Arturo Elí Cubas Rodriguez

**Toward GPU-Based Ground
Structures for Large Scale Topology
Optimization**

DISSERTAÇÃO DE MESTRADO

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Arturo Elí Cubas Rodriguez

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Large Scale Topology Optimization**

Dissertação de Mestrado

Thesis presented to the Programa de Pós-Graduação em Engenharia Mecânica of the Departamento de Engenharia Mecânica do Centro Técnico Científico da PUC-Rio, as partial fulfillment of the requirements for the degree of Mestre em Engenharia Mecânica.

Advisor: Prof. Ivan Fábio Mota de Menezes

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Prof. Ivan Fábio Mota de Menezes

Advisor

Departamento de Engenharia Mecânica — PUC-Rio

Prof. Waldemar Celes Filho

Departamento de Informática – PUC-Rio

Prof. Anderson Pereira

Tecgraf – PUC-Rio

Profa. Sylvia Regina Mesquita de Almeida

Universidade Federal de Goiás

Prof. José Eugenio Leal

Coordinator of the Centro Técnico Científico — PUC-Rio

Rio de Janeiro — October 9th, 2014

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Arturo Elí Cubas Rodriguez

The author graduated from Universidad Nacional de Ingeniería – UNI, Lima, Peru in Mechatronic Engineering.

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Abstract

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. **Toward GPU-Based Ground Structures for Large Scale Topology Optimization.** Rio de Janeiro, 2014. 82p. MSc. Thesis
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Topology optimization aims to find the most efficient material distribution in a specified domain without violating user-defined design constraints. When applied to continuum structures, topology optimization is usually performed by means of the well-known density methods. In this work we focus on the application of its discrete formulation where a given domain is discretized into a *ground structure*, i.e., a finite spatial distribution of nodes connected using truss members. The ground structure method provides an approximation to optimal Michell-type structures, composed of an infinite number of members, by using a reduced number of truss members. The optimal least weight truss for a single load case, under linear elastic conditions, subjected to stress constraints can be posed as a linear programming problem. The aim of this work is to provide a scalable implementation for the optimization of least weight trusses embedded in any domain geometry. The method removes unnecessary members from a truss that has a user-defined degree of connectivity while keeping the nodal locations fixed. We discuss in detail the scalable implementation of the ground structure method using an efficient and robust interior point algorithm within a parallel computing environment (involving Graphics Processing Units or GPUs). The capabilities of the proposed implementation is illustrated by means of large scale applications on practical problems with millions of members in both 2D and 3D structures.

Keywords

Topological optimization; High performance computing; GPU computing; Finite element method; Mathematical programming; Linear equations solvers.

Resumo

Cubas Rodriguez, Arturo Elí; Menezes, Ivan Fábio Mota de. **Otimização Topológica de Estruturas de Grande Porte utilizando o Método de Ground Structures em GPU**. Rio de Janeiro, 2014. 82p. Dissertação de Mestrado — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

A otimização topológica tem como objetivo encontrar a distribuição mais eficiente de material em um domínio especificado sem violar as restrições de projeto definidas pelo usuário. Quando aplicada a estruturas contínuas, a otimização topológica é geralmente realizada por meio de métodos de densidade, conhecidos na literatura técnica. Neste trabalho, daremos ênfase à aplicação de sua formulação discreta, na qual um determinado domínio é discretizado na forma de uma *estrutura base*, ou seja, uma distribuição espacial finita de nós conectados entre si por meio de barras de treliça. O método de estrutura base fornece uma aproximação para as estruturas de Michell, que são compostas por um número infinito de barras, por meio de um número reduzido de elementos de treliça. O problema de determinar a estrutura final com peso mínimo, para um único caso de carregamento, considerando um comportamento linear elástico do material e restrições de tensão, pode ser formulado como um problema de programação linear. O objetivo deste trabalho é fornecer uma implementação escalável para o problema de otimização de treliças com peso mínimo, considerando domínios com geometrias arbitrárias. O método remove os elementos que são desnecessários, partindo de uma treliça cujo grau de conectividade é definido pelo usuário, mantendo-se fixos os pontos nodais. Propomos uma implementação escalável do método de estrutura base, utilizando um algoritmo de pontos interiores eficiente e robusto, em um ambiente de computação paralela (envolvendo unidades de processamento gráfico ou GPUs). Os resultados apresentados, em estruturas bi e tridimensionais com milhões de barras, ilustram a viabilidade e a eficiência computacional da implementação proposta.

Palavras-chave

Otimização Topológica; Computação de alto desempenho; Computação em GPU; Método dos elementos finitos; Otimização linear; Resolvedores de sistemas.

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List of Symbols

Upper-case Roman

DIM	Dimension of the problem, can be two or three
$NODE$	Matrix of nodes coordinates
BAR	Matrix of bars nodes
V	Truss volume
$NODE$	Matrix of nodes coordinates
$ELEM$	Matrix of elements in the base mesh
$LOAD$	Matrix of forces in the base mesh
$SUPP$	Matrix of supports in the base mesh
N_e	Number of elements in the base mesh
N_n	Number of nodes in the base mesh
N_f	Number of nodes with fixities
N_l	Number of nodes with loads
N_b	Number of bars
N_x, N_y, N_z	Domain number of nodes in x, y, z
L_x, L_y, L_z	Domain lengths in x, y, z
N_{dof}	Number of degree of freedom
Lvl	Specified ground structure connectivity level
$ColTol$	Tolerance in the collinearity check
$Cutoff$	Cross-section cutoff value for plotting
E	Young's module
$\mathbf{B}$	Force equilibrium (geometrical) matrix
$\mathbf{K}$	Stiffness matrix
$\mathbf{M}$	Matrix of normal equations
$\mathbf{X}, \mathbf{Z}$	Diagonal matrices
$\mathbf{J}$	Jacobian
$\mathbf{M}$	Diagonal matrix from interior point method
$\mathbf{B}_r$	Column of $\mathbf{B}$
$\mathbf{K}_r$	Bar stiffness in global coordinates

Lower-case Roman

r	Bar index
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i	First node of a bar
j	Second node of a bar
l	Bar length
a	Bar transversal area
t	Directional cosines vector
l	Bar length vector
n	Internal (axial) forces vector
s, z	Slack variables
x	Primal variable
y	Dual variable
x^*, y^*, z^*	Primal dual solution
Δx	x search direction
Δy	y search direction
Δz	z search direction
r_x	Residue on x variable
f	Force vector
u	Node displacement vector
k_r	Bar stiffness in local coordinates

Lower-case Greek

σ_T	Stress limit in tension
σ_C	Stress limit in compression
σ_0	Stress limits mean
α	Search direction factor
μ	Duality measure
σ	Centering parameter
τ	Central path parameter
δ	Bar elongation
ϵ	Bar strain

Abbreviations

TO	Topological Optimization
IPM	Interior Point Method
FEM	Finite Element Method
GPU	Graphics Processing Unit

GPGPU	General-Purpose Computing on Graphics Processing Units
PC	Predictor Corrector algorithm
LS	Long Step algorithm