8.

Gibraltar Bridge

In this section, the theory presented in the previous chapters is applied to a real bridge, the Gibraltar Bridge, described by Larsen et al [35] and Larsen et al. [37], considering the following alternatives:

- Bridge deck alone (see item 8.1);
- Bridge deck with two stationary wings (see item 8.2);
- Bridge deck with two control surfaces.(see item 8.3).

To calculate the critical velocity of the Gibraltar bridge deck starting from its dynamical properties and the rational functions representing its aerodynamic derivatives, the state matrix is assembled, and its eigenvalues, damping ratios and frequences are extracted for increasing wind velocities. When one damping ratio is negative, the critical velocity is found. For Gibraltar Bridge, the state matrix has three aerodynamic states, to achieve better approximating functions, so its dimensions are 7 x 7. The critical velocity was determined as 48.2 m/s, close to the value 43.9 m/s found experimentally by Thiesemann [88].

To calculate the critical velocity of the Gibraltar Bridge with stationary wings, i.e., with no controls applied, the state matrix to assemble is a [15 x 15] square matrix, composed of 8 structural states, 3 aerodynamic states for the deck, to achieve better approximating rational functions, and 2 aerodynamic states for each wing, considered as flat plates. The same procedure stated above for the bridge deck alone is applied to the [15x15] state matrix and the critical velocity is found. Its magnitude depends on how wide the wings are with respect to the deck width. For 6m wide wings, the critical velocity increased from 48.2 to 66.7 m/s while for 3m wide wings, an increase to 53.4 m/s was found. Therefore, the concept of a variable-gain output feedback control is applied to the Gibraltar bridge, with appendages consisting of 2 wings, 3m wide, representing 5% of the deck width. The operating range was selected as $40 < U < 80$ m/s.

Table 8-1 shows deck characteristics of the Gibraltar and Akasho-Kayko bridges. The characteristics of the Gibraltar bridge, reported by Thiesemann [88] were transformed into a 1:200 scaled model, using similitude laws outlined in Appendix A, in order to compare the results obtained for the Akasho-Kayko in Wilde [98] and the results obtained for the Gibraltar bridge by the author.

System Characteristics	Gibraltar (1:1) (acc. to [88])	Gibraltar Scaled Model (1:200) (acc. to similitude laws)	Akashi-Strait Scaled Model (1:150) (acc. to [95])
Deck width	60m	60/200 = 0.30	0.2927
Mass of the deck(kg/m)	39.5×10^3	$39.5 \times 10^3/200^2 = 0.9875$	1.91
mass rotational moment of inertia (kg/m) x m2	26700×10^3	$26.7 \times 10^6/200^4 = 0.0167$	0.019345
natural frequency of heaving motion	0.383 rd/s	0.383x200=76.6	7.88
natural frequency of pitching motion	0.509 rd/s	0.509x200=101.8	25.06
damping ratio ofheaving	0.003	0.03	0.0011
damping ratio of pitching	0.0015	0.0015	0.001
Critical velocity [88] (experimental)	U_{crit}=43.9m/s		
Reduced frequency	$K_{crit}=B.w/U_{crit}=0,66$		
Critical velocity [88]	U_{crit}=34m/s		
Reduced frequency	$K_{crit}=B.w/U_{crit}=0,80$		
Critical velocity (RFA)	U_{crit}=47.9m/s	U_{crit}=47.9m/s	U_{crit}=10.2m/s
Reduced frequency	$K_{crit}=B.w/U_{crit}=0,57$	Kcrit=B.w/Ucrit=0,57	$K_{crit}=B.w/U_{crit}=0,51$

Table 8-1 - Comparison of Gibraltar and Akashi-Kayko Bridge deck characteristics.

The FORTRAN program written by Masukawa [43] was used to calculate the rational functions corresponding to the aerodynamic derivatives reported by Starossek [81] and Thiesemann [88] for 0.38 < K < 1.13. The complete input and output for the FORTRAN program are shown in item 9.4. The approximate functions read:

$$\mathbf{A}_0 = \begin{bmatrix} 0.3058600\text{E}+00 & 0.3093970\text{E}+01 \\ 0.1373977\text{E}+01 & -0.5468954\text{E}+00 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0.2459046\text{E}+01 & -0.1403846\text{E}+00 \\ -0.2400085\text{E}+00 & -0.2141308\text{E}+00 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0.1021054\text{E}+02 & 0.1787100\text{E}+01 & -0.4713930\text{E}+00 \\ -0.4298823\text{E}+00 & 0.1149574\text{E}+01 & 0.1083369\text{E}+01 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} -0.1027703\text{E}-01 & 0.4017426\text{E}-01 \\ -0.3480843\text{E}+00 & -0.2610986\text{E}+00 \\ -0.1946212\text{E}+01 & 0.2132517\text{E}+01 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.5164275\text{E}+00 & 0 & 0 \\ 0 & 0.1346202\text{E}+01 & 0 \\ 0 & 0 & 0.1971455\text{E}+01 \end{bmatrix}$$

These results are valid for 0.38 < K < 1.13. Note in item 9.4.2 that the total approximation error “min_err” is 0.021, and “err1” to “err4” are smaller than 0.3 x 10^{-3}, which denote good approximations between the rational functions and the tabular data.

MATLAB programs presented in Chapter 10 were used to calculate the critical velocity of the 1:200 scaled model of Gibraltar bridge.

8.1. Gibraltar Bridge (deck without wings).

The circles in Figure 8-1 represent experimental values of the aerodynamic derivatives of the Gibraltar profile and the curves denote the approximation functions for 3 lag terms for 0.38 < K < 1.13.

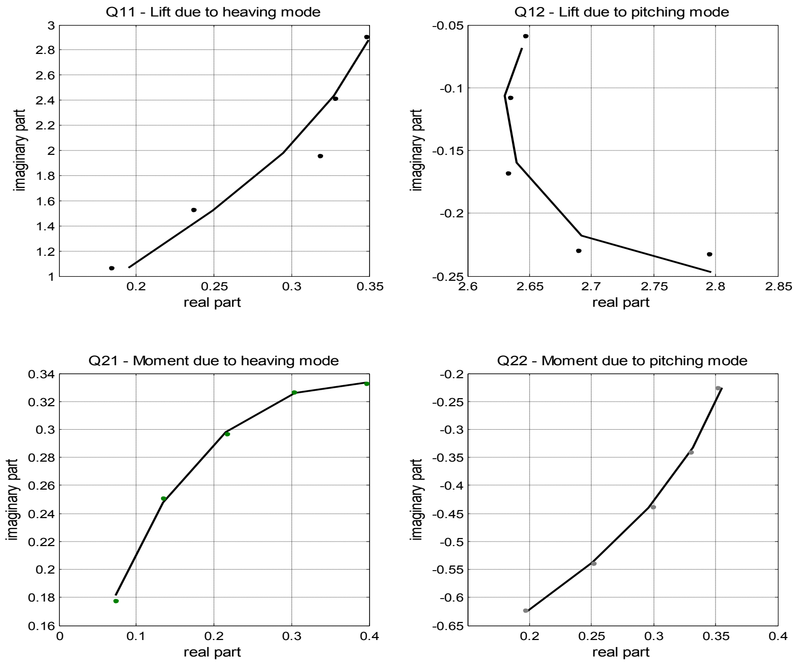

Figure 8-1 –Plots of experimental data (points) and the approximating curves corresponding to rational functions (0.38 < K < 1.13) for the Gibraltar profile.

Next, the [7x7] state matrix is assembled for increasing wind velocity, using the approximation functions and dynamic data belonging to the Gibraltar Bridge, as reported by Thiesemann. The state matrix for U = Ucrit = 47.95 m/s is shown below. Results of the program "Main_Program_Gibraltar" are:

Critical wind speed Uc = 47.95 m/s

State matrix for Uc = 47.95 m/s, as shown in Table 8-2.

$$A = \begin{vmatrix} -22.4 & 1.2526 & -6303.7 & -4412.3 & -14561 & -2548.6 & 672.25 \\ -11.396 & -10.473 & 10428 & -14514 & -3262.6 & 8724.6 & 8222.2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.6426 & 6.4212 & -82.542 & 0 & 0 \\ 0 & 0 & -55.635 & -41.732 & 0 & -215.17 & 0 \\ 0 & 0 & -311.07 & 340.85 & 0 & 0 & -315.1 \end{vmatrix}$$

Table 8-2 - State Matrix corresponding to U = 47.95 m/s.

Eigenvalues, damping factors and frequencies of the state matrix for Ucrit, as shown in Table 8-3.

Eigenvalue	Damping	Freq. (rad/seg)
1.43E-03 + 9.18e+001i	-1.56E-05	9.18E+01
1.43E-03 -9.18e+001i	-1.56E-05	9.18E+01
-2.92E+01 +7.20e+001i	3.76E-01	7.77E+01
-2.92E+01 -7.20e+001i	3.76E-01	7.77E+01
-7.96E+01	1.00E+00	7.96E+01
-2.25E+02	1.00E+00	2.25E+02
-2.83E+02	1.00E+00	2.83E+02

Table 8-3 - Eigenvalues, damping factors and frequences of the state matrix A for Ucrit = 47.95 m/s.

Notice that the critical frequence (pitching) is 91.8 rad/s, corresponding to zero damping. The real part of the eigenvalue is zero and the imaginary part is the frequency. The heaving frequency is 77.7 rad/s. Real eigenvalues correspond to the aerodynamic states and are meaningless.

Figure 8-2 shows these frequences for the critical velocity of 47.95 m/s.

The reduced frequency for the 1:200 scaled model or the 1:1 bridge is obviously the same:

K = B.w / U = 0.3 x 91.8 / 47.95 = 0.5743, or

K = B.w / U = (0.3 x 200) x (91.8 / 200) / 47.95 = 0.5743.

The critical frequency is found between $0.38 < K < 1.13$, where the rational functions were approximated to the experimental values.

Plots of the damping factors and structural frequencies of Gibraltar Bridge versus wind velocity are presented in Figure 8-2.

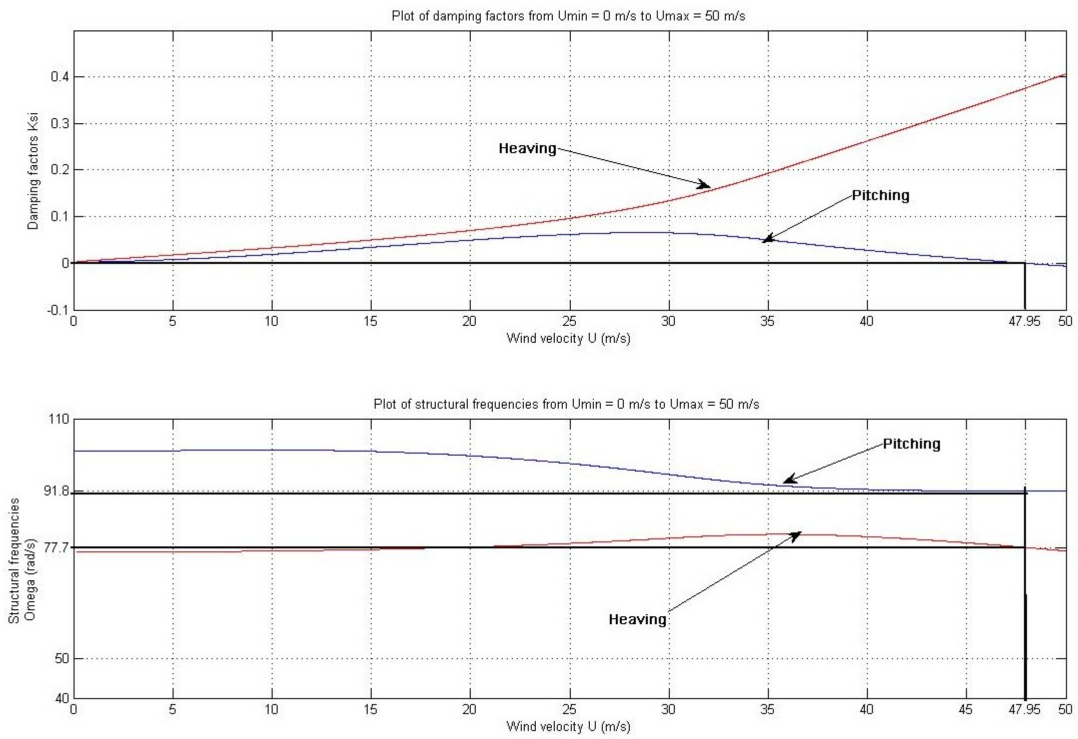

Figure 8-2 - Variation of damping factors and structural frequencies of the Gibraltar Bridge deck versus wind velocity .

8.2. Gibraltar Bridge (deck with stationary wings).

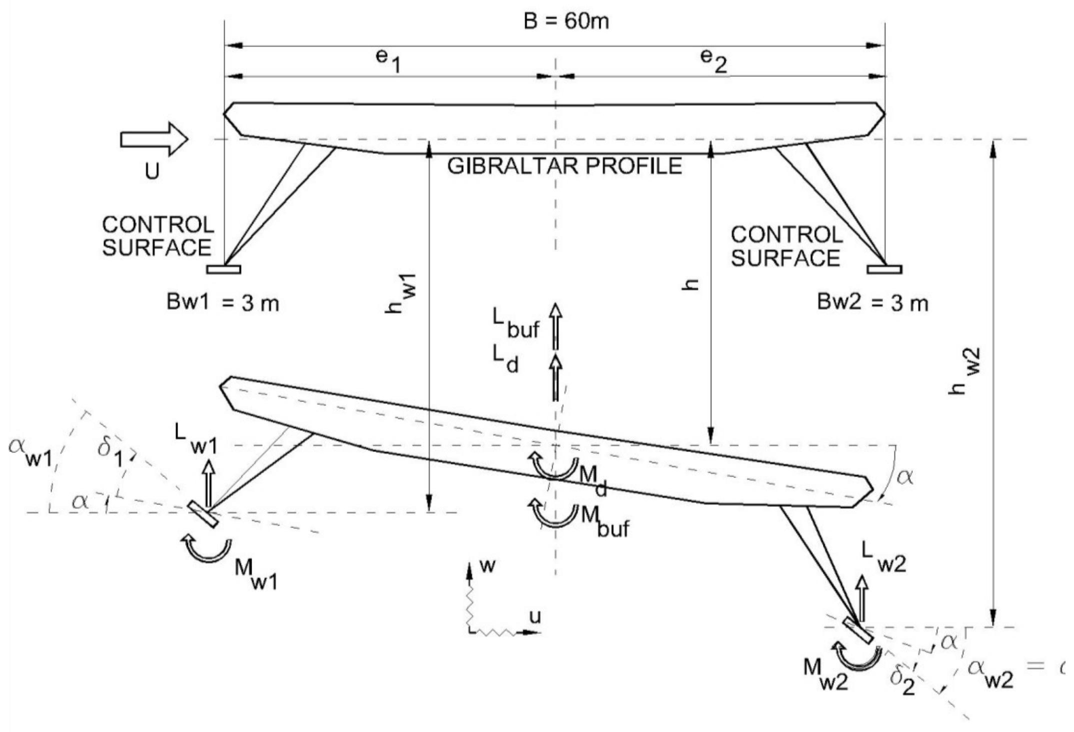

Figure 8-3 - Gibraltar bridge with leading and trailing control surfaces.

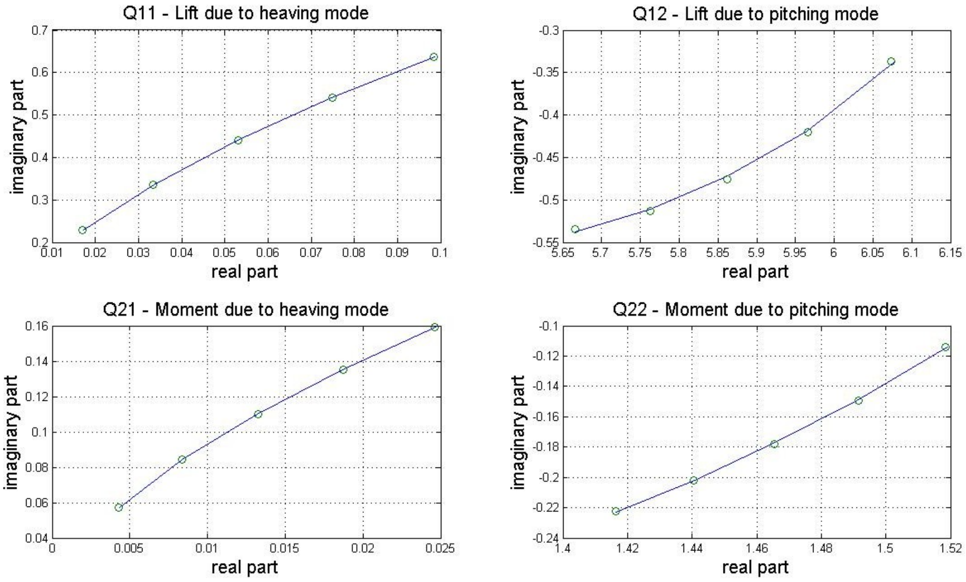

Figure 8-4 - Plots of tabular data (points) and the approximating curves corresponding to rational functions (0.038 < K < 0.13) for the wings 6m wide.

If K_{deck} is supposed to vary between 0 and 1.0, and B(wings) = 0.1 B(deck), then 0.038 < [K = (0.1Bdeck) x w) / U] < 0.113. Therefore, the approximation functions for the aerodynamic derivatives of the wings are sought for 0.038 < K < 0.113, considering the wings as flat plates 6m wide. Input and output of the FORTRAN program delivering the approximate rational functions of the aerodynamic derivatives are shown in item 9.4. The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, etc are:

$$\mathbf{A}_0 = \begin{bmatrix} 0.5742289\mathrm{E}+00 & 0.4010688\mathrm{E}+01 \\ 0.1435572\mathrm{E}+00 & 0.1002672\mathrm{E}+01 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0.3817801\mathrm{E}+01 & 0.2337816\mathrm{E}+01 \\ 0.9544502\mathrm{E}+00 & -0.2009442\mathrm{E}+00 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0.5719828\mathrm{E}+01 & 0.2422333\mathrm{E}+01 \\ 0.1429957\mathrm{E}+01 & 0.6055831\mathrm{E}+00 \end{bmatrix}$$

$$\mathbf{E}' = \begin{bmatrix} -0.1395796\mathrm{E}-03 & -0.6151422\mathrm{E}-01 \\ 0.2926502\mathrm{E}-02 & 0.2082658\mathrm{E}+00 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.4898931\mathrm{E}-01 & 0 \\ 0 & 0.2672425\mathrm{E}+00 \end{bmatrix}$$

Plots of experimental data (shown by points) and the approximating curves with rational functions for 0.038 < K <0.113 are shown in Figure 8-4.

Next, the [15x15] state matrix is assembled for increasing wind velocity, using the approximation functions of the deck and wings, as well as dynamic data belonging to the Gibraltar Bridge and the wings 6m wide.

The state matrix A for U = Ucrit from which the eigenvalues, damping factors and frequences were extracted (see Table 8-4) is a square matrix, [15x15], composed by 8 structural and 7 aerodynamic states. From the latter, 3 belong to the deck and 2 x 2 to the wings.

-30.698	1.4808	0.013698	0.013698	-7260.7	-6943.6	-501.29	-501.29	-19606	-3431.6	905.16	-460.67	-174.87	-460.67	-174.87
-12.916	-18.488	-0.042793	0.030159	14105	-16962	1368.3	-1301.5	-4396.2	11756	11079	1257.4	477.31	-1196.1	-454
0	0	-175.93	0	0	0	-15791	0	0	0	0	0	0	0	0
0	0	0	-176.02	0	0	0	-15807	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1.906	7.451	0	0	-95.78	0	0	0	0	0	0
0	0	0	0	-64.558	-48.425	0	0	0	-249.68	0	0	0	0	0
0	0	0	0	-360.96	395.51	0	0	0	0	-365.64	0	0	0	0
0	0	0	0	0.10919	5.6009	5.6555	0	0	0	0	-129.83	0	0	0
0	0	0	0	-1122.1	780.14	219.11	0	0	0	0	0	-489.63	0	0
0	0	0	0	0.10919	5.7101	0	5.6555	0	0	0	0	0	-129.83	0
0	0	0	0	-1122.1	-341.92	0	219.11	0	0	0	0	0	0	-489.63

Table 8-4 – State matrix for $\mathbf{A_c} = \mathbf{A} - \mathbf{BKC}$ for Ucrit =66.66 m/s (Gibraltar Bridge with stationary wings).

The critical velocity for the open loop system, which corresponds to the deck and stationary wings with bw = 6m, is 66.66m/s, 40% bigger than the critical velocity of the bridge deck alone.

Eigenvalue	Damping	Freq. (rad/s)
1.39e-003 + 9.49e+001i	-1.46E-05	9.49E+01
1.39e-003 - 9.49e+001i	-1.46E-05	9.49E+01
-1.70e+001 + 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 - 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 + 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 - 1.74e+001i	7.00E-01	2.43E+01
-4.42E+01	1.00E+00	4.42E+01
-9.31e+001 + 2.96e+001i	9.53E-01	9.77E+01
-9.31e+001 - 2.96e+001i	9.53E-01	9.77E+01
-1.09E+02	1.00E+00	1.09E+02
-1.11E+02	1.00E+00	1.11E+02
-3.19E+02	1.00E+00	3.19E+02
-3.80E+02	1.00E+00	3.80E+02
-5.88e+002 + 1.03e+000i	1.00E+00	5.88E+02
-5.88e+002 - 1.03e+000i	1.00E+00	5.88E+02

Table 8-5 - Eigenvalues, damping factors and frequences of the state matrix $\mathbf{A_c}$ for U = Ucrit = 66.66 m/s.

Considering now wings 3m wide, the approximation functions for the aerodynamic derivatives of the wings are sought for 0.019 < [K = (0.05 Bdeck x w) / U] < 0.056. Input and output of the FORTRAN program delivering the approximate functions of the aerodynamic derivatives are shown in item 9.4. The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, etc are in this case:

$$\mathbf{A}_0 = \begin{bmatrix} 0.2098244\text{E}+00 & 0.5221252\text{E}+01 \\ 0.5251235\text{E}-01 & 0.1305108\text{E}+01 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0.4615840\text{E}+01 & -0.5292162\text{E}+00 \\ 0.1153550\text{E}+01 & -0.9164623\text{E}+00 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0.4798194\text{E}+01 & 0.1821348\text{E}+01 \\ 0.1197957\text{E}+01 & 0.4559187\text{E}+00 \end{bmatrix}$$

$$\mathbf{E}' = \begin{bmatrix} 0.1471817\text{E}-05 & -0.1512478\text{E}-01 \\ 0.1524671\text{E}-02 & 0.5906955\text{E}-01 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.3500\text{E}-01 & 0 \\ 0 & 0.1320\text{E}+00 \end{bmatrix}$$

Plots of tabular data and the rational functions for 0.019<K<0.056 are shown in Figure 8-5. Next, the [15x15] state matrix is assembled for increasing wind velocity, using the approximation functions of the deck and wings, as well as dynamic data belonging to the Gibraltar Bridge and the wings 3m wide.

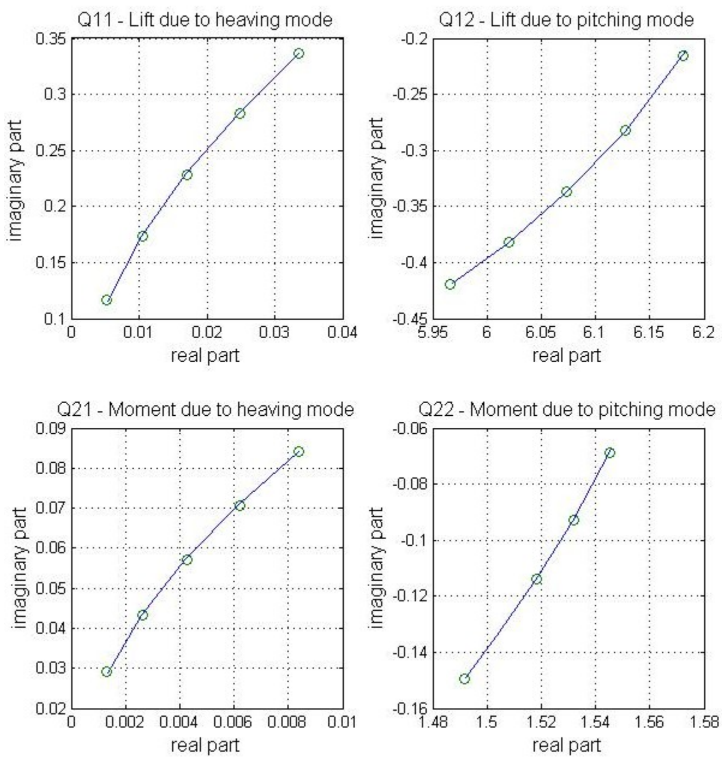

Figure 8-5- Plots of tabular data (points) and the approximating curves corresponding to rational functions (0.019 < K < 0.056) for the wings 3m wide.

The state matrix A for U = Ucrit = 55.64 m/s from which the eigenvalues, damping factors and frequences were extracted is shown in Table 8-6. The critical velocity of the open loop system, which corresponds to the deck and stationary wings and bw = 6m, is 55.64m/s, 17% bigger than the critical velocity of the bridge deck alone.

-30.698	1.4808	0.013698	0.013698	-7260.7	-6943.6	-501.29	-501.29	-19606	-3431.6	905.16	-460.67	-174.87	-460.67	-174.87
-12.916	-18.488	-0.04279	0.030159	14105	-16962	1368.3	-1301.5	-4396.2	11756	11079	1257.4	477.31	-1196.1	-454
0	0	-34.024	0	0	0	-590.63	0	0	0	0	0	0	0	0
0	0	0	-34.024	0	0	0	-590.63	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1.906	7.451	0	0	-95.78	0	0	0	0	0	0
0	0	0	0	-64.558	-48.425	0	0	0	-249.68	0	0	0	0	0
0	0	0	0	-360.96	395.51	0	0	0	0	-365.64	0	0	0	0
0	0	0	0	0.10919	5.6009	5.6555	0	0	0	0	-129.83	0	0	0
0	0	0	0	-1122.1	780.14	219.11	0	0	0	0	0	-489.63	0	0
0	0	0	0	0.10919	5.7101	0	5.6555	0	0	0	0	0	-129.83	0
0	0	0	0	-1122.1	-341.92	0	219.11	0	0	0	0	0	0	-489.63

Table 8-6 - State matrix for $\mathbf{A_c} = \mathbf{A} - \mathbf{BKC}$ for Ucrit =55.64 m/s (Gibraltar Bridge with stationary wings).

Figure 8-6 shows plots of damping ratios and structural frequencies for Gibraltar bridge with 3m wide stationary wings. Note that the critical frequency (pitching) corresponds to the first and second eigenvalue, while the heaving frequency occurring for the same wind velocity corresponds to the seventh and eighth eigenvalue. Both frequencies are marked in the axis of structural frequencies and correspond to the critical velocity of 55.64 m/s also highlighted in the velocity axis.

Eigenvalue	Damping	Frequency. (rad/s)
7.24e-004 + 9.29e+001i	-7.80E-06	9.29E+01
7.24e-004 - 9.29e+001i	-7.80E-06	9.29E+01
-1.70e+001 + 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 - 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 + 1.74e+001i	7.00E-01	2.43E+01
-1.70e+001 - 1.74e+001i	7.00E-01	2.43E+01
-4.37e+001 + 5.95e+001i	5.92E-01	7.38E+01
-4.37e+001 - 5.95e+001i	5.92E-01	7.38E+01
-8.98E+01	1.00E+00	8.98E+01
-1.30E+02	1.00E+00	1.30E+02
-1.30E+02	1.00E+00	1.30E+02
-2.63E+02	1.00E+00	2.63E+02
-3.24E+02	1.00E+00	3.24E+02
-4.88e+002 + 3.22e-001i	1.00E+00	4.88E+02
-4.88e+002 - 3.22e-001i	1.00E+00	4.88E+02

Table 8-7 - Eigenvalues, damping factors and frequences of the state matrix $\mathbf{A}_c$ for U = 55.64 m/s.

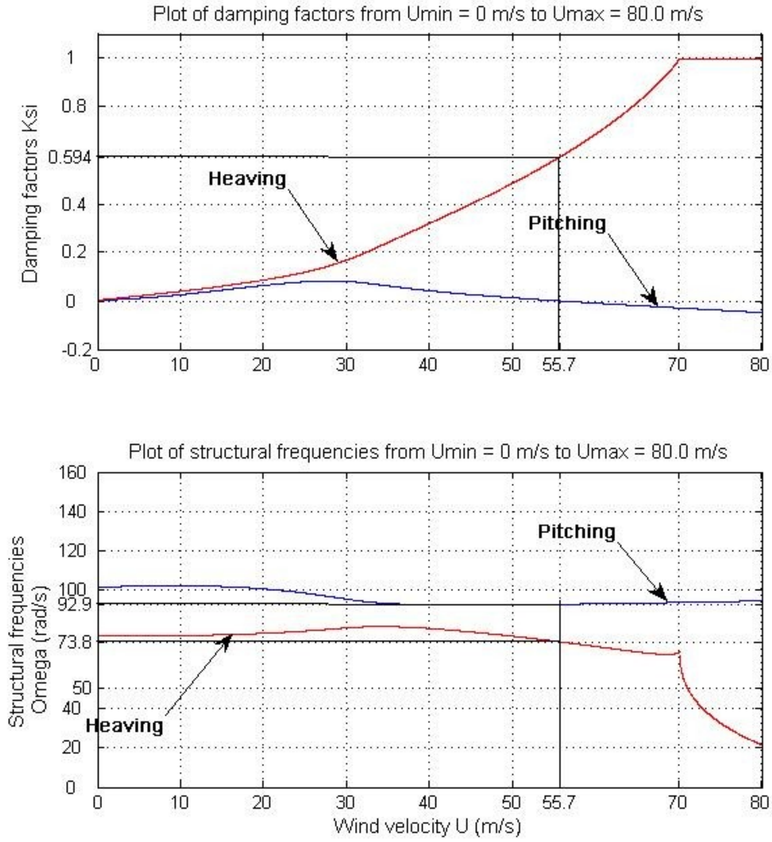

Figure 8-6 - Plots of damping ratios and structural frequencies for Gibraltar bridge with stationary wings, 3m wide.

8.3. Gibraltar Bridge and 3m wide wings, regulated by a variable-gain control system.

The conclusions drawn in last section were:

For the bridge deck alone, Ucrit = 47.5m/s;

For stationary wings, 6m wide, Ucrit = 66.67m/s

For stationary wings, 3m wide, Ucrit = 55.64m/s.

The next sections are dedicated to the study of Gibraltar bridge with 3 m wide wings, regulated by a variable-gains system.

Table 8-8 shows data adopted for the characteristics of the wings, produced by the actuators. The rotational frequency of the wings attached to the scaled model 1:200 of Gibraltar bridge was taken as 20 Hz. The period of the oscillation of the wings in the real bridge corresponds to 200 / 20 = 10s, which seems plausible for a 3m wide wing. If a mechanism is able to rotate the wings faster, this results in a better performance of the system. The damping ratios are equal in both models.

System Characteristics (WINGS)	Gibraltar - Scaled Model (1:200)	Akashi-Strait Scaled Model (1:150)
Bw	B x 1/10	B x 1/10
Mass rotational momentof inertia of the wings	1 kg.m^2/m	1 kg.m^2/m
Rotational frequency of the wings	20Hz x 2π =40 π rad/s	6Hz x 2π = 12π
Damping ratio of the wings	0.7	0.7

Table 8-8 - Comparison of Gibraltar and Akashi-Kayko Bridge wing characteristics

8.3.1. Definition of the operating points and weights

The operating points and weigths are described by the following vectors:

$\mathbf{U\_p} = [\,0\ 40\ 49\ 58\ 67\ 76]'$ operating points

$\mathbf{fr\_p} = [\,1\ 50\ 100\ 300\ 500\ 925]'$ weight on $\boldsymbol{R}$ (on the control signals)

$\mathbf{q\_p} = [\,0.1\ 1.0\ 5.0\ 10.0\ 30.0\ 68.0]'$ weight on $\boldsymbol{Q}$.

$\mathbf{fw\_p} = [\,0.1\ 200\ 175\ 160\ 150\ 100]'$ weight on the wind velocity U.

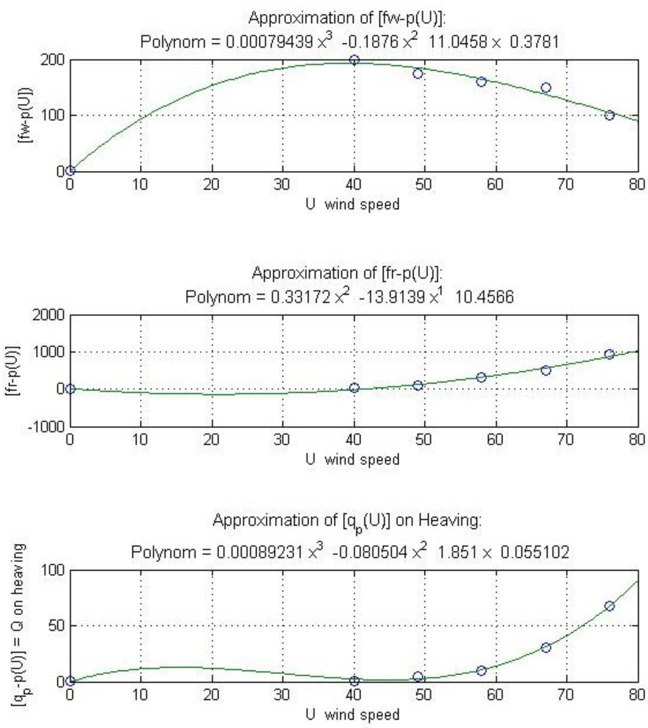

Figure 8-7 - Plots of the operating points and weights on Q , R and W (wind)

Figure 8-7 shows plots of the various weights defined by algebric functions of the second and third degree. The approximations are produced by the MATLAB routines poly_qp = polyfit(U_p, q_p, 3), poly_fr = polyfit(U_p, fr_p, 2) and poly_fw = polyfit(U_p, fw_p,3), see item 10.3.1.

8.3.2. Sistematic method to obtain the matrix of control gains

The next step is to obtain the matrices $\mathbf{KI}$ and $\mathbf{K0}$, where:

$$\mathbf{K}_0 = [\ [\mathbf{KI}]\ [\mathbf{K0}]\ ] \tag{7.45}$$

$$\mathbf{KI} = \begin{bmatrix} \mathrm{KI}(1,1) & \mathrm{KI}(1,2) & \mathrm{KI}(1,3) & \mathrm{KI}(1,4) & \mathrm{KI}(1,5) & \mathrm{KI}(1,6) \\ \mathrm{KI}(2,1) & \mathrm{KI}(2,2) & \mathrm{KI}(2,3) & \mathrm{KI}(2,4) & \mathrm{KI}(2,5) & \mathrm{KI}(2,6) \end{bmatrix}$$

$$\mathbf{K0} = \begin{bmatrix} \mathrm{K0}(1,1) & \mathrm{K0}(1,2) & \mathrm{K0}(1,3) & \mathrm{K0}(1,4) & \mathrm{K0}(1,5) & \mathrm{K0}(1,6) \\ \mathrm{K0}(2,1) & \mathrm{K0}(2,2) & \mathrm{K0}(2,3) & \mathrm{K0}(2,4) & \mathrm{K0}(2,5) & \mathrm{K0}(2,6) \end{bmatrix}$$

In order to obtain $\mathbf{P}_\mathrm{j}$ and $\mathbf{L}_\mathrm{j}$ as positive definite matrices, the vector $\mathbf{p}$ of operating points is chosen as $\mathbf{p}' = [50\ 51\ 52\ 53]'$, because all 4 velocities are lower than the critical velocity of the open loop system "deck with stationary wings", and therefore produce state matrices whose eigenvalues always have negative real parts. The initial $\mathbf{K0}$ is set to zero. The output of the program Main_Program_GIBRALTAR.m (item 9.3.1)reads:

K0 (inicial) =	K1=	0	0	0	0	0	0
		0	0	0	0	0	0
	K0=	0	0	0	0	0	0
		0	0	0	0	0	0
K0 (after 15 iterations) =	K1=	0.00022244	-0.00051501	-0.076016	0.015872	-0.0090158	-0.023352
		-0.00086561	0.042311	5.0983	-0.91432	0.74553	0.92479
	K0=	0.0011373	-0.00013952	-0.0022974	-0.059854	-0.021289	-0.0042956
		-0.081782	0.0061477	-0.13269	4.4809	1.1521	0.88956
Improvement dJ				5.5551			
Trace J				108281.6628			

If the last $\mathbf{K}_0$ is input as the initial $\mathbf{K}_0$ matrix of the next try, the output would be:

K0 (inicial) =	K1=	0.00022244	-0.00051501	-0.076016	0.015872	-0.0090158	-0.023352
		-0.00086561	0.042311	5.0983	-0.91432	0.74553	0.92479
	K0=	0.0011373	-0.00013952	-0.0022974	-0.059854	-0.021289	-0.0042956
		-0.081782	0.0061477	-0.13269	4.4809	1.1521	0.88956
K0 (after 1 iteration) =	K1=	0.00022663	-0.00053535	-0.076927	0.01523	-0.0095826	-0.023505
		-0.0010799	0.043285	5.1411	-0.88472	0.77367	0.93392
	K0=	0.001163	-0.0001765	-0.0030343	-0.063619	-0.021568	-0.0032822
		-0.083096	0.0079215	-0.10089	4.6622	1.1681	0.84604
Improvement dJ				1.7111			
Trace J				108279.9641			

The next vector $\mathbf{p}$ of operating points is chosen as $[50\ 52\ 54\ 56]'$, and the initial matrix $\mathbf{KI}$ is chosen as the last $\mathbf{KI}$ of the previous loop.

The output of the program Main_Program_GIBRALTAR.m reads now:

K0 (inicial) =	K1=	0.00022663	-0.00053535	-0.076927	0.01523	-0.0095826	-0.023505
		-0.0010799	0.043285	5.1411	-0.88472	0.77367	0.93392
	K0=	0.001163	-0.0001765	-0.0030343	-0.063619	-0.021568	-0.0032822
		-0.083096	0.0079215	-0.10089	4.6622	1.1681	0.84604
K0 (after 2 iterations) =	K1=	0.00016698	-0.00043938	-0.061731	0.017164	-0.005351	-0.022521
		0.0019925	0.038343	4.3583	-0.98607	0.55342	0.88395
	K0=	0.00090902	-0.00015351	-0.0017753	-0.055705	-0.019505	0.0012389
		-0.070055	0.0066941	-0.16942	4.2523	1.06	0.6159
Improvement dJ		2.8415					
Trace J		114733.8811					

The next vector $\mathbf{p}$ of operating points is chosen as $[50\ 58\ 64\ 72]'$, and the initial initial matrix $\mathbf{KI}$ is chosen as the last $\mathbf{KI}$ of the previous loop. The output of the program Main_Program_GIBRALTAR.m reads now:

K0 (initial) =	K1=	0.00016698	-0.00043938	-0.061731	0.017164	-0.005351	-0.022521
		0.0019925	0.038343	4.3583	-0.98607	0.55342	0.88395
	K0=	0.00090902	-0.00015351	-0.0017753	-0.055705	-0.019505	0.0012389
		-0.070055	0.0066941	-0.16942	4.2523	1.06	0.6159
K0 (after 5 iterations) =	K1=	0.000054548	-0.000263	-0.032757	0.020793	0.0048028	-0.01859
		0.0079457	0.029044	2.8055	-1.1824	-0.0061058	0.68313
	K0=	0.00046737	-0.000068814	0.0017267	-0.040987	-0.013516	0.0082545
		-0.04651	0.0021743	-0.36131	3.4756	0.72394	0.22625
Improvement dJ		6.432					
Trace J		100185.5129					

Finally, the vector $\mathbf{p}$ of operating points is chosen as $[50\ 60\ 70\ 80]'$, and the initial matrix $\mathbf{KI}$ is chosen as the last $\mathbf{KI}$ of the previous loop. The output of the program Main_Program_GIBRALTAR.m reads now:

K0 (initial) =	K1=	0.000054548	-0.000263	-0.032757	0.020793	0.0048028	-0.01859
		0.0079457	0.029044	2.8055	-1.1824	-0.0061058	0.68313
	K0=	0.00046737	-0.000068814	0.0017267	-0.040987	-0.013516	0.0082545
		-0.04651	0.0021743	-0.36131	3.4756	0.72394	0.22625
K0 (after 5 iterations) =	K1=	1.01E-05	-0.00013364	-0.016203	0.025297	0.01181	-0.014927
		0.010369	0.021369	1.7632	-1.4495	-0.47051	0.49159
	K0=	0.00025394	-3.78E-05	0.0036974	-0.036226	-0.010415	0.011387
		-0.033388	0.00015963	-0.49006	3.1879	0.50661	0.01074
Improvement dJ		8.8399					
Trace J		150383.102					

If the initial matrix **KI** is chosen as the last **KI** of the previous loop, the output would read as:

K0 (initial) =	K1=	0.000010072	-0.00013364	-0.016203	0.025297	0.01181	-0.014927
		0.010369	0.021369	1.7632	-1.4495	-0.47051	0.49159
	K0=	0.00025394	-0.000037765	0.0036974	-0.036226	-0.010415	0.011387
		-0.033388	0.00015963	-0.49006	3.1879	0.50661	0.01074
K0 (after 1 iteration) =	K1=	9.42E-06	-0.0001522	-0.016647	0.024445	0.010494	-0.01612
		0.010496	0.022488	1.7937	-1.399	-0.39172	0.55871
	K0=	0.00025054	-3.23E-05	0.0038901	-0.035307	-0.0097233	0.011929
		-0.033178	-0.00016454	-0.49878	3.1332	0.4689	-0.022304
Improvement dJ				3.2343			
Trace J				150379.8828			

This procedure can be repeated after the operating points and weights are defined differently from the initial assumptions.

The results that follow were based in the following output of the program Main_Program_GIBRALTAR.m:

K0 (initial) =	K1=	9.4273E-06	-0.00015221	-0.016647	0.024445	0.010493	-0.016121
		0.010496	0.022489	1.7937	-1.3989	-0.39166	0.55875
	K0=	0.00025054	-0.000032332	0.0038904	-0.035306	-0.0097225	0.011928
		-0.033179	-0.00016458	-0.49878	3.1332	0.46887	-0.022267
K0 (after 1 iteration) =	K1=	9.15E-06	-0.00014133	-0.016381	0.024998	0.011349	-0.015291
		0.010457	0.021831	1.7753	-1.4318	-0.44322	0.51146
	K0=	0.00025244	-3.53E-05	0.0037303	-0.035848	-0.010175	0.011589
		-0.033297	0.00001494	-0.49098	3.1655	0.49351	-0.0015636
Improvement dJ				1.2045			
Trace J				150378.687			

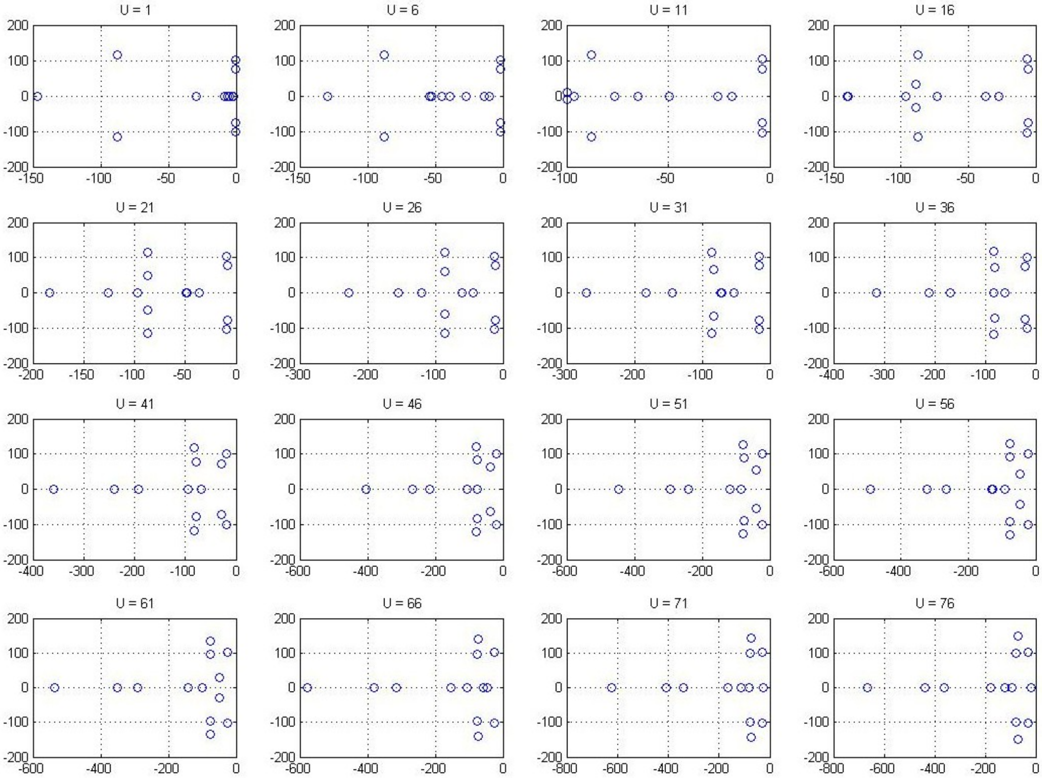

Figure 8-8 -Plot of the eigenvalues of the state matrix from 1 to 76 m/s in steps of 5 m/s.

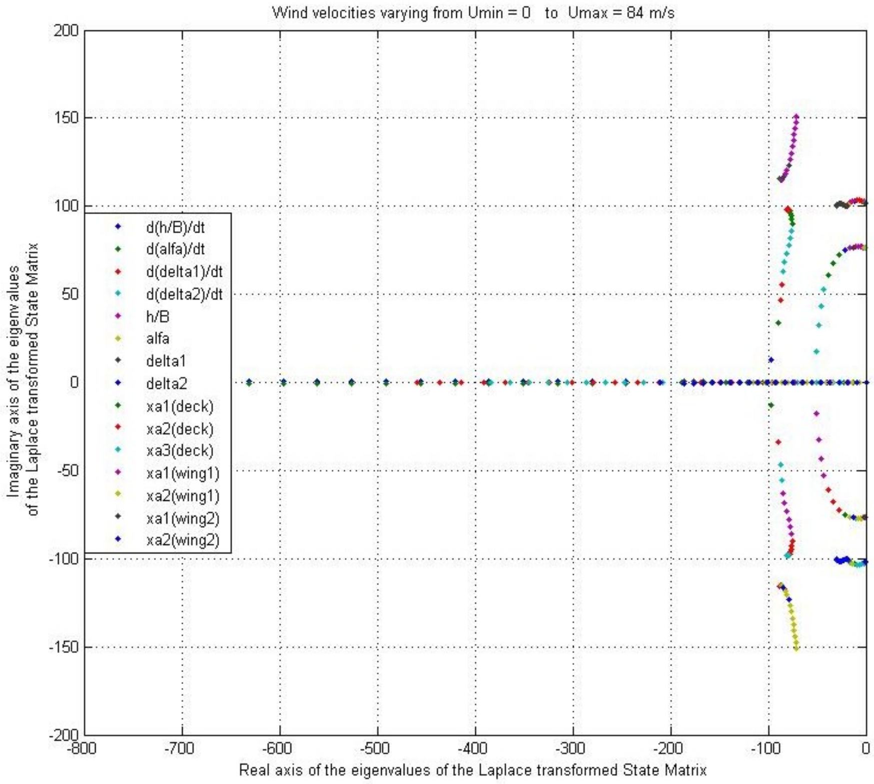

Figure 8-9 - Plots of the eigenvalues of the state matrix from 1 to 76 m/s put together.

Figure 8-9 shows no system instability from 1 to 76 m/s. Next,

Figure 8-10 shows plots of damping factors and structural frequencies from 0 to 80 m/s. The damping plot confirms this statement, as there is no crossing of the zero damping line.

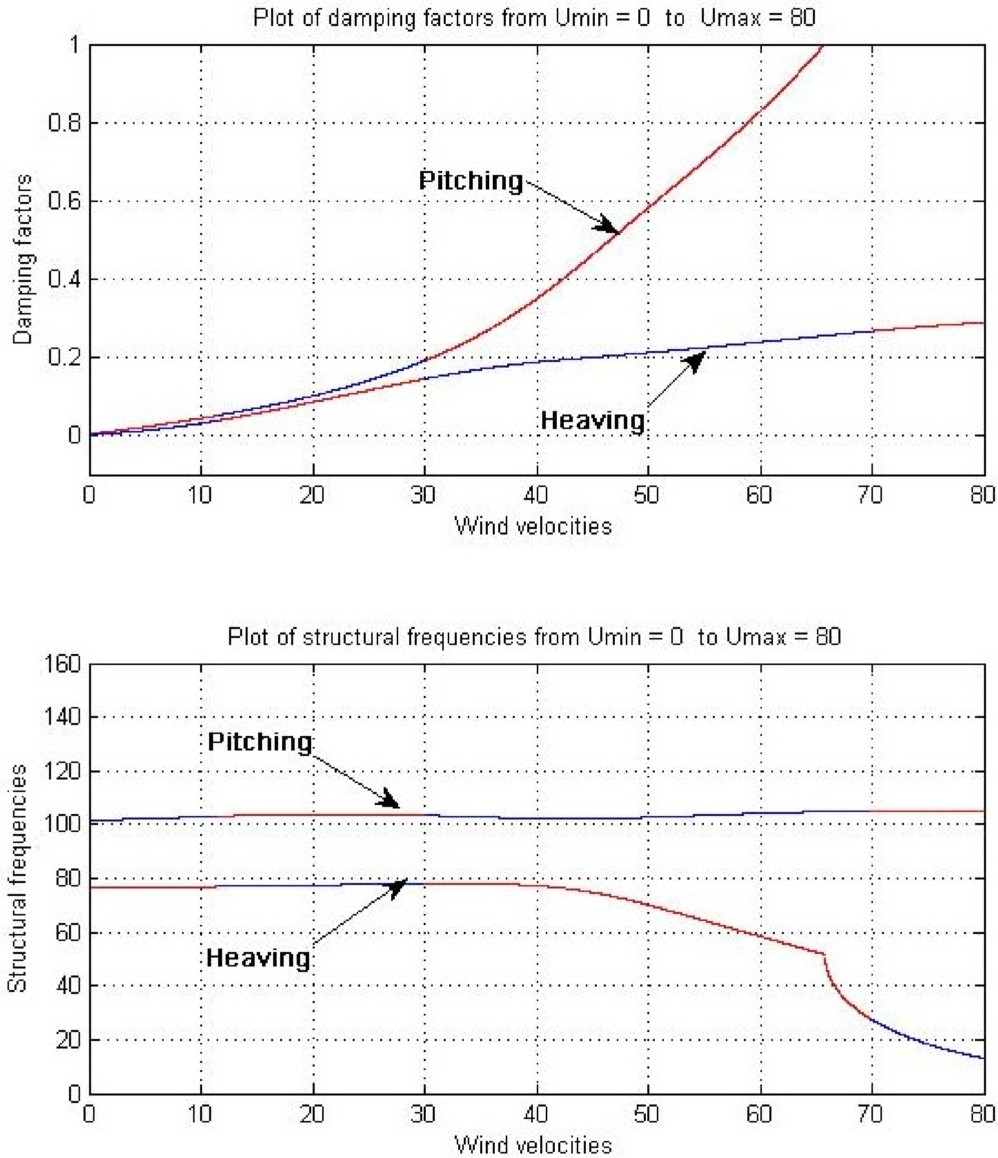

Figure 8-10 - Plots of damping factors and structural frequencies from 0 to 80 m/s.

The frequencies of the pitching and the heaving modes do not change significantly until the velocity of 40 m/s. From this velocity onwards the wings add a large amount of aerodynamic damping to the system. The pitching frequences do not change substantially and the heaving frequency drops to low levels. The active control of the bridge flutter with variable gains is considered successful.

The next figures show the behaviour of the system when subjected to a unit impulse at t=0, while the wind is blowing at different velocities. The system is stable for all velocities.

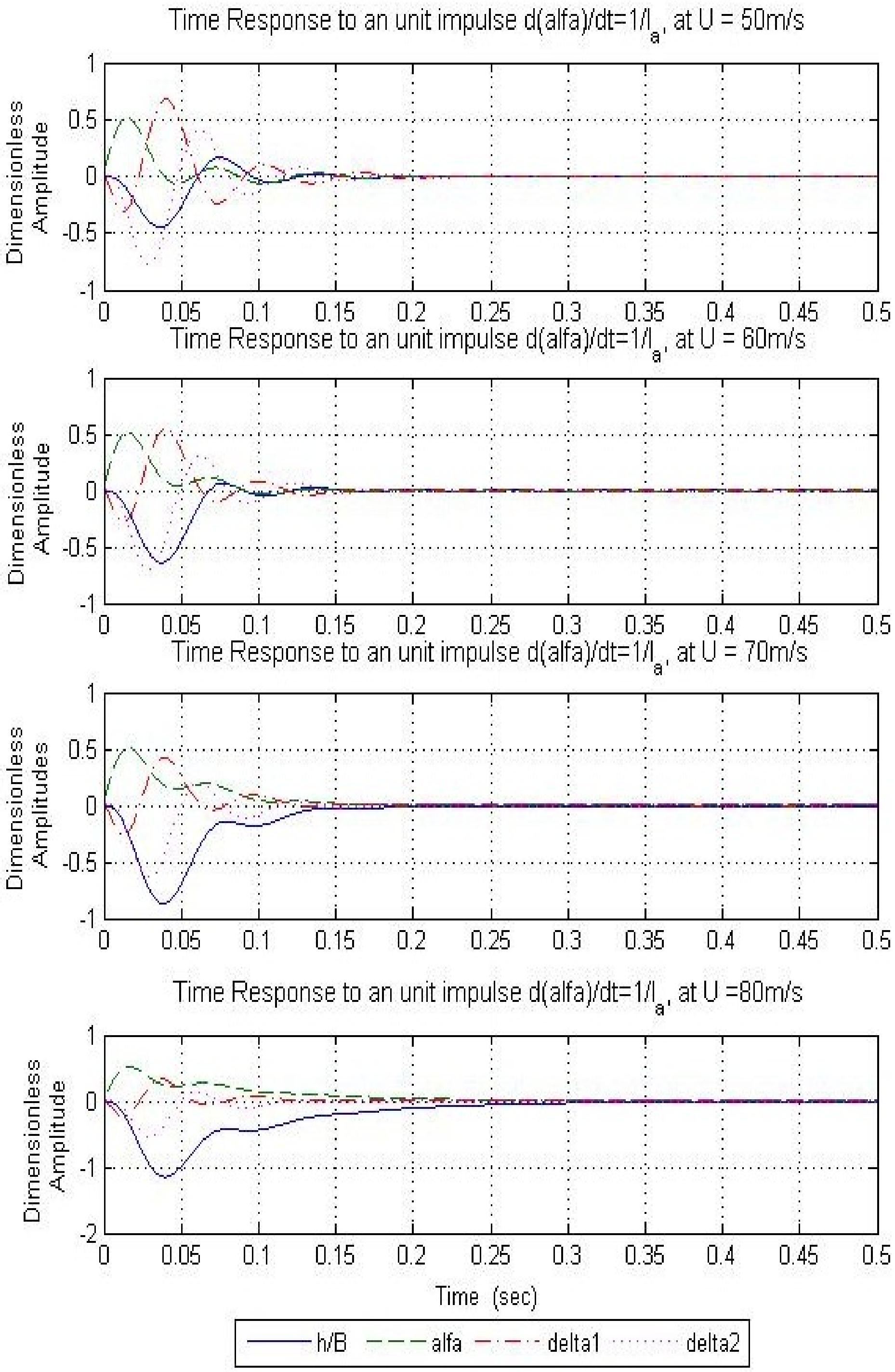

Figure 8-11 - Plots of amplitudes due to an unit impulse at t=0 at the operating points.

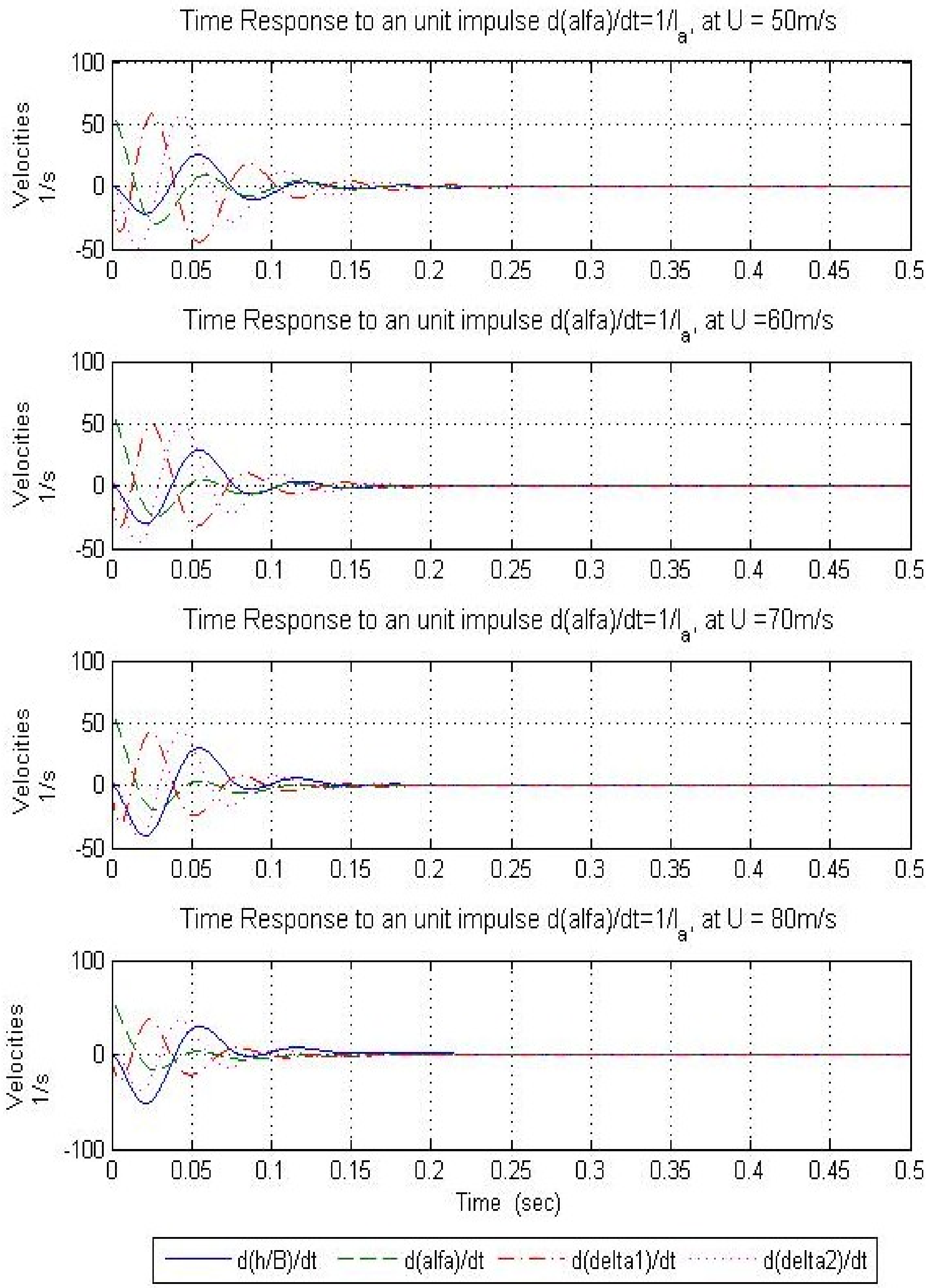

Figure 8-12 - Plots of velocities due to an unit impulse at t=0 at the operating points.