3
Dynamics of the Platform

An example of application of the proposed procedure will be given where the equipment to be designed is similar to the drilling tower mounted on a platform shown on Fig. 3.1.

Some simplifications on the geometry of the platform have been made, each leg of the platform was considered to have a cylindrical shape and the pontoons between the legs were removed. The draft of the platform, the depth of the submerged volume of the body measured from undisturbed sea surface, has been modified in order to compensate the differences on the geometry. A closed-form solution for the wave loads over a cylinder is available on literature. A sketch of the simplified platform is shown on Fig. 3.2.

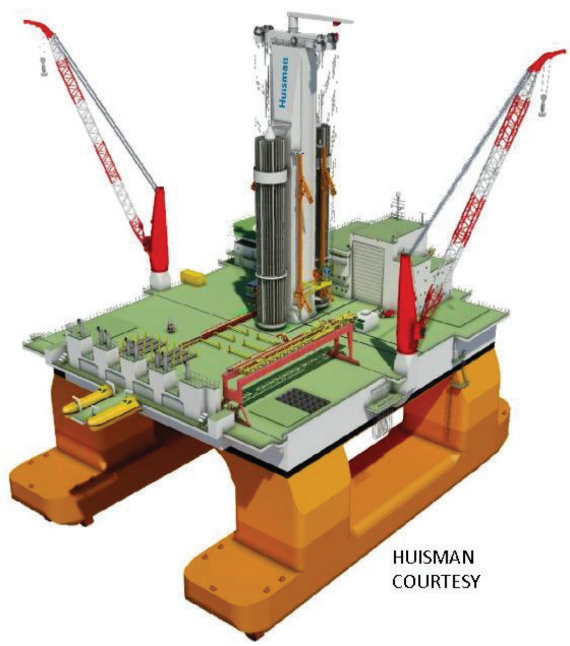

Figure 3.1: Drilling tower mounted on a platform

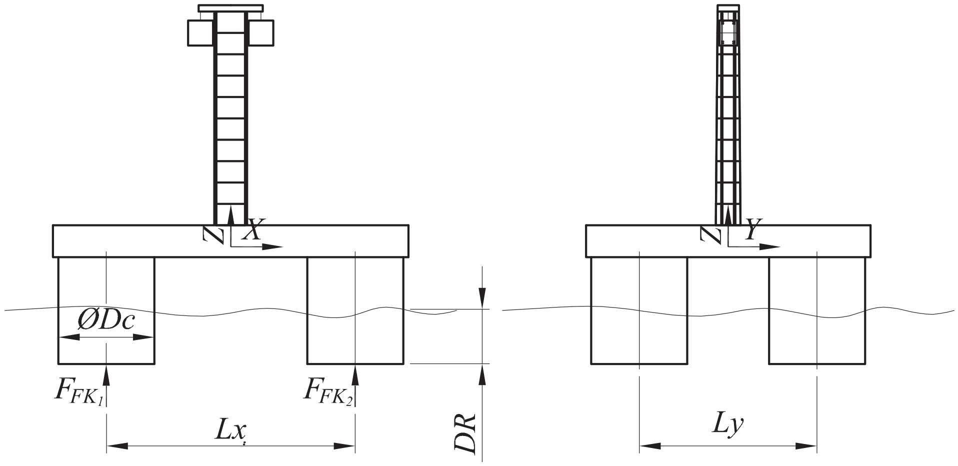

Figure 3.2: Sketch of the platform

3.1 Equation of Motion

The motions of the platform can be split into three mutually perpendicular translations of the center of gravity G and the three rotations about G shown on Fig. 3.3. When obtaining the dynamics of the platform, the global coordinate system will be used.

The equations of motion for the six degrees of freedom of the platform are given by

$$\sum_{j=1}^{6} \{(M_{ij} + A_{ij})\, \ddot{x}_j + B_{ij}\dot{x}_j + C_{ij}x_j\} = F_i \quad \text{for } i = 1, \ldots, 6 \tag{3.1}$$

where $i = 1$ to 6 are the surge, sway, heave, roll, pitch and yaw motions, x_j is the displacements of harmonic oscillation in or about direction j, M_{ij} are solid mass or inertia coefficients, A_{ij} are hydrodynamic mass or inertia coefficients, B_{ij} are hydrodynamic damping coefficients, C_{ij} are restitution coefficients and F_i is the harmonic exciting wave force or moment in direction i. The surge, sway and yaw motions are considered to be restricted by the mooring system or dynamic positioning system of the platform. Therefore, in this work only the the heave, roll and pitch motions of the platform will be considered when determining the base excitation for the drilling tower.The solid mass matrix of the platform is given by

$$[M^{(p)}] = \begin{bmatrix} M_p & 0 & 0 \\ 0 & I_{xx} & -I_{xy} \\ 0 & -I_{xy} & I_{yy} \end{bmatrix} \tag{3.2}$$

where M_p is the mass of the platform, I_{xx} is the mass moment of inertia around

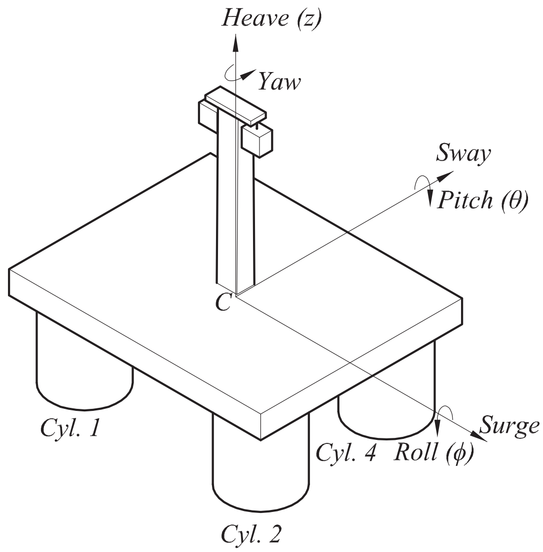

Figure 3.3: Movements of the platform

X axis, I_{yy} is the mass moment of inertia around Y axis and I_{xy} is the mass product of inertia. The hydrodynamic mass matrix of the platform is given by

$$[A^{(p)}] = \begin{bmatrix} 4a & 0 & 0 \\ 0 & 2aL_y^2 & 0 \\ 0 & 0 & 2aL_x^2 \end{bmatrix} \tag{3.3}$$

where a is the hydrodynamic mass coefficient per cylinder of the platform, and L_x and L_y are the distances between the cylinders along X and Y direction respectively. The hydrodynamic damping matrix is given by

$$[B^{(p)}] = \begin{bmatrix} 4b & 0 & 0 \\ 0 & 2bL_y^2 & 0 \\ 0 & 0 & 2bL_x^2 \end{bmatrix} \tag{3.4}$$

where b is the hydrodynamic damping coefficient per cylinder of the platform. The restitution matrix is given by

$$[C^{(p)}] = \begin{bmatrix} 4c & 0 & 0 \\ 0 & 2cL_y^2 & 0 \\ 0 & 0 & 2cL_x^2 \end{bmatrix} \tag{3.5}$$

where c is the restitution coefficient per cylinder of the platform.

Considering that the platform motions have a linear behavior and the sea state have a known wave spectrum, the resulting motions of the platform can be obtained by the superposition of the motions of the platform in still water and under the action of regular waves. The following two types of loads are considered to be acting on the platform [18]

1. The hydromechanical forces and moments induced by the harmonic oscillations of the rigid body moving in the undisturbed surface of the fluid

2. The wave exciting forces and moments produced by the action of the waves over the restrained body

3.2 Hydromechanical Loads

The geometry of the platform was simplified considering that the legs of the platform have the shape of a cylinder. The hydromechanical loads over a vertical cylinder will be discussed in the following. The dynamics of a heaving cylinder is given by [18]

$$m\ddot{z} = -W_c + \rho g(DR - z)A_w - b\dot{z} - a\ddot{z} \tag{3.6}$$

where m is the solid mass of the cylinder, z is the vertical displacement, P is the weight of the cylinder, ρ is the specific mass of the water, DR is the draft of cylinder at rest, A_w is the water plane area of the cylinder, b is the hydrodynamic damping coefficient and a is the hydrodynamic mass coefficient. According to Archimedes' law

$$W_c = \rho g DR A_w \tag{3.7}$$

and Eq. (3.6) becomes

$$(m + a)\ddot{z} + b\dot{z} + cz = 0 \tag{3.8}$$

where c is the restoring spring coefficient given by

$$c = \rho g A_w \tag{3.9}$$

The vertical oscillations of the cylinder will generate waves which propagate radially from it. Since these waves transport energy they withdraw energy from the free cylinder oscillations causing its motion die out. This so-called wave damping is proportional to the velocity of the cylinder and the coefficient b is called the wave or potential damping coefficient.

The other part of the hydromechanical force, $a\ddot{z}$, is caused by the accelerations that are given to the water particles near to the cylinder. This part of the force does not dissipate energy and manifests itself as a standing wave system near the cylinder. The coefficient a is called the hydrodynamic mass or added mass.

After experiments it could be noted that both the acceleration and the velocity terms have a sufficiently linear behavior at small amplitudes [18]. The term cz is the restoring force and the total reaction forces of the fluid on the oscillating cylinder, $a\ddot{z} + b\dot{z} + cz$, are called hydromechanical forces.

3.3 Wave Loads

In this section the steps for obtaining the loads over platform will be explained

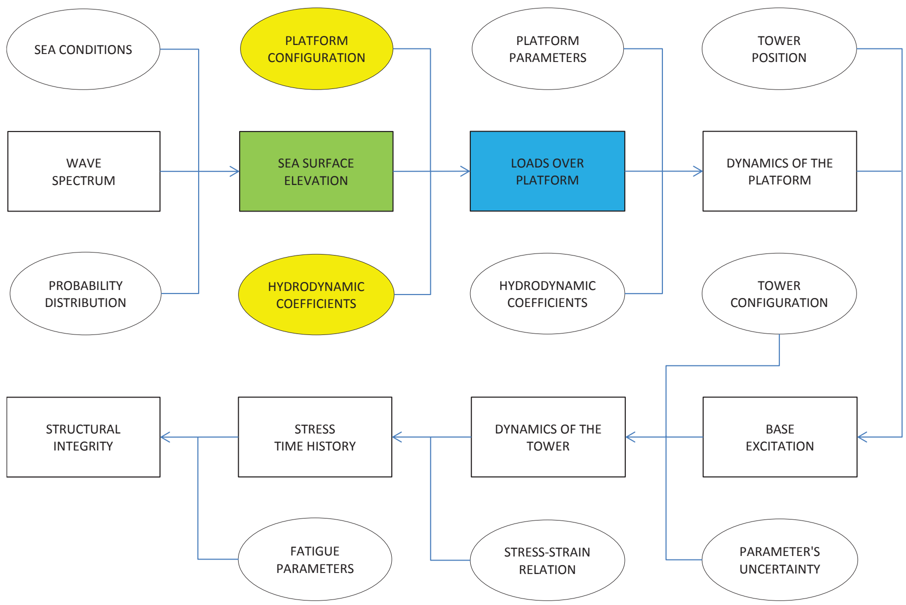

Figure 3.4: Obtaining the loads over platform

The loads due to the waves over the cylinders that represent the legs of the platform will be determined from the potential theory based on classic theory of deep water. This classic theory is based on following assumptions [18]

– The water surface slope is small, therefore terms in the equations of the waves with magnitude in the order of the steepness-squared can be ignored

– Harmonic displacements, velocities, accelerations of the water particles and also harmonic pressures will have a linear relation with the wave surface elevation, therefore the theory is considered linear

For a single regular wave traveling on x direction, the wave potential is written as [18]

$$\Phi_w(x, z, t) = P(z)\sin(kx - \omega t) \tag{3.10}$$

where z is the distance below the still water level (positive upwards), k is the wave number and ω is the wave frequency. P is a function yet to be defined. This velocity potential has to fulfill four requirements:

1. Continuity, or Laplace, condition
2. Sea bed boundary condition
3. Free surface dynamic boundary condition
4. Free surface kinematic boundary condition

From the definition of the velocity potential, the velocity of the water particles in the three translational directions is given by [18]

$$\begin{aligned} u = v_x &= \frac{\partial \Phi_w}{\partial x} \\ v = v_y &= \frac{\partial \Phi_w}{\partial y} \\ w = v_z &= \frac{\partial \Phi_w}{\partial z} \end{aligned} \tag{3.11}$$

The continuity condition states that [18]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3.12}$$

and since the fluid is homogeneous and incompressible, this condition results in the Laplace Equation for potential flows [18]

$$\nabla^2 \Phi_w = \frac{\partial^2 \Phi_w}{\partial x^2} + \frac{\partial^2 \Phi_w}{\partial y^2} + \frac{\partial^2 \Phi_w}{\partial z^2} = 0 \tag{3.13}$$

Considering that water particles move in the $x - z$ plane only and substituting Eq. (3.10) into (3.13) yields a homogeneous solution of this equation [18]

$$\frac{d^2 P(z)}{dz^2} - k^2 P(z) = 0 \tag{3.14}$$

One of the solutions for P is given by

$$P(z) = C_1 e^{kz} + C_2 e^{-kz} \tag{3.15}$$

Considering the first boundary condition, the wave potential can be written now with two unknown coefficients as [18]

$$\Phi_w(x, z, t) = \left(C_1 e^{kz} + C_2 e^{-kz}\right) \sin(kx - \omega t) \tag{3.16}$$

The vertical velocity of water particles at the sea bed is zero (no-leak condition) [18]

$$\left.\frac{\partial \Phi_w}{\partial z}\right|_{z=-h} = 0 \tag{3.17}$$

where h is the sea depth at considered location. Substituting this boundary condition in Eq. (3.16) it is obtained

$$C_1 e^{-kh} = C_2 e^{kh} \tag{3.18}$$

and Eq. (3.15) can be written as

$$P(z) = \frac{C}{2}\left(e^{k(h+z)} + e^{-k(h+z)}\right) = C\cosh[k(h+z)] \tag{3.19}$$

and the wave potential with only one unknown becomes

$$\Phi_w(x,z,t) = C\cosh[k(h+z)]\sin(kx-\omega t) \tag{3.20}$$

where C is a constant to be determined. The pressure at the free surface of the fluid is equal to the atmospheric pressure. This requirement for the pressure is called the dynamic boundary condition at the free surface. The Bernoulli equation for an unsteady irrotational flow is in its general form [18]

$$\frac{\partial \Phi_w}{\partial t} + \frac{1}{2}\left(u^2+v^2+w^2\right) + \frac{p}{\rho} + gz = C^* \tag{3.21}$$

In two dimensions $v = 0$ and considering that waves have a small steepness Eq. (3.21) turns into

$$\frac{\partial \Phi_w}{\partial t} + \frac{p}{\rho} + gz = C^* \tag{3.22}$$

At the free surface this condition becomes

$$\frac{\partial \Phi_w}{\partial t} + \frac{p_0}{\rho} + g\zeta = C^* \quad \text{for } z = \zeta \tag{3.23}$$

where p_0 is the atmospheric pressure. Since $p_0/\rho - C^*$ is a constant the Eq. (3.23) can be written as

$$\frac{\partial \Phi_w}{\partial t} + g\zeta = 0 \quad \text{for } z = \zeta \tag{3.24}$$

since this equation is valid for all values of ζ it is valid for $z = 0$ as well and the wave profile becomes

$$\zeta = -\frac{1}{g}\frac{\partial \Phi_w}{\partial t} \quad \text{for } z = 0 \tag{3.25}$$

Substituting the Eq. (3.20) into (3.25) it is obtained

$$\zeta = \frac{\omega C}{g}\cosh(kh)\cos(kx-\omega t) \tag{3.26}$$

Eq. (3.26) can be written as

$$\zeta = \zeta_a \cos(kx - \omega t) \tag{3.27}$$

where

$$\zeta_a = \frac{\omega C}{g} \cosh(kh) \tag{3.28}$$

Therefore, the corresponding wave potential, as a function of the water depth, is given by

$$\Phi_w = \frac{\zeta_a g}{\omega} \frac{\cosh[k(h+z)]}{\cosh(kh)} \sin(kx - \omega t) \tag{3.29}$$

For deep water $h \to \infty$ (short waves) and the wave potential becomes

$$\Phi_w = \frac{\zeta_a g}{\omega} e^{kz} \sin(kx - \omega t) \tag{3.30}$$

The pressure on the bottom of the cylinder ($z = -DR$) can be obtained from Eq. (3.22) [18]

$$p = \rho g \zeta_a e^{-kDR} \cos(\omega t - kx) + \rho g DR \tag{3.31}$$

where DR is the draft, the distance from undisturbed sea surface to the bottom of the cylinder, see Fig. 3.2 . Since the diameter of the cylinder is small compared to the wave length the pressure distribution on the bottom of the cylinder can be considered uniform and Eq. (3.31) turns into

$$p = \rho g \zeta_a e^{-kDR} \cos(\omega t) + \rho g DR \tag{3.32}$$

and the vertical force on the bottom of the cylinder is given by

$$F = \left[\rho g \zeta_a e^{-kDR} \cos(\omega t) + \rho g DR\right] \frac{\pi}{4} D_c^2 \tag{3.33}$$

where D_c is the diameter of the cylinder. The harmonic part of this force is the regular harmonic wave force and it can be expressed as a spring coefficient times a reduced or effective wave elevation

$$F_{FK} = c\zeta^* \tag{3.34}$$

This wave force is called the Froude-Krilov force and the spring coefficient is given by

$$c = \rho g \frac{\pi}{4} D_c^2 \tag{3.35}$$

and the reduced or effective wave elevation for deep water is given by

$$\zeta^* = e^{-kDR} \zeta_a \cos(\omega t) \tag{3.36}$$

where k is the wave number, given by

$$k_i = \frac{\omega_i^2}{g} \quad \text{for } i = 1, \ldots, N \tag{3.37}$$

The Froude-Krilov forces are obtained from an integration of the pressures on the body in the undisturbed wave. It can be noted that only the harmonic components of the sea surface elevation with lower frequencies have significant contribution to the Froude-Krilov forces.

As part of the waves will be diffracted, there are two additional force components, one proportional to the effective vertical acceleration and one proportional to the effective vertical velocity, therefore the total wave force on the bottom of the cylinder is given by

$$F_w = a\ddot{\zeta}^* + b\dot{\zeta}^* + c\zeta^* \tag{3.38}$$

where a is the hydrodynamic mass coefficient and b is the hydrodynamic damping coefficient. The terms $a\ddot{\zeta}^*$ and $b\dot{\zeta}^*$ are considered to be corrections on the Froude-Krilov force due to diffraction of the waves by the presence of the cylinder in the fluid. Substituting the Eq. (3.36) into 3.38 it is obtained

$$F_w = \zeta_a e{-}kDR\left(c - a\omega^2\right)\cos(\omega t) - \zeta_a e{-}kDR(b\omega)\sin(\omega t) \tag{3.39}$$

This wave force can be written independently in terms of the in-phase and out-of-phase terms

$$F_w = F_a\cos(\omega t + \varepsilon_{F\zeta}) = F_a\cos(\varepsilon_{F\zeta})\cos(\omega t) - F_a\sin(\varepsilon_{F\zeta})\sin(\omega t) \tag{3.40}$$

Equating Eqs. (3.39) and (3.40), the following equations are obtained

$$F_a\cos(\varepsilon_{F\zeta}) = \zeta_a e{-}kDR\left(c - a\omega^2\right) \tag{3.41}$$

and

$$F_a\sin(\varepsilon_{F\zeta}) = \zeta_a e{-}kDR\,(b\omega) \tag{3.42}$$

Adding the square of these two equations results in the wave force amplitude

$$\frac{F_a}{\zeta_a} = e^{-kDR}\sqrt{\left(c - a\omega^2\right)^2 + (b\omega)^2} \tag{3.43}$$

and the division of the in-phase and the out-of-phase term in Eq. (3.41) results in the phase shift

$$\varepsilon_{F\zeta} = \arctan\left\{\frac{b\omega}{c - a\omega^2}\right\} \quad \text{for } 0 < \varepsilon_{F\zeta} 2\pi \tag{3.44}$$

Therefore, the equation of motion of a heaving cylinder under the action of hydromechanical and wave load is given by

$$(m + a)\ddot{z} + b\dot{z} + cz = a\ddot{\zeta}^* + b\dot{\zeta}^* + c\zeta^* \tag{3.45}$$

3.4
Response in Regular Waves

In this section and in the next one, the steps for obtaining the dynamics of the platform will be explained.

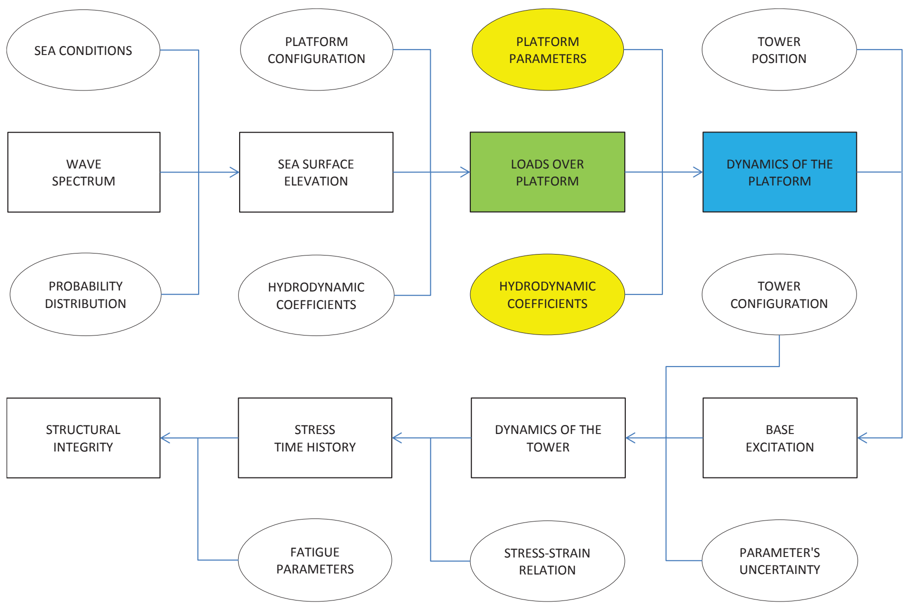

Figure 3.5: Obtaining the dynamics of the platform

The heave response to the regular wave excitation is given by [18]

$$z = z_a \cos\left(\omega t + \varepsilon_{z\zeta}\right) \tag{3.46}$$

and substituting the Eqs. (3.36) and (3.46) into (3.45) yields

$$\begin{aligned} z_a\left[c-(m+a)\omega^2\right]\cos\left(\omega t+\varepsilon_{z\zeta}\right)-z_a b\omega\sin\left(\omega t+\varepsilon_{z\zeta}\right) = \\ = \zeta_a e^{-kDR}\left(c-a\omega^2\right)\cos(\omega t)-\zeta_a e^{-kDR}b\omega\sin(\omega t) \end{aligned} \tag{3.47}$$

By equating the two out-of-phase terms and the two in-phase terms, the following two equations are obtained

$$z_a\left\{\left[c-(m+a)\omega^2\right]\cos\left(\varepsilon_{z\zeta}\right)-b\omega\sin\left(\varepsilon_{z\zeta}\right)\right\} = \zeta_a e^{-kDR}\left(c-a\omega^2\right) \tag{3.48}$$

and

$$z_a\left\{\left[c-(m+a)\omega^2\right]\sin\left(\varepsilon_{z\zeta}\right)+b\omega\cos\left(\varepsilon_{z\zeta}\right)\right\} = \zeta_a e^{-kDR}b\omega \tag{3.49}$$

Adding the squares of these two equations results in the heave amplitude characteristics

$$\frac{z_a}{\zeta_a} = e^{-kDR}\sqrt{\frac{(c - a\omega^2)^2 + (b\omega)^2}{[c - (m+a)\omega^2]^2 + (b\omega)^2}} \tag{3.50}$$

and eliminating the term $z_a/\zeta_a e^{-kDR}$ from Eqs. (3.48) and (3.49) yields the phase shift characteristics

$$\varepsilon_{z\zeta} = \arctan\left(\frac{-mb\omega^3}{(c - a\omega^2)\left[c - (m+a)\omega^2\right) + (b\omega)^2}\right) \quad \text{for } 0 \leq \varepsilon_{z\zeta} \leq 2\pi \tag{3.51}$$

It can be noted that the requirements of linearity are fulfilled, namely, the heave amplitude is proportional to the wave amplitude and the phase shift is not dependent on the wave amplitude. The amplitude and phase characteristics are called the frequency characteristics of the vessel. The amplitude characteristic is also called the Response Amplitude Operator (RAO).

3.5 Response in Irregular Waves

The heave response spectrum can be found by using the transfer function of the motion and the wave spectrum [18]

$$S_z(\omega) = \left|\frac{z_a}{\zeta_a}(\omega)\right|^2 S_\zeta(\omega) \tag{3.52}$$

The moments of the heave response are given by

$$m_{nz} = \int_o^\infty \omega^n S_z(\omega)\mathrm{d}\omega \quad \text{for } n = 0, 1, 2, \ldots \tag{3.53}$$

The significant heave amplitude, that is the mean value of the highest one-third part of the amplitudes, is given by

$$\bar{z}_{a1/3} = 2RMS = 2\sqrt{m_{0z}} \tag{3.54}$$

where RMS is the Root Mean Square value. A mean period can be found from the centroid of the spectrum

$$T_1 z = 2\pi \frac{m_{0z}}{m_{1z}} \tag{3.55}$$

The average zero-crossing period is given by

$$T_{2z} = 2\pi \sqrt{\frac{m_{0z}}{m_{1z}}} \tag{3.56}$$

Wu and Hermundstad [43] presented a nonlinear time-domain formulation for ship motions and wave loads and a nonlinear long-term statistics method. Initially they presented the theoretical long-term probability of exceedance per unit time, assuming the linearity of the ship-fluid system and that the short-term response is a stationary Gaussian narrow-band process with zero mean and therefore the peak values are distributed according Rayleigh distribution. In this case the probability of exceedance per unit time is given by

$$P_R(y > y_1) = \int_R \int_\beta \int_H \int_T \mathrm{e}^{-y_1^2/2R} np(\beta, H, T)\mathrm{d}R\mathrm{d}\beta\mathrm{d}H\mathrm{d}T \tag{3.57}$$

where y are the wave-induced loads, R is the *zeroth* spectral moment representing the mean square of each short-term response, n is the average number of maxima or minima per unit time in each short-term response, β is the wave heading, H is the wave height and T is the wave period . A completely independent calculation using Eq. (3.57) was carried out for each loading condition. Since the joint probability p is not available in advance a few simplifications were necessary and the Eq. (3.57) can be approximated by the following summation

$$P_R(y > y_1) \approx \sum_\beta \sum_{H_s} \sum_{T_1} \mathrm{e}^{-y_1^2/2R} nP_1(\beta)P_2(H_s, T_1) \tag{3.58}$$

where the joint probability P_2 is presented for a given ocean area in the form of a scatter diagram. Since the nonlinear response is no longer Gaussian, the distribution of peak values is not according Rayleigh distribution, and Wu and Hermundstad used an alternative probability density function

$$f_g(y) = \frac{c}{\Gamma(r)}\mu^{cr} y^{cr-1} \mathrm{e}^{-(\mu y)^c} \qquad 0 \leq y \leq \infty \tag{3.59}$$

where Γ is the Gamma function and μ, c and r are parameters of the distribution that can be evaluated through certain moments of the histogram or by a weighted curve fitting. The histogram of peak values, together with the average number of maxima or minima for each wave heading and sea state, are obtained from the nonlinear time-domain simulation. The probability distribution function is given by

$$F_g(y) = \int_0^y f_g(x)\mathrm{d}x = \frac{\Gamma_l\left(r, (uy)^c\right)}{\Gamma(r)} \tag{3.60}$$

where Γ_l is the Lower Incomplete Gamma function given by

$$\Gamma_l\left(r, (uy)^c\right) = \int_0^{(uy)^c} u^{r-1}\mathrm{e}^{-u}\mathrm{d}u \tag{3.61}$$

After some manipulation the long-term probability of exceedance for nonlinear responses is given by

$$P(y > y_1) \approx \sum_\beta \sum_{H_s} \sum_{T_1} \frac{(\mu y_1)^{c(r-1)}\,\mathrm{e}^{-(\mu y_1)^c}}{\Gamma(r)} nP_1(\beta)P_2(H_s, T_1) \tag{3.62}$$

Wu and Hermundstad compared the long-term obtained bending moments over the ship with those given by classification societies and a good agreement has been obtained and intend to use the method for accurately evaluating the extreme wave loads and other nonlinear responses in ship design.