APPENDIX A
Generalization with respect to second-stage problems

An essential part of the methodology proposed in this work – namely the linearization of the product of binary variables using the properties of the logarithm – lies on the piecewise linear approximation of the exponential function which may be represented in the optimization problem as linear constraints. However, if the objective function value of the optimal solution of a second stage problem is negative, the first stage problem clearly becomes unbounded and this would represent a limitation to the applicability of the concepts discussed afterwards.

This Appendix describes how to address this situation so that the methodology remains valid, regardless of the values of the optimal solutions of the second-stage problems. In summary, the next two sections discuss that a constant term may be added to the value of the optimal solution of all second stage problems without affecting the solution of the original problem.

A.1
Problems solved with full scenario enumeration

In the case where one is able to enumerate all the possible scenarios of network configuration, the objective function (3.12) may be re-written as follows:

$$\sum_{e \in E} r_e x_e + \sum_{s \in S} \bar{g}_s \hat{p}_s - \bar{g} \tag{9.1}$$

where $\bar{g}_s = g_s + \bar{g}$ and $\bar{g}$ is such that $\bar{g}_s > 0, \forall s \in S$. Expression (9.1) is clearly equivalent to:

$$\sum_{e \in E} r_e x_e + \sum_{s \in S} g_s \hat{p}_s + \sum_{s \in S} \bar{g} \cdot \hat{p}_s - \bar{g} \tag{9.2}$$

On the one hand, the algebraic sum of the third and fourth terms of expression (9.2) will always amount to zero since $\sum_{s \in S} \hat{p}_s = 1$. On the other hand, the sum of the first and second terms above is the exact expression of the original objective function. The alternate objective function (9.1) assumes the exact same values as the original one (3.12) for all feasible values of the decision variables and, consequently, problem (3.12) – (3.17) is equivalent to problem (9.1) – (3.13) – (3.17).

A.2
Problems solved with a sample of scenarios

Following the methodology proposed in the thesis, large-scale problems are solved using the formulation (5.3) – (5.8). Using the same rationale as above, the objective function (5.3) may be re-written as:

$$\sum_{e \in E} r_e x_e + \frac{1}{|S|} \sum_{s \in S} \bar{g}_s \left(\frac{\hat{p}_s}{p_s^{INI}} \right) - \bar{g} \tag{9.3}$$

which is equivalent to:

$$\sum_{e \in E} r_e x_e + \frac{1}{|S|} \sum_{s \in S} g_s \left(\frac{\hat{p}_s}{p_s^{INI}} \right) + \frac{1}{|S|} \sum_{s \in S} \bar{g} \left(\frac{\hat{p}_s}{p_s^{INI}} \right) - \bar{g} \tag{9.4}$$

If we denote by D the set of distinct scenarios in the sample and by n_d the number of occurrences of each one of them, the third term may be written as:

$$\bar{g} \sum_{d \in D} \frac{n_d}{|S|} \left(\frac{\hat{p}_d}{p_d^{INI}} \right) \tag{9.5}$$

As $|S| \to \infty$, $\frac{n_d}{|S|} \to p_d^{INI}$ and expression (9.5) converges to $\bar{g} \cdot \sum_{d \in D} \hat{p}_d$. Since $\sum_{d \in D} \hat{p}_d \to 1$, the result is analogous to that obtained in the previous Section.

APPENDIX B

Solution robustness

The solution of two-stage stochastic programs depends, essentially, on balancing the trade-off between deterministic first-stage costs and the expected value of probabilistic second-stage costs. It is thus imperative that we have a reasonable estimate of second stage costs in order to be able to have confidence in the quality of the solution obtained.

On the one hand, the larger the set of sampled scenarios, the better the estimate of second stage costs will be. On the other hand, having fewer scenarios makes the problem smaller and solution times are usually faster. Anyhow, once a solution is found for a given set of scenarios, a Monte Carlo simulation – in which the probability distribution of the edges' availabilities takes into account the determined first-stage decisions – may then provide a confidence interval against which the estimate of the expected costs of the second-stage can be compared in order to assess the need for a larger number of samples. This suggests the following algorithm, detailed below:

```
1   Initialize the set of cuts $K = \emptyset$, define the maximum percentage error $\varepsilon$ and
    the confidence level for the estimator of the mean second stage costs
    (expressed in terms of the number of standard deviations $\theta$)
2   Initialize the lower bound $LB = -inf$, upper bound $UB = +inf$
3   While $\hat{\mu}_S \notin \{\hat{\mu}_M - \theta \cdot \sigma_{\hat{\mu}_M}, \hat{\mu}_M + \theta \cdot \sigma_{\hat{\mu}_M}\}$
        Generate a new sample $S_{aux}$ of network configuration scenarios based
        on the initial probability distribution of the edges' availabilities
        For each scenario $s \in S_{aux}$
            Solve problem (3.1) – (3.4) and obtain the corresponding value $g_s$
        End For
4       Set $S = S \cup S_{aux}$
5       While $|(UB - LB)/UB| > \varepsilon$
6           Solve problem $P_3$ defined by (5.3) – (5.18) with the currently
            defined set of cuts $K$ and scenario sample set $S$
```

```
        Set $LB = v(P_3)$
        Set $UB_{aux} = \sum_{e \in E} r_e x_e^* + \frac{1}{|S|} \sum_{s \in S} g_s \left( \frac{\exp(w_s^*)}{p_s^{INI}} \right)$
        If $UB_{aux} < UB$, set $UB = UB_{aux}$
        For each scenario $s \in S$
            Add the cut defined by $\alpha_k = \exp(w_s^*) \cdot (1 - w_s^*)$ and
            $\beta_k = \exp(w_s^*)$ to the cut set $K$
        End For
    End While
    Compute $\hat{\mu}_S = \frac{1}{|S|} \sum_{s \in S} g_s \left( \frac{\exp(w_s^*)}{p_s^{INI}} \right)$
    Generate a sample $M$ ($|M| \gg |S|$) of network configuration scenarios
    based on the probability distribution of the edges' availabilities which
    results from the determined first stage decisions
    For each scenario $m \in M$
        Solve problem (3.1) – (3.4) and obtain the corresponding value $g_m$
    End For
    Obtain the estimator of the mean of second stage costs and its standard
    deviation: $\hat{\mu}_M$ and $\sigma_{\hat{\mu}_M}$, respectively
End While
```

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