



Bruno da Costa Flach

**Stochastic Programming with Endogenous Uncertainty:
An Application in Humanitarian Logistics**

Tese de Doutorado

Thesis presented to the Postgraduate Program in Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC–Rio in partial fulfillment of the requirements for the degree of Doutor em Engenharia Elétrica.

Advisor: Prof. Álvaro de Lima Veiga Filho

Co-Advisor: Prof. Marcus Vinicius Soledade Poggi de Aragão

Rio de Janeiro

April, 2010



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To my wife, Lavinia

To my parents, Otto and Lânia, and my sister Luciana

To my grandma “vovó Nysa” and my uncle Marcelo

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To God, for the opportunity to live and experience His universe.

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Abstract

Flach, Bruno da Costa; Veiga Filho, Álvaro de Lima (Advisor); Aragão, Marcus Vinicius Soledad Poggi de (Co-Advisor). **Stochastic Programming Models with Endogenous Uncertainty: An Application in Humanitarian Logistics**. Rio de Janeiro, 2010. 74 p. PhD Thesis – Electrical Engineering Department, Pontifical Catholic University, Rio de Janeiro (PUC-Rio).

In this work we study a class of stochastic programming problems with endogenous uncertainty – i.e., those in which the probability distribution of the random parameters is decision-dependent – which is formulated as a mixed integer non-linear programming (MINLP) problem. Although discussed in the context of the humanitarian logistics problem, the proposed methodology and obtained results are also valid for a more general class of problems which comprehends a variety of applications. In particular, we propose (i) a convexification technique for polynomials of binary variables, (ii) an efficient cut-generation algorithm and (iii) the incorporation of importance sampling concepts into the stochastic programming framework so as to allow the solution of large instances of the problem. Computational results demonstrate the effectiveness of the proposed methodology by solving instances significantly larger than those reported in related works.

Keywords

Stochastic programming; Endogenous uncertainty; Convexification; Importance sampling; Humanitarian logistics.

Resumo

Flach, Bruno da Costa; Veiga Filho, Álvaro de Lima (Orientador); Poggi de Aragão, Marcus Vinicius Soledad (Co-Orientador). **Modelos de Programação Estocástica com Incertezas Endógenas: Uma Aplicação em Logística Humanitária**. Rio de Janeiro, 2010. 74 f. Tese de Doutorado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Neste trabalho estudamos uma classe de problemas de otimização estocástica com incertezas endógenas que é formulado como um problema de programação não-linear inteira (MINLP). Esta classe de problemas difere dos problemas de otimização estocástica geralmente estudados na literatura pelo fato de que a distribuição de probabilidade dos parâmetros aleatórios depende das decisões tomadas. Apesar de discutido dentro do contexto do problema de logística humanitária, a metodologia proposta e os resultados obtidos são válidos para uma classe geral de problemas que agrega uma variedade de aplicações. Em particular, propõe-se (i) uma técnica de convexificação de polinômios de variáveis binárias, (ii) um algoritmo de geração de cortes e (iii) a incorporação dos conceitos de *importance sampling* dentro do contexto de otimização estocástica de modo a permitir a solução de grandes instâncias do problema. Os resultados computacionais apresentados demonstram as vantagens da metodologia proposta ao permitir a solução de instâncias significativamente maiores que aquelas atualmente apresentadas em trabalhos relacionados.

Palavras-chave

Otimização estocástica; Incertezas endógenas; Convexificação; Logística Humanitária.

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