4
Dynamics of the Drilling Tower

In this section the steps to obtaining the base excitation over the tower will be explained

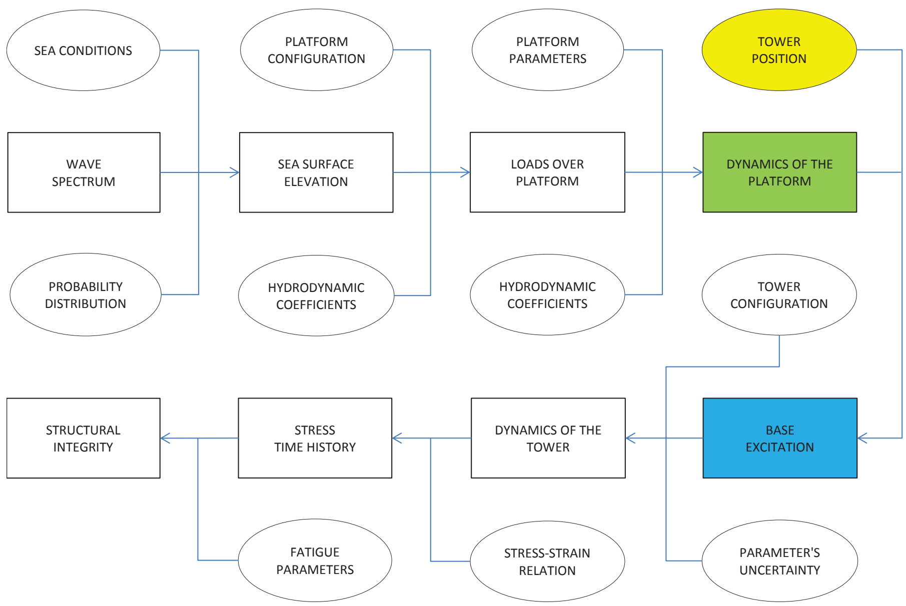

Figure 4.1: Obtaining the base excitation

The drilling tower mounted on the platform consists of a vertical beam shaped structure used to support two lifting systems. The base of the tower is welded to the platform and this weld is critical for fatigue. The base excitation on the structure is obtained by means of a coordinate transformation of the dynamic response of the platform to the x, y and z local coordinate system located at the base of the tower. The Fig. 4.2 shows the model of the tower.

4.1
Partial Differential Equation

The tower will be considered a beam clamped to the platform and free on the other end and the normal stress due to the bending about the y and z

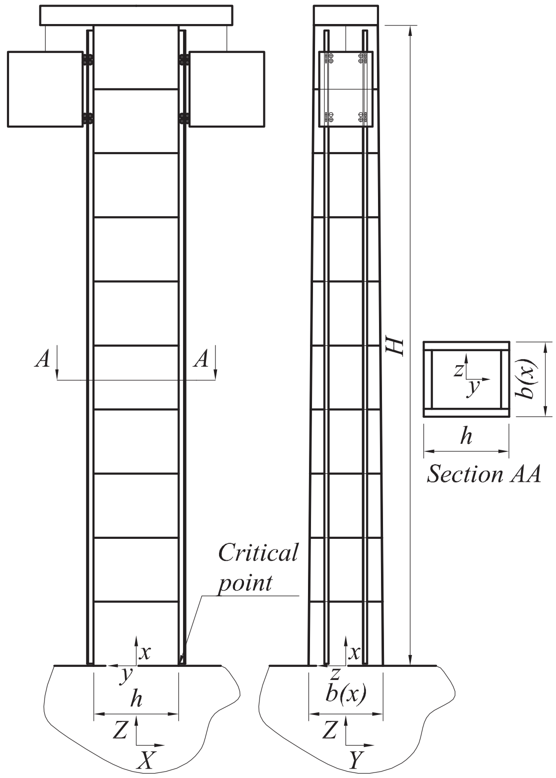

Figure 4.2: Sketch of the tower

directions will be calculated. As the mass of the tower is much smaller than the mass of the platform, it will be considered that the dynamics of the tower does not affect the dynamics of the platform. The differential equation for a beam in bending around the z direction is given by

$$-\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right] + f_y(x,t) = m(x)\frac{\partial^2 v(x,t)}{\partial t^2} \quad \text{for } 0 < x < L \tag{4.1}$$

where $v(x,t)$ is the displacement on y direction of any point x and instant t, $f_y(x,t)$ is the inertia load per unit length and $I_z(x)$ is the inertia area moment about the z direction, the direction x cross the geometric center of transverse sections. The Euler-Bernoulli theory has been used. The boundary conditions for a clamped-free beam are given by

$$v(0,t) = 0 \tag{4.2}$$

$$\left.\frac{\partial v(x,t)}{\partial x}\right|_{x=0} = 0 \tag{4.3}$$

$$\left.EI_z(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right|_{x=L} = 0 \tag{4.4}$$

and

$$\frac{\partial}{\partial x}\left[EI_z(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right]\bigg|_{x=L} = 0 \tag{4.5}$$

It is necessary to solve the eigenvalue problem associated to this system. As the solution for Eq.(4.1) is splittable on space and time it can be given by

$$v(x,t) = V(x)F_v(t) \tag{4.6}$$

where H is an harmonic function. Considering the frequency of H as ω_y, the associated eigenvalue problem can be given by the following differential equation

$$\frac{d^2}{dx^2}\left[EI_z(x)\frac{d^2 V(x)}{dx^2}\right] = \omega_y^2 m(x)V(x) \quad \text{for } 0 < x < L \tag{4.7}$$

together with the following boundary conditions for a clamped-free beam

$$V(0) = 0 \tag{4.8}$$

$$\frac{dV(x)}{dx}\bigg|_{x=0} = 0 \tag{4.9}$$

$$EI_z(x)\frac{d^2V(x)}{dx^2}\bigg|_{x=L} = 0 \tag{4.10}$$

and

$$\frac{d}{dx}\left[EI_z(x)\frac{d^2V(x)}{dx^2}\right]\bigg|_{x=L} = 0 \tag{4.11}$$

4.2 Approximation to the Solution

In this section the steps for obtaining the dynamics of the tower will be explained.

As the tower has a variable cross section, it will be necessary to obtain an approximation for the solution to the dynamics of the structure. One of the possible ways to obtain such approximation is through the discretizing of the equations that describe the dynamics of the structure using the Finite Element Method (FEM). The equations will be discretized using one-dimensional elements with two nodes and six degrees of freedom per node as shown on Fig. 4.4.

In this work only the dynamic response of the tower on y and z direction will be investigated. An approximation to the displacement on y direction within an element is given by

$$v(x,t) \approx L_2(x)u_2(t) + L_6(x)lu_6(t) + L_8(x)u_8(t) + L_{12}(x)lu_{12}(t) \tag{4.12}$$

where

$$\begin{array}{ll} L_2(x) = (1 - 3\xi^2 + 2\xi^3) & L_6(x) = (-\xi - 2\xi^2 + \xi^3) \\ L_8(x) = (3\xi^2 - 2\xi^3) & L_{12}(x) = (-\xi^2 + \xi^3) \end{array} \tag{4.13}$$

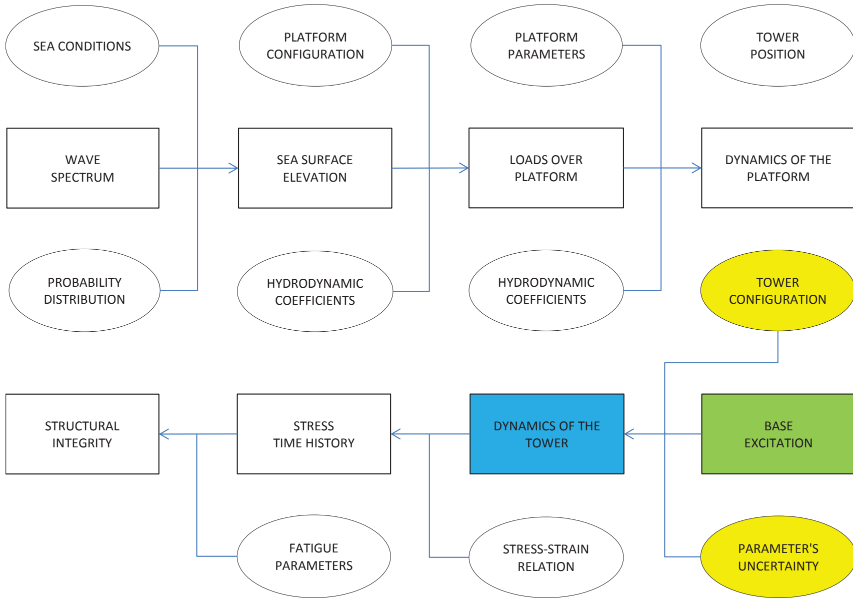

Figure 4.3: Obtaining the dynamics of the tower

$\xi = x/l$ and l is the length of the element. The mass and stiffness matrix obtained considering such approximations are given by

$$[M_y^{(e)}] = \frac{\rho \bar{A} l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{4.14}$$

and

$$[K_y^{(e)}] = \frac{E\bar{I}_z}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \tag{4.15}$$

where ρ is the mass density of the material of the tower, $\bar{A}$ is the average cross section of the tower, E is the elasticity modulus and $\bar{I}_z$ is the average inertia

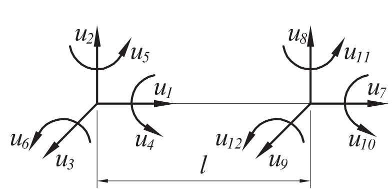

Figure 4.4: One-dimensional element

area moment about the z direction.

As in real systems there is always some level of energy dissipation, a damping matrix can be used. This matrix can be considered proportional to mass and stiffness matrix and is given by

$$[C_y^{(e)}] = \alpha[M_y^{(e)}] + \phi[K_y^{(e)}] \tag{4.16}$$

where α and ϕ are damping parameters. The assembly of the elements can be seen on Fig. 4.5

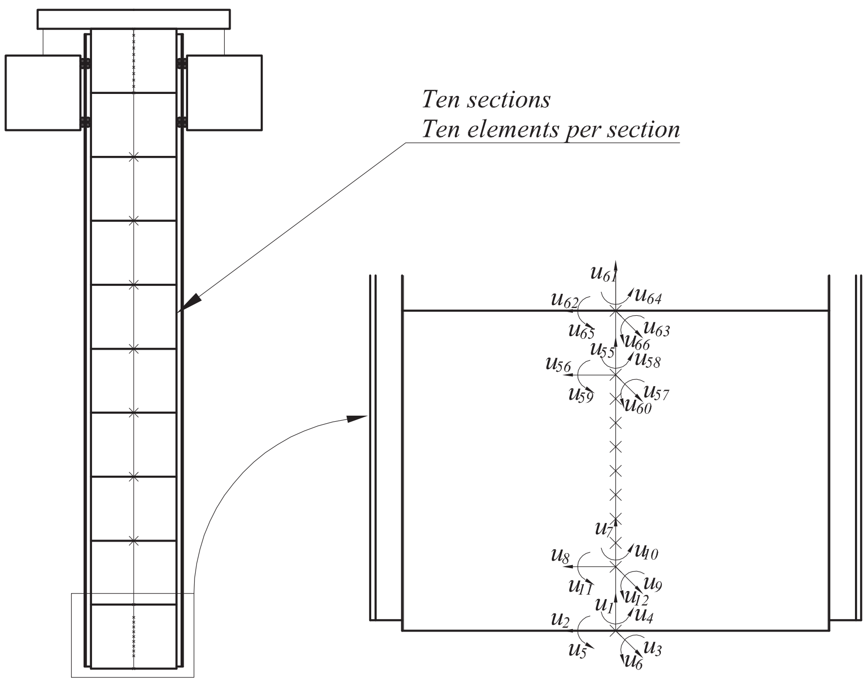

Figure 4.5: Assembly of the elements

and the approximation to the dynamics of the structure is given by

$$[M]\ddot{\mathbf{X}} + [C]\dot{\mathbf{X}} + [K]\mathbf{X} = \mathbf{F} \tag{4.17}$$

where $[M]$, $[C]$ and $[K]$ are the global matrices of the assembly of elements, $\mathbf{X}$ are the degrees of freedom of the approximation to the dynamics and $\mathbf{F}$ are the external loads over the tower.

4.3 Reduced-order Model for the Dynamics

In the section 2.6 it has been proposed to use a reduced-order model to represent the sea surface elevation since the use of a full-order model to calculate the nonlinear wave body interactions is a time-consuming computational task. In the same way, the use of the complete finite element model for the structure of the tower for all necessary simulations to evaluate the fatigue resistance of the equipment will make this task prohibitively expensive and an alternative reduced-order model becomes necessary.

Considering that the matrices $[M]$, $[C]$ and $[K]$ have dimensions $m \times m$ and a basis composed by the n elements that constitute the columns of the matrix $[\Psi]$ with dimension $m \times n$ with $n \ll m$, the dynamic response of the system represented in this basis is given by [35]

$$\mathbf{X}(t) = [\Psi]\mathbf{a}(t) \tag{4.18}$$

Substituting Eq. (4.18) into (4.17) it is obtained

$$[M][\Psi]\ddot{\mathbf{a}}(t) + [C][\Psi]\dot{\mathbf{a}}(t) + [K][\Psi]\mathbf{a}(t) = \mathbf{F}(t) \tag{4.19}$$

Matrix $[\Psi]$ is composed by orthogonal vectors, ψ_i, that generate a reduced subspace. The projection of the dynamics of the system, Eq. (4.19), into this reduced subspace is given by

$$[M_r]\ddot{\mathbf{a}}(t) + [C_r]\dot{\mathbf{a}}(t) + [K_r]\mathbf{a}(t) = \mathbf{f}_r(t) \tag{4.20}$$

where

$$[M_r] = [\Psi]^T[M][\Psi] \tag{4.21}$$

is the reduced mass matrix,

$$[C_r] = [\Psi]^T[C][\Psi] \tag{4.22}$$

is the reduced damping matrix,

$$[K_r] = [\Psi]^T[K][\Psi] \tag{4.23}$$

is the reduced stiffness matrix and

$$[\mathbf{f}_r] = [\Psi]^T\mathbf{f} \tag{4.24}$$

is the reduced vector of external loads. The system has now order $n \times n$ and it is expected that the necessary simulations to evaluate the fatigue resistance of the drilling tower will demand a reduced computational effort.

It is necessary to choose an efficient basis to represent the dynamics of the system. One of the options is to use a basis composed of the normal modes of the system. This is the best choice when linear systems are being analyzed [35]. The modes are obtained solving the following eigenvalue problem

$$\left(-\omega^2[M] + [K]\right)\phi = 0 \tag{4.25}$$

where ω are the natural frequencies and ϕ are the normal modes associated.

Since the base of the drilling tower is excited by the displacement of the platform and the tower is considered to be clamped to the deck of the platform, the degrees of freedom of the finite element node that represent the section of the tower close to the deck have prescribed displacements and rotations.

If the complete finite element model of the tower is being used, it is necessary to prescribe only the displacements and rotations of the degrees of freedom of the node at the bottom of the tower. If, in turn, a reduced-order model is being used, it is necessary to associate a prescribed mode for the entire model for each prescribed degree of freedom. The finite elements used to obtain the approximation of the dynamics of the tower have two nodes and six degrees of freedom per node. Therefore, when constructing the reduced-order model using the normal modes obtained from finite element model, six additional prescribed modes have to be included on the basis before accomplishing the projection of the approximation to the dynamics of the tower.

In general, the prescribed modes are given by

$$\chi = \begin{bmatrix} U_1 & U_2 & \dots & U_m \end{bmatrix}^T \tag{4.26}$$

where U_i are the prescribed values for each degree of freedom of the finite element model. The prescribed mode for the displacement of the tower on x direction is given by

$$\chi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{4.27}$$

The prescribed mode for the displacement of the tower on y direction is given by

$$\chi_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{4.28}$$

The prescribed mode for the displacement of the tower on z direction is given by

$$\chi_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \tag{4.29}$$

The prescribed mode for the torsion of the tower around x direction is given by

$$\chi_4 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \tag{4.30}$$

The prescribed mode for the bending of the tower around y direction is given by

$$\chi_6 = \begin{bmatrix} 0 & 0 & -X_1 & 0 & 1 & 0 & 0 & 0 & -X_2 & 0 & 1 & 0 & \dots & 0 & 0 & -X_{nn} & 0 & 1 & 0 \end{bmatrix}^T \tag{4.31}$$

where X_i is the coordinate of the node i on x direction and nn is the number of nodes.

The prescribed mode for the bending of the tower around z direction is given by

$$\chi_5 = \begin{bmatrix} 0 & X_1 & 0 & 0 & 0 & 1 & 0 & X_2 & 0 & 0 & 0 & 1 & \dots & 0 & X_{nn} & 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{4.32}$$

The basis for the reduced-order model is given by

$$[\Phi] = \begin{bmatrix} \chi_1 & \chi_2 & \chi_3 & \chi_4 & \chi_5 & \chi_6 & \phi_\mathbf{1} & \phi_\mathbf{2} & \dots & \phi_\mathbf{n} \end{bmatrix} \tag{4.33}$$

and the dynamic response is given by

$$\mathbf{X}(t) = [\Phi]\mathbf{q}(t) \tag{4.34}$$

where $\mathbf{q}$ are the modal coordinates. The approximation to the dynamics of the system projected on this basis is given by

$$[M_r]\ddot{\mathbf{q}}(t) + [C_r]\dot{\mathbf{q}}(t) + [K_r]\mathbf{q}(t) = \mathbf{f}_r(t) \tag{4.35}$$

where

$$[M_r] = [\Phi]^T[M][\Phi] \tag{4.36}$$

$$[C_r] = [\Phi]^T[C][\Phi] \tag{4.37}$$

$$[K_r] = [\Phi]^T[K][\Phi] \tag{4.38}$$

$$\mathbf{f}_r = [\Phi]^T\mathbf{f} \tag{4.39}$$

4.4
Stress at Critical Points

In this section the steps for obtaining the stress time history on required points of the structure will be explained.

The cross section of the tower is shown on Fig. 4.2. The normal stress-deformation relation for the bending about the z direction for a variable cross-section beam is given by

$$\sigma(x,t) = Ey_p\frac{\partial^2 v(x,t)}{\partial x^2} \tag{4.40}$$

where y_p is the distance from required point to the neutral line of the cross section. A similar relation for the bending about the y direction can be obtained.

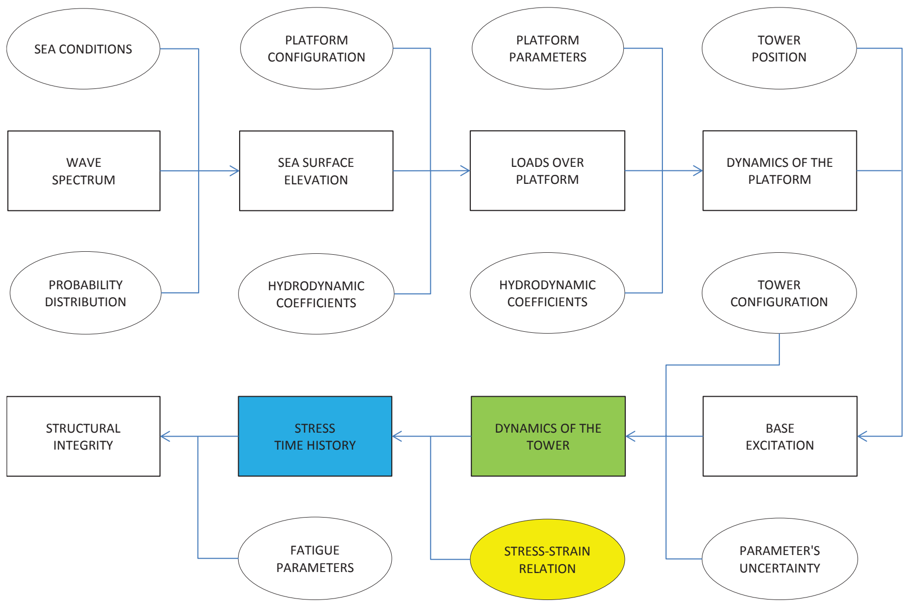

Figure 4.6: Obtaining the stress time history

As an approximation to the dynamics of the structure was obtained using the Finite Element Method it is necessary to obtain an approximation to the normal stress on required points as well. By substituting the Eq. (4.12) into (4.40) an approximation to the normal stress due to the bending about the z direction is obtained

$$\sigma(x,t) \approx E y_p \left(L_2''(x) u_2(t) + L_6''(x) l u_6(t) + L_8''(x) u_8(t) + L_{12}''(x) l u_{12}(t) \right) \frac{\bar{I}_z}{I_z(x)} \tag{4.41}$$

where double primes indicate a double differentiation with respect to the spatial variable x. A similar approximation for the normal stresses due to the bending about the y direction can be obtained as well.

In this work the critical for fatigue point to be investigated is located at the base of the tower, on the weld between the tower and the platform. It is expected that the highest bending moments take place at this section. The total stress at the critical point is a summation of the normal stresses due to the bending about the y and z directions. Only the steady-state part of the dynamic response of the tower should be considered when evaluating the stresses at critical points.

4.5 Parametric Uncertainty

The inherent uncertainties on the parameters or operators of any mechanical system must be considered when such system is being analyzed. Since the probability density function for the random variables that represent such parameters or operators are not always available for the designer in advance, an strategy to obtain such functions becomes necessary.

If there is not enough data available about the random variables to be studied, the Principle of Maximum Entropy can be used to obtain an approximation to the required probability density function [38], [15] and [16]. This principle states that:

"Among all the probability distributions consistent with the prescribed conditions the one that maximizes the uncertainty (entropy) should be chosen"

Being n the number of the welds of the tower, W a random vector with n components and pW the probability density function of W, the entropy related to pW is given by

$$S\left(pW\right) = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}...\int_{-\infty}^{+\infty} pW(w)\ln\left(pW(w)\right)\mathrm{d}w \tag{4.42}$$

The only available information about the random variables is the fabrication tolerance, $W_{min} \preceq W \preceq W_{max}$. By using the Principle of Maximum Entropy the obtained probability density function is

$$pW(w) = \mathbb{1}_{[W_{min},W_{max}]}(w)\prod_{i=1}^{n}\frac{1}{W_{maxi} - W_{mini}} \tag{4.43}$$

therefore, the random variables are independent with uniform probability density function. The same distribution applies to the thickness of the plates.

Batous and Soize [3] proposed a methodology for construction and identification of a probabilistic model of random fields in presence of modeling errors in high stochastic dimension and presented in context of computational structural dynamics. They presented two ways to construct the prior stochastic model of a random field $\mathbf{H}$.

The first way is using an algebraic stochastic representation of the random field $\mathbf{H}$

$$\mathbf{H}((x)) = f_r(\mathbf{G}(\mathbf{x})) \quad \text{for } \mathbf{x} \in \Omega \tag{4.44}$$

where $\mathbf{x}$ is a vector representing any point in the open bounded domain Ω of $\mathbb{R}^3$, f_r is a given nonlinear deterministic mapping and where $\{\mathbf{G}(\mathbf{x}), \mathbf{x} \in \Omega\}$ is a given random field for which the probability law (system of marginal probability distributions) is completely defined and for which a generator of independent realizations is available.

For the second way it is necessary that the mean function and the covariance function of random field $\mathbf{H}$ are known functions, what is the case when an algebraic stochastic representation of $\mathbf{H}$ has been constructed or if experimental data are available for estimating these two functions with a sufficient accuracy. Then under certain hypotheses a statistical reduction can be constructed using the Karhunen-Loève expansion. The first way has been used in this work for the constructing the stochastic model for the thickness of the welds and for the thickness of the plates of the drilling tower.