# 5
# Fatigue Analysis

In this chapter the steps for calculating the structural integrity will be explained.

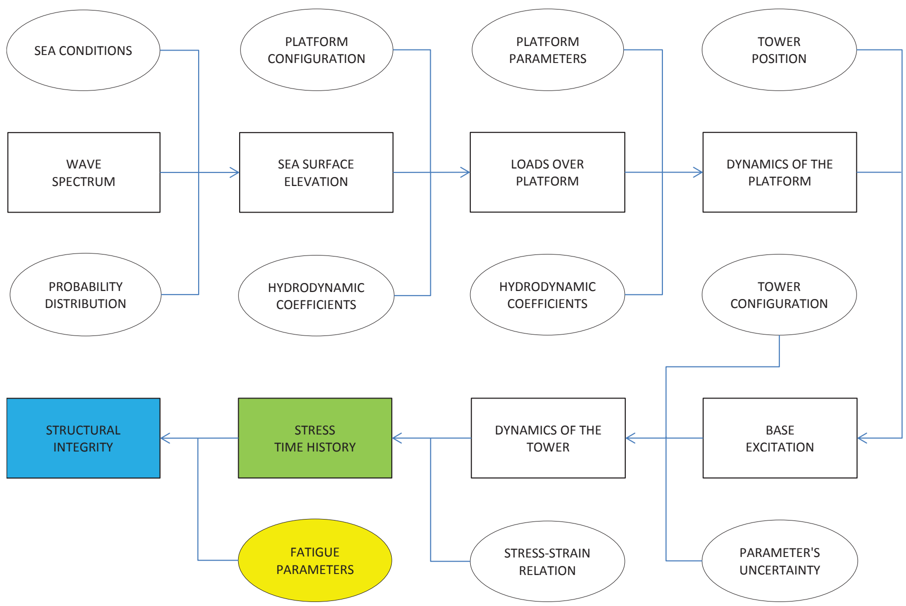


Figure 5.1: Calculating the structural integrity

In order to determine the fatigue strength of any equipment, it is necessary to calculate the cumulative damage on its structure caused by cyclic loads. The expected cumulative damage for the total working life of the equipment at every point of the structure considered critical for fatigue should not exceed a limit level, [19]. In this work the fatigue life will be calculated based on the S-N fatigue approach under the assumption of linear cumulative damage.

## 5.1
## Palmgren-Miner rule

The Palmgren-Miner rule for calculating the fatigue damage is given by

$$D = \frac{n}{N} \tag{5.1}$$

where $n$ is the number of stress cycles in a constant stress range $S$ and $N$ is the number of cycles to failure at the same constant stress range. The S-N curve for a given material and structural joint is then given by

$$NS^{m_f} = C_f \tag{5.2}$$

where $m_f$ is the fatigue strength exponent and $C_f$ is the fatigue strength coefficient. Stiftelsen Det Norske Veritas, DNV, is a classification society that provides services for managing of risk. DNV states that [5] the mean stresses can be neglected for fatigue assessment of welded connections and only the ranges of cyclic stress should be considered in determining the fatigue endurance. The chosen S-N curve for a given joint takes into account the local stress concentrations created by the joint itself and by the weld profile and the design stress can be considered the stress adjacent to the weld. If the weld is situated in a region of stress concentration, the nominal stress should be multiplied by an appropriate stress concentration factor [5].

In this work the structural integrity of the welded connection of the base of the tower to the deck of the platform will be investigated as it is critical for fatigue and a failure in this connection would be catastrophic. Due to this criticality, [6] recommends the use of a full penetration weld and a non-destructive examination after the welding process in order to check for the existence of cracks or bubbles on the weld. As the stresses were calculated using a classical beam theory, a nominal stress S-N curves will be used.

## 5.2 Stress Range Distribution Evaluation

The use of Miner's rule together with required S-N curve to determine the fatigue strength of the structure makes necessary the knowledge of the number of stress cycles at every stress ranges for all critical points of the structure during the working life of the structure. As the fatigue strength has to be determined during the design phase of the structure, it is necessary to know in advance the expected sea and loading condition, short term condition, as well as their probability distribution. In this work it is proposed to do a numerical simulation for each expected short term condition and construct a stress histogram to express the stress range distribution using a rainflow procedure. An approximation to the accumulated damage per each short term condition can then be given by

$$D_j = \frac{T_j}{t_j}\sum_{i=1}^{R}\frac{n_i}{\bar{N}_i} \quad \text{for } j = 1,\ldots,o \tag{5.3}$$

where $o$ is the number of expected short term conditions, $T_j$ is the expected working time under each short term condition, $t_j$ is the period of the simulation, $n_i$ is the number of stress cycles in stress block $i$, $R$ is the number of stress blocks and $\bar{N}_i$ is the number of cycles to failure given by

$$\bar{N}_i = C_f \bar{S}_i^{-m_f} \quad \text{for } i = 1,\ldots,R \tag{5.4}$$

where $\bar{S}_i$ is an average stress range attributed to each stress range block. The choice of this average stress range may have a significant influence on the calculated fatigue life [5] and can be given by

$$\bar{S}_i = \lambda_i\left(S_{i-1} + S_i\right) \quad \text{for } i = 1,\ldots,R \tag{5.5}$$

where $S_{i-1}$ and $S_i$ are the limits for each stress block and $\lambda_i$ are coefficients to be obtained from related S-N curve in order to $\bar{N}_i$ be an average number of cycles to failure at that stress block. An approximation to the probability of occurrence of the average stress range $\bar{S}_i$ is given by

$$P\left(\bar{S}_i\right) = \frac{n_i}{N_T} \quad \text{for } i = 1,\ldots,R \tag{5.6}$$

where $N_T$ is the total number of stress cycles obtained during the simulation of the given short term condition. After substituting the Eqs. (5.4) and (5.6) into (5.3) and rearranging the accumulated damage per each short term condition can be given by

$$D_j = \frac{T_j}{t_j}\sum_{i=1}^{M}\frac{N_T P\left(\bar{S}_i\right)\bar{S}_i^m}{C} \quad \text{for } j = 1,\ldots,o \tag{5.7}$$

The summation of the product $P\left(\bar{S}_i\right)\bar{S}_i^m$ can be considered an approximation to the expected value of $\bar{S}^m$ and Eq. (5.7) can be rewritten as

$$D_j = \frac{T_j}{t_j}\frac{N_T}{C}E\left[\bar{S}^m\right] \quad \text{for } j = 1,\ldots,o \tag{5.8}$$

The total expected damage for the working life of the structure is then given by

$$D = \sum_{j=1}^{o} D_j \tag{5.9}$$

A simplified approach for fatigue analysis where the stress range distribution may be presented as a two-parameter Weibull distribution is proposed on [5]. The two-parameter Weibull probability distribution function is given by

$$F_W(s) = 1 - \mathrm{e}^{-(s/q)^h} \tag{5.10}$$

where $s$ is the stress stress range, $q$ is a scale parameter and $h$ is a shape parameter.

The scale parameter can be obtained from the largest expected stress range. In this case, it is given by [5]

$$q = \frac{s_m}{\left[(\ln(n_0))^{1/h}\right]} \tag{5.11}$$

where $n_0$ is the number of cycles and $s_m$ is the largest expected stress range during the working life. The shape parameter, $h$, can be determined by least-squares methods, provided that stress range data distribution is available. In reference [5] a maximum value of $h = 1.2$ is recommended for steel structures under offshore environmental conditions.

The probability density function of the stress ranges is given by

$$f_W(s) = \frac{\mathrm{d}F_W(s)}{\mathrm{d}s} = h\frac{s^{h-1}}{q^h}\mathrm{e}^{-(s/q)^h} \tag{5.12}$$

The fatigue damage for finite stress range is given by Eq. (5.1) and the number of cycles to failure at a given stress range can be obtained from Eq. (5.2)

$$N(s) = C_f S^{-m_f} \tag{5.13}$$

By using Eqs. (5.12) and (5.13) a differential fatigue damage can be obtained as

$$\mathrm{d}D = \frac{n_0 f(s)}{N(s)}\mathrm{d}s \tag{5.14}$$

and an estimation of fatigue damage can be obtained as

$$\tilde{D} = \int_0^{s_m} \mathrm{d}D \tag{5.15}$$

In general the S-N curves are two slope curves. Considering that the turning point is at $s = s_1$ and $s_m > s_1$ the integration on Eq. (5.15) has to be split into

$$\tilde{D} = \tilde{D}_l + \tilde{D}_u - \tilde{D}_m \tag{5.16}$$

where

$$\tilde{D}_l = \int_0^{s_1} \frac{n_0 f(s)}{C_{f2} s^{-m_{f2}}}\mathrm{d}s \tag{5.17}$$

and

$$\tilde{D}_u = \int_{s_1}^{\infty} \frac{n_0 f(s)}{C_{f1} s^{-m_{f1}}}\mathrm{d}s \tag{5.18}$$

and

$$\tilde{D}_m = \int_{s_m}^{\infty} \frac{n_0 f(s)}{C_{f1} s^{-m_{f1}}}\mathrm{d}s \tag{5.19}$$

Evaluating the integral on Eq. (5.17) it is obtained

$$\tilde{D}_l = \frac{n_0}{C_{f2}}\left\{q^{m_{f2}}\Gamma_l\left(\frac{h+m_{f2}}{h}, \left(\frac{s_1}{q}\right)^h\right)\right\} \tag{5.20}$$

and evaluating the integral on Eq. (5.18) it is obtained

$$\tilde{D}_u = \frac{q^{m_{f1}} n_0}{C_{f1}} \left\{ \Gamma\left(\frac{h+m_{f1}}{h}\right) + \Gamma_u\left(\frac{h+m_{f1}}{h}, \left(\frac{s_1}{q}\right)^h\right) - \frac{h}{h+m_{f1}} \Gamma\left(\frac{2h+m_{f1}}{h}\right) \right\} \quad (5.21)$$

where $\Gamma_u$ is the Upper Incomplete Gamma function given by

$$\Gamma_u(h, s_1) = \int_{s_1}^{\infty} s^{h-1} \mathrm{e}^{-s} \mathrm{d}s \quad (5.22)$$

The integration on Eq. (5.19) can be obtained by replacing $s_1$ by $s_m$ on Eq. (5.21).

This estimation of fatigue damage may be used within an optimization strategy on intermediate calculating steps in order to reduce the computational effort.

Low, [24], presented a closed form solution to estimate the fatigue damage when the stress ranges are a narrowband stochastic process. For a narrowband process the average frequency of the peaks may be approximated by the zero mean upcrossing rate which, for a Gaussian process, is given by

$$v_X^+(0) = \frac{1}{2\pi} \frac{\sigma_{\dot{X}}}{\sigma_X} \quad (5.23)$$

where $X$ is the stochastic process, $\sigma$ denotes the standard deviation and a dot the time derivative. In this case the probability density function of the peaks follows a Rayleigh distribution and is given by

$$f_R r = \frac{r}{\sigma_X^2} \exp\left(-\frac{r^2}{2\sigma_X^2}\right) \quad (5.24)$$

The number of stress cycles during a period $T$ is given by

$$n = v_X^+(0) T \quad (5.25)$$

Integrating over all the stress ranges the expected damage can be calculated as

$$\bar{D} = v_X^+(0) T \int_0^{\infty} \frac{1}{N(s)} f_S(s) \mathrm{d}s \quad (5.26)$$

substituting the S-N relationship it is obtained

$$\bar{D} = \frac{v_X^+(0) T}{C_f} \int_0^{\infty} s^{m_f} f_S(s) \mathrm{d}s \quad (5.27)$$

for a narrowband process the stress amplitude and the peak may be assumed to be identical an are conveniently designated by the same variable $r$. Considering

$$s = 2r \quad (5.28)$$

and substituting the Eq. (5.28) into (5.27) it is obtained

$$\bar{D} = \frac{2^{m_f} v_X^+(0)T}{C_f} \int_0^\infty r^{m_f} f_R(r)\mathrm{d}r \tag{5.29}$$

and substituting the Eq. (5.24) into (5.29) and integrating, the Rayleigh approximation is obtained

$$\bar{D} = \frac{v_X^+(0)T}{C^{m_f}} \left(2\sqrt{2}\sigma_X\right)^{m_f} \Gamma\left(1 + \frac{m}{2}\right) \tag{5.30}$$

If the process can not be considered narrowbanded, the Rayleigh approximation is a conservative estimate to the fatigue damage.

Fricke et al [13] compared the results obtained for the fatigue resistance of a detail of a containership according several classification societies and concluded that a variation on the predicted fatigue lives is significant, mainly due to considered loads, local stresses and chosen S-N curves. A direct calculation of loads using a spectral method was performed and the variation of the predicted life was reduced but was still significant. The results obtained by direct calculation were considered to be too conservative. It can be concluded that even using simplified approaches recommended by classification societies or direct calculation of expected fatigue lives a lot of uncertainty is presented on results.

Sutherland and Veers [39] examined the effects of using various models for the distribution of stress cycles over the structure of wind turbine components. They used a generalized Weibull fitting technique and obtained good results for matching the body of the distribution and extrapolating the tail of the distribution.

Tasdemir and Nohut [40] investigated the fatigue resistance of primary supporting members of a ship structure. They used a global finite element model for the ship and a local finite element model to obtain the stress concentration factors for the weld details. For the long term stress range distribution they used the procedure recommended by a classification society based on the Weibull distribution.

Dong et al [7] performed a long-term fatigue analysis of welded multiplanar tubular joints for a fixed jacket offshore wind turbine. They investigated the influence of the wave loads, the wind loads and the combined effect of wind and wave loads over the fatigue resistance of the welds of the structure. For the distribution of the stresses due to the wave and wind loads they used a two-parameter Weibull distribution and for the combination of wind and wave loads they used the generalized gamma function whose probability density function is given by

$$f_g(s) = \frac{|h|}{\Gamma(a)} \frac{s^{ah-1}}{q^{ah}} \mathrm{e}^{-(s/q)^h} \tag{5.31}$$

Low and Cheung [26] proposed a customized approach for assessing the fatigue resistance of mooring lines and risers. They used the JONSWAP spectrum for calculating the sea surface elevation. Since this spectrum is a function of a shape parameter, the significant wave height, $H_S$ and the spectral peak period, $T_p$, and they selected the value of the shape parameter, the joint probability density function of $H_S$ and $T_p$ is expressed as

$$f_j\left(H_S, T_p\right) = f_H\left(H_S\right) f_{TH}\left(T_p | H_S\right) \tag{5.32}$$

Considering $d\left(H_S, T_p\right)$ as the damage function for a given $H_S$ and $T_p$ pair, Low and Cheung proposed to calculate the expected long-term damage accumulated over a period T as

$$E[D] = T \int_0^\infty \int_0^\infty d\left(H_S, T_p\right) f_j\left(H_S, T_p\right) \mathrm{d}H_S \mathrm{d}T_p \tag{5.33}$$

and they proposed to use a multipeaked third-order asymptotic approximation for the integrand in order to evaluate this probability integral.

In most of the cases the stresses on structural components are a combination of two or more stresses due to different loads. Leira [22] investigated the fatigue damage of welds subjected to multiple stress components. Despite of the stress cycles of each individual component being distributed according Weibull distribution even a linear combination of two or more Weibull components will, in general, not be Weibull distributed [22]. Leira proposes that for the linear combination of two stress components with Weibull cycle distributions the fatigue damage which is accumulated during a time period $T$ for a one-slope S-N curve expressed according Eq. (5.2) can be expressed as

$$E[D(T)] = \frac{N(T)}{C_f} \int_0^\infty \int_0^\infty \left[\sqrt{s_1^2 + c s_2^2}\right]^m \times f_{s_1 s_2}(s_1, s_2) \mathrm{d}s_1 \mathrm{d}s_2 \tag{5.34}$$

where $N(T)$ is the number of stress cycles that occur during the period $T$, $s_1$ and $s_2$ are the stress components, $c$ is a constant to obtain the combined stress and $f_{s_1 s_2}$ is the joint probability density function. Leira investigated the effect of correlation between the two stress components on fatigue damage.

Ang et al [1] developed a technical procedure for a reliability-based approach to fatigue analysis and fatigue-resistant design. They considered that the stress cycles can be distributed according a Beta distribution whose probability density function is given by

$$f_b(s) = \frac{s^{q-1}}{\beta(q, r)} \frac{\left(s_u - s\right)^{r-1}}{s_u^{q+r-1}} \qquad 0 \leq s \leq s_u \tag{5.35}$$

where $s$ are the stress ranges, $s_u$ is a upper limit for the stress ranges and

$$\beta(q,r) = \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)} \tag{5.36}$$

where $\Gamma$ is the gamma function and $q$ and $r$ are parameters of distribution given by

$$q = \frac{\mu}{s_u}\left[\Omega^{-2}\left(\frac{s_u}{\mu}-1\right)-1\right] \tag{5.37}$$

and

$$r = \left(\frac{s_u}{\mu}-1\right)q \tag{5.38}$$

where $\mu$ is the mean and $\Omega$ is the covariance of the applied stress range.

Wang [42] calculated the fatigue life of a ship structural detail using a spectral approach. Assuming that the wave-induced bending stress variation in a ship structural element in a specific sea state is a narrow band Gaussian random process, and consequently the peak values of the stress has a Rayleigh probability density function, Wang presented the following formula to calculate the fatigue damage in a specific sea state

$$D_i = \frac{T}{C_f}\left(2\sqrt{(2)}\right)^{m_f}\Gamma\left(\frac{m_f}{2}+1\right) f_{0i} p_i \left(\sigma_i\right)^{m_f} \tag{5.39}$$

where $T$ is design life of a ship in seconds, $C_f$ is the fatigue strength coefficient, $m_f$ is the fatigue strength exponent, $\Gamma$ is the gamma function, $f_{0i}$ is zero-up crossing frequency of the stress response in Hz, $p_i$ is the probability of occurrence of the sea state $i$ and $\sigma_i$ is the standard deviation of the stress process in the specific sea state.

Since for a wide band random process the Rayleigh distribution for the stress peak values will result in a conservative estimation of the fatigue damage a cycle counting correction factor in damage calculation should be introduced in order to reduce the conservatism due to the narrow band assumption. In this case the formula for fatigue damage, Eq. (5.39), has to be written as

$$D_i = \frac{T}{C_f}\left(2\sqrt{(2)}\right)^{m_f}\Gamma\left(\frac{m_f}{2}+1\right)\lambda\left(m_f,\epsilon_i\right) f_{0i} p_i \left(\sigma_i\right)^{m_f} \tag{5.40}$$

where $\lambda$ is the damage correction factor. This factor is a function of the fatigue strength exponent and of $\epsilon_i$ that can be either a bandwidth parameter or a regularity factor depending on the chosen formula for calculating the cycle counting correction factor. Wang [42] presented three different formulas for calculating this factor. Using one of these formulas, Wang obtained a fatigue life of 25.16 years for a structural ship detail. For comparison, Wang also calculated the fatigue life according the recommendation of a classification society that assumes that the long-term distribution of the stresses follows a Weibull distribution. In this case the calculated fatigue life was 18.765 years.

## 5.3 Uncertainties in Fatigue Life Prediction

According [5] large uncertainties are associated with fatigue life prediction. One of the sources of uncertainty are the S-N curves. Such curves are determined by mean of experiments on specimen and the two slope exponential curves are obtained from measured points by using curve fitting techniques. Further, the design curves recommended on standards are the mean curve minus two standard deviations.

Veldkamp on [41] presents the uncertainties on the results of fatigue strength obtained by experiments with identical specimens under constant and variable amplitude loading. He concluded that the fatigue life under variable amplitude loading is shorter than the fatigue life under constant amplitude loading. Therefore, when the designer chooses the S-N curve to be used for the fatigue resistance evaluation of a structural component it is necessary to be aware of whether the curve was obtained using constant or variable amplitude loading. If the available curves were obtained under constant amplitude loading a reduction factor for the values of the curve have to be used. Veldkamp concluded on the same reference that the uncertainty of the fatigue parameters dominate the overall uncertainty of the fatigue resistance of the studied equipment.

Since the design of offshore equipments has to attend to the required standards, the use of Design Fatigue Factors when determining critical parameters for the structure has a big influence in the predicted fatigue life. Such design factors are intended to overcome the uncertainty on loading, on S-N data and on the Palmgren-Miner damage accumulation rule and to avoid the need of a probabilistic analysis of the problem.

When the weld details require the use of hot spot stress factors, the derivation of these factor is a source of uncertainty as well. The critical details that present reduced fatigue life have to be inspected in-service to check for existence of fatigue cracks.

Others sources of uncertainties are the choice of parameters for the simulation of sea surface elevation, the model for the interaction between the platform and the sea waves, the choice of hydrodynamics coefficients for evaluation of the dynamics of the platform and the finite element model to be used to obtain the stresses on required critical points.

Sarkar et al [36] proposed an approach based on Wiener chaos expansions to estimate the fatigue damage in structural systems with parameter uncertainties. They used the Hermite polynomial expansion to describe the dependence of the damage rate on some uncertain parameters

$$d(z) = \sum_{j=0}^{\infty} c_j H_j(z) \approx \sum_{j=0}^{n} c_j H_j(z) = d_n(z) \tag{5.41}$$

where $d$ is the damage rate, $z$ is a random variable, $H_j$ are Hermite polynomials and

$$\begin{aligned} c_j = E\left[d(z)H_j(z)\right] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d(z)H_j(z)(e)^{-z^2/2}\mathrm{d}z \\ &\approx \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{n} h_i d(z_i) H_j(z_i) \mathrm{e}^{-z_i^2/2} \end{aligned} \tag{5.42}$$

Sarkar et al used this expansion to quantify how the uncertainty of one of the parameters of the Morison's equation used to model the force acting on the pile of an offshore structure. They calculated the damage rate using a three term truncated Hermite expansion and compared with the results obtained using rainflow technique. A good agreement between the two estimates was obtained, even in the tails of the distribution of $z$.

Low [25] presented a method for analyzing the variance of the damage for any narrow-band Gaussian process. The covariance of the damage is given by

$$c_D^2 = \frac{N + 2\chi}{N^2} \left( \frac{\Gamma(1 + m_f)}{\Gamma(1 + m_f/2) - 1} \right) \tag{5.43}$$

where $N$ is the number of half-cycles, $\Gamma$ is the Gamma function and $m_f$ is the fatigue strength exponent and

$$\chi = \sum_{k-1}^{N-1} (N - k) \left[ \alpha_{m_f} \rho_{ss}^2(k) + \beta_{m_f} \rho^4 ss(k) \right] \tag{5.44}$$

where $\rho_{ss}$ is the autocorrelation of the stochastic process for the stress half-cycles, $\alpha_{m_f}$ and $\beta_{m_f}$ are coefficients depending on $m_f$ obtained by curve-fitting techniques. The variance of the damage can then be obtained as

$$\sigma_D^2 = c_D^2 \bar{D}^2 \tag{5.45}$$

where $\bar{D}$ is the total expected damage. Low concludes that the proposed method is nearly exact up to around $m_f = 6$ and the method when applied to processes that are less narrowband presents some minimal errors.

Garbatov and Soares [14] studied the effect of various factors related to fatigue damage assessment of a welded ship structural component. The considered factors were the model of the ship, scatter diagram, heading and wave spectra. The fatigue damage was calculated using a spectral approach, considering the long-term stress range distribution as a series of short-term Rayleigh distributions for different sea states and headings.

They concluded that there are significant differences between all the pairs of fatigue damage means as function of the model of the ship, there are significant differences between the mean fatigue damage pairs of some of the heading directions, the mean fatigue damages as function of the scatter diagrams for North Atlantic and World Wide Trade are similar but for all the others tested scatter diagrams there were significant differences, and finally that for the three considered wave spectra there were also significant differences on the obtained fatigue damage.