2
HUMANITARIAN LOGISTICS PROBLEM

2.1
Introduction

The impact of natural or man-made disasters can be very significant in terms of death toll and damages to affected regions. Earthquakes, hurricanes and floods have recently proven their catastrophic potential and concerns over global warming and climate change worsen the perspective in years to come. Besides the immediate loss of lives and destruction of infra-structure, the effects of these calamities usually last long after the initial strike. When an earthquake strikes a city, for example, utility services such as water, electricity and gas may have to be interrupted for weeks before necessary repairs are carried out. On top of that, several roads and bridges are usually affected, rendering the transportation network severely impaired. It has been pointed out [64] that more casualties actually happen due to the isolation to which many residents are forcefully put to rather than by the event itself. This has also been the experience reported by humanitarian organizations in the aftermath of the recent earthquake in Haiti [28].

In face of that, regions that are prone to the occurrence of natural disasters must take preventive measures in order to mitigate potential damages, and devise emergency plans so that they are able to provide care for those affected by such events. It is clear that it is very important to assess the vulnerability of the transportation network and to take steps aimed at guaranteeing that it will be possible to either evacuate people to safe locations or to provide them with basic resources in post-disaster days.

The objective of the humanitarian logistics problem is to determine the optimal set of investments on the seismic retrofit of the links of a transportation network so as to minimize the sum of (deterministic) investment costs and expected (probabilistic) costs incurred when transporting people and/or resources after a catastrophic event. Investment in bridges and tunnels, for example, may increase their resilience so that an earthquake is less likely to render them unusable – Cooper et al. (1994) [19]. Such investments usually involve very large sums of money and a limited budget must thus be optimally allocated.

2.2
Literature review

The literature on the humanitarian logistics problem is very limited. To the best of our knowledge, there are only three papers that deal with the same (or a very similar) problem as the one studied in this thesis.

Viswanath, Peeta and Salman (2004) [66] were the first to state the problem, motivated by the risks of an earthquake hitting Istambul, the capital of Turkey. They limit the scope of their model to the case where one is interested in maintaining connectivity between origin (O) and destination (D) pairs. Their approach relies on the enumeration of the paths O-D (which, for practical purposes and due to computational difficulties is limited to listing a pre-defined number of paths by using a *k*-shortest path algorithm). Next, they propose an approximation of the objective function based on the first order terms of its Taylor series expansion. As they recognize in their article, the disadvantage of this approach is that by ignoring higher order terms they neglect the potential synergies of simultaneously investing in more than one link.

Liu, Fan and Ordonez (2006) [43] and Fan and Liu (2009) [24] also study the stochastic network protection problem. In the former, the problem follows the same outline as that described above [66] and they propose an extension of the L-Shaped method of Van Slyke and Wets by using generalized Benders decomposition. In the latter, the second-stage problem involves the determination of a Nash equilibrium by solving an MPEC (mathematical program with equilibrium constraints) which results from the consideration that users may choose their own best-perceived routes along the network. Their solution method relies on the application of the Progressive Hedging algorithm of Rockafellar and Wets (1991).

Both articles, however, make the explicit assumption that the decision to invest on the reinforcement of a link eliminates the probability that it might become unavailable after the disaster. They argue that it would be preferable and more realistic to maintain a probabilistic view on link failures but doing so would lead the problem to fall under the class of stochastic programming problems with decision-dependent uncertainties for which "mathematical analysis (…) is very sparse, and is only limited to convex problems of special structures" thus relying "heavily on heuristic methods to solve problems with realistic sizes due to computational difficulties".

Although not dealing with the same problem, there are some related works on the investment in links of a stochastic network. Wollmer (1980) [69] focused on a generalized multicommodity network in which links have random capacities. He formulated the problem as a two-stage stochastic program – where first-stage decisions are the amounts to be invested on the increase of link capacities, and second-stage variables represent the flows of each commodity through the links – and proposed a cutting plane technique that exploits network structure. In 1987, Wallace [67] studied the problem of investing in new links in a network where existing link capacities are random. He also formulated it as a standard two-stage stochastic program and suggested decomposition strategies to solve it. Again in 1991, Wollmer [70] worked on a problem in which one seeks to optimize the tradeoff between first-stage investment costs and second-stage expected maximum flow between a pair of vertices. The formulation follows the regular two-stage stochastic programming framework and was solved using an algorithm based on cutting planes.

Finally, there is also a significant body of work on the development of plans for disaster preparedness and response which adopt a different perspective from that of mathematical programming. Instead, these works usually take a somewhat heuristic view to determine critical links of a network based on a set of pre-defined criteria. Sohn et al. (2003) [59] and Sohn (2006) [58] study the prioritization of links which may become unavailable due to earthquakes in the Midwest states in the US or due to floods in Maryland, US. Based on a disaster scenario, they analyze the potential disruptions and their consequences with respect to travel delays, reconstruction costs and accessibility to affected cities/counties. This is also in line with the approach of Basoz and Kiremidjian (1995) [6] and Bana e Costa, Oliveira and Vieira (2008) [7] who use Palo Alto, CA and Lisbon, respectively, as case studies for their methodologies which consider the physical characteristics of bridges and the social and economical aspects which may be adversely affected by disasters.

2.3 Mathematical formulation

Mathematically, the problem is formulated by assuming we are given an undirected graph $G = (N, E)$ with vertex set N and edge set E. Vertices represent locations where survivors and/or resources may be located, and edges represent the roads, bridges and tunnels which comprise the transportation network. For ease of presentation, a deterministic supply or demand h_i is associated with each vertex i. Edges have non-negative transportation costs c_e, capacity u_e and are assumed to be available after the occurrence of the disastrous event with probabilities p_e^C. As also stated in related works [66], it is assumed that each edge fails independently of the others – although this is not a necessary assumption for the methods proposed in this work. The survival probability of an edge may be increased to p_e^I if an amount r_e is invested in it. We associate the availability status of an edge (*i.e.*, whether the edge remains operational or not) to the value of a random variable ξ_e, which is equal to 1 if the edge e is operational and 0 otherwise.

Assuming that we are able to enumerate all the possible scenarios S of network configuration, the problem may be formulated as follows:

$$(\boldsymbol{P}) \quad Min \quad \sum_{e \in E} r_e x_e + \sum_{s \in S} p_s \left(\sum_{e \in E} c_e y_{es} + \sum_{i \in N} d_i z_{is} \right) \qquad (2.1)$$

$$subject\ to: \quad Ax \leq b \qquad (2.2)$$

$$W_s y_s + z_s = h_s \qquad \forall s \in S \qquad (2.3)$$

$$p_s = \prod_{e \in E} (\mathrm{p}_{es}^C + (\mathrm{p}_{es}^I - \mathrm{p}_{es}^C) \cdot x_e) \qquad \forall s \in S \qquad (2.4)$$

$$y_{es} \leq u_e \xi_{es} \qquad \forall s \in S, \forall e \in E \qquad (2.5)$$

$$x \in \{0,1\}^{|E|};\ y, z \in \mathbb{R}^+ \qquad (2.6)$$

where:

ξ_{es} realization of random variable ξ_e in scenario s

p_{es}^C probability of the availability status of edge e in scenario s, given

	that no investment is made on it (i.e., $P(\xi_e = \xi_{es} \mid x_e = 0)$ or, alternatively, $q_e^C \cdot \xi_{es} + (1 - q_e^C) \cdot (1 - \xi_{es})$
p_{es}^I	probability of the availability status of edge e in scenario s, given that a reinforcement investment is made on it (i.e., $P(\xi_e = \xi_{es} \mid x_e = 1)$ or, alternatively, $q_e^I \cdot \xi_{es} + (1 - q_e^I) \cdot (1 - \xi_{es})$
d_i	penalty cost for the non-fulfillment of demand of vertex i
p_s	continuous variable equal to the probability of scenario s
x_e	binary variable which is equal to 1 if an investment is to be made on edge e, 0 otherwise
y_s	vector of continuous flow variables of scenario s
z_s	vector of continuous slack variables for the demand and supply of each vertex in scenario s

The objective function (2.1) to be minimized provides the sum of deterministic costs incurred in the first stage due to decisions of reinforcement investments and expected second-stage costs of routing commodities through the network and demand curtailment. Expressions (2.2) and (2.3) represent, respectively, the sets of first-stage constraints (such as budget limitations, minimum investment in each region, etc.) and second-stage constraints (such as mass-balance equations on the realized network configuration of each scenario). Expression (2.4) defines variables p_s as a function of investment decision variables x_e and constraint (2.5) determines the upper bound of the flow in edge e, according to the realization of the random variable ξ_e in scenario s.

Problem (2.1) – (2.6) is a mixed-integer nonlinear program for which solution methods are usually not guaranteed to find a global optimal solution. In particular, there are three main difficulties associated with this formulation that prevent existing algorithms to obtain global optimal solutions. These obstacles are briefly described below; following that, Chapter 0 presents a reformulation scheme that overcomes the first two difficulties and Chapter 5 proposes a solution to the third.

1) **Non-linearity due to product of first and second stage variables.** In standard stochastic programming problems the probability of a scenario is

known and it thus usually becomes a coefficient of the objective function. In the case of the class of problems being studied in this work, the expression for the expected value of second stage costs – $\sum_{s\in S} p_s(\sum_{e\in E} c_e y_{es} + \sum_{i\in N} d_i z_{is})$ – involves the product of first stage variables p_s – since, as described earlier, first stage decisions affect the probability of occurrence of each possible outcome – and second stage variables y_{es} and z_{is}.

2) **Non-linearity due to the expression for the scenarios' probabilities.** A second source of non-linearity arises from the expression that defines variables p_s themselves, which represent the probabity of occurrence of each possible network configuration after taking into account first stage investment decisions. In this case, the expression involves non-linear terms of order up to $|E|$ due to products of binary variables x_e : $p_s = \prod_{e\in E}(\mathrm{p}^C_{es} + (\mathrm{p}^I_{es} - \mathrm{p}^C_{es}) \cdot x_e)$. These non-linear terms arise from the product of the probability of occurrence of the outcome of each random variable that composes a scenario.

3) **Scenario generation.** As previously mentioned, most stochastic programming models deal with random variables whose probability distribution is independent of the decision variables. This *a priori* knowledge of the joint probability distribution allows one to obtain scenarios for the realization of the random variables and their respective probabilities of occurrence – either by sampling from it in a Monte Carlo fashion or by constructing them based on a given criteria (e.g., moment matching such as in Kaut and Wallace (2007) [41] and Kaut, Wallace and Hoyland (2003) [42] or minimization of distances between probability measures – Romisch (2009) [50], Heitsch and Romisch (2005) [35] and Hochreiter and Pflug (2007) [37]) – which may then be used to numerically compute the expectation of second stage costs, as described in Chapter 1. Since the probability distribution of the random variables is not known beforehand in the class of problems being studied in this thesis (i.e., it can only be computed after first stage decisions are determined), one cannot rely on existing scenario generation methods.