José Teixeira Cal Neto

Numerical Analysis of Ambrosetti-Prodi Type Operators

Tese de Doutorado

Thesis presented to the Postgraduate Program in Mathematics of the Departamento de Matemática do Centro Técnico Científico da PUC-Rio as partial fulfillment of the requirements for the degree of Doutor

Advisor: Prof. Carlos Tomei

Rio de Janeiro
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To Celina and Bruna.

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Abstract

Cal Neto, José T.; Tomei, Carlos. **Numerical Analysis of Ambrosetti-Prodi Type Operators**. Rio de Janeiro, 2010. 49p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Berger and Podolak obtained a geometric interpretation of the seminal result of Ambrosetti and Prodi regarding the behavior of solutions of certain semilinear elliptic partial differential equations. We consider extensions of such interpretation to develop a stable numerical algorithm that solves the equations.

Keywords

Nonlinear Functional Analysis; Semilinear elliptic PDE's; Numerical Analysis.

Resumo

Cal Neto, José T.; Tomei, Carlos. **Análise Numérica de Operadores do Tipo Ambrosetti-Prodi**. Rio de Janeiro, 2010. 49p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Berger e Podolak apresentaram uma interpretação geométrica do resultado seminal de Ambrosetti e Prodi sobre o comportamento das soluções de certas equações diferenciais parciais elípticas semi-lineares. Consideram-se extensões deste ponto de vista, a partir das quais se desenvolve um algoritmo numérico para resolver as equações.

Palavras-chave

Análise Funcional Não Linear; EDP's elípticas semi-lineares; Análise Numérica.

Contents

1	Introduction	10
2	Basic Theory	14
2.1	Notation	14
2.2	Differentiability of Nemytskii Operators	15
3	The Problem	17
3.1	Some basic geometry	17
3.2	The Nonlinear Operator	19
4	Numerical Analysis	26
4.1	An Inversion Algorithm	26
4.2	Finite Elements and Linear Algebra	29
5	Numerical Examples	33
5.1	Moving Horizontally	34
5.2	Moving along a fiber	35
5.3	Interacting With Two Eigenvalues	40
	Bibliography	43
A	A Quick Survey of the Finite Element Method	46
A.1	Variational Formulation	46
A.2	Triangulation and $\mathcal{P}_1$ Elements	47
A.3	Error Bounds, Convergence	48

Alea Jacta Est

Gaius Julius Cæsar, *Crossing the Rubicon River.*