Pontifícia Universidade Católica do Rio de Janeiro

Claudia Interlandi

Safety assessment of reinforced concrete structures with a global resistance approach

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Engenharia Civil of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Engenharia Civil.

Advisor: Prof. Luiz Fernando Campos Ramos Martha
Co-advisor: Eng. Luís Miguel Pina de Oliveira Santos

Rio de Janeiro
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To Angelo and Olivia,
my everything

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Abstract

Interlandi, Claudia; Martha, Luiz Fernando Campos Ramos; Santos, Luís Oliveira (Advisor). **Safety assessment of reinforced concrete structures with a global resistance approach.** Rio de Janeiro, 2020. 154p. Tese de Doutorado – Departmento de Engehnaria Civil e Ambiental, Pontifícia Universidade Católica do Rio de Janeiro.

Structural safety assessment has evolved from the traditional allowable stress design to more rational methods based on limit states. Current practice on the limit states verifications are based on the partial factor method. However, since the beginning of this century, the arbitrariness in the definition of the partial factors used to increase the values of the loads and to reduce the properties of the material has been recognized. This simplified procedure are often conservative and may lead to expensive upgrades. This situation has motivated the development of methods for safety verification based on a probabilistic approach. Parallel to this, it is sought to develop, within deterministic methods, processes more compatible with real situations of rupture of structures, such as the global safety approach. Structural design standards are progressively introducing these new methods. However, a systematic analysis for the quantification of probabilistic safety resulting from the application of these methods has not yet been found in the literature. The objective of this thesis is to contribute in this safety assessment of reinforced concrete structures, analyzed by the conventional methods of Limit States, by Global Safety methods and by complete probabilistic analyses. Within the probabilistic analyses, it is tried to overcome a great difficulty in the application of the methods that is the evaluation of the probabilistic characteristics of the variables, by the application of a Bayesian approach, that allows a continuous revaluation of the variables, as well as the application in this definition of "engineering judgment" of the analyst. . An important point of the analysis that the reliability analysis usually done in isolated sections of a structure can lead to conservative and misleading results, since the structures behave as a whole.

Keywords

Reliability; reinforced concrete; Global safety approach.

Resumo

Interlandi, Claudia; Luiz Fernando Campos Ramos Martha; Luís Oliveira Santos. **Avaliação da segurança de estruturas de concreto armado com uma abordagem de resistência global.** Rio de Janeiro, 2020. 154p. Tese de Doutorado - Departamento de Engenharia Civil e Ambiental, Pontifícia Universidade Católica do Rio de Janeiro.

A avaliação da segurança das estruturas vem evoluindo do formato tradicional das Tensões Admissíveis para métodos mais racionais baseados em Estados Limites a partir de meados do século passado. A partir de finais deste século, se reconheceu a arbitrariedade na definição de coeficientes de majoração de cargas e minoração de resistências implícitas nestes métodos e ferramentas de análise de segurança baseadas em métodos probabilísticos têm sido desenvolvidos. Em paralelo a isso, busca-se desenvolver, dentro dos métodos determinísticos, processos mais compatíveis com as situações reais de ruptura das estruturas, como a abordagem por Segurança Global. As normas de projeto de estruturas vão progressivamente introduzindo estes novos métodos. Porém, ainda não se encontra na literatura uma análise sistemática para quantificação da segurança probabilística consequente da aplicação destes métodos. O objetivo deste trabalho é contribuir nesta avaliação de segurança em estruturas típicas de concreto armado, analisadas pelos métodos convencionais dos Estados Limites, pelos métodos de Segurança Global e por análises probabilísticas completas. Dentro das análises probabilísticas, procura-se superar uma grande dificuldade na aplicação dos métodos, que é a avaliação das características probabilísticas das variáveis, pela aplicação de uma abordagem Bayesiana, que permite uma reavaliação contínua das variáveis, assim como a aplicação nesta definição do "julgamento de Engenharia" do analista. Um ponto importante da análise de confiabilidade, quando é feita em seções isoladas de uma estrutura pode levar a resultados conservadores e enganosos, uma vez que as estruturas se comportam como um todo.

Palavras-chave

Confiabilidade; concreto armado; segurança global.

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Table of contents

List of figures

List of tables

List of abbreviations, symbols and acronyms

Roman and Latin characters

A	cross section area
A_c	concrete area
A_s	steel area
b	section width
b_w	section web width
c	reinforcement cover
Cov()	covariance between two random variables
d’	distance from the reinforcement cg to the nearest edge of the section
E(X)	expected value (or mean value) of the random variable X
E_s	longitudinal elasticity modulus of steel
f_c	compressive strength of concrete
f_{cd}	design value of the compressive strength of concrete
f_{ck}	characteristic value of the compressive strength of concrete
f_{cm}	average compressive strength of concrete
$F_X(x)$	cumulative probability function
$f_X(x)$	probability density function
f_y	yield strength of steel
f_{yd}	*design* yield stress of steel
f_{yk}	characteristic value of the tension strength of steel
f_{ym}	average value of tension strength of steel
G	failure function or behavior function
H	cross section height
M	bending moment
N	normal strength
P_f	failure probability
s	sample standard deviation
V(X)	coefficient of variation of variable X
Var(X)	variance of a random variable X
x	sample mean

Greek characters

β	reliability index; $\beta = -\Phi^{-1}(p_f)$
ε_0	axial deformation in the center of gravity
ε_c	specific deformation of concrete
ε_s	specific deformation of steel
λ	Global safety factor
ϕ	diameter of steel bar
μ	average value, or the mean, of a random variable
σ_c	compressive stress in concrete
σ_s	normal tension in steel; service stress in the reinforcement
χ	curvature at the center of gravity

Acronyms and Abbreviations

ABNT	Associação Brasileira de Normas Técnicas
DEC	Departamento de Engenharia Civil
FEA	Finite Element Analysis
FORM	First Order Reliability Method
MCS	Monte Carlo Simulation
MPP	Most Probable Point
PDF	Probability Density Function
SORM	Second Order Reliability Method
SLS	Service Limit State
ULS	Ultimate Limit State

"Tudo é ousado para quem a nada se atreve."
Fernando Pessoa

1
Introduction

1.1
Initial considerations and motivation

The importance of assessing the safety of existing structures has grown significantly over the years. The number of existing structures, such as bridges, which are sometimes very old and do not receive the necessary maintenance and repair services over years or decades, and sometimes end in serious accidents, such as the collapse of the Genoa bridge (2018), with dead and wounded, makes clear and immediate the need for increased investment in investigations and studies in the area of structural safety. In the meetings and congresses of the main groups of civil engineering around the world, this is undoubtedly one of the most discussed topics, given its importance. Figure 1.1 and Table 1.1 show some major serious accidents around the world, in the past two decades.

Figure 1.1 - Some major bridge accidents in the past two decades

Table 1.1 - Some major bridge accidents in the past two decades in numbers

Date	Accident	Local	Death	Injured
14/08/2018	Viaduct fall	Genoa, Italy	38	> 10
15/03/2018	Collapse of pedestrian bridge	Miami, USA	6	> 10
02/01/2018	Collapse of pedestrian bridge, over Apurímac River	Cuzco, Peru	5	> 10
17/10/2016	Bridge fall	Bali, Indonesia	8	> 30
27/07/2010	Bridge fall	Luanchuan, China	37 + 19 missing	N/I
26/09/2007	Bridge fall	Mekong, Vietnam	49	> 10
01/08/2007	Bridge collapse over the Mississippi River	Minnesota, USA	11	N/I
07/11/2005	Fall from a suspended platform on an overpass	Granada, Spain	6	> 5
23/12/2003	Bridge fall	Cochabamba / Santa Cruz, Bolivia	> 50	N/I
05/03/2001	Bridge fall	Entre-os-Rios/ Castelo de Paiva, Portugal	> 70	N/I

Ensuring that accidents of this type do not occur is a priority. Special bridges are vital in the context of national infrastructure and must be reevaluated in order to assure high coefficients of structural reliability, assuring their physical integrity and especially, the safety and well-being of people.

The codes aim to ensure that the structures have acceptable risks and minimum total costs during the design life. With the introduction of the concept of Global Safety in the *fib* Model Code 2010 (MC2010), in its item 4.6, from the conceptual aspects exposed, for example, by Cervenka (2013), new studies for the assessment of the safety of structures associated with the design, following these new concepts, it is necessary. The appropriate approach for this assessment is the Reliability Analysis, as exposed, for example, by Melchers and Beck (2018).

For the Global Safety Analysis, it is necessary to re-evaluate the maximum resistant forces in the structural sections using, instead of the design values of the resistance, their average values. Then, a factor λ must be found, which increases the forces until the final situation of collapse. Reliability analyses can then be performed. Reliability indices are found for a conventional sectional analysis and for the final collapse situation of the Global Analysis. The results obtained are analysed and compared. The conclusions about the Global Safety Analysis may or may not be applied without jeopardizing structural safety, are presented at the end of this work.

Perhaps, the most significant point of this work is the comparison of the results obtained in the localized analyzis, beam and column separately, and the results obtained when analyzing the structure as a whole, and proving the efficiency of global safety checks of existing structures is an important verification to performed.

1.2
History and bibliographic review

The Monte Carlo simulation method (MC) and the First Order Analytical Method (FORM) among the most used in Reliability Analysis. The first ideas about the Monte Carlo method arose in 1946 when the mathematician Stanislaw Ulam played the Solitary card game and together with John Von Neumann, and after years of research on random events, published the article The Monte Carlo Method (1949). The FORM (First Order Reliability Method) method allows for the incorporation of probability functions as well as the correlation between the random variables in the problem under the analysis. According to Melchers (2002), its concepts were initially disseminated by Mayer in 1926, although they were only better accepted after Cornell's work as Benjamin & Cornell (1970).

Over time with the implementation of the formulations of the mathematical methods FORM and Monte Carlo, Rackwitk and Fiessler (1978), Elligwood et al. (1995) and Melchers (2002) presented some important works.

Frangopol et al. (1996) carried out studies on the reliability of slender and short reinforced concrete columns. The reliability analysis was performed using the Monte Carlo simulation method and it was verified that the reliability of these structures would depend on the loading sequence and the correlation between the loads.

Henriques (1998) applied the concepts of safety for the design of concrete structures, using a reinforced concrete viaduct as a case study. In conclusion, the application of probabilistic methods in the verification of structural safety for non-linear models is conditioned by its rigor and effectiveness, which is assuring in the application of classic reliability techniques. However, a large number of samples is required for assuring a small margin of error. Given these requirements, the calculation time can be extremely long and prohibitive, when using very sophisticated nonlinear models, by the finite element method.

Neves (2001) presented a paper on simplified probabilistic analysis of structural safety, a synthesis of the new model code for probabilistic structure verification. The use of such a regulation is much more complex than that of semi-probabilistic regulations. Thus, its use will be restricted, in the coming years, to special structures or existing structures, where the use of nonlinear analysis models makes the application of semi-probabilistic models problematic. It has also been demonstrated that some current engineering practices, such as the adoption of higher safety factors in elements with fragile rupture or the use of quality control to increase the safety of structures by reducing the variability of the various actors, have a valid theoretical basis.

According to Soares et al. (2002), the first work to apply reliability methods to reinforced concrete structures dates from 1947, when Freudenthal published his work on reliability concepts applied to structural designs. Since then, the main international codes have proposed to incorporate these concepts.

Szerszen and Nowak (2005), reliability analyses of reinforced concrete columns subjected to eccentric load were developed. These reliability analyses were performed using Monte Carlo simulations. The study proposes a new model for the strength reduction factor of reinforced concrete columns subjected to eccentric load.

Lopes (2009) presents a methodology for evaluating the safety of existing structures using a reliability analysis by probabilistic methods. The chosen structure was an overpass over a railway line, and for the analysis, two critical sections were adopted, for two loading combinations on this bridge. The Latin Hipercube method was used, which is a simulation method similar to the Monte Carlo method, which proved to be effective for the evaluation of this existing structure. Of the two sections, which were considered unsafe, it was found that one of them on one of the columns still had a large reserve of resistance, becoming, after the analysis, considered as safe.

Jacinto (2011) contributed with his safety assessment of existing bridges using a Bayesian probabilistic approach, and as a case study, evaluated the safety of an existing bridge in an advanced state of deterioration, which needed to be demolished. The probabilistic analysis by the FORM method allowed, with a sensitivity analysis, to show which variables whose uncertainty most contributed to the estimated reliability. The potentialities of the Bayesian approach were also evidenced in the scope of the safety assessment of existing bridges, once collected additional information related to the previously selected variables. This approach offers the mechanism that allows adding this new information to the one initially had, updating the respective probability distributions and consequently the reliability of the structure. The Bayesian approach assures that statistical uncertainty is always considered.

Matos (2015) evaluated the methods for designing reinforced concrete columns, according to EC2, adopting a safety assessment based on a non-linear analysis. To enable the simulation of the real behavior of the structure to the second order effects, it adopted the existing simplification in EC2 for columns, by the Nominal Stiffness and Nominal Curvature Methods. When comparing the analysed frames, it was concluded that there is a great proximity to the safety levels found for the first and second order designs. This proximity allowed him to positively assess the quality of the design of the 2^{nd} order simplified methods of EC2, since it removes the subjectivity of the existing simplifications in the safety assessment methodology, being possible to calibrate the 2^{nd} order results by the 1^{st} order results consensually and widely used.

Regarding the assessment of safety in structures, analysed by the methods of Global Safety and by complete probabilistic analyses, there are few studies found, as those of Cervenka (2013), Allaix *et al* (2013) and Silva (2013).

Cervenka (2006) introduces a new safety format suitable for the design of reinforced concrete structures using non-linear analysis. The safety format is based on the overall assessment of structural safety. The new method is called ECOV (coefficient of variation estimate). According to the author, the advantage of the proposed method is that it can capture the sensitivity of resistance to random variation of the input variables and, thus, reflect the effect of the Safety failure mode. The method requires two non-linear analyses with average and characteristic values of the input parameters, respectively. Other safety formats suitable for non-linear analyses based on Global Safety are presented: an approach proposed by EN 1992-2, with a fully probabilistic analysis and a simple one based on design values of input parameters, that is, characteristic parameters reduced by factors partial safety measures.

For the MC2010, it is important to validate the model for its application in engineering practice. The verification of the ultimate limit states through numerical simulations and the introduction of the Global Safety format suggested for this purpose, become very attractive, especially for existing structures. The importance of these concepts is increasing, although there are still few references and studies in this specific subject.

This work aims to contribute in some way to the continuity of research and studies on Global Safety analysis.

1.3
Objectives and main contributions

In view of the above, one of the main objectives of this thesis is to contribute for researching and for updating the design standards in relation to the Global Safety analysis, with the reassessment of the maximum resistant forces in the critical structural sections and the assessment of safety, quantified in terms of reliability indexes.

The fundamentals of the methods for the safety evaluation of new and existent structures are presented in this Thesis, showing the evolution from the deterministic methods of Allowable Stresses and Ultimate Limit States to the modern methods, based on the probabilistic evaluation of the structural safety.

A novel methodology is proposed herein, based on a Global Safety approach, where applied loads are progressively increased up to the final structural collapse, being the corresponding failure probabilities evaluated in each step, allowing for establishing a relationship between global safety factors and failure probabilities.

The methodology also includes a practical procedure for updating the evaluation of the resistances, applying a Bayesian approach. After the analysis of several dozens of actual concrete tests, a typical behavior pattern was found, allowing for proposing a very simple procedure for the updating concrete strengths.

The Thesis presents also a description of the available probabilistic methods for the evaluation of structural safety. A very complete and detailed description and definition the probabilistic characteristics of each of the variables involved in the safety evaluation of concrete structures is presented. These definitions are based on data found in the international literature, but also in the Brazilian construction experience. The definitions are also adjusted to the requirements of Brazilian Standards, regarding for instance the recurrence period defined for each of the variable loads.

This methodology will be particularly useful for the evaluation of existing structures, for estimating the probabilities of failure, along the remaining working life of the structure. However, in the presente work, it was first applied to the design of a new structure: the analysis is for the central frame of a hypothetical building

The methodology is also applied for the analysis of an existing bridge, located in a region of medium seismicity, designed in a time when there was not any requirement for seismic resistance in Brazil. Bayesian updating of concrete strength was applied and also the Global Resistance approach.

In summary, it can be said that the main objectives of this work are: make a review of reliability concepts and their applications, development of a methodology for the verification of structural security based on the "Global Resistance approach", demonstrate the applicability to building structures, demonstrate its usefulness in assessing the safety of existing structures and contribute for researching and for updating the design standards.

1.4
Organization of work

This work is organized in the chapters described in a summary form as follows.

Chapter 1

Deals with the initial considerations and motivation for this work, including the history and bibliographic review that show the need for studies referring to Global safety coefficients for structures. Also inludes de main objectives presented in this work.

Chapter 2

Presents, in a general way, the basic concepts of structural safety. The ultimate limit state criterion is briefly addressed, subdivided into local and Global analyses. Finally, probabilistic criteria are addressed, focusing on local analyses, Global analyses and aspects of the Bayesian approach.

Chapter 3

Presents structural reliability applications, showing some of the main methods of analysis, Monte Carlo, FORM and SORM. It also brings up the discussion of what are acceptable safety indices and the aspects considered in safety assessments of structures. Finally, the criteria adopted in the choice of variables for the modelling of structures and actions are discussed, as well as how to implement the updating of variables using the Bayesian method.

Chapter 4

Describes the analysis of a frame with a global approach, referring to a building of 13 floors in reinforced concrete. Analyses are performed using the Ultimate Limit States and Global Safety methods and the Reliability indices corresponding to the two processes are evaluated. The main variable action considered is the wind.

Chapter 5

Describes the analysis of a typical bridge located in the Northeast of Brazil, with a global approach, the specific example being the Bridge over the Madeira River, located in Sobral, Ceará. Analysis is performed using the Ultimate Limit States and Global Safety methods and the Reliability indices corresponding to the two processes are evaluated. The main variable action considered is the earthquake.

Chapter 6

Presents the relevant conclusions registered during the development of this work, as well as the final considerations and suggestions for the continuation of future studies on the subject.

2
Structural safety

2.1
Basic considerations and concepts

The concept of structural safety is based on the idea that every structure is designed to perform in the best possible way, for all the purposes for which it was designed, in order to assure maximum ecconomy, efficiency and durability and, mainly, withstanding the actions for which is exposed throughout its life without rupture.

The numerous sources of uncertainty present in the structures are responsible for the variability of the parameters that characterize their behavior and, therefore, structural evaluation problems have a fundamentally non-deterministic character. In the past, before regulations existed, safety depended on the prior experience and intuition of those who designed and built it. With the development of the Strength of Materials theory in the 20th century, the first scientifically based safety assessment rules emerged, with the admissible stress method, where the basic principle was to assure that in the critical zones, the maximum stress did not exceed the resistance of the materials divided by a safety factor, fixed arbitrarily. This was the safety criterion adopted for almost a century.

With the need for more refined safety assessments in relation to the risk involved in this type of problem, studies and safety analyses were developed within a probabilistic approach and, thus, the concept of probability of failure and the definitions of the risks involved and identified emerged for various situations.

New criteria for the verification of Safety on a probabilistic basis have emerged, considering the following points (Henriques 1998):

- definition of limit states or situations to avoid;

- estimation of the severity of the consequences resulting from these limit states being disrespected,

- definition of safety coefficients and appropriate devices so that the probability of these limit states being disregarded is small enough to be considered acceptable.

These concepts provide the basis for the Limit State Methods, which are adopted currently on design.

In recent years, significant studies have been carried out in the field of evaluating the safety of structures and in the safety formats used in the design of new structures, using reliability methods. In relation to existing structures, development has been advancing in recent years due to the extreme need to assure and prolong the safety and working life of these structures.

Significant experience and intuition have been acquired over the past four decades in relation to the use of modern methods of structural reliability. These methods now comprise important instruments in advanced engineering assessments, as well as in the context of the development of formats for design and evaluation of structures. Despite these very positive developments, the modelling and probabilistic analysis of structural performances still includes challenges in the area of applied structural reliability analysis.

On the one hand, the wider application of modern methods of structural reliability can be of great value for a more concise image regarding how different classes of practical reliability problems should be modelled and analysed, and which tools are most effective for this. On the other hand, the rapid increase in computational capacity opens possibilities for probabilistic modelling and more efficient assessments. The last years of development of specialized probabilistic analysis tools can, in fact, facilitate the assessment of how the practical problems of reliability analysis can be dealt with much more efficiently than before and how to explore and understand the potentials for this.

The latest advances in structural design studies based on reliability analysis allow the development of new design methodologies. The Global Safety approach (Cervenka, 2013) appears with the aim of enabling the design of more economical structures, without compromising the level of structural safety. In the Global Safety approach, structural design is performed based on normative recommendations, aiming at further optimization.

2.1.1 Failure function (G(X))

A failure function or equation for any system corresponds to a G (X) function of all *n* random variables involved in the problem. When this function is equal to zero, this is a critical situation in which the limit state configuration recommended in the analysis is reached, written as follows (Beck, 2015):

$$G(X) = g(X_1, X_2, \dots, X_n) = 0 \tag{1}$$

This function has two distinct domains. When g (X) ≤ 0 the function is in the failure domain (Ω_f); in the situation where g (X) > 0 the function is in the safety domain (Ω_S).

$$\Omega_f = \{x | g(x) \leq 0\} \tag{2}$$

$$\Omega_s = \{x | g(x) > 0\} \tag{3}$$

The limit state equation adopted in the structural reliability analyses, which expresses the fundamental reliability problem, is a function that relates the resistance and solicitation variables considered in a design, as will be discussed later.

$$g(R, S) = Z = R - S \quad (4)$$

R is the random variable corresponding to the resistance and S is the random variable corresponding to the request. Therefore, the failure situation is reached in the situation where S> R, that is, when the load is greater than the structural strength, which corresponds to Z (safety margin) < 0.

The failure domain for the fundamental reliability problem is presented in the Figure 2.1.

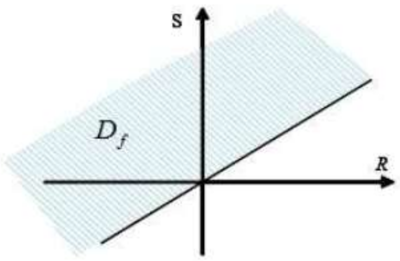


Figure 2.1 - Failure domain for R-S problem (Beck, 2015)

2.1.2
Probability of failure (P_f)

The probability of failure of a system represents the chance that the results obtained in the analyses are contained in the failure domain (X ∈ Ω_f).

$$P_f = P[\{X \in \Omega_f\}] = P[\{g(x) \leq 0\}] \quad (5)$$

If the joint probability function $f_x(x)$ is known, the failure probability can be obtained by the following integral in the failure domain (Ω_f):

$$P_f = \int f_x(x)dx \quad (6)$$

For the fundamental problem of reliability, we have:

$$P_f = P[\{R \leq S\}] = [\{(r, s) \in \Omega_f\}] \quad (7)$$

Considering the probability of resistance (R) and load (S) distributions, as known and independent, the probability of failure for the fundamental reliability problem is:

$$P_f = \int_{-\infty}^{\infty} \int_{-\infty}^{s} f_r(r) f_s(s)\, drds = \int_{-\infty}^{\infty} F_r(s) f_s(s) ds \tag{8}$$

It is perceived that the probability of failure is related to the area of overlap between the density probability and resistance functions. That is, the more distant the mean values for resistance and load, the lower the probability of failure.

Another factor that influences the area of overlap of the curves is the degree of dispersion of the random variables, which is related to the standard deviation of the distributions. The lower the degree of dispersion of the distributions, the lower the probability of failure.

2.1.3
Reliability index (β)

The reliability index β is a reference factor that expresses the degree of confidence of a structural model in relation to a failure, considering a certain mode of behavior for the structure and respecting the premises adopted in the design.

When analyzing the fundamental problem of reliability, in which $\mathrm{Z} = \mathrm{R} - \mathrm{S}$, if R and S are random variables, Z will also be a random variable. Therefore, Z has a probability density function $f_z(z)$.

The probability of failure of the Z function can be expressed as follows (Beck, 2015):

$$Pf = P[\{Z \leq 0\}] = \int_{-\infty}^{0} f_z(z) dz = F_z(0) \tag{9}$$

Transforming the Z variable into a Y variable with standard normal distribution:

$$\mu_z = \mu_R - \mu_S \tag{10}$$

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_{RS}^2} \tag{11}$$

The probability of failure can then be described as:

$$P_f = P[\{Z \leq 0\}] = P[\{Y\sigma_Z + \mu_Z \leq 0\}]$$

$$P_f = P\left[\left\{Y \leq -\frac{\mu_Z}{\sigma_Z}\right\}\right] = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right) = \Phi(-\beta) \tag{12}$$

Being β defined as:

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{13}$$

The reliability index can then be interpreted as the distance between the average value of Z and the failure situation, measured in units of standard deviation, see Figure 2.2. This definition can be used even when the Z variable has a different distribution from the normal.

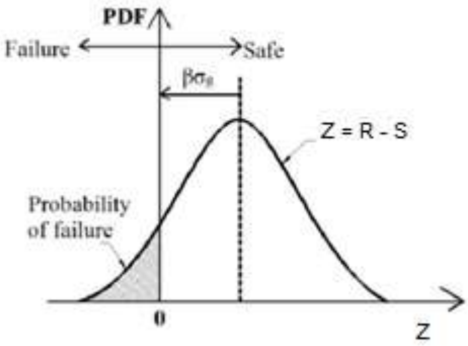


Figure 2.2 - Reliability index

The reliability index of the structure is directly proportional to its degree of confidence for the failure, and consequently, inversely proportional to its failure probability. The degree of confidence of a structure is directly related to the average value of the safety margin function (Z), the safer the structure is the higher this value. The higher the standard deviation, the greater the dispersion of the safety margin function is, which reduces structural reliability.

Once the reliability index (β) is defined, limits must be set for its values and for the probability of failure of the structures, in order to assure safety, according to the values presented in item 3.4 (Acceptable Reliability Indices).

The various uncertainties present in the structural design are then considered through the adoption of a Global safety factor (λ). This single factor is adopted for the joint consideration of all uncertainties present in the design, unlike the usual semi-probabilistic design method, in which partial safety factors are adopted for each of the variables in the design, as shown in Figure 2.3.

Semi-probabilistic method			Global safety approach	
concrete strength →	γ_c		concrete strength →	
steel strength →	γ_s	→	steel strength →	λ
permanent load →	γ_g		permanent load →	
variable loads →	γ_q		variable loads →	

Figure 2.3 - Global Safety approach x semi-probabilistic method (Santos, 2018)

In this evaluation, the factor λ is used to increase one or more loads acting on the structural model, until the collapse of the structure is reached, that is, the numerical value of λ that causes the structure to collapse is considered as the overall safety of the performed analysis.

$$\lambda = \frac{Action\ (structural\ collapse)}{Average\ action} \qquad (14)$$

In order that the analysis to be free of arbitrary definitions of characteristic values, and to be possible to determine the probability of failure and the reliability index (β) associated with the Global safety factor (λ), the resistance and solicitation variables are taken with their average values, avoiding the influence of the arbitrary definition of the characteristic design variables.

2.2
Limit state criteria

2.2.1
Local analysis

The safety verification methods should consider the appropriate form as uncertainties related to the variables that interfere in the characterization of the actions and in the structural response. A design solution applied to applications that use removal methods is a safety margin in relation to the different limit states, according to the probabilities of occurrence (Henriques, 1998).

The allowable stresses method was the first scientific-based structural design criterion, which considers the stresses that can act on the structure admitting a linear elastic behavior. The allowable stresses due to the design loads are deterministically admitted as the maximum values presented during the life of the structure. The maximum stresses that may occur in the structure must not exceed the value of the corresponding rupture, yielding or instability stresses of the materials, divided by an internal safety factor, γi, greater than one. The quotient of the rupture the yielding stress of the material by the internal safety factor, is called the allowable stress.

The principle of the method consists in calculating the stresses σ in the linear elastic regime for the maximum expected actions and comparing it with the allowable stress σ_{adm}, which is a fraction of the limit stress related to a rupture. In summary, the allowable stress method can be defined by eq. (15).

$$\sigma \leq \sigma_{adm} = \frac{\sigma_{lim}}{F.S.} = \frac{(\sigma_e \; ou \; \sigma_r \; ou \; \sigma_{inst})}{F.S.} \tag{15}$$

Limit State is a condition from which the structure no longer meets one or more requirements, being in some way impaired in the performance of the functions for which it was built. In other words, in this method, the level of Safety of a structure is determined by the capacity that it has to support the several actions that come to request it during its working life without collapsing or reaching any ultimate limit state or service imit state.

The safety verification methods must consider, in the most appropriate possible way, the uncertainties associated with the variables that intervene in the characterization of the actions and of the structural response. The design solution by applying these methods should ensure a safety margin in relation to each limit state, according to its probability of occurrence. This assessment can be considered as a decision-making process, at the discretion of the engineer's experience and intuition.

Structural quality and levels of reliability in relation to limit states to be required of structures is a decision problem that involves areas outside the exclusive jurisdiction of the engineer (Ferry Borges, 1982).

Depending on the severity of the damage, the limit states are classified into two types:

- Ultimate limit states, which are associated with serious damage and which imply the finalization of the use of the structure, which can reach collapse. The ultimate limit states concern not only the safety of the structure itself (its physical integrity), but also (and, above all) the safety of individuals. This means that any adverse state in a structure that compromises the safety of people is serious enough to be classified as the ultimate limit state
- Serviceability limit states, which states that correspond to conditions beyond which specified service requirements for a structure or structural member are no longer met.

Some examples of ultimate limit states: loss of equilibrium of the structure considered as a rigid body; limit state of resistance or excessive deformation; loss of resistance due to fatigue phenomena; instability of the structure or one of its parts.

Some examples of serviceability limit states: local damage that is likely to accelerate deterioration or impair appearance (cracking, for example); deformation incompatible with the smooth functioning of non-structural elements, or that impairs the appearance of the structure; excessive vibrations, susceptible to causing discomfort in people.

The safety verification of a structure in relation to a certain limit state is carried out using a model describing the limit state in terms of a function (called the limit state function) whose value depends on all relevant design parameters.

The verification of the limit states must be performed by a method based on probabilities. MC2010 recommends the verification of limit states by some Safety approaches, among them:

- probabilistic safety format;
- partial safety factor format;

- global safety format.

For each specific limit state, the relevant basic variables must be identified, that is, the variables that characterize environmental actions, material and soil properties, geometric parameters, etc.

The variability of the basic variables must be analysed based on the available information. In the case of a probabilistic approach, the basic variables are considered as random variables. In the case of the partial safety factors, the basic variables are treated as deterministic quantities. In the case of the Global Safety check, the Global resistance is treated as a random variable.

For each limit state, the models must be established in order to adequately describe the behavior of the structure. These models include mechanical models, which describe structural behavior, as well as other physical or even chemical models, which can describe the effects of environmental influences on the properties of materials. In principle, the parameters of such models should be treated in the same way as the basic variables and model uncertainties are considered.

The severity differentiation of the limit states should be reflected in the required levels of reliability for those limit states. More severe limit states should have lower probabilities of occurrence and, therefore, higher reliability values.

2.2.1.1 Principle of design

The verification of safety by the limit state method consists, in general, of comparing an acting quantity E with a resistant variable R, and trying to assure that E ≤ R. If this condition is fulfilled, it is said that the safety is attained. The acting quantity E is related to the actions that act on the structure and can correspond to an imposed load or displacement. The magnitude R refers to the resistance of the structure in relation to the acting action, and its quantification generally involves some kind of conventional hypothesis.

The condition E ≤ R, sometimes referred to as a safety condition, can be approached in a semi-probabilistic or probabilistic manner. Here we will deal with the probabilistic approach.

In the probabilistic approach, the quantities E and R are modelled as random variables, that is, they are represented not by a single value, but by a distribution of probabilities. Once probability distributions are assigned to E and R, the probability of the E> R event can be assessed, probability denoted here by p_f (probability of failure).

Therefore, p_f = P (E > R) denotes the probability that the safety condition E ≤ R, is not attained.

The probability p_f refers to the probability of exceeding a certain limit state, here called failure probability, being understood that structural failure simply denotes the occurrence of structural damage and encompasses not only ultimate limit states, but also service limit states.

In the structural safety probabilistic approach, it is sought that the probability p_f is less than a value previously accepted as the maximum allowable, or target value, here denoted by p_{fT}. Thus, from a probabilistic point of view, the safety check consists of assuring the attendance of the condition $p_f \leq p_{fT}$.

The following condition must be satisfied according to the *fib* Model Code 2010: $\varepsilon < \varepsilon_u$, where, ε is a generic tension of the structure and ε_u is its limit value. As a matter of operational simplicity, the condition becomes:

$E_d < R_d$ if an action effect of a component is considered,

$E_d < R_d^*$ if a multi-component action effect is considered

where, Ed denotes a design loading effect and Rd is the design resistance (and Rd* the design resistance domain).

2.2.1.2 Application in the approach by partial safety factors

On the loading side, at least the following variables should be considered: the structure's own weight, other permanent loads, wind, prestressing, other variable loads (earthquake, fire, accidental loads) and uncertainties in the modelling of the loads.

On the resistance side, at least the following parameters must be considered: concrete strength, steel strength, geometric uncertainties and resistance modelling uncertainties.

For reasons of simplification, the uncertainties related to some variables may be incorporated into other partial variable factors (for example, some geometrical uncertainties are incorporated into γ_m).

For materials, the following relationships apply:

$$\gamma_M = \gamma_{Rd} \cdot \gamma_m \tag{16}$$

$$\gamma_{Rd} = \gamma_{Rd1} \cdot \gamma_{Rd2} \tag{17}$$

where:

γ_{Rd1} = approach by partial safety factors representing the uncertainties of the model

γ_{Rd2} = partial safety factors approach representing the geometric uncertainties

2.2.2 Global analyses

With the introduction of the concept of Global Safety in the *fib*'s Model Code 2010, specifically in its item 4.6, from the conceptual aspects exposed, for example, by Cervenka (2013), new studies to assess the safety associated with the design following these new concepts are necessary. The appropriate approach for this assessment is the Reliability Analysis, as explained, for example, by Melchers and Beck (2018).

For the Global Safety Analysis, it is necessary to re-evaluate the maximum strength in the structural sections using, instead of the design values of the resistance, their average values. Then, a factor λ can be found, which increases the forces until the final situation of collapse. Reliability analyses are made in the several intermediate situations using computer programs.

Reliability indices can be found for a conventional sectional analysis and for the final collapse of the Global Analysis. The results obtained can be analysed and compared, analyzing whether the Global Safety Analysis can be applied without compromising structural safety.

The Global Safety approach treats the uncertainties of structural behavior symbolically according to the condition of limit state, expressed by eq. (4), at the level of structural resistance. The effects of the various uncertainties (of material properties, geometric quantities, etc.) are integrated into a Global design resistance and can be expressed by a Global Safety equation.

$$r\,(\mathbf{r}) \leq e\,(\mathbf{e}) \tag{18}$$

The representative values of the Global resistance variables and the Global safety factors must be chosen so that the reliability requirements for the design of new structures, in terms of β reliability indexes related to the design reference period are met.

2.2.2.1 Basic rules for Global Safety approach

The representative variable for global resistance is the structural strength R. Resistance uncertainty is expressed by the following resistance values:

R_m – average resistance value,

R_k – characteristic resistance value, (corresponds to the probability of failure of 5%),

R_d – design value of resistance.

The basic variables, defined for the partial factors in item 2.3.4 of MC2010, are used to calculate the resistance values. The values of these variables must be chosen according to the safety formats described below. The load value F is considered in the same way as in the partial safety factor method.

Design conditions

The design condition derived from eq. (18) for the Global format takes the following form:

$$e\ (F_d,..) \leq r\ (R_{d,...}) \tag{19}$$

In terms of representation of forces, this form can be assumed:

$$F_d \leq R_d \tag{20}$$

The mean and design values of the resistance are related as:

$$R_d = R_m / \gamma_R^* \tag{21}$$

where γ_R^* is the Global safety factor for average resistance.

The Global safety factor γ_R^* considers the random uncertainties of the model parameters, related to the material properties. The uncertainty due to the formulation of the model must be addressed by a separate safety factor by the model's uncertainty γ_{Rd}. This can be applied to both loads and resistances. In the latter case, the resistance of the structure takes the form:

$$R_d = \frac{R_m}{\gamma_R^* \gamma_{Rd}} \tag{22}$$

The model's uncertainty factor value depends on the quality of the formulation of the resistance model. The MC2010 recommended values are:

$\gamma_{Rd} = 1{,}0$ for no uncertainty

$\gamma_{Rd} = 1{,}06$ for models with few uncertainties

$\gamma_{Rd} = 1{,}1$ for models with high uncertainties

The sequence of steps adopted here for the structural verification in the Global Safety approach consists of designing the structural elements using the semi-probabilistic/deterministic design method, following the normative recommendations, then performing a non-linear analysis of the structural model using a commercial computer program, considering average values of actions and resistance. The next step concerns the depenalization of the global safety factor (λ), referring to the load that leads the structure to collapse. Finally, a reliability analysis is performed for the factor λ found, considering the global behavior of the structural model.

2.3
Probabilistic criteria

The main objective of a reliability analysis using the probabilistic approach is to assess the safety of the structure by estimating its probability of failure (or the reliability index β). It is also an appropriate approach for assessing the performance of existing structures.

The return period is defined as the average or expected interval between two consecutive statistically independent events. As a probabilistic measure of structural reliability, this concept allows for quickly consider the variability in time, but on the other hand, it ignores the fact that for a given point in time, the value of the variable is uncertain. Both the structural resistance R and the solicitation S applied to the structure are functions of time and space. In general, the variability of R and S increases with time, causing the probability density curves, f_R and f_S, to take on widest and flattest shapes. The average values of R and S can also vary over time. Actions tend to increase and resistance to decrease (Melchers, 1987). The general reliability problem is presented in the Figure 2.4.

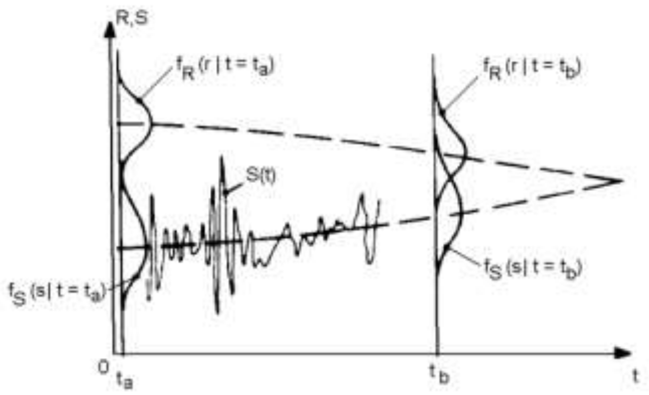


Figure 2.4 - General problem of reliability versus time (Henriques, 1998)

The verification of a limit state is performed by estimating the probability of failure occurring in a specified reference period.

In general terms, the limit state can be expressed by: g (**e**, **r**) = 0, where, g (**e**, **r**) is the function of the limit state, **e** represents the loading and **r** the resistance. Conventionally, the failure criterion is formulated according to:

$$g(\mathbf{e}, \mathbf{r}) \leq 0 \tag{23}$$

The probability of failure occurring can generally be expressed as:

$$p_f = \text{Prob}\{g(\mathbf{e}, \mathbf{r}) \leq 0\} = \text{Prob}\{M \leq 0\} \tag{24}$$

where M = g (**e**, **r**) represents the safety margin, if the function of the Limit State is expressed as $r(\mathbf{r}) - e(\mathbf{e}) = 0$. If the parameters that characterize actions, environmental, material and geometry parameters are represented in the random variables E and R, the probability of failure occurring can be expressed as:

$$p_f = \text{Prob}\ \{ r\ (\text{R}) \le e\ (\text{E})\} = \text{Prob}\ \{\text{R} \le \text{E}\} \qquad (25)$$

where E = e (**E**) and R = r (**R**) are the basic random variables associated with loading and resistance, respectively, and:

$$r\ (\mathbf{r}) \le e\ (\mathbf{e}) \qquad (26)$$

2.3.1 Characterization of uncertainties

Among the uncertainties present in current engineering problems, they can be classified into intrinsic (or physical) uncertainty, epistemic (or modelling) uncertainty, statistical uncertainty and uncertainties due to human error.

Intrinsic uncertainty, or physical uncertainty, is the one associated to the very nature of the involved processes. Some examples are earthquake-induced loads; variation in strength of structural materials; variation of loads such as wind, waves, etc. This type of uncertainty cannot be eliminated, but as new information becomes known, this type of uncertainty can be reduced and can be represented and incorporated into the analysis through random variables and stochastic processes.

Epistemic or modelling uncertainty originates from representing structural behaviour through simplified models. When the strength of a reinforced concrete element is determined according to the strength of the steel, the concrete and the dimensions of the element, a model error is immediately introduced. This type of uncertainty could be represented by a random variable, and its probability distribution can be determined, for instance, by making comparisons between experimental tests and the resistances determined by the model. This type of uncertainty can be considered by a variable that represents the relationship between the true response and the response predicted by the model.

Statistical uncertainty is determined based on samples of probabilistic distribution curves for a random variable or its parameters. When the mean of a variable is determined from a sample, the variance of the result corresponds to a statistical uncertainty of that mean, for example. It is associated with statistical inference where the estimation of the parameters that characterize the probabilistic models is performed from a limited number of available data. Statistical uncertainty can be considered through a probability distribution function. It is possible to use a Bayesian approach (Henriques, 1998) to redefine this distribution function in order to incorporate new information obtained, at any time, increasing the available database.

The type of uncertainty considered herein is related to material, geometric, load properties, is represented in mathematical models that use simplification processes in its modelling, and is contains he lack of knowledge of aspects that may be important in the phenomenon, but which are not covered in the model. Decision uncertainty is related to whether a given event has already occurred or not.

The numerous sources of uncertainty in the design can lead to extreme situations. Even when there is experimental or previous data still, usually, these are not sufficient to eliminate uncertainties and do not provide an absolute safety of the structure.

2.3.2
Probabilistic distributions

Herein some types of the most common continuous random distributions that are usually used in structural reliability analyses are presented. To characterize them, their main parameters and their respective functions, probability density and cumulative distribution are presented.

2.3.2.1 Normal Distribution N (μ, σ)

The Normal distribution, also known as Gaussian distribution, is the most important continuous distribution. Its importance is due to several factors. Among them, it can be mentioned the Central Limit Theorem, which is of fundamental importance in practical and theoretical applications, as it assures that even if the data are not distributed according to a normal curve, the superposition of these data converges to a normal distribution aslong as the number of data increases. In addition, several practical phenomena result in a normal distribution.

A continuous random variable *x* has a Normal distribution if its probability density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], x \in (-\infty, \infty) \tag{27}$$

Its cumulative distribution function has no analytical expression and can be only assessed using numerical integration processes or using statistical tables. It is defined by:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma_x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right] dx \tag{28}$$

If a random variable has a normal distribution, with zero mean and standard deviation equal to one, it is said that this variable has a standard normal distribution (Pinheiro, *et al.*, 2011). Figure 2.5 shows a normal distribution of standard probability, which has a zero mean (μ = 0).

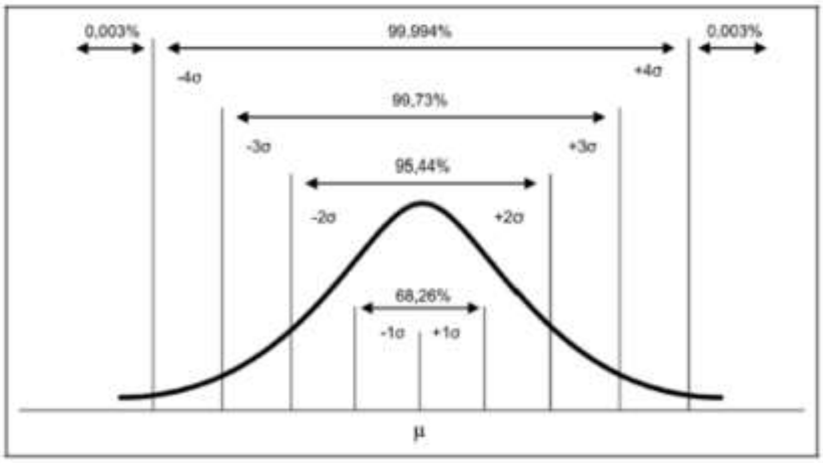


Figure 2.5 - Normal distribution chart

Also shown in this figure are accumulated probability ranges with mean ± one, two, three and four standard deviations. The greater the dispersion in relation to the mean, the broader the basis of the probability density (PDF) function will be, which leads to distributions of greater dispersion.

When μ and σ are unknown, these values will be estimated by $\mu\overline{X}$and σ, respectively, from a sample, in which:

$$\bar{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{29}$$

and

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{X})^2} \tag{30}$$

For each value of μ and/or σ there is a probability distribution curve. However, for applying in specific problems, it can be used this standardized normal distribution, also called standard or reduced, which is the normal distribution with μ = 0 and σ = 1.

To obtain such distribution, that is, the variable *X* presents a normal distribution with average μ other than zero and/or standard deviation σ other than one, the variable *X* should be reduced it to a variable *Y*, applying the following expression:

$$Y = \frac{X - \mu_x}{\sigma_x} \tag{31}$$

Thus, this distribution has an average μ = 0 and standard deviation σ = 1. Because the distribution is symmetric in relation to the average μ = 0, the area on the right is equal to the area on the left of μ, as shown in Figure 2.6. Because it is a widely used distribution, there are tables in which we find the resolution of its integrals.

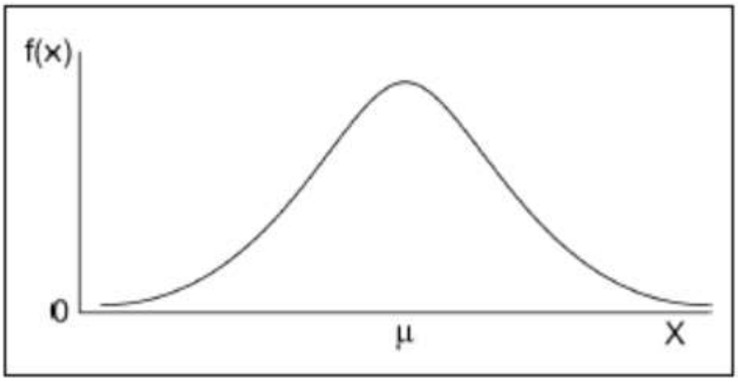


Figure 2.6 - Standard normal distribution graph

Thus, the probability density function of the standard normal distribution is given by:

$$f(y) = \phi(y) = \frac{1}{\sqrt{2\pi}} exp\left[-\frac{1}{2}y^2\right] \quad (32)$$

The cumulative function of the standard normal distribution is defined as:

$$\phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} exp\left[-\frac{1}{2}y^2\right] \quad (33)$$

The Central Limit Theorem states that if there is a sum of a large number of random variables, the probability distribution of that sum is close to the Normal distribution, regardless of the nature of distribution of these variables. (Melchers, 1999).

In the reliability analyses, the normal distribution is used to represent variables such as the strength of the materials.

2.3.2.2 T-Student distribution

The t-Student distribution is one of the most used distributions in statistics, with applications ranging from statistical modelling to hypothesis testing, and is the distribution used in the Bayesian approach.

A continuous random variable X has a t-Student distribution with n degrees of freedom (number of samples) if its probability density function is given by:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} . \left(1 + \frac{x^2}{\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}, x \in (-\infty, \infty) \quad (34)$$

The density function of the t-Student distribution has a similar shape to that of the Normal distribution, but reflects greater variability (with wider curves), which is to be expected in small samples, as shown in Figure 2.7.

The greater the number of degrees of freedom (samples), the more the t-Student distribution approaches the Normal distribution.

Below, as an example, is a graph of the t-Student density function with 10 degrees of freedom.

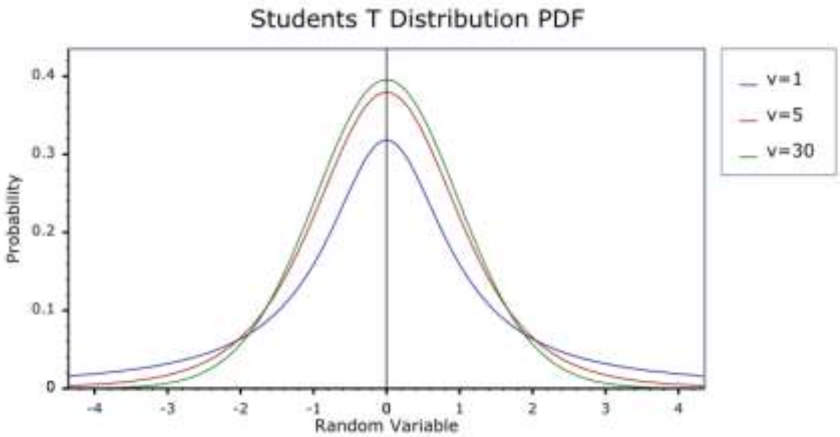


Figure 2.7 - T-Student distribution graph (typ.)

2.3.2.3 Log-Normal distribution

The Lognormal distribution is widely used to characterize the working life of products and materials. This includes fatigue of metals, semiconductors, diodes and electrical insulation.

A random variable has a lognormal distribution if its probability density function is given by:

$$f(x; \mu, \sigma) \left\{ \frac{1}{x\sigma\sqrt{2\pi}} exp\left[\frac{-(\log(x) - \mu)^2}{2\sigma^2}\right] \right\}, se\ x > 0 \quad (35)$$

where $\mu \in \mathbb{R}$ is the average of the logarithm of the variable and $\sigma \geq 0$ is its standard deviation.

2.3.2.4 Weibull distribution

W. Weibull originally proposed the Weibull distribution in 1954 in studies related to failure due to metal fatigue, as in the example shown in Figure 2.8. It is often used to describe the life span of industrial products. Its popularity in practical applications is because it has a wide variety of shapes, all with a basic property: its failure rate function is monotonous, that is, it is strictly increasing, strictly decreasing or constant. Its probability density function is given by:

$$f(t) = \frac{\delta}{\alpha^{\delta}} t^{\delta-1} exp\left[-\frac{t^{\delta}}{\alpha}\right],\ t > 0 \tag{36}$$

where $\delta > 0$ and $\alpha > 0$ are the shape and scale parameters, respectively.

If the form parameter is considered $\delta = 1$ in eq. (36), an exponential probability density is obtained with parameter α. That is, the Weibull distribution is a generalization of the Exponential distribution.

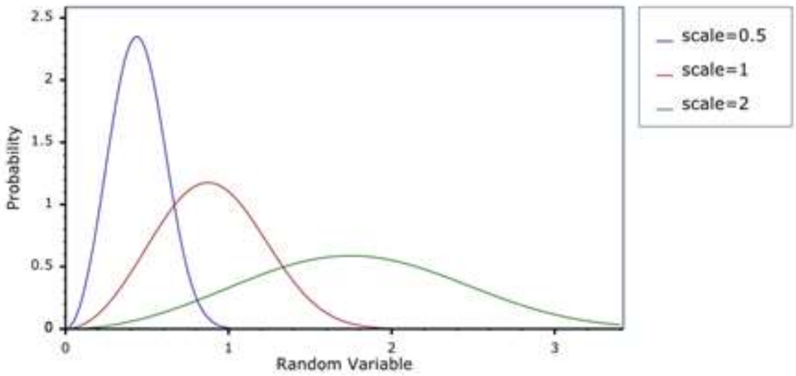


Figure 2.8 - Weibull distribution probability density function graph (typ.)

2.3.2.5 Gumbel distribution

The Gumbel or extreme value distribution appears when you take the logarithm of a variable with the Weibull distribution. That is, if the variable X has a Weibull distribution, then the variable $Y = \log(X)$ has a Gumbel distribution. In other words, the Gumbel distribution is classified as a distribution of extreme values (Figure 2.9).

This type of distribution is used when it is intended to represent the maximum or minimum values of the occurrence of a phenomenon. A variable has a Gumbel distribution if it has a probability density function given by:

$$f(x) = \frac{1}{\sigma} exp\left[\frac{x-\mu_x}{\sigma_x} - exp\left(\frac{x-\mu_x}{\sigma_x}\right)\right],\ x\ \epsilon(-\infty, \infty) \tag{37}$$

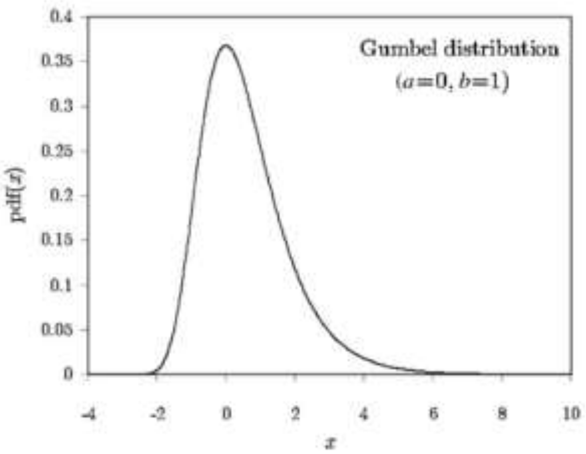


Figure 2.9 - Graph of the density function of the Gumbel distribution (typ.)

The average is given by:

$$\mu_x = b + \frac{y}{a} \tag{38}$$

The variance is given by:

$$a^2 = \frac{n^2}{6a^2} \tag{39}$$

where *a* represents the scale parameter; *b* represents the position parameter; γ = 0,57721566 (Euler constant).

The cumulative function of the Gumbel distribution is defined as:

$$F(x) = exp\left[-exp\left(-\frac{x-\mu_x}{\sigma_x}\right)\right] \tag{40}$$

The Gumbel distribution has many applications in Engineering, being used mainly to describe extreme values of natural phenomena such as rainfall volume, wind speeds, magnitude of earthquakes and waves height. With regard to structural reliability analysis, the Gumbel distribution is used herein to represent the extreme values of accidental loads and effects due to the action of wind and earthquakes.

2.3.3
Local analysis

2.3.3.1 Approach by partial safety factors

The partial safety factors approach separates the treatment of uncertainties and variability caused by the various causes through design values attributed to the various variables. According to MC2010, the representative values of the variables and the partial safety factors are chosen in such a way that the reliability requirements for the design of new structures in terms of β reliability indexes related to the defined reference period, are known.

In the case of existing structures, the same principles of the partial safety factor approach can be applied. However, the values of the design variables (ie characteristic values and partial factors) for existing structures need to be updated to assure that the reliability requirements for the assessment of these structures are satisfied.

For basic variables, the design values include margins due to uncertainties. For other variables, whose dispersion can be neglected or covered by a set of partial factors, these values are normally considered equal to their most likely values.

In MC2010, the following variables are considered as basic:

– actions (F) unless otherwise indicated in specific clauses,

– materials and their properties (X), unless otherwise specified in specific clauses (e.g. forces (f), deformations (φ) and friction coefficients (μ))

– geometrical (*a*),

– variables that represent the uncertainties of the model (θ).

Occasionally, other variables should be considered as basic variables. For example, the number of load repetitions in fatigue checks.

With reference to the failure representation given in eq. (14), the design condition can be expressed in terms of values of basic variables such as:

$$g\ (F_d, X_d, a_d, \theta_d) \geq 0 \qquad (41)$$

where

F_d are the design values of the loads

X_d are the design values of the materials and properties of the soil

a_d are the design values of the geometrical variables

θ_d are the design values of the variables that represent the uncertainties of the model

The relationship given in eq. (19) makes the following representation of the partial safety factors approach:

$$e\ (F_d, \ldots) \leq r\ (X_d, \ldots) \qquad (42)$$

In MC2010, the design values of the basic variables are expressed as follows.

The loading design values are:

$$F_d = \gamma_F\, F_{rep} \qquad (43)$$

where

F_{rep} is the representative value of the loading

γ_F is the partial safety factor

The design values of materials and their properties are thus considered:

$$f_d = f_k / \gamma_m \quad (44)$$

or in case of uncertainty in the model, as given below:

$$f_d = f_k / \gamma_M = f_k / \gamma_m * \gamma_{Rd} \quad (45)$$

where

f_k is the characteristic value of the resistance

γ_m it is the partial safety factor for material properties

γ_{Rd} is the partial safety factor associated with the uncertainty of the (resistance) model plus geometric deviations, if these are not explicitly modelled.

$\gamma_M = \gamma_m * \gamma_{Rd}$ it is the partial safety factor for a material property that also accounts for model uncertainties and dimensional variations.

The design value of geometrical data, to be considered as basic variables, are generally expressed directly by its design value

The design value of the variables representing the model uncertainty is expressed by γ_d or $1/\gamma_d$, where γ_d are the partial factors for the model's uncertainties (e.g. γ_{Rd} associated with the model's resistance uncertainties).

In the design of new structures, the calculation values of the basic variables must be determined using representative values of the basic variables and adequate partial safety factors for representing the loads, representing the material properties and representing the geometric quantities.

When evaluating existing structures, a reconsideration of the design values of the basic variables may be necessary.

In relation to the representative value of the basic variables, the actions can be classified as:

– direct or indirect,

– permanent, temporary or accidental,

– static, quasi-static or dynamic,

– confined or non-confined,

– fixed or free.

For each action, different load conditions settings must be defined.

When evaluating existing structures, the load characteristics must be defined with values corresponding to the real situation.

Each permanent load is represented by a single representative value G if at least one of the following conditions is met:

– the variability of the load over time and in relation to the structure is small.

– the influence of the load on the total effect of the loads is small.

– it is evident that one of the two representative values (upper or lower) governs the design of the structure.

In the other cases, two representative values (upper and lower, G_{sup} e G_{inf}) shall be defined, taking into account the variations that can be predicted.

Each variable load can be represented by:

– characteristic value Q_k

– combined value Ψ_0 Q_k

– frequent value Ψ_1 Q_k

– quasi-permanent value Ψ_2 Q_k.

where

Ψ_0 coefficient for the combination of values of a variable load, taking into account the reduced probability of simultaneous occurrence of the most unfavourable values of several independent actions.

Ψ_1 coefficient for the frequent value of a variable action, usually representing the value that is exceeded with a given frequency during a year.

Ψ_2 coefficient for the quasi-permanent value of a variable stock, usually representing the value that is exceeded with a given frequency or its average value over a period of time.

Accidental loads can be given by a single representative value, which is usually the design value A_d.

To conform with the basic definition of representative seismic action, each individual component history must be defined so that the values of its elastic response spectrum for standard damping are at least 90% of the specified spectrum values over the period of interest.

A sufficient number of independent seismic events (in terms of time) should be used to derive meaningful and robust statistics from the effects of actions.

In the design for partial safety factors, it must be proved that the structure, given the design values for the basic variables, does not reach the relevant limit states for loads below the design load. The basic design rules defined next are applicable to the limit states.

In principle, all relevant limit states should be considered, as well as all relevant design conditions, load arrangements and load combinations.

The numerical values of the safety factors γ are applicable to the design of new structures and for existing structures, reduced values can be considered. After assessing the updated design values, the check of the structural reliability of existing structures using standard procedures for new structures can be made.

The numerical values of the factors γ are applicable to the design of buildings and civil engineering structure not subject to variable actions with exceptional variability.

2.3.4
Global analysis

The global format considers the various uncertainties present in the structural behaviour through a pre-defined limit state in which one or more loading variables are increased by a factor λ, until a collapse situation is reached. In this assessment, the variables related to actions and resistance are taken with their average values. The values obtained for the factor λ must be compatible, from the point of view of safety, with the reliability factors β corresponding to the required safety levels. A conventional structure will be analysed herein, and the reliability factors obtained corresponding to the two approaches will be presented. It is intended to show that the application of the Global Safety format can lead to structures that are more economical.

For the reliability analysis in the collapse situation corresponding to the Global Safety Approach in a structure, it is initially necessary to relate, using equilibrium equations, the vertical and horizontal forces acting, the maximum bending moments resisted by the beams and the normal forces and the bending moments resisted in a critical section at the base of the columns. The reliability index β related to factor λ, must correspond to that of a conventional design, associated with the ultimate limit state.

After defining the equations, F_h and F_v, corresponding respectively to numerical values of the horizontal and vertical forces acting at each point of the structure, appropriate substitutions are made until the final equation of the ultimate limit state function is reached. The definition of the probabilistic variables is done previously, based on these values; it is possible to define which probabilistic variables are to be considered in the beam and column reliability analyses. The considered values are always average values.

If the value obtained for the reliability index β, for the structure designed according to the criteria of the ultimate limit state is higher than the normative limits for the 50-year reference period, this means that, following the Global Safety Approach, the structure can be further optimized, leading to a more economical solution. This can be seen graphically in Figure 2.10 (typical in an analysis), where a correlation between the reliability index β with the parameter λ, Global safety factor, is shown.

2.3.5
Bayesian Approach

The Bayesian approach to statistical inference proposes to combine data obtained from observations with subjective assessments or judgments. In many reliability problems, the use of Bayesian methods is not an option but a necessity. In the reliability study, especially when the samples are available in a very small number - sometimes one or two items - the classic statistical inference does not provide adequate answers, as it does not allow the use of previous experience with similar models, nor the opinion of specialists from area. Thus, Bayesian theory emerges as the tool indicated for the use of all available information, be it objective, provided by test results, or subjective, dictated by experience. Through Bayesian methods, it is possible to incorporate statistical analysis, convictions, prior knowledge and opinions.

According to Jacinto (2011), the Bayesian interpretation indicates that the probability of an event occurring should be seen simply as a measure on a scale between 0 and 1 of the confidence that one has in the eventual occurrence of that event, regardless of whether it is repeated or not. This confidence, or degree of certainty, depends fundamentally on the avaiable level of information about this event, which can be greater or lesser, and not so much about the event itself. This interpretation, as a measure of trust conditional on available information, is often referred to as subjective or personalist interpretation (Paulino et al., 2004). The probability is always conditional on the information available about the event in question and, of course, it is susceptible to be changed whenever new evidences and new information appear that modify the expectations that were initially had.

Considering S as a sample space, that is, the set of all possible results of an experiment. Considering A as any event in this sample space.

Probability of A, denoted by P (A), is called a real number that satisfies the following axioms:

1) $0 \leq P(A) \leq 1$

2) P(B) = 1

3) if A and B are two disjoint or mutually exclusive events, that is, events that cannot occur simultaneously, then:

$$P\ (A \cup B) = P(A) + P(B) \tag{46}$$

The first axiom states that the probability is a real number between 0 and 1. The second asserts that the sample space is a certain event, that is, if the experiment is carried out, at least one event in the sample space occurs. The third axiom states that the probabilities of events that cannot occur simultaneously are additive. Note that these axioms, which are accepted without discussion, constitute properties of the concept of relative frequency, and are therefore intuitive.

Since events can be interpreted as subsets of a larger set, the sample space, the probability can be seen as a set function - a function that assigns to each event a real number between 0 and 1, which, as said , measures the confidence (or degree of certainty) that one has regarding the possibility of this event occurring. The probabilities p = 0 and p = 1 correspond to the two extreme situations in which there is no uncertainty: null probability indicates certainty of no occurrence and unitary probability indicates certain occurrence. The probability p = 0.5 corresponds to the maximum uncertainty and indicates that there is no inclination either towards the non-occurrence or towards the occurrence of the event.

The Bayesian interpretation of probability (as a measure of confidence or certainty) is compatible with experiences that are not random in nature. For example, suppose it is necessary to measure the height of the column of an existing bridge. Now, the height of the column is not in itself a random variable, since the height value, although unknown, is a fixed quantity. It happens, however, that successive measurements of the height of that column systematically lead to different values, creating uncertainty about the true value of the height of the column. If the probability is interpreted from a Bayesian point of view, it makes sense to model the uncertainty at the height of the column using a probability distribution.

That is, all uncertainty, even if it is not of intrinsically random origin, should be modelled as a random variable, characterized by a certain distribution of probabilities. For example, the height of the column mentioned above, in the light of the Bayesian paradigm, is modelled as a random variable. The probability distribution attributed to this variable simply characterizes the uncertainty about its true value. The true value of a fixed but unknown quantity is often referred to as a state of nature, which, being unknown, makes sense to assign a probability to it - as a conditional measure of uncertainty.

2.3.5.1 Bayes' Theorem and probability distribution

From the expression that defines conditional probability:

$P[A\text{I}B] = \frac{P[A\cap B]}{P[B]}$, it follows that, $P[A \cap B] = P[A\text{I}B].P[B]$

Since $P[A \cap B] = P[A \cup B]$, that is obtained:

$P[A\text{I}B].P[B] = P[B\text{IA}].P[A]$

or

$$P[A\text{I}B] = \frac{P[B\text{IA}].P[A]}{P[B]} \quad (47)$$

Considering N events, A_i being mutually exclusive and such that their sum corresponds to Ω:

$P[A_i \cap A_j] = 0, \qquad i \neq j$

$P[A_1 + A_2 + \cdots + A_N] = 1$

By the total probability theorem:

$$P[A_i|B] = \frac{P[B|A_i].P[A_i]}{P[B|A_1].P[A_1]+P[B|A_2].P[A_2]+\cdots+P[B|A_N].P[A_N]} \quad (48)$$

2.3.5.2 Bayesian Update

The probability distribution of a random variable can be updated whenever relevant information appears. Three situations will be distinguished. One arises when the new information is of the sampling type, that is, when a sample has been observed $\varepsilon = \{x_1,.....,x_n\}$ of variable X and if you want to update your probability distribution. Another situation arises when the new information is in the form X ≥ **a** or X ≤ **b**. Finally, the third situation arises when it is required to update the distribution of a variable from the observation of another variable that is correlated with it.

In all these situations, Bayes' theorem is the basis for updating, considering the total number of samples.

2.4
Bayesian analysis in concrete samples

This example has already been presented by Interlandi et al. (2017) and is reproduced because it fits in the context of the development of this thesis.

There are real results of laboratory tests carried out for 12 series of executed concrete on a bridge, where, in each series or concreting, tests were carried out on a maximum of 3 samples, according to Table 2.1. Due to the small number of samples, for the evaluation of the characteristic strengths of concrete f_{ck}, item 6.2.3.1 of NBR 12655 is applicable, with regard to the concrete control by total sampling, where all launchings are sampled and represented by one sample that defines the compressive strength of that concrete lauching. The value of the estimated characteristic concrete compressive strength ($f_{ck,est}$) is given by the higher strength value of these concrete samples, that is: $f_{ck,est} = f_{c,conc}$.

A Bayesian formulation is presented for the reassessment of concrete strength. This formulation is defined as Bayesian, as it considers knowledge of the problem prior of testing new samples and to this knowledge is given a "weight", defined from an "engineering judgment". The concepts of Bayesian approach, normal and t-Student probabilistic distributions are applied for comparison with the NBR 12655 criterion.

In the studied specific case it is considered that the contractors have a great deal of previous knowledge in the production of concrete for the specified strengths. The approach is Bayesian in the sense that it is associated with the weight that is arbitrarily attributed to this prior knowledge. This is mathematically materialized in variable n_0, which represents this weight in relation to the number of tests performed on current samples. The analyst, based on his “Engineering judgment”, will define the value of variable n_0. A parametric analysis is recommended, in relation to this variable, in order to evaluate how its definition affects the results.

The mathematical expressions that will be used are those found in Jacinto (2011), reproduced below and whose development is presented in item 3.8.

a) Previous knowledge:

- number of samples: n_0 (arbitrary in the Bayesian sense);
- average: μ_0 ; standard deviation: s_0
- auxiliary parameters: $\alpha_0 = \frac{(n_0 - 1)}{2}$; $\beta_0 = (\frac{n_0}{2}) s_0^2$

b) Actual tests:

- number of samples: n

- average: $\bar{x}$; standard deviation: *s*

c) Distribution "a posteriori":

- number of samples: $n_n = n_0 + n$
- average: $\mu_n = \frac{n_0\,\mu_0 + n\mu}{n_0 + n}$; $\alpha_n = \alpha_0 + \frac{n}{2}$; $\beta_n = \beta_0 + \frac{n-1}{2}s^2 + \frac{n_0 n(\mu_0 - \bar{x})}{2(n_0+n)}$
- *T-student* distribution (*St*): $f_x(x) = St(x \mid \mu_n, \sqrt{(1+\frac{1}{n_n})\frac{\beta_n}{\alpha_n}}, 2.\alpha_n)$

As previous knowledge, it is admitted that each mix has been properly dosed to provide the required f_{ck}, considering a variation coefficient of 10%.

Jacinto (2011) states that for n_0 (confidence index associated with previous knowledge) a range between 3 and 50 is reasonable; here $n_0 = 10$ is addopted.

The following numerical data were considered, with respect to "prior knowledge":

- number of arbitrated samples: $n_0 = 10$
- average resistance: $\mu_0 = 43{,}3$ MPa;
- standard deviation: $s_0 = 4{,}33$ MPa

Note that these values were chosen to correspond to the required characteristic resistance $f_{ck} = 35$ MPa of t-Student distribution.

In the samples:

- number of samples: n = 2
- average resistance: $\bar{x}$ = 36 MPa
- standard deviation: s = 0,57 MPa

In Figure 2.10, results are presented for one of the analysed series, obtained with the Mathcad application, where:

- The red curve, refers to the concrete of the actual tests, in a Normal probabilistic distribution;
- The blue curve, refers to prior knowledge, in a t-Student probabilistic distribution;
- The green curve, refers to the adjusted probabilistic distribution, that is, for the test samples + 10 samples of prior knowledge, in a t-Student probabilistic distribution.

Combined results:

- average resistance: $\bar{x}$ = 42 MPa

- standard deviation: s = 5,1 MPa

Note that the coefficient of variation rose to 0,121.

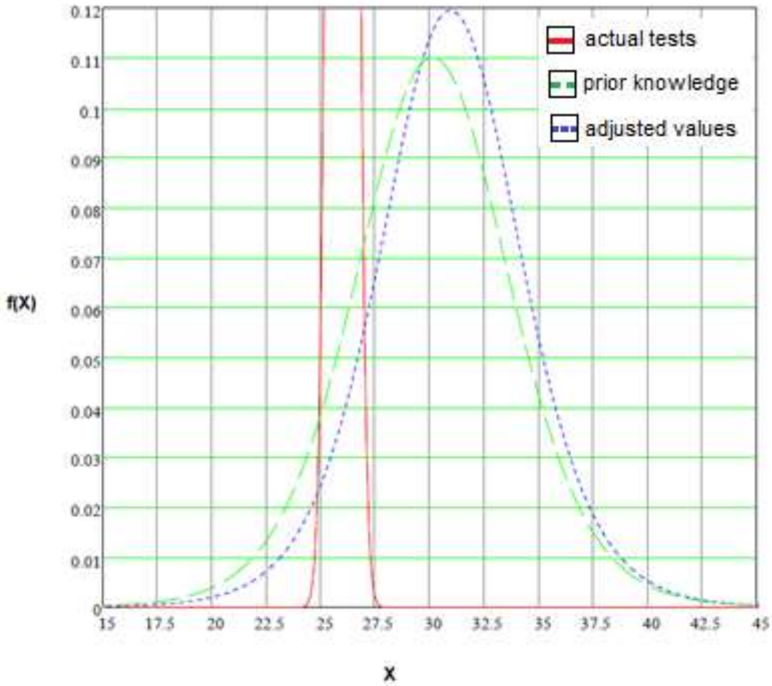


Figure 2.10 - Resistance distribution graph (kN/m^2)

For each batch, the adjusted values of $f_{ck,\mathrm{Bayes}}$ is presented in Table 2.1 It is shown that the NBR 12655 criterion is clearly not conservative.

Table 2.1 - Concreting map – $f_{ck\ \mathrm{NBR}}$ and $f_{ck\ \mathrm{Bayes}}$ (adapted in Interlandi, 2017)

Laboratório

Mapa Concretagem - Rio Roncador/ Rio Guaraí

						CP1		CP2		CP3			
RMCP	SÉRIE	Data Concret.	Vol (m^3)	*fck*	Idade (dias)	*Kgf*	*fck*	*Kgf*	*fck*	*Kgf*	*FCk*	fck NBR	fck BAYES
002	009	11/05/16	8,0	35,0	28	0,00	35,80	0,00	36,10	0,00	0,00	36,1	33,1
002	*010*	11/05/16	8,0	35,0	28	0,00	36,30	0,00	36,70	0,00	0,00	36,7	33,4
002	*011*	11/05/16	8,0	35,0	28	0,00	36,60	0,00	36,90	0,00	0,00	36,9	33,5
002	*012*	11/05/16	8,0	35,0	28	0,00	37,10	0,00	37,20	0,00	0,00	37,2	33,7
004	014	18/05/16	8,0	35,0	28	0,00	36,60	0,00	37,40	0,00	0,00	37,4	33,6
004	015	18/05/16	8,0	35,0	28	0,00	35,70	0,00	36,40	0,00	0,00	36,4	33,1
004	016	18/05/16	8,0	35,0	28	0,00	35,60	0,00	36,70	0,00	0,00	36,7	33,1
005	022	24/05/16	8,0	35,0	28	0,00	40,30	0,00	40,40	0,00	40,70	40,4	35,2
005	024	24/05/16	8,0	35,0	28	0,00	41,30	0,00	41,80	0,00	42,10	41,8	35,6
009	043	14/06/16	5,5	35,0	28	0,00	43,70	0,00	44,20	0,00	45,30	45,3	35,6
037	128	09/08/16	6,0	40,0	28	37,59	47,02	35,95	44,97	35,86	44,86	44,97	40,03
037	129	09/08/16	6,0	40,0	28	35,39	44,27	34,98	43,76	36,34	45,46	45,46	39,42

3
Structural Reliability Application

3.1
Initial considerations

Currently, two issues have been investigated in terms of structural reliability. The first one concerns the identification of different classes of practical reliability problems. That is, how reliability problems should be formulated in different cases of practical problems, in order to reflect the effects of random and epistemic uncertainties on the structures or "performance" of systems, in the domain of time-space, in line with the available knowledge about the phenomenological characteristics of the problem. This usually results in non-trivial probabilistic models involving a mixture of random variables with random processes and random fields - with hierarchical and stochastic dependencies.

The second issue concerns the identification of suitable techniques and tools for probabilistic analyses that individually or in combinations facilitate the effectiveness of probabilistic analyses of different classes of practical reliability problems. For this purpose, classic reference studies are of interest. However, and much more importantly, these studies need to be adapted to the underlying assumptions and robustness of the different techniques and it should be studied how these aspects affect their relevance in the context of different classes of reliability problems.

Many reliability problems in Structural Engineering can be formulated using relatively simple limit state functions. The classic way to find the corresponding probability of failure is using the FORM analytical methods. The effects of system and time can be resolved by the crossing approach or similar techniques.

Given the increasing capacity of computers, Monte Carlo simulation methods are becoming increasingly popular in many fields of application. They are often used in combination with classical methods and with simplified descriptions of structural behavior.

The reliability analyses in this work are performed using in the VAP® program and the methods adopted for simulation are, Monte Carlo and analytical, FORM.

3.2
Simulation Methods

3.2.1
Monte Carlo Method

The Monte Carlo method comprises a class of probabilistic methods that rely on massive random sampling to obtain numerical results, that is, repeating successive simulations a high number of times, for calculating probabilities, as if, in fact, the results were recorded in casino games (the name is derived from the city of the same name, also famous for its casinos). This type of method is used in stochastic simulations in several applications in Engineering. The Monte Carlo method has been used for a long time as a way to obtaining numerical approximations of complex functions in which it is not feasible, or even impossible, to obtain an analytical solution.

In 1946, the mathematician Stanislaw Ulam during a game of solitaire tried to calculate the probabilities of success of a given move using the traditional combinatorial analysis. After spending a lot of time doing calculations, he realized that a more practical alternative would be to simply make countless moves, for example, a hundred or a thousand, and count how many times each result occurred. Ulam knew that statistical sampling techniques, like this one, were not widely used because they involved extremely lengthy, tedious and error-prone calculations. However, at that time, the first electronic computer, developed during the Second World War, ENIAC, was avaiable; before him, mechanical devices were used to make calculations. The versatility and speed of ENIAC, unprecedented at the time, impressed Ulam, who suggested the use of statistical sampling methods to solve the problem of neutron diffusion in material subject to nuclear fission, thus spreading its application. Later, this method became known as the Monte Carlo Method, a name inspired by Ulam's uncle, who played constantly in the Monte Carlo casino, whose random aspect of his roulette wheels is also closely linked to the method. The Monte Carlo Method was formalized in 1949, through the article entitled “Monte Carlo Method”, published by John Von Neumann and Stanislaw Ulam (Eckhardt ,1987).

In practice, when faced with a problem involving uncertainties, performing a simulation with Monte Carlo to approximate your solution consists of four standard steps: modelling the problem, generating random values for the uncertainties of the problem, replacing the uncertainties with values to calculate the result and for finally, obtain an estimate for the solution of the problem (Figure 3.1).

Note that this method only provides an approximation of the solution; therefore, it is essential to analyze the approximation error, considering the sample standard deviation and the sample size. Therefore, it is evident that the larger the sample size, the smaller the approximation error.

In case of having a problem with more than one variable, several samples of these variables are generated. Starting from the generation of the *N* sets of random numbers to the *n* random variables used to determine the probability of failure, this failure function is evaluated for each of the generated random sets and the failure function is tested several times, being the probability of failure expressed as follows:

$$Pf = \frac{n(g(X) \leq 0)}{N} \tag{49}$$

where $Pf = \frac{n}{N}$ represents the number of times the failure function falls in the failure region for a number N of evaluations.

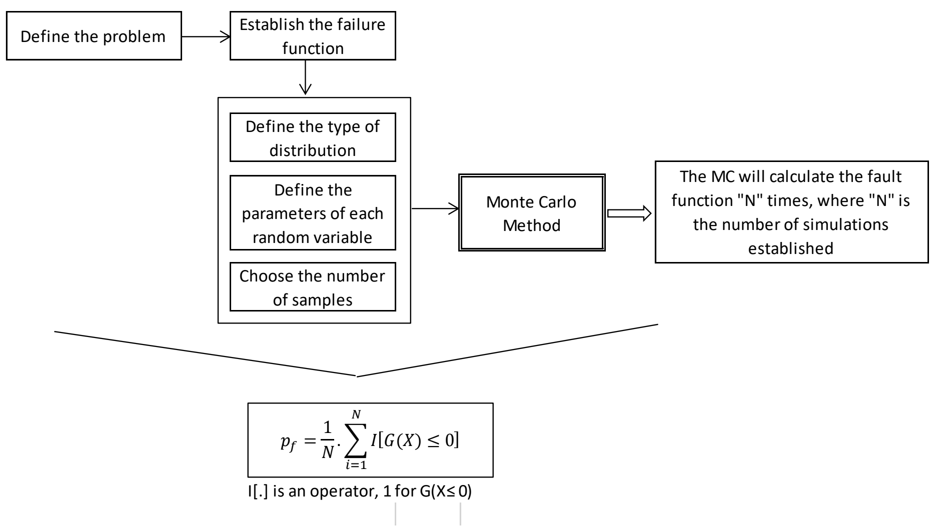


Figure 3.1 - Monte Carlo Method diagram

3.2.2
Latin Hypercube Sampling

Latin Hypercube Sampling is a simulation method, similar to the Latin Square Method in several two dimensions.

The Latin Square sampling method, consists of dividing the space (in this case, a square) formed by the two variables on a *n* x *n* chessboard, and choosing, in the sampling, the *n* points in each sub-square, but so that no two points occupy the same row or column. Latin Square, a name coined by Leonhard Euler, is an *n x n* box in which each box has a letter (from the Latin alphabet, hence the name) arranged so that two identical letters do not occupy the same row or column.

The Latin Hypercube is the generalization of this method for *k* dimensions: the Sample Space of each of the *k* variables is divided into n equiprobable intervals, and a point is chosen in each of these intervals; this generates a matrix of *k* columns, formed by the samples of each variable, which are then randomly exchanged, generating the final sample of *n* vectors of *k* dimensions.

In Nowak and Collins (2000), the basic steps that describe the method were defined, namely: first, the probability distribution of each random variable is divided into N intervals, with each interval having the same probability (1 / N); then a random value is chosen that is representative of the random variables in each interval, obtaining N representative values for each of the K random variables. There are N.K possible combinations of these representative values. The objective of the method is to obtain N combinations in such a way that each representative value appears only once in those N combinations.

Subsequently, to obtain the first combination, a value representative of the N values for each random variable is randomly selected. For the second combination, a random value is selected from the remaining N-1 values for each random variable, and so on. Finally, the limit state function is evaluated in each combination and the probability of failure is obtained by the formulation expressed in eq. (50), with n being the number of times the limit state function reached a value less than or equal to zero (G≤0):

$$Pf = \frac{n}{N} \tag{50}$$

The technique of the LHS method is illustrated in Figure 3.2.

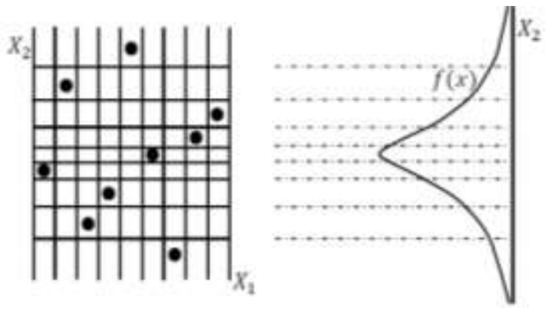


Figure 3.2 - Representation of the LHS Method (Chakraborty, 2014)

3.3
Analytical methods

3.3.1
FORM (First Order Reliability Method)

The FORM method has been widely used when one wants to reduce the total computational time of the analyses. For the depenalization of the reliability index, it has been well accepted due to its efficiency, being recommended by the JCSS (Yang et al., 2006). The FORM method, calculates the reliability index β as the distance of the failure function to the origin in the space of the normal uncorrelated equivalent standard variables *Y*. Thus, the failure function *g(X)* is written in terms of the variables *Y* as *g(Y).* Next, we look for the point *Y**, called design point, whose distance to the origin is the minimum and the value of the reliability index is determined, which is equal to the distance of *Y** to the origin. So:

$$\beta = |Y^*| \tag{51}$$

When using the FORM method, a process of transformation from normal space to standard space is necessary. This transformation is carried out using equivalent normal distributions. The design point is obtained in the form of an optimization problem (Figure 3.3).

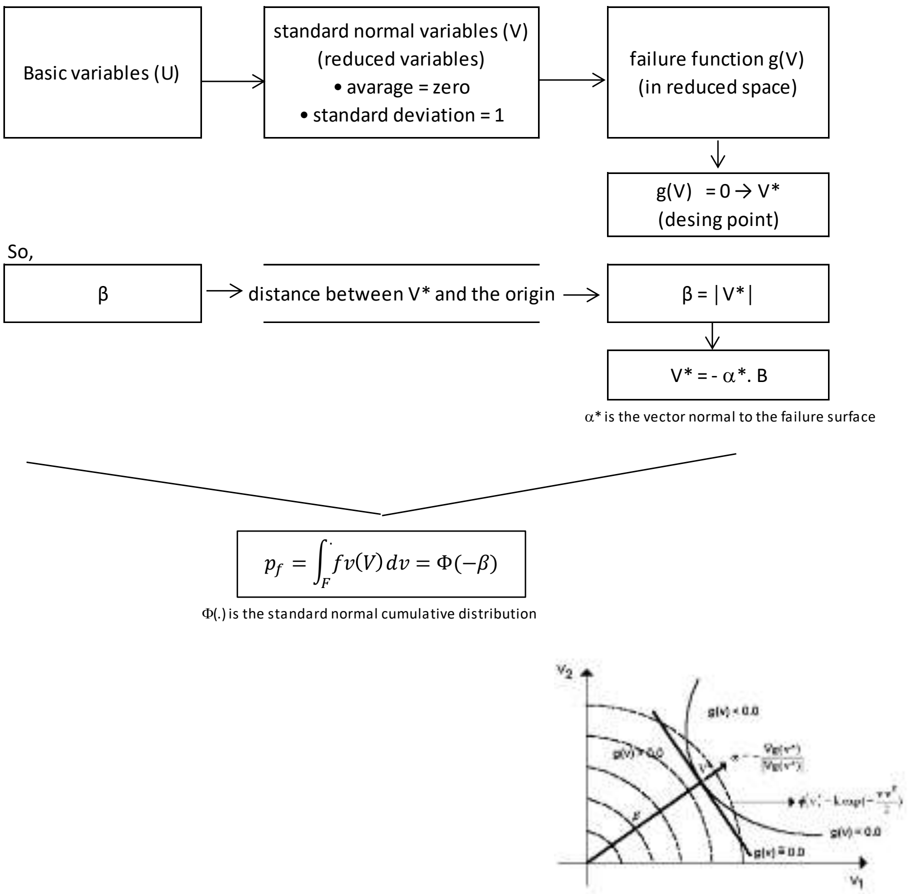


Figure 3.3 - FORM method diagram

3.3.2
SORM (Second Order Reliability Method*)*

For a Limit State Function consisting of one or more non-normal random variables, it can be said that this is a highly non-linear function in standard normal space, which leads to the conclusion that a first order approximation can cause a significant error in safety assessment of a given structure. For this type of situation, the use of the SORM method allows a more precise analysis, compared to the FORM method, due to the implementation of second order terms in the approach of the Limit State Function. A Limit State Function *g* expressed in terms of a second order Taylor Series is presented in eq. (52):

$$g(X_1, X_2, \dots, X_n) = g(x_1^*, x_2^*, \dots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(x_i - x_i^*)$$

$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 g}{\partial X_i \partial X_j}(x_i - x_i^*)(x_j - x_j^*) + \cdots \qquad (52)$$

To estimate the probability of failure using second-order expansion, Breitung (1984) suggested the asymptotic approximation shown in eq. (53), which starts from the reliability index estimated using the FORM method (β_{FORM}):

$$P_{f.SORM} \approx \phi(-\beta_{FORM}) \prod_{i=1}^{n-1} (1 + \beta_{FORM} k_i)^{-\frac{1}{2}} \quad (53)$$

where k_i is the curvature of the boundary state surface at the design point.

For the calculation of the reliability index β_{FORM}, the Rackwitz-Fiessler iterative process can be implemented (Haldar and Mahadevan, 2000). This process makes possible to calculate, in addition to the reliability index, also the point on the failure surface where the highest probability density is obtained, called the design point. Once the design point is obtained, the reliability index is calculated using eq. (54):

$$\beta_{FORM} = \frac{\{G\}^t\{u^*\}}{\sqrt{\{G\}^t\{G\}}} \quad (54)$$

where {G} is the vector of the partial derivatives of the Limit State Function evaluated at the design point, and {u^*} is the vector of random variables in normal space standardized at the design point. The suffix **t** denominates the transpose of the vector. The vector {G} is defined by eq. (55):

$$\{G\} = \left\{\frac{\partial g}{\partial^*_{xi}}\right\} \quad (55)$$

3.4
Acceptable reliability index

Once the reliability index associated with a given reference period is known, it is relatively simple to convert it to another reference period. In effect, designating the reliability index for a unit of time (1 year, for example) by β_1 and the reliability index for **n** units of time per β_n, the following relationship applies (NP EN 1990, 2009):

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \quad (56)$$

that allows to obtain β_n from β_1 and vice versa. This relationship is valid within the hypothesis of a probability of failure in the period$[0, n]$, by the eq. (57).

$$\mathrm{P}_f(0,n) = 1 - P\left[non - occurrence\ of\ failure\ in[0,n]\right] = 1 - \left[1 - P_f(\Delta t)\right]^n \quad (57)$$

Indeed, considering that $\mathrm{p}_f = \Phi(-\beta)$, eq. (57) turns into $\Phi(-\beta_n) = 1 - [1 - \Phi(-\beta_1)]^n$. Noting that $\Phi(-\beta) = 1 - \Phi(\beta)$, given the symmetry of the FDP of the Normal model, the eq. (56) is immediately obtained. Annex B of EN 1990 (CEN, 2002) establishes limitations for the reliability indexes, according to consequence classes, according to Table 3.1.

Each consequence class (CC) is related to a reliability class (RC), whose reliability indexes are limited see Table 3.1.

Table 3.1 - Definition of consequence classes (EN, 2002)

Consequences Class	Description	Examples of buildings and civil engineering works
CC3	**High** consequencefor loss of human life, *or* economic, social or environmental consequences **very great**	Grandstands, public buildings where consequence of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequence of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life, *and* economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

The reliability index is related to the working life of the construction, which can be classified in the categories defined in Table 3.2, by the Model Code of fib (2013).

Table 3.2 - Working life of the structures (fib, 2013)

Type of structure	Specified (desgn) service life
Temporary structure	1 to 5 years
Replaceable components of structures e.g. gantry girders, bearings	25 years
Buildings and other common structures of avarage importance	50 years
Structures of greater importance e.g. monumental bildings, large bridges, other specail or important structures	100 years or more

Finally, it must also be considered the limit state that is being analyzed to limit the values of β, according to Tables 3.3 and 3.4.

Table 3.3 shows reliability indexes for two reference periods: 1 and 50 years. For example, for ultimate limit states and reliability class RC2, NP EN 1990 recommends a reliability of 3,8 in 50 years and 4,7 in 1 year. These recommendations must be interpreted as follows: for the structure to have a reliability of 3,8 in 50 years, it must have a reliability of 4,7 in the first year. The key value is the value of 3,8, which should be interpreted as the recommended reliability for the working life of a structure (Jacinto, 2011).

Table 3.3 - Recommended Minimum Values for β - Ultimate Limit State (EN, 2002)

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Table 3.4 - Minimum values of β for structural elements of class RC2 (EN, 2002)

Limit state	Target reliability index	
	1 year	50 years
Ultimate	4,7	3,8
Fatigue		1,5 to 3,8
Serviceabilily (irreversible)	2,9	1,5

Therefore, for the reliability analysis of reinforced concrete bridges (RC2), in the ultimate limit state, as studied in this work, the reliability index must be $\beta \geq 3{,}8$.

A more modern reference is in ASCE / SEI 7-16, 2017 (Table 3.5).

Table 3.5 - Values Target Reliability (Annual Probability of Failure, pf) and Associated Reliability Indices (β) (ASCE / SEI 7-16, 2017)

Basis	Risk Category			
	I	II	III	IV
Failure that is not sudden and does not lead to widespread progression of damage	P_f = 1,25x10^{-4}/yr β = 2,5	P_f = 3,0x10^{-5}/yr β = 3,0	P_f = 1,25x10^{-5}/yr β = 3,25	P_f = 5,0x10^{-6}/yr β = 3,5
Failure that is either sudden or leads to widespread progression of damage	P_f = 3,0x10^{-5}/yr β = 3,0	P_f = 5,0x10^{-6}/yr β = 3,5	P_f = 2,0x10^{-6}/yr β = 3,75	P_f = 7,0x10^{-7}/yr β = 4,0
Failure that is sudden and results in widespread progression of damage	P_f = 5,0x10^{-6}/yr β = 3,5	P_f = 7,0x10^{-7}/yr β = 4,0	P_f = 2,5x10^{-7}/yr β = 4,25	P_f = 1,0x10^{-7}/yr β = 4,5

The standards NP EN 1990 (2009) and the Probabilistic Model Code (JCSS, 2001a) recommend values for the reliability indexes and designate these values by target values, or minimum values, generally denoted by β_T. They can also be seen as values to be achieved as an objective whenever they are used for calibrating partial safety factors. Reliability indices significantly different from these target values, whether due to lack or excess, will tend to penalize the structure's economy in a long-term perspective (Jacinto, 2011).

Reliability ratios that are too high penalize initial costs and values that are too low penalize expected long-term costs due to the increased risk of structural failure. Safety values of the reliability index depend fundamentally on two quantities: the severity of the consequences of not respecting the limit state in question and the relative cost of the measures to increase Safety. The problem of optimization of reliability is not easy to solve and is by no means consensual, as it depends on sensitive values such as the value of human life (Diamantidis & Bazzurro, 2007).

Different references recommend different target values. On the other hand, since one of the intervening variables is the relative cost of measures to increase safety and this cost is generally higher in existing structures (compared to structures still in the design phase), it is understood that there are documents that recommend distinct reliability indices for new and existing structures.

The recommendations of NP EN 1990 (2009) are summarized in Table 3.1. The values are presented for different classes of reliability (Reliability Classes), RC1, RC2 and RC3 that correspond to failure consequences, respectively, low, medium and high. The consequences of failure, that is, the damage that results if the limit states are reached, can include loss of human life, economic loss, environmental and social damage.

The Probabilistic Model Code (PMC) (JCSS, 2001) recommends the values shown in Table 3.6, obtained based on cost-benefit analyses. Regarding the consequence classes (low, moderate and high), the PMC establishes that the category of low consequences applies to situations in which the risk of human losses (given the occurrence of failure) is low and the economic losses are low - ρ less than 2, with ρ = (construction cost + cost in case of failure)/(construction cost). Such is the case for example of agricultural buildings and silos. The category of moderate consequences applies to situations with an average risk of human loss or considerable economic damage (ρ between 2 and 5), such as office, industrial and residential buildings. Finally, it is defined as high consequences when there are situations with a high risk of loss of human life or very high economic losses (ρ between 5 and 10), as for example in bridges, theatres and hospitals (Jacinto, 2011).

Table 3.6 - Values of βT (minimum reliability index) recommended by JCSS (2001)

1	2	3	4
Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large (A)	$\beta = 3{,}1$ ($p_f \approx 10^{-3}$)	$\beta = 3{,}3$ ($p_f \approx 5\text{x}10^{-4}$)	$\beta = 3{,}7$ ($p_f \approx 10^{-4}$)
Normal (B)	$\beta = 3{,}7$ ($p_f \approx 10^{-4}$)	$\beta = 4{,}2$ ($p_f \approx 10^{-5}$)	$\beta = 4{,}4$ ($p_f \approx 5\text{x}10^{-6}$)
Small (C)	$\beta = 4{,}2$ ($p_f \approx 10^{-5}$)	$\beta = 4{,}4$ ($p_f \approx 5\text{x}10^{-6}$)	$\beta = 4{,}7$ ($p_f \approx 10^{-6}$)

Table 3.7 compares the recommendations from the two sources presented above. These recommendations concern new structures and, for these, the relative cost of increasing Safety can be considered low.

Table 3.7 - Values of βT comparison between EN 1990 and JCSS: ultimate limit states and reference period equal to the working life of the structure

Source	Consequences of failure		
	Low	Medium	High
EN 1190	3,3	3,8	4,3
JCSS	3,2	3,5	3,8

3.5
Assessment of existing structures

In recent decades, many efforts have been made to develop a set of recommendations for assessing the safety of existing structures on a probabilistic basis. There are many issues in the assessment and management of existing structures that must be treated differently from those adopted for the design of new structures.

Probabilistic methods have a wide field of application in the evaluation of existing structures, for example, in the application of statistical techniques in the treatment of data measured in real structures, such as the traffic jam on a bridge, allowing a more rigorous assessment of the variability of quantities involved in characterizing the response. It is also possible to define more concretely the various parameters for the assessment of the level of real risk.

The reliability techniques can be adopted in the definition of adequate values for the safety coefficients and for the representative values of the quantities involved in the safety assessment of existing structures. The safety factors used for the design of new structures in general are too conservative and this can lead to excessive safety or expensive solutions, if rehabilitation is necessary.

For existing structures, the feed continues with new probabilistic information over time, added to the existing data, generating more realistic and objective data. Some of the pertinent actions in order to consolidate this update and, consequently, generate more realistic safety coefficients can be, for example, the objective depenalization of the parameters that quantify the risk level of the structure and the resilts of regular tests and inspections with the update of these new data within an existing database.

The values of β presented in Table 3.4 can also be used for the evaluation of existing structures, however the differentiation of the level of reliability for the new and existing structures must be considered. The decision to choose a different level of reliability for existing structures can be made based on a well-founded analysis of the consequences of failure and the cost of safety measures for a specific case.

Some suggestions for the reliability index for existing structures are presented by fib 2010, as reproduced in Table 3.8 for the specified reference periods.

Table 3.8 - Values of β for existing structures suggested by *fib* MC2010

Limit states	**Target reliability inbex β**	**Reference period**
Serviceability	1,5	Residual Service Life
Ultimate	in the range of 3,1 - 3,8	50 years
	in the range of 3,4 - 4,1	15 years
	in the range of 4,1 - 4,7	1 year

3.6
Modelling of resistance

In this work, the analysis of variables related to the strength of the structure can be classified into two types: strengths of reinforced concrete materials and variability of dimensions, as shown next. The descriptions of resistance and action modelling broadens and develops the concepts exposed by Rodrigues (2019).

3.6.1 Resistances of reinforced concrete materials

The depenalization of the values of the Bias factor (average value / characteristic value) and the coefficient of variation (COV) of the resistance of concrete and steel are addressed in this item.

3.6.1.1 Concrete strength

The compressive strength of concrete has significant randomness. This considerable uncertainty stems from the inaccuracies in the concrete mix, the fact that this material is not homogeneous and the variability of the materials that compose it. Different specimens of the same concrete mix show variations in the results of the compression test. Concretes produced in loco have a strength variability, in general, greater than precast concretes (Beck, 2014).

As for permanent loads, concrete strength could be represented by the lognormal distribution, since their values cannot be negative. For the same reasons given for permanent loads (see item 3.7.1), including compatibility with the normative definitions of characteristic resistances, it is decided to consider the normal distribution.

The characteristic strength is directly related to the probabilities of not attained unfavorable. The normative definition is that of the 5% quantile, defining the characteristic resistance of the materials, as described in NBR 6118 (2014). In the case of concrete, it is considered that the compressive strength follows a normal distribution and, therefore, to determine its characteristic value, it is necessary to shift the average of 1.645 times the standard deviation to reach the characteristic strength value, with probability 95% of not being negatively exceeded, as represented schematically in Figure 3.4.

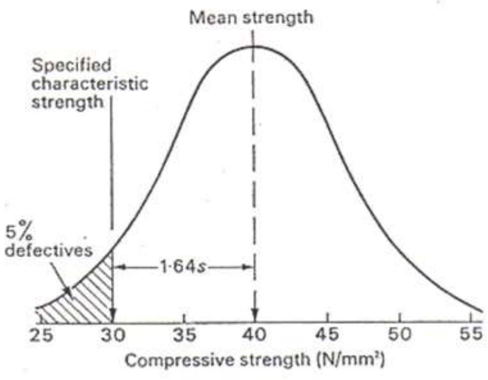


Figure 3.4 - Gaussian curve of compressive strength of concrete (typical)

According to Novak and Rakoczy (2007), the coefficient of variation of common concrete varies around the value of 0.15, as shown in Figure 3.5 (Recommended V for NWC curve, common concrete).

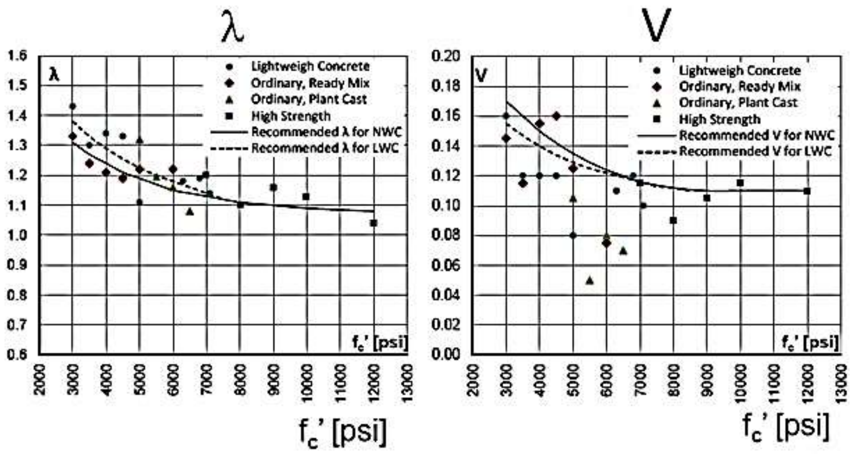


Figure 3.5 - Variation of means and variation coefficients of the concrete compressive strength (Novak and Rakozy, 2007)

Considering the information in Figures 3.4 and 3.5, it is possible to make the relationship between average and characteristic values of the concrete strength. The standard deviation is equivalent to multiplying the coefficient of variation, COV, by the average value, f_{cm}.

$$f_{ck} = f_{cm} - 1{,}645 \cdot (COV \cdot f_{cm}) \tag{58}$$

Through eq. (58) it is possible to determine the value of the Bias factor for the compressive strength of concrete, as shown below.

$$Bias = \frac{1}{1-1{,}645 \cdot COV} = \frac{1}{1-1{,}645 \cdot 0{,}15} = 1{,}328$$

Therefore, the values 1,328 and 0,15 are adopted, respectively, for the Bias factor and for the coefficient of variation of the strength of the concrete. Figure 3.6 shows the graph of the strength density function for C40 concrete, while Figure 3.7, its shows cumulative function.

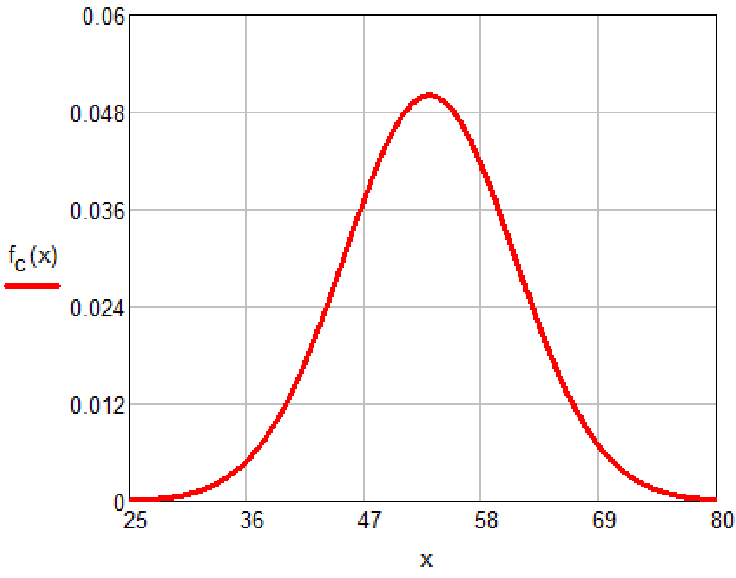


Figure 3.6 - C40 concrete strength density function

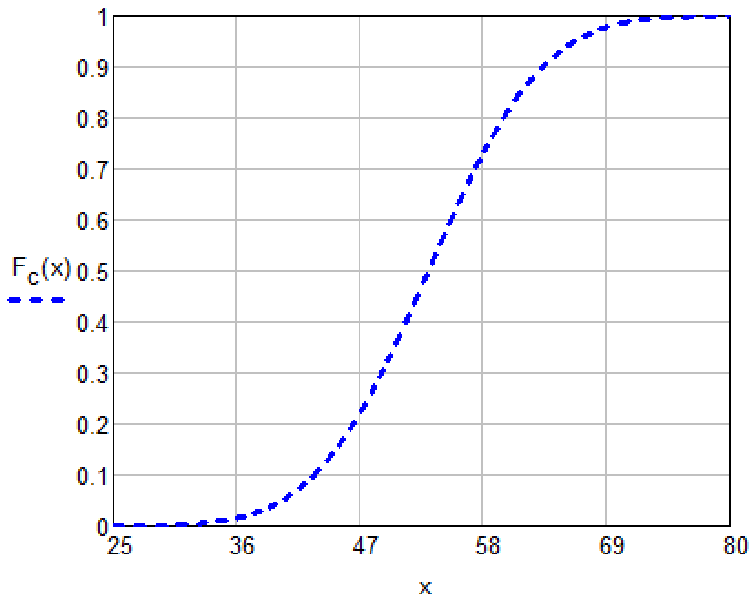


Figure 3.7 - Cumulative function of C40 concrete strength

3.6.1.2 Steel strength

The strength of steel does not show as significant variability as that of concrete, due to the greater control in its manufacture when compared to the concrete. There is also a greater homogeneity in this material, thus reducing its variability. Consequently, the coefficient of variation of steel will be considerably less than that of concrete.

Nowak and Rakoczy (2007) established a coefficient of variation of 0,05 to be adopted for the strength of steel, a value adopted by all researchers. In the same way that the Bias of the concrete strength was determined. This factor can be determined for the steel strength, applying eq. (59).

$$f_{yk} = f_{ym} - 1{,}645 \cdot (COV \cdot f_{ym}) \tag{59}$$

$$Bias = \frac{1}{1-1{,}645 \cdot COV} = \frac{1}{1-1{,}645 \cdot 0{,}05} = 1{,}089$$

The values 1,089 and 0,05 are then adopted for the Bias factor and for the coefficient of variation of the steel resistance, respectively. Figure 3.8 presents the graph of the density function of the resistance of CA-50 steel, in the same way in Figure 3.9, the cumulative function of the same material.

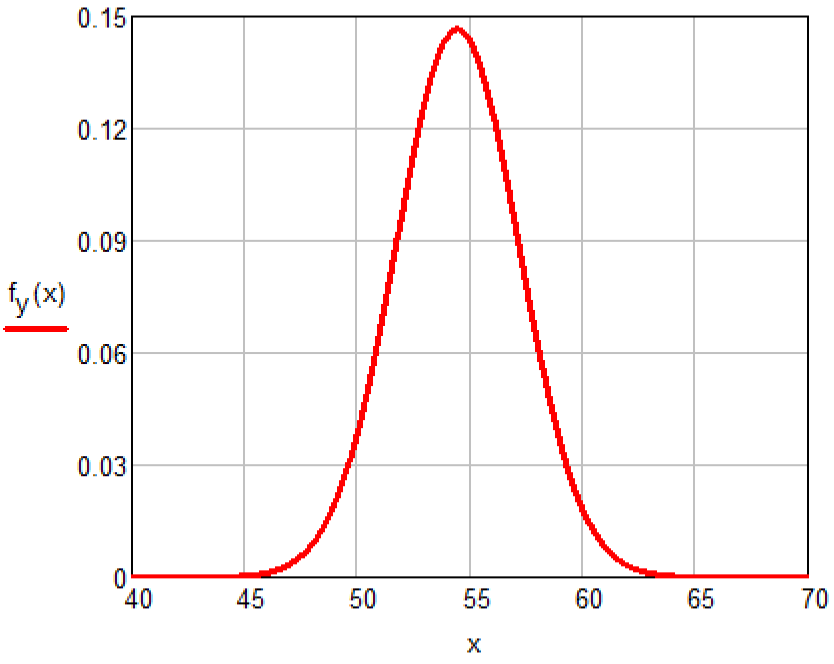


Figure 3.8 - Strength density function of CA-50 steel

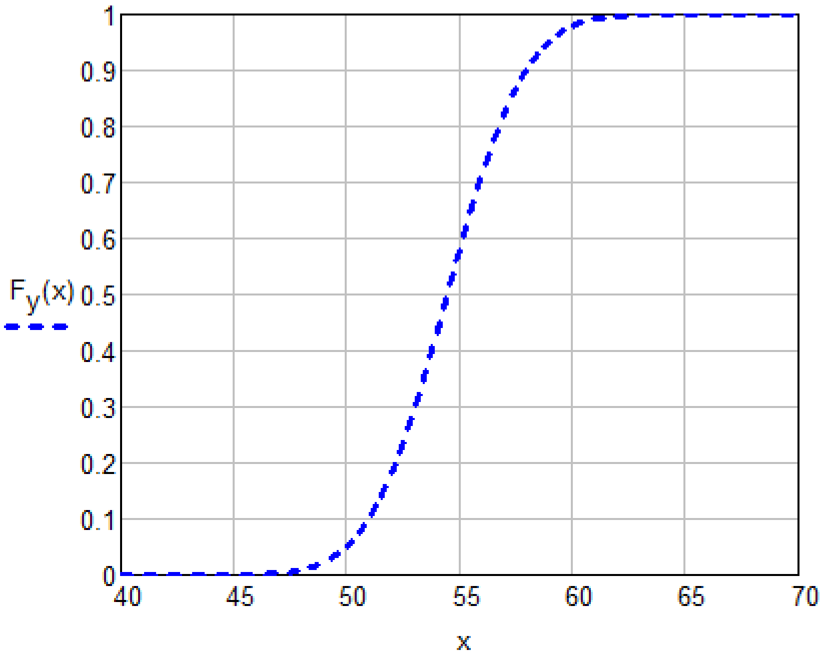


Figure 3.9 - Cumulative strength function of CA-50 steel

3.6.2
Variability of dimensions

All structures are subject to variations, even small, due to the lack of precision and tolerance allowed in the dimensions of the produced structural parts. These variations are independent of the effects of time. The dimensions of the concrete and of the covering may vary in relation to the values foreseen in the design due to inaccuracy in the construction, which cannot be avoided. These variations are considered in the design standards through constructive tolerances, which establish limits for them.

Steel reinforcements, on the other hand, may not have precisely the exact area considered in the calculations, although, as already mentioned, due to the greater control of steel manufacture, the variability of this parameter tends to be less in relation to that of concrete. In this item, the Bias factor and the variation coefficient of dimensions, concrete cover and reinforcement area are defined.

3.6.2.1 Concrete dimensions

The variation of a certain dimension of a structural part *X* is described through the value *Y* that this value presents with relation to the nominal value of the dimension, X_m.

$$Y = X - X_m \tag{60}$$

According to JCSS (2006), the dimensional variations of the concrete are little affected by the type of the element (whether it is reinforced or prestressed), by the shape of the section (rectangular, I, L or T), by the strength class of the concrete, by the fact whether it is a width or a length or even by the position in the cross section. The dimensions of the pieces are really influenced only by the production method: precast or *in situ*.

The expected values for the mean and for the standard deviation of the constructive deviations are within the following limits, considering that these deviations follow a normal distribution:

$$0 \leq \mu_y = 0{,}003 \cdot X_{nom} \leq 3mm \tag{61}$$

$$\sigma_y = 4mm + 0{,}006 \cdot X_{nom} \leq 10\ mm \tag{62}$$

In this work, a Bias of 1.0 (μ_y = 0), will be adopted, with the mean of the deviation being neglected due to its very small value, and a standard deviation that follow eq. (62).

3.6.2.2 Concrete cover

Unlike concrete dimensions, which do not show significant differences between beams, slabs and columns, the coverings have some characteristics that vary according to the type of element (JCSS, 2006). Table 3.9, adapted from the JCSS, provides indications of means and standard deviations for each type of element.

Table 3.9 - Avarage and standard deviations of concrete cover for each type of element

Element type	**μ**	**σ**
Column and wall	0 a 5 mm	5 a 10 mm
Below section of steel slab	0 a 10 mm	5 a 10 mm
Section below the steel beam	0 a 10 mm	5 a 10 mm
Section above steel slab and beam	0 a 10 mm	10 a 15 mm

Nowak and Rakoczy (2007), on the other hand, indicated that the standard deviation for coverings should vary between 7 to 11 mm. However, it was decided to adopt herein a standard deviation that is compatible with the constructive tolerances suggested by NBR6118, around 5 mm. Therefore, an average value equal to the nominal one is adopted, that is, a Bias equal to 1.0, and a standard deviation of 5.0 mm for the concrete coverings.

3.6.2.3 Steel area

Little is described in the references on variations in the areas of steel bars. Nowak and Rakoczy (2007) established that this variability can be represented by a normal distribution. Furthermore, from this reference, it is possible to conclude that the adoption of a Bias factor of 1.00 and a variation coefficient of 0.015 is reasonable for the areas of steel bars.

3.6.3 Resistance modelling

In addition to the intrinsic uncertainties in the material properties and geometric characteristics, it is necessary to consider the uncertainties inherent in the mathematical models adopted in the safety checks, recognizing that there are inaccuracies in these models.

Table 3.10 reproduces the averages and coefficients of variation presented by JCSS (2006) for the coefficients of resistance modelling.

Table 3.10 - Averages and coefficients of variation of the resistance modelling defined by the JCSS (2006).

Model type / capacity	distribution	avarage	COV
Steel resistance			
Flexion by moment	log-normal	1,0	0,05
Shear	log-normal	1,0	0,05
Welded connection	log-normal	1,2	0,15
Bolted connection	log-normal	1,25	0,15
Concrete resistance			
Flexion by moment	log-normal	1,2	0,15
Shear	log-normal	1,4	0,25
Connection	log-normal	1,0	0,10

For the value of the average of the coefficient of resistance modelling the value of 1,0, is defined for application in this work, and for its standard deviation the value of 0,05, with normal distribution.

3.7
Modelling of actions

The structures are subject to imposed loads, the influence of the environment and the effective behavior of the structural properties, which can vary over time. Such variations, which occur over the working life of the structure, must be considered in the design, each one represented in a specific time interval for the associated load cases and other risks, conditions and limit states defined for each structure. The condition considered in the design must include all situations that may occur during the execution and use of the structure.

In the design criteria, the following loads shall be defined as relevant, according to *fib* MC2010, an international reference for the design of concrete structures:

- Type 1 - permanent loads, which refer to normal conditions of use of the structure and related to the designed working life of the structure;
- Type 2 - accidental loads, which refer to exceptional conditions of the structure or its exposure;
- Type 3 - temporary loads, which refer to temporary conditions of the structure, in terms of use or exposure;
- Type 4 - seismic loads, which refer to conditions of the structure under the event of an earthquake.

In many cases, the designer's experience is necessary to assess and complement existing regulatory conditions, in order to identify which design loads shall be considered for a specific structure.

According to *fib* MC 2010, for permanent loads, generally a reference period tR is considered, equal to the working life specified in the proeject for new structures or equal to the residual life, in the case of existing structures. Normally, for this type of loading in the case of new structures, a reference period of 50 years is adopted for usual buildings and 100 years for more important structures, such as bridges and tunnels.

Type 1 loading has as main characteristic its low variability during the operation period, which gives the appearance of having a constant character. This type of loading, schematically, is illustrated in Figure 3.10.

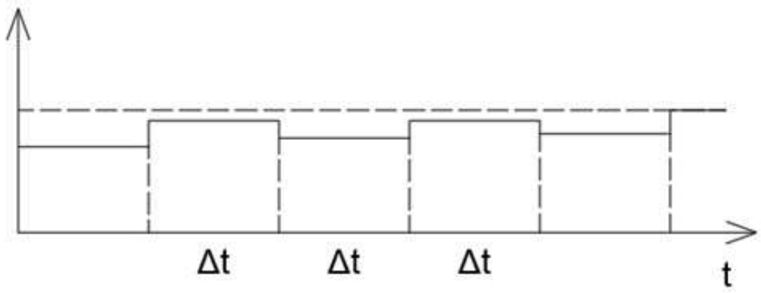


Figure 3.10 - Example of type I loading behavior over time

In general, load events occur as shown in Figure 3.11. To enable the temporal analysis of loads, the FBC model (Ferry Borges and Castanheta) is used, which considers the division of the time series into time intervals and, in each interval, the average value of the load intensity during this interval. This significantly facilitates probabilistic analyses (JCSS, 2006). Figure 3.12 shows an example of a FBC model.

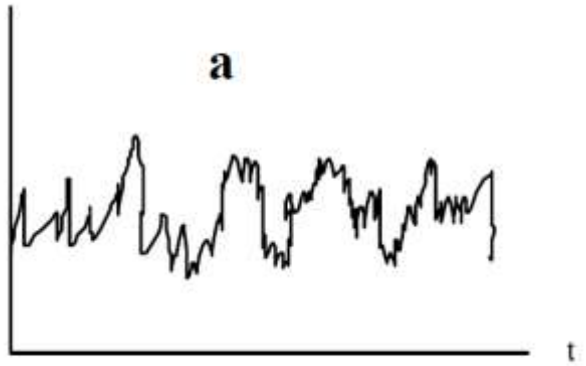


Figure 3.11 - Actual variation of load events over time (JCSS, 2006)

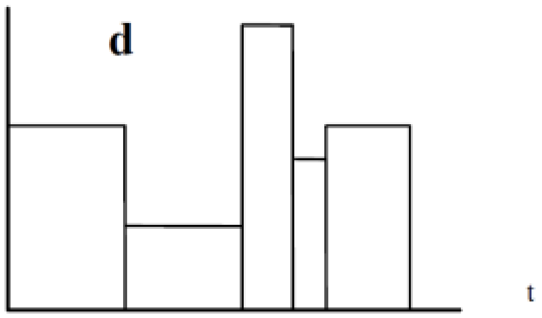


Figure 3.12 - Example of the FBC model (JCSS, 2006)

Type 2 and 3 loadings, unlike the first, include loads that have the property of occurring with isolated extreme values within time intervals. This type of loading, as it presents isolated peaks of occurrence, is studied through extreme statistics. Figure 3.13 shows an example of the behavior of a Type II loading over time.

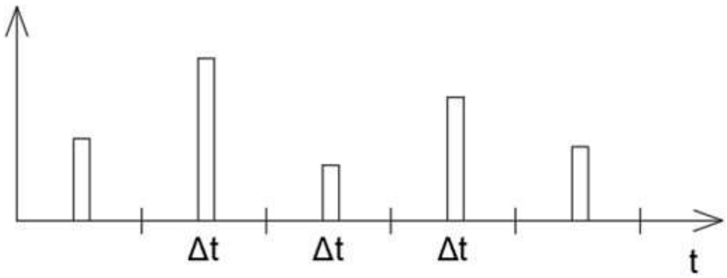


Figure 3.13 - Example of type II load behavior over time

Type 4 loadings are those that are unikely to occur, such as earthquake and tornado loads. These may or may not occur in the lifetime of a structure. This way they behave as shown in Figure 3.14.

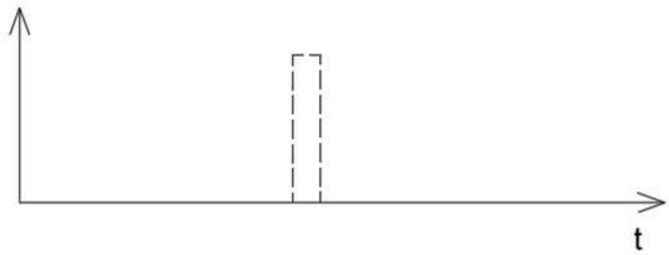


Figure 3.14 - Model referring to type IV load

3.7.1
Permanent loads

Permanent loads, classified as type I, refer to the dead weight of the structures and the weight of the fixed building elements and permanent installations (NBR 6120, 2019). When evaluating any of the permanent forces in the timeline, the probability of occurrence of this loading is 100%.

Permanent loading has slow and insignificant variation over time. The uncertainties regarding the magnitude of this load can be justified by the dimensional or specific weights variability in a structural component, by the variability of structural parts in the same structure and by the variability between different structures (JCSS, 2006). In other words, columns with the same dimensions may present different weights, due to possible inaccuracies in their manufacture. Changes such as replacing architectural components and coatings with different weights from the originals should also be considered.

In general, permanent loads are determined through the nominal value of the dimensions (or volume) and the average density. Assuming a homogeneous material, the material's dead weight, G, can be determined using the formula:

$$G = \gamma_m \cdot V \tag{63}$$

with:

γ_m: average material density.

V: volume of material.

It is considered that both density and volume follow a normal distribution and thus, it is assumed that the weight itself also has such a distribution.

As there is a probability, albeit extremely low, in the normal distribution, of presenting negative values of self-weigh. Therefore, it would be not consistent to consider that this load could follow the normal distribution. Thus, a lognormal distribution would be more appropriate, as it would eliminate this unrealistic possibility. However, due to the greater complexity of the calculation in the lognormal distribution, the low probability of negative values being reached in the normal distribution and the need for compatibility with the normative definitions of characteristic values, the permanent load is considered to have a normal distribution.

This leads to the discussion if the discrepancies between the two distributions, normal, N, and lognormal, LN, in the representation of permanent loads, are small. Figure 3.15 shows the density function and the cumulative function of these two distributions in one example, with a characteristic load value of 5,0 kN / m².

Figure 3.16 shows a zoom of the density functions of the two distributions in the previous figure, explaining the probability of negative values in the normal distribution, the same not occurring in the lognormal.

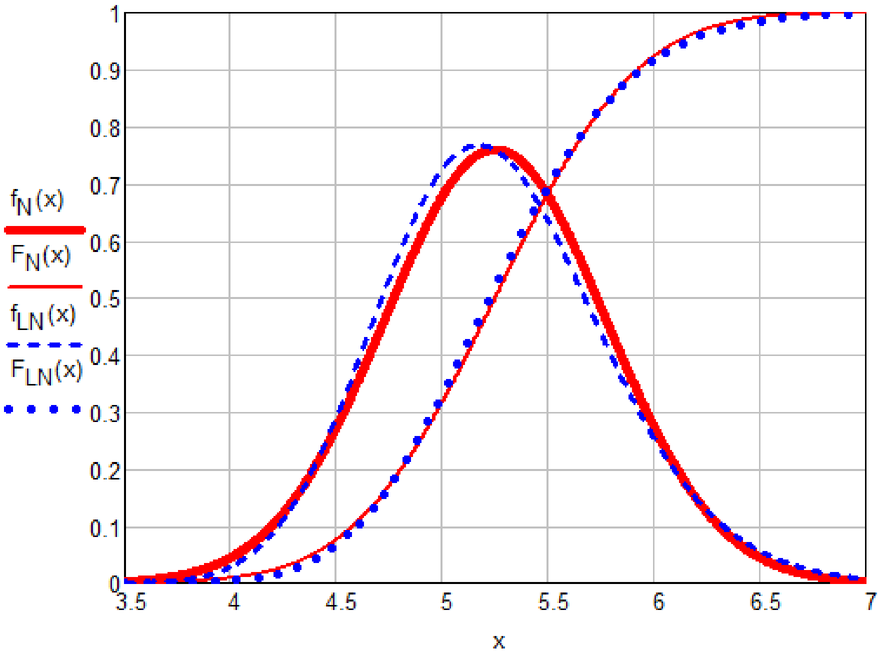


Figure 3.15 - Comparison of density and cumulative functions of normal and lognormal distribution (typical)

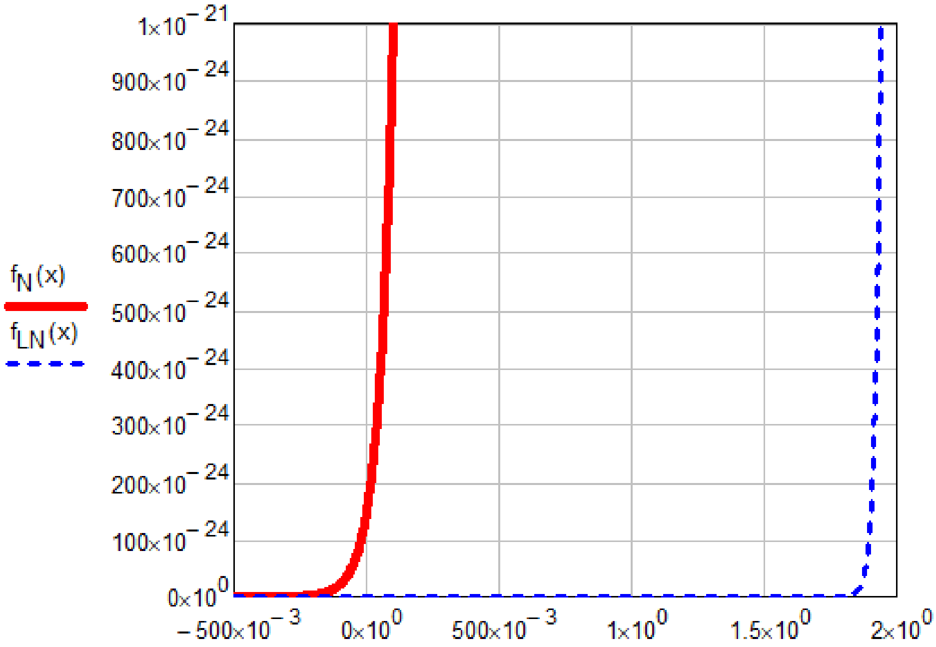


Figure 3.16 - Comparison between normal and lognormal distributions (zoom)

The average values of the permanent load, in general, are close to the nominal values. Due to this consideration, some authors consider that the Bias factor (average value / nominal or characteristic value) of permanent loads shall be 1,00. However, other authors adopt the value of 1,05, since they consider as common that small changes, in relation to the design data, occur in the final structural dimensions, normally with an increase in relation to the nominal values, and also that small future changes after construction phase can occur as adding cladding, tiles or flooring, for example.

Using the values of the variation coefficients available in the JCSS (2006) for volumes and densities, it is possible to evaluate the variation coefficients of the permanent loads of several construction materials, using eq. (64), thus generating, as a result, Table 3.11 presented by Holicky and Sykora (2011).

$$COV_G = \sqrt{COV_V^2 + COV_\gamma^2 + COV_V^2 \cdot COV_\gamma^2} \quad (64)$$

Table 3.11 - Indicated values of coefficients of variation for permanent loads (Holicky and Sykora, 2011)

Coefficient of variation			
Material	V (m^3)	γ (kN/m^3)	G (kN)
Laminated steel	0,03	0,01	0,0316
Concrete	0,02	0,04	0,0447
Masonry	0,04	0,05	0,0641
Wood	0,01	0,10	0,1005

The conclusion obtained from Table 3.11 is that the values of the coefficients of variation of the permanent load of the most common materials of civil construction vary between the values of 0,03 to 0,10.

Nowak and Rakoczy (2007) considered, in their studies, that the average value of permanent loads varies between 1,03 and 1,05 of the characteristic value, while the coefficient of variation varies between 0,08 and 0,10.

Considering all this information, the adoption of 1,5 for the Bias factor for permanent loads and 0,10 for its coefficient of variation (COV) was defined in this work.

In order to expose a practical example, a characteristic permanent load of 7,0 kN/m² is considered and the calculations and commands of the Mathcad application are presented to reach the graphs of the density function and the cumulative function with normal distribution of this load (Figure 3.17).

$$\mu = BIAS_g \cdot q_k = 1{,}05 \times 7{,}00 = 7{,}35\ kN/m^2$$

$$\sigma = COV_g \cdot \mu = 0{,}10 \times 7{,}35 = 0{,}735\ kN/m^2$$

$$f_N(x) = dnorm(x, 7.35, 0.735)$$

$$F_N(x) = pnorm(x, 7.35, 0.735)$$

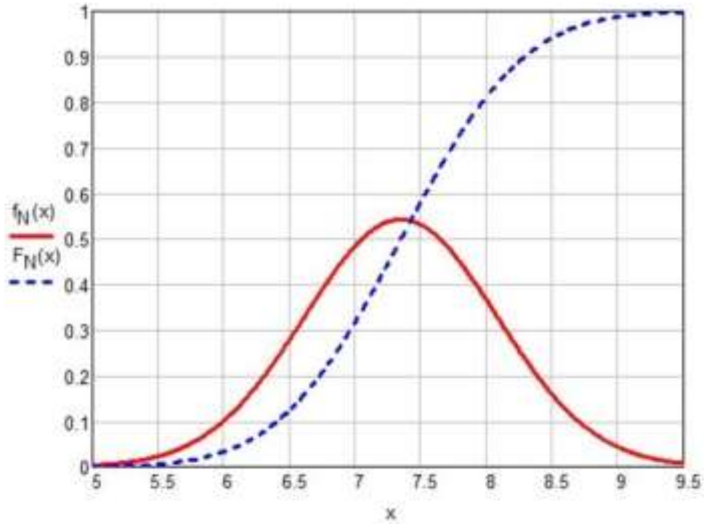


Figure 3.17 - Result of the practical example of permanent loads (kN/m²)

3.7.2 Accidental loads

Accidental loads are directly related to the function of building use, being generated by the weights of furniture, people, vehicles, etc. not including structural and non-structural elements (NBR 6120, 2019).

Variations over time of accidental weights occur at random (JCSS, 2006). These loads can be considered as formed by two parcels classified as long-term loads, arising for example from the weight of furniture and heavy equipment, and of short duration, characterized by intermittent jumps of these loads over the life of the structure, including all the other cases of accidental loads that are not long-lasting.

Figure 3.18 shows an example of an intermittent process, where $\boldsymbol{\lambda}$ is the expected number of load renewals over a period of time and $\mathbf{1}/\boldsymbol{\mu}$ is the average load duration (Hollicky and Sykora, 2011). In a classroom, for example, the of long-term loads consists of the weight of the tables and chairs present there and the average weight of the students who normally attend it, while the short-term loads are those occurring at atypical moments, such as at lecture events, or when the room is used for other activities, thus generating an unusual overweight in the compartment.

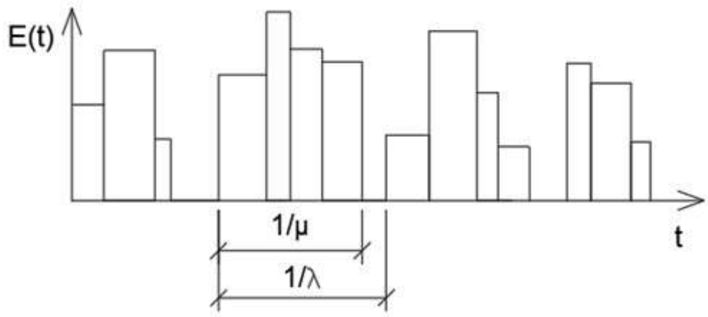


Figure 3.18 - Example of intermittent process

Variations over time in long-term loads are commonly justified by changes in the form of use of the building or its users.

A building previously used by a consulting company, which at some point is replaced by another company specialized in drawings, for example, will have its furniture, machinery and equipment altered, thus generating changes in weights and, consequently, in loads on the floor when compared to the values considered in the design.

It is possible to determine the probability of exceedance of accidental loads by considering a Poisson process (or Poisson distribution). This process considers the average number of occurrences of a certain event in a certain time interval (or space). This distribution is based on the premise that the event can occur at any moment of time and / or any point in space and is characterized by having no memory, that is, the occurrence of an event in a certain time interval is independent from the occurrence in any other time non-coincident interval (Beck, 2019).

The Poisson process has two parameters: the time window to be analyzed, T, which for determining the probability of failure is the working life of the structure, and the average load occurrence (recurrence time), TM, according to the equation below.

$$pf(T) = 1 - e^{-(T/TM)} \quad (65)$$

The average recurrence time considered here for the accidental load is 140 years, an average value following the definitions of NBR 6118 (2014). That means that the value of the accidental load is expected to reach or exceed the characteristic value once, in average, every 140 years. As the expected working life for usual building structures is 50 years, the following failure probability calculation is made:

$$pf_q(50) = 1 - e^{-(50/140)} = 0{,}300$$

It is concluded, therefore, that the probability of failure (overtaking) of the accidental load during the working life is 30%. Extending this concept to characteristic wind loads, whose average occurrence is 50 years, it is obtained:

$$pf_v(50) = 1 - e^{-(50/50)} = 0{,}632$$

Analyzing the probability of exceeding the wind load, it is possible to observe that, even with the recurrence time being equal to the working life, there is still the possibility that this load will not occur during the expected 50 years, since the probability of overtaking is 63,2% and not 100%.

A Gamma distribution is used to represent both the long-term portion and the short-term portion of accidental loads.

The following equation is used to determine the variance of each part (JCSS, 2001):

$$VAR_q = \sqrt{\sigma_V^2 + \frac{\sigma_U^2 \cdot A_0 \cdot \kappa}{A}} \qquad (66)$$

In eq. (63), $\boldsymbol{\sigma_V}$ is the magnitude of the standard deviation of the accidental load, and for the short term part, its value is equal to 0, $\boldsymbol{\sigma_U}$ is the standard deviation of the load's influence area of operation, A_0 is a reference area that, in general, varies between 20 to 100 m ^{2}, A is the influence area of the loading and $\boldsymbol{\kappa}$, a influence factor according to the loading arrangement. The factor $\boldsymbol{\kappa}$ it is directly related to the difference in the way the loads act on the structure. Considering a line of influence on a slab, for example, and that the load on that slab is distributed, the effect generated on it is equivalent to half the effect generated by a concentrated load equivalent to that distributed in the center of the slab.

Table 3.12 describes the parameters for accidental loads for different categories of construction use. Among them are the parameters already described in eq. (63), in addition to the jump rate values of long-term accidental loads, $\boldsymbol{\lambda}$, the jump rate of short-term accidental loads and $\boldsymbol{\nu}$ and the days of the short-term uploads, **d**.

Table 3.12 - Parameters for accidental loads according to category (Holick and Sykora, 2011)

Category	Long-term loads					short-term loads			
	A0 (m^2)	μq (kN/m^2)	σv	σu (kN/m^2)	1/λ (years)	μp (kN/m^2)	σu	1/ν (years)	d (dias)
Office	20	0,50	0,30	0,60	5	0,20	0,40	0,30	1-3
Lobby	20	0,20	0,15	0,30	10	0,40	0,60	1,00	1-3
Residence	20	0,30	0,15	0,30	7	0,30	0,40	1,00	1-3
Hotel room	20	0,30	0,15	0,10	10	0,20	0,40	0,10	1-3
Hospital room	20	0,40	0,30	0,60	5-10	0,20	0,40	1,00	1-3
Laboratory	20	0,70	0,40	0,80	5-10	-	-	-	-
Library	20	1,70	0,50	1,00	10	-	-	-	-
Classroom	100	0,60	0,15	0,40	10	0,50	1,40	0,30	1-5
Stores	100	0,90	0,60	1,60	1-5	0,40	1,10	1,00	1-14
Warehouse	100	3,50	2,50	6,90	0,10-1	-	-	-	-
Light industry	100	1,00	1,00	2,80	5-10	-	-	-	-
Heavy industry	100	3,00	1,50	4,10	5-10	-	-	-	-
Crowd	20	-	-	-	-	1,25	2,50	0,02	0,5

With the values given in Table 3.12 and with eq. (63) it is possible to determine the Gamma functions that represent both the accidental load of long duration and the short duration, in order to determine the characteristic value of the total accidental load.

A practical example is presented in order to determine the characteristic accidental load on an office slab of a 30 m², considering the working life of 50 years.

First, the Gamma function is calculated for the long-term part equivalent to one year and, soon after, for the 50-year equivalent.

$$\upsilon_s = \frac{1}{\lambda} = \frac{1}{5} = 0{,}2$$

$$\sqrt{\sigma_q} = VAR_q = \sqrt{0{,}3^2 + 0{,}6^2 \cdot \frac{20}{30} \cdot 2} = 0{,}755$$

$$\upsilon_q = \frac{\mu_q}{\sigma_q} = \frac{0{,}50}{0{,}755^2} = 0{,}877$$

$$s_q = \mu_q \cdot \upsilon = 0{,}50.0{,}877 = 0{,}439$$

The function input data in the Mathcad application to generate the graphs of the density and accumulated Gamma functions for the period of one year and 50 years for the long-term portion of the accidental letters are presented as follows. Results are given in Figure 3.20:

- For one year:

$$q_q(x) = dgamma(x, s_q) \tag{67}$$

$$Q_q(x) = pgamma(x, s_q) \tag{68}$$

- For 50 years:

$$f_q(x) = q_q(x) \cdot \left(1 + \upsilon_s \cdot T \cdot Q_q(x)\right) \cdot e^{\left[-\upsilon_s \cdot T \cdot \left(1 - Q_q(x)\right)\right]} \tag{69}$$

$$F_q(x) = Q_q(x) \cdot e^{\left[-\upsilon_s \cdot T \cdot \left(1 - Q_q(x)\right)\right]} \tag{70}$$

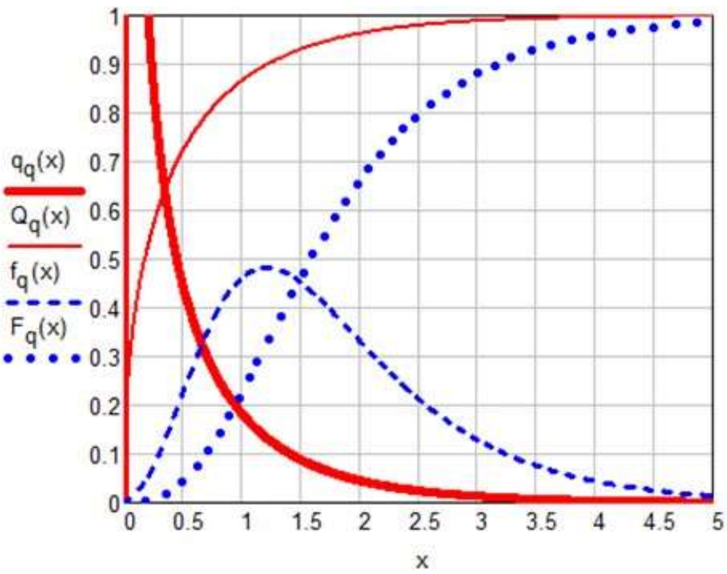


Figure 3.19 - Gamma functions of density and accumulated long-term accidental load for periods of one year and 50 years (kN/m^2)

Then, the Gamma function (Figure 3.19) is calculated for the short-term part equivalent to one year and the equivalent to 50 years.

$$\sqrt{\sigma_p} = VAR_p = \sqrt{0{,}4^2 \cdot \frac{20}{30} \cdot 2} = 0{,}462$$

$$\upsilon_p = \frac{\mu_p}{\sigma_p} = \frac{0{,}20}{0{,}462^2} = 0{,}937$$

$$s_p = \mu_p \cdot \upsilon = 0{,}20.0{,}937 = 0{,}188$$

The function input data in the Mathcad application to generate the graphs of the density and accumulated Gamma functions for the period of one year and 50 years for the short-term portion of accidental loads are given below. Results are given in Figure 3.21:

- For one year:

$$q_p(x) = dgamma(x, s_p) \tag{71}$$

$$Q_p(x) = pgamma(x, s_p) \tag{72}$$

- For 50 years:

$$f_p(x) = q_p(x) \cdot \left(1 + \upsilon_r \cdot T \cdot Q_p(x)\right) \cdot e^{\left[-\upsilon_r \cdot T \cdot \left(1 - Q_p(x)\right)\right]} \tag{73}$$

$$F_q(x) = Q_p(x) \cdot e^{\left[-\upsilon_r \cdot T \cdot \left(1 - Q_p(x)\right)\right]} \tag{74}$$

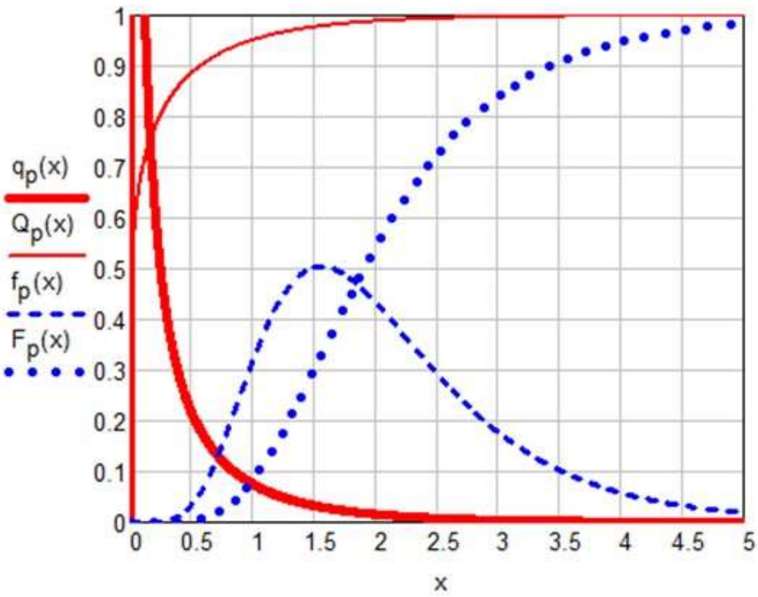


Figure 3.20 - Gamma functions of density and accumulated short-term accidental load for periods of one year and 50 years (kN/m²)

In order to obtain the equivalent function for the total accidental load, that is, to superimpose the two parts, it is necessary to combine the information of the density and cumulative functions of the long and short parts referring to 50 years through the convolution method (see Figure 3.20).

This method is expressed in the following equations, to determine the density and cumulative functions of the total accidental load:

$$F_t(x) = \int_0^x (F_q(x - y) \cdot f_p(y) dy \tag{75}$$

$$f_t(x) = \frac{dF_t(x)}{dx} \tag{76}$$

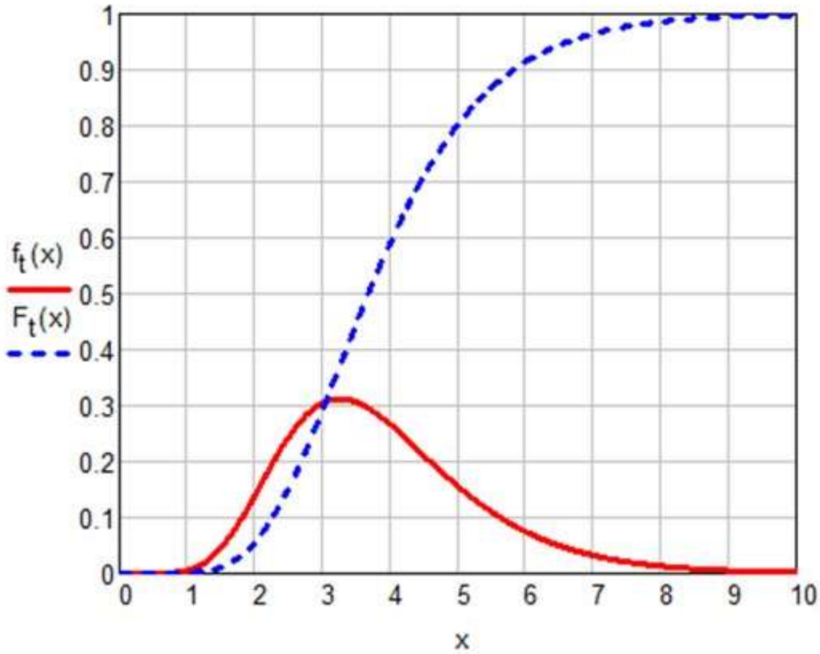


Figure 3.21 - Density and accumulated functions of the total accidental load (kN/m²)

It can be seen in Figures 3.20 and 3.21 that, in 50 years, both in the long-term portion of the load and in the short-term portion, the density functions present characteristics of extreme functions, thus differing significantly from the density functions of the period of one year. In long intervals, therefore, the Gamma distributions converge to Gumbel distributions.

In this way, it is possible to determine the cumulative and density function through the mean and standard deviation calculated with the obtained density function, as shown below.

$$\mu_t = \int_{-\infty}^{\infty} x \cdot f_t(x)dx = 3{,}879$$

$$\sigma_t = \sqrt{\int_{-\infty}^{\infty} (x-\mu_t)^2 \cdot f_t(x)dx} = 1{,}32$$

From the values of the mean and standard deviation, the parameters $\boldsymbol{\alpha}$ and $\boldsymbol{u}$ of the equivalent Gumbel distribution are calculated to then establish the density and cumulative function of the distribution. These calculations are shown below:

$$\alpha = \frac{\pi}{\sigma_t \cdot \sqrt{6}} = \frac{\pi}{1{,}32 \cdot \sqrt{6}} = 0{,}972$$

$$u = \mu_t - \frac{0{,}577216}{\alpha} = 3{,}285$$

$$F_G(x) = e^{-e^{-\alpha(x-u)}} \tag{77}$$

$$f_G(x) = \frac{dF_G(x)}{dx} = \alpha \cdot e^{[-\alpha\cdot(x-u)-e^{-\alpha\cdot(x-u)}]} \tag{78}$$

Figure 3.22 shows the comparison between the density and cumulative functions for the accidental loads of the two distributions, combined and Gumbel equivalent. Note that the difference is not very significant, which therefore validates the use of the Gumbel distribution for the case of extreme accidental load.

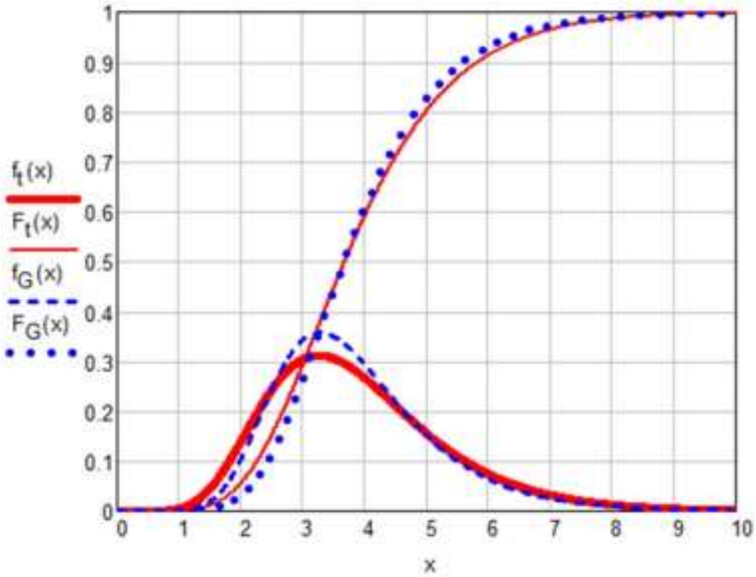


Figure 3.22 - Comparison between Gamma and Gumbel distributions (typical)

From the accumulated function and the probability of the load not being exceeded, it is possible to determine the characteristic value of the accidental load. The probability of non-failure is determined through a Poisson process, as mentioned above, considering the 140-year recurrence time of the accidental load, as shown below.

$$PNF = e^{-T/TM} = e^{-50/140} = 0{,}70$$

Through the probability of non-failure, the corresponding characteristic load is verified in the accumulated function found for the total accidental load. For the example in question, for a 30 m² office, the characteristic accidental load of 4.347 kN / m² was determined. Note that this value is much higher than that prescribed by NBR 6120 (2019) for offices (2,5 kN / m2).

$$F_G(4{,}347) = 0{,}70 \rightarrow q_k = 4{,}347\ kN/m^2$$

Once the value of the characteristic load is found, the Bias factor is determined, dividing it from the average accidental load, $\boldsymbol{\mu_t}$. The coefficient of variation is determined by the ratio of the standard deviation, $\boldsymbol{\sigma_t}$, and the average, $\boldsymbol{\mu_t}$.

$Bias = \frac{\mu_t}{q_k} = \frac{3,879}{4,347} = 0,89$

$COV = \frac{\sigma_t}{\mu_t} = \frac{1,32}{3,879} \approx 0,35$

The values of the Bias factor and the variation coefficient that will be adopted for accidental loads, therefore, will be 0,89 and 0,35, respectively. These Bias values and coefficient of variation are valid for the total accidental loads. The value of the coefficient of variation (0,35) is in line with the value defined by Holicky and Sykora (2011).

3.7.3 Wind loads

The effects of wind in buildings vary according to the ambient climate, with the way in which the structure and its elements are exposed to the wind, with the dynamic properties and with the shape and dimensions of the construction (JCSS, 2006). As with accidental loads, the wind load can be represented by a Gumbel distribution, as it has extreme function characteristics.

Table 3.13 - Wind load variation coefficients indicated

References	COV
JCSS	0,26 to 0,49
Holicky	0,33
Novack	0,35

Table 3.13 gathers the data for the variation of the wind load indicated by the three main references. Considering the given values, the value adopted for the variation coefficient will be 0,35.

Considering an average nominal wind load of 1,0 kN / m², calculations are made to obtain the value of the Bias factor in a manner similar to those performed in determining the Bias for the accidental load. For the calculation of the probability of non-failure, a 50-year recurrence period for this load is considered. The parameters of the Gumbel function that represents the wind are also calculated below.

$PNF = e^{-T/TM} = e^{-50/50} = 0,368$

$\delta_w = \mu_w \cdot COV = 1,0.0,35 = 0,35$

$$\alpha = \frac{\pi}{\delta_w \cdot \sqrt{6}} = \frac{\pi}{0,35 \cdot \sqrt{6}} = 3,664$$

$u = \mu_w - \frac{0,577216}{\alpha} = 1,0 - \frac{0,577216}{3,664} = 0,8429$

$$f_w(x) = \alpha \cdot e^{\left[-\alpha \cdot (x-u) - e^{-\alpha \cdot (x-u)}\right]} \quad (79)$$

$$F_w(x) = e^{-e^{-\alpha(x-u)}} \quad (80)$$

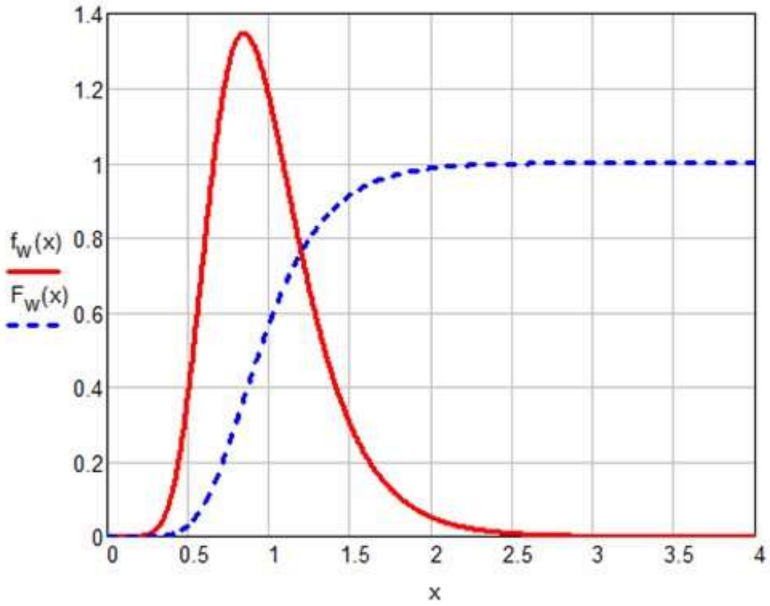


Figure 3.23 - Density and cumulative wind load function (x =kN/m^2)

With the cumulative function and the probability of non-failure of the wind, shown in Figure 3.23, the characteristic wind load for a unit pressure can be determined and, consequently, the Bias factor for wind loads be determined.

$F_G(0,8425) = 0,368 \rightarrow w_k = 0,8425\ kN/m^2$

$$Bias = \frac{\mu_w}{w_k} = \frac{1,0}{0,8425} = 1,187$$

The values of the Bias factor and the variation coefficient adopted, therefore, will be 1,177 and 0,35, respectively.

3.7.4
Load modelling

The variation of the load modelling is generated by the difficulty of perfectly representing the physical reality in a calculation model. As a result, there are significant probabilities that there are differences, however small, between the loads on the model in the design and those that actually act on the structure.

Table 3.14 - Averages and variation coefficients for load modelling determined adapted by JCSS (2006)

Model	Distribution	Avarage	COV
Moment in the section	Lognormal	1,0	0,1
Axial force in the section	Lognormal	1,0	0,05
Shear force in the section	Lognormal	1,0	0,1
Moment in plates	Lognormal	1,0	0,2
Plate forces	Lognormal	1,0	0,1
Stress in 3D solids	Normal	0,0	0,5
Stress in 2D solids	Normal	0,0	0,05

Table 3.14 presents the values of means and variation coefficients of the load modelling indicated by the JCSS. According to Holicky and Sykora (2011), the average for the load modelling established is 1,0 and the standard deviation is from 0,05 to 0,10 value of 1,0. From the average it is adopted herein the value of 0,10, since it is expected a good degree of precision in the creation of the calculation model by the designer.

3.8
Bayesian update of variables

One of the functionalities of the Bayesian approach is the possibility of inserting new information into the existing one, Jacinto (2011) expresses this concept objectively.

Suppose that the safety assessment of a structure is performed and that a basic variable of the problem is variable *X*, is known and removed from an existing database, as:

$$f_X(x) = N(x|\mu_{X0}.\sigma_{X0}^2) \quad (81)$$

Suppose, moreover, that it was possible to obtain from the structure under evaluation a sample $\varepsilon = \{x_1, \ldots x_n\}$ of this sampling (through consultation of work records or tests). It is intended to combine the information contained in this sample with the new information, presented in eq. (81).

When the previous information is materialized in the normal conjugate of the Normal model, the distribution of the parameters μ and σ^2 is:

$$f(\mu, \sigma^2) = N\left(\mu\middle|\mu_0, \frac{\sigma^2}{n_0}\right).GI(\sigma^2|\alpha_0, \beta_0) \quad (82)$$

Thus, the problem is to determine the parameters μ_0, n_0, α_0 and β_0 consistent with eq. (81). From a purely mathematical point of view, this problem can be seen as unsolved, as the parameters μ and σ^2 are modelled as random variables and eq. (82) sets the value of these parameters (respectively in μ_{X0} and σ_{X0}^2).

In order to find a solution to the above problem, note that eq. (81), whose average is μ_{X0} , is comparable with the initial equation of X given by:

$$f_X(\mathrm{x}) = St\left(x \middle| \mu_0 . \sqrt{\left(1+\frac{1}{n_0}\right)\frac{\beta_0}{\alpha_0}}, 2\alpha_0\right) \tag{83}$$

Thus, it appears that the initial average of X coincides with the parameter μ_0, so it can be considered that $\mu_0 = \mu_{X0}$. Regarding the parameters α_0 and β_0, when the only information available is of the sample type, the posterior distribution of μ and σ_2 is given by:

$$f(\mu, \sigma^2|\varepsilon) = N\left(\mu \middle| \bar{x}, \frac{\sigma^2}{n}\right) . GI\left(\sigma^2 \middle| \frac{n-1}{2}, \frac{n-1}{2}s^2\right) \tag{84}$$

Comparing this equation with eq. (80) it seems reasonable to attribute the α_0 and β_0 the following values:

$$\alpha_0 = \frac{n_0-1}{2} \tag{85}$$

$$\beta_0 = \frac{n_0-1}{2}\sigma_{X0}^2 \tag{86}$$

Where n_0 designates the size of the initial equivalent sample and represents the size of a hypothetical sample containing information equivalent to *a priori* information.

Thus, the problem of specifying parameters is solved once a value is assigned to n_0 . According to Jacinto (2011), one possibility is to assign a value to n_0 through the experience of the engineer (*engineering judgement*), considering that n_0 represents the relative weight that μ_0 has on average *a posteriori* $\mu_n = (n_0\mu_0 + n\bar{x}/n_0 + n)$, where x and n represent, respectively, the sample mean and size $\varepsilon = \{x_1,...,x_n\}$ available. Thus, the parameter n_0 it constitutes the weight, or credibility, that is intended to be given to the previous information. For example, by adopting $n_0 = n$, this means that a priori information is being given the same weight as the sample information. Another possibility is to assign a value to n_0 a value such that p of eq. (81) is equal to p of eq. (87). Then matching the quantiles p given by these equations, we obtain:

$$\mu_0 + \sqrt{\left(1+\frac{1}{n_0}\right)\frac{\beta_0}{\alpha_0}}t(p, 2\alpha_0) = \mu_{X0} + \sigma_{Xo}.z(p) \tag{87}$$

where $t(.,.)$ and **z**(.)are the inverse of the accumulated t-Student and Normal distributions, respectively. As already said, $\mu_0 = \mu_{X0}$, so:

$$\sqrt{\left(1+\frac{1}{n_0}\right)\frac{\beta_0}{\alpha_0}t(p,2\alpha_0)=\sigma_{X0^z}(p)} \tag{88}$$

Dividing eq. (87) by eq. (88), $\beta_0/\alpha_0 = \sigma_{X0}^2$, so:

$$\sqrt{\left(1+\frac{1}{n_0}\right)\sigma_{X0}^2 t(p,2\alpha_0)=\sigma_{X0^z}(p)} \tag{89}$$

Finally:

$$\sqrt{\left(1+\frac{1}{n_0}\right)}t(p,n_0-1)=z(p) \tag{90}$$

The only solution to this eq. (90) is $n_0 = \infty$,for any value of p. This is due to the fact that, since eq. (58) sets the parameters μ and σ_2, that is to say there is no uncertainty in these parameters or that the parameters were estimated from an infinite sample, so it is logical that this recommendation leads to $n_0 = \infty$.

The value $n_0 = \infty$ creates some numerical difficulties. However, for practical purposes, one could adopt $n_0 = 50$, which represents an important reduction in statistical uncertainty. It should be emphasized, however, that placing a high value on n_0 is equivalent to giving high credibility to the previous information. The bigger n_0, the greater the weight of the previous information in the intended estimates, and consequently less is the weight of the sample information in these same estimates.

In summary, when the previous information is as presented in eq. (58), it is possible to specify the parameters consistent with that information, being necessary to use the experience of the engineer to assign the value of n_0, whereas this is a measure of the credibility of that information. The remaining parameters are obtained by deduction, using the expressions:

$$\mu_0=\mu_{X0}; \qquad \alpha_0=\frac{n_0-1}{2}; \qquad \beta_0=\frac{n_0-1}{2}\sigma_{X0}^2 \tag{91}$$

3.9
Methodology Chart

This item describes the methodology proposed for new and existent structures and later wil be applied in the sudy of cases presented in this work.

3.9.1 Methodology for New Structures

The methodology proposed for the new structures is described below.

- Analysis of the structural elements, considering the application of the critical actions, concentrated in the nodes and self-weight loads. The loads that are applied to the model acting on each structural element are extracted.
- Structural design following the normative recommendations, considering the semi-probabilistic approach with the standardized safety factors and the characteristic resistance and load values in the ultimate limit state.
- After determining the dimensions and reinforcement of the structural elements, a global safety analysis is then performed. A non-linear analysis of the structure is executed through a structural analysis program, where the application of the mean values of the critical load of the model and the self weight is considered
- The average resistance of the structural elements and the formation of plastic hinges are considered when the respective resistance capacities are reached.
- Then, the global safety factor is determined, through the ratio between the load applied at the time of the collapse and the average load value. After the structural elements are designed in the ultimate limit state and the global safety factor is determined by the global safety approach, the reliability analyzes are performed.
- Determination of the safety levels associated with each structural element, done through a probabilistic analysis in the ultimate limit state. The determination of the level of safety associated with the global behavior of the structure element is done through a probabilistic analysis.
- Reliability analyses are performed using the failure function associated with the adopted parameters λ, their respective mean, standard deviation and type of probabilistic distribution as input, and as a result, the reliability indices and failure probabilities for each case studied.
- - With the results obtained in the reliability analyses, comparisons can be made between the reliability indices. In this comparison of results, it can be analyzed whether the results obtained in the global safety approach are greater than the values obtained in the semi-probabilistic analysis, for the adopted structural elements sections.

This methodology could lead to an optimization of the design, with more economical structures, without affecting its safety.

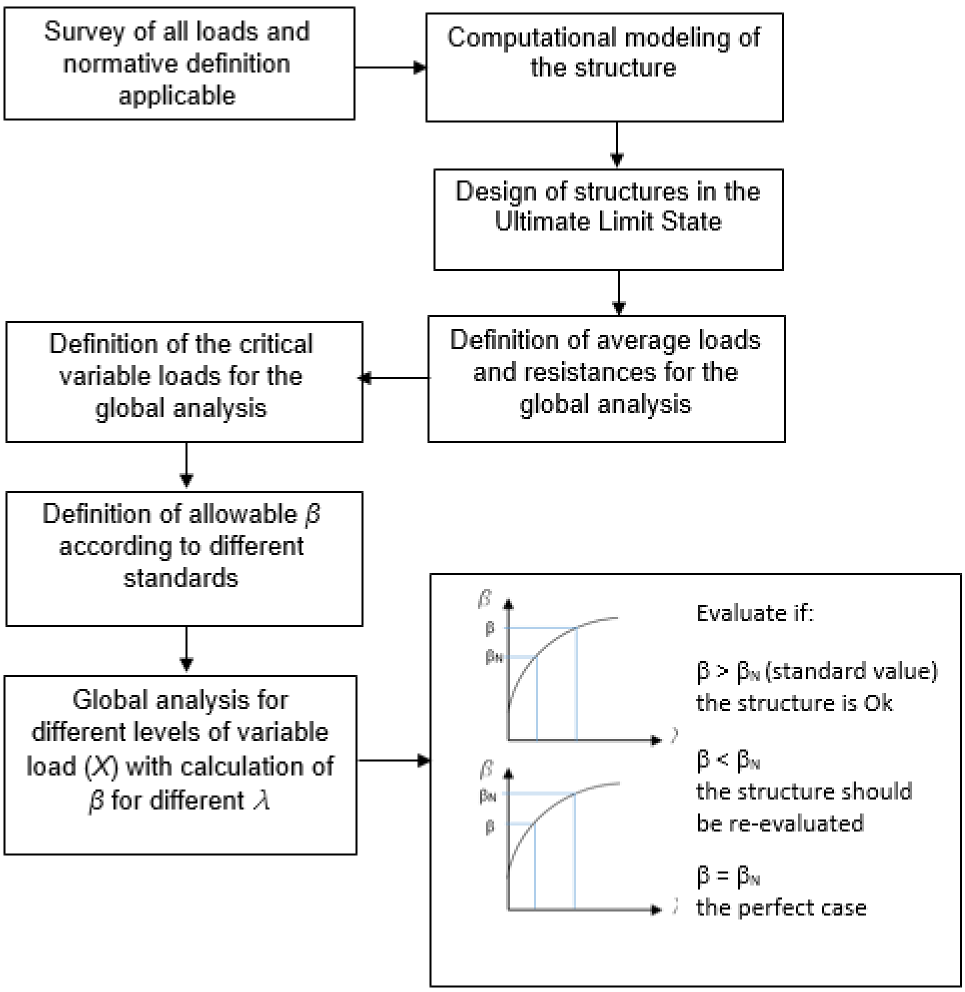


Figure 3.24 – Methodology for a new structure

3.9.2
Methodology for Existent Structures

The methodology proposed for the existent structures is described below:

- The first and not trivial question to do is “is there a design for this structure?”. For a positive answer, the next step is the knowledge of the level of degradation of this structure, in case of a negative answer, a on the field survey as detailed as possible should be executed.
- After, it is mandatory obtaining the actual structural material data, through concrete testings, for example, doing a detailed verification in all

standards applicable for this case, with special attention for possible normative changes, especially regarding for applied loadings.

- The next step is an execution of a Bayesian updating for the materials strength. All these studies will provide the necessary information for decision-making: elaborating a new structural model or updating the existing one.
- The next steps are similar for the methodology for new structures, which are: the definition of average loads and resistances for the global analysis, the definition of the critical variable loads for this analysis and execution of the global analysis for different levels of variable loads with the respective calculation of β for different values of λ.
- In the final analysis, the evaluation if for the existent lo if the βo is acceptable or some kind of reinforcement is necessary for this structure, for increasing $\lambda o \rightarrow \lambda 1$.

This methodology will indicate whether the level of safety that the structure will present in its remaining working life is acceptable.

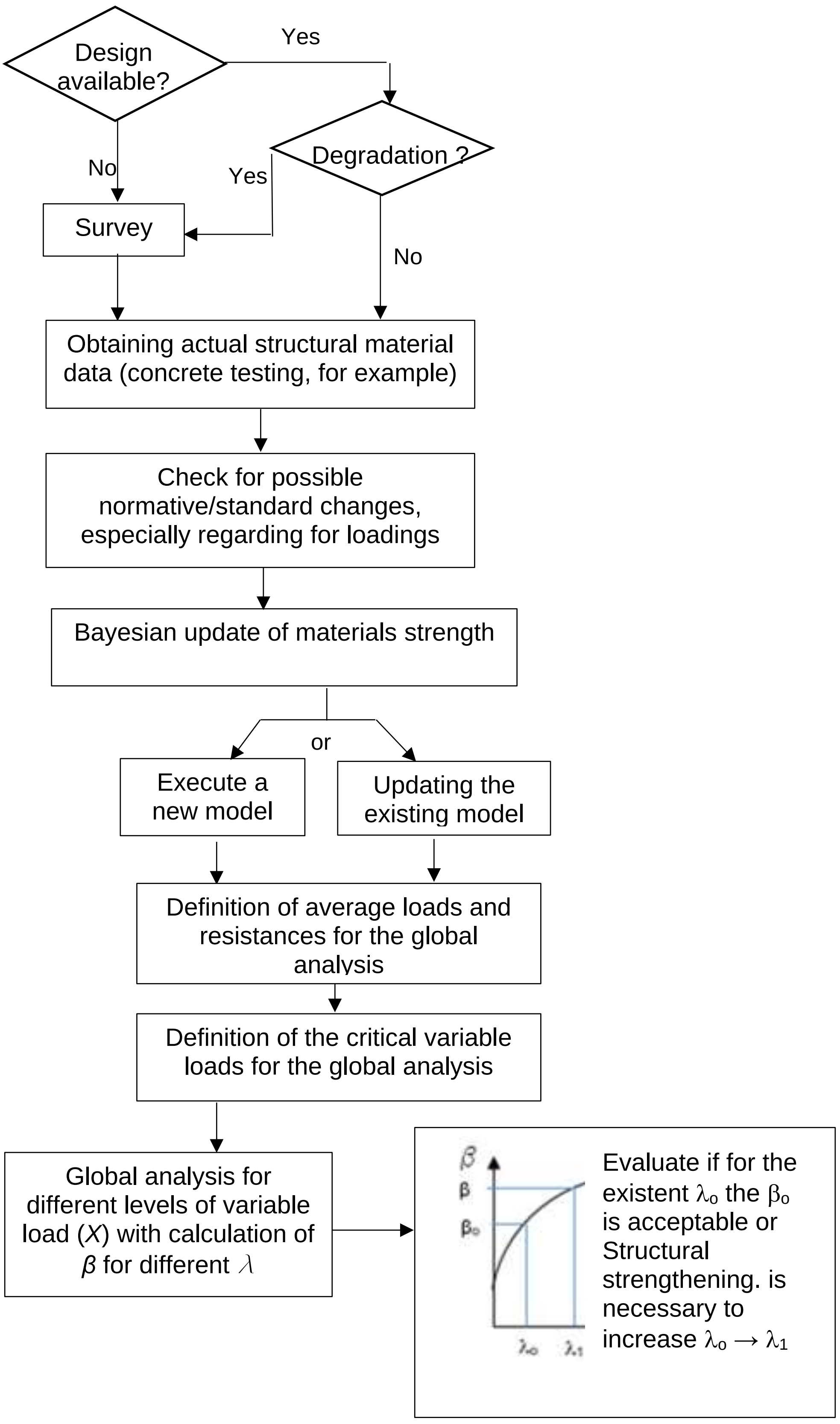


Figure 3.25 – Methodology for a existent structure

4
Analysis of a frame using a global approach

4.1
Structure analysed

Aiming to present two different procedures to assess the safety of a concrete structure, the concepts presented in the previous chapters are going to be applied to the central frame of a 13-story building. For this purpose the usual verification criterion in the Ultimate Limit States based on the partial factor method and the Global Safety approach are applied. A probabilistic approach also is applied to the beams and columns analysis and to a global safety procedure. Here the results presented by Santos *et al*. (2019) and Monteiro Jr. (2019) are developed.

A conventional symmetrical structure was selected for the analysis, instead of a real one, to facilitate the analysis of results. The building is subject to a loading situation compatible with a real situation, with the simultaneous application of permanent loads and wind.

For the analysis of Global Safety, it is necessary to re-evaluate the maximum resistant forces in the structural sections, using, instead of the calculation values of resistance, their average values. A factor λ must be found, which will increase the loads until the situation of collapse.

Figure 4.1 represents a typical building floor, with 10m $\times$ 8m slabs in plan and a frontal view of the central frame.

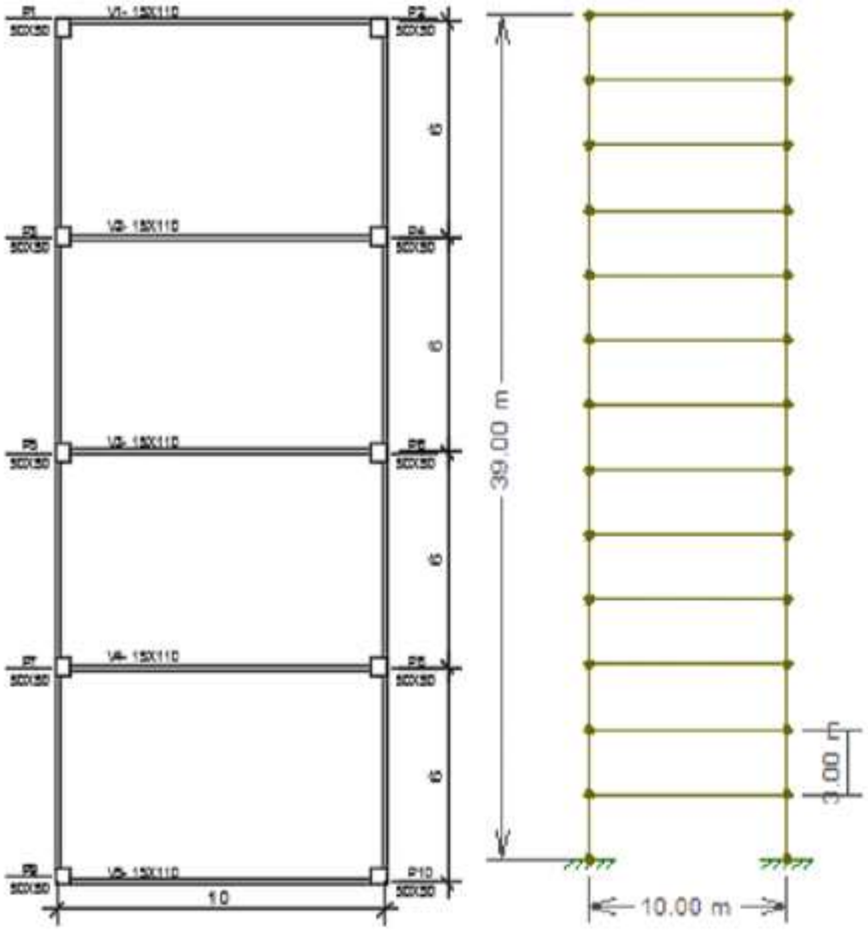


Figure 4.1 - View of a typical floor and cross section of the analyzed building

The loads considered correspond to a permanent load of 8,0 kN/m^2 and to a total wind pressure of 1,0 kN/m^2. The resulting nodal loads applied to the model are 18,0 kN (wind, on the left facade of the building) and a linear load of 48,0 kN/m as a permanent load on the beams.

The characteristic values of the strength of concrete and steel are f_{ck} = 30 MPa (Class C30) and f_{yk} = 500 MPa (CA-50).

The structural dimensions of the beams and the necessary reinforcement bars are adjusted floor to floor to strictly resist the forces obtained in the elastic analysis for the design in the Ultimate Limit State. This is necessary for the subsequent analysis by Global Safety to be consistent, as plastic hinges apperar in each floor.

For example, on the first floor, the dimensions of the beams are 15 cm × 110 cm and the reinforcement 22,73 cm^2; on the eleventh floor, the dimensions are 15 cm × 85 cm and the reinforcement of 16,7 cm^2.

The columns have a constant dimension of 50 cm × 50 cm in the plane. The 72 cm^2 reinforcement bars, as shown schematically in Figure 4.2, are required to support the stresses at the base of the columns.

Wind action: V = 1,187 . 18 = 21,37 kN

Floor load: P = 8 x 5 x 6 = 240 kN

The following calculation data are considered:

d = h – cover = 1,10 – 0,05 = 1,05 m

b = h = 0,50 m (column transversal section)

d' = d'' = 0,03 m (reinforcement cover)

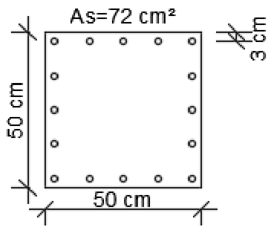


Figure 4.2 - Typical column section

4.2
Deterministic analysis – Ultimate Limit States

The most relevant results of the analysis are shown in Figure 4.3: maximum moment in the beam of the first floor maximum moment, and axial force at the base of the column of the first floor (characteristic values).

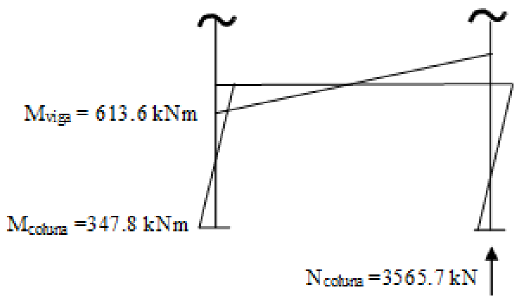


Figure 4.3 - Relevant results of the elastic analysis

The verification of the reinforcement of beams and columns is done with the spreadsheets developed by Santos (2020). The corresponding interaction curves are shown in Figures 4.4 and 4.5. The forces are increased by the factor $\gamma_f = 1{,}4$.

transversal section				**Concrete**		**Steel**	CA50A
b (m)	0,15	d'(m)	0,05	fck (MPa)	30	fyk (kN/cm	50
h (m)	1,1	d''(m)	0,05	fcd (kN/m^2)	21429	Es (kN/cm	21000
		d(m)	1,05			ε_{yd} (‰)	2,070
Reinforcement position						fyd (kN/cm	43,48
layers	A_{si} (cm^2)	d_i (m)		Nd(kN) =	0		
1	22,73	0,05		Md(kN.m) =	859		
2	0	1,05					
3							
4							
5							
6							
7							
SUM	22,73						

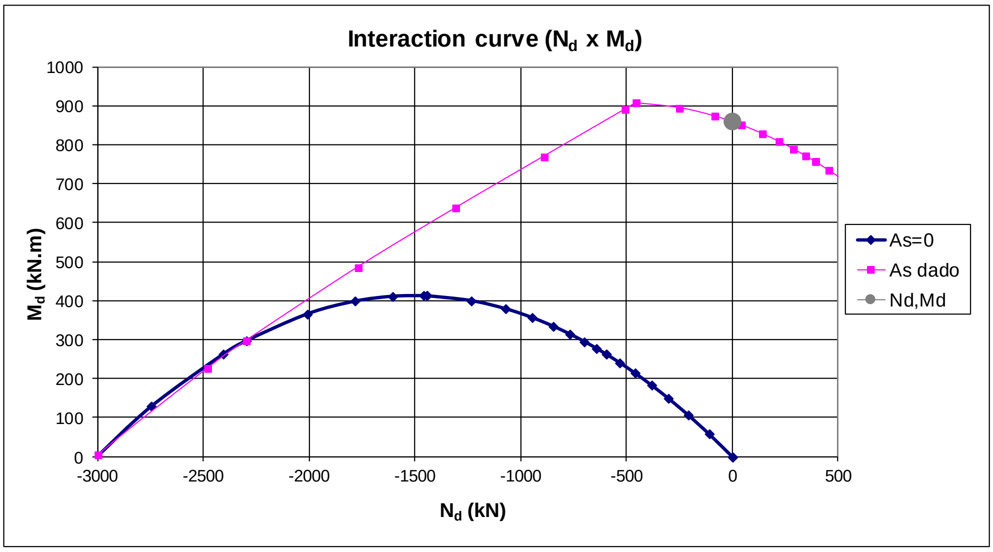


Figure 4.4 - Interaction curve (N,M) for a beam

transversal section				**Concrete**		**Steel**	CA50A
b (m)	0,5	d'(m)	0,03	fck (MPa)	30	fyk (kN/cm	50
h (m)	0,5	d''(m)	0,03	fcd (kN/m^2)	21429	Es (kN/cm	21000
		d(m)	0,47			ε_{yd} (‰)	2,070
Reinforcement position						fyd (kN/cm	43,48
Layers	A_{si} (cm^2)	d_i (m)	Nd(kN) =	-4992			
1	22,5	0,03	Md(kN.m) =	467			
2	22,5	0,47					
3	9	0,14					
4	9	0,25					
5	9	0,36					
6							
7							
SUM =	72						

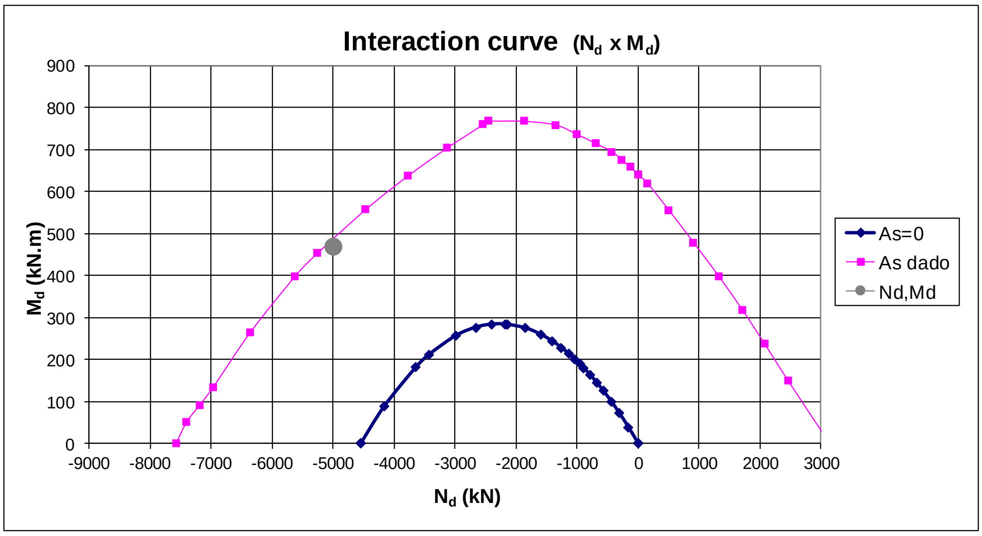


Figure 4.5 - Interaction curve for column verification

4.3

Deterministic Analysis - Global Safety

In the Global Safety Analysis, the section strengths are evaluated with the average values of the material resistance, according to item 4.6.2.1 of the *fib* Model Code (2010). Thus, interaction curves of the sections previously analyzed are presented to evaluate the bending moments resisted by them in this condition.

In order to obtain the mean values of the resistances, it should be considered that for defining the characteristic values of the resistances of the materials, the quantile of 5% was considered,

Considering for the strength of concrete and steel the coefficients of variation (COV = standard deviation / mean value) respectively equal to 0,15 and 0,05, the relationships between mean and characteristic values (or bias factors) result in 1,328 and 1,089. So, the average values to be considered are:

Concrete: f_{cm} = 1,328 × 30000 = 39840 kPa

Steel: f_{ym} = 1,089 × 500000 = 544500 kPa

For the beam of the first floor, considering the interaction curve shown in Figure 4.6, obtained with the mean values of the resistances, the maximum resistant moment of 1151 kNm is obtained. The procedure is repeated for the beams of all floors. Similarly, for the column, the interaction curve 4.7 is plotted, with the mean values of the resistances. The maximum resistant moment is 1016 kNm, obtained after an iterative process, in which the final value of the axial force is 5114 kN.

The analysis for Global Safety assumes an ultimate situation of rupture, in which both the beams and the columns of the frame reach their resistant capacities. In the case of beams, the situation of the formation of plastic hinges is considered when they reach their resistant capacity with the increase of the loads applied to the analyzed structure (wind loads, in the analyzed case).

After the formation of plastic hinges on the beams, the increase in stresses applied to the frame is supported exclusively by the columns. The critical moment of the analysis occurs when plastic hinges are formed at the bases of the columns, initiating a kinematic chain.

In the exemplified case, the analysis consists of maintaining constant the applied value of permanent load (floor load) and progressively increasing the wind loads applied to the structure, multiplying them by a factor λ (Global safety factor) until the moment of collapse of the analyzed frame.

transversal section				**Concrete**		**Steel**	CA50A
b (m)	0,15	d'(m)	0,05	fck (MPa)	30	fyk (kN/cm	50
h (m)	1,1	d''(m)	0,05	fcd (kN/m^2)	39840	Es (kN/cm	21000
		d(m)	1,05			ε_{yd} (‰)	2,070
Reinforcement position						fyd (kN/cm	54,43
Layers	A_{si} (cm^2)	d_i (m)	Nd(kN) =	0			
1	22,73	0,05	Md(kN.m) =	1151			
2	0	1,05					
3							
4							
5							
6							
7							
SUM=	22,73						

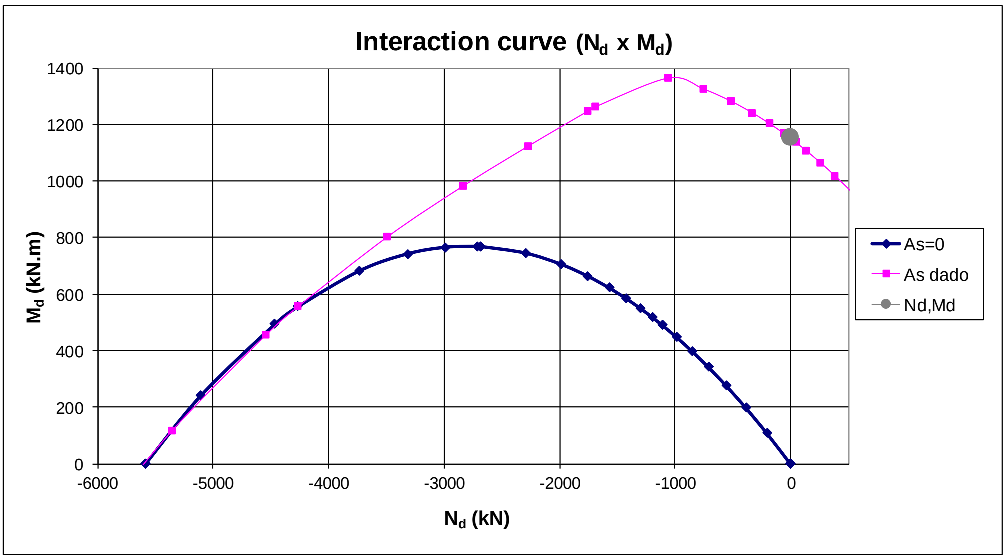


Figure 4.6 - Interaction curve for checking the beam with mean values of resistances

transversal section				Concrete		Steel	CA50A
b (m)	0,5	d'(m)	0,03	fck (MPa)	30	fyk (kN/cm	50
h (m)	0,5	d''(m)	0,03	fcd (kN/m^2)	39840	Es (kN/cm	21000
		d(m)	0,47			ε_{yd} (‰)	2,070
Reinforcement position						fyd (kN/cm	54,43
Layers	A_{si} (cm^2)	d_i (m)	Nd(kN) =	-5114			
1	22,5	0,03	Md(kN.m) =	1016			
2	22,5	0,47					
3	9	0,14					
4	9	0,25					
5	9	0,36					
6							
7							
SUM =	72						

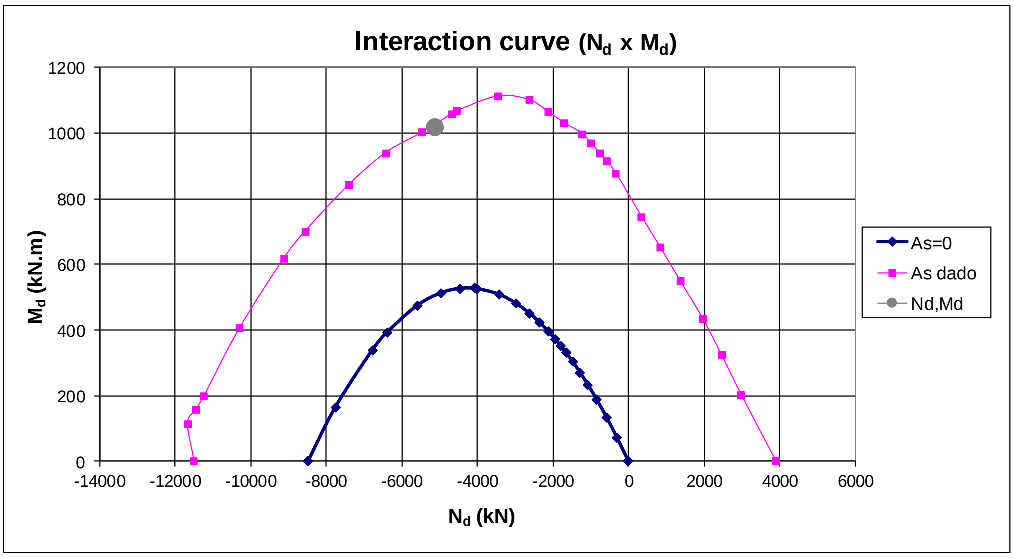


Figure 4.7 - Interaction curve for the verification of the column with average resistance values

To obtain the required results in the Global Safety Analysis, the average values of the applied loads are considered, as well as the average values of the considered resistances. This consideration aims to exempt the results found from being influenced by arbitrary definitions of characteristic and design stresses and resistances, which vary according to the standard considered in the project.

In addition, in order to assure the symmetry of the analysis, allowing for a simple of the mechanisms of formation of plastic hinges, the loads are only applied on the nodes.

For obtaining the average values of the applied loads, the relationships between mean and characteristic values (“bias factors”) of 1,05 for permanent load and 1,177 for wind load (corresponding to a 50-year recurrence period and the coefficient of variation of 0,35) are considered. Thus, the following average values are obtained for the applied nodal loads.

Permanent loads: $D_m = 1{,}05 \times 8 \times 6 \times 5$ = 252 kN

Wind loads : $W_m = 1{,}187 \times 9$ = 10,68 kN

The numerical model is defined in such a way that, as the horizontal force is increased, plastic hinges appear successively in the beams, as long as the respective resistant moments are reached. The process ends when the last plastic hinge appears at the base of the columns.

The relevant results, which appear at the end of the Global Safety Analysis process, are shown in Figure 4.8.

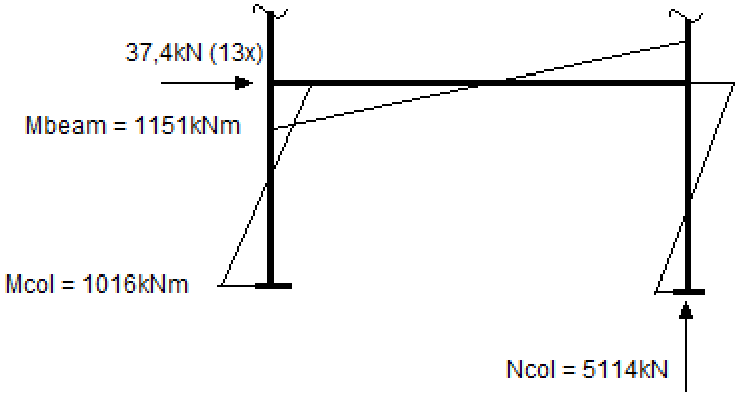


Figure 4.8 - Relevant results of the Global Safety Analysis

The obtained overall safety factor (λ) is:

$$\lambda = \frac{37{,}40}{10{,}68} = 3{,}50$$

Note that this relatively high Global safety coefficient indicates that, considering Global Safety concepts, the design of a building can be optimized, allowing for a reduction in the cost of the structure.

4.4
Probabilistic analysis - Beams

The probabilistic safety assessment is carried out in the critical sections of beams and columns.

For the probabilistic definition of the random variables, the values presented in the previous items are considered, and summarized in Table 4.1.

The normal force on the column is considered to be due to the permanent load and the horizontal load due to the wind.

Table 4.1 - Probabilistic characteristics for reliability analysis

Parameter	Distribution	BIAS factor	COV ou σ
Permanet load	Normal	1,050	0,10
Accidental load (50 years)	Gumbel	0,890	0,35
Wind (50 years)	Gumbel	1,187	0,35
Modeling of loads	Normal	1,000	0,10
Concrete resistance	Normal	1,328	0,15
Steel resistance	Normal	1,089	0,05
Sections dimension	Normal	1,000	4mm+0,006L≤10mm
Bar area	Normal	1,000	0,015
Covering	Normal	1,000	5mm
Modeling of resistances	Normal	1,000	0,05

For beams, in simple bending, the sectional equilibrium is considered through the equivalent stress rectangular block, defined in NBR 6118 (ABNT, 2014), as shown in Figure 4.9.

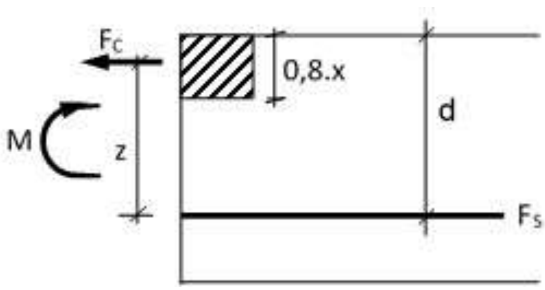


Figure 4.9 - Equilibrium in the concrete section

Considering the equilibrium:

$$F_S = \mathrm{A_S}.\mathrm{f_y} \tag{92}$$

$$F_C = 0{,}85.f_c.b.0{,}8x \tag{93}$$

$$z = d - 0{,}4x \tag{94}$$

Considering $F_S = F_C$:

$$M = A_S.f_y.z = A_S.f_y.(d - 0{,}4x) \tag{95}$$

$$M = A_S.f_y\left(d - \frac{0{,}5}{0{,}85}.\frac{A_S.f_y}{f_c.b}\right) \tag{96}$$

For the probabilistic analysis, the following equation is finally considered:

$$F_{lim} = A_S.f_y.\left(h - cob - 0{,}588.A_S.\frac{f_y}{b.f_c}\right) - W \tag{97}$$

The variables related to the modelling of loads and resistances are considered, according to Table 4.1, but not explicitly expressed in the equation above, for simplicity. They are considered in the VAP (Variables Processor program is adopted as the reability analysis software in this work, PETSCHACHER SOFTWARE, 2016) run, Figure 4.10.

The variables in this equation not yet explicitly defined are: *b* (section width), *h* (section height), *cob* (distance between the axis of the reinforcement and the face closest to the section) and *W* (bending moment in the beam caused by the wind).

Based on Table 4.1, the variables considered in the analysis of the first-floor beam are given in Table 4.2.

Table 4.2 - Probabilistic characteristics for the beam reliability analysis

Parameter	Distribution	Avarage	Standard deviation
h (m)	Normal	1,100	0,01
b (m)	Normal	0,150	0,0049
cob (cm^2)	Normal	0,050	0,005
As (cm^2)	Normal	22,730	0,341
fc (kN/m^2)	Normal	39840	5976
fy (kN/m^2)	Normal	54,45	2,7225
W (kNm)	Gumbel	728,3	254,91
Modeling of loads	Normal	1,00	0,10
Modeling of resistances	Normal	1,00	0,05

The Reliability Analysis is performed with the VAP program, applying the FORM method. Figure 4.10 reproduces a program screen.

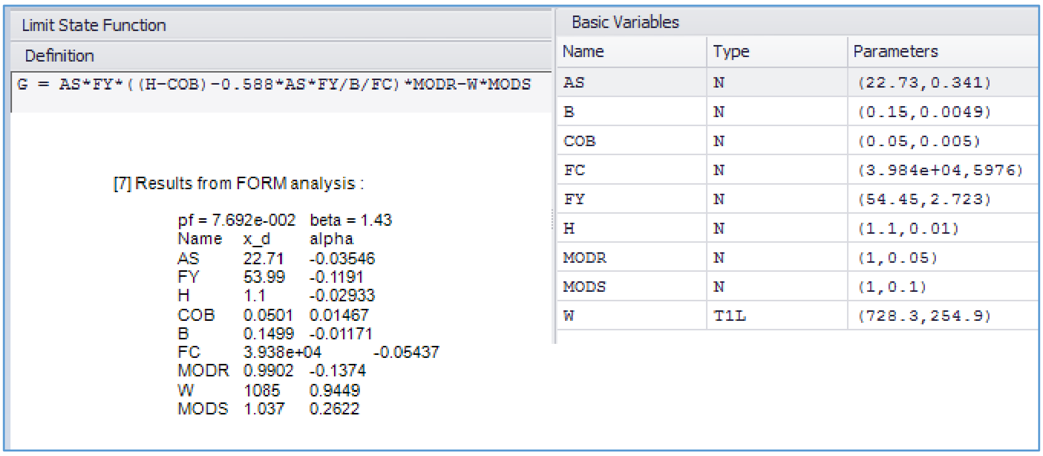


Figure 4.10 - Probabilistic beam analysis

The main results of the analysis are:

- Reliability index: $\beta = 1{,}43$
- Probability of failure: $p_f = 7{,}692 \times 10^{-2}$

This very low value obtained for the reliability index β is compatible with the situation of a beam subject only to the variable wind load, which has a high coefficient of variation (COV = 0,35).

For the beams on the other floors very similar results are obtained (p_f up to $7{,}85 \times 10^{-2}$).

4.5
Probabilistic analysis - Columns

For the columns, the following variables are initially defined, mechanical ratio of reinforcement ω, reduced normal strength η and reduced moment μ:

$$\omega = \frac{A_S.f_{yd}}{b.h.f_{cd}} \tag{98}$$

$$\eta = \frac{N}{b.h^2 f_{cd}} \tag{99}$$

$$\mu = \frac{M}{b.h^2 f_{cd}} \tag{100}$$

N and *M* are the normal force and the moment acting on a column base *b* and height *h*, being f_{cd} the design compressive strength of concrete and f_{yd} the design yield strength of reinforcement.

It is assumed that, around the design point, the variables ω, η and μ can be related through a linear relationship:

$$\mu = A + B\eta + C\omega \tag{101}$$

This relationship is shown graphically in Figure 4.11, drawn on a dimensionless diagram taken from Santos (2020).

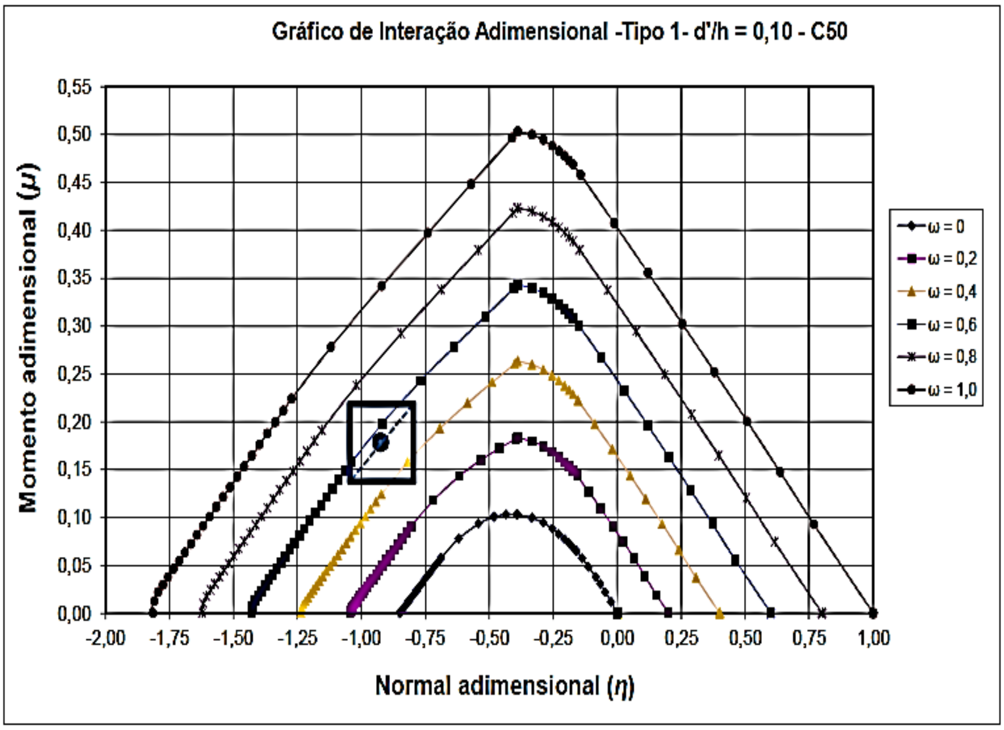


Figure 4.11 - Linear relationship between dimensionless values in the columns

For the analyzed column, the obtained values are:

- A = -0,6129 ; B = 2,8802 ; C = 0,7268

(positive signal for compression)

After some substitutions, the following limit state equation is obtained:

$$F_{lim} = A_s \cdot f_y + 0{,}6129 \cdot b \cdot h \cdot f_c - \frac{2{,}8802 \cdot M}{h} - 0{,}7268 \cdot N \qquad (102)$$

Based on Table 4.1, the variables considered in the analysis at the base of the first-floor column are given in Table 4.3.

Table 4.3 - Probabilistic characteristics for the column reliability analysis

Parameter	Distribution	Avarage	Standard deviation
h (m) = b (m)	Normal	0,500	0,007
As (cm^2)	Normal	72,000	1,08
fc (kN/m^2)	Normal	39840	5976
fy (kN/m^2)	Normal	54,45	2,7225
N (kN)	Normal	3744	187,20
W (kNm)	Gumbel	412,9	144,5
Modeling of loads	Normal	1,00	0,10
Modeling of resistances	Normal	1,00	0,05

The Reliability Analysis is performed with the VAP program, applying the FORM method. Figure 4.12 reproduces a program screen.

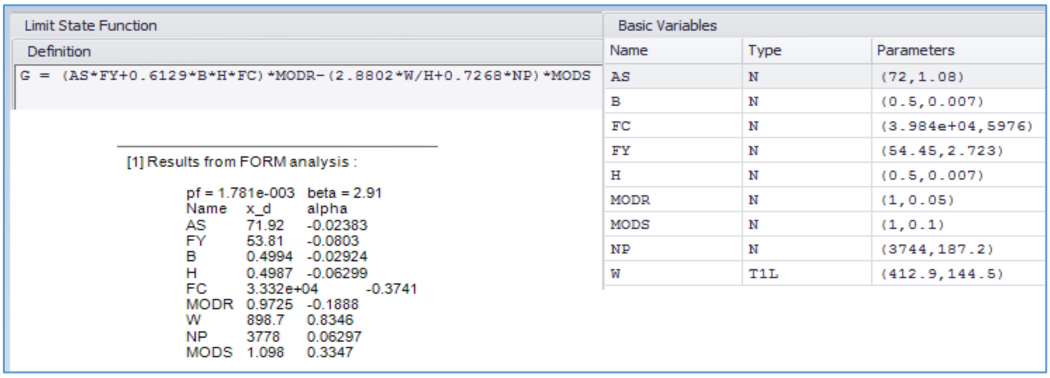


Figure 4.12 - Probabilistic analysis of the column

The main results of the analysis are:

- Reliability Index: $\beta = 2{,}91$
- Probability of failure: $p_f = 1{,}781 \times 10^{-3}$

This relatively low value obtained for the reliability index β can be explained, because considering that permanent and variable loads acting in the column simultaneously, the latter one (wind) presents high coefficient of variation and high participation in the total load.

4.6
Probabilistic Analysis - Global Safety

The analysis is done for the collapse situation shown in Figure 4.8, corresponding to the Global Safety Analysis.

A relationship is sought between the value of the reliability index *β* and the Global safety coefficients λ.

Initially, it is necessary to correlate, using equilibrium equations, the acting vertical and horizontal forces, the maximum bending moment resisted by the beams with the normal forces and bending moments acting at the base of the critical column.

By making these considerations of equilibrium in the studied frame, the following expressions are found:

$$M_{col} = 273 \times F_H - \sum M_{beam} \tag{103}$$

$$N_{col} = \frac{(2 \times 273 \cdot F_H - 2 \times M_{col})}{10} + 13. F_V \tag{104}$$

In these equations, M_{col} e N_{col} are, respectively the bending moment and the normal force acting at the base of the column and F_H and F_V are, respectively, the numerical values of the horizontal and vertical forces acting on the frame nodes.

After some suitable substitutions, the following limit state equation is obtained:

$$\begin{aligned} F_{lim} &= A_s \cdot f_y + 0{,}6129 \cdot b \cdot h \cdot f_c - \frac{2{,}8802 \cdot (273 \cdot F_H - \sum M_{beams})}{h} \\ &\quad - 0{,}7268 \times \left(13 \cdot F_V + 0{,}2 \sum M_{beams}\right) \end{aligned} \tag{105}$$

Based on the basic values given in Table 4.1, the probilistic variables to be considered in the Reliability Analysis for Global Safety are defined in Table 4.4. The values considered for mean and standard deviation of the sum of the resistant moments in the total of thirteen beams are found applying the eq. (106):

$$\mu = \sum \mu_{beams}; \ \sigma = \sqrt{\sum \sigma^2_{beams}} \tag{106}$$

Table 4.4 - Probabilistic characteristics for the global safety reliability analysis

Parameter	Distribution	Avarage	Standard deviation
h (m) = b (m)	Normal	0,500	0,007
As (cm^2)	Normal	72,000	1,08
fc (kN/m^2)	Normal	39840	5976
fy (kN/m^2)	Normal	54,45	2,7225
F_V (kN)	Normal	252,0	7,48
F_H (kN)	Gumbel	21,37	7,48
M_{vigas} (kNm)	Gumbel	9193	267,1
Modeling of loads	Normal	1,00	0,10
Modeling of resistances	Normal	1,00	0,05

The Reliability analysis is made using the VAP program, applying the FORM method. A screen of the program is shown in Figure 4.13.

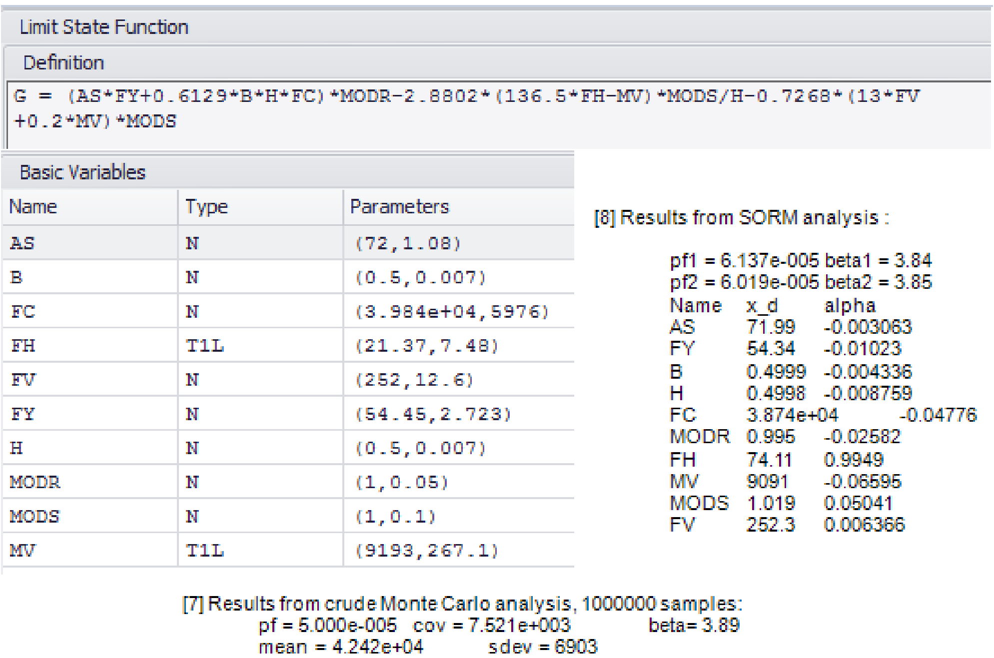


Figure 4.13 - Global probabilistic analysis of the structure

The main results of the analysis are:

- Reliability index: $\beta = 3{,}84$
- Probability of failure: $p_f = 6{,}137 \times 10^{-5}$

The value obtained for the reliability index β, for this structure, designed with the criteria of the Ultimate Limit State, is higher than the usual limit $\beta = 3,8$ for the 50-year reference period.

This means that, following the Global Safety approach, the design can still be optimized, leading to a more economical structure.

5
Seismic assessment of a bridge using a global safety approach

5.1
General consideration

According the ISO 13822:2010 (ISO, 2010), the structural assessment can be initiated under the following circumstances: an anticipated change in use or extension of design working life; a reliability check (e.g. for earthquakes, increased traffic actions) as required by authorities, insurance companies, owners, etc.; structural deterioration due to time-dependent actions (e.g. corrosion, fatigue); structural damage by accidental actions.

The ongoing revision of the Brazilian Standard NBR 7187 is going to establish new requirements regarding the seismic resistance of reinforced and prestressed concrete bridges. This situation will raise doubts doubts about meeting these new requirements by the numerous existing bridges, designed and built according to previous norms without seismic requirements. This is an important issue since the possible failure to meet these new requirements will imply the strengthening of these structures, with major economic consequences.

This is an excellent circumstance to show the utility of the advance assessment techniques, particularly the global approach. For this purpose, the seismic assessment of a bridge placed in the Brazilian Northeast is carried out in this chapter. The bridge is located in an area with significant seismic effects, requiring a seismic assessment based on the recommendations of the Brazilian Earthquake Standard, NBR 7187. It is intended to verify whether, with the application of Global Safety concepts, the bridge would meet normative requirements for seismic resistance. In the analysis by Global Safety only the probabilistic approach will be considered, since for the deterministic approach there is no physical sense in an "average load" of earthquakes.

5.2
Bridge description

The road bridge over the Madeira River is part of the Avenida Perimetral, in the city of Sobral, State of Ceará, Brasil.

The bridge was designed by Hugo Mota and Joaquim Mota in early 2018. The bridge owner was the Secretariat of Infrastructure of the State of Ceará (SEINFRA).

The superstructure is composed by a deck 10,80 m wide, supported on two continuous main beams, 40,00 m long. The mesostructure consists of three pairs of columns of variable height, with circular section, 0,80 m diameter. The concrete specified for the columns was fck = 25MPa. The steel used is CA-50.

Drawings of formwork and reinforcements of the bridge over the Madeira River are presented in Appendix B. A general view of this bridge is presented in Figure 5.2.

Figure 5.1 - Bridge over river Madeira in Sobral, Ceará. General view.

Figure 5.2 - Bridge over Madeira River in Sobral, Ceará - side view

Figure 5.3 - Bridge over Madeira River in Sobral, Ceará (Google Maps, 2019)

5.3
Brief commentary on seismic forces

Seismic forces have proven to be one of the most destructive forces in nature, frequently causing tragedies worldwide, causing the loss of many lives and significant economic losses. In Brazil, scientifically based studies on this subject started in the 1970s and since then a seismological network has been set up that is in continuous operation.

Traditionally, the effects of earthquakes were not considered in the design of reinforced concrete structures in the country. This is only required for structures of greater importance such as nuclear power plants, for instance. However, probabilistic analyses of the available data shown that the Brazilian territory is not free from such natural manifestations, even having regions with great seismic potential. There are few scientific studies in Brazil for the evaluation the damage caused to the structures and the only standard related to this issue in Brazil, deals with ordinary buildings, the NBR15421 (ABNT, 2006).

This standard was issued in 2006 and represented an evolution in the country's technical-scientific culture. Some technical terms related to Seismic Engineering are presented here. More details could be found in Souza Lima and Santos (2008)

Magnitude is a measure of the amount of energy released by an earthquake. Normally earthquakes of magnitude less than 5 cause little damage. The most common magnitude scale is the Richter Scale, limited to a magnitude of about 9.

Intensity is a qualitative measure of the damage caused by an earthquake, but because it is an qualitative measure, it does not provide a great quantitative contribution to Engineering. The intensity scale most used today is the Modified Mercalli Scale. This scale has 12 graduations (from I - Imperceptible to people, up to XII - Virtually total destruction).

The acceleration on the ground caused by earthquakes is its most important characteristic for Structural Engineering, being normally measured in the north-south, east-west and vertical directions. Acceleration is the basis for all standards that deal with earthquake. The standard normally divides the territories that they cover into regions called seismic zones.

5.4
Seismic zoning of Brazil

The Brazilian seismic zoning divides the national territory into five zones, as shown in Figure 5.4 and Table 5.1.

In Table 5.1, a_g means the maximum horizontal acceleration expected in a region on the surface of a Class B ground (“Rock”).

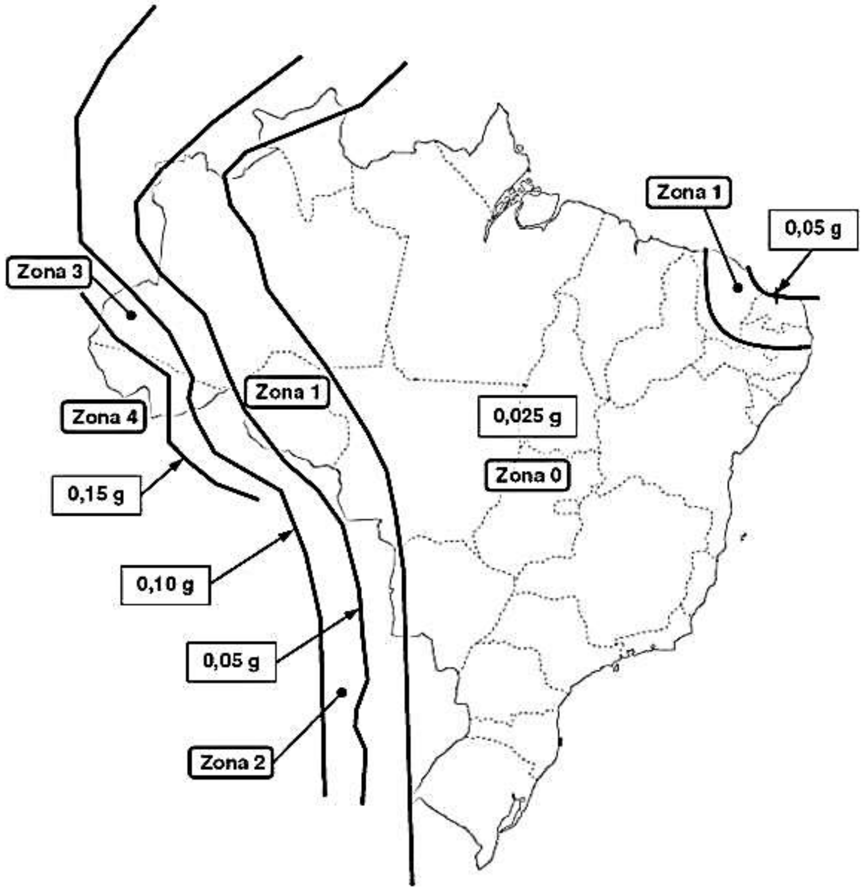


Figure 5.4 - Brazilian Seismic Zoning - NBR 15421

Table 5.1 - Brazilian seismic zones and accelerations - NBR 15421

Seismic zones	Values of a_g
Zone 0	$a_g = 0{,}025g$
Zone 1	$0{,}025g \le a_g \le 0{,}05g$
Zone 2	$0{,}05g \le a_g \le 0{,}10g$
Zone 3	$0{,}10g \le a_g \le 0{,}15g$
Zone 4	$a_g = 0{,}15g$

In the zoning defined by NBR 15421, it is observed that most of Brazil has low seismic activity, but in some regions of the Northeast and on the Northwest border, the accelerations are high and cannot be neglected.

In the Northeast, the higher acceleration curves are explained by the proximity of the region to the Central Atlantic failure, and in the Northwest, by the proximity to the edges of the tectonic plates that follow the Pacific coast and the Andes Mountains.

Normative recommendations that include seismic resistance requirements for bridges are being presently studied by the ABNT Commission, which is revising NBR 7187. These recommendations will form Annex B to this Standard, and is reproduced in Appendix A to this Thesis.

These specific requirements complement, for reinforced and prestressed concrete bridges, the general requirements regarding seismic resistance for structures established in NBR 15421.

5.5
Deterministic Bridge Analysis - Ultimate Limit States

The bridge was considered as usual and according to Table A.1 of Appendix A.1. Then, the Use Importance Factor I = 1,0 should be considered. The bridge is located in a Seismic Zone 1, with a_g = 0,05g.

5.5.1
Bridge seismic analysis

Here is presented the seismic analysis of the bridge, following the proposal for revision of NBR 7187.

Considering the characteristic acceleration a_g = 0,05g and Type E soil (soft soil), according to the SPT tests presented in the bridge formworks drawings (in Appendix A), the design response spectrum shown in Figure 5.5 is constructed. The non-linear behaviour, factor R, is conservatively taken as R=1,0.

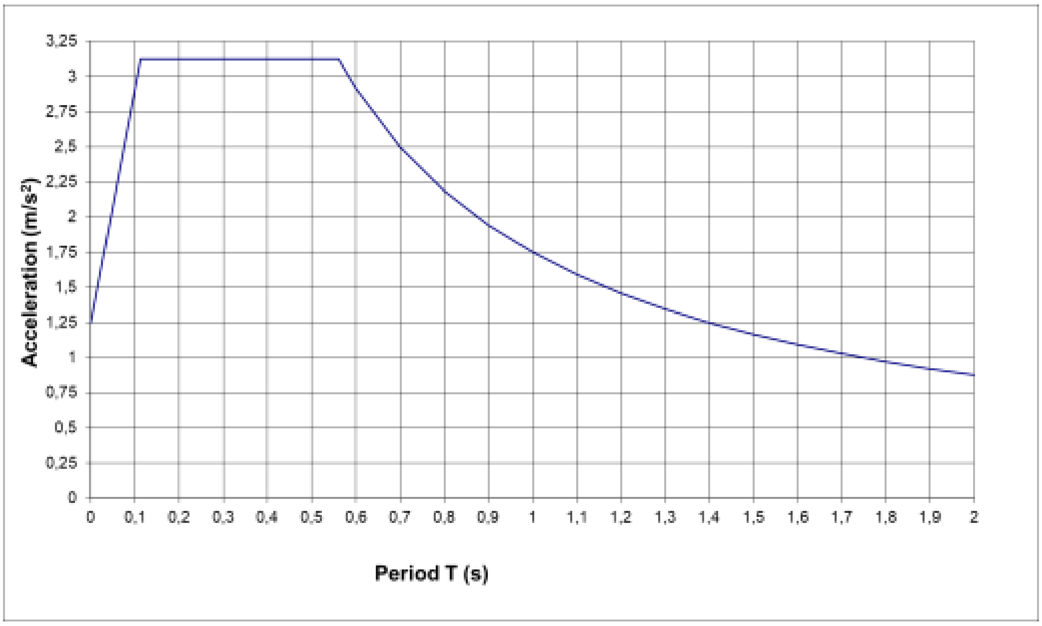


Figure 5.5 - Design response spectrum

5.5.2 Three-dimensional numerical model

A finite element model was developed for the spectral seismic analysis of the bridge, in the SOFISTIK program. Figure 5.6 illustrates a view of this model, described in more detail in Santos et al. (2020).

The model is composed by xx bar elements, …

The connection between the stringer and the slab is made using rigid connections, in order to represent the eccentric connection between these elements. THE soil-structure interaction is represented through the use of linear behavior springs, arranged along the length of the driven pile-supported foundations, in the global directions X and Y. For study purposes, the SOFiSTiK program, version 18. Figures 5.7 and 5.8 show the first vibration mode in the longitudinal direction of the bridge, with a frequency of 3,378 Hz and the second vibration mode with a frequency of 3,935 Hz.

Although the bridge, being located in Seismic Zone 1, does not require a dynamic analysis, this is done, inclusive for confirming the adequacy of this consideration defined by NBR 15421.

Appendix D to this work presents the complete results of the structural analysis of the bridge, for permanent loads, moving loads and seismic loads.

Table 5.2 presents a summary the forces for verifying the most critical sections of Column 3 (central), in the Ultimate Limit State:

- Combination of actions 1: $F_d = 1{,}5\ F_g + 1{,}5\ F_q$ (normal situation)
- Combination of actions 2: $F_d = 1{,}2F_g + 1{,}0\ F_e$ (seismic situation)

The inferior section of Column 3 is considered as the most critical point in this study. The combination of moments is done by the square root of the sum of squares.

Table 5.2 - Combination of action (kN, kNm)

N_d	M_{dy}	M_{dz}	M_d
Load combination 1			
-2633,9	17,18	14,70	22,61
Load combination 2			
-1625,1	633,9	920,4	1117,6

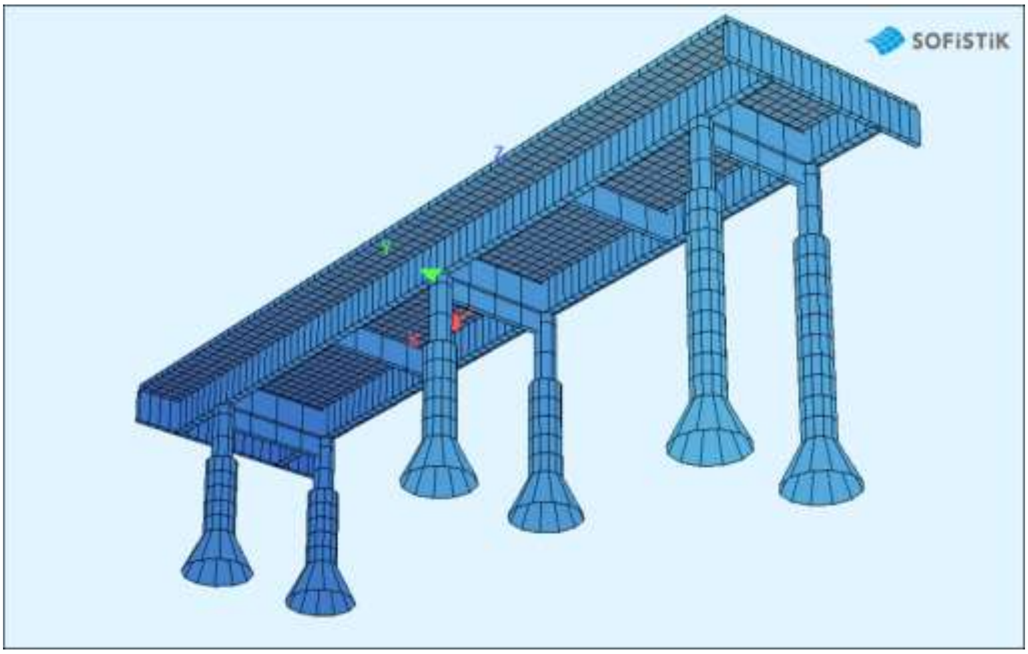


Figure 5.6 - Three-dimensional model of the bridge

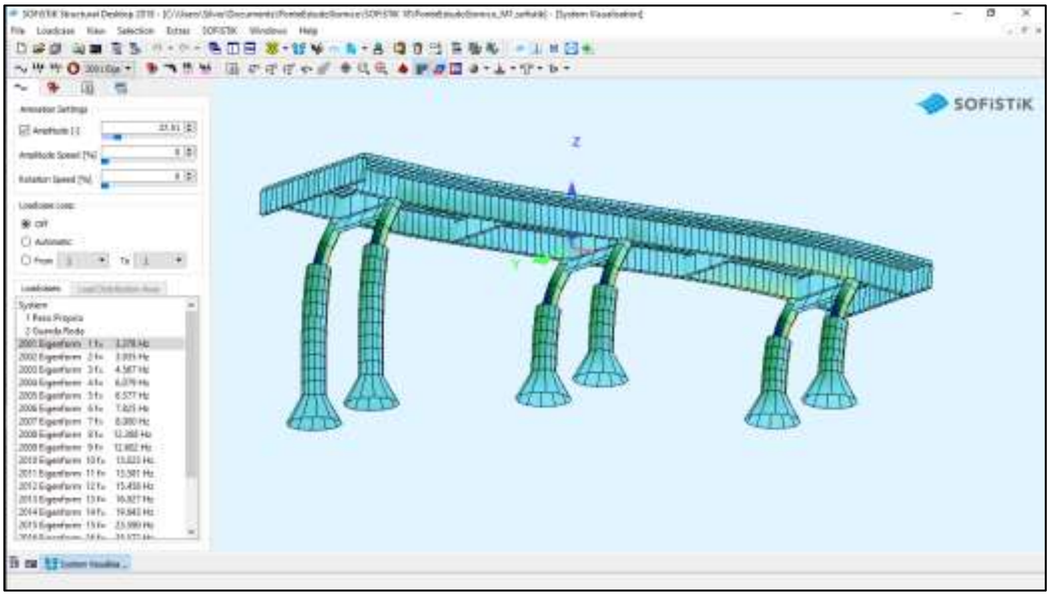


Figure 5.7 - First vibration mode - longitudinal direction

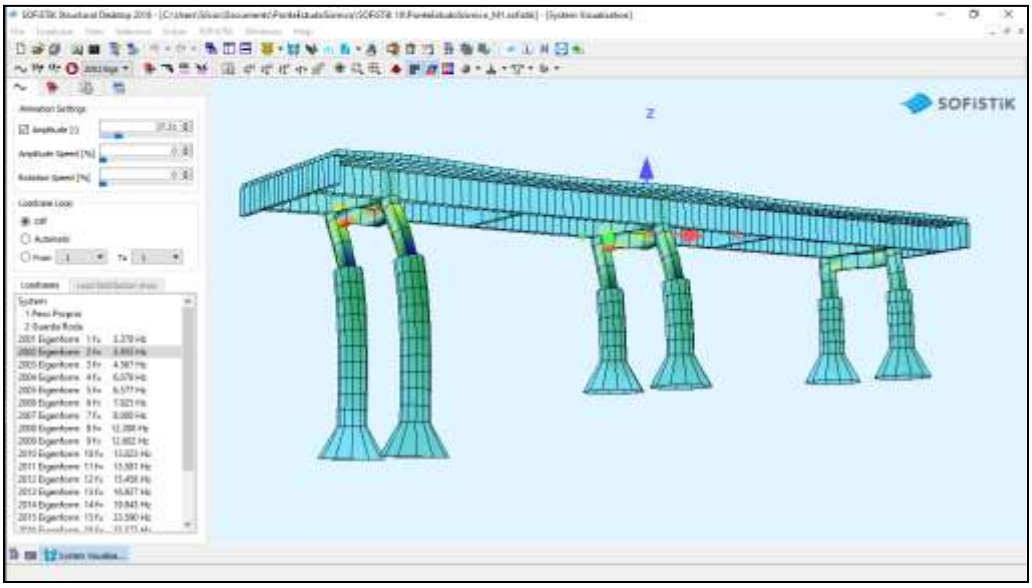
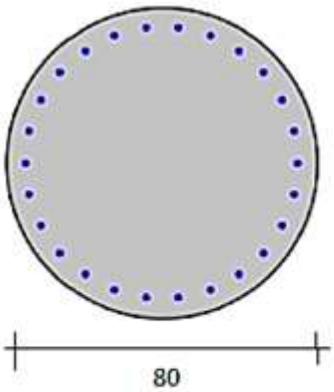


Figure 5.8 - Second vibration mode - transverse direction

5.5.3
Bending verification of the critical section of column 3

The verification of the column with the earthquake forces and with the existing reinforcement is done with the P-CALC program (TQS Store, 2020). It is shown that the section, in this verification of the Ultimate Limit State, practically resists exactly the applied forces.



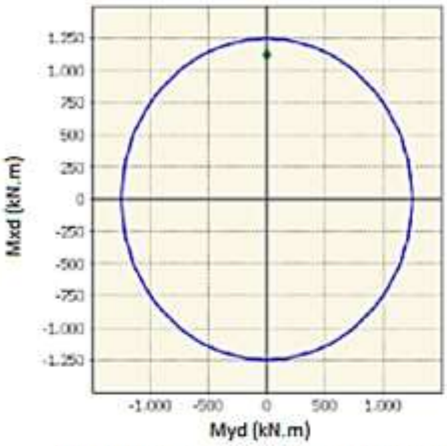


Figure: Interaction diagram (Comb.2)

Reinforcement: 26ϕ20 mm(As= 81,68cm2)
Materials: Concrete fck= 25 MPa
Steel fyk= 500 MPa

Combination	Na	Mxd	Myd
1	-2634	22	0
2	-1625	1118	0

Table: Loads combination (kN, kNm)

Figure 5.9 - Checking the critical section at the base of the columns

5.6
Probabilistic bridge analysis - Global Safety

5.6.1
Bayesian update of concrete strength

For the Bayesian update of the concrete strength, the methodology presented in item 2.4 is followed, which uses the formulation presented by Jacinto (2011).
a) Previous knowledge:

As previous knowledge, it is admitted that each mix has been properly dosed to provide the required f_{ck}, considering a variation coefficient of 10%.

For n_0 (confidence index associated with previous knowledge), it is adopted $n_0 =$ 10.

The following numerical data were considered, with respect to “prior knowledge”:

- arbitrated number of samples: n_0 = 10
- average resistance: μ_0 = 30,9 MPa;
- standard deviation: s_0 = 3,09 MPa

These values were chosen to correspond to the required characteristic resistance f_{ck} = 25 MPa of the t-Student distribution.

b) Actual tests:

From the results of the Technological Control of Concrete, partially presented in Appendix C, the values 25,7 MPa and 26,3 MPa are obtained, obtained in tests for 28 days in the concrete of Columns 3 and 4. For the analysis:

- number of samples: $n = 2$
- average strenght $\bar{x}$ = 26 MPa
- standard deviation: s = 0,42 MPa

c) “A posteriori” distribution (obtained with the Mathcad application):

- average resistance: $\bar{x}$ = 30,1 MPa
- standard deviation: s = 3,54 MPa
- characteristic resistance: f_{ck} = 23,77 MPa

d) Results shown in Figure 5.10

- Red curve, refers to the concrete of the actual tests, in a Normal probabilistic distribution;
- Blue curve, refers to prior knowledge, in a t-Student probabilistic distribution;
- Green curve, refers to the adjusted probabilistic distribution, that is, for the test samples + 10 samples of prior knowledge, in a t-Student probabilistic distribution.

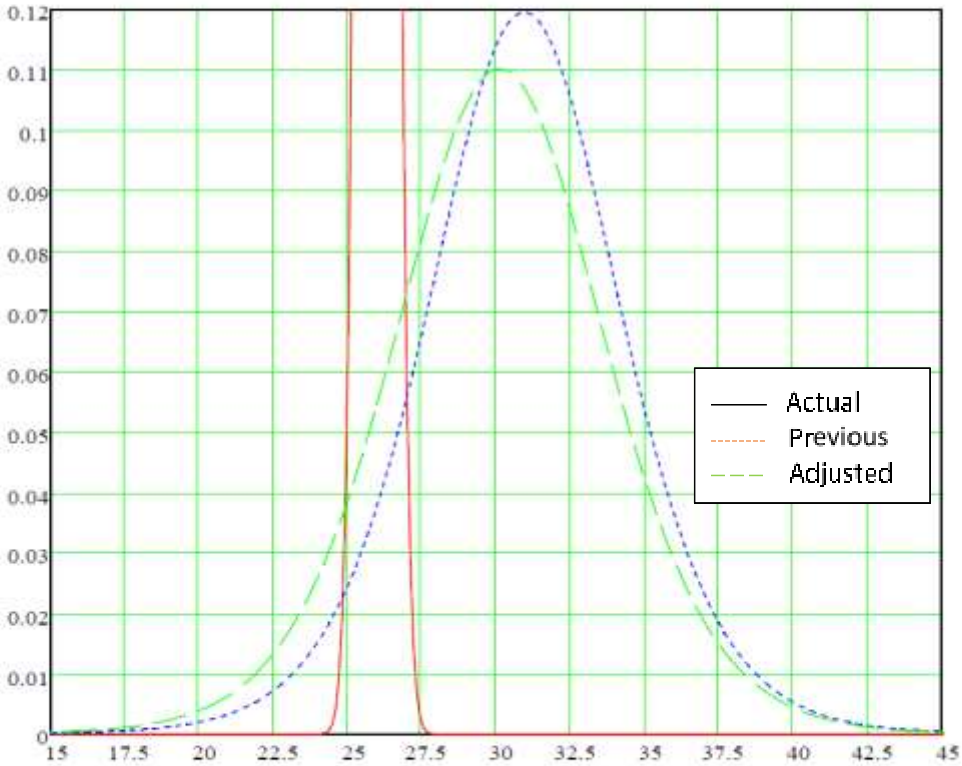


Figure 5.10 - Resistance distribution graph

The consideration of the Bayesian update, applied to the available tests, leads to a reduction of the fck from 25 MPa to 24 MPa (value to be adopted).

5.6.2 Definition of probabilistic variables

The probabilistic analysis is done in terms of resistant and acting moments:

$$F_{lim} = M_{res} - M_{atuante} \tag{107}$$

For the calculation of the acting moments, a relationship between maximum moments in the column and acceleration in the base is considered:

$$M_{atuante} = FATOR.acel \tag{108}$$

The proportionality factor is found considering that, in the analysis presented in item 5.4, the total seismic moment of 1107kN corresponds to an acceleration of 0,05 g:

$$FATOR = \frac{1107}{0,05} = 22140 \tag{109}$$

The acceleration function is defined based on the relationship between recurrence periods and horizontal accelerations for the Northeast Region that was presented by Santos *et al*. (2010). The curve that represents this relationship is reproduced in Figure 5.11 (“PGA”).

Also in this figure is represented the Gumbel function that is used in the probabilistic analysis for representing the Recurrence Function (“Gumbel”). Also shown are the recurrence periods of 475 years and 2475 years that were used as the basis for adjusting the curve.

Gumbel function:

$$p_f(a_h) = 1 - \exp[-exp(-\alpha(a_h - u))] \quad (110)$$

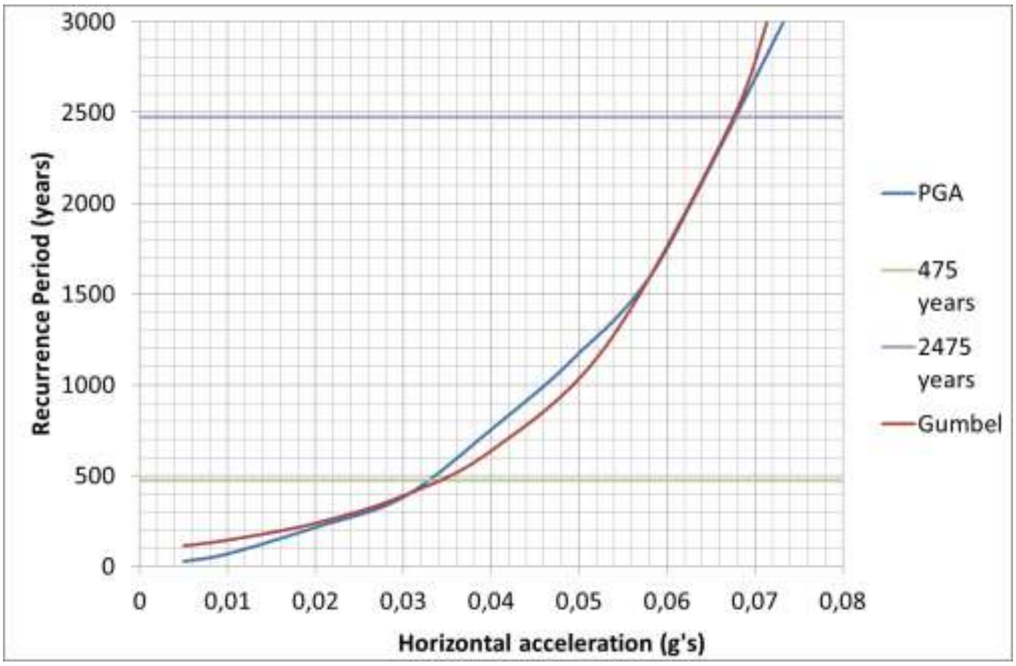


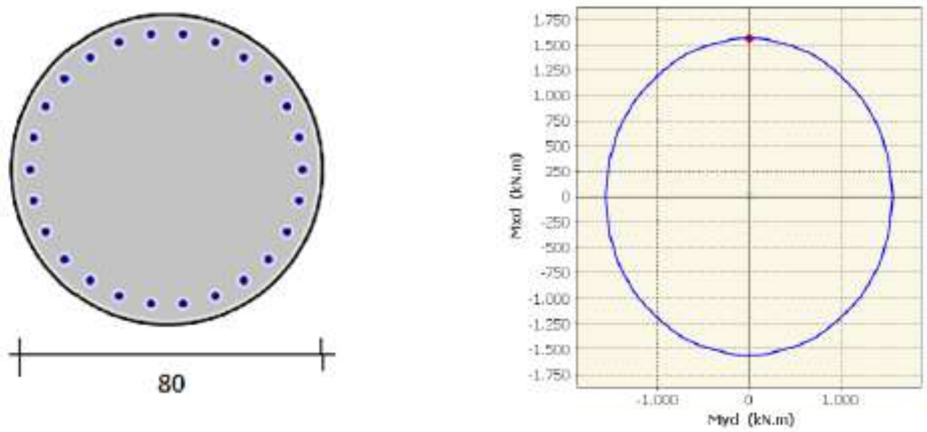
Figure 5.11 - Gumbel approximation for the recurrence function

5.6.3
Definition of probabilistic resistance variables

For defining the probabilistic resistance variable, in relation to the moment in the base, the PCALC program must initially be reprocessed with the average values of the variables. Following the sequence of item 4.3 and the updating of the strength of the concrete, it is obtained:

- Concrete: f_{cm} = 1,328 × 24000 = 31872 kPa
- Steel: f_{ym} = 1,089 × 500000 = 544500 kPa

The normal load is considered to have its characteristic value as N = 1390,6 kN. With these data at their average values, the average resulting moment is 1560 kNm, see Figure 5.12.



Reinforcement: 26ϕ20 mm (As = 81.68 cm²)

Materials: Concrete fck = 31 MPa
Steel fyk = 545 MPa

Figure: Interaction diagram (Comb.1)

Combination	Na	Mxd	Myd
1	-1391	1560	0

Figure 5.12 - Analysis of the column with mean values of the variables

For the probabilistic analysis, the following equation is finally considered:

$$F_{lim} = MRES.MODRES - 22140.ACEL.MODCAR.FACTOR \quad (111)$$

Based on Table 4.1, the variables considered in the bridge analysis are defined in Table 5.3

A coefficient of variation of 0,1 is adopted for the resistant moment and the variable FACTOR serves for inputting the factors λ.

Table 5.3 - Probabilistic variables for bridge reliability analysis

VARIABLE	DISTRIBUTION	AVERAGE	STANDARD DEVIATION
Resistant moment	Normal	1560	156
Resistance modelling	Normal	1	0,05
Acceleration	Gumbel	-0,079	0,026
Modelling of loads	Normal	1	1

The Reliability Analysis is performed with the VAP program, applying the FORM method. Figure 4.10 reproduces a program screen. This screen corresponds to λ =1,0.

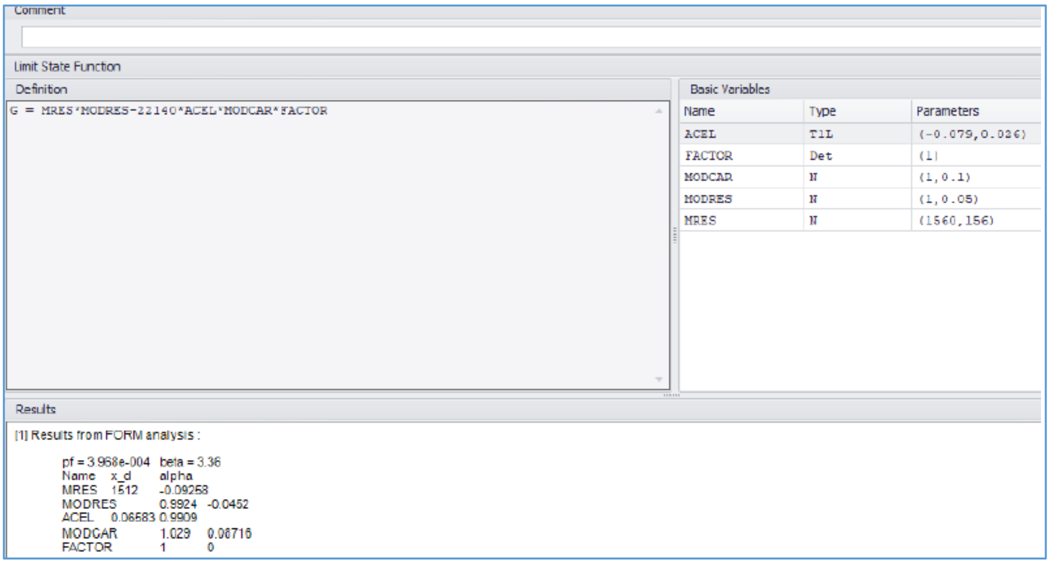


Figure 5.13 - Probabilistic column analysis

This processing, which corresponds to considering all variables with their average values, presented the results (using the FORM Method):

- Reliability index : $\beta = 3{,}36$
- Probability of failure: $p_f = 3{,}968 \times 10^{-4}$

This value is quite reasonable for a rupture of the ductile type and compatible with the Recurrence Periods defined for the earthquake in NBR 15421.

Figure 5.14 shows the β values obtained with different λ values, where this variable represents “Global safety factor” for the seismic load.

Acceptable values of β (above 3) are obtained with the increase coefficient equal to 1, which is the one usually defined in the seismic standards.

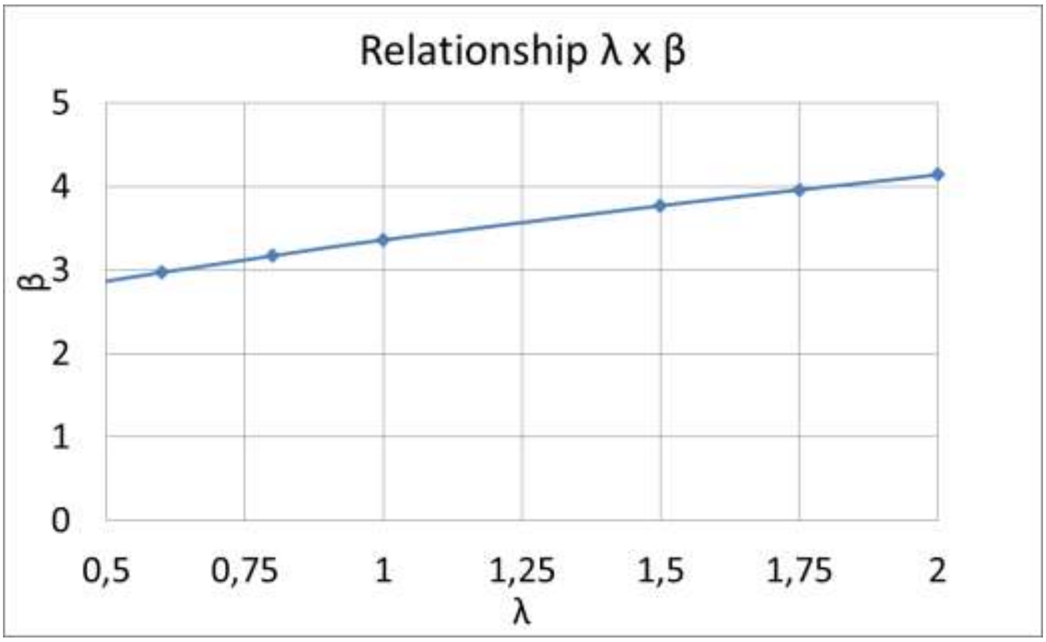


Figure 5.14 - Values $\lambda \times \beta$

6
Conclusions and final remarks

6.1
Conclusions

Initially the fundamentals of the methods for the safety evaluation of new and existent structures were presented in this Thesis, showing the evolution from the deterministic methods of Allowable Stresses and Ultimate Limit States to the modern methods, based on the probabilistic evaluation of the structural safety. It was shown that the Ultimate Limit States methods presently used are based on an arbitrary definition of safety factors.

A novel methodology is proposed herein, based on a Global Safety approach, where applied loads are progressively increased up to the final structural collapse, being the corresponding failure probabilities evaluated in each step, allowing for establishing a relationship between global safety factors and failure probabilities.

The methodology also includes a practical procedure for updating the evaluation of the resistances, applying a Bayesian approach. The application of this approach is fully illustrated using data obtained in actual structures of two different types: buildings and bridges. After the analysis of several dozens of actual concrete tests, a typical behavior pattern was found, allowing for proposing a very simple procedure for the updating concrete strengths.

The Thesis presented also a description of the available probabilistic methods for the evaluation of structural safety. A very complete and detailed description and definition the probabilistic characteristics of each of the variables involved in the safety evaluation of concrete structures is presented. These definitions are based on data found in the international literature, but also in the Brazilian construction experience. The definitions are also adjusted to the requirements of Brazilian Standards, regarding for instance the recurrence period defined for each of the variable loads.

This methodology is particularly useful for the evaluation of existing structures, for estimating the probabilities of failure, along the remaining working life of the structure.

The methodology is first applied to the design of a new structure. This analysis is for the central frame of a hypothetical building. An important conclusion of the analysis that the Reliability Analysis usually done in isolated sections of a structure can lead to conservative and misleading results, since the structures behave as a whole. The obtained relationship between actual safety factors and probabilities of failure can show situations where criteria of the design standards can lead to situations in which the safety of the structures, assessed from a probabilistic point of view, can be insufficient, or inversely, that the design of the structural elements can be further optimized.

The methodology is also applied for the analysis of an existing bridge, located in a region of medium seismicity, designed in a time when there was not any requirement for seismic resistance in Brazil. Bayesian updating of concrete strength was applied and also the Global Resistance approach. Two interesting conclusions were found. First, is that, at least for the analyzed bridge, the design considering only permanent and variable loads, can also covers seismic loads in this region. Second, is that, for an existent structure, the decision for accepting the actual design or for the rehabilitation, if necessary, of the structure can be supported by the evaluated relationship between actual safety factors and probabilities of failure.

A normative definition for the minimum values for the parameter λ to be accepted in the design is necessary. The global safety factors should have higher values in situations where a rupture of fragile type can occur and lower in cases of ductile rupture.

6.2
Suggestions for future work

For the effective application of the concepts exposed here, more studies are needed.

More data is needed to consolidate the concepts of the Bayesian updating of concrete strengths exposed here, including data from the different regions of Brazil.

In the aspect of Global Safety, more analyses need to be done, including with real structures, so that an adequate normative criterion can be proposed.

Another interesting point for the continuation of this study is the performance of economic and financial analyzis for existing structures, in order to help in the future decisions on how to proceed in the residual useful life of that structure, through cost comparison based also on global safety analysis.

7
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Appendix A
Proposed revision of NBR 7187 (Seismic Actions) (in Portuguese)

A.1 Escopo

Este item fixa os requisitos mínimos exigíveis na verificação da segurança de pontes de concreto armado e protendido relativamente às ações de sismos. Estes requisitos específicos complementam, para pontes de concreto armado e protendido, os requisitos gerais relativos à resistência sísmica para estruturas estabelecidos na NBR 15421,

Em princípio, os requisitos apresentados são aplicáveis a pontes de concreto armado e protendido em que a resistência às ações horizontais é conferida primariamente por flexão nos pilares e/ou pelos encontros, ou seja, pontes em que pilares verticais suportam o tráfego aplicado no tabuleiro. Estes requisitos podem ser também aplicados, mas complementados por requisitos específicos, a outros tipos de pontes.

Requisitos gerais estabelecidos na NBR 15421, relativos a referências normativas, definições, simbologia, requisitos gerais de segurança e valores característicos das ações sísmicas, respectivamente em seus itens 2 a 6, aqui se aplicam sem alteração. Adicionalmente, define-se:

d_g: deslocamento horizontal máximo do solo nas condições sísmicas de projeto

L_{lim}: distância limite entre juntas para a não consideração da variabilidade espacial da ação sísmica

L_g: distância a partir da qual os movimentos sísmicos do solo são considerados como não correlacionados

w_x: peso efetivo para a análise, valor do peso da ponte a ser considerado na análise sísmica

A.2 Categorização das pontes para a análise sísmica

Para cada ponte deve ser definida uma categoria sísmica, de acordo com o item 7.3 da NBR 15421. As categorias sísmicas são utilizadas para definir os tipos de análise que devem ser realizadas.

Para cada ponte deve também ser definida uma categoria de utilização e um correspondente fator de importância de utilização (I), conforme a tabela A.1. As estruturas necessárias ao acesso às pontes de categoria II ou III, também devem ser categorizadas como tal.

Tabela A.1 – Definição das categorias de utilização e dos fatores de importância de utilização (*I*)

Categoria de utilização	Natureza da utilização	Fator *I*
I	Pontes usuais, todas aquelas não classificadas como de categoria II ou III	1,0
II	Pontes essenciais, aquelas que devem estar operacionais após a ocorrência do sismo de projeto, para os veículos necessários às atividades ligadas a emergência, segurança e Defesa Nacional.	1,25
III	Pontes críticas, aquelas que devem estar operacionais para todo o tráfego após a ocorrência do sismo de projeto	1,50

A.3 Requisitos sísmicos para as estruturas de pontes

A.3.1 Requisitos de análise para pontes de categoria sísmica *A*

Para as pontes localizadas na zona sísmica 0, nenhum requisito de resistência sísmica é exigido.

As pontes localizadas na zona sísmica 1 devem apresentar sistemas estruturais resistentes às ações sísmicas horizontais em duas direções ortogonais, inclusive com um mecanismo de resistência a esforços de torção. Devem resistir a cargas horizontais aplicadas simultaneamente à toda a estrutura e independentemente em cada uma de duas direções ortogonais, com valor numérico igual a:

$F_x = 0{,}01\ w_x$

onde:

F_x - força sísmica de projeto em uma dada direção.

w_x - peso efetivo para a análise, que deve considerar as cargas permanentes atuantes, incluindo o peso do tabuleiro e metade do peso dos pilares, além de 20% da carga móvel em pontes rodoviárias e 30% da carga móvel em pontes ferroviárias.

A.3.2 Requisitos de análise para pontes de categoria sísmica *B* e *C*

As pontes de categoria sísmica *B* e *C* poderão ser analisadas pelo método espectral ou pelo método dos históricos de acelerações no tempo, conforme definido nos itens 7.3.2.4 e 7.3.2.5.

A.3.3 Coeficientes de modificação de resposta

A tabela A.2 define coeficientes de modificação de resposta *R* em função do tipo de elemento estrutural analisado, a serem utilizados para a determinação das forças de projeto nestes elementos estruturais.

Tabela A.2 – Coeficientes *R* de modificação de resposta

Sistemas sismo-resistentes	Pontes com detalhamento usual	Pontes com detalhamento especial
Estruturas em geral	1,5	2,5
Estruturas rigidamente ligadas ao solo, como encontros e pontes com tabuleiro rigidamente ligado aos encontros	1,0	1,0
Pontes em arco	1,2	2,0
Fundações	1,0	1,0

Os requisitos de detalhamento especial para pontes deverão ser definidos em documentos complementares a esta Norma.

A.3.4 Efeitos do sismo vertical

Os efeitos do sismo vertical podem ser dispensados na verificação dos pilares. Na verificação de apoios e ligações, estes efeitos devem ser considerados e determinados de acordo com a expressão abaixo:

$E_v = 0,5\ (a_{gs0}\,/g).G$

onde:

E_v e G são, respectivamente, os efeitos estruturais do sismo vertical e das cargas gravitacionais.

a_{gs0} é a aceleração espectral para o período de 0,0s, já considerado o efeito da amplificação sísmica no solo, conforme definido no item 6.3 da NBR 15421.

A.3.5 Critérios de modelagem da fundação e da estrutura

Para a modelagem da fundação, o item 8.7.1 da NBR 15421 deve ser seguido. Para a verificação da resistência das estruturas de fundação, o coeficiente *R* deve ser tomado igual a 1,0.

Deverá ser utilizado um modelo tridimensional para a ponte, que considere a distribuição espacial de massa e rigidez de todos os elementos significativos para a adequada distribuição de forças e deslocamentos na estrutura. Nas estruturas de concreto, o modelo deverá considerar a perda de rigidez devida à fissuração, conforme a NBR 6118.

A.3.6 Limitações para deslocamentos

Caso o sistema estrutural seja divididos em partes, separadas por juntas, estas devem apresentar entre si distâncias que garantam que não haja contato entre as partes, considerando a superposição dos deslocamentos devidos às cargas operacionais, aos efeitos térmicos e aos efeitos sísmicos.

Deve ser verificado se os deslocamentos avaliados podem implicar em danos ou risco de perda de estabilidade para os elementos estruturais.

A.3.7 Efeitos de segunda ordem

Os efeitos de segunda ordem devidos aos sismos em pilares, em uma combinação de cálculo, podem ser avaliados de forma aproximada, considerando um momento adicional igual ao produto da força axial de cálculo pelo deslocamento relativo das respectivas extremidades.

A.4 Análise sísmica pelo método espectral

A.4.1 Número de modos a ser considerado

O número de modos a ser considerado na análise espectral deve ser suficiente para capturar ao menos 90% da massa total em cada uma das direções ortogonais consideradas na análise.

A.4.2 Respostas modais para o projeto

O espectro de projeto conforme o item 6.3 da NBR 15421, deve ser considerado nas direções ortogonais analisadas.

Todas as respostas modais obtidas em termos de forças, momentos e reações de apoio devem ser multiplicadas pelo fator *I*/*R*.

As respostas em termos de deslocamentos absolutos e relativos serão as obtidas diretamente da análise espectral.

A.4.3 Combinação das respostas modais e nas diferentes direções ortogonais

O item 10.3 da NBR 15421 deve ser seguido.

A.5 Análise sísmica com históricos de acelerações no tempo

A.5.1 Requisitos da análise

A análise com históricos de acelerações no tempo deve consistir da análise dinâmica de um modelo definido de acordo com os requisitos estabelecidos em A.3.5, submetido a históricos de acelerações no tempo aplicados à sua base, compatíveis com o espectro de projeto definido para a estrutura, de acordo com o item A.4.2. Pelo menos três conjuntos de acelerogramas devem ser considerados na análise.

A.5.2 Requisitos para os acelerogramas

As análises deverão considerar a aplicação simultânea de um conjunto de acelerogramas, independentes entre si, em duas direções ortogonais relevantes para a ponte. Os acelerogramas podem ser registros de eventos reais, compatíveis com as características sismológicas do local de estrutura, ou podem ser acelerogramas gerados artificialmente.

Os acelerogramas a serem aplicados devem ser afetados de um fator de escala, de forma que os espectros de resposta em uma direção considerada, para o amortecimento de 5%, tenham valores de aceleração não inferiores a 10% dos valores correspondentes no espectro de projeto ema uma faixa entre $0{,}2T$ e $1{,}5T$, sendo T o período fundamental da ponte nesta direção.

A.5.3 Definição dos efeitos finais da análise

Para cada acelerograma analisado, as respostas obtidas em termos de forças, momentos e reações de apoio devem ser multiplicadas pelo fator I/R.

Os efeitos finais obtidos na análise correspondem às envoltórias dos efeitos máximos obtidos com cada um dos conjuntos de acelerogramas considerados.

As respostas em termos de deslocamentos absolutos e relativos serão as obtidas diretamente da análise dinâmica.

A.6 Variabilidade espacial da ação sísmica

A.6.1 Consideração da variabilidade espacial da ação sísmica

Para a aplicação deste item, deve-se considerar a classificação de classes de terreno dada na Tabela 2 da NBR 15421.

A variabilidade espacial da ação sísmica deverá ser considerada se, em um trecho da ponte entre juntas:

- as propriedades do solo variarem em mais de uma classe de terreno;

- o comprimento entre juntas exceder o valor L_{lim} definido abaixo:.

$L_{lim} = L_g / 1{,}5$

onde:

L_g é a distância a partir da qual os movimentos sísmicos do solo são considerados como não correlacionados.

L_g é definido na tabela A.3 em função da classe de terreno.

Tabela A.3 – Distância L_g

Classe de terreno	A	B	C	D	E
L_g (m)	600	600	500	400	300

A.6.2 Efeitos sísmicos inerciais

Os efeitos sísmicos inerciais considerarão a envoltória dos espectros de projeto, definidos conforme o item 6.3 da NBR 15421 para as diferentes classes de terreno presentes no trecho considerado.

A.6.3 Consideração aproximada da variabilidade espacial da ação sísmica

A variabilidade espacial da ação sísmica pode ser considerada de forma aproximada, pela aplicação pseudo-estática de deslocamentos horizontais nos apoios dos pilares, separadamente nas duas direções de análise.

As configurações pseudo-estáticas de deslocamentos deverão ser definidas de forma a se obter os máximos esforços nos elementos estruturais da ponte.

Nestas configurações os deslocamentos relativos máximos entre dois apoios dos pilares estão limitados a;

$$\Delta d_i = \sqrt{2}\, d_g\, \beta\, \frac{L_i}{L_g}$$

onde:

d_g = 0,0119 . a_{gs0}

d_g é o deslocamento máximo do solo

a_{gs0} e d_g são expressos, respectivamente, em m/s^2 e m

β = 0,5 se os dois apoios estão na mesma classe de terreno

β =1,0 se os dois apoios estão em classes diferentes de terreno

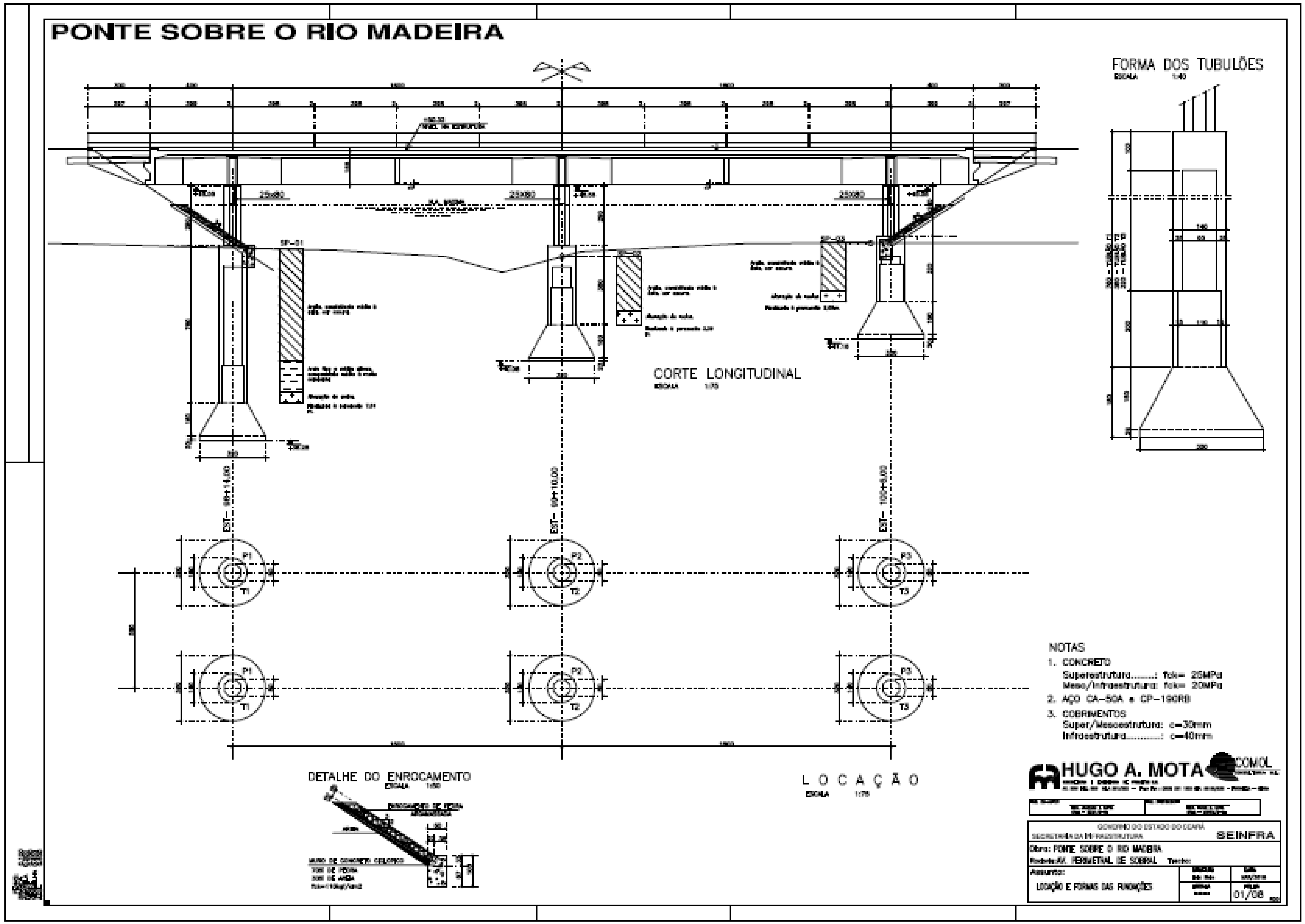
L_i é a distância entre os dois apoios, medida perpendicularmente à direção dos deslocamentos impostos

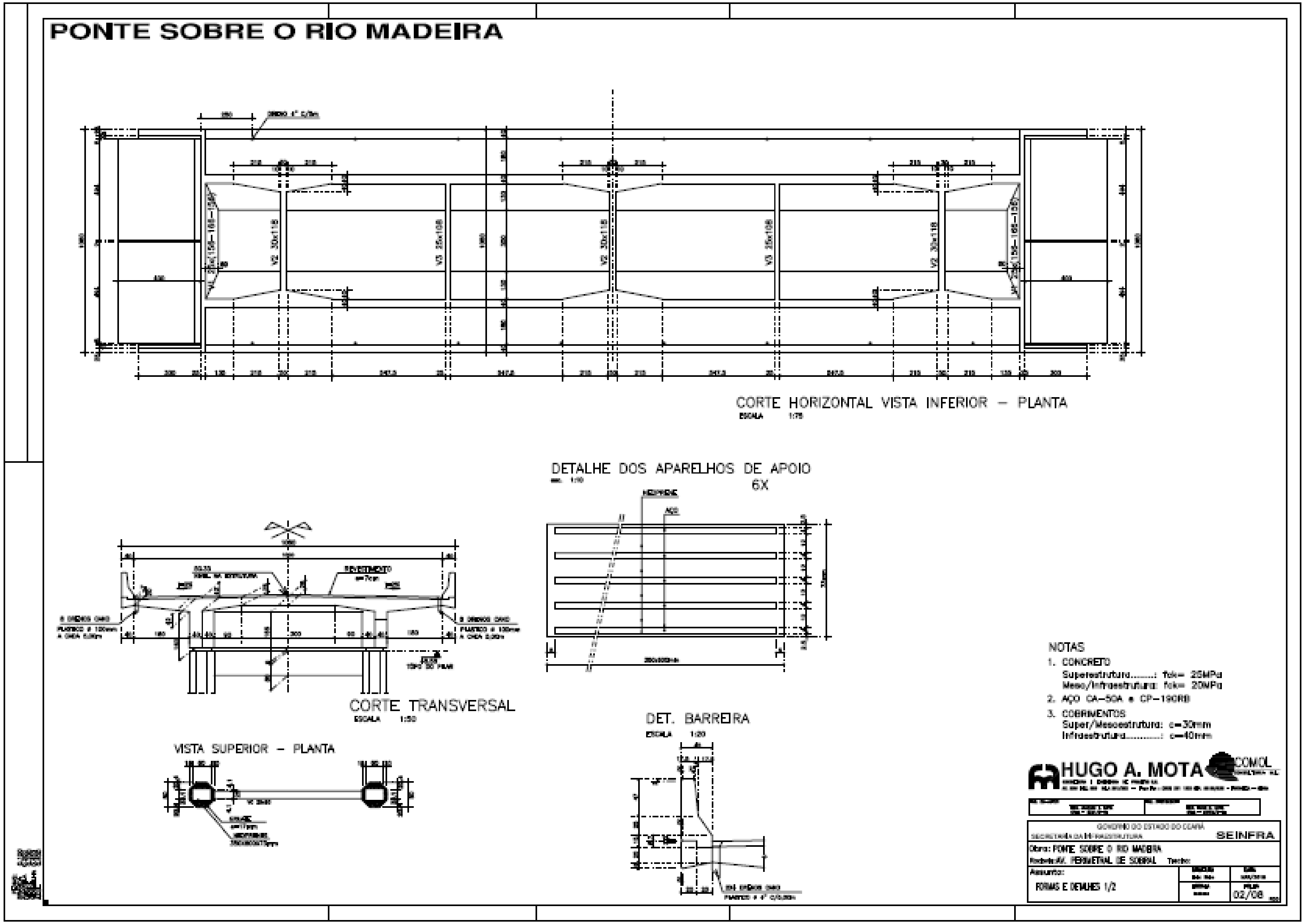
Os deslocamentos relativos entre dois pontos quaisquer estão limitados a:

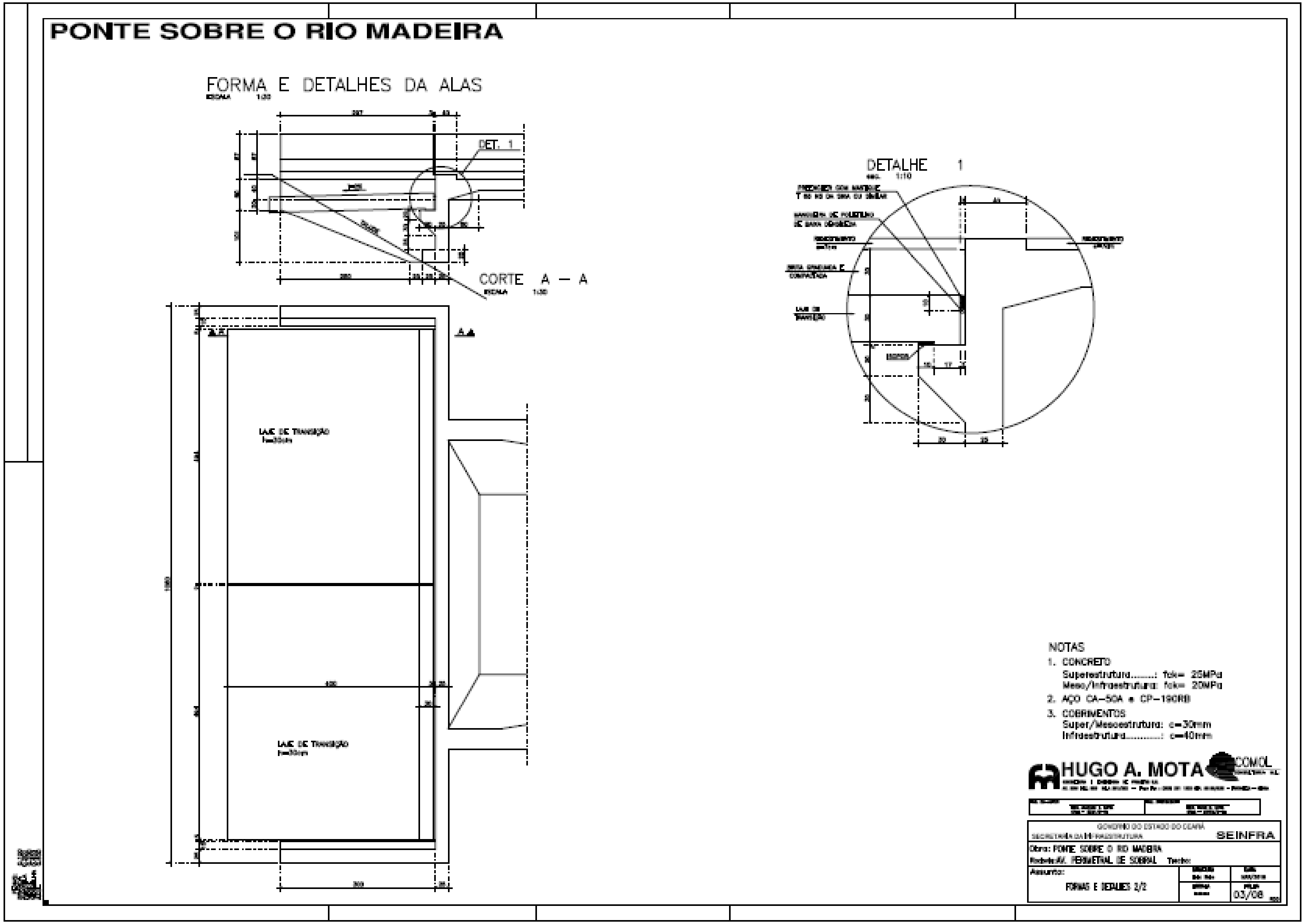
$$\Delta d_{i,max} = \sqrt{2}\, d_g$$

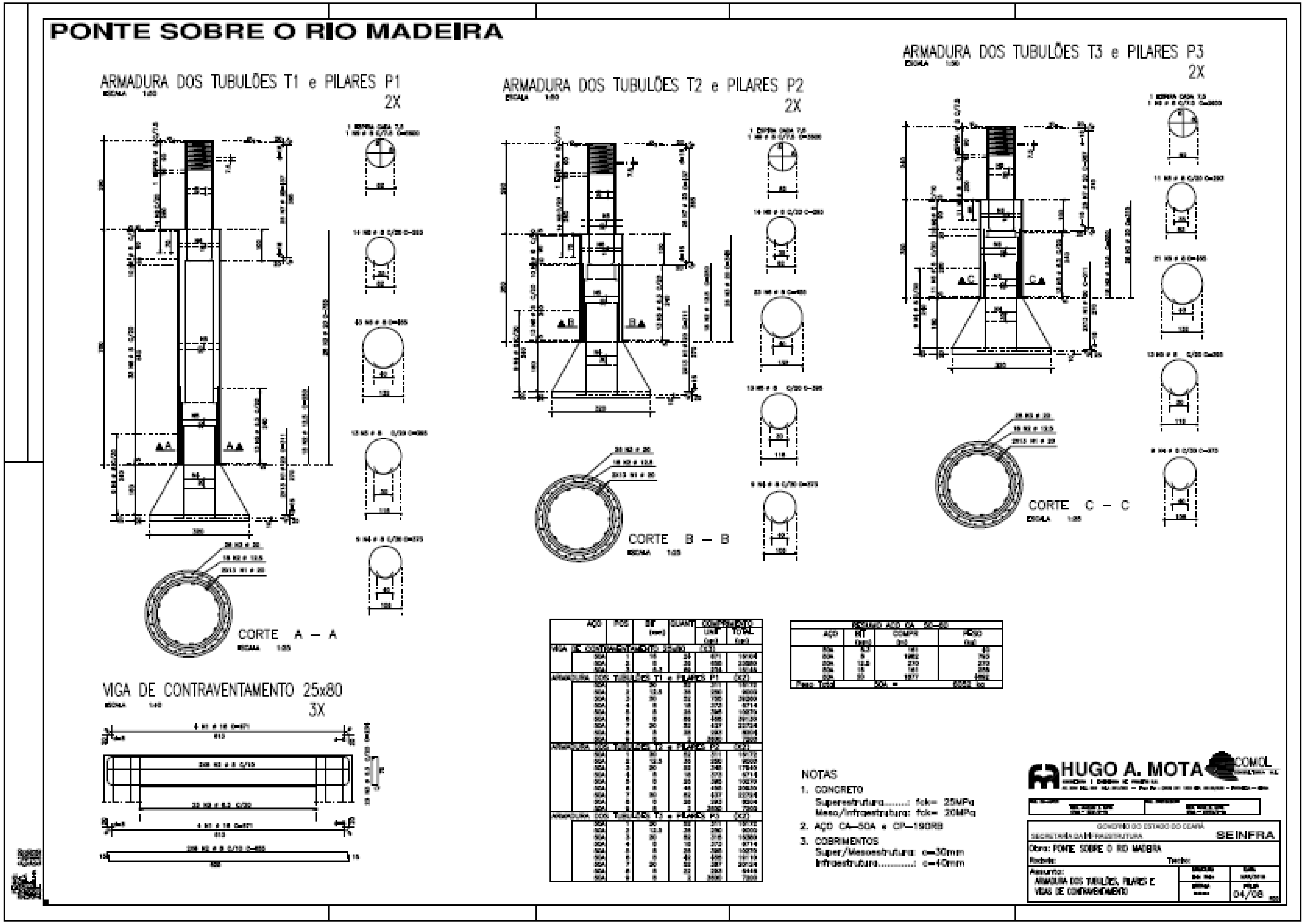
Os efeitos finais da variabilidade espacial da ação sísmica serão obtidos pela combinação, pela regra da raiz quadrada da soma dos quadrados, dos efeitos sísmicos inerciais com os efeitos da aplicação pseudo-estática de deslocamentos horizontais.

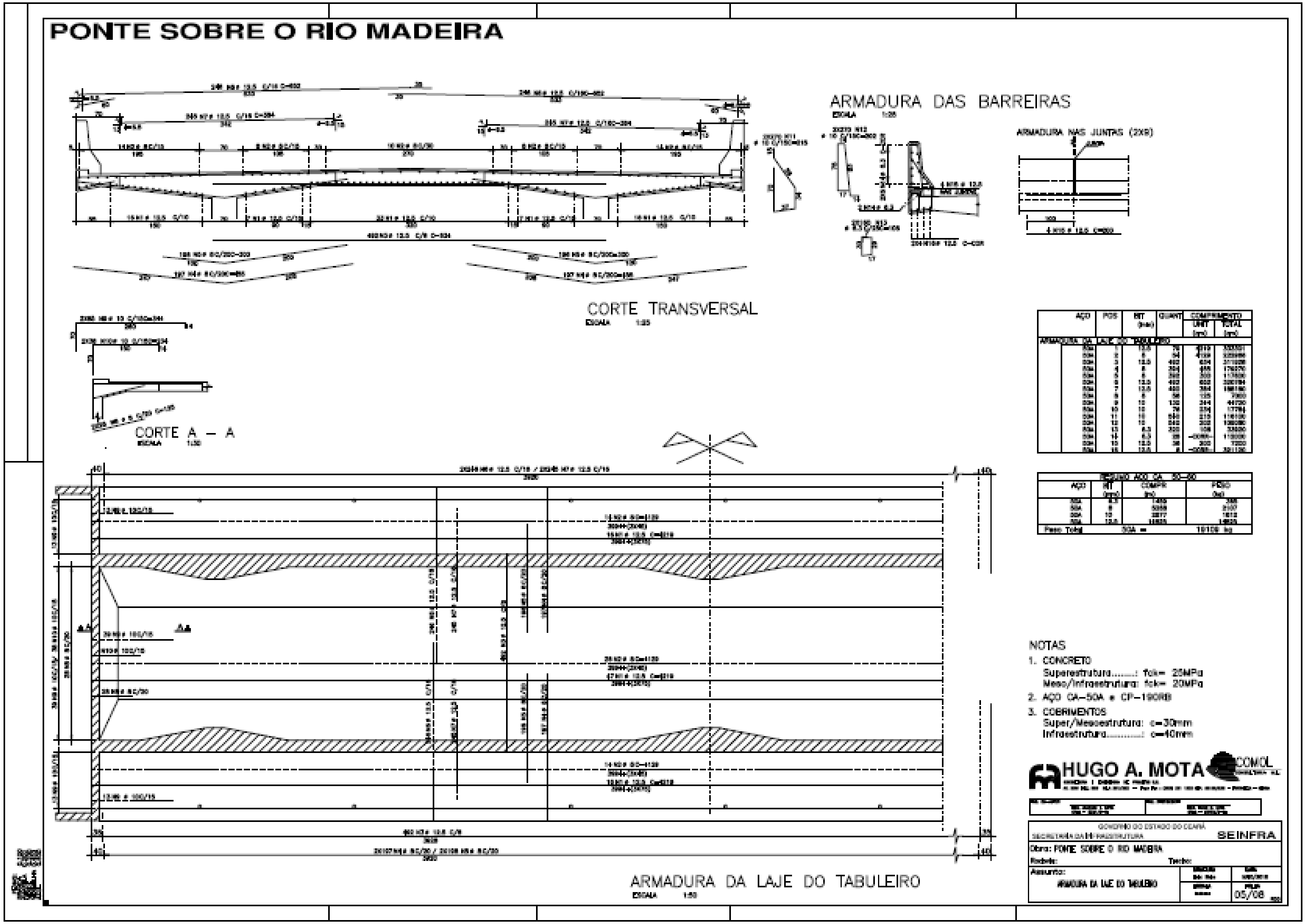
Appendix B
Formwork and reinforcement Drawings - Bridge over the Madeira River - Sobral

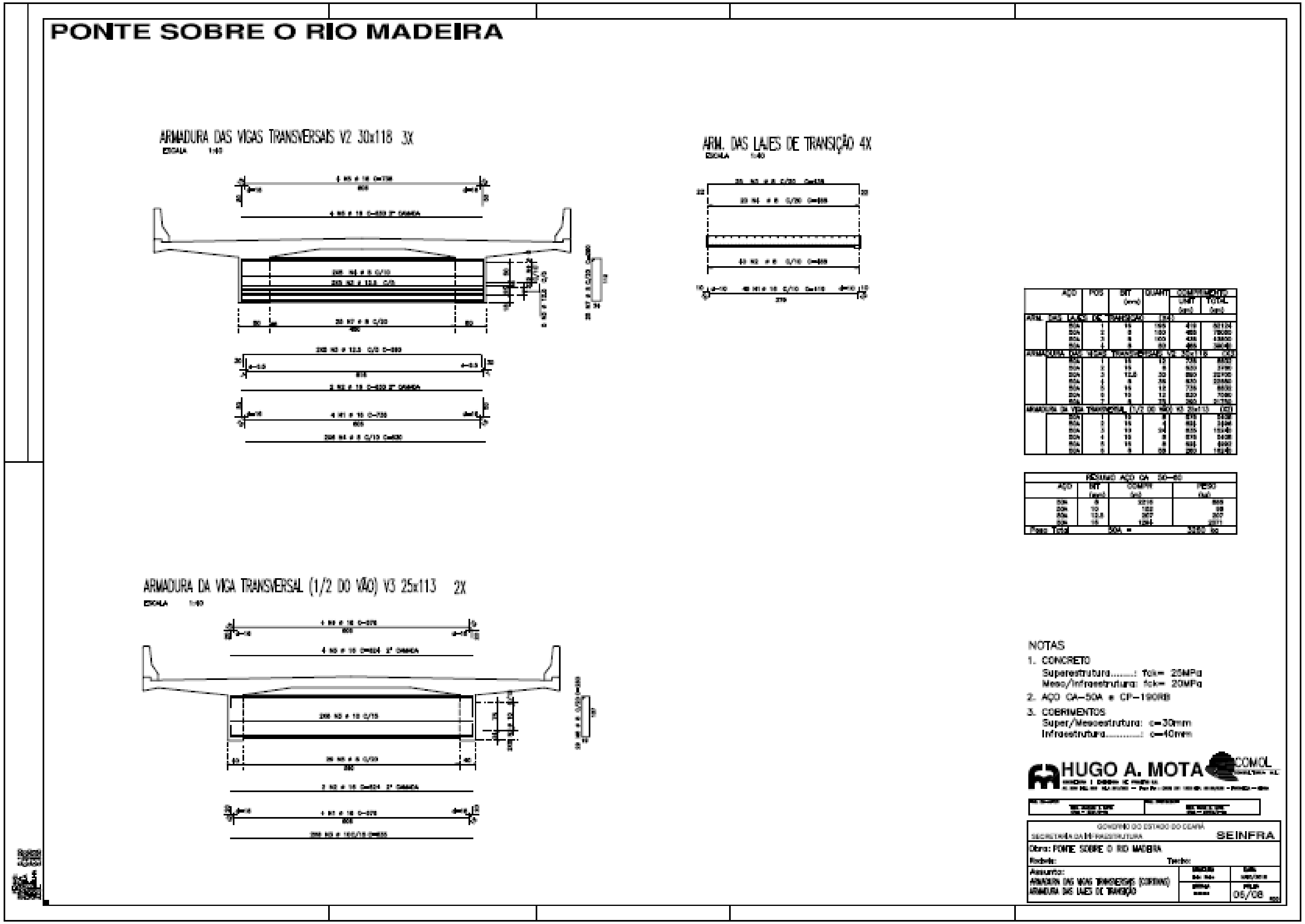

PONTE SOBRE O RIO MADEIRA
FORMA DOS TUBULÕES
CORTE LONGITUDINAL
P1
P2
P3
T1
T2
T3
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA-50A e CP-190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura...........: c=40mm
DETALHE DO ENROCAMENTO
L O C A Ç Ã O
HUGO A. MOTA
COMOL
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
LOCAÇÃO E FORMAS DAS FUNDAÇÕES
01/08

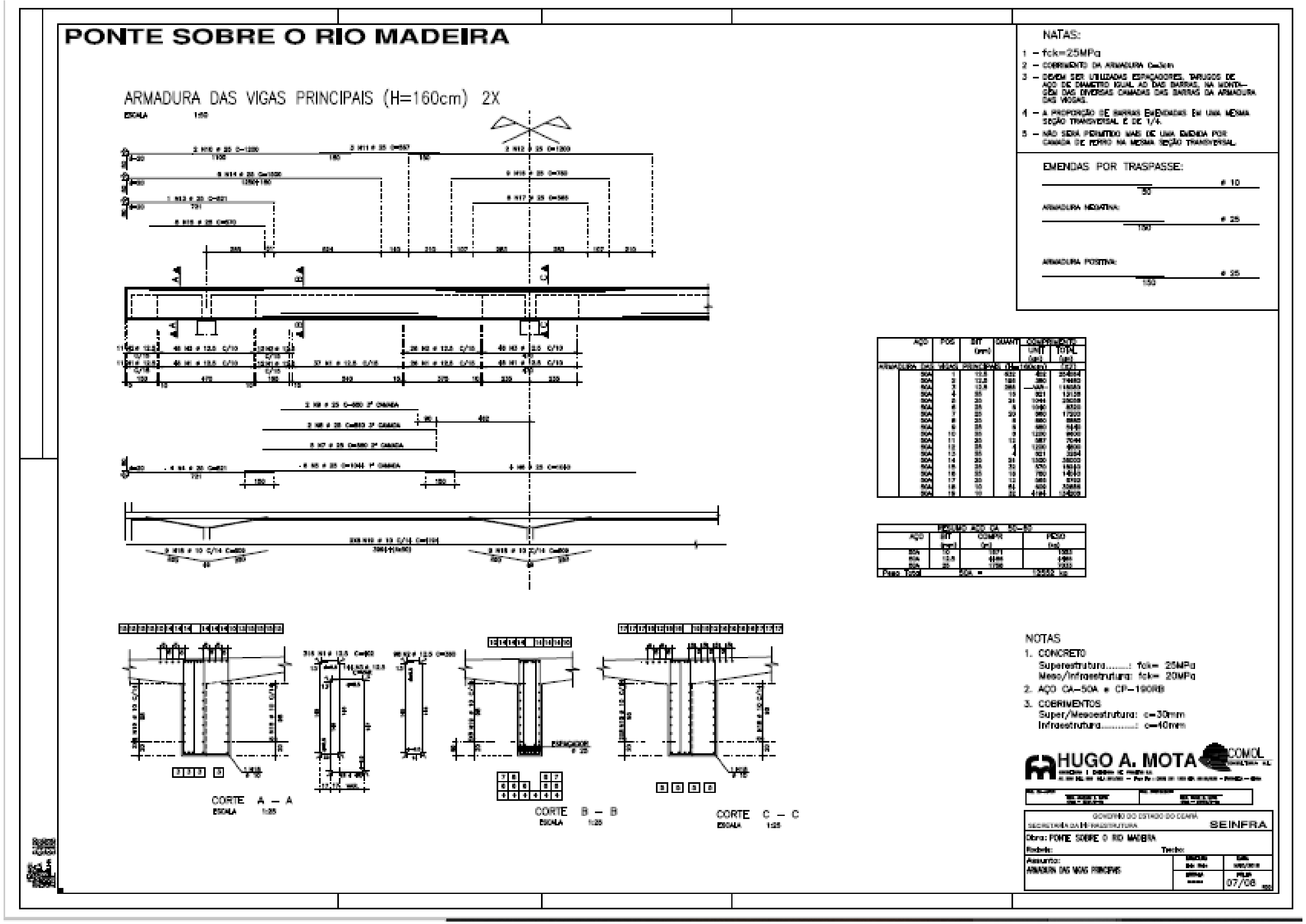

PONTE SOBRE O RIO MADEIRA
CORTE HORIZONTAL VISTA INFERIOR – PLANTA
ESCALA 1:75
DETALHE DOS APARELHOS DE APOIO
6X
CORTE TRANSVERSAL
ESCALA 1:50
DET. BARREIRA
ESCALA 1:20
VISTA SUPERIOR – PLANTA
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA-50A e CP-190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura..........: c=40mm
HUGO A. MOTA
COMOL
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
Rodovia: AV. PERIMETRAL DE SOBRAL
Assunto:
FORMAS E DETALHES 1/2
02/08

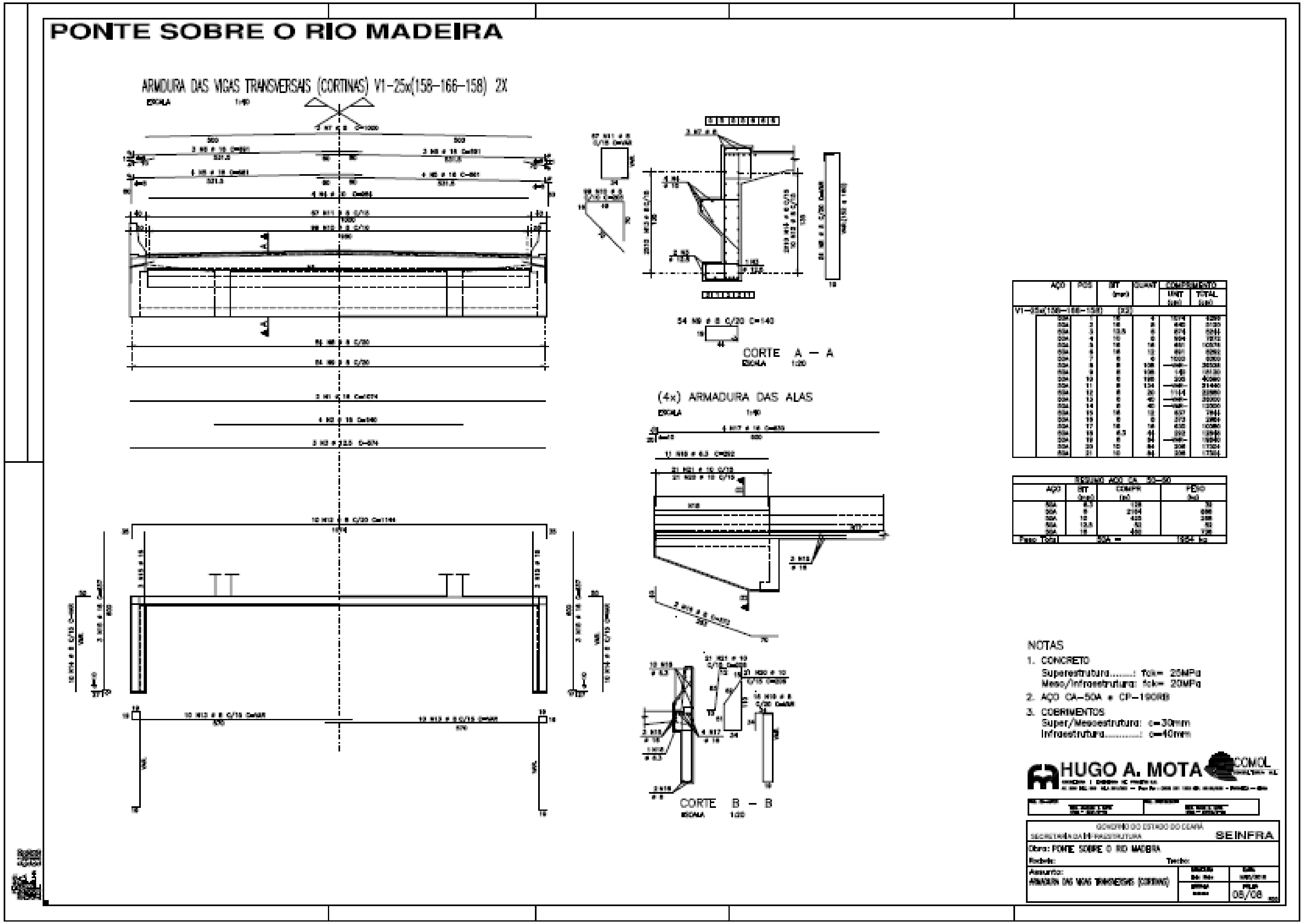

PONTE SOBRE O RIO MADEIRA
FORMA E DETALHES DA ALAS
DET. 1
CORTE A — A
DETALHE 1
LAJE DE TRANSIÇÃO
LAJE DE TRANSIÇÃO
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA-50A e CP-190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura...........: c=40mm
HUGO A. MOTA
COMOL
GOVERNO DO ESTADO DO CEARÁ
SECRETARIA DA INFRAESTRUTURA
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
Rodovia: AV. PERIMETRAL DE SOBRAL Trecho:
Assunto:
FORMAS E DETALHES 2/2
03/08


PONTE SOBRE O RIO MADEIRA
ARMADURA DOS TUBULÕES T1 e PILARES P1
2X
ARMADURA DOS TUBULÕES T2 e PILARES P2
2X
ARMADURA DOS TUBULÕES T3 e PILARES P3
2X
CORTE A — A
CORTE B — B
CORTE C — C
VIGA DE CONTRAVENTAMENTO 25x80
3X
NOTAS
1. CONCRETO
Superestrutura.......: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA—50A e CP—190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura...........: c=40mm
HUGO A. MOTA
COMOL
GOVERNO DO ESTADO DO CEARÁ
SECRETARIA DA INFRAESTRUTURA
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
ARMADURA DOS TUBULÕES, PILARES E VIGAS DE CONTRAVENTAMENTO
04/08


PONTE SOBRE O RIO MADEIRA
ARMADURA DAS BARREIRAS
ARMADURA NAS JUNTAS (2X9)
CORTE TRANSVERSAL
CORTE A – A
ARMADURA DA LAJE DO TABULEIRO
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA–50A e CP–190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura..........: c=40mm
HUGO A. MOTA
COMOL
GOVERNO DO ESTADO DO CEARÁ
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
Assunto:
ARMADURA DA LAJE DO TABULEIRO
05/08


PONTE SOBRE O RIO MADEIRA
ARMADURA DAS VIGAS TRANSVERSAIS V2 30x118 3X
ARM. DAS LAJES DE TRANSIÇÃO 4X
ARMADURA DA VIGA TRANSVERSAL (1/2 DO VÃO) V3 25x113 2X
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA-50A e CP-190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura...........: c=40mm
HUGO A. MOTA
COMOL
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA


PONTE SOBRE O RIO MADEIRA
ARMADURA DAS VIGAS PRINCIPAIS (H=160cm) 2X
NATAS:
1 – fck=25MPa
EMENDAS POR TRASPASSE:
ARMADURA NEGATIVA:
ARMADURA POSITIVA:
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA–50A e CP–190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura..........: c=40mm
CORTE A – A
CORTE B – B
CORTE C – C
HUGO A. MOTA
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
07/08


PONTE SOBRE O RIO MADEIRA
ARMADURA DAS VIGAS TRANSVERSAIS (CORTINAS) V1-25x(158-166-158) 2X
CORTE A - A
(4x) ARMADURA DAS ALAS
CORTE B - B
NOTAS
1. CONCRETO
Superestrutura........: fck= 25MPa
Meso/Infraestrutura: fck= 20MPa
2. AÇO CA-50A e CP-190RB
3. COBRIMENTOS
Super/Mesoestrutura: c=30mm
Infraestrutura..........: c=40mm
HUGO A. MOTA
COMOL
GOVERNO DO ESTADO DO CEARÁ
SECRETARIA DA INFRAESTRUTURA
SEINFRA
Obra: PONTE SOBRE O RIO MADEIRA
Assunto:
ARMADURA DAS VIGAS TRANSVERSAIS (CORTINAS)
08/08

Appendix C
Technological Control - Bridge over the Madeira River - Sobral-CE-BR

CONTROLE DE MOLDAGEM

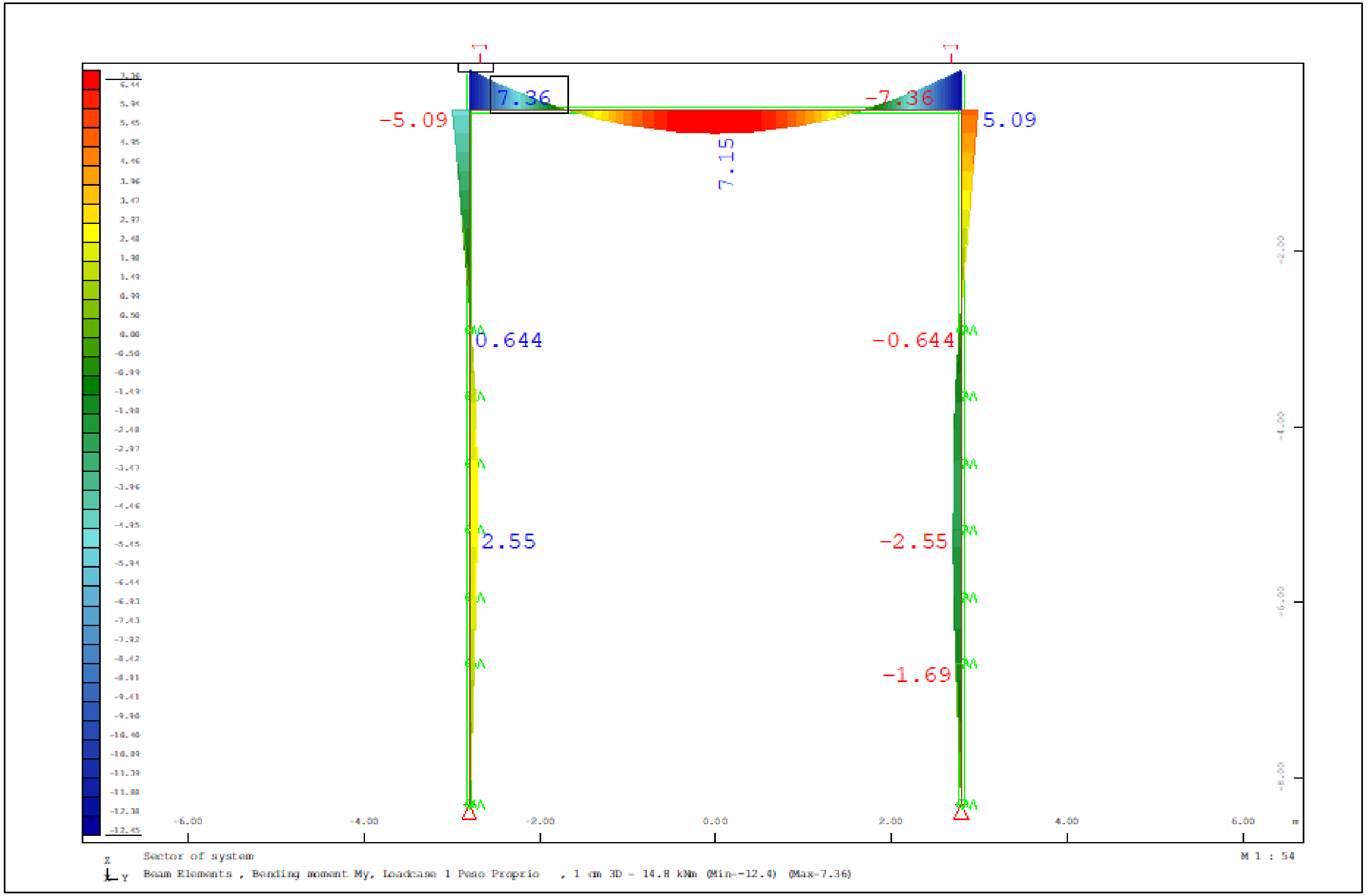
SETOR DE TECNOLOGIA DO CONCRETO - DIVISÃO DE CONTROLE DE QUALIDADE

SÉRIE	CP	DATA DE	DATA DE	APLICAÇÃO	SLUMP	TRAÇO	FATOR	CONSUMO	RESISTÊNCIA À COMPRESSÃO			
(Nº)	(Nº)	MOLDAGEM	RUPTURA	TIPO-DENOMINAÇÃO-LOCAL	(cm)	(Nº)	A/C	kg/m³	IDADE (dias)	CARGA (Kg.)	TENSÃO (FCK)	MPA
25	025	08/08/18	15/08/18		10,0				7	24.178	307,8	30
26	026	08/08/18	15/08/18		10,0				7	23.845	303,6	30
27	027	08/08/18	22/08/18	PILAR 5 E 6 ESTACA 98+10 PILAR RIO MADEIRA	10,0				14	25.656	326,7	32
28	028	08/08/18	22/08/18		10,0				14	25.982	330,8	32
29	029	08/08/18	05/09/18		10,0				28	27.928	355,6	34,9
30	030	08/08/18	05/09/18		10,0				28	25.814	328,7	32,2
31	031	17/08/18	24/08/18		10,0				7	17.492	222,7	22
32	032	17/08/18	24/08/18		10,0				7	16.829	214,3	21
33	033	17/08/18	31/08/18	PILAR 3 E 4 DO RIO MADEIRA	10,0				14	19.030	242,3	24
34	034	17/08/18	31/08/18		10,0				14	18.339	233,5	23
35	035	17/08/18	14/09/18		10,0				28	20.572	261,9	25,7
36	036	17/08/18	14/09/18		10,0				28	21.060	268,1	26,3
37	037	23/08/18	30/08/18		8,0				7	13.505	172,0	16,9
38	038	23/08/18	30/08/18		8,0				7	13.194	168,0	16,5
39	039	23/08/18	06/09/18	CONTINUAÇÃO DO PONTILHÃO 1 E 2 DO RIO MADEIRA	8,0				14	17.381	221,3	21,7
40	040	23/08/18	06/09/18		8,0				14	18.721	238,4	23,4
41	041	23/08/18	20/09/18		8,0				28	18.159	231,2	22,7
42	042	23/08/18	20/09/18		8,0				28	18.319	233,2	22,9
43	043	28/08/18	04/09/18		10,0				7	16.456	209,5	20,5
44	044	28/08/18	04/09/18		10,0				7	15.659	199,4	19,6
45	045	28/08/18	11/09/18	CONCRETAGEM DA BASE DO TUBO T1 DO RIO MADEIRA	10,0				14	19.080	242,9	23,8
46	046	28/08/18	11/09/18		10,0				14	19.593	249,5	24,5
47	047	28/08/18	25/09/18		10,0				28	24.856	316,5	31,0
48	048	28/08/18	25/09/18		10,0				28	26.211	333,7	32,7
49	049	30/08/18	06/09/18		8,0				7	26.211	333,7	32,7
50	050	30/08/18	06/09/18		8,0				7	20.500	261,0	25,6
51	051	30/08/18	13/09/18	CONCRETAGEM DA BASE DO TUBO T2 DO RIO MADEIRA	8,0				14	24.100	306,9	30,1
52	052	30/08/18	13/09/18		8,0				14	22.500	286,5	28,1
53	053	30/08/18	27/09/18		8,0				28	20.831	265,2	26,0
54	054	30/08/18	27/09/18		8,0				28	22.318	284,2	27,9

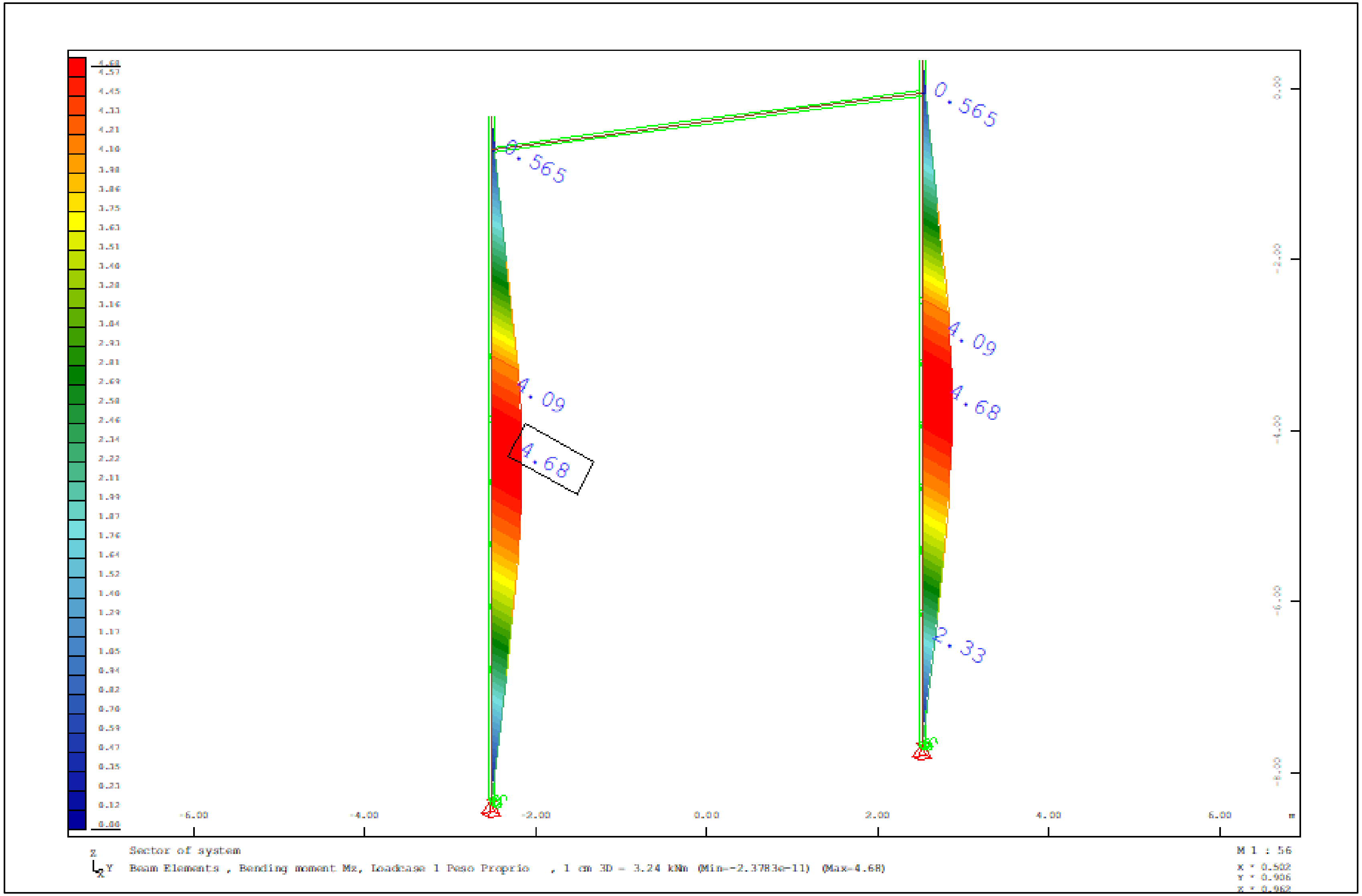
Appendix D

Structural Analysis – Results (Santos *et al*., 2020)

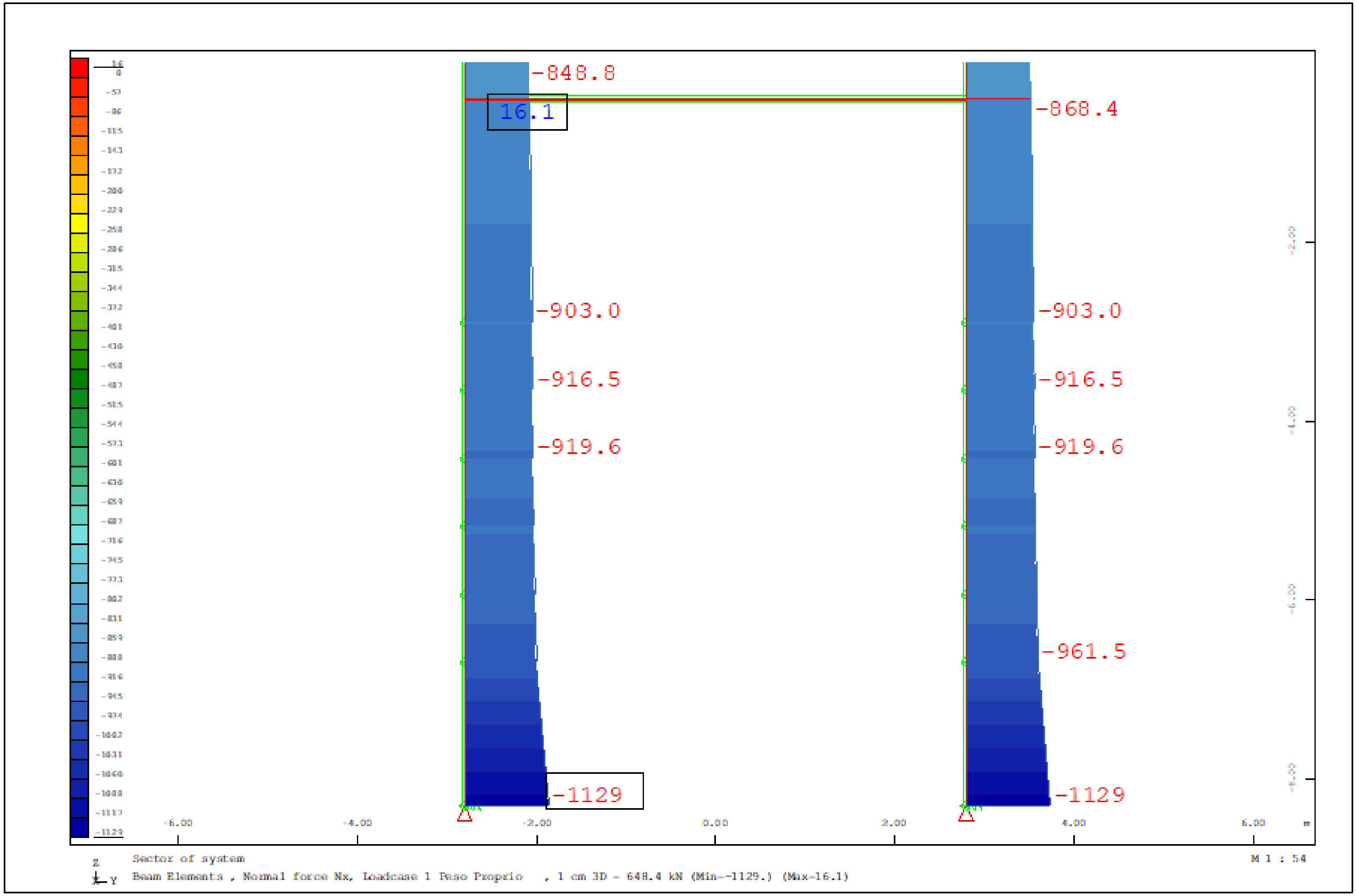
Dead Weight - Moment My



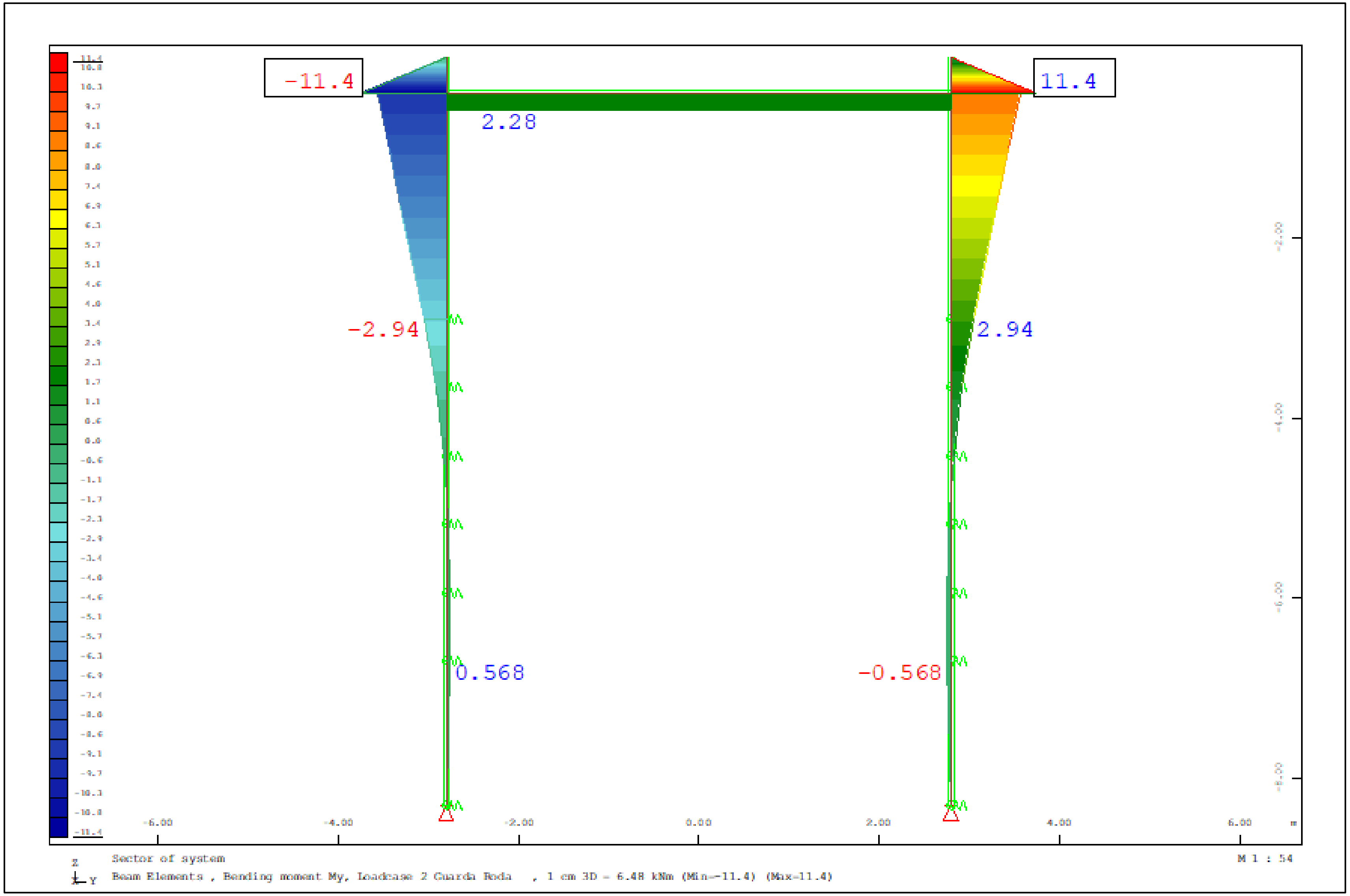
Dead Weight - Moment Mz



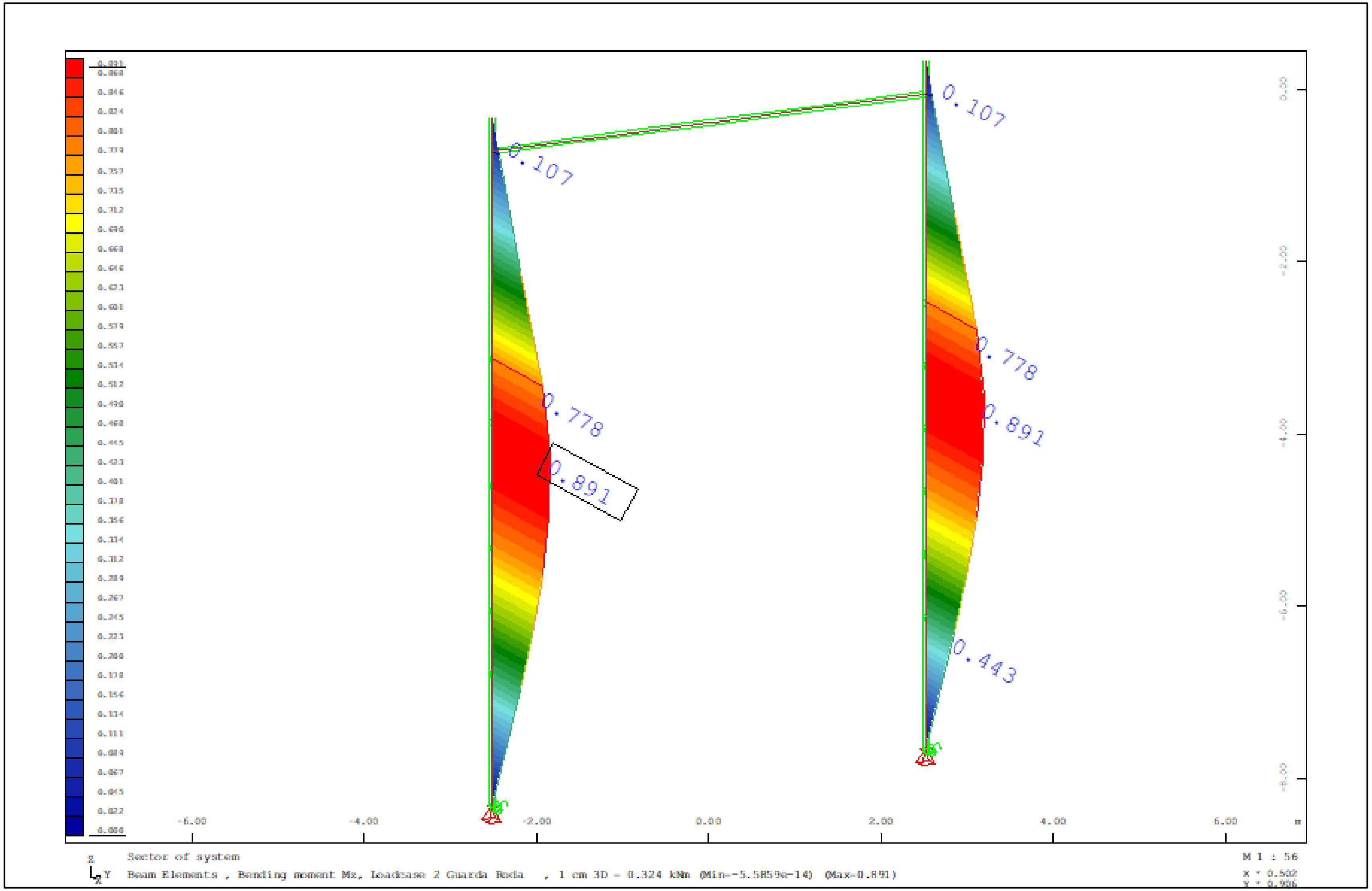
Dead Weight - Normal Force N



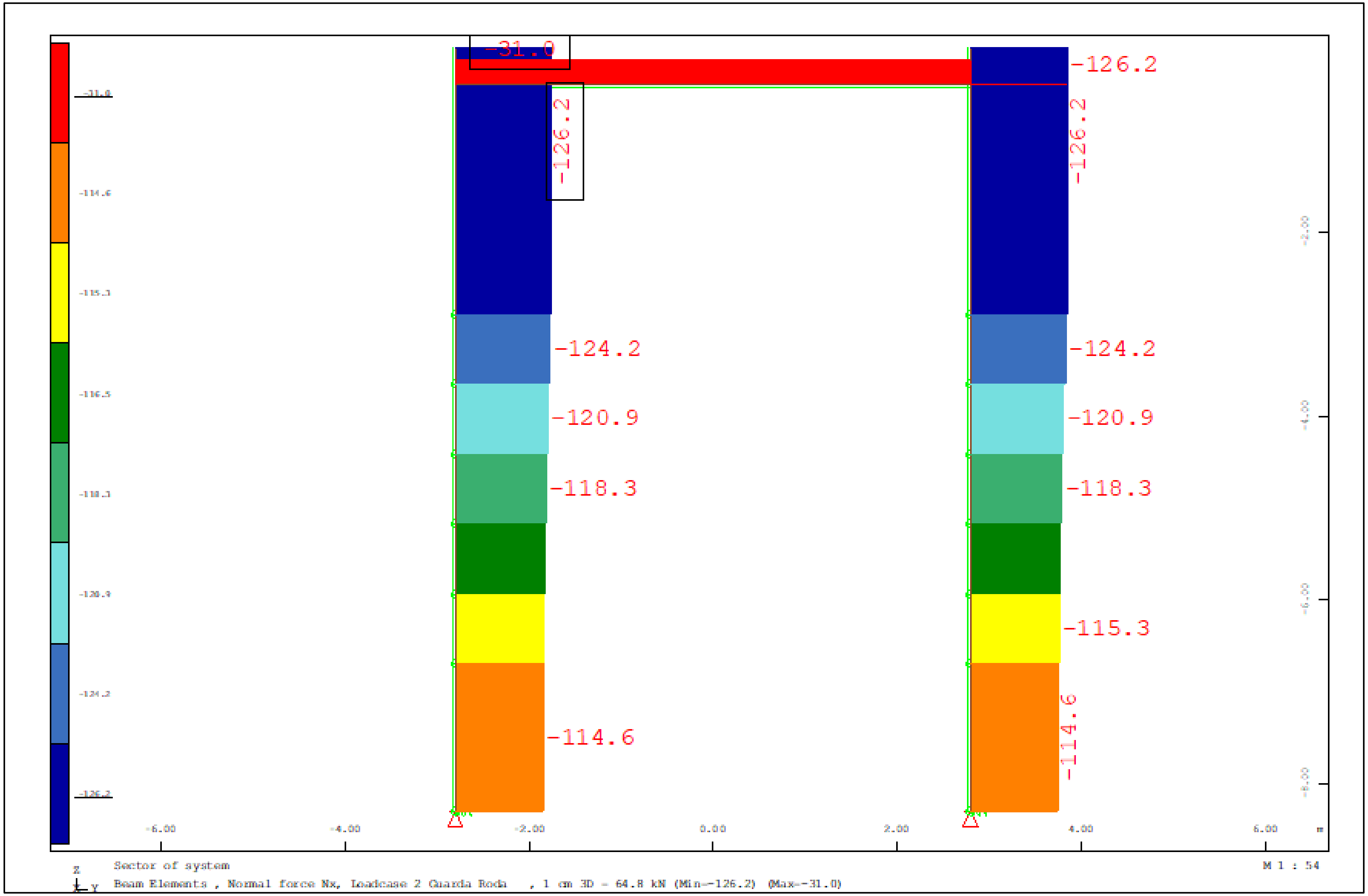
Wheel Guard - Moment My



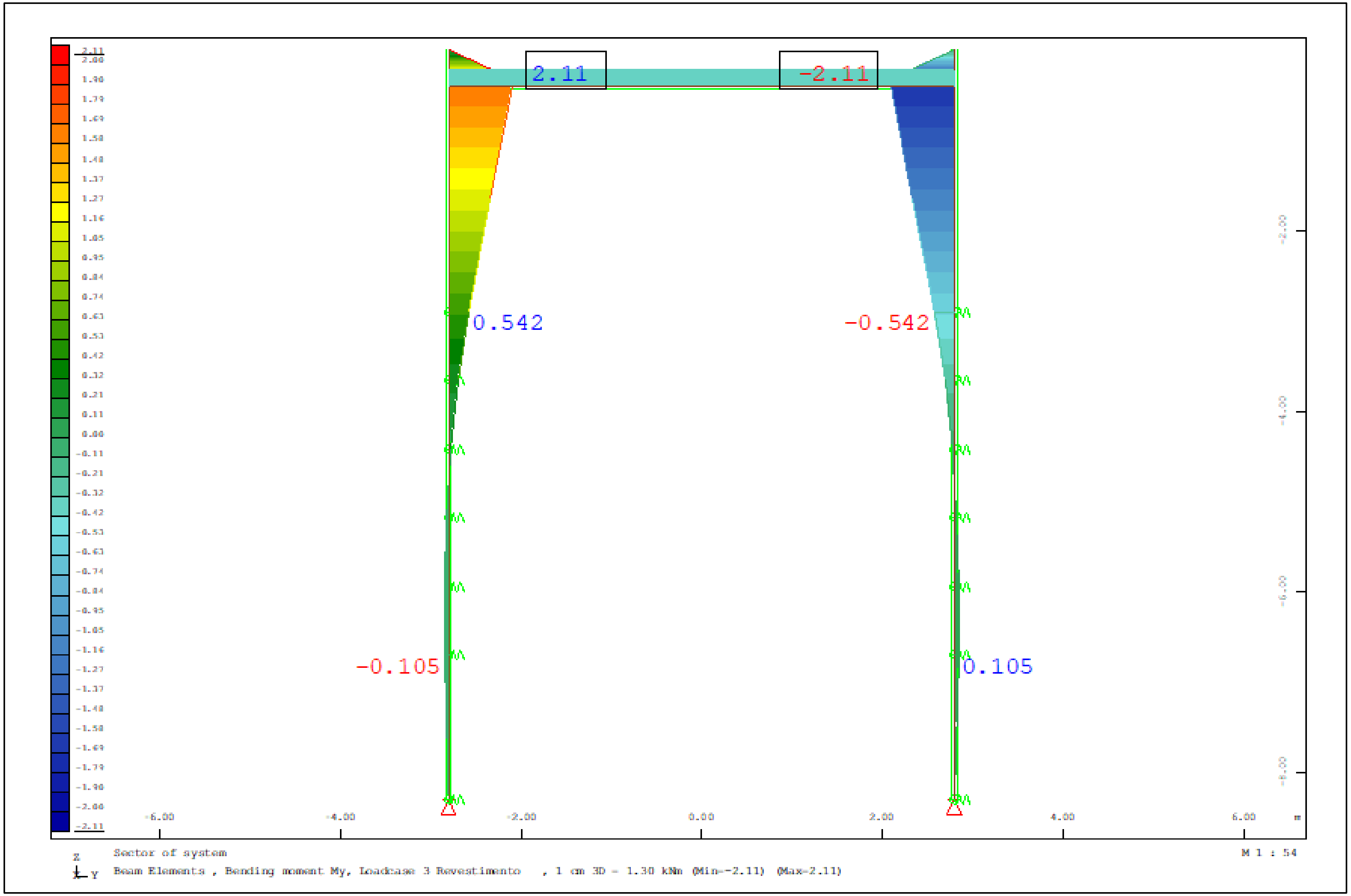
Wheel Guard - Moment Mz



Wheel Guard – Normal Force N



Covering - Moment My



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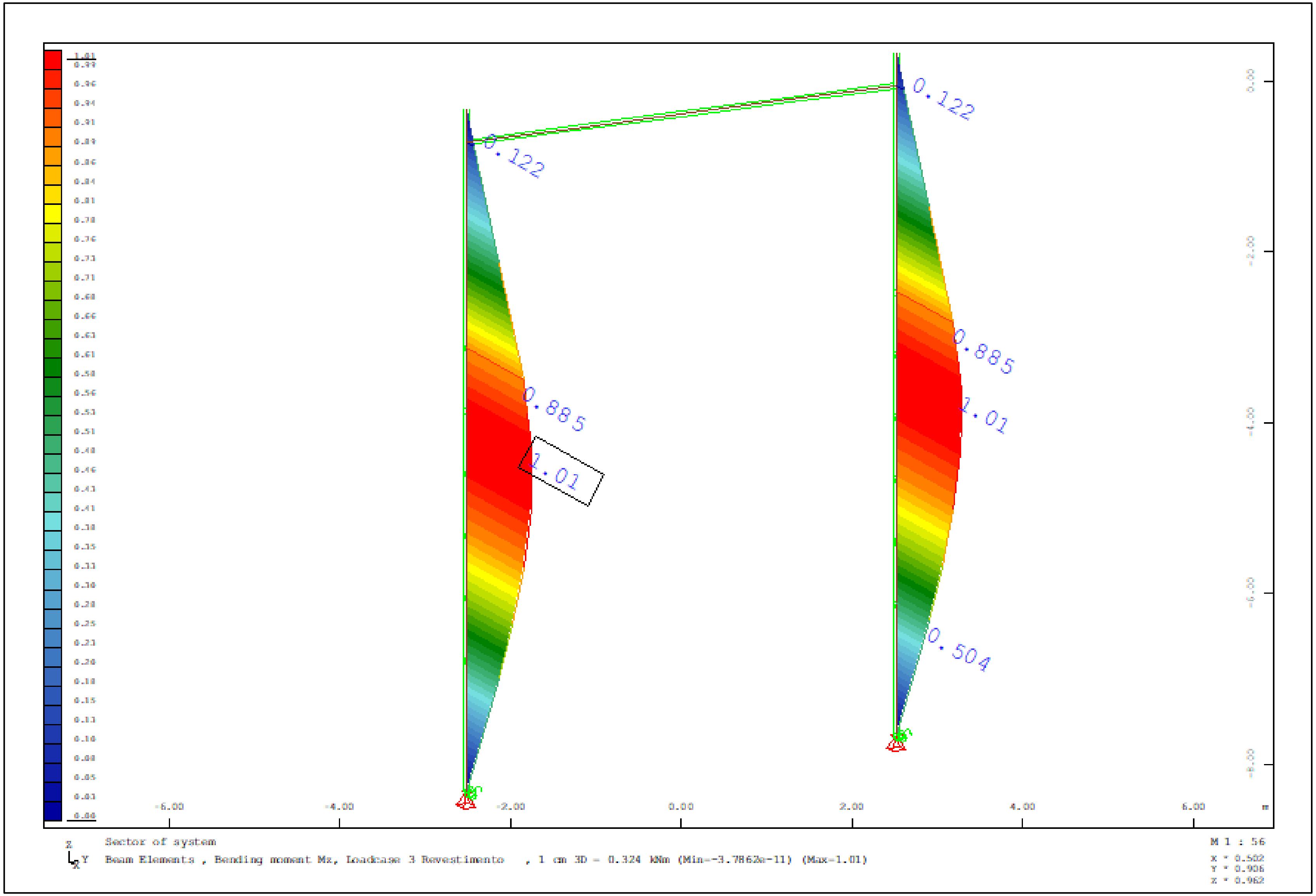
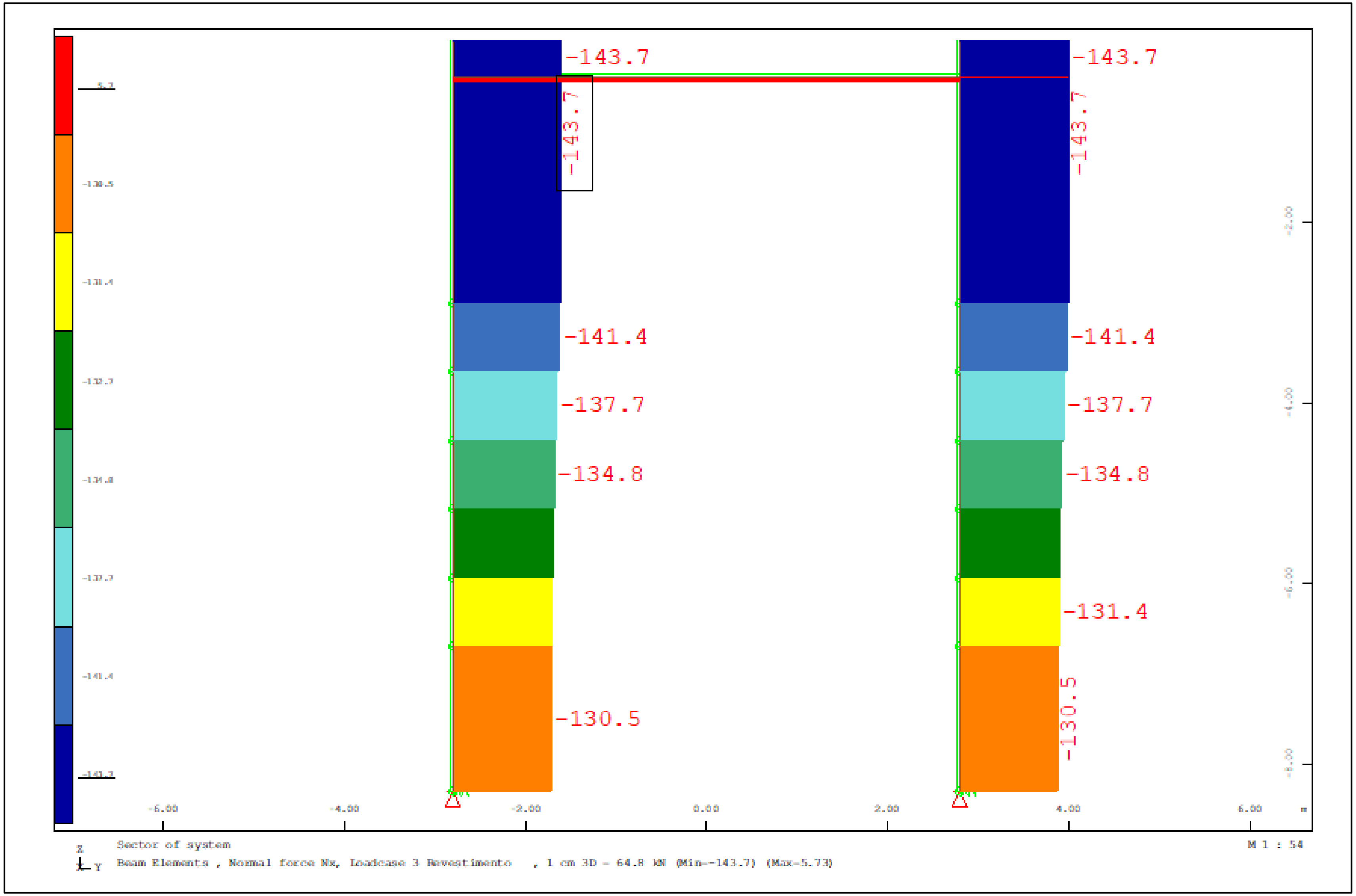
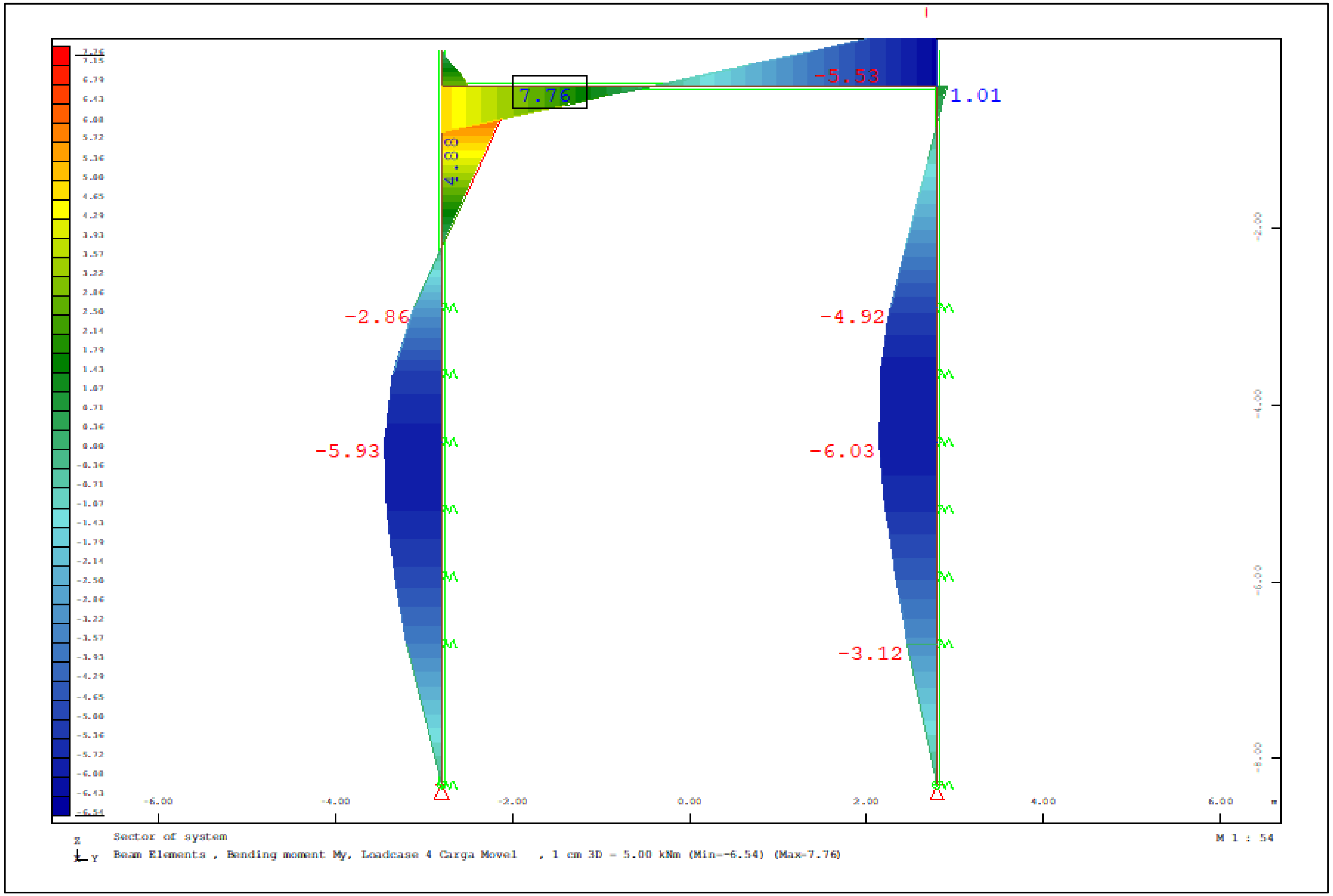


Covering – Normal Force N



Live Load - Moment My



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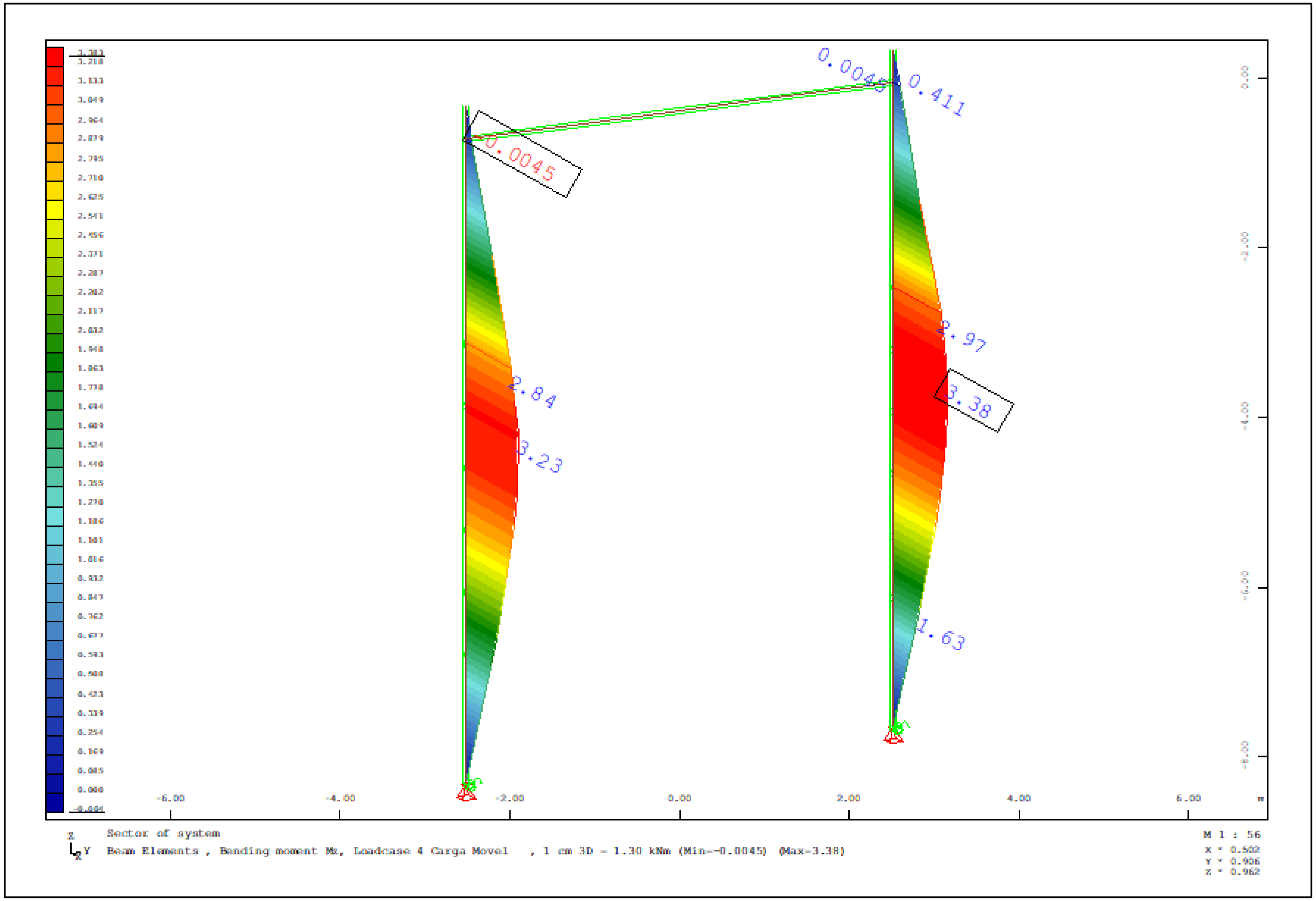

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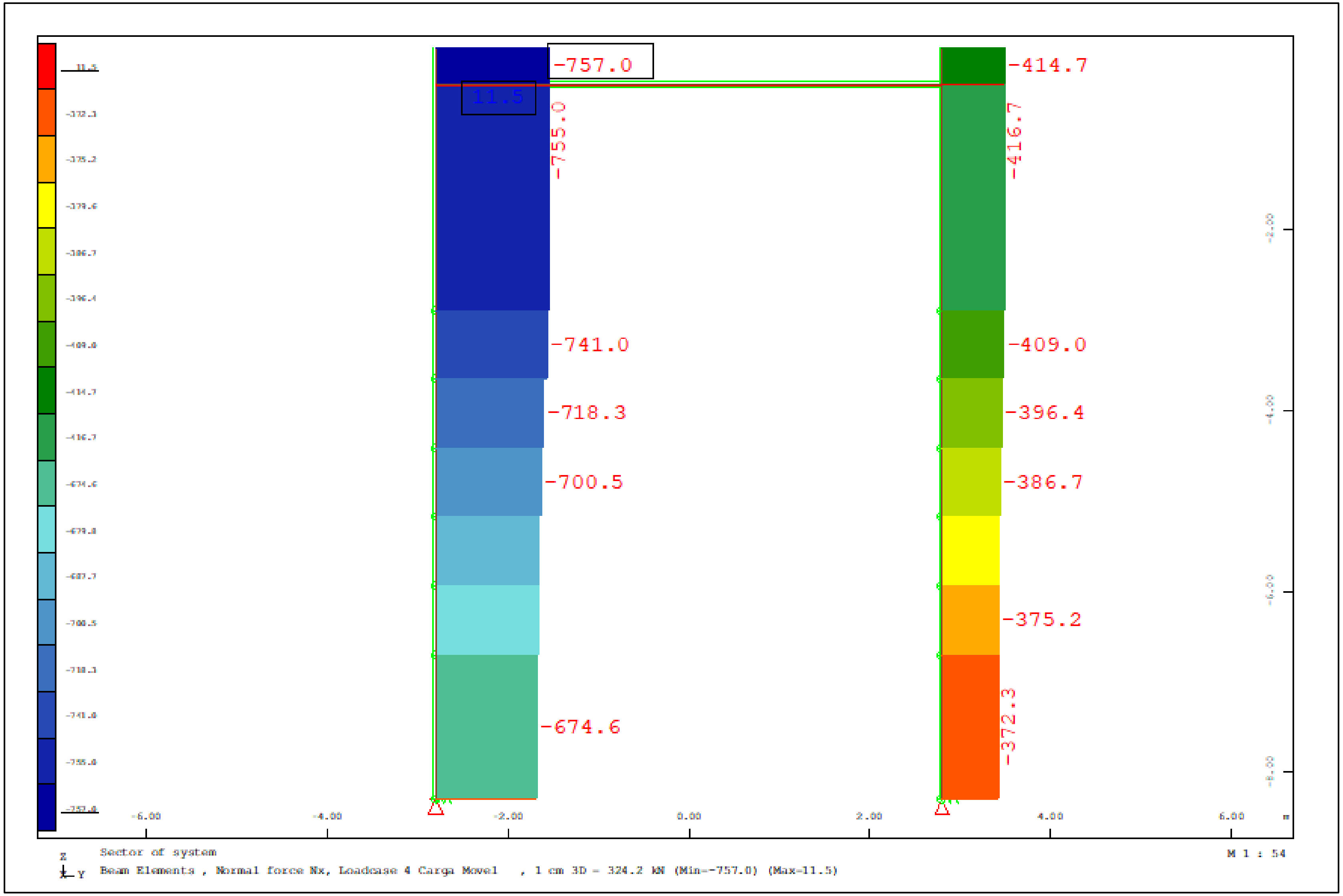
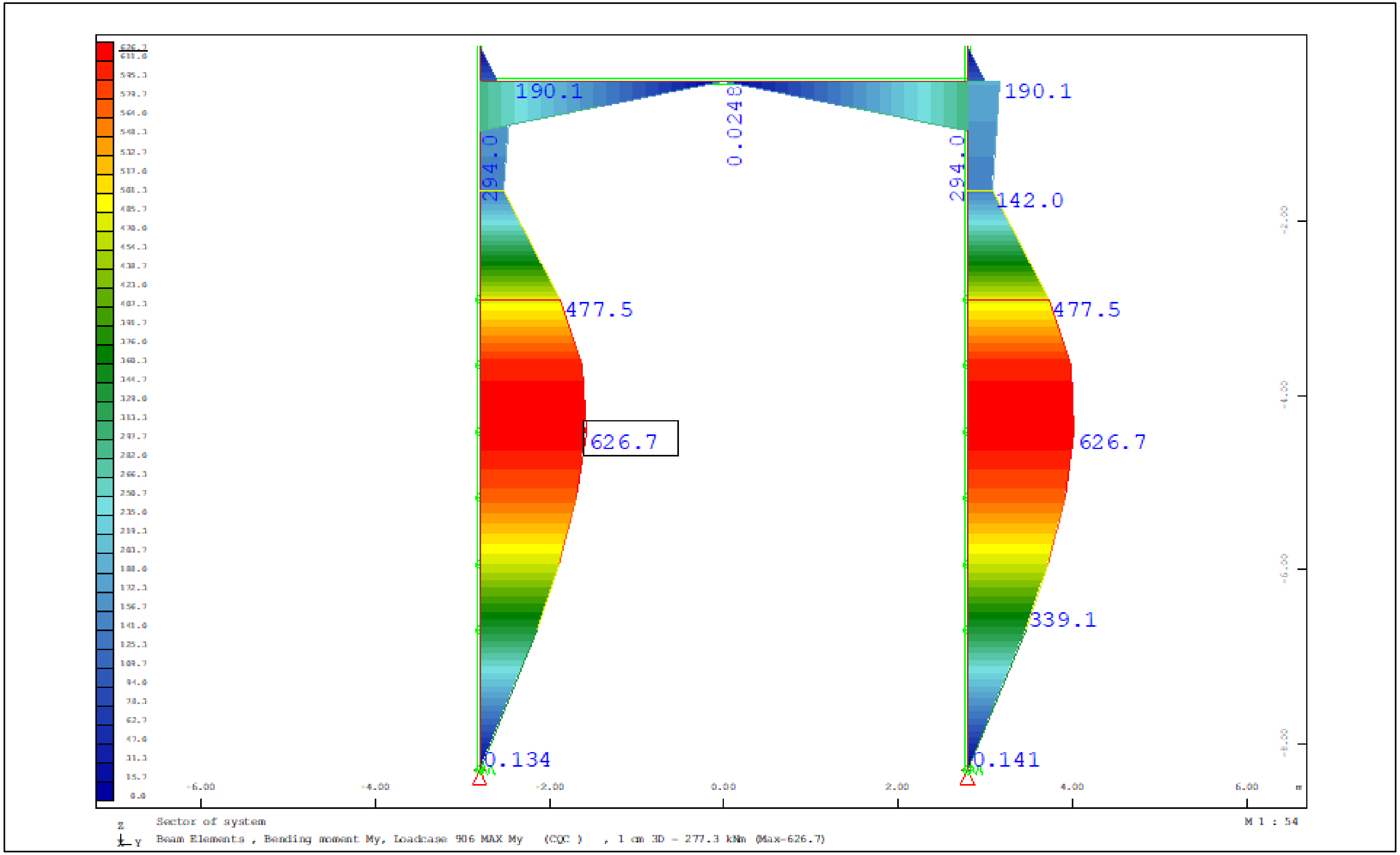
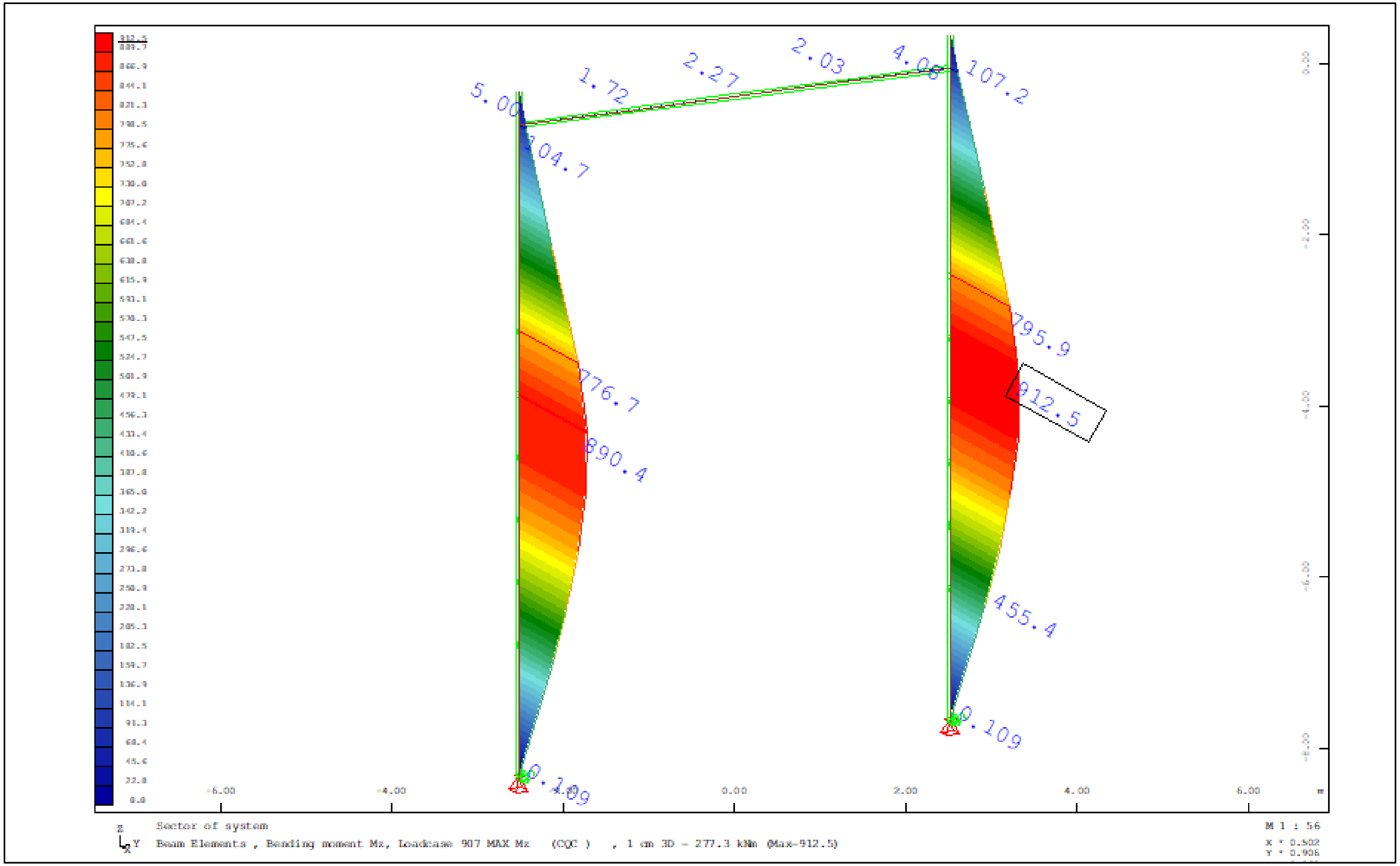


Seismic – Moment My



Seismic – Moment Mz



Seismic – Normal Force N