5.

Equation of motion of the bridge deck and wings system with active aerodynamic control

5.1. Formulation

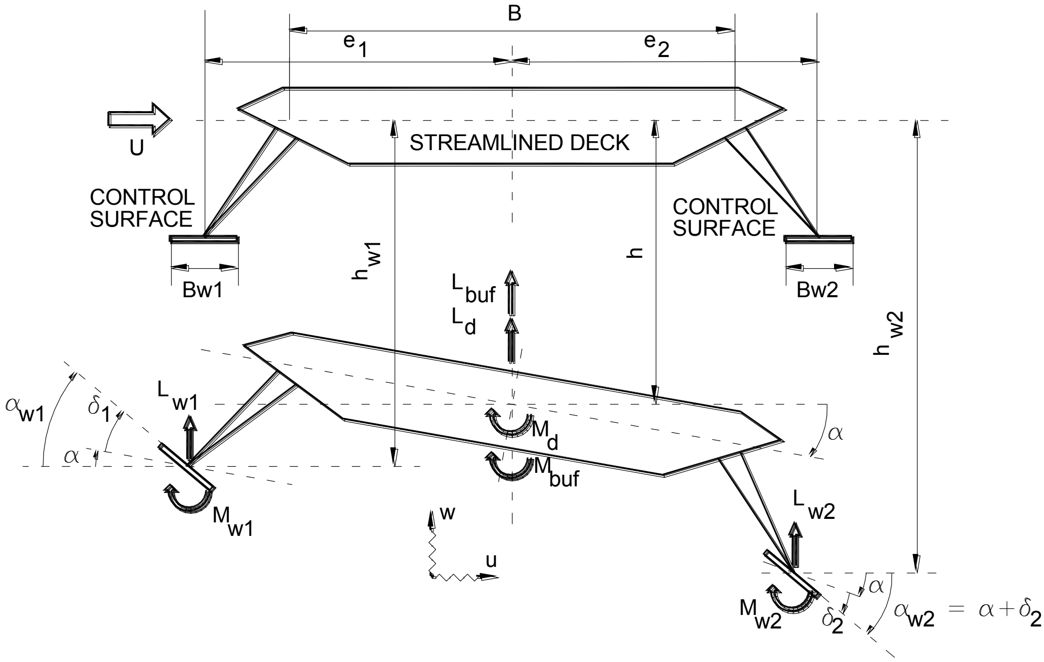

Figure 5-1 - Cross section of bridge deck with control surfaces

A section of a streamlined bridge deck with two additional surfaces attached below the leading and trailing edges of the bridge deck is shown in Figure 5-1. The coordinate system (the global system) has its origin at the center of gravity of the deck. The positive vertical displacement is downwards, while positive angles of rotation are clockwise. The equation of motion of the entire system is:

$$\mathbf{M}\ddot{\mathrm{q}}(\mathrm{t}) + \mathbf{C}\dot{\mathrm{q}}(\mathrm{t}) + \mathbf{K}\mathrm{q}(\mathrm{t}) = \mathbf{B}_{\mathrm{d}}\mathbf{F}_{\mathrm{d}}(\mathrm{t}) + \mathbf{B}_{\mathrm{w1}}\mathbf{F}_{\mathrm{w1}}(\mathrm{t}) + \mathbf{B}_{\mathrm{w2}}\mathbf{F}_{\mathrm{w2}}(\mathrm{t}) + \mathbf{B}_{\mathrm{b}}\mathbf{F}_{\mathrm{buf}}(\mathrm{t}) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(\mathrm{t}) \tag{5.1}$$

where $\mathbf{B}_{\mathrm{d}}$, $\mathbf{B}_{\mathrm{w1}}$, $\mathbf{B}_{\mathrm{w2}}$, $\mathbf{B}_{\mathrm{b}}$, $\mathbf{B}_{\mathrm{u}}$ are transfer matrices which transfer forces from a local to the global system of coordinates. The variables describing the motion are selected as:

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$$\mathbf{q} = \begin{bmatrix} h/B \\ \alpha \\ \delta_1 \\ \delta_2 \end{bmatrix} \tag{5.2}$$

The heaving motion is described by the dimensionless variable h/B , pitching is denoted by α, and δ_1, δ_2 denote rotations of the leading and trailing surfaces relative to the deck. As the width and thickness of the control surfaces are very small compared to the deck, all the terms describing the inertial coupling between deck and surfaces are ignored. The mass matrix **M** , the damping matrix **C** and the stiffness matrix **K** are diagonal, with dimensions [4x4].

The total aerodynamic forces are supposed to be a superposition of self-excited forces $\mathbf{F}_d(t)$ acting on the deck, $\mathbf{F}_{w1}(t)$, $\mathbf{F}_{w2}(t)$ acting on the control surfaces, and related to a mean speed U, as well as buffeting forces $\mathbf{F}_{buf}(t) = \begin{bmatrix} \mathbf{L}_{buf} \\ \mathbf{M}_{buf} \end{bmatrix}$ induced by along-wind and vertical-wind directions u and w respectively. This assumption is valid when the oscillations of the structure in each mode are small, according to Simiu and Scanlan [67].

The formulation of the self-excited forces for the deck-control surfaces system assumes that the flow around the deck or trailing surfaces is unaffected by the wake created by the upstream members. Thus, the description of unsteady aerodynamics of the deck and surfaces can be formulated independently.

Using Karpel's RFA minimum state formulation (see Chapter 3), the self-excited lift L_d and moment M_d acting on the deck are:

$$\mathbf{F}_d = \begin{bmatrix} L_d \\ M_d \end{bmatrix} = \mathbf{V}_f^d\left(\frac{B}{U}\mathbf{A}_1\dot{\mathbf{q}}_d + \mathbf{A}_0\mathbf{q}_d + \mathbf{D}_d[\mathbf{x}_a]_d\right)U^2 \tag{5.3}$$

where

$$\mathbf{q}_d = \begin{bmatrix} h/B \\ \alpha \end{bmatrix} \tag{5.4}$$

The aerodynamic states of the deck forces $[\mathbf{x}_a]_d$ are governed by the equations:

$$[\dot{\mathbf{x}}_a]_d = \begin{bmatrix} 0 & (U/B)\mathbf{E} & -(U/B)\mathbf{R} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_d \\ \mathbf{q}_d \\ [\mathbf{x}_a]_d \end{bmatrix} \tag{5.5}$$

By analogy, the aerodynamic lift L_{w1} , moment M_{w1} and the governing equation of the aerodynamic states of the leading surface can be formulated as:

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$$\mathbf{F}_{w1} = \begin{bmatrix} L_{w1} \\ M_{w1} \end{bmatrix} = \mathbf{V}_f^w \left(\frac{B_w}{U} \mathbf{A}_{1w} \dot{\mathbf{q}}_{w1} + \mathbf{A}_{0w} \mathbf{q}_{w1} + \mathbf{D}_w [\mathbf{x}_a]_{w1}\right) U^2 \tag{5.6}$$

$$[\dot{\mathbf{x}}_a]_{w1} = \begin{bmatrix} 0 & (U/B_w)\mathbf{E_w} & -(U/B_w)\mathbf{R_w} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_{w1} \\ \mathbf{q}_{w1} \\ [\mathbf{x}_a]_{w1} \end{bmatrix} \tag{5.7}$$

where

$$\mathbf{q}_{w1} = \begin{bmatrix} h_{w1}/B_w \\ \alpha_{w1} \end{bmatrix} \quad \text{and} \quad [\mathbf{x}_a]_{w1} = \begin{bmatrix} x_{aw1}^1 \\ x_{aw1}^2 \end{bmatrix} \tag{5.8}$$

The aerodynamic lift and moment and the governing equation of the aerodynamic states of the trailing surface can be also formulated as:

$$\mathbf{F}_{w2} = \begin{bmatrix} L_{w2} \\ M_{w2} \end{bmatrix} = \mathbf{V}_f^w \left(\frac{B_w}{U} \mathbf{A}_{1w} \dot{\mathbf{q}}_{w2} + \mathbf{A}_{0w} \mathbf{q}_{w2} + \mathbf{D}_w [\mathbf{x}_a]_{w2}\right) U^2 \tag{5.9}$$

$$[\dot{\mathbf{x}}_a]_{w2} = \begin{bmatrix} 0 & (U/B_w)\mathbf{E_w} & -(U/B_w)\mathbf{R_w} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_{w2} \\ \mathbf{q}_{w2} \\ [\mathbf{x}_a]_{w2} \end{bmatrix} \tag{5.10}$$

where

$$\mathbf{q}_{w2} = \begin{bmatrix} h_{w2}/B_w \\ \alpha_{w2} \end{bmatrix} \quad \text{and} \quad [\mathbf{x}_a]_{w2} = \begin{bmatrix} x_{aw2}^1 \\ x_{aw2}^2 \end{bmatrix} \tag{5.11}$$

The diagonal matrices $\mathbf{R}$ and $\mathbf{R}_w$ in the equations above contain lag terms $\lambda > 0$.

The width of the control surfaces is denoted B_w. The self-excited forces of the control surfaces are computed with respect to absolute vertical displacements h_{w1} and h_{w2} and absolute angles of rotation α_{w1} and α_{w2} The linearized relations between the absolute degrees of freedom of the surfaces $\mathbf{q}_{w1}$ and $\mathbf{q}_{w2}$ and the variables describing the motion are:

$$\alpha_{w1} = \alpha + \delta_1 \quad \text{and} \quad \alpha_{w2} = \alpha + \delta_2 \tag{5.12a}$$

$$h_{w1} = h - e_1\alpha \quad \text{and} \quad h_{w2} = h + e_2\alpha \tag{5.12b}$$

Where e_1 and e_2 are the distances from the surface hinge lines to the center of the deck.

Vector $\mathbf{F}_{buf}(t) = \begin{bmatrix} \mathbf{L}_{buf} \\ \mathbf{M}_{buf} \end{bmatrix}$ of size [2x1], represents the buffeting lift and moment acting on the bridge deck. Buffeting forces acting on the control surfaces are neglected. The formulation of the buffeting forces is simplified to quasi steady-state theory of the flat plate by Scanlan and Jones [63].

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5.2. Derivation of the equation of motion for the deck-wings system

From equations (5.12), the transport of the global vector of unknowns $\mathbf{q}$ to the local $\mathbf{q}_{\mathrm{d}}$, $\mathbf{q}_{\mathrm{w1}}$, $\mathbf{q}_{\mathrm{w2}}$ is given by:

$$\mathbf{q_d} = \mathbf{T}_{\mathrm{d}}\,\mathbf{q} \qquad \mathbf{q_{w1}} = \mathbf{T}_{\mathrm{w1}}\,\mathbf{q} \qquad \mathbf{q_{w2}} = \mathbf{T}_{\mathrm{w2}}\,\mathbf{q} \tag{5.13}$$

where the transfer matrices are:

$$\mathbf{T_d} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{5.14}$$

$$\mathbf{T}_{\mathrm{w1}} = \begin{bmatrix} \mathrm{B/B_w} & -\mathrm{e_1/B_w} & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \tag{5.15}$$

$$\mathbf{T}_{\mathrm{w2}} = \begin{bmatrix} \mathrm{B/B_w} & +\mathrm{e_2/B_w} & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{5.16}$$

The matrices expressing the aerodynamic forces in terms of

$$\mathbf{q} = \begin{bmatrix} \mathrm{h/B} \\ \alpha \\ \delta_1 \\ \delta_2 \end{bmatrix} \qquad \text{and} \qquad \mathbf{x}_{\mathrm{a}}{}' = \left[\, \mathrm{x}_{\mathrm{ad}}^1 \;\; \mathrm{x}_{\mathrm{ad}}^2 \;\; \mathrm{x}_{\mathrm{aw1}}^1 \;\; \mathrm{x}_{\mathrm{aw1}}^2 \;\; \mathrm{x}_{\mathrm{aw2}}^1 \;\; \mathrm{x}_{\mathrm{aw2}}^2 \right] \qquad \text{are}$$

$$\underbrace{\begin{bmatrix} \mathbf{F}_{\mathrm{d}} \\ \mathbf{F}_{\mathrm{w1}} \\ \mathbf{F}_{\mathrm{w2}} \end{bmatrix}}_{\mathbf{F}_{\mathrm{all}}} = \mathrm{U}^2 \underbrace{\begin{bmatrix} \frac{\mathrm{B}}{\mathrm{U}}\,\mathbf{V}_{\mathrm{f}}\mathbf{A}_1\mathbf{T}_{\mathrm{d}} \\ \frac{\mathrm{B_w}}{\mathrm{U}}\,\mathbf{V}_{\mathrm{fw}}\mathbf{A}_{1\mathrm{w}}\mathbf{T}_{\mathrm{w1}} \\ \frac{\mathrm{B_w}}{\mathrm{U}}\,\mathbf{V}_{\mathrm{fw}}\mathbf{A}_{1\mathrm{w}}\mathbf{T}_{\mathrm{w2}} \end{bmatrix}}_{\mathbf{A}_1^{\mathrm{a}}} \dot{\mathbf{q}} + \mathrm{U}^2 \underbrace{\begin{bmatrix} \mathbf{V}_{\mathrm{f}}\mathbf{A}_0\mathbf{T}_{\mathrm{d}} \\ \mathbf{V}_{\mathrm{fw}}\mathbf{A}_{0\mathrm{w}}\mathbf{T}_{\mathrm{w1}} \\ \mathbf{V}_{\mathrm{fw}}\mathbf{A}_{0\mathrm{w}}\mathbf{T}_{\mathrm{w2}} \end{bmatrix}}_{\mathbf{A}_0^{\mathrm{a}}} \mathbf{q}$$

$$+\mathrm{U}^2 \underbrace{\begin{bmatrix} \mathbf{V}_{\mathrm{f}}\mathbf{D}_{\mathrm{d}} & 0 & 0 \\ 0 & \mathbf{V}_{\mathrm{fw}}\mathbf{D}_{\mathrm{w}} & 0 \\ 0 & 0 & \mathbf{V}_{\mathrm{fw}}\mathbf{D}_{\mathrm{w}} \end{bmatrix}}_{\mathbf{D}^{\mathrm{a}}} \mathbf{x}_{\mathrm{a}} \tag{5.17}$$

And the matrices expressing the aerodynamic states are:

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}}_{\mathrm{ad}} \\ \dot{\mathbf{x}}_{\mathrm{aw1}} \\ \dot{\mathbf{x}}_{\mathrm{aw2}} \end{bmatrix}}_{\dot{\mathbf{x}}_{\mathrm{a}}} = \underbrace{\begin{bmatrix} \frac{\mathrm{U}}{\mathrm{B}}\mathrm{ET_d} \\ \frac{\mathrm{U}}{\mathrm{B_w}}\mathrm{E_w T_{w1}} \\ \frac{\mathrm{U}}{\mathrm{B_w}}\mathrm{E_w T_{w2}} \end{bmatrix}}_{\mathbf{G}_{\mathrm{s}}} \mathbf{q} - \underbrace{\frac{\mathrm{U}}{\mathrm{B}} \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mathrm{B}}{\mathrm{B_w}}\lambda_{\mathrm{w1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mathrm{B}}{\mathrm{B_w}}\lambda_{\mathrm{w2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mathrm{B}}{\mathrm{B_w}}\lambda_{\mathrm{w1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mathrm{B}}{\mathrm{B_w}}\lambda_{\mathrm{w2}} \end{bmatrix}}_{\mathbf{F}_{\mathrm{s}}} \tag{5.18}$$

The total aerodynamic forces $\mathbf{F}_{\mathrm{tot}}$ acting on the bridge deck and on the control surfaces must be transferred to the center of the deck by applying the transfer matrix **S** to (5.17):

$$\mathbf{F}_{\mathrm{tot}} = \mathbf{S}\,(\mathbf{A}_1^{\mathrm{a}}\dot{\mathbf{q}} + \mathbf{A}_0^{\mathrm{a}}\mathbf{q} + \mathbf{D}^{\mathrm{a}}\mathbf{x}_{\mathrm{a}}) \tag{5.19}$$

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -\mathrm{e}_1 & 1 & \mathrm{e}_2 & 1 \end{bmatrix} \tag{5.20}$$

Next, the relationship between the relative angular displacements of the control surfaces δ_1, δ_2 and the computer signals $\mathrm{u}_1, \mathrm{u}_2$, with dimensions of [Force x Length / Length] , representing the control input, must be established. This relationship is chosen in the form of a second order differential equation:

$$\ddot{\delta}_1 + 2\,\xi_{\delta_1}\,\omega_{\delta_1}\,\dot{\delta}_1 + (\omega_{\delta_1})^2\,\delta_1 = (\omega_{\delta_1})^2\,(\frac{\mathrm{u}_1}{\mathrm{I}_{\mathrm{w}}}) \tag{5.21}$$

$$\ddot{\delta}_2 + 2\,\xi_{\delta_2}\,\omega_{\delta_2}\,\dot{\delta}_2 + (\omega_{\delta_2})^2\,\delta_2 = (\omega_{\delta_2})^2\,(\frac{\mathrm{u}_2}{\mathrm{I}_{\mathrm{w}}}) \tag{5.22}$$

Expressed in matrix form, and considering $I_{\mathrm{w}} = 1$, equations (5.21-2) read:

$$\mathbf{M}_\delta\ddot{\boldsymbol{\delta}} + \mathbf{C}_\delta\dot{\boldsymbol{\delta}} + \mathbf{K}_\delta\boldsymbol{\delta} = \mathbf{K}_\delta\mathbf{u} \tag{5.23}$$

$$\mathbf{d} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \qquad \text{and} \qquad \mathbf{u} = \begin{bmatrix} \mathrm{u}_1 \\ \mathrm{u}_2 \end{bmatrix} \tag{5.24}$$

$\mathbf{M}_\delta$, $\mathbf{C}_\delta$, $\mathbf{K}_\delta$ are mass, damping and stiffness matrices. Equation (5.23) does not represent the actuator's dynamics, as it is designed to model the behavior of the control surfaces responding to the command of the computer signal $\mathbf{u}$. Following Wilde [95], the stiffness and damping ratio in (5.23) are assumed to be identical , taking values f_δ = 6 Hz and ξ_δ.=0.70.

The equation of motion of the bridge deck and wings system under active aerodynamic control reads:

$$\underbrace{\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{M}_\delta \end{bmatrix}}_{\mathbf{M}_{\mathrm{s}}}\ddot{\mathbf{q}} + \underbrace{\begin{bmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{C}_\delta \end{bmatrix}}_{\mathbf{C}_{\mathrm{s}}}\dot{\mathbf{q}} + \underbrace{\begin{bmatrix} \mathbf{K} & 0 \\ 0 & \mathbf{K}_\delta \end{bmatrix}}_{\mathbf{K}_{\mathrm{s}}}\mathbf{q} =$$

$$= \underbrace{\begin{bmatrix} \mathbf{SA}_1^{\mathrm{a}} \\ 0 \end{bmatrix}}_{\mathbf{A}_1^{\mathrm{s}}}\dot{\mathbf{q}} + \underbrace{\begin{bmatrix} \mathbf{SA}_0^{\mathrm{a}} \\ 0 \end{bmatrix}}_{\mathbf{A}_0^{\mathrm{s}}}\mathbf{q} + \underbrace{\begin{bmatrix} \mathbf{SD}^{\mathrm{a}} \\ 0 \end{bmatrix}}_{\mathbf{D}_{\mathrm{s}}}\mathbf{x}_{\mathrm{a}} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{K}_\delta \end{bmatrix}}_{\mathbf{B}_{\mathrm{s}}}\mathbf{u} + \underbrace{\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}_{\mathrm{s}}^{\mathrm{buf}}}\mathbf{F}_{\mathrm{buf}} \tag{5.25}$$

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Equations (5.25) representing the structural states and (5.18) representing the aerodynamic state are joined in a matrix representing the equations of motion of the deck and wing systems in the state space format:

$$\underbrace{\begin{bmatrix}\ddot{\mathbf{q}}\\ \dot{\mathbf{q}}\\ \dot{\mathbf{x}}_a\end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix}\mathbf{M}_s^{-1}(-\mathbf{C}_s + \mathbf{S}\,\mathbf{A}_1^s) & \mathbf{M}_s^{-1}(-\mathbf{K}_s + \mathbf{S}\,\mathbf{A}_0^s) & \mathbf{M}_s^{-1}\mathbf{D}_s\\ \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{G}_s & -\mathbf{F}_s\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}\dot{\mathbf{q}}\\ \mathbf{q}\\ \mathbf{x}_a\end{bmatrix}}_{\mathbf{x}} +$$

$$+\underbrace{\begin{bmatrix}\mathbf{M}_s^{-1}\mathbf{B}_s\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}}_{\mathbf{B}_u}\mathbf{u} + \underbrace{\begin{bmatrix}\mathbf{M}_s^{-1}\mathbf{B}_s\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}}_{\mathbf{B}_{buf}}\mathbf{F}_{buf} \tag{5.26}$$

The equation of motion (5.26) of the entire aeroelastic system is summarized in the state space form and reads:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(U)\mathbf{x}(t) + \mathbf{B}_b\mathbf{F}_{buf}(t) + \mathbf{B}_u\mathbf{u}(t) \tag{5.27}$$

where $\mathbf{u}(t) = \begin{bmatrix}u_1\ (t)\\ u_2\ (t)\end{bmatrix}$ is the control vector and $\mathbf{x}(t)$ is the state vector:

$$\mathbf{x}(t) = \begin{bmatrix}\dot{\mathbf{q}}\\ \mathbf{q}\\ [\mathbf{x}_a]_d\\ [\mathbf{x}_a]_{w1}\\ [\mathbf{x}_a]_{w2}\end{bmatrix} \tag{5.28}$$

$$\mathbf{x}(t)' = \Big[\underbrace{\tfrac{\dot{h}}{B}\ \ \dot{\alpha}\ \ \dot{\delta}_1\ \ \dot{\delta}_2\ \ \tfrac{h}{B}\ \ \alpha\ \ \delta_1\ \ \delta_2}_{\text{8 structural states}}\ \ \underbrace{x_{ad}^1\ \ x_{ad}^2\ \ x_{aw1}^1\ \ x_{aw1}^2\ \ x_{aw2}^1\ \ x_{aw2}^2}_{\text{6 aerodynamic states (2 lag terms each, for the deck and control surfaces).}}\Big] \tag{5.29}$$

where $\mathbf{x}(t)'$ = transpose of $\mathbf{x}(t)$.

The state vector is composed by eight structural states and three pairs of aerodynamic states. The latter correspond to two lag terms per rational function representing the unsteady aerodynamics of the deck and the control surfaces.

Equation (5.26) models the dynamics of the entire aeroservoelastic system. The elements of state matrix **A** [14x14] are dependent on mean wind speed U.

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For a fixed wind speed U the equations of motion describe a time-invariant system.

5.3. The open loop system

The open loop system corresponds physically to the system composed by the bridge deck and the wings at rest, i.e., in a stationary condition. The open loop system is described by equation (5.26), for the control input $\mathbf{u} = \mathbf{0}$. Considering the buffeting forces also zero, the open loop system reads:

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \dot{\mathbf{q}} \\ \dot{\mathbf{x}}_a \end{bmatrix} = \begin{bmatrix} \mathbf{M}_s^{-1}(-\mathbf{C}_s + \mathbf{S}\,\mathbf{A}_1^s) & \mathbf{M}_s^{-1}(-\mathbf{K}_s + \mathbf{S}\,\mathbf{A}_0^s) & \mathbf{M}_s^{-1}\mathbf{D}_s \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_s & -\mathbf{F}_s \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \\ \mathbf{x}_a \end{bmatrix}$$

Performing a complex eigenvalue analysis of the state matrix of the bridge deck and wings system for increasing wind velocities, similar to the one already done for the bridge deck alone, the critical velocity of 10.663 m/s is found. This is approximately 5% higher than the critical velocity of the bridge deck alone.

Deck characteristics	
Bd = 0.2927	deck width in m
m = 0.1910	deck mass in kgf.s^2 / m^2
I_a = 1.9345e-03	deck rotational mass moment of inertia in kgf.s^2.
δ_h = delta_h = 0.007	damping log. decrement of h mode $\delta_h = \xi_h / (2*\pi)$
delta_a = 0.006	damping log. decrement of a mode $\delta_a = \xi_a / (2*\pi)$
fre_h = 1.254;	natural frequency of h mode in cycles per sec
fre_a = 3.988	natural frequency of α mode in cycles per sec
Control wings (winglets) characteristics	
Bw1 = 0.1*Bd; Bw2 = 0.1*Bd	flap width of wings 1,2 in m
e1,2 = 0.5*Bd;	location of C.G. of flaps 1,2 from center of deck in m
m11 = 1; m22 = 1	actuators 1, 2 masses in kg/m
ksi_1 = 0.7, ksi_2 = 0.7	actuators 1, 2 damping ratios
ome_1 = 6*2*pi; ome_2 = 6.01*2*pi	frequencies of actuators 1,2 rad/s

Table 5-1 - Geometric properties of the bridge deck and wings

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The geometric and dynamical properties of the bridge deck and wings are shown in Table 5-1. The state matrix **A** for the critical velocity of 10.663 m/s is shown in Table 5-2. Eigenvalues, damping and frequencies of **A** for 10.663 m/s are shown in Table 5-3.

State Matrix A for U = 10.663 m/s.

A

	1	2	3	4	5	6	7
1	-4.325	-2.4414	-0.01683	-0.0168	-139.13	-164.8	-16.67
2	7.0816	-3.4749	0.067968	-0.0744	111.59	-406.15	74.027
3	0	0	-52.779	0	0	0	-1421.2
4	0	0	0	-52.867	0	0	0
5	1	0	0	0	0	0	0
6	0	1	0	0	0	0	0
7	0	0	1	0	0	0	0
8	0	0	0	1	0	0	0
9	0	0	0	0	-0.5271	2.8479	0
10	0	0	0	0	-8.3949	9.4553	0
11	0	0	0	0	-0.1037	0.4878	0.4359
12	0	0	0	0	-127.21	115.45	51.848
13	0	0	0	0	-0.1037	0.3841	0
14	0	0	0	0	-127.21	-11.758	0

A =

	8	9	10	11	12	13	14
1	-16.67	-129.07	-121.55	-25.138	-7.9793	-25.138	-7.9793
2	-66.98	268.33	271.93	111.63	35.435	-101.01	-32.061
3	0	0	0	0	0	0	0
4	-1426	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0
9	0	-6.9649	0	0	0	0	0
10	0	0	-27.239	0	0	0	0
11	0	0	0	-12.293	0	0	0
12	0	0	0	0	-72.382	0	0
13	0.4359	0	0	0	0	-12.293	0
14	51.848	0	0	0	0	0	-72.382

Table 5-2 - State matrix for 10.663 m/s.

Eigenvalue	Damping	Freq. (rad/s)
4.96e-004 + 1.72e+001i	**-2.88E-05**	**1.72E+01**
4.96e-004 - 1.72e+001i	-2.88E-05	1.72E+01
-1.45E+00	1.00E+00	1.45E+00
-1.08e+001 + 9.49e+000i	**7.50E-01**	**1.43E+01**
-1.08e+001 - 9.49e+000i	7.50E-01	1.43E+01
-1.21E+01	1.00E+00	1.21E+01
-1.23E+01	1.00E+00	1.23E+01
-2.04E+01	1.00E+00	2.04E+01
-2.64e+001 + 2.69e+001i	7.00E-01	3.77E+01
-2.64e+001 - 2.69e+001i	7.00E-01	3.77E+01
-2.64e+001 + 2.70e+001i	7.00E-01	3.78E+01
-2.64e+001 - 2.70e+001i	7.00E-01	3.78E+01
-7.15E+01	1.00E+00	7.15E+01
-7.20E+01	1.00E+00	7.20E+01

Table 5-3 - Eigenvalues, damping and frequencies of **A** for 10.663 m/s

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The diagonal matrix of the 14 eigenvalues, and the eigenvectors corresponding to the eigenvalues (3,3) and (6,6) are shown in Table 5-3:

Diagonal matrix of the Eigenvalues		State vector	Eigenvector corresponding to Eigenvalue (3,3)	Eigenvector corresponding to Eigenvalue (6,6)
(1,1)	-72.015	d[(h/B)] / dt	0.60323 + 0.29272i	-0.81137
(2,2)	-71.499	dα / dt	0.73541	0.28682 - 0.46242i
(3,3)	0.00049599 + 17.2i	d(δ1) / dt	4.27e-017 -8.1657e-017i	1.0755e-016 -8.1127e-017i
(4,4)	0.00049599 - 17.2i	d(δ2) / dt	9.874e-018 +6.1572e-017i	-1.4743e-016 +1.2535e-017i
(5,5)	-1.4484	**h/B**	0.01702 - 0.035071i	0.042429 + 0.037406i
(6,6)	-10.76 + 9.486i	**α**	1.233e-006 - 0.042757i	-0.036317 + 0.010958i
(7,7)	-10.76 - 9.486i	δ1	-3.9869e-018 -4.4916e-018i	-1.0536e-017 +6.6632e-018i
(8,8)	-20.444	δ2	4.2569e-018 +3.7932e-018i	5.1622e-018 -4.5513e-018i
(9,9)	-12.146	x1	-0.0053401 - 0.0016412i	0.0056175 + 0.011013i
(10,10)	-12.285	x2	-0.0055704 - 0.00051563i	-0.037407 + 0.0087644i
(11,11)	-26.433 + 26.967i	x_{w11}	-0.00071113 - 0.00040577i	-0.00021644 + 0.002296i
(12,12)	-26.433 - 26.967i	x_{w12}	-0.029787 + 0.00051719i	-0.16056 - 0.031974i
(13,13)	-26.389 + 26.923i	x_{w21}	-0.00054061 - 0.00028388i	-0.0002706 + 0.0018904i
(14,14)	-26.389 - 26.923i	x_{w22}	-0.012887 + 0.071646i	-0.090721 - 0.065347i

Table 5-4 - Diagonal matrix of the 14 eigenvalues, and eigenvectors corresponding to the eigenvalues (3,3) and (6,6).

The phase angle φ reads :

$\Phi = 90^o - \arctan(0.035071 / 0.01702) = 90^o - 64.11^o = 25.887^0$.

The amplitude ratio reads:

$$a = 0.042757 / (0.01702^2 + 0.035071^2)^{1/2} = 1.0968.$$

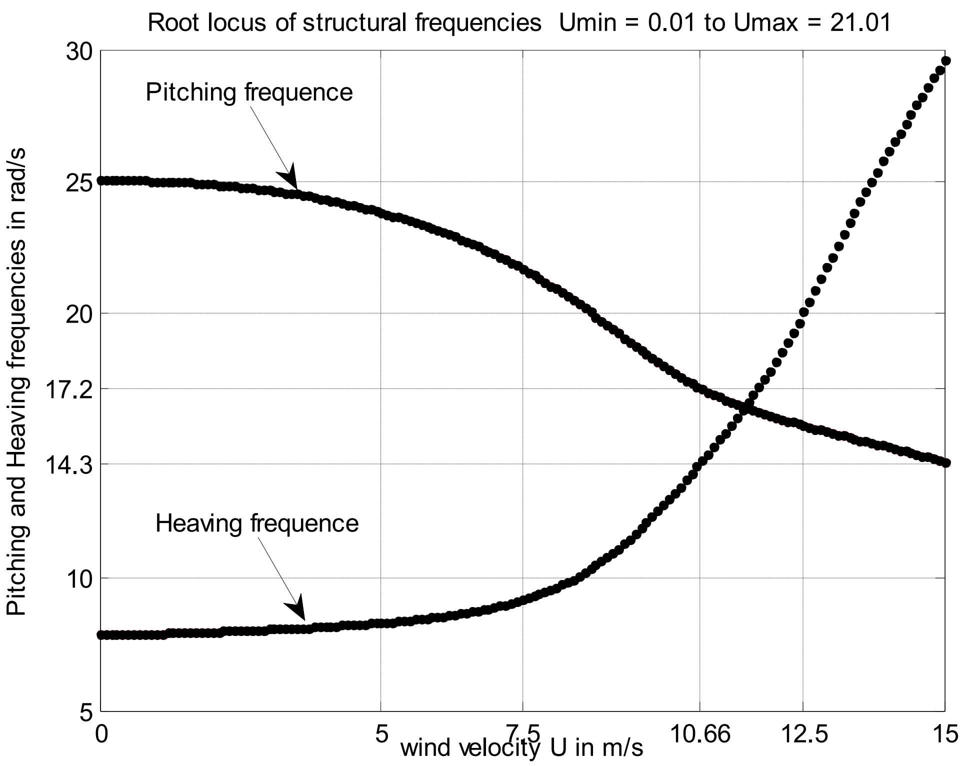

Figure 5-2 - Pitching and heaving frequencies versus wind velocity.

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The pitching and heaving frequencies corresponding to the critical velocity of 10.663 m/s are 17.2 rad/s and 14.3 rad/s, as Figure 5-2 shows.

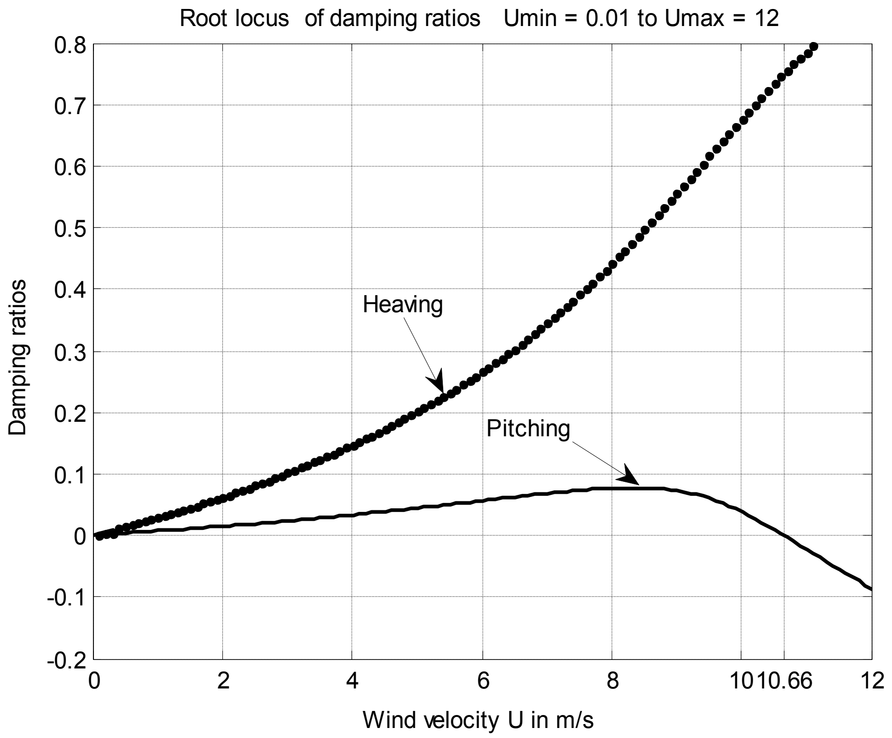

Figure 5-3 - Damping ratios versus wind velocity.

The critical velocity is found when the pitching damping ratio reaches zero, as Figure 5-3 shows.

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