol{\pi}, ...)$ and ordinary letters for scalars numbers $(\tau, n, ...)$. Vectors and matrices dimensions are shown in brackets $([n \times 1], [n \times n], ...)$. The symbols ˆ and ˜ are used to denote estimates and random variables respectively. The subscript $_t$ is used to denote that the variable is on time step t. We use the Black-Litterman notation as presented in Satchell and Scowcroft [2000], Cheung [2009] and Walters [2011].

For application purposes, we assume the portfolio investment decision is made before knowing the values taken by uncertain parameters. For instance, let $\hat{\boldsymbol{m}}_t$ be the vector of expected returns of the securities on the investment universe between step time t and $t+1$; it shall be based on the information available up to time t.

We consider a daily allocation based on one-period step in a rolling horizon scheme. Let $\boldsymbol{w}_t$ be the allocation vector made at the beginning of day t. Considering $\hat{\boldsymbol{m}}_t$ (which is a one-step ahead forecast) for maximizing the portfolio return $\boldsymbol{w}_t^T \hat{\boldsymbol{m}}_t$. Afterwards, we test this allocation in an out of sample one-step-ahead analysis. Let $\boldsymbol{r_t}$ be the realized return on time step t.The time frame diagram is presented in Figure 1.1.

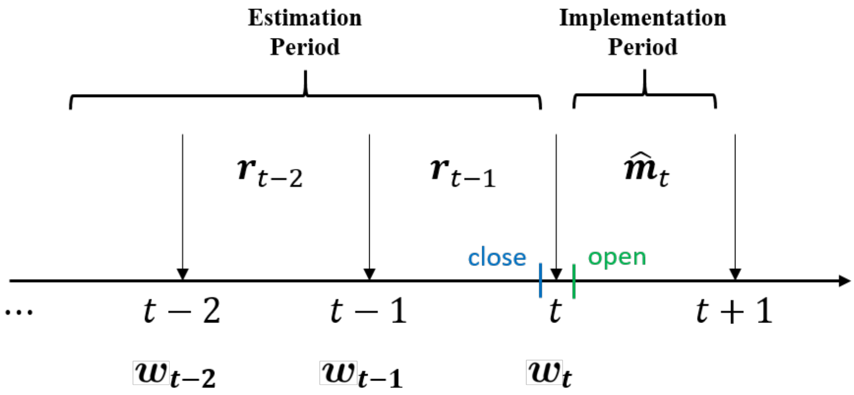

Figure 1.1: Time frame diagram

2
The Black-Litterman model

The model, which was first published by Black and Litterman [1992], focuses on helping investors input their views on asset returns to their allocations using the framework of mixed estimation proposed by Theil and Goldberger [1961] to be blended with prior returns estimates based on market return assumptions. This process is analogous to a Bayesian update[1].

Black and Litterman [1992] did not describe a formulation of the model in their work, but gave instead a rationale and the mathematical intuition beneath it. He and Litterman [2002] provide further detail on the model and present a simple and intuitive working example to show the logic of the results obtained.

Satchell and Scowcroft [2000] explain the model and derive a full formulation to it based on the available information. However, they did not consider the prior returns to be distributions and found a little different final formulation for the model. Firooyze and Blamont [2003] presented the full formulation and discussed the intuition of the parameter τ. This parameter was removed in the extension proposed by Fusai and Meucci [2003] and later developed by Meucci [2006] where he presents his alternate copula-opinion pooling approach. Idzorek [2002] presented another extension of the original model which suggested the use of a confidence parameter C to mix the prior and the posterior estimates, and thus avoiding the use of the views covariance matrix $\mathbf{\Omega}$.

Full reviews of the model and some of its extensions are presented in the works of Walters [2011] and Cheung [2009]. Zhou [2009] proposed mixing historical data with the posterior returns to improve his estimates; Fernandes et al. [2018] presented an investment strategy for the Brazilian stock index setting views based on past returns and price-earnings ratio; and Cheung [2013] presented a mix of the Black-Litterman model with multi-factor return modelling enabling a factor instead of security view to be incorporated.

[1]The model could also be derived from the Bayesian perspective.

2.1 Modelling the securities returns

Suppose there are n securities in the universe with a random return vector given by $\tilde{\boldsymbol{r}}_{[n\times 1]}$. Considering the returns follow a normal distribution, $\tilde{\boldsymbol{r}}_{[n\times 1]}$ has the distribution:

$$\tilde{\boldsymbol{r}} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{2-1}$$

where $\boldsymbol{\mu}_{[n\times 1]}$ is the vector of means and $\boldsymbol{\Sigma}_{[n\times n]}$ is the covariance matrix. Consider one shall not have $\boldsymbol{\mu}_{[n\times 1]}$ nor $\boldsymbol{\Sigma}_{[n\times n]}$, and use the estimates $\hat{\boldsymbol{\pi}}_{[n\times 1]}$ (which covariance matrix is $\hat{\boldsymbol{\Sigma}}_{\pi[n\times n]}$) and $\hat{\boldsymbol{\Sigma}}_{[n\times n]}$ instead. As an estimate, $\boldsymbol{\mu}_{[n\times 1]}$ has the following distribution:

$$\boldsymbol{\mu} \sim N(\hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\pi}}) \tag{2-2}$$

Therefore, we have the following distribution for $\tilde{\boldsymbol{r}}_{[n\times 1]}$:

$$\tilde{\boldsymbol{r}} \sim N(\hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\Sigma}} + \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\pi}}) \tag{2-3}$$

2.2 Incorporating investors views

Assuming an investor has k views regarding market securities based on his private perception, let $\tilde{\boldsymbol{y}}_{[k\times 1]}$ be the vector of investors updated view on returns estimates, which can be expressed as a linear combination of the returns $\tilde{\boldsymbol{r}}_{[n\times 1]}$

$$\boldsymbol{P}\tilde{\boldsymbol{r}} = \tilde{\boldsymbol{y}} + \tilde{\boldsymbol{\epsilon}} \tag{2-4}$$

where $\boldsymbol{P}_{[k\times n]}$ is the views structure matrix and $\tilde{\boldsymbol{\epsilon}}_{[k\times 1]}$ is the vector of view estimation errors. Suppose investors have $\hat{\boldsymbol{q}}_{[k\times 1]}$ as their best estimation of $\tilde{\boldsymbol{y}}_{[k\times 1]}$ subject to unbiased and normally distributed errors, then

$$\tilde{\boldsymbol{\epsilon}} \sim N(\boldsymbol{0}, \boldsymbol{\Omega}) \tag{2-5}$$

where $\boldsymbol{0}_{[k\times 1]}$ is a vector of zeros and $\boldsymbol{\Omega}_{[k\times k]}$ is the variance matrix of view estimation errors. One can combine this new information on $\tilde{\boldsymbol{r}}_{[n\times 1]}$, and using Theil's mixed estimation obtain the posterior return estimates mean vector $\hat{\boldsymbol{m}}_{[n\times 1]}$ is given by

$$\hat{\boldsymbol{m}} = \left[\hat{\boldsymbol{\Sigma}}_{\pi}^{-1} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}\left[\hat{\boldsymbol{\Sigma}}_{\pi}^{-1}\hat{\boldsymbol{\pi}} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\hat{\boldsymbol{q}}\right] \tag{2-6}$$

and posterior return estimates covariance matrix $\hat{\boldsymbol{V}}_{[n\times n]}$ is given by

$$\hat{\boldsymbol{V}} = \left[\hat{\boldsymbol{\Sigma}}_{\pi}^{-1} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1} \tag{2-7}$$

For a detailed derivation on 2-6 and 2-7 see Walters [2011] or Cheung [2009]. Alternatively $\hat{\boldsymbol{m}}$ and $\hat{\boldsymbol{V}}$ can be written as a Bayesian correction of $\boldsymbol{\pi}$ and $\boldsymbol{\Sigma}$ respectively:

$$\hat{\boldsymbol{m}} = \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\boldsymbol{P}^T[(\tau\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T) + \boldsymbol{\Omega}]^{-1}[\boldsymbol{q} - \boldsymbol{P}\boldsymbol{\pi}] \tag{2-8}$$

$$\hat{\boldsymbol{V}} = \boldsymbol{\Sigma} + [(\tau\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P}]^{-1} \tag{2-9}$$

2.3
Further assumptions and optimization framework

For $\hat{\boldsymbol{\pi}}_{[n\times 1]}$, the original Black-Litterman model assumes the validity of CAPM, where the expected returns for each asset $\hat{\pi}_i$ in equilibrium is

$$\hat{\pi}_i = r_f + \hat{\beta}_i(\mu_m - r_f), \forall i = 1, ..., n \tag{2-10}$$

$$\hat{\beta}_i = \frac{Cov(\boldsymbol{r}_i, \boldsymbol{r}_{mkt})}{\sigma^2_{mkt}} \tag{2-11}$$

where $\boldsymbol{r}_{mkt\ [t\times 1]}$ is market return over a time interval, given by the market-weighted return of the securities[2], thus $\boldsymbol{r}_{mkt} = \boldsymbol{r}^T\boldsymbol{w}_{mkt}$, where $\boldsymbol{w}_{mkt\ [n\times 1]}$ is determined by the market value of N securities and σ^2_{mkt} is the variance of the market return. As for the covariance matrix we have $\hat{\Sigma} = Cov(\boldsymbol{r}, \boldsymbol{r}^T)$, then we can set $\hat{\boldsymbol{\pi}}_{[n\times 1]}$ to:

$$\hat{\boldsymbol{\pi}} = r_f\mathbf{1} + \delta\hat{\boldsymbol{\Sigma}}\boldsymbol{w}_{mkt} \tag{2-12}$$

where δ is a positive constant known as the risk aversion parameter and $\mathbf{1}_{[n\times 1]}$ is a matrix of elements one. For a more detailed explanation, see Satchell and Scowcroft [2000] and Walters [2011].

The model also assumes that $\hat{\boldsymbol{\Sigma}}_{\pi\ [n\times 1]}$ and $\hat{\boldsymbol{\Sigma}}_{[n\times 1]}$ are proportional, therefore:

$$\hat{\boldsymbol{\Sigma}}_\pi = \tau\hat{\boldsymbol{\Sigma}} \tag{2-13}$$

where τ is a scalar representing investor uncertainty about mean estimation. The original model presented in Black and Litterman [1992] did not present any methodology for the estimation of τ. Still, if we set $\tau = 1/ss$ where ss the sample size, τ would be the maximum likelihood estimator, and if we set $\tau = 1/(ss - n)$, where n is the number of securities considered, τ would be the best quadratic unbiased estimator.

Given the complexity related to the estimations of $\boldsymbol{\Omega}_{[k\times k]}$, for simplicity reasons it is often assumed that the same τ is related to the uncertainty

[2]The market weights $\boldsymbol{w}_{mkt}$ are determined by the market values of the securities on the investment universe.

regarding view estimation errors, and that each view estimation error is unrelated, resulting in a diagonal $\boldsymbol{\Omega}_{[k\times k]}$ matrix as:

$$\boldsymbol{\Omega} = diag(\boldsymbol{P}\hat{\boldsymbol{\Sigma}}_{\pi}\boldsymbol{P}^T) = diag(\boldsymbol{P}\tau\hat{\boldsymbol{\Sigma}}\boldsymbol{P}^T) = \tau diag(\boldsymbol{P}\hat{\boldsymbol{\Sigma}}\boldsymbol{P}^T) \tag{2-14}$$

Other possibility for setting $\boldsymbol{\Omega}$ as presented in Walters [2011] is using the variances of the views estimation.

Figure 2.1 shows the framework of Black-Litterman.

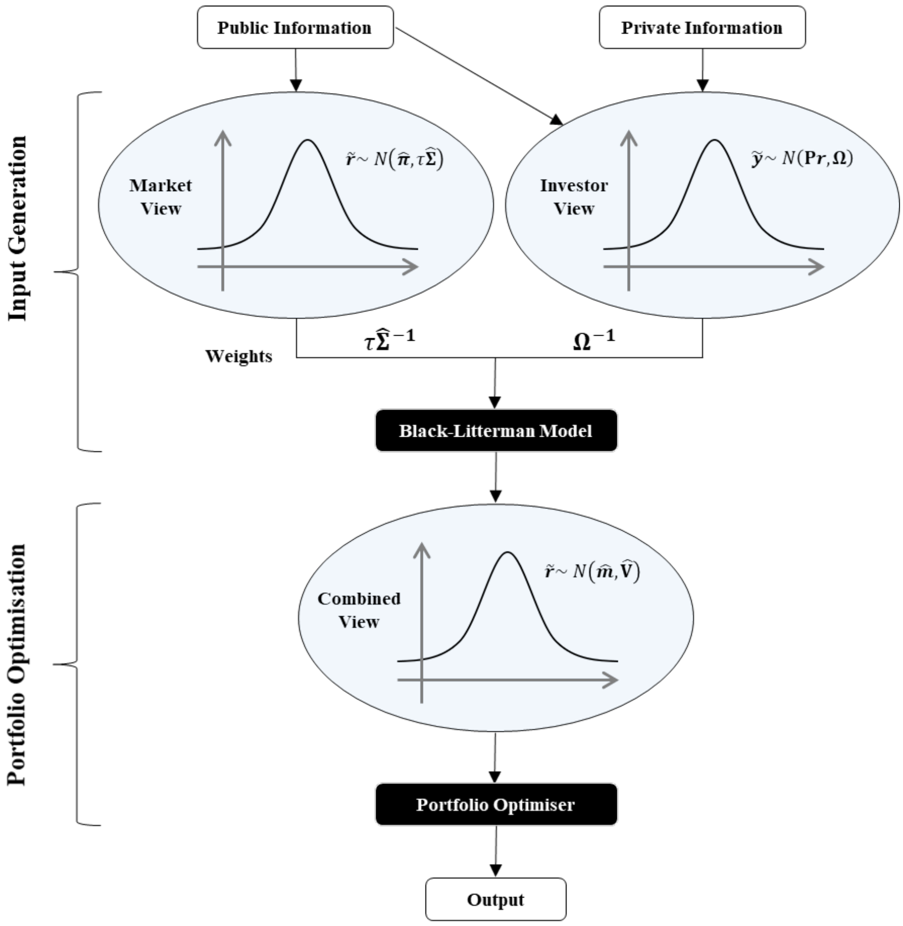

Figure 2.1: The Black-Litterman optimization framework

In the following section it will be presented the novel framework for the Black-Litterman model proposed in this work.

3
The Proposed MBL Model

3.1
The Value of Macroeconomic Predictions

Trying to predict the future is an intrinsic desire for financial market investors, improving the overall returns based on predictions is a two step procedure: 1) Achieving good predictions and; 2) Being able to invest accordingly.

Related to the first step, it is credited to Niels Bohr the quote: "*It is difficult to predict, especially the future.*" and indeed it gets more difficult to predict correctly the more specific the prediction is, and the further in time it is. A bunch of bold sell-side analyst may even try to predict specific stock prices, but it is more likely for players to venture to predict broader macroeconomic indicators, as the complexity and number of variables related to them tend to be significantly smaller. We can point out as examples, surveys conduced by central banks, studies performed by consulting companies and even semi-annual prospects from World Bank. It is important for investors to have many sources of information because it tends to improve the quality of predictions they use which widens investment opportunities to more sectors.

Related to the second step, our goal with the Macrofactor Black-Litterman model (MBL) is to provide a simple yet effective framework, that will enable investors to use the macroeconomic factors predictions at hand to update parameters of any set of securities they feel comfortable working with. And by simple framework we mean one that avoids the use of complex econometric multi-factor models.

3.2
Incorporating Macroeconomic Views

We propose a MBL model where investors views on factors affect securities' expected returns and covariance matrix, due to its intrinsic relation with such securities (their correlations), even when those factors are not explicit linked to the securities by any factor model.

Let us split the n securities within the views' universe in s tradable securities and f factors, such as $n = s + f$. The returns of our views' universe

$\tilde{\boldsymbol{y}}_{t\,[n\times 1]}$ are:

$$\tilde{\boldsymbol{y}}_t = \begin{bmatrix} \tilde{\boldsymbol{r}}_{s,t} \\ \tilde{\boldsymbol{r}}_{f,t} \end{bmatrix} \tag{3-1}$$

Considering separate views for tradable securities and non-tradable factors[1], we could sort the model inputs: $\hat{\boldsymbol{\Sigma}}_{t\,[n\times n]}$, $\hat{\boldsymbol{\pi}}_{t\,[n\times 1]}$, $\boldsymbol{P}_{t\,[k\times n]}$, $\hat{\boldsymbol{q}}_{t\,[k\times 1]}$ and $\boldsymbol{\Omega}_{t\,[k\times k]}$ in tradable and non-tradable components such as:

$$\hat{\boldsymbol{\Sigma}}_t = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{s,t} & \hat{\boldsymbol{\Sigma}}_{sf,t} \\ \hat{\boldsymbol{\Sigma}}_{fs,t} & \hat{\boldsymbol{\Sigma}}_{f,t} \end{bmatrix} \tag{3-2}$$

$$\hat{\boldsymbol{\pi}}_t = \begin{bmatrix} \hat{\boldsymbol{\pi}}_{s,t} \\ \hat{\boldsymbol{\pi}}_{f,t} \end{bmatrix} \tag{3-3}$$

$$\boldsymbol{P}_t = \begin{bmatrix} \boldsymbol{P}_{s,t} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{f,t} \end{bmatrix} \tag{3-4}$$

$$\hat{\boldsymbol{q}}_t = \begin{bmatrix} \hat{\boldsymbol{q}}_{s,t} \\ \hat{\boldsymbol{q}}_{f,t} \end{bmatrix} \tag{3-5}$$

$$\boldsymbol{\Omega}_t = \begin{bmatrix} \boldsymbol{\Omega}_{s,t} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}_{f,t} \end{bmatrix} \tag{3-6}$$

where k is the total number of views and $k = k_s + k_f$.

The Black-Litterman model outputs mean vector $\hat{\boldsymbol{m}}_{t\,[n\times 1]}$ and covariance matrix $\hat{\boldsymbol{V}}_{t\,[n\times n]}$ would also be sorted:

$$\hat{\boldsymbol{m}}_t = \begin{bmatrix} \hat{\boldsymbol{m}}_{s,t} \\ \hat{\boldsymbol{m}}_{f,t} \end{bmatrix} \tag{3-7}$$

$$\hat{\boldsymbol{V}}_t = \begin{bmatrix} \hat{\boldsymbol{V}}_{s,t} & \hat{\boldsymbol{V}}_{sf,t} \\ \hat{\boldsymbol{V}}_{fs,t} & \hat{\boldsymbol{V}}_{f,t} \end{bmatrix} \tag{3-8}$$

and in possession of $\hat{\boldsymbol{m}}_{s,t}$ and $\hat{\boldsymbol{V}}_{s,t}$, which are the parameters affected by investors views, we could obtain the tradable assets allocation $\boldsymbol{w_t}$ using the following optimizatiom problem:

$$\begin{array}{lll} \max\limits_{\boldsymbol{w_t}} & \boldsymbol{w}_t^T \hat{\boldsymbol{m}}_{s,t} & \\ \text{s.t.} & \frac{1}{2}\boldsymbol{w}_t^T \hat{\boldsymbol{V}}_{s,t} \boldsymbol{w}_t & \leq \sigma^2 \\ & \boldsymbol{w}_t^T \boldsymbol{1} & = 1 \\ & w_{i,t} & \geq 0 \quad \forall i = 1, ..., s \end{array} \tag{3-9}$$

where σ^2 is the target variance for the portfolio.

Through the use of $\hat{\boldsymbol{m}}_{s,t}$ and $\hat{\boldsymbol{V}}_{s,t}$, opposed to $\hat{\boldsymbol{\pi}}_{s,t}$ and $\hat{\boldsymbol{\Sigma}}_{s,t}$, we ensure that $\boldsymbol{w}_t$ is a product of investors' views. The conceptual workflow would be:

1. Choose the tradable securities and obtain prior parameters;

[1]The model also allows the use of combined security/factor views, but it is more practical for investors to set separate views.

2. Select the macroeconomic factors related to tradable securities;

3. Set views regarding the macroeconomic factors returns (may include views on securities returns);

4. Set further model parameters and obtain posterior tradable assets parameter;

5. Define allocation; and

6. Compare results to chosen benchmarks.

3.3 Assessing Performance Using Historical Data

We use the historical average for both factors and the tradable securities to estimate their prior expected returns, following the work of Zhou [2009], which enables us to avoid the use of δ, τ and the heuristic for estimate $\mathbf{\Omega}$. These are the three issues most criticized on Black-Litterman literature (see Fusai and Meucci [2003], Walters [2011] and Michaud et al. [2013]). By doing so, its easier to estimate prior returns or macroeconomic factors, which many times can not be marked to market, and thus create a more robust model that avoids the use of these parameters.

It is important to estimate the appropriate length for the rolling window estimation, weighting the tradeoff between using a bigger sample to reduce the estimation error and avoiding using older data as the distribution changes over time. We achieve the proper windows for expected returns and covariance matrix estimation through successive out-of-sample evaluations using different combinations of expected returns estimation length l_r and covariance matrix estimation length l_c. We understand that using different windows is important as the effect of estimation errors in covariances might be greater than the estimation errors on expected values (see Chopra and Ziemba [2013]). Figure 3.1 presents a conceptual flowchart used to optimize estimation windows lengths.

3.4 Guided Example

Let us consider an example where we have two tradable securities: (i) an Oil & Gas stocks index e (ii) S&P 500 index; and also one macroeconomic factor: Brent oil price, which our market reference is the future price one month forward.

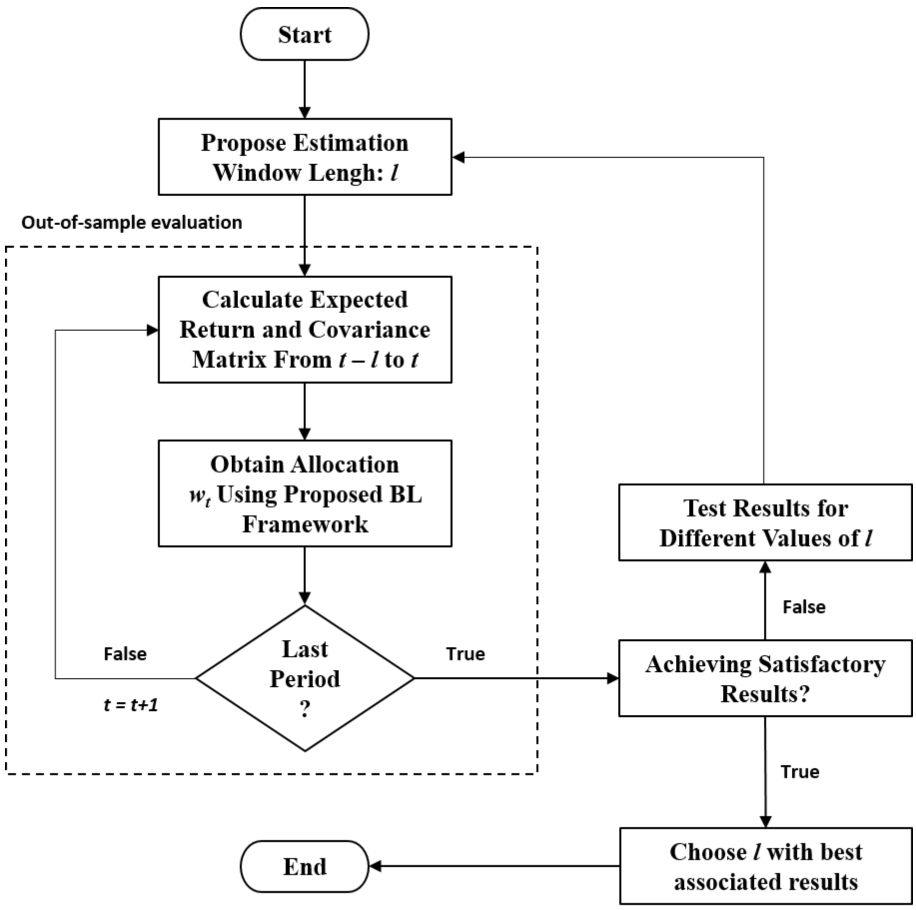

Figure 3.1: Flowchart for backtesting and optimizing estimation windows lengths

We would use historical data to estimate the priori expected returns array $\hat{\boldsymbol{\pi}}$ and covariance matrix $\hat{\boldsymbol{\Sigma}}$. We could use a sample size of $ss = 30$ months and use the maximum likelihood estimator to estimate $\hat{\boldsymbol{\Sigma}}_\pi$ ($\hat{\boldsymbol{\Sigma}}_\pi = \frac{1}{ss}\hat{\boldsymbol{\Sigma}}$). The array of prior expected returns $\hat{\boldsymbol{\pi}}$ and the prior covariance matrix $\hat{\boldsymbol{\Sigma}}$[2] in this example would be:

$$\hat{\boldsymbol{\pi}} = \begin{bmatrix} 3\% \\ -1\% \\ 2\% \end{bmatrix} \tag{3-10}$$

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} 10^{-3} & 0,8^{-3} & 6^{-3} \\ 0,8^{-3} & 6,4^{-3} & 2,8^{-3} \\ 6^{-3} & 2,8^{-3} & 14,4^{-3} \end{bmatrix} \tag{3-11}$$

Considering one only absolute view that Brent returns will be 6%. As we have 3 securities this view would be represented as:

[2] Asset 1 is the Oil & Gas stocks index, 2 is the S&P 500 index and 3 Brent.

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_{OG} \\ \boldsymbol{\mu}_{SP} \\ \boldsymbol{\mu}_{B} \end{bmatrix} = 6\% \tag{3-12}$$

Where $\boldsymbol{\mu}_{OG}$ is Oil & Gas stocks index expected return, $\boldsymbol{\mu}_{SP}$ is S&P 500 index expected return and $\boldsymbol{\mu}_{B}$ is Brent expected return.

Considering this view, the $\boldsymbol{P}_{1\times3}$ and $\hat{\boldsymbol{q}}_{1\times1}$ matrix are $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ and [6%], these matrix line count is the number of views, and $\boldsymbol{P}$ column count is the number of securities.

Using the heuristic proposed by Black e Litterman on their model to set the views covariance matrix $\boldsymbol{\Omega}$ as in equation 2-14 we have:

$$\begin{aligned} \boldsymbol{\Omega} &= \text{diag}(\tau \boldsymbol{P} \boldsymbol{\Sigma} \boldsymbol{P}^T) \\ &= \tfrac{1}{30} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \times 10^{-3} & 0,8 \times 10^{-3} & 6 \times 10^{-3} \\ 0,8 \times 10^{-3} & 6,4 \times 10^{-3} & 2,8 \times 10^{-3} \\ 6 \times 10^{-3} & 2,8 \times 10^{-3} & 14,4 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \tfrac{1}{30} \begin{bmatrix} 6 \times 10^{-3} & 2,8 \times 10^{-3} & 14,4 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \tfrac{1}{30} 14,4 \times 10^{-3} = 4,32 \times 10^{-3} \end{aligned} \tag{3-13}$$

Having $\hat{\boldsymbol{\pi}}$, $\hat{\boldsymbol{\Sigma}}$, $\boldsymbol{P}$, $\hat{\boldsymbol{\Sigma}}_{\pi}$, $\hat{\boldsymbol{q}}$ e $\boldsymbol{\Omega}$, we can use equations 2-8 and 2-9 to obtain $\hat{\boldsymbol{m}}$, which is the posterior expected returns array and $\hat{\boldsymbol{V}}$, which is the posterior covariance matrix.

$$\hat{\boldsymbol{m}} = \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\boldsymbol{P}^T[(\tau\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T) + \boldsymbol{\Omega}]^{-1}[\boldsymbol{q} - \boldsymbol{P}\boldsymbol{\pi}]$$

$$\boldsymbol{\pi} = \begin{bmatrix} 3\% \\ -1\% \\ 2\% \end{bmatrix}$$

$$\begin{aligned} \tau\boldsymbol{\Sigma}\boldsymbol{P}^T &= \tfrac{1}{30}\begin{bmatrix} 10\times10^{-3} & 0,8\times10^{-3} & 6\times10^{-3} \\ 0,8\times10^{-3} & 6,4\times10^{-3} & 2,8\times10^{-3} \\ 6\times10^{-3} & 2,8\times10^{-3} & 14,4\times10^{-3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 2\times10^{-4} \\ 0,93\times10^{-4} \\ 4,8\times10^{-4} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} [(\tau\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T) + \boldsymbol{\Omega}]^{-1} &= \left\{ \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\times10^{-4} \\ 0,93\times10^{-4} \\ 4,8\times10^{-4} \end{bmatrix} + 4,32\times10^{-3} \right\}^{-1} \\ &= [4,8\times10^{-3}]^{-1} = 208,3 \end{aligned} \tag{3-14}$$

$$[\boldsymbol{q} - \boldsymbol{P}\boldsymbol{\pi}] = \left\{ 6\% - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3\% \\ -1\% \\ 2\% \end{bmatrix} \right\} = [6\% - 2\%] = 4\%$$

$$\begin{aligned} \hat{m} &= \begin{bmatrix} 3\% \\ -1\% \\ 2\% \end{bmatrix} + \begin{bmatrix} 2\times10^{-4} \\ 0,93\times10^{-4} \\ 4,8\times10^{-4} \end{bmatrix} \times 208,3 \times 4\% \\ &= \begin{bmatrix} 3\% \\ -1\% \\ 2\% \end{bmatrix} + \begin{bmatrix} 0,17\% \\ 0,08\% \\ 0,4\% \end{bmatrix} = \begin{bmatrix} 3,17\% \\ -0,92\% \\ 2,4\% \end{bmatrix} \end{aligned}$$

$$\hat{\boldsymbol{V}} = \boldsymbol{\Sigma} + [(\tau\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P}]^{-1}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} 10\times10^{-3} & 0,8\times10^{-3} & 6\times10^{-3} \\ 0,8\times10^{-3} & 6,4\times10^{-3} & 2,8\times10^{-3} \\ 6\times10^{-3} & 2,8\times10^{-3} & 14,4\times10^{-3} \end{bmatrix}$$

$$(\tau\boldsymbol{\Sigma})^{-1} = \begin{bmatrix} 4012 & 251,2 & -1720 \\ 251,2\times10^{-3} & 5139 & -1104 \\ 1720 & -1104 & 3015 \end{bmatrix}$$

$$\boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [4,32\times10^{-3}]^{-1} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 231,5 \end{bmatrix} \quad (3\text{-}15)$$

$$[(\tau\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^T\boldsymbol{\Omega}^{-1}\boldsymbol{P}]^{-1} = \begin{bmatrix} 4012 & 251,2 & -1720 \\ 251,2\times10^{-3} & 5139 & -1104 \\ 1720 & -1104 & 3246 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} 0,33\times10^{-3} & 0,023\times10^{-3} & 0,18\times10^{-3} \\ 0,023\times10^{-3} & 0,21\times10^{-3} & 0,084\times10^{-3} \\ 0,18\times10^{-3} & 0,084\times10^{-3} & 0,432\times10^{-3} \end{bmatrix}$$

$$\hat{\boldsymbol{V}} = \begin{bmatrix} 10,33\times10^{-3} & 0,823\times10^{-3} & 6,18\times10^{-3} \\ 0,823\times10^{-3} & 6,61\times10^{-3} & 2,884\times10^{-3} \\ 6,18\times10^{-3} & 2,884\times10^{-3} & 14,832\times10^{-3} \end{bmatrix}$$

With $\hat{\boldsymbol{m}}$ and $\hat{\boldsymbol{V}}$ we obtain the reduced matrix $\hat{m}_s$ removing the lines associated to macrofactor returns from $\hat{\boldsymbol{m}}$, and $\hat{\boldsymbol{V}}_s$ removing the lines and columns associated to macrofactor returns from $\hat{\boldsymbol{V}}$.

We can then define our allocation with $\hat{\boldsymbol{m}}_{\boldsymbol{s}}$ and $\hat{\boldsymbol{V}}_s$.

$$\hat{\boldsymbol{m}}_s = \begin{bmatrix} 3,17\% \\ -0,92\% \end{bmatrix}$$
$$\hat{\boldsymbol{V}}_s = \begin{bmatrix} 10,33\times10^{-3} & 0,823\times10^{-3} \\ 0,823\times10^{-3} & 6,61\times10^{-3} \end{bmatrix} \quad (3\text{-}16)$$

4
Case Study

We analyze the performance of the MBL model considering a daily investment strategy that sets the views with data collected from BCB[1]. The study is performed with daily data from March 2010 to October 2018[2].

4.1
The Views

The views are constructed with the median of the top 5 predictors estimates for:

1. The Brazilian monthly inflation rate prediction (BIR);
2. The Real to US Dollar exchange rate, obtained from end of the month prediction (BRLUS) and;
3. The target Brazilian interest rate (Selic).

We convert the end of the month daily exchange rate into returns by calculating the uniform daily returns that would take the present exchange rate into the end of the month prediction. With these information we set $\boldsymbol{P}_t$ and $\hat{\boldsymbol{q}}_t$. The variances of the predictions are used to set $\boldsymbol{\Omega}_t$.

4.2
The Securities

Table 4.1 shows the securities available for investment, Table 4.2 presents their correlations and Figure 4.1 presents their cumulative return during the full sample period.

As presented in chapter 2, in the absence of market weights for the securities or macroeconomic factors, we use historical averages (instead of using CAPM to estimate the value of $\hat{\boldsymbol{\pi}}_t$).

The windows lengths chosen for mean ($\hat{\boldsymbol{\pi}}_t$) and covariance ($\hat{\boldsymbol{\Sigma}}_t$) estimation were 60 and 90 business days respectively, as discussed before, different

[1]Brazilian Central Bank holds a system called Market Expectations System to gather data from market players.

[2]The starting of the period was chosen given the BCB system data availability.

Table 4.1: List of available investment securities

Security	**Ticker**
US Dollar	BRLUS
Brazilian stock index	IBOV
Interbank deposit rate	CDI
Brazilian inflation-linked bonds with constant duration of 3 years	iDkA I3
Brazilian fixed income bonds with constant duration of 3 years	iDkA P3

Table 4.2: Securities correlation (full sample period)

	CDI	**BRLUS**	**IBOV**	**iDkA I3**	**iDkA P3**
CDI	1.000	-0.009	0.001	0.040	0.030
BRLUS	-0.009	1.000	-0.340	-0.232	-0.330
IBOV	0.001	-0.340	1.000	0.256	0.313
iDkA I3	0.040	-0.232	0.257	1.000	0.831
iDkA P3	0.030	-0.330	0.313	0.831	1.000

windows are important as the estimation errors on means and covariances are different. As discussed, we estimate $\hat{\mathbf{\Sigma}}_{\pi,\,t}$ as $\frac{1}{90}\,\hat{\mathbf{\Sigma}}_t$.

4.3 The Results

Once with $\hat{\mathbf{\Sigma}}_t$, $\hat{\boldsymbol{\pi}}_t$, $\hat{\mathbf{\Sigma}}_{\pi,\,t}$, $\boldsymbol{P}_t$, $\hat{\boldsymbol{q}}_t$ and $\mathbf{\Omega}_t$, we could use 2-8 and 2-9 to obtain $\hat{\boldsymbol{m}}_t$ and $\hat{\boldsymbol{V}}_t$ and then, with $\hat{\boldsymbol{m}}_{s,t}$ and $\hat{\boldsymbol{V}}_{s,t}$, run the optimizer. The optimization given by 3-9 is carried out setting σ^2 to $\frac{0.05^2}{250}$. By doing so, we set a 5% target for annualized standard deviation considering 250 business days in a year. This value represents a common benchmark for Brazilian hedge funds[3]. The constraint $w_{i,t} \geq 0$ was used to avoid short selling or further leverage.

We consider six portfolios as described below:

1. Mean-variance optimization using historical data (MVO);

2. MBL model considering BCB views on **BRLUS**, **BIR** and **Selic** (MBL BCB);

3. MBL model considering perfect t+1 views[4] on **BRLUS**, **BIR** and **Selic** (MBL PV), used to evaluate the effect of perfect views on the returns;

4. MBL model considering perfect t+1 views on **BIR** and **Selic** (MBL PV -BRLUS), used to evaluate the effect of perfect views on macroeconomic

[3]It is roughly the standard deviation of a portfolio consisting of 20% IBOV and 80% CDI

[4]Perfect t+1 views means we set the views at day D predicting the return of the security as the t+1 return of the security, for all the time interval.

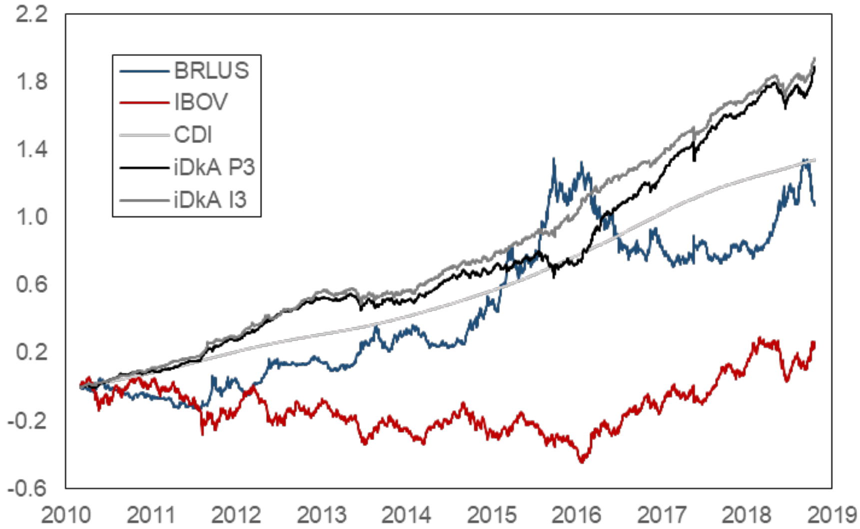

Figure 4.1: Securities cumulative returns (March 2010 to October 2018)

factors on the returns as **BRLUS** is also a security available for investment;

5. MBL model considering BCB views on **BIR** and **Selic** (MBL PV - BRLUS), used to compare the effects of BCB views only on macroeconomic factors;

6. Invested in interbank deposit rate (CDI).

Table 4.3 and Figure 4.2 shows the out-of-sample results for the optimized portfolios, its annualized returns, annualized volatility[5] and maximum drawdown, from July 2010 to October 2018.

Table 4.3: Out-of-sample portfolios returns, volatility and drawdown

Portfolio	**Ticker**	**Ann. Ret.**	**Ann. σ**	**Drawdown**
1. MVO	MVO	14.70%	6.99%	6.24%
2. MBL using BCB views on BRLUS, BIR and Selic	MBL BCB	17.02%	6.92%	6.11%
3. MBL using t+1 perfect views on BRLUS, BIR and Selic	MBL PV	40.41%	6.71%	6.21%
4. MBL using t+1 perfect views on BIR and Selic	MBL PV -BRLUS	16.33%	6.86%	5.72%
5. MBL using only BCB views on BIR and Selic	MBL BCB -BRLUS	16.54%	6.86%	5.34%
6. Interbank deposit rate	CDI	10.31%	0.14%	0.02%

The higher cumulative returns for MBL PV reflect the use of the perfect view for the BRLUS, which is a security included in the investment universe. Although its results are not useful to validate the MBL model, we present the results to show the impact of a perfect security view on the out-of-sample cumulative portfolio returns.

[5]Assuming 250 business days on the Brazilian calendar year.

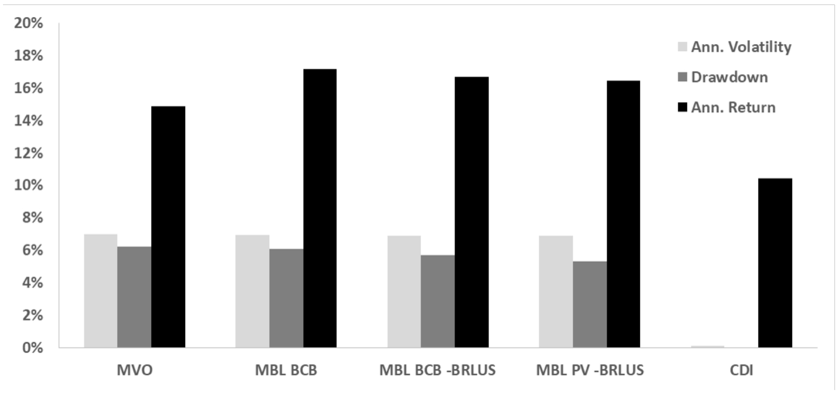

Figure 4.2: Out-of-sample portfolios returns, volatility and drawdown

All portfolios surpass the 5% target for annualized standard deviation, which is expected given that results are out-of-sample and the covariances estimates change over time. As covariances in $\hat{\boldsymbol{V}}_t$ are greater than covariances in $\hat{\boldsymbol{\Sigma}}_t$ due to the model correction, the MVO portfolio which is the only one that uses $\hat{\boldsymbol{\Sigma}}_t$ in its risk constraint has a higher standard deviation.

The use of perfect views on BIR and Selic[6] (MBL PV -BRLUS) increases the return and lowers the risk, compared to MVO even though there are not any known factor model that relates the securities' returns to such factors. The use of BIR and Selic views from BCB (MBL BCB -BRLUS) provides very similar returns, and using BRLUS views (MBL BCB) further increase the overall outcome. Figure 4.3 presents the cumulative returns for all portfolios, except MBL PV, compared to the CDI benchmark.

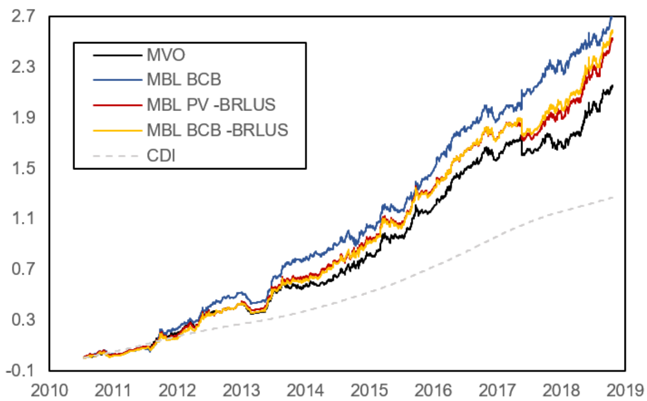

Figure 4.3: Cumulative returns of studied strategies

One can see that portfolios MVO, MBL PV -BRLUS and MBL BCB -BRLUS have a very similar profile and that portfolio MBL BCB, with BCB views on BRLUS, outperforms other portfolios on downfalls, since the BCB

[6]The two macroeconomic factors considered in this study.

view on BRLUS seems to be effective to avoid more downfalls. The proposed MBL model seems to enhance the regular MVO using either good views on the factors, as the returns using perfect views outperforms, or using the BCB public views[7].

Figure 4.4 and Figure 4.5 present the dynamic asset allocation and cumulative returns for portfolios MVO and MBL BCB respectively.

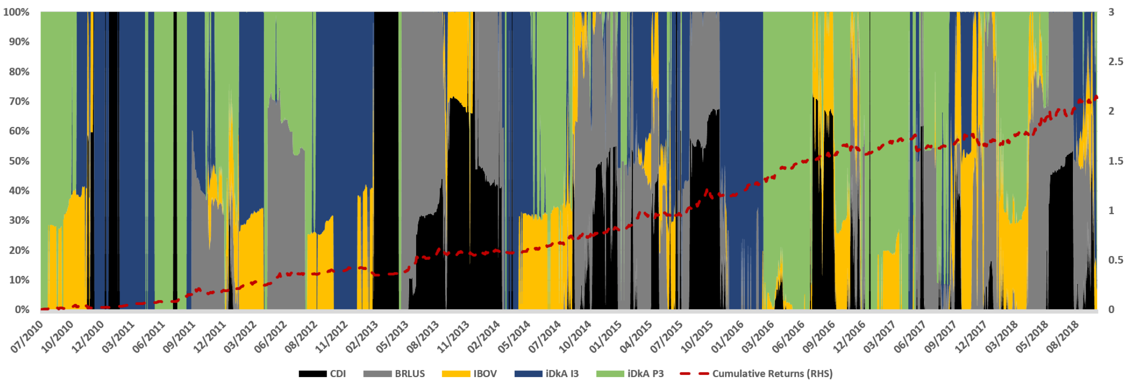

Figure 4.4: Allocations and cumulative return for Portfolio MVO

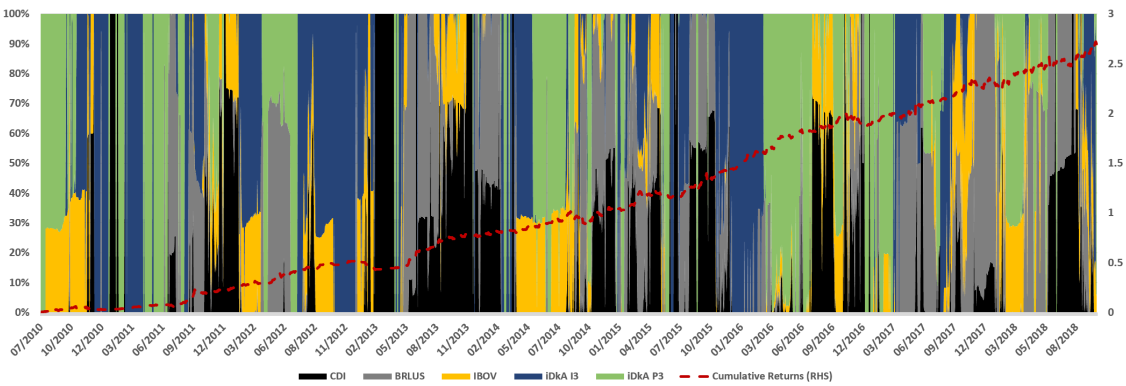

Figure 4.5: Allocations and cumulative return for Portfolio MBL BCB

One can see that the MBL is more dynamic, since not only the views on returns but also the views' variances may change suddenly, and both information are relevant. For instance the change on BRLUS view variance in May 2017 (presented in Figure 4.6) made it possible to the MBL BCB portfolio to avoid the downfall suffered by other portfolios as seen on Figure 4.3.

[7]RHS means the right hand side axis

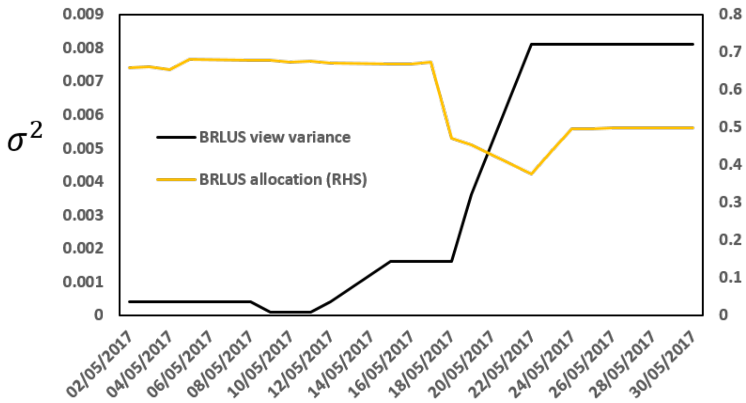

Figure 4.6: BRLUS view risk May 2017

5
Conclusions

Our contributions are: (i) the Macrofactor Black-Litterman model, which enables the investors to enhance their investments using views on macroeconomic factors that, despite not being explicit linked to securities through a full linear model, affect somehow securities returns in a simple and intuitive way; (ii) a framework for setting views on returns based on information disclosed by BCB; and (iii) a case study to test the contributions (i) and (ii). Using our model we were also able to eliminate the parameters τ and δ of the original Black-Litterman model, which are many times criticized, by relying on historical information and variances of the securities.

Within the case study presented, focused on investment in Brazilian assets, we blended BCB views on one tradable security and two macroeconomic factors using historical data. The optimized portfolios outperformed the MVO portfolio with historical data on every studied scenario, implying that the use of the proposed model setting views using both the BCB database or good estimates have good outcomes for the tested example. As we used two synthetic securities with constant duration (iDkA I3 and iDkA P3), which in practice might be built using derivatives, we limited the use of leverage in our example.

5.1
Future Works

As noticed in section 4.3, using the MBL model presents a more dynamic allocation, and that is a reason why we understand that, for more realistic results, an extension of this thesis would be to consider transaction costs.

We use only the next time step prediction for the macroeconomic factors, in a future work one could mix the multiple views for different time horizons to try to obtain the effect of more subtle and future change on the predictions.

In a broader application it is possible to also consider a larger number of macroeconomic factors and securities, for instance, indexes related to different sectors and segments on the Brazilian Stock Exchange that are more sensitive to the macroeconomic factors predicted on the BCB survey. An alternative would be searching data on the American market for daily, weekly or monthly predictions (for example the New York Federal Reserve Bank's Survey of Market Participants[1]) and related securities.

[1] See https://www.newyorkfed.org/markets/survey_market_participants for more information

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