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Gilberto de Barros Rodrigues Lopes

Aerodynamic Control of Flutter of Suspension Bridges

Tese de Doutorado

Thesis presented to the Postgraduate Program in Civil Engineering of the Departamento de Engenharia Civil, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Engenharia Civil.

Advisor: Prof. Raul Rosas e Silva

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Fluvium ponte jungere.

To throw a bridge over a river.

Lançar uma ponte sobre um rio.

Lanzar un puente sobre un rio.

At slaa en bro over en flod.

Eine Brücke über einen Fluß zu schlagen.

川に橋を架ける.

Abstract

Lopes, Gilberto de Barros Rodrigues; Silva, Raul Rosas e. **Aerodynamic Control of Flutter of Suspension Bridges**. Rio de Janeiro, 2010. 220p. Doctor Thesis - Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro.

Long span bridges, with main spans beyond 2.000 m become highly sensitive to wind action, particularly to flutter. An active aerodynamic control method of suppressing flutter of very long span bridges is studied in this thesis. Analytical design techniques for active control of the aeroelastic system consisting of the bridge deck and two control surfaces are presented. These techniques are based on a rational approximation of the unsteady aerodynamic loads in the entire Laplace domain, which yieds matrix equations of motion with constant coefficientes. The first part of this thesis is dedicated to the matrix formulation of the rational functions known as "Minimum State" and to applications to aerodynamic data obtained experimentally for various types of bridge profiles. The precision of the approximations iscalculated, and plots of the approximation functions compared to the available tabular data are drawn. Next, the state-space equations of motion describing the aeroelastic behaviour of a section of a bridge deck is presented. Given the dynamic data of a bridge structure (mass, rotational mass moment of inertia, natural frequencies, stiffness and damping ratios), and assuming that a geometric similitude exists between the profiles of the full-scale bridge deck and the sectional model from which the frequency dependent aerodynamic data was extracted, it is possible to calculate the critical velocity of that particular bridge. This part of the thesis shows that it is possible to build up a catalog of several profiles, characterized by frequency dependent aerodynamic data and the corresponding rational functions. The second part is dedicated to the formulation of the state-space equations of motion describing the aeroelastic behaviour of the entire system consisting of the bridge deck and control surfaces. The resulting equation includes new aerodynamic states which model the air flow influence on the moving deck. The equation of motion is a function of the mean velocity of the incoming wind. The dependence of the equation of motion on the wind velocity motivated the application of a constant and a variable-gain feedback concept to the problem of flutter suppressing, which are presented separatelly. The output variable-gain approach is formulated in terms of minimizing a performance index dimensionally proportional to the sum of the work done by the rotating control surfaces and the kinetic energy of the heaving velocity. A sistematic method to determine the matrix of variable control gains is shown in detail, as applied to the hypothethical case of Gibraltar bridge. Application of the variable-gain feedback concept was found to be very effective in suppressing flutter of the bridge deck. Different geometric and dynamic characteristics can be introduced in the MATLAB programs included in this work, in order to obtain the critical velocities of a bridge deck alone, a bridge deck with stationary wings and a bridge with moving wings activelly controled.

Keywords

Aerodynamics; Aeroelasticity; Approximations with rational functions; Active control of flutter; Control surfaces; Long-span suspension bridges; Optimal feedback control; Critical velocity.

Resumo

Lopes, Gilberto de Barros Rodrigues; Silva, Raul Rosas e. **Controle Aerodinâmico de Tabuleiros de Pontes com Uso de Superfícies Ativas.** Rio de Janeiro, 2010. 220p. Tese de Doutorado - Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro.

Pontes com vãos superiores a 2.000 m tornam-se muito sensíveis à ação do vento, particularmente ao drapejamento. Nesta tese é estudado um método para a supressão do drapejamento em pontes de grandes vãos através de um controle aerodinâmico ativo. Apresentam-se técnicas analíticas de projeto para o controle ativo do sistema aero elástico constituído pelo tabuleiro e por duas superfícies de controle. Estas técnicas são baseadas em aproximações racionais das cargas aerodinâmicas não permanentes (ou auto-excitadas) no domínio Laplaciano, no qual as equações de movimento são representadas por equações matriciais de coeficientes constantes. A primeira parte da tese é dedicada à formulação matricial das funções racionais conhecida como "Minimum State", assim como a aplicações a dados aerodinâmicos obtidos experimentalmente para vários tipos de seções transversais de pontes. A precisão das aproximações é calculada. Desenhos dos derivativos aerodinâmicos, dados sob forma de tabelas, e das respectivas aproximações, são elaborados para fins de comparação. Em seguida, são apresentadas as equações em espaço de estado descrevendo o comportamento aeroelástico de uma seção transversal de ponte. A partir dos dados geométricos e características dinâmicas de uma determinada ponte, (massa, momento de inertia polar, frequências naturais e fatores de amortecimento), e assumindo a semelhança geométrica entre as seções transversais da ponte em verdadeira grandeza e do modelo em escala do qual os derivativos aerodinâmicos foram extraídos, é possível calcular a velocidade crítica desta ponte, utilizando os programas em linguagem MATLAB apresentados no corpo deste trabalho.

Esta parte da tese mostra ser possível construir um catálogo com vários perfis de pontes, caracterizados por derivativos aerodinâmicos variáveis em função de frequências reduzidas adimensionais, e das funções racionais correspondentes. A segunda parte é dedicada à fomulação das equações de movimento em espaço de estado, descrevendo o comportamento aeroelástico do sistema “tabuleiro - superfícies de controle”. As equações resultantes são ampliadas com novos estados aerodinâmicos responsáveis pela modelagem da influência do fluxo de ar sobre o tabuleiro e sobre as superfícies de controle em movimento. As equações de movimento são função da velocidade média do vento incidente. A dependência da equação de movimento à velocidade do vento motivou a aplicação dos conceitos de realimentação de ganhos, constante e variável, ao problema da supressão do drapejamento, os quais são apresentados separadamente em dois capítulos.O enfoque de ganho variável de saída é formulado em termos de minimização de um índice de desempenho dimensionalmente proporcional à soma do trabalho realizado pelas superfícies de controle e da energia cinética proporcional à velocidade vertical do tabuleiro. Apresenta-se também em detalhe um método sistemático para determinar a matriz de controle de ganhos variável, aplicada ao caso hipotético da ponte de Gibraltar. Neste caso, o conceito de realimentação de ganhos variável mostrou-se muito efetivo em suprimir o drapejamento do tabuleiro da ponte. Diferentes características geométricas e dinâmicas de outras pontes podem ser introduzidas nos programas MATLAB apresentados no Apêndice, para obtenção da velocidade crítica nos casos de tabuleiros isolados, tabuleiros com asas estacionárias e tabuleiros com asas giratórias ativamente controladas, para supressão do drapejamento do tabuleiro.

Palavras-chave

Aerodinâmica; Aeroelasticidade; Aproximações com funções racionais; Controle ativo de drapejamento; Superfícies de controle; Pontes suspensas de grandes vãos; Controle ótimo de realimentação; Velocidade Crítica.

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Notation

$\mathbf{A}(\mathrm{U})$	state-space coefficient matrix.
$\mathbf{A}_c$	state-space closed-loop system matrix.
$\mathbf{A}_0,\ \mathbf{A}_1,\ \mathbf{D},\ \mathbf{E},\ \mathbf{R}$	coefficient matrices of rational function approximations.
$a_i,\ b_i$	upper and lower limits of parameter p_i
$B,\quad b = B/2$	bridge deck width, half width.
$\mathbf{B}_d,\ \mathbf{B}_{w1},\ \mathbf{B}_{w2},\ \mathbf{B}_u,\ \mathbf{B}_{buf}$	force distribution matrices.
$B_{w1},\ B_{w2}$	width of leading and trailing surfaces.
$\mathbf{C}$	damping matrix / output matrix.
$C(k)$	Theodorsen function.
$F(k),\ G(k)$	real and imaginary parts of the Theodorsen function.
$c(\bullet)$	damping coefficient associated with $(\bullet)$ coordinate.
$E(\bullet)$	expectation.
$e_1,\ e_2$	distance from control surface hinge lines to center of the deck.
$\mathbf{F}_{buf}$	buffeting forces acting on deck.
$\mathbf{F}_d,\ \mathbf{F}_{w1}, \mathbf{F}_{w2}$	aerodynamic forces acting on deck, leading and trailing surfaces.
$\mathbf{H}_i^*,\ \mathbf{A}_i^*$	flutter derivatives.
h	vertical displacement = heaving displacement.
$\mathbf{I}$	identity matrix.
$\mathbf{J}$	performance matrix.
$K,\ k$	reduced frequency.
$\mathbf{K}$	stiffness matrix / feedback gain matrix.
$\mathbf{K_c}$	coefficient matrices of variable gain.
$k(\bullet)$	stiffness coefficient associated with $(\bullet)$ coordinate.

$\mathbf{L}$, $\mathbf{M}$	aerodynamic forces per unit length.
$\mathbf{L}$, $\mathbf{P}$	auxiliary matrices to solve Riccatti's equation.
$\mathscr{L}(\bullet)$	Laplace operator.
L_i, U_i	low and upper limits of lag coefficients.
$\mathbf{M}$	mass matrix.
m , $I_{\boldsymbol{\alpha}}$	mass and polar mass moment of inertia per unit length.
p	dimensionless Laplace variable.
$\mathbf{p}$	vector of size q describing system operating point.
$\mathbf{Q}(p)$, $\widehat{\mathbf{Q}}(p)$	aerodynamic matrix , approximate aerodynamic matrix.
$\mathbf{q}(\bullet)$	vector of unknowns of system $(\bullet)$
s	Laplace variable.
$tr(\bullet)$	trace of a matrix.
U	mean wind speed.
$\mathbf{u}$	control vector.
$\mathbf{V_f}$	matrix of wind forces.
w_{ij}	weighing factors.
$\mathbf{X}$	state-space vector
$\mathbf{Y}$	output vector.
$\mathbf{x_a}$	vector of aerodynamic states.
α, δ_1, δ_2	rotations of the deck, leading and trailing surfaces.
ε_{ij}	approximation error of RFA.
λ_i	lag coefficients.
ρ	air density.
ρ_r, ρ_q, $f(p)$	weighing functions.
$\mathbf{\Omega}$	transition matrix.