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Apêndice A
Cálculo Analítico das Integrais *INT*₁(*x*) a *INT*₁₂(*x*)

Neste apêndice serão apresentados os resultados das integrais envolvendo funções trigonométricas, que surgem da determinação dos elementos a_{ij} e b_{ij}, indicadas no Capítulo 3.

As funções *FSS*, *FCC*, *FSX* e *FCX*, utilizadas nas soluções das integrais $INT_1(x)$ a $INT_{12}(x)$, são dadas por:

$$FSS(\alpha,\beta,\delta)=\int_0^{\delta} sen(\alpha y)\, sen(\beta y)\, dy \tag{A.1a}$$

$$FCC(\alpha,\beta,\delta)=\int_0^{\delta} cos(\alpha y)\, cos(\beta y)\, dy \tag{A.1b}$$

$$FSX(\alpha,\beta,\delta)=\int_0^{\delta} y^{\alpha}\, sen(\beta y)\, dy \tag{A.1c}$$

$$FCX(\alpha,\beta,\delta)=\int_0^{\delta} y^{\alpha}\, cos(\beta y)\, dy \tag{A.1d}$$

Abaixo seguem os resultados das integrais $INT_1(x)$ a $INT_{12}(x)$, onde:

$$f_b(x)=b_0\left[1-\left(\frac{x}{a_0}\right)^{\gamma}\right]^{1/\gamma}.$$

$$INT_1(x)=\int_0^{f_b(x)} cos\left(\frac{n_1\pi y}{2b}\right) cos\left(\frac{n_2\pi y}{2b}\right) dy$$

- Caso $n_1=n_2=0$

$$INT_1(x)=1 \tag{A.2a}$$

- Caso $n_1 = n_2 \neq 0$

$$INT_1(x) = \frac{x}{2} + \frac{sen\left[2\left(\frac{n_1 \pi y}{2b}\right) f_b(x)\right]}{4\left(\frac{n_1 \pi y}{2b}\right)} \tag{A.2b}$$

- Caso $n_1 \neq n_2$

$$INT_1(x) = \frac{sen\left[\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right) f_b(x)\right]}{2\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right)} + \frac{sen\left[\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right) f_b(x)\right]}{2\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right)} \tag{A.2c}$$

$$INT_2(x) = \int_0^{f_b(x)} sen\left(\frac{n_1 \pi y}{2b}\right) sen\left(\frac{n_2 \pi y}{2b}\right) dy$$

- Caso $n_1 = n_2 = 0$

$$INT_2(x) = 0 \tag{A.3a}$$

- Caso $n_1 = n_2 \neq 0$

$$INT_2(x) = \frac{x}{2} - \frac{sen\left[2\left(\frac{n_1 \pi y}{2b}\right) f_b(x)\right]}{4\left(\frac{n_1 \pi y}{2b}\right)} \tag{A.3b}$$

- Caso $n_1 \neq n_2$

$$INT_2(x) = \frac{sen\left[\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right) f_b(x)\right]}{2\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right)} - \frac{sen\left[\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right) f_b(x)\right]}{2\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right)} \tag{A.3c}$$

$$INT_3(x) = \int_0^{f_b(x)} \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 sen\left(\frac{n_1 \pi y}{2b}\right) sen\left(\frac{n_2 \pi y}{2b}\right) dy$$

- Caso $n_1 = n_2$

$$INT_3(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 FSS\left(\frac{n_1 \pi y}{2b}, \frac{n_2 \pi y}{2b}, f_b(x)\right) -$$

$$-\frac{2\gamma}{b_0}\left\{\left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{b_0}\right)^{\gamma-1}\left[\frac{f(x)^{\gamma+1}}{2(\gamma+1)} - \frac{FSX\left(\gamma-1, 2\left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)}{4\left(\frac{n_1 \pi y}{2b}\right)}\right] +\right.$$

$$\left. + \left(\frac{1}{b_0}\right)^{2\gamma-1}\left[\frac{f(x)^{2\gamma+1}}{2(2\gamma+1)} - \frac{FSX\left(2\gamma-1, 2\left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)}{4\left(\frac{n_1 \pi y}{2b}\right)}\right]\right\}$$

(A.4a)

- Caso $n_1 \neq n_2$

$$INT_3(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 FSS\left(\frac{n_1 \pi y}{2b}, \frac{n_2 \pi y}{2b}, f_b(x)\right) -$$

$$-\frac{2\gamma}{b_0}\left\{\left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{b_0}\right)^{\gamma-1}\left[\frac{FSX\left(\gamma-1, \frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right)} - \frac{FSX\left(\gamma-1, \frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right)}\right] +\right.$$

$$\left. + \left(\frac{1}{b_0}\right)^{2\gamma-1}\left[\frac{FSX\left(2\gamma-1, \frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}\right)} - \frac{FSX\left(2\gamma-1, \frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}\right)}\right]\right\}$$

(A.4b)

$$INT_4(x) = \int_0^{f_b(x)} \left[\left(\frac{x}{a_0} \right)^{\gamma} + \left(\frac{y}{b_0} \right)^{\gamma} - 1 \right] sen\left(\frac{n_1 \pi y}{2b} \right) sen\left(\frac{n_2 \pi y}{2b} \right) dy$$

- Caso $n_1 = n_2$

$$INT_4(x) = \left[\left(\frac{x}{a_0} \right)^{\gamma} - 1 \right] FSS\left(\frac{n_1 \pi y}{2b}, \frac{n_2 \pi y}{2b}, f_b(x) \right) +$$

$$+ \left(\frac{1}{2b_0} \right)^{\gamma} \left[\frac{f(x)^{\gamma+1}}{\gamma + 1} - FCX\left(2\left(\frac{n_1 \pi y}{2b} \right), \gamma, f_b(x) \right) \right] \tag{A.5a}$$

- Caso $n_1 \neq n_2$

$$INT_4(x) = \left[\left(\frac{x}{a_0} \right)^{\gamma} - 1 \right] FSS\left(\frac{n_1 \pi y}{2b}, \frac{n_2 \pi y}{2b}, f_b(x) \right) +$$

$$+ \left(\frac{1}{2b_0} \right)^{\gamma} \left[FCX\left(\frac{n_1 \pi y}{2b} - \frac{n_2 \pi y}{2b}, \gamma, f_b(x) \right) - FCX\left(\frac{n_1 \pi y}{2b} + \frac{n_2 \pi y}{2b}, \gamma, f_b(x) \right) \right] \tag{A.5b}$$

$$INT_5(x) = INT_4(x) = \int_0^{f_b(x)} \left[\left(\frac{x}{a_0} \right)^{\gamma} + \left(\frac{y}{b_0} \right)^{\gamma} - 1 \right] sen\left(\frac{n_1 \pi y}{2b} \right) sen\left(\frac{n_2 \pi y}{2b} \right) dy \tag{A.6}$$

$$INT_6(x) = INT_2(x) = \int_0^{f_b(x)} sen\left(\frac{n_1 \pi y}{2b} \right) sen\left(\frac{n_2 \pi y}{2b} \right) dy \tag{A.7}$$

$$INT_7(x) = \int_0^{f_b(x)} \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 cos\left(\frac{n_1\pi y}{2b}\right) cos\left(\frac{n_2\pi y}{2b}\right) dy$$

- Caso $n_1 = n_2$

$$INT_7(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 FCC\left(\frac{n_1\pi y}{2b}, \frac{n_2\pi y}{2b}, f_b(x)\right) -$$

$$-\frac{2\gamma}{b_0}\left\{\left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{b_0}\right)^{\gamma-1}\left[\frac{f_b(x)^{\gamma+1}}{2(\gamma+1)} + \frac{FSX\left(\gamma-1, 2\left(\frac{n_1\pi y}{2b}\right), f_b(x)\right)}{4\left(\frac{n_1\pi y}{2b}\right)}\right] + \right.$$

$$\left. + \left(\frac{1}{b_0}\right)^{2\gamma-1}\left[\frac{f_b(x)^{2\gamma+1}}{2(2\gamma+1)} + \frac{FSX\left(2\gamma-1, 2\left(\frac{n_1\pi y}{2b}\right), f_b(x)\right)}{4\left(\frac{n_1\pi y}{2b}\right)}\right]\right\}$$

(A.8a)

- Caso $n_1 \neq n_2$

$$INT_7(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right]^2 FCC\left(\frac{n_1\pi y}{2b}, \frac{n_2\pi y}{2b}, f_b(x)\right) -$$

$$-\frac{2\gamma}{b_0}\left\{\left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{b_0}\right)^{\gamma-1}\left[\frac{FSX\left(\gamma-1, \frac{n_1\pi y}{2b} - \frac{n_2\pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1\pi y}{2b} - \frac{n_2\pi y}{2b}\right)} + \frac{FSX\left(\gamma-1, \frac{n_1\pi y}{2b} + \frac{n_2\pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1\pi y}{2b} + \frac{n_2\pi y}{2b}\right)}\right] + \right.$$

$$\left. + \left(\frac{1}{b_0}\right)^{2\gamma-1}\left[\frac{FSX\left(2\gamma-1, \frac{n_1\pi y}{2b} - \frac{n_2\pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1\pi y}{2b} - \frac{n_2\pi y}{2b}\right)} + \frac{FSX\left(2\gamma-1, \frac{n_1\pi y}{2b} + \frac{n_2\pi y}{2b}, f_b(x)\right)}{2\left(\frac{n_1\pi y}{2b} + \frac{n_2\pi y}{2b}\right)}\right]\right\}$$

(A.8b)

$$INT_8(x) = \int_0^{f_b(x)} \left(\frac{y}{b_0}\right)^{\gamma-1} \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right] cos\left(\frac{n_1 \pi y}{2b}\right) sen\left(\frac{n_2 \pi y}{2b}\right) dy$$

Caso $n_1 = n_2$

$$INT_8(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{2b_0}\right)^{\gamma-1}\left[FSX\left(\gamma-1, 2\left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)\right] +$$

$$+\left(\frac{1}{2b_0}\right)^{2\gamma-1}\left[FSX\left(2\gamma-1, \left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)\right] \tag{A.9a}$$

Caso $n_1 \neq n_2$

$$INT_8(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{2b_0}\right)^{\gamma-1}\left[FSX\left(\gamma-1, \frac{n_2 \pi y}{2b} - \frac{n_1 \pi y}{2b}, f_b(x)\right) + FSX\left(\gamma-1, \frac{n_2 \pi y}{2b} + \frac{n_1 \pi y}{2b}, f_b(x)\right)\right] +$$

$$+\left(\frac{1}{2b_0}\right)^{2\gamma-1}\left[FSX\left(2\gamma-1, \frac{n_2 \pi y}{2b} - \frac{n_1 \pi y}{2b}, f_b(x)\right) + FSX\left(2\gamma-1, \frac{n_2 \pi y}{2b} + \frac{n_1 \pi y}{2b}, f_b(x)\right)\right] \tag{A.9b}$$

$$INT_9(x) = \int_0^{f_b(x)} \left(\frac{y}{b_0}\right)^{\gamma-1} \left[\left(\frac{x}{a_0}\right)^{\gamma} + \left(\frac{y}{b_0}\right)^{\gamma} - 1\right] sen\left(\frac{n_1 \pi y}{2b}\right) cos\left(\frac{n_2 \pi y}{2b}\right) dy$$

Caso $n_1 = n_2$

$$INT_9(x) = \left[\left(\frac{x}{a_0}\right)^{\gamma} - 1\right]\left(\frac{1}{2b_0}\right)^{\gamma-1}\left[FSX\left(\gamma-1, 2\left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)\right] +$$

$$+\left(\frac{1}{2b_0}\right)^{2\gamma-1}\left[FSX\left(2\gamma-1, 2\left(\frac{n_1 \pi y}{2b}\right), f_b(x)\right)\right] \tag{A.10a}$$

Caso $n_1 \neq n_2$:

$$INT_9(x)=\left[\left(\frac{x}{a_0}\right)^{\gamma}-1\right]\left(\frac{1}{2b_0}\right)^{\gamma-1}\left[FSX\left(\gamma-1,\frac{n_1\pi y}{2b}-\frac{n_2\pi y}{2b},f_b(x)\right)+FSX\left(\gamma-1,\frac{n_1\pi y}{2b}+\frac{n_2\pi y}{2b},f_b(x)\right)\right]+$$

$$+\left(\frac{1}{2b_0}\right)^{2\gamma-1}\left[FSX\left(2\gamma-1,\frac{n_1\pi y}{2b}-\frac{n_2\pi y}{2b},f_b(x)\right)+FSX\left(2\gamma-1,\frac{n_1\pi y}{2b}+\frac{n_2\pi y}{2b},f_b(x)\right)\right]$$

(A.10a)

$$INT_{10}(x)=\int_0^{f_b(x)}\left(\frac{y}{b_0}\right)^{2\gamma-2} sen\left(\frac{n_1\pi y}{2b}\right) sen\left(\frac{n_2\pi y}{2b}\right)dy$$

Caso $n_1 = n_2$

$$INT_{10}(x)=\left(\frac{1}{2b_0}\right)^{\gamma}\left[\frac{f_b(x)^{2\gamma-1}}{2\gamma-1}-FCX\left(2\gamma-2,2\left(\frac{n_1\pi y}{2b}\right),f_b(x)\right)\right]$$

(A.11a)

Caso $n_1 \neq n_2$

$$INT_{10}(x)=\left(\frac{1}{2b_0}\right)^{\gamma}\left[FCX\left(2\gamma-2,\frac{n_1\pi y}{2b}-\frac{n_2\pi y}{2b},f_b(x)\right)-FCX\left(2\gamma-2,\frac{n_1\pi y}{2b}+\frac{n_2\pi y}{2b},f_b(x)\right)\right]$$

(A.11b)

$$INT_{11}(x)=INT_1(x)=\int_0^{f_b(x)} cos\left(\frac{n_1\pi y}{2b}\right) cos\left(\frac{n_2\pi y}{2b}\right)dy \qquad (A.12)$$

$$INT_{12}(x)=INT_3(x)=\int_0^{f_b(x)}\left[\left(\frac{x}{a_0}\right)^{\gamma}+\left(\frac{y}{b_0}\right)^{\gamma}-1\right]^2 sen\left(\frac{n_1\pi y}{2b}\right) sen\left(\frac{n_2\pi y}{2b}\right)dy \qquad (A.13)$$