PONTIFÍCIA UNIVERSIDADE CATÓLICA
DO RIO DE JANEIRO

Gustavo Souto dos Santos Diz

Maritime Inventory Routing:

a practical assessment and robust optimization approach

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Engenharia de Produção of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Engenharia de Produção.

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Co-Advisor: Prof. Fabrício Oliveira

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To my wife Alessandra,
my son Théo and my daughter Nina.

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Abstract

Diz, Gustavo Souto dos Santos; Hamacher, Silvio (advisor); Oliveira, Fabrício (co-advisor). **Maritime inventory routing: a practical asessment and robust optimization approach**. Rio de Janeiro, 2017. 115p. Tese de Doutorado – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

Maritime inventory routing (MIR) problem is an academic name for a practical logistic problem that represents the routing or scheduling of vessels to carry product(s) between ports. Meanwhile, the product(s) inventory levels in these ports must remain between operational bounds during the entire planning horizon. This thesis focus on how to support decision on a real-life MIR problem faced by a Brazilian petroleum company. To do so, we structure a set of tests to compare different formulation from literature and identify which is more adherent to real problem. Due to computational complexity of the problem, we present an heuristic approach that provides reasonably good solutions when compared to deterministic mixed integer linear programming (MILP) formulations and reduces considerably the computational time of solving real-life instances. However, uncertainty events have great impact in the ship scheduling planning. Therefore, we propose a robust optimization approach that considers uncertainty in the time spent at ports in each ship visit. Our approach is able to determine the probability of infeasibility and the impact in the objective function for each level of robustness, helping to measure the uncertain aversion of the decision maker. Our experiments identified that, for a certain instance, varying the level of robustness one may reduce the probability of infeasibility from 87% (of deterministic solution) to 2% and it represents an increase in the transportation costs of about 13%.

Keywords

maritime inventory routing; mixed-integer linear programming; valid inequalities; relax-and-fix; fix-and-optimize; and robust optimization

Resumo

Diz, Gustavo Souto dos Santos; Hamacher, Silvio; Oliveira, Fabrício. **Roteamento de navios com gestão de estoques: Uma avaliação prática e uma abordagem robusta.** Rio de Janeiro, 2017. 115p. Tese de Doutorado – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

O problema de roteamento de navios com gestão de estoques (conhecido pelo termo em inglês *Maritime inventory routing* ou MIR) representa um problema prático de logística onde o transportador da carga também é responsável pela manutenção dos estoques do produto transportado nos portos de carga e descarga. Esta tese estuda um caso real do problema MIR. Um conjunto de testes é apresentado de modo a comparar diferentes formulações matemáticas da literatura, a fim de encontrar aquela mais aderente ao problema real. Em função da complexidade computacional do problema, é apresentada uma abordagem heurística que consegue encontrar soluções similares e reduz consideravelmente o tempo computacional quando comparadas com as formulações baseadas em PLIM. No entanto, problemas reais são muito influenciados por aspectos incertos. Sendo assim, é apresentada uma abordagem robusta para a otimização do problema MIR, que considera incerteza no tempo de estadia do navio nos portos. A abordagem apresentada produz soluções para diferentes níveis de robustez. Em outras palavras, considera o risco de variação no tempo de estadia do navio em um porto durante uma operação de carga ou descarga. Assim, é capaz de determinar a probabilidade de inviabilidade da solução encontrada para cada nível de robustez oferecido, além do impacto no custo de transporte à medida que soluções mais robustas são apresentadas. Esta abordagem oferece ao tomador de decisão a medida do *trade-off* entre robustez e custo de transporte. Desta forma, o mesmo pode determinar qual o nível de conservadorismo irá adotar em sua programação de navios e quanto isto irá impactar o custo de transporte. Os experimentos apresentados identificaram que, aumentos sutís no nível de robustez (com pequeno impacto no custo de transporte) podem reduzir consideravelmente a probabilidade de inviabilidade de uma solução.

Palavra-chave

maritime inventory routing; programação linear inteira mista; inequações válidas; *relax-and-fix; fix-and-optimize;* e otimização robusta

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1. Introduction

Maritime transportation is one of the most important mode for global trading. UNCTAD (2015) reports that approximately 80% of the total volume traded around the world is carried by sea. In 2014, 9.8 billion tons of goods were loaded at ports worldwide, and tanker trade (crude oil, petroleum products, and gas) contributed to nearly 40% of this total. Logistics problems involving maritime transportation have received increased attention from industry and academia over the past few years (Fagerholt, 2004; Christiansen, Fagerholt, and Ronen 2004; Christiansen et al. 2013). In particular, the petroleum industry and crude oil transportation problems have receiving considerable attention (Bausch, Brown, and Ronen 1998; Perakis and Bremer 1992; Bremer and Perakis 1992; Furman et al. 2011; Rocha et al. 2013).

This thesis focus on a maritime transportation problem known as maritime inventory routing (MIR). In problems like MIR, the player responsible for carrying the products from supplier ports to consumer ports is also in charge of managing the inventory level at these ports. Some authors also named this problem inventory-constrained maritime routing problem (Al-Khayyal and Hwang, 2007 and Stanzani, 2017), as inventory costs are not included in objective function and the model is required to maintain inventory levels between certain limits at the ports. Nevertheless, hereinafter we will use the name MIR.

MIR problems feature the supply chain of many bulk product industries, such as grain, coal, iron ore, and petrochemicals. In particular, the petroleum industry offers an interesting inventory-routing environment for maritime transportation due to the need for transportation of huge volumes combined with limited storage capacities at consumer and production sites.

We study the case of a vertically integrated Brazilian petroleum company. In this case, the company needs to carry its crude oil production from production sites to onshore terminals, where the crude oil will be consumed by refineries. In addition, it is vital to maintain crude oil inventory levels between certain operational limits at production and consumption ports to avoid interruption of production processes because of storage capacity (at production sites) and/or inventory (at onshore terminals) shortages.

This practical MIR problem has some important characteristics that motivate our study. It is highly capital intensive, greatly influenced by uncertain events, and computationally complex to solve, offering a wide range of improvement opportunities through academic research. As an illustration of these characteristics, in the case we study, the company spends approximately US$ 1.2 million per day on fuel. The average crude oil inventory level along the entire supply chain is approximately 40 million barrels, which is responsible for high inventory costs. The violation of inventory physical limits at ports can represent interruptions in the crude oil production or disruptions in the refining process. A single day of interruption of average crude oil production may represent more than US$ 5.0 million (using US$ 60.00 as the price of a crude oil barrel), which is more than four times the daily fuel expense of the company's entire fleet. Despite the great potential of monetary benefits related to the optimization of MIR problems, researchers recognize how complex is to solve this type of problem for real-life instances (Furman et al., 2011; Agra et al., 2013; and Papageorgiou et al., 2014). Some authors reported that the use of heuristics to overcome the computational complexity of real-life MIR problems have resulted good results (Dauzère-Pérès et al., 2007, Christiansen et al., 2011, Rodrigues et al., 2016 and Uggen et al., 2016).

Besides the enormous opportunity of reducing maritime transportation costs and the computational complexness of solving such problems, another important issue that influences MIR problems decision making is uncertainty. Uncertain events are responsible for a great part of transportation costs. In order to be protected against uncertainty and ensure that all crude oil production will be streamlined, the fleet is slightly oversized, in order to prevent against the worst occurrences of some uncertainty events. Some examples of uncertainty parameters that influence ship-scheduling decisions are bad weather, vessel degradation, bureaucracy, variations in production and consumption rates, and delays in time spent at port. Uncertainty increases inefficiency of the fleet, raises maritime transportation costs, and makes ship scheduling decisions even harder, pushing inventory levels to the limits and raising the risk of violating inventory bounds.

1.1. Objectives and contributions

In front of such a challenging problem, which involves high transportation costs, uncertainty events and computational complexity, this thesis has the following objectives:

- Examine the academic literature and identify the mathematical formulations that is more adherent to a real-life MIR problem;
- Propose a heuristic approach to improve solution quality and reduce computational time;
- Propose a robust approach to solve MIR problems considering uncertain parameters in the time spent by vessels at port; and
- Demonstrate how a robust approach can leverage fleet efficiency, reducing the risk of violating inventory limits, while keeping maritime transportation costs controlled.

The main contributions of this research provides are:

- Propose a set of real-life instances that are more difficult than the ones found in literature;
- For the first time in literature, a structured framework that compares continuous time and discrete time formulations for MIR problems is presented, indicating which is the most appropriate for real-life MIR problems;
- Propose a constructive and improvement heuristic approach based on *relax-and-fix* and *fix-and-optimize* heuristics to solve MIR problems that reduces computational time considerably and reaches similar solutions that the ones obtained using conventional MILP formulations;
- Compare different ways of selecting variables to fix and relax during the heuristic execution and demonstrate which combination is the best performing for a real-life MIR problem;
- Quantify the risks of failing service level (infeasible solutions) and impacts of level of conservativeness admitted in transportation costs, considering

uncertainty in the time spent by vessels at ports. As far as we know, this is the first time in literature that such trade-off assessment is presented in literature; and

- Provide a decision support tool that helps the decision maker on scheduling decisions based on the risk of failing level of service and, at the same time, minimize transportation costs. Such contribution is of great value to the company once, they did not count with any decision support system to ship scheduling decisions that considers uncertainty.

1.2. Thesis organization

In the second Chapter, we describe the real-life MIR problem we used as case of study here, the assets involved and ship scheduling decision-making process. We also focus on uncertain aspects that influence the ship scheduling decision-making process.

In Chapter 3, we review literature about MIR problem and present the three deterministic formulations found in literature that we consider that are more adherent to the real-life MIR problem we study here. We also present valid inequalities found in literature that were used to strengthen the formulations. Moreover, we propose some reformulations in order to make them meaningfully comparable among themselves.

In Chapter 4, we present an experimental framework that compares the formulations of literature. We also describe how to use lower bounds of the continuous time model to close the optimality gap of some instances and present some conclusions draw from the tests we performed.

In Chapter 5, we propose a heuristic approach based on *relax-and-fix* and *fix-and-optimize* in order to improve quality of solution and reduce computational time.

In Chapter 6, we describe the robust optimization approach to solve a MIR problem considering uncertainty in the time spent by vessels at ports.

In Chapter 7, we discuss the computational experiments and the results of the robust optimization approach implementation in a practical case.

Finally, in Chapter 8 we present main conclusions of this study and future research.

2. A practical MIR problem

To support numerical experiments throughout this thesis, we use a real-life MIR problem as a case of study based PETROBRAS, a Brazilian petroleum company. PETROBRAS is the largest Brazilian petroleum company. Most of its production is offshore with the use of a vessel called Floating Production Storage and Offloading (FPSO), making the company highly dependent on maritime transportation to offload its production. As a vertically integrated company, PETROBRAS is responsible for every step of its supply chain, from the exploration and production of the crude oil, including transportation, storing, refining, and distribution of refined products to customer. Such supply chain offers an important MIR problem that we will use as a case of study here.

2.1. MIR problem: Brazilian case

PETROBRAS crude oil supply chain involves a great effort to store, manage inventory, transport, assure its quality and deliver the crude oil in time at the right place. Managing all these tasks without relying on a decision support system is prone to inefficiency and waste of economic resources. Figure 1 gives an overview in the crude oil flow, from production to refining.

After the production in the FPSOs, the oil flows to onshore terminals. A fleet of dedicated vessels carries most of the crude oil, but a small part is pumped through

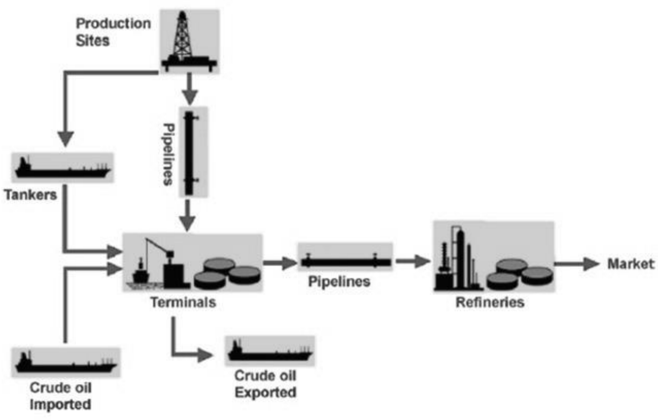

Figure 1: Crude oil flow in the company's supply chain.

pipelines directly to onshore terminals. From the onshore terminals, the crude oil either follows to refineries through pipelines, or is exported to foreign customers.

It is also necessary to import light-weight oil to complement the ideal oil blend of refineries.

We focus our attention on the part of the process immediately from the production of crude oil in the FPSOs to the supply of onshore terminals. The transportation of oil through pipelines is not considered in the problem under study, neither from the FPSOs to the terminals nor from terminals to refineries. Therefore, the problem consists of managing crude oil inventory level in the FPSOs, transporting it by a heterogeneous fleet of vessels from FPSOs to the onshore terminals while ensuring that inventory level at terminals will not fall under a minimum stock limit.

Next, we present the PETROBRAS MIR problem in terms of ship scheduling activity, planning horizon, number of assets (FPSOs, vessels and terminals) and the loading and discharging operations.

2.1.1. Ship scheduling and planning horizon

The ship scheduling activity at PETROBRAS is a short-term activity responsible to define on a daily basis which ship will carry the crude oil from FPSOs to refineries. Usually the schedulers decide the voyages for the next day and the offloading operations that must occur at the most 5 days ahead. Every day, the schedule of the previous day is updated and incremented with those vessels that will be available for a new voyage in the next day. When a ship schedule order is sent, it is important to define some aspects: which FPSO(s) the vessel will visit (vessel can offload up to two FPSO in sequence); the amount of crude oil must be loaded at each FPSO; and which onshore terminal(s) the vessel must follow to discharge its load (up to two terminals in sequence). Voyages from FPSOs to onshore terminals takes from 1 to 3 days of sailing time, while the time spent in the FPSOs and in the terminals to load and discharge operations can varies from 1 to 5 days, or even longer, depending on where the ship is operating. We can split time spent at ports in two parts: waiting and operating time. Waiting time depends on berth congestion at onshore terminals, on the onshore tank availability to receive a new load, bad weather, crude oil quality and many other unexpected situations. Operating time is proportional to the quantity to be (un)loaded and the discharge rate used in the operation plus berthing and unberthing time. It also has some variation depending on the loading or discharging rates of the terminal's or vessel's pump, respectively.

The planning horizon is short due to the times involved in each round voyage for each vessel (about 7 days, on average) and to increase the accuracy of forth-coming vessels activities estimative. Maritime transportation is subject to a great number of unforeseen occurrences that can cause delays in the operations or sailing times (see Diz et al., 2014). To increase the chance of success in their voyage plans and to reduce the risk of facing these unexpected events, schedulers wait until the last moment to release a ship scheduling. They also must check every day the position of its vessels and the inventory levels at FPSOs and onshore terminals. If necessary, the scheduler can always change a preview schedule to ensure that inventory levels stay inside operational range, to overcome unexpected occurrences or to reduce transportation costs.

2.1.2. FPSOs

In 2016, PETROBRAS was producing oil using about 32 FPSO vessels. Most of them are located in Brazilian waters in the Atlantic Ocean, about 100 miles from the coast of southeast region of country. Each of them have a specific storage capacity, a minimum stock level that works as a ballast volume[1] and a production rate. In real life, all of these three parameters can suffer variations along the time, mainly the production rate.

2.1.3. Vessels (or Ships)

PETROBRAS counts on a fleet of more than 35 vessels dedicated to the offloading and supply operation. This number may vary according to vessel availability for each planning horizon. As the Brazilian shore is long and PETROBRAS have production sites and onshore terminals all over the cost, the fleet is divided in groups to serve each sub system. Each group of ships can be re-sized according to transportation demand of each subsystem, but physical and operational characteristics of the fleet, onshore terminals and production sites may restrict interchanging vessels between subsystems. Additionally, some voyages between subsystems may take several days, what further limits these interchanges.

[1] Ballast volume is the volume of crude oil used to keep the vessel's balance in the sea. In other words, the ballast volume of crude oil helps to maintain stability of the vessel.

The vessels are classified according to three classes depending on its storage capacity: Suezmax, Aframax and MR. On average, Suezmax vessels load up to 1.0 million barrels of crude oil, Aframax vessels usually load up to 650 thousand barrels and MR has the capacity to transport about 350 thousand barrels. However, it does not mean that vessels from the same class have exactly the same storage capacity, as it varies from ship to ship. Each vessel has its own bunker consumption rate for each type of operation: sailing loaded, sailing in ballast, discharging, loading and on standby mode. Bunker consumption represents the biggest part of the variable transportation cost and another important parcel is given by port fees. The fixed part of maritime transportation cost is hiring costs. This is known in shipping industry as *hire*, which is a daily fee payed monthly to vessel's owners for every day of the time chartered party agreed between PETROBRAS and vessel's owner. As the fleet is time-chartered, there are no demurrage costs incurred when the vessel experience delays at a port. However, as waiting time is considered an inefficiency, it may be penalized. We estimate such penalty using the hiring fee of the vessel and bunker consumption costs.

2.1.4. Onshore terminals

There are 8 onshore terminals that send oil through pipeline to 12 refineries located from the south to the north of Brazil, as it can be seen in Figure 2.

Each terminal has a tank farm capacity, a consumption rate and a minimum stock level that guarantees the continuous oil supply to the refinery(ies). Tank farm capacity was projected to guarantee an inventory able to supply refineries continually. The consumption rate is given by the sum of processing rates of the

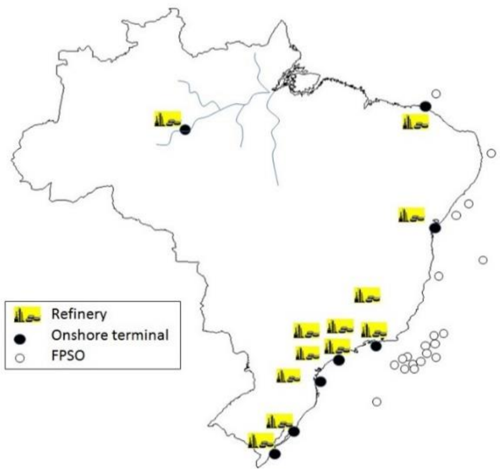

Figure 2: Map of Brazilian onshore terminals, refineries and FPSOs.

refineries linked to each terminal, while the minimum stock level defines the inventory level before refineries reduce their processing loads. For example, São Sebastião Terminal supplies 4 refineries, and therefore its consumption rate is equal to the sum of the processing rates of these 4 refineries. If the inventory level at São Sebastião Terminal reaches a total under the minimum level, one or more refineries fed by this terminal will have to reduce their processing rate or, even worse, stop their process due to a lack of crude oil inventory. Although it is not a frequent occurrence, parameters like consumption rate, storage capacity and minimum stock level can suffer variation along the time. A refinery can increase or reduce its consumption rate, a tank can stop for maintenance or repairing or other unforeseen events can happen anytime.

2.1.5. Loading and discharging operations

The main aspects of loading and discharging operations are the quantity of oil to be loaded/discharged and the moment in time when these operations occur. The moment to offload a FPSO or to supply an onshore terminal is determined based on inventory levels at the ports and vessel arrival times. The operation must occur such that the inventory levels do not exceed its maximum limit at FPSOs or its minimum limit at onshore terminals. The quantity to be loaded or discharged must respect ship and FPSO capacities and must be such that minimize transportation costs.

Notice that in most parts of the crude oil supply chain we have mentioned some type of uncertainty aspect that influences the scheduling activity. Besides the usual complexity of MIR problems, the uncertainty aspects and the quantity of assets in PETROBRAS supply chain make the problem more complex. In the next section, we describe in more details some of the main uncertainty aspects involved in the MIR problem faced by PETROBRAS.

2.2. Uncertainty aspects

Uncertainty events are something inherent to any real life problem. In maritime transportation, they are very frequent and much of them are related to environment conditions, bad weather, currency, wind and waves are just some of conditions that may delay, interrupt or even abort some operation.

In MIR problems such as the one described above, most of uncertainty events will have some impact in the time spent by the vessel in some activity (sailing, loading or discharging). The time aspect is vital for an efficient ship scheduling. A precise prediction of the time that a vessel will spend in each activity of the current voyage is fundamental for the scheduling of the next voyage of each ship. In addition, a good prediction of production and consumption rates, in FPSOs and onshore terminals, respectively, will determine the moment in time of the next offloading or discharging operation, as discussed in Section 2.1.

Due to the characteristics of the MIR problem faced by PETROBRAS, the time spent by the fleet on sailing is about 25% of the total available time of the fleet. While sailing, vessels may be fully loaded, partially loaded or in ballast condition (when it is empty). In this case, most voyages are short, and then variation of speed has a little impact in total sailing time. Hence, unexpected event also has little impact in prediction of sailing time for each vessel. Meanwhile, the time spent by the vessels at ports (FPSO or an onshore terminal) represents the remaining 75% of available time. During the time spent at ports, a vessel may be waiting for berthing, performing berthing or unberthing operation, loading or discharging, or even waiting for the next voyage. As the time spent by vessels at ports represents a much larger fraction of the available time of the fleet, we will focus our attention in the uncertainty events the impact this parcel of time.

The time spent at ports may be divided in three parts: pre-operation (from the time vessel arrives at port until it starts berthing; operation (from the start of berthing until unberthing, including the time spent during loading or discharging operation); and post-operation (from end of unberthing until the time vessel leaves the port for a new voyage). Next, we analyze each part of the time spent by vessels at port and the uncertain events that influence them.

Pre-operation time

Theoretically, when the vessel arrives at a port, it should immediately start berthing operation. However, it often does not happen in practice. The delay between the arrival of a vessel and the start of berthing operation may happen due to a diverse range of causes. At a production site, it may occur due to bad weather, vessel or FPSO unreliability, waiting for a batch formation to load and many other reasons. At onshore terminals, where the vessels discharge the crude oil, besides bad weather and equipment degradation, the waiting time before start of berthing operation must happen due to berth congestion, lack of tank capacity, bureaucracy reasons (vessel clearance) and pilot delays, for example.

Operation time

Operational time should be equal to maneuvering time (berthing and unberthing operations) plus the time spent to load or discharge product on the vessel. Nevertheless, in real-life, operation time is usually increased by many sorts of events, such as vessel's or port's pump unreliability, vessel clearance, lack of tank capacity, lack of crude oil inventory, bad weather (that might interrupt or delay a pumping operation), and many other unexpected reasons.

Post-operation time

The post-operation time is the time spent by the vessel at port after the end of unberthing operation. At this point, the vessel has finished loading or discharging operation, and is ready to follow to another port or start a new voyage and thus, has no reason for remaining at port. Nevertheless, in the MIR problem we are studying, there is a previously sized and dedicated fleet to carry all the crude oil production. Sometimes a vessel is free to start a new voyage, but there is no available cargo to load. When it happens, the vessel remains idle, on standby mode, waiting for a new voyage. Hence, the post-operation time is an idle time and should not exist in a hypothetically ideal world. However, as the fleet is fixed and previously dimensioned, the post-operation time is inherent to activity. In other words, post-operation time is related to the availability of vessels (fleet dimension) and transportation demand. In real-life, vessel's owners usually take advantage of this stand by time to perform some little maintenance, provisions supplying, crew changing or some other support work that must happen at ports.

Shortly, unforeseen occurrences influence directly to the time spent by the vessels in their operations, causing delays in sailing, discharging and/or loading operations. They provoke some undesirable effects: an inefficient use of the fleet; higher transportation costs; a higher risk of violating inventory bounds and, consequently, causing interruption in crude oil production or refining processes.

With exception to the post-operation time, which is inherent to the ship scheduling activity, the pre-operation time and operation time are highly impacted by unforeseen occurrences. In Section 8.1, we present some historical data that we use to estimate this parameter and a robust approach that addresses the uncertainty in the time vessels spent at ports.

3. Literature review

In this Chapter, we examine the literature to review the most recent practice related to MIR problem, how researchers approached uncertainty in MIR problems and deterministic mathematic formulations to solve MIR problems.

3.1. Literature review in maritime inventory routing

According to Christiansen (1999), MIR problem combine an inventory management problem and a routing problem. In her pioneering study, a fleet of ships transports a single product between production and consumption harbors, while the inventory level at these harbors should be kept between maximum and minimum levels. The objective function of her model minimizes transportation costs. Flatberg et al. (2000) studied the same problem, but made use of an iterative improvement heuristic combined with an LP solver for finding arrival times and quantities for given routes. These were the two pioneer studies concerning MIR problems.

Since these pioneer studies, researchers have been struggling to overcome the challenges posed by MIR problems. According to Furman et al. (2011), Agra et al. (2013) and Papageorgiou et al. (2014), MIR problems are very hard to solve to optimality for real-life instances. Usually, this kind of problem has weak linear relaxation bounds, what makes it harder for commercial solvers (that uses branch-and-bound algorithm) to find optimal solutions. To overcome the challenges of solving MIR problems to optimality, researchers have developed and implemented several approaches, most of them based on mixed-integer linear programming (MILP), although heuristics and hybrid models can also be found in literature.

3.1.1. Heuristics and hybrid methods for MIR problems

Dauzère-Pérès et al. (2007) presents the implementation of a decision support system (DSS) applied to a calcium carbonate slurry maritime supply chain for European paper manufactures. The proposed DSS is based on a meta-heuristic called memetic algorithm (also known as a genetic local search or hybrid genetic algorithm). Authors claimed its implementation has saved about US$ 7 million.in production and transportation costs during a year. Christiansen et al. (2011) presents a heuristic based on genetic algorithm to deal with the cement transportation problem. A heterogeneous fleet of bulk ships is assigned to transport multiple non-mixable cement products from producing factories to regional silo stations along the coast of Norway, while, inventory levels must remain between upper and lower levels at all factories and silos. Siswanto et al. (2011) treated a variation of MIR problem that they called ship inventory routing and scheduling problem with undedicated compartments. They based their tests in a real-world problem of oil products transportation in the South East of Asia. In this specific problem, a heterogeneous fleet is scheduled and ships compartments are considered in the loading and discharging decisions. They identified four sub-problems (route selection, ship selection, loading and discharging activities), proposed a set of heuristics to deal with these sub-problems, and choose one heuristic to solve each one of them.

The studies cited herein are based on heuristic methods and hybrid approaches, which are very useful for solving real-life MIR problems. As real-life MIR problems usually involves large sized instances, with many vessels, ports and/or longer planning horizon, heuristics and hybrid approaches are one alternative to find good solutions. Nevertheless, it is important to highlight that these methods usually cannot guarantee optimality.

3.1.2. MILP approach for MIR problems

When it comes to MILP approaches, Christiansen (1999) based her model on a continuous-time planning horizon and the concept of harbor arrival. Every time a ship arrives in a harbor, it is called harbor arrival (also known as port visit). A ship may visit a port how many times it is necessary. Al-Khayyal and Hwang (2007) studied a multi-product MIR problem, where the products require dedicated compartments in the ship. They formulated the problem as a MILP. Christiansen et al. (2007) also presented a MILP to model a multi-product MIR problem. In the multi-product MIR problem, they assumed that the shipper (cargo owner) does not control and operate the fleet of ships. Transportation is carried out by ships that are chartered for performing single voyages from a loading to a discharging port at known cost (spot charters). This means that the focus of the problem is to determine the quantity and timing of shipments to be shipped, while the routing of the ships is not an important part of the problem. Furman et al. (2011) reported one of the few successful implementations in industry of the use of MILP to model a real-life MIR problem. In their problem, a major petroleum company had to plan the transportation of significant volumes of vacuum gas oil (VGO) from supply points in Europe to refineries in the United States. As a real problem, they included important aspects, such as draft restrictions, and complex transportation costs, such as fixed-leg cost, variable overage rates[2], and demurrage rates.

As the problems became more complex, Song and Furman (2013) claimed that their model and algorithm framework is flexible and effective enough to be a choice of model and solution method for practical inventory routing problems. Papageorgiou et al. (2014) also proposed a general formulation that incorporates assumptions and families of constraints that are most prevalent in practice. They claim that their model is a core model that can features many practical situations found in real-life problems. Papageorgiou et al. (2014) also presented a review of MIR problems and the first library of MIR problems of literature.

3.1.3. Improving MILP formulations

According to Papageorgiou et al. (2014), most of studies found in literature succeed with small instances. According to Dauzère-Pérès et al. (2007), one well-known difficulty encountered when solving MIR problems is the combination of large continuous decision variables (shipment quantities and inventory levels) and discrete decision variables (choice of routes and ships). As consequence, one obtains poor bounds with linear relaxation of the mixed-linear programming model. Moreover, general cutting planes in the standards solvers are not effective for this type of problem. To enhance modeling of the MIR problem, Christiansen (1999) propose to decompose it in two subproblems: ship routing and inventory management. For this approach, the author used a Dantzig-Wolfe decomposition technique. Rocha et al. (2013) also considered a crude oil offloading and supply problem, but in a tactical perspective, showing that the natural formulation of the problem has weak lower bounds. To strengths their formulation, they proposed a reformulation of the model with some valid inequalities that they called Cascading Knapsack inequalities. In order to tighten linear relaxation bounds, Agra et al.

[2] Overage rate is an extra fee payed to vessel's owner when the vessel loads more cargo then it was contracted in a voyage charter party.

(2013) propose a reformulation of a discrete time MIR model that they called Fixed Charge Network Flow (FCNF) formulation and implement some important valid inequalities that make the model formulation stronger. Apart from these studies, many authors propose valid inequalities to improve their models (Persson and Gothe-Lundgren, 2005, Al-Kayyal and Hwang, 2007, Gronhaug et al., 2010, Engineer et al. 2012, Song and Furman, 2013, Agra et al., 2013b and Papageorgiou et al. 2014).

Solving large instances of MIR problems using MILP is still a challenge. Hence, in next chapter we present three different formulations and some valid inequalities that model MIR problems and help to strength the formulation, respectively.

4. Deterministic maritime inventory routing problem

One of the main characteristic that differentiate MIR formulations concerns how to model the time dimension, either using continuous or discrete time representation. Continuous-time models treat time as a continuum and do not restrict events to take place at fixed time points, providing more accuracy in the time-lapse representation (Agra et al., 2013). In contrast, discrete-time models consider that the planning horizon is discretized into time periods that are uniform in length and assume that events (e.g., loading product onto vessels) can only take place at these fixed points in time (Papageorgiou et al., 2014). This implies that waiting and sailing times must be an integer number multiple of the time period length, which often leads to rounding-caused simplifications.

Although the cost parameters of both types of formulations are identical, such simplifications in the aspect time may eventually lead to slight differences between objective function values of both types of formulation for the same instance. The difference between objective function values, if it happens, is associated to the discretization of planning horizon in time periods that limits the occurrence of events in fixed points in time and simplifies parameters as sailing, waiting and operation times.

According to Christiansen et al. (2013), Agra et al. (2013) and Papageorgiou et al. (2014) continuous time is usually used when consumption and/or production rates are given and fixed over the planning horizon and discrete formulations are used to overcome the complicating factors associated with variable consumption and production rates.

Next, we present three basic formulations to model a MIR problem, namely a continuous-time model, a discrete-time model and an extension of the discrete-time model named Fixed Charge Network Flow (FCNF) model. The continuous-time formulation is similar to those first presented in Christiansen (1999), Christiansen et al. (2007), and Christiansen and Fagerholt (2009). In order to make the continuous time model meaningfully comparable to other discrete time models, we implemented some reformulations on it. These reformulations comprehend multiberthing operations at the same port, proposed by Stanzani (2017), and waiting costs at objective function. The discrete-time model is encountered in Dauzère-Pérès et al. (2007), Christiansen et al. (2013), and Agra et al. (2013). Moreover, the third one is the FCNF model that Agra et al. (2013) claimed that it provides better linear relaxation bounds for the integer programming problem. For the sake of completeness, next we describe these three formulations.

4.1. Continuous time model

In continuous time models, the time representation of the planning horizon is continuous and the events (loading or discharging operations, or ship arrival at a port, for example) may occur at any time. These events determine the movements and activities of the ships and they only happen at nodes. A node represents the pair (port, visit), which means the port where the operation occurs and the visit is a counter of how many times each port is visited.

Figure 3 exemplifies the movements of two vessels. In this example, Ship A starts operating at node (1,1) (port 1, visit number 1) at the beginning of planning horizon, when the vessel loads 40.000m^3 of product. It sails directly to node (3,1), where it arrives at moment 2.5 (2.5 days after the beginning of the time horizon) to discharge 20,000m^3 of product. After discharging, Ship A sails half-loaded to node (4,1) to

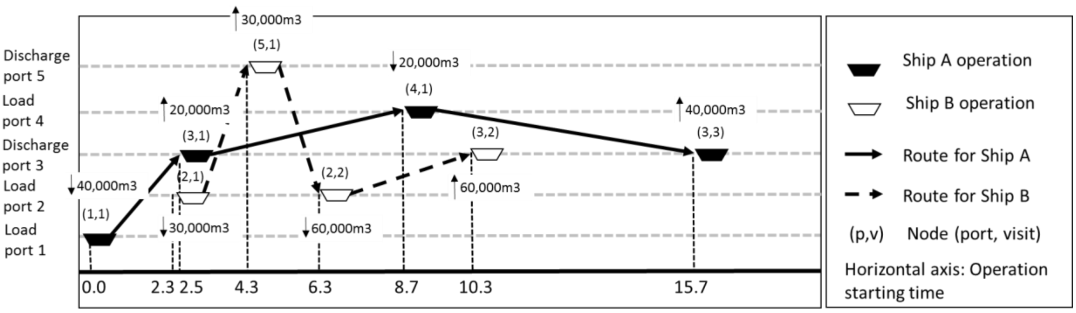

Figure 3: Possible routes example.

load more 20,000m^3 of product that will be discharge at node (3,3), starting at moment 15.7 days. Note that this is the third visit at port 3. Meanwhile, Ship B starts at a point in the sea and arrives at node (2,1) at moment 2.3, where it loads 30,000m^3 of product. After that, it sails to node (5,1) to discharge all cargo on board at moment 4.3. Later, Ship B sails to port 2 for the second visit (2,2) to load 60,000m^3. Finally, Ship B sails to node (3,2) to discharge the remaining of the cargo.

Next, we present the notation used in the formulation of continuous time model.

Sets:

N set of ports, indexed by i and j,

V set of available ships (or vessels) indexed by v,

MT_i set of possible visits at port i, indexed by m, n and τ. It represents the number of possible visits for each port.

H_v set of ports that can be visited by ship v

M_{iv} set of arrivals at i that can be made by ship v

I set of internal ports, which are the ones that the inventory level must be controlled during the entire planning horizon, $I \subset N$

Parameters:

A_{vimjn} 1 if the ship v can be scheduled in the route that leads from the node (i,m) to the node (j,n), where $i,j \in N$ and $m,n \in MT_{i,j}$, 0 otherwise.

C^T_{ijv} sailing cost from port i to port j using ship v.

C^P_{iv} port fee at port i when visited by ship v.

$WCost_v$ waiting costs of ship v, correspond to hire costs plus bunker consumption at standby mode per time-period.

$ORIG_v$ represents an artificial origin port of the ship v.

$L0_v$ cargo on board ship v at the beginning of the planning period,

QMX_{ivm} maximum load limit of each ship v in the port visit (i,m),

QMN_{im} minimum load limit for each port visit (i,m),

CAP_v ship capacity,

J_i equal to 1 if it is a load ports, -1 if it is a discharge port, and 0 if port is $ORIG_v$,

TQ_i the time required to load a unit of product at port i,

TS_{ijv} sailing time from port i to port j with the ship v,

TB_i minimum time required after every berthing at port i (correspond to maneuver time),

$TIME$ represents the length of the planning period,

$T0_v$ time ship v leaves the artificial origin port

R_i the product production/consumption rate. It is positive if is a production port and negative if is a consumption port,

SMN_i lower bound for stock level at port i,

SMX_i upper bound for stock level at port i,

SI_i initial stock level at port i,

MMX_i maximum number of visits at port I during the planning horizon,

B_i number of berths in port i.

Variables:

x_{imjnv} where $x_{imjnv} \in \{0,1\}$, $v \in V$ and $A_{vimjn} = 1$;

1 if the ship v sails directly from node (i,m) to node (j,n); 0 otherwise.

y_{im} where $y_{im} \in \{0,1\}$, $i \in I$ and $m \in MT_i$

1 if no ship visits node (i,m); 0 otherwise.

z_{imv} where $z_{imv} \in \{0,1\}$, $v \in V, i \in H_v$ and $m \in MT_i$

indicates the last operation of vessel v during planning horizon; is equal to 1, if the ship v ends its route at port i and during visit m; 0, otherwise.

q_{imv} where $q_{imv} \in R^+ \setminus \{ORIG_v\}$

represents the quantity loaded or discharged in each node (i,m) for each ship v.

l_{imv} where $l_{imv} \in R^+$, $l_{(ORIG_v,1,v)} = L0_v$

represents the load on board the ship v after leaving the node (i,m).

t_{im} where $t_{im} \in R$

the start time at node (i, m).

te_{im} where $te_{im} \in R$, $te_{(ORIG_v,1)} = T0_v$

the end time at node (i,m).

w_{imjnv} where $w_{imjnv} \in R$,

the time vessel v waits at port, visit (j, n) after leaving port, visit (i, m) from it arrival to beginning of loading or discharging operation. It may correspond to waiting time in queue in case of berth congestion, for example.

s_{im} where $s_{im} \in R$

represents the stock level at the port when the service starts at node (i, m).

se_{tm} where $se_{im} \in R$

represents the stock level at the port at the end of service at node (i, m).

$\sigma_{im\tau}$ where $\sigma_{im\tau} \in \{0,1\}$

0 if visit m superposes previous visit τ, 1 otherwise.

Objective function:

$$\min z = \sum_{v \in V} \sum_{(i,m,j,n)|A_{vimjn}=1} (C_{ijv}^{T} + C_{iv}^{P}) x_{imjnv} + WCost_v w_{imjnv} \quad (1)$$

The minimization function (1) contains transportation costs and port fee (contained in the parameter C_{ijv}) and the variable x_{imjnv} indicates ships movements through nodes (port, visit). It also minimizes waiting costs at ports.

Constraints:

$$\sum_{j \in Hv} \sum_{n \in Mjv} x_{jnimv} - \sum_{j \in Hv} \sum_{n \in Mjv} x_{imjnv} - z_{imv} = 0,$$

$$\forall v \in V, \forall i \in Hv \backslash \{ORIG_v\}, m \in Miv \quad (2)$$

$$\sum_{j} \sum_{n} x_{ORIG_v,1,jnv} + z_{ORIG_v,1,v} = 1, \ \forall v \in V \quad (3)$$

$$\sum_{i} \sum_{m} z_{imv} = 1, \quad \forall v \in V \quad (4)$$

$$\sum_{v \in V} \sum_{j \in Hv} \sum_{n \in Mjv} x_{jnimv} + y_{im} = 1, \forall i \in N\{ORIG_v\}, m \in MT_i \quad (5)$$

$$y_{im} - y_{i(m-1)} \geq 0, \quad \forall i \in I, m \in MT_i \quad (6)$$

Constraints (2) ensure that the ship v that arrives at port i leaves the same port towards another port j or to its route end. Constraints (3) ensure that a ship can be idle during the planning horizon. Constraints (4) ensure route ending. Constraints (5) ensure that each (port, visit) node must be visited either by a real ship or a dummy ship and (6) ensure that after been visited by a dummy ship, a node cannot be visited for real ships anymore.

$$x_{imjnv}\left[l_{imv} + J_j q_{jnv} - l_{jnv}\right] = 0, \quad \forall v \in V, \forall i, m, j, n |\ A_{vimjn} = 1 \qquad (7)$$

Constraints (7) state the relationship between the binary flow variable and the ship load/discharge at each node. These constraints are linearized as in (7') and (7''). The ship capacity $CAP(v)$ is the largest value that $(l_{imv} + J_j q_{jnv} - l_{jnv})$ can take, so constraints (7) are redundant if x_{imjnv} is equal to 0. Similarly, $(l_{imv} + J_j q_{jnv} - l_{jnv})$ will never be less than $-CAP(v)$ (for details, please refer to Christiansen, 1999).

$$l_{imv} + J_j q_{jnv} - l_{jnv} + CAP_v x_{imjnv} \leq CAP_v, \forall v \in V, \forall i, m, j, n |\ A_{vimjn} = 1 \qquad (7')$$

$$l_{imv} + J_j q_{jnv} - l_{jnv} - CAP_v x_{imjnv} \geq -CAP_v, \forall v \in V, \forall i, m, j, n |\ A_{vimjn} = 1 \qquad (7'')$$

$$0 \leq l_{imv} \leq \sum_{j \in H_v} \sum_{n \in M_{jv}} CAP_v x_{jnimv}, \quad \forall v \in V, \forall i \in H_v, \forall m \in M_{iv} \qquad (8)$$

$$q_{imv} - \sum_{j \in H_v} \sum_{n \in M_{jv}} Q_{MXimv} x_{jnimv} \leq 0, \forall v \in V, \forall i \in H_v, \forall m \in M_{iv} \qquad (9)$$

$$\sum_{v \in V} q_{imv} + Q_{MNim} y_{im} \geq Q_{MNim}, \qquad \forall i \in H_v, m \in M_{iv} \qquad (10)$$

Constraints (8) give the bounds for the product amount on board a vessel. Constraints (9) and (10) give the limits for loading/discharging quantity.

$$t_{im} \geq t_{im-1} \qquad \forall i \in Hv, \forall m \in MT_i, m \neq 1 \qquad (11)$$

$$t_{im} + \sum_{v \in V} TQ_i q_{imv} + TB_i - te_{im} = 0, \qquad \forall i \in Hv, \forall m \in MT_i \qquad (12)$$

$$(te_{im} + TS_{ijv} - t_{jn}) + TIME x_{imjnv} \leq TIME, \quad \forall v \in V, \forall (i, m, j, n) | A_{vimjn} = 1 \qquad (13)$$

$$t_{im} - te_{i\tau} \geq (\sigma_{im\tau} - 1) TIME, \forall i \in Hv, \forall m \in MT_i, \tau = 1, \dots, m - 1 \qquad (14)$$

$$t_{im} - te_{i\tau} \leq (\sigma_{im\tau})TIME, \quad \forall i \in Hv, \forall m \in MT_i, \tau = 1, \dots, m-1 \qquad (15)$$

$$1 + \sum_{\tau=1,\dots,m-1}(1 - \sigma_{im\tau}) \leq B_i \; \forall i \in Hv, \forall m \in MT_i \qquad (16)$$

$$(t_{jn} - te_{im} - TS_{ijv}) + TIMEx_{imjnv} \leq w_{imjnv} + TIME$$
$$\forall v \in V, \forall i, m, j, n | \; A_{vimjn} = 1 \qquad (17)$$

Constraints (11) impose that start time at visit m cannot occurs earlier than start time at visit $m-1$; additionally, they allow the overlapping of visit in a port i. with more than one berth. Constraints (12) define the relationship between the operating start time t_{im} and the operating ending time te_{im} at the same port i. Constraints (13) give the time relationship between subsequent ports. Start time $t_{(j,n)}$at port j must be greater than or equal to the end time $te_{(i,m)}$ on the previews port i plus the sailing time between ports i and j. Note that the ship may arrive before $t_{(j,n)}$ in node j for visit n. In this case, it waits until $t_{(j,n)}$ to start service. It may happen due to berth congestion at the port. Constraints (14) and (15) define the relationship between variables $\sigma_{im\tau}$ and variables t_{im} and $te_{i\tau}$. Note that they correctly ensure that $\sigma_{im\tau} = 0$ if the m_{th} visit to port i superposes a previous visit τ ($t_{im} \leq te_{i\tau}$) in (15), and that $\sigma_{im\tau} = 1$ otherwise ($t_{im} \geq te_{i\tau}$) in (14). Constraints (16) limits the number of overlapping operations in a port i to the number of berths of such port. Constraints (17) define waiting time for every visit i of a vessel v to a port j.

$$s_{im} - \sum_{v \in V} J_i q_{imv} + R_i te_{im} - R_i t_{im} - se_{im} = 0, \quad \forall i \in I, \forall m \in MT_i \qquad (18)$$

$$se_{i(m-1)} + R_i t_{im} - R_i te_{i(m-1)} - s_{im} = 0, \quad \forall i \in I, \forall m \in MT_i \qquad (19)$$

$$SI_i + R_i t_{im} - s_{im} = 0, \quad \forall i \in I, m = 1 \qquad (20)$$

$$SMN_i \leq s_{im} \leq SMX_i \quad \forall i \in I, \forall m \in MT_i \qquad (21)$$

$$SMN_i \leq se_{im} \leq SMX_i \quad \forall i \in I, \forall m \in MT_i \qquad (22)$$

$$SMN_i \leq se_{im} + R_i(TIME - te_{im}) \leq SMX_i \quad \forall i \in I, m = MMX_i \qquad (23)$$

Constraints (18) calculate the stocks at the end of a visit in an internal port and (19) give the inventory level between visits for each port. Constraints (20) give the inventory level at the beginning of the first visit for each port. Constraints (21), (22) and (23) give the inventory level bounds for the stock at the beginning of the service, after the service and at the end of planning horizon, respectively.

4.2. Discrete time model

Differently from the continuous time model, in the discrete time model the planning horizon is divided into uniform time periods, which becomes an index of the model. The events (loading or discharging operations, or ship arrival at a port for example) must occur in one of these defined time periods. Sailing, operating and waiting time of each ship also must start and end at one time period. We defined one time period as one day based on the scheduling team recommendation. They considered that one day represents most vessel activities.

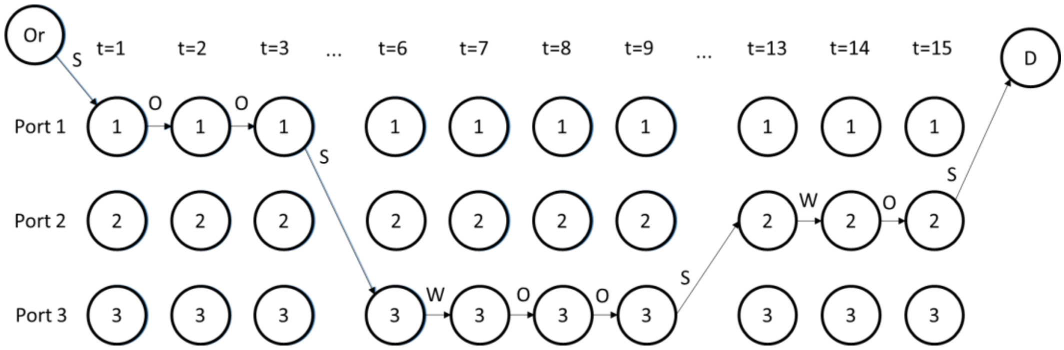

Figure 4: Time-space network representing a movement of a ship in an artificial problem.

Figure 4 presents a time-space network diagram that describes the movements of a vessel between ports along the planning horizon. Ships navigate through nodes that represent pairs of time periods and ports. At each of these nodes, the inventory is controlled and the vessel performs one of the following activities: waiting, operating or leaving a port. Usually, there is at least one variable to represent vessel activity at each node. In this example, the vessel leaves an artificial origin port (Or) and sail (S) to port 1, where it arrives during time period 1 and operates (O) for 2 periods. Thereafter, the vessel sails to port 3 where it arrives during time period 6, waits (W) for 1 time period and operates for 2 more time periods. Finally, during time period 9, the ship sails to port 2, waits for 1 time period, operates for another period, and finishes its voyage at an artificial destination port (D).

The artificial origin and destination nodes have different roles in the mathematical formulation. The origin node defines where the vessel is positioned in the beginning of the planning horizon. It represents the point in which the ship is ready to start its first voyage in the planning horizon. This point may be a onshore terminal or a production site, for example. On the other hand, the artificial destination port is used to indicate that a vessel finished its last voyage during that planning horizon. It does not denote any actual location, simply denoting the end of its tasks for that planning horizon. From the moment the vessel enters its destination port, it will not perform any other service during that planning horizon. The sailing time from the last real port visited to artificial destination port is zero and it does not add any cost to objective function either.

Next, we present the notation used in the discrete time formulation.

Sets:

N set of all ports indexed by i and j

T set of time periods indexed by t

V set of vessels indexed by v

NP set of loading ports indexed by i and j

ND set of discharge ports indexed by i and j

Parameters:

C_{ijv}^{T} sailing cost of vessel v between ports i and j

C_{v}^{W} waiting cost of vessel v for each time period

C_{iv}^{P} port cost of port i for vessel v

O_v position of vessel i in the beginning of planning horizon

DE_v artificial end node for each vessel v

T_{ijv} sailing time of vessel v between ports i and j

B_{it} number of berths available in port i during time period t

Q_v maximum amount of product to be (un)loaded at one time period of ship v

$L0_v$ inventory on board of vessel v in the beginning of the planning horizon

K_v	vessel capacity
D_{it}	demand rate in the discharge port i for each time period
P_{it}	production rate in the load port i for each time period
SMX_{it}	upper bound of inventory level in port i for each time period
SMN_{it}	lower bound of inventory level in port i for each time period
$S0_i$	inventory level at port i in the beginning of the planning horizon

Variables:

o_{ivt} where $o_{ivt} \in \{0,1\}$

equal to 1 if vessel is operating (loading/discharging) in port during time period t and 0 otherwise,

x_{ijvt} where $x_{ijvt} \in \{0,1\}$

equal to 1 if vessel v left port i to port j during time period t and 0 otherwise,

w_{ivt} where $w_{ivt} \in \{0,1\}$

equal to 1 if vessel v is waiting outside berth in port i during time period t and 0 otherwise,

l_{vt} where $l_{vt} \in$ R

inventory level on board vessel v during time period t,

q_{ivt} where $q_{ivt} \in$ R

quantity (un)load from/to vessel v in port i during time period t,

s_{it} where $s_{it} \in$ R

inventory level at port i during time period t.

Objective function:

$$min \sum_{v\in V}\sum_{i\in N\{O_v\}}\sum_{j\in N\cup\{DE_v\}}\sum_{t\in T} C^T_{ijv}x_{ijvt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C^P_{iv}x_{ijvt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C^W_{v}w_{ivt} \quad (24)$$

The minimization function (24) contains transportation costs, operation costs and waiting costs.

Constraints

$\sum_{j\in N\cup\{DE_v\}}\sum_{t\in T} x_{O_v jvt} = 1,\ \forall v \in V,$ (25)

$\sum_{i\in N\cup\{O_v\}}\sum_{t\in T} x_{iDE_v vt} = 1,\ \forall v \in V,$ (26)

$o_{iv,t-1} \leq \sum_{j\in N\cup\{DE_v\}} x_{ijvt} + o_{ivt}\ ,\ \forall v \in V, i \in N, t \in T,$ (27)

$o_{iv,t-1} \geq \sum_{j\in N\cup\{DE_v\}} x_{ijvt}\ ,\ \forall v \in V, i \in N, t \in T,$ (28)

Constraints (25) and (26) guarantee that every ship leaves from its artificial origin port and finishes the voyage at its artificial destination port. Constraints (27) determine that after an operation period, a vessel can continue to operate or sail the port in the next period and constraints (28) ensure that before leaving a port, the vessel must have operated there in the time period immediately before.

$\sum_{j\in N\cup\{O_v\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} + o_{iv,t-1} = \sum_{j\in N\cup\{DE_v\}} x_{ijvt} + w_{ivt} + o_{ivt}\ ,$

$\forall v \in V, i \in N, t \in T,$ (29)

$\sum_j x_{ijvt} + w_{ivt} + o_{ivt} \leq 1, \forall v \in V,\ i \in N,\ t \in T$ (29')

$\sum_{v\in V} o_{ivt} \leq B_{it}\ ,\ \forall i \in N, t \in T,$ (30)

Constraints (29) represent vessel's movement along the time-space network structure. At every time period, the vessel must be leaving a port, waiting to operate or operating at a port. These constraints also connect the time when a vessel leaves one port j to the time when the same vessel arrives at a port i. However, constraints (29) allow both sides of the equality to be equal to 2 or 3, for example, which is an unexpected solution. Then, we add constraints (29') that works as a valid inequality, removing unexpected solutions from the feasible solution space and strengthening lower bounds. However, preliminary tests were inconclusive about the effectiveness of constraints (29') in improving computational performance. Constraints (30) give berth restriction at each node and consequently waiting time.

$0 \leq q_{ivt} \leq Q_v o_{ivt}\ , \forall v \in V, i \in N, t \in T,$ (31)

$l_{v,t-1} + \sum_{i\in NP} q_{ivt} - \sum_{i\in ND} q_{ivt} - l_{vt} = 0\ ,\ \forall v \in V, t \in T,$ (32)

$$0 \leq l_{vt} \leq K_v, \forall v \in V, t \in T, \quad (33)$$

$$l_{v0} = L0_v, \forall v \in V, \quad (34)$$

The quantity to be (un)loaded (q_{ivt}) must be less than the maximum amount of product to be loaded to or discharged from ship v (constraints 31). The load onboard the vessel onboard is controlled in constraints (32) and it must always observe ship's capacity (constraints 33). Initial inventory level onboard the vessel is given in constraints (34).

$$s_{i,t-1} + \sum_{v \in V} q_{ivt} = D_{it} + s_{it}, \forall i \in ND, t \in T, \quad (35)$$

$$s_{i,t-1} + P_{it} = \sum_{v \in V} q_{ivt} + s_{it}, \forall i \in NP, t \in T, \quad (36)$$

$$SMN_{it} \leq s_{it} \leq SMX_{it}, \forall i \in N, t \in T, \quad (37)$$

$$s_{i0} = S0_i, \forall i \in N, \quad (38)$$

$$x_{ijvt} \in \{0,1\}, \forall v \in V, i \in N \cup \{O_v\}, j \in N \cup \{DE_v\}, t \in T, \quad (39)$$

$$o_{ivt}, w_{ivt} \in \{0,1\}, \forall v \in V, i \in N, t \in T, \quad (40)$$

$$l_{vt}, s_{it}, q_{ivt} \in \mathbb{R} \quad \forall v \in V, i \in N, t \in T. \quad (41)$$

In the discrete time model, the inventory level at each port is controlled during every time period of the planning horizon at load and discharge ports (constraints 35 and 36). Constraints (37) give the operational range for inventory levels and constraints (38) define initial inventory levels at each port. Constraints (39), (40) and (41) define variable domains.

4.3. Fixed charge network flow formulation (FCNF)

The third formulation considered in this study is the FCNF. It is an extension of the discrete time model presented in 3.2.2. According to Agra et al. (2013), FCNF provides linear relaxation bounds that are better than those obtained using the discrete time model. They model the problem as a single-commodity FCNF problem, which allows us to take advantage of known inequalities for such problems (Agra et al., 2013). As the FCNF model is an extension of the discrete time model, we have used the same notation of the previews model. In this sense, we state only the additional variables needed to complete this formulation.

In an extended network, the variables o_{ivt} are split into two variables o_{ivt}^{A} and o_{ivt}^{B}. The first indicates the start of an operation in a port and the second variable indicates the continuation of an operation in the same port. Figure 5 represents a discharge operation at port i by ship v in an extended network, considering two

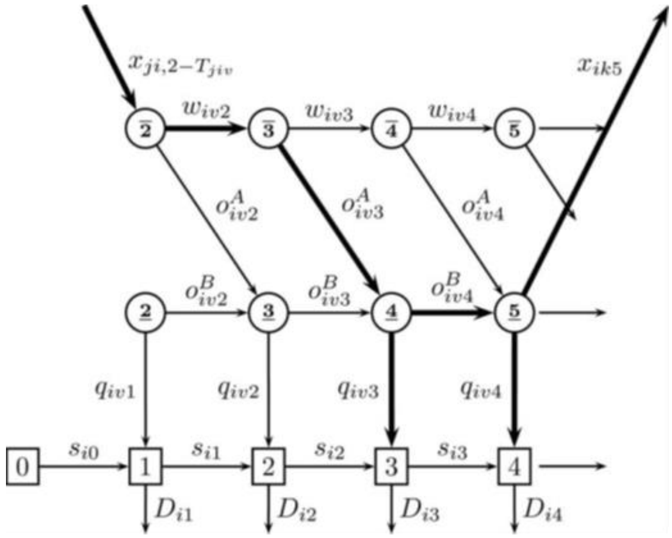

Figure 5: Discharging operation of ship v at port i in the extended network structure. Source: Agra et al. 2013

layers, one for the beginning of the operation (variable o_{ivt}^{A}) and a second one for an eventual continuation of the operation in the same port (variable o_{ivt}^{B}). The ship arrived at port i at time period 2, waits for one period, starts operating (discharging) in period 3, continues operating in period 4, and then leaves for port k in period 5.

Variables:

o_{ivt}^{A} where $o_{ivt}^{A} \in \{0,1\}$

indicates whether ship v starts to operate at port i in period t

o_{ivt}^{B} where $o_{ivt}^{B} \in \{0,1\}$

indicates whether ship v continue to operate at port i in period t

With these new variables, the ship flow-conservation constraints (29) can now be reformulated as:

$$\sum_{j \in N \cup \{O_v\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} = w_{ivt} + o_{ivt}^{A}, \ \forall v \in V, \ i \in N, \ t \in T, \quad (42)$$

$$\sum_j x_{ijvt} + w_{ivt} + o^A_{ivt} + o^B_{ivt} \leq 1 \quad \forall v \in V, \ i \in N, \ t \in T, \qquad (42')$$

$$o^A_{ivt-1} + o^B_{ivt-1} = o^B_{ivt} + \sum_{j \in N \cup \{DE_v\}} x_{ijvt}, \forall v \in V, \ i \in N, \ t \in T, \qquad (43)$$

$$o^A_{ivt}, o^B_{ivt} \in \{0,1\}, \ \forall v \in V, \ i \in N, \ t \in T, \qquad (44)$$

Constraints (42) indicate the ship arrival at one port and (42') is a disaggregation of (42) following the same reasoning of (29). Constraints (43) show when the ship sailing from one port. Constraints (44) indicate the variables o^A_{ivt} and o^B_{ivt} are binary.

$$o^A_{ivt} + o^B_{ivt} = o_{ivt}, \quad \forall v \in V, \ i \in N, \ t \in T, \qquad (45)$$

Constraints (45) provide the coordination between the path of the ships and the loading or discharging of crude oil in a port.

To complete the FCNF formulation, variable l_{vt} must be replaced with the following variables. They represent the flow amount of product for each arc in the extended network presented in Figure 5.

Variables:

f^X_{ijvt} that indicates the load on board ship v when traveling from port i to port j, leaving port i in period t,

f^{OA}_{ivt} that indicates the load on board ship v when starting to operate at port i in period t and has not operated in period $t-1$,

f^{OB}_{ivt} that indicates the load on board ship v before continuing to operate at port i in period t after having operated in time period $t-1$,

f^W_{ivt} that indicates the load on board ship v while waiting during time period t at port i.

Hence, constraints (32) – (34) that represent the ship's onboard quantity are replaced with the flow conservation constraints below (46) – (49):

$$\sum_{j \in N \cup \{O_v\}} f^X_{jiv,t-T_{jiv}} + f^W_{iv,t-1} = f^W_{ivt} + f^{OA}_{ivt}, \ \forall v \in V, \ i \in N, \ t \in T, \qquad (46)$$

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} + q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j \in N \cup \{DE_v\}} f^X_{ijvt},$$

$$\forall v \in V, \ i \in NP \cup \{O_v\}, \ t \in T, \qquad (47)$$

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} - q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j \in N \cup \{DE_v\}} f^{X}_{ijvt},$$
$$\forall v \in V,\ i \in ND \cup \{O_v\},\ t \in T, \quad (48)$$

$$f^{X}_{O_v jvt} = L0_v\, x_{O_v jvt}, \quad \forall v \in V,\ j \in N \cup \{DE_v\},\ t \in T \quad (49)$$

The variables upper bounds and non-negativity constraints are expressed in (50) – (54):

$$0 \le f^{X}_{ijvt} \le K_v x_{ijvt},\ \forall v \in V, i \in N \cup \{O_v\}, j \in N \cup \{DE_v\},\ t \in T \quad (50)$$

$$0 \le f^{OA}_{ivt} \le K_v o^{A}_{ivt},\ \forall v \in V, i \in N,\ t \in T, \quad (51)$$

$$0 \le f^{OB}_{ivt} \le K_v o^{B}_{ivt},\ \forall v \in V, i \in N,\ t \in T, \quad (52)$$

$$0 \le q_{ivt} \le Q_v o_{ivt},\ \forall v \in V, i \in N,\ t \in T, \quad (53)$$

$$0 \le f^{W}_{ivt} \le K_v w_{ivt},\ \forall v \in V, i \in N,\ t \in T. \quad (54)$$

Finally, the FCNF formulation is defined by (24) – (26), (30) – (31), (35) – (38) from the discrete time formulation and the additional constraints (42) – (54).

Additionally, to further strengthen the FCNF formulation, we have considered some valid inequalities proposed in Agra et al. (2013). Using the names proposed in Agra et al. (2013), we have implemented:

- Knapsack inequalities;
- Mixed Integer rounding inequalities;
- Wagner-Whitin constant capacity lot-sizing reformulations; and
- Inequalities for lot-sizing with start-up relaxations.

For a complete demonstration about such valid inequalities, see Agra et al. (2013).

In next chapter, we present computational experiments to compare the formulations presented here using a set of real-life instances.

5. Computational experiments

As MIR problems are known as hard to solve, especially for real-life instances, and there are different formulations available in literature, we developed a test bed to compare their computational performance. However, it is important to highlight that there are a few differences between the formulations. The discrete time formulations allow variation in production and/or consumption rates along planning horizon, while these rates are fixed in the continuous time formulation. On the other hand, in discrete time formulations the time required for each operation (sailing, loading or discharging) is always multiple of one time period, while the continuous time formulation provides more accuracy to model regarding the time spent at each operation. The continuous time formulation enables one to consider time between berthing (maneuvering time), for example, and the operation times is calculated according to loading rates and vessel speed.

To overcome the differences between the formulations and make them meaningfully comparable, we have created instances with fixed production and consumption rates. Maneuvering time were set to zero in every port, sailing time were rounded to be multiple of one time period, and loading and discharging rates were set such that the load/discharge of a typical cargo quantity would last one time period.

To compare the different formulations found in literature and presented in the previous Chapter, we structured some computational tests. We implemented all models using AIMMS 3.13 and solved them using the GUROBI 5.5. The computational experiments were performed on an Intel Core i7 CPU with 8.0 GB RAM. The advantages of the developed test bed are that we use the same computational recourses and the same set of instances, which ultimately means that the results, as well as he model performance, are comparable

We performed the experiments in two phases. First, we compared the computational performance of three formulations presented and the FCNF formulation associated with the valid inequalities proposed in Agra et al. (2013). The four groups of valid inequalities are denoted hereinafter as: Knapsack inequalities (K), Mixed-integer rounding inequalities (M), Wagner-Whitin constant capacity lot-sizing reformulation (W), and the Lot-sizing with start-up relaxation (D).

The FCNF formulation is compared in 5 different forms: without any valid inequality and associating one valid inequality at time (FCNF + K; FCNF + K + M; FCNF + K + M + W; and FCNF + K + M + W + D). In this phase, we used a set of ten instances based on the real-life problem of the Brazilian petroleum company. The instances were named according to the number of assets considered. For example, 4fpso3term4ves15days means that there are 4 FPSOs, 3 onshore terminals and 4 vessels during a 15 day-long planning horizon. As the complete problem is large in terms of assets (40 production sites, 40 ships and 9 onshore terminals) and we did not find any record in the academic literature that solves a MIR problem of such size, we have segregated a sub-system of the whole problem to create our instances. We elect the southeast region of Brazilian coast, where most part of Brazilian crude oil production and refineries are located. We started with smaller instances and gradually added new FPSOs, terminals and ships in order the increase the complexity of the problem. We followed this procedure up to the computational performance limit of the models.

In the second phase of experiments, we enlarged some of these instances with a longer planning horizon in order to stress the performance of the best formulations of the first phase. For all experiments, it was set a time limit of 3600 seconds as stop criterion.

5.1. First phase

In Table 1, we summarize for each formulation the computational time to reach optimality (when it was reached) and the optimality gap when the optimization stopped due to time limit. When the solver was not capable of finding any integer solution within the time limit, we reported the gap as “not applicable” (N/A).

No	Instances	Continuous time		Discrete time		FCNF		FCNF + K		FCNF + K + M		FCNF + K + M + W		FCNF + K + M + W + D		FCNF + K + D + Br Oa	
		gap	Time(s)	gap	Time(s)	gap	Time(s)	gap	Time(s)	gap	Time(s)	gap	Time(s)	gap	Time(s)	gap	Time(s)
1.01	**03fpso2term3ves15days**	0.0%	1	0.0%	9	0.0%	1	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0
1.02	**03fpso3term3ves15days**	0.0%	2	0.0%	50	0.0%	6	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0
1.03	**04fpso2term4ves15days**	0.0%	95	39.0%	3600	6.9%	3600	0.0%	28	0.0%	15	0.0%	1	0.0%	1	0.0%	24
1.04	**04fpso3term4ves15days**	0.0%	2	0.0%	154	0.0%	102	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0
1.05	**05fpso2term5ves15days**	0.0%	176	60.8%	3600	29.9%	3600	18.9%	3600	18.5%	3600	7.9%	3600	0.0%	1607	14.7%	3600
1.06	**06fpso3term6ves15days**	0.0%	210	63.8%	3600	18.9%	3600	16.6%	3600	15.2%	3600	8.7%	3600	0.0%	1594	8.0%	3600
1.07	**07fpso3term6ves15days**	7.5%	3600	71.1%	3600	24.7%	3600	18.3%	3600	21.5%	3600	4.7%	3600	7.1%	3600	21.4%	3600
1.08	**08fpso4term6ves15days**	N/A	3600	74.9%	3600	34.0%	3600	21.4%	3600	20.4%	3600	12.7%	3600	12.6%	3600	21.8%	3600
1.09	**09fpso5term6ves15days**	N/A	3600	N/A	3600	N/A	3600	N/A	3600	N/A	3600	0.0%	2875	4.9%	3600	32.0%	3600
1.10	**10fpso5term7ves15days**	N/A	3600	N/A	3600	N/A	3600	N/A	3600	N/A	3600	14.1%	3600	14.1%	3600	N/A	3600

Table 1: Minimum gap obtained and time required for each formulation at each instance.

Table 1 provides us some insights about the comparison between formulation. Note that, even for the smallest instance, the discrete time model takes considerably more time to reach optimality than continuous time model and FCNF. It suggests that this is the weakest of the formulations tested. This behavior is repeated for the other instances, in which the discrete time formulation provides worst integrality gaps or takes more time to reach optimality. We can also observe that for the easier instances (the first 6 instances), the continuous time model presented significantly better performance, reaching the optimal solution faster than discrete time and FCNF models. However, even for these easier instances, the continuous time model performs worse than the FCNF plus valid inequalities, in accordance with the findings of Agra et al. (2013). They claim that the use of valid inequalities would improve computational performance of the FCNF model. With the addition of the valid inequalities to the FCNF formulation, the computational performance increased. Note that when the instances become larger (for example, in the last 3 instances), the addition of valid inequalities to the FCNF formulation were essential to find feasible solutions or reach optimality. It is important to highlight that we did not develop or implement any valid inequality to improve the computational performance of the continuous time formulation. The reformulations we implemented in the continuous time model were made in order to represent operational aspects such as multi-berth port (Stanzani, 2017), waiting time before start operation and waiting costs at objective function. Such reformulations let formulations (discrete time, FCNF and continuous time) comparable.

Table 2 presents a summary of the computational results. In the first column, we list the formulations tested, the second presents how many times each formulation provided at least one integer solution. Third column presents how many times each formulation reached the best solution among the other formulations (by "best solution" we refer to the formulation that encountered the solution with minimum objective function value). The last column shows how many times each formulation proved optimality.

Formulations	Integer Solution up to time limit	Best Solution	Proved Optimality
Continuous time	7	6	6
Discrete time	8	4	3
FCNF	8	4	3
FCNF + K	8	4	4
FCNF + K + M	8	4	4
FCNF + K + M + W	10	7	5
FCNF + K + M + W + D	10	7	6

Table 2: Computational results summary table

Notice that the continuous model and the FCNF+K+M+W+D were the models that proved optimality more times. In six of them, the optimality gap was closed. On the other hand, only formulations FCNF+K+M+W and FCNF+K+M+W+D could find feasible solutions for all instances within the time limit of 3600 seconds. The continuous time and discrete time models did not find feasible solutions in 3 and 2 of the instances, respectively. The formulations that reached the best solutions more times were FCNF+K+M+W and FCNF+K+M+W+D, 7 times for each of them.

We have calculated the average optimality gap only for those formulations that could find feasible solutions for all instances (FCNF+K+M+W and FCNF+K+M+W+D). The average optimality gap using FCNF+K+M+W+D was 3.9% against 4.8% obtained with the FCNF+K+M+W formulation.

5.2. Second phase

From the results of the first phase of experiments, the continuous time, FCNF+K+M+W+D, and FCNF+K+M+W formulations were the ones that presented best performance. Therefore, they were tested further considering larger (and presumably harder) instances. The first 6 instances used in the previews phase were used again but considering a longer planning horizon of 30 days. It should force an increasing number of cargoes transported due to a longer period of crude oil production and refining, which makes instances much harder (as the numerical results suggests) to be solved to optimality.

Table 3 gives the optimality gap and time required by each formulation for each instance tested. Notice that the continuous time model did not find any feasible solution in any of the instances tested, while the other formulations could reach feasible solutions in 4 of the 6 instances tested.

No	Instances	Continuous time		FCNF + K + M + W		FCNF + K + M + W + D	
		gap	Time (s)	gap	Time (s)	gap	Time (s)
2.01	03fpso2term3ves30days	N/A	3600	0.0%	1525	0.0%	1317
2.02	03fpso3term3ves30days	N/A	3600	14.9%	3600	16.2%	3600
2.03	04fpso2term4ves30days	N/A	3600	10.4%	3600	10.4%	3600
2.04	04fpso3term4ves30days	N/A	3600	14.3%	3600	18.3%	3600
2.05	05fpso2term5ves30days	N/A	3600	N/A	3600	N/A	3600
2.06	06fpso3term6ves30days	N/A	3600	N/A	3600	N/A	3600

Table 3: Minimum gap obtained in the second phase of tests

Table 4 is similar to Table 2 with an additional column informing the average gap. It suggests that the performance of formulations FCNF+K+M+W and FCNF+K+M+W+D are very similar. The first one has a slight advantage in the average gap, while continuous time formulation was not able to solve any of the instances tested.

Formulations	**Integer solution up to time limit**	**Best solution**	**Optimality proved**	**Average gap**
Continuous time	0	0	0	N/A
FCNF + K + M + W	4	2	1	9.9%
FCNF + K + M + W + D	4	2	1	11.2%

Table 4: Computational results summary table in the second phase of tests

5.3. Using the Lower bound of continuous time model

We observed a noteworthy aspect regarding the lower bounds provided by the linear relaxation problem. However, prior to go into this subject, it is important to highlight an important relation between continuous time and discrete time formulations (including the FCNF). As discrete time formulations divide the planning horizon in time periods, they only provide solutions where the events (e.g. load or discharge operations, port arrivals or leavings) are constrained to happen in fixed points in time. This requirement does not exist in the continuous formulation. As the MIR problem at hand considers fixed production/consumption rates, the continuous time formulation can be seen as a relaxation of the discrete time formulations. This perception becomes important once the continuous time formulation has obtained, in several instances strong lower bounds.

In 13 instances, the lower bound obtained for the continuous time model was equal to or better than the ones obtained in the discrete time models. It means that, with exception of the cases where optimality gap was completely closed, the lower bound of the continuous time model can be used to reduce the optimality gap obtained for the other models. Table 5 presents the best solution and (best lower bound) obtained for the formulations: continuous time, FCNF+K+M+W and FCNF+K+M+W+D for each of the instances. The lower bound in bold caption in the column of the continuous time model identifies the ones equal to or better (higher) than the ones obtained using the FCNF plus inequalities models. Note that in 6 of them, the lower bound proved the optimality of the integer solution and in the other 7 they were better (i.e., presented higher values) than the bounds obtained in the discrete time model.

No	Instances	Continuous time	FCNF + K + M + W	FCNF + K + M + W + D
1.01	03fpso2term3ves15days	726,045 (**726,045**)	726,045 (726,045)	726,045 (726,045)
1.02	03fpso3term3ves15days	913,260 (**913,260**)	913,260 (913,260)	913,260 (913,260)
1.03	04fpso2term4ves15days	1,535,460 (**1,535,460**)	1,535,460 (1,535,460)	1,535,460 (1,535,460)
1.04	04fpso3term4ves15days	1,030,730 (**1,030,730**)	1,030,730 (1,030,730)	1,030,730 (1,030,730)
1.05	05fpso2term5ves15days	2,232,730 (**2,232,730**)	2,233,560 (2,056,130)	2,233,560 (2,233,560)
1.06	06fpso3term6ves15days	2,309,635 (**2,309,635**)	2,309,635 (2,109,235)	2,309,635 (2,309,635)
1.07	07fpso3term6ves15days	2,589,810 (2,394,780)	2,569,010 (2,448,525)	2,642,900 (2,455,680)
1.08	08fpso4term6ves15days	N/A (**2,879,985**)	2,915,410 (2,544,910)	2,915,410 (2,547,520)
1.09	09fpso5term6ves15days	N/A (3,590,145)	3,893,375 (3,893,375)	3,893,375 (3,701,415)
1.10	10fpso5term7ves15days	N/A (**3,224,380**)	3,226,085 (2,769,945)	3,225,905 (2,769,895)
2.01	03fpso2term3ves30days	N/A (3,396,145)	3,397,100 (3,397,100)	3,397,100 (3,397,100)
2.02	03fpso3term3ves30days	N/A (**3,557,830**)	3,832,185 (3,259,290)	3,832,185 (3,210,790)
2.03	04fpso2term4ves30days	N/A (**5,297,655**)	5,502,695 (4,930,830)	5,501,730 (4,929,685)
2.04	04fpso3term4ves30days	N/A (**4,504,095**)	4,908,275 (4,205,935)	5,168,900 (4,223,280)
2.05	05fpso2term5ves30days	N/A (**6,292,870**)	N/A (5,931,880)	N/A (5,922,875)
2.06	06fpso3term6ves15days	N/A (**7,017,010**)	N/A (6,990,465)	N/A (6,991,035)

Table 5: Best solutions and (linear relaxation lower bounds) obtained for each formulation.

Observing Table 5, among the 16 instances tested, in 6 of them we found feasible solutions but the optimality gap was not completely closed (instances 1.07, 1.08, 1.10, 2.02, 2.03, and 2.04). We used the lower bound of the continuous time model to calculate the optimality gap of this group of 6 instances. As the continuous time model is a relaxation of the FCNF model, its lower bound is also a lower bound for the FCNF model. Using this insight, we can recalculate the optimality gap in 5 of 6 instances of this group. Only for instance 1.07 we could not reduce the gap obtained by the branch-and-bound algorithm. We summarize these results in Table 6. In this set of instances, we reduced the average gap in 65%. It went from 13.6% to 4.8%. In practice, this procedure allows one to verify if the solution obtained using the FCNF model is closer to optimum or not.

No	Instance	Formulation	Best Solution	Discrete time lower bound	Discrete gap	Continuous time lower bound	Combined gap
1.07	07fpso3term6ves15days	FCNF + K + M + W	2,569,010	2,448,525	4.9%	2,394,780	7.3%
1.08	08fpso4term6ves15days	FCNF + K + M + W + D	2,915,410	2,547,520	14.4%	2,879,985	1.2%
1.10	10fpso5term7ves15days	FCNF + K + M + W + D	3,225,905	2,769,895	16.5%	3,224,380	0.0%
2.02	03fpso3term3ves30days	FCNF + K + M + W	3,832,185	3,259,290	17.6%	3,557,830	7.7%
2.03	04fpso2term4ves30days	FCNF + K + M + W + D	5,501,730	4,929,685	11.6%	5,297,655	3.9%
2.04	04fpso3term4ves30days	FCNF + K + M + W	4,908,275	4,205,935	16.7%	4,504,095	9.0%

Table 6: Reducing optimality gap, using continuous time model lower bound.

It is interesting to note how fast the lower bound of the continuous time model evolves. It takes little time for the solver to obtain a lower bound that overcome those obtained using discrete time (FCNC) models. Figure 6 shows the evolution of the lower bound of the continuous time model in the first 300 seconds of the solver execution. We used the same group of 6 instances described in Table 6. The cross mark in the diagram represents the moment when the lower bound of the continuous time model overcomes the best lower bound obtained among the FCNF plus valid inequalities models. It means that from this moment on the continuous time model started to reduce the optimality gap proved for the branch-and-bound algorithm. Notice that this moment happened before the end of the first 2 minutes in 5 of the 6 instances evaluated. Only in the instance 1.07 the lower bound of the

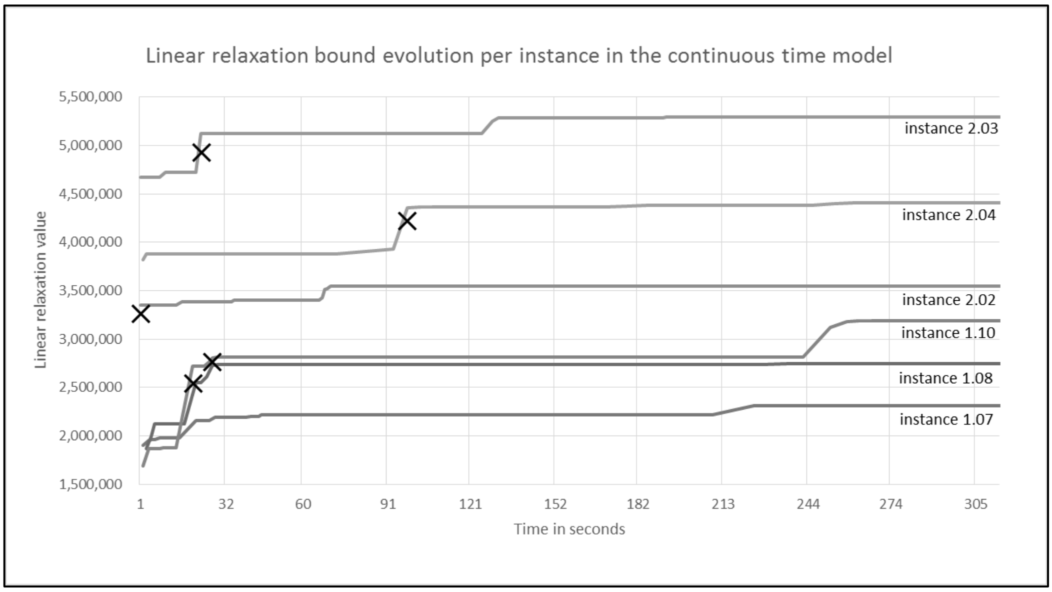

Figure 6: Lower bound evolution per instance in the continuous time model.

continuous time model did not improved the optimality gap. For these instances, the linear relaxation bound of the continuous time model was lower (i.e., worse) than the one obtained in the FCNF+K+M+W and FCNF+K+M+W+D formulations.

5.4. Computational tests conclusion

In this Chapter, we proposed an experimental framework to assess several mathematic MIR formulations when solving a set of real-life based instances of a MIR problem. We tested three formulations, namely, continuous-time, discrete-time, and FCNF formulations. Additionally, we have also considered valid inequalities proposed in Agra et al. (2013), creating four additional formulations.

The first conclusions of the tests were that the FCNF formulation associated with the valid inequalities obtained the best results in terms of computational performance. These formulations, especially FCNF+K+M+W and FCNF+K+M+W+D, were capable to solve more instances (14 instances solved in the 16 tested). However, they are still far from solving instances that could represent a complete problem such as the one we discuss in this study, involving tens of ports and ships, for example.

The second conclusion of the tests is that, even though the continuous time formulation did not achieve good results in terms of computational performance, we identified that it could be used to provide strong linear relaxation bounds (lower bounds). In 13 of the 16 instances, these lower bounds were equal or better than the ones obtained for the discrete-time formulations (including de FCNF plus valid inequalities). Once continuous time formulation is a relaxation of the discrete time formulation, we were able to reduce the optimality gap provided for the branch-and-bound algorithm. In a set of 6 instances, we reduced the optimality gap in 5 of them, reducing the average gap in 65%, from 13.6% to 4.8%.

In practice, this approach allows one to assess how good a solution provided for the FCNF plus valid inequalities is when it does not reach optimality. In our case, we found out that the group of 6 instances was at least 65% closer to optimum then the branch-and-bound encountered. It means the analysts could implement these solutions with confidence that they were doing an efficient scheduling.

Despite the results obtained using MILP formulations, MIR problems are still a wide source of opportunities for improvement, especially regarding its computation complexity. In next Chapter, we present a heuristic approach that intends to improve quality of solution and reduce computational time.

6. Heuristic approach for MIR problems

In order to improve the computational performance and quality of the solutions obtained, we implemented an approach that combines the *relax-and-fix* and *fix-and-optimize* heuristics in a two-phase setting. The *relax-and-fix* phase aims to find feasible solutions while the *fix-and-optimize* phase seeks to improve the solution obtained.

Dillenberger et al. (1994) first presented the *relax-and-fix* heuristic. Since then, it has been widely applied, most notably in production planning, in particular lot-sizing and scheduling problems. Examples are Dillenberger et al. (1994), Stadtler (2003), Kelly and Mann (2004), Beraldi et al. (2006, 2008), Absi and Kedad-Sidhoum (2007), Federgruen et al. (2007), De Araujo et al. (2007, 2008), Akartunali and Miller (2009), Pochet and Warichet (2008), Mohammadi et al. (2008), Ferreira et al. (2009).

We found few studies that applied *relax-and-fix* heuristic to MIR problems. Uggen et al. (2013) extended a *relax-and-fix* heuristic and implemented to solve a general MIR problem. Their results indicate that the computational time was considerably reduced while objective function value was only slightly worse when compared to a general MILP solver. Rodrigues et al. (2016) also implemented *relax-and-fix* heuristic to a ship scheduling in a maritime oil transportation problem. They claim that the method finds reasonable good solutions for instances of moderate size in a relatively small computational time. Our implementation differentiates itself from Uggen et al. (2013), Rodrigues et al. (2016) due to the modifications implemented in *relax-and-fix* heuristic in order to avoid infeasible solutions, and the use of *fix-and-optimize* heuristic associated to improve quality of solution. Additionally, both Uggen et al. (2013) and Rodrigues et al. (2016) used the index time t to select variables in the *relax-and-fix* heuristc. We implemented two alternative versions of the heuristic, one using the index time t and other using index ports i. In this Chapter, we present how this implementation contributes to find good solutions for

MIR problems instances and reduces the computation time when compared to conventional MIP solving.

6.1. Relax-and-fix heuristic

The basic idea of the *relax-and-fix* heuristic is to divide the planning horizon into a finite number of time intervals *n*, as illustrated in Figure 7. The problem is then decomposed into *n*-sub-problems, and solved in iterations corresponding to the time intervals.

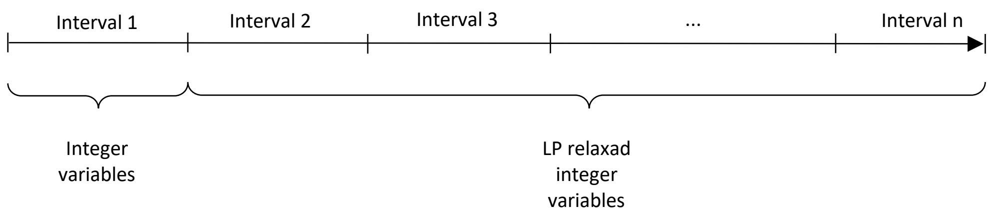

Figure 7: Relax-and-fix iteration one.

In the first iteration, the iteration counter *it* is set to 1 and the problem is solved considering integer variables in the first interval (named Integer Block), while integrality constraints are relaxed for remaining intervals. The block of intervals where integer variables are relaxed is named Continuous Block.

In the next iteration, *it* is increased by 1. From the second iteration on, we have an additional block, named Fixed Block, in which integer variables from interval $it - 1$ are fixed at the solution values obtained in the previous iteration. Integrality constraints are reintroduced for the integer variables in the interval *it* ($it = 2$), while all other variables are kept non-fixed and continuous. Figure 8 illustrate the intervals in the planning horizon in the second iteration.

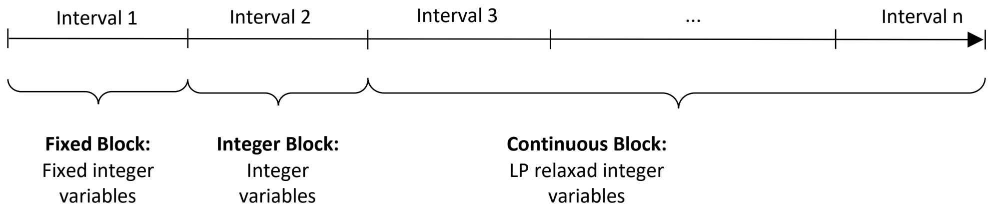

Figure 8: Relax-and-fix: iteration two.

The heuristic finishes when Integer Block reaches the end of planning horizon. At this point, the Continuous Block is suppressed, the planning horizon is divided in two blocks, Fixed Block and Integer Block, and all integer variables have their integrality constraint preserved. A pseudocode for the *relax-and-fix* heuristic is given in Algorithm 1. Here *P* represents the complete problem defined over t := {0, . . . , *Horizon* } time periods with integer variables *x* and continuous variables *y*; *n* is the number of intervals that the planning horizon is divided into and *it* is at the same time an iteration counter and an interval counter (*it={1, …, n}*).

Algorithm 1: Relax-and-fix (P(x(t), y(t)), n, it)

```
RelaxIntegralityCondition (x(t));
it=1;
while it <= n do
  RestoreIntegralityCondition (x(t|t ϵ it));
  SolveMIP P(x(t), y(t));
  FixIntegerVariables (x(t| t ϵ {1, ..., it));
  it:= it + 1;
endwhile;
GetLastSolution of P
```

Algorithm 1: Relax-and-fix pseudo code.

However, a basic implementation of the *relax-and-fix* heuristic, as described in Algorithm 1, may fail to return feasible solution, even when such a solution is guaranteed to exist. This will be the case whenever the fixed portion of the solution leads to an infeasible sub-problem, which does not necessarily mean that the original problem is infeasible. Several proposals have been made to mitigate or eliminate this issue. Dillenberg et al. (1994) fixed the second best solution of the previous iteration in case a sub-problem (with exception of the first) is infeasible. Stadler (2003) used overlapping between the intervals so that some variables with integrality requirement in the previous iteration are kept unfixed, and Escudero and Salmeron (2005) unfix the variables of previous intervals and re-optimise it until the problem becomes feasible.

In order to avoid the occurrence of infeasibility, we implemented two modifications in the basic *relax-and-fix* heuristic. First, we follow Stadler (2003) and used overlapping between the intervals so that not every integer variable of the Integer Block have their values fixed in the next iteration. The second modification is based on Escudero and Salmeron (2005). When an infeasible solution is obtained during an iteration of the heuristic (with exception of the first), we unfix part of the integer variables of the Fixed Block and re-optimize until it finds a feasible solution. Next, we present the proposed version of the *relax-and-fix* heuristic and the mechanisms implemented to avoid infeasible solutions.

6.2. Modified relax-and-fix heuristic

The first strategy we adopt to avoid infeasibility is using overlapping between Integer Blocks of successive iterations. A pseudocode for this mechanism is given in Algorithm 2. Here, *IB* represents *Integer Block*, *FB* is the *Fixed Block* (both are subsets of set *TIME*), *iIB* is the first time period of *IB*, *fIB* is the last time period of *IB*, *sizeIB* defines how many time periods compose *IB* and *pace* determines the displacement of *IB* towards the end of the planning horizon.

Algorithm 2: Relax-and-fix with overlapping procedure (P(x(t), y(t)), iIB, fIB, sizeIB pace)

```
RelaxIntegralityCondition (x(t));
iIB:=1;
fIB:=iIB + sizeIB -1;
IB = {iIB, ..., fIB}
FB = { }
it=1;
RestoreIntegralityCondition (x(t|t ϵ IB));
SolveMIP P(x(t), y(t));
while fIB<= Horizon and P(x(t), y(t)) is feasible do
  iIB:=iIB + pace;
  fIB:=fIB + pace;
  IB = {iIB, ..., fIB}
  FB = {1, ..., iIB - 1}
  FixIntegerVariables (x(t| t ϵ FB);
  RestoreIntegralityCondition (x(t|t ϵ IB));
  SolveMIP P(x(t), y(t));
  it:= it + 1;
endwhile;
GetLastSolution of P
```

Algorithm 2: Pseudo code of relax-and-fix heuristic with overlap procedure.

The first step of Algorithm 2 relaxes integrality condition of all integer variables, and then *IB* and *FB* are defined. After restoring integrality condition of variables in the subset *IB*, problem *P* is solved using MIP. After the first iteration, *IB* and *FB* are modified according to the *pace* defined, integer variables in the subset *FB* are fixed to the values obtained in the previous iteration, integrality restrictions of the variables in subset *IB* is restored, and *P* is solved using MIP. This procedure is repeated while feasible solutions are successively obtained and the last time period of *IB* does not reach the end of planning horizon.

According to Stadler (2003), the overlapping mechanism described reduces the chance of reaching infeasible subproblems. However, it does not assure that the heuristic will return a feasible solution (even when the problem is guaranteed to have at least one feasible solution). Hence, we have implemented another mechanism based on Escudero and Salmeron (2005) to make sure that, if a feasible solution exists, the heuristic will be able to find it (unless it stops due to time limit). We name this phase of the algorithm the *return phase.*

A pseudocode of the complete heuristic (*relax-and-fix* with overlapping procedure and the return phase) is presented in Algorithm 3. An auxiliary parameter *return* is added in order to modify the *FB*, unfixing some integer variables and solving *P* repeatedly until a feasible solution is found. The part between brackets in Algorithm 3 represents the *return phase* of the heuristic. With exception of the first iteration, after every call to SolveMIP, we check the feasibility of solution. If the solution is infeasible, the *return* parameter modifies *IB* and *FB*, unfixing some integer variables from *FB* and including them to *IB*, which reduces the size of *FB* and increases *IB*. The variables in the subset *FB* remain fixed to values obtained before, the integrality condition of variables in the subset *IB* (now modified) are restored, and problem *P* is solved again. This procedure is then repeated until a feasible solution is found or the beginning of *IB* reaches the beginning of planning horizon. If a feasible solution is found during this phase, the heuristic returns to the overlapping procedure. If the solution of *P* is still infeasible after the beginning of *IB* reaches the beginning of planning horizon, then *P* is infeasible.

Algorithm 3: Relax-and-fix with overlapping and return phase procedures (P(x(t), y(t)), iIB, fIB, pace, sizeIB, return, it)

```
RelaxIntegralityCondition (x(t));
iIB:=1;
fIB:=iIB + sizeIB -1;
IB = {iIB, ..., fIB}
FB = { }
it=1;
RestoreIntegralityCondition (x(t|t ϵ IB));
SolveMIP P(x(t), y(t));
while fIB<= Horizon and P(x(t), y(t)) is feasible do
  iIB:=iIB + pace;
  fIB:=fIB + pace;
  IB = {iIB, ..., fIB}
  FB = {1, ..., iIB - 1}
  FixIntegerVariables (x(t| t ϵ FB);
  RestoreIntegralityCondition (x(t|t ϵ IB));
  SolveMIP P(x(t), y(t));
  [if P(x(t), y(t)) is infeasible then
    while P(x(t), y(t)) is infeasible and iIB>1 do
      iIB:= iIB -return;
      IB = {iIB, ..., fIB}
      FB = {1, ..., iIB - 1}
      FixIntegerVariables (x(t| t ϵ FB);
      RestoreIntegralityCondition (x(t|t ϵ IB));
      SolveMIP P(x(t), y(t));
    endwhile;
  endif;]
  it:= it + 1;
endwhile;
GetLastSolution of P
```

Algorithm 3: Pseudo code of relax-and-fix heuristic with overlap procedure and return phase.

In the extreme case, the *return phase* might lead to the original problem. It may happen when the last time period of *IB* reaches the end of planning horizon and the solution found is infeasible. After the heuristic entering in the *return phase* trying to find a feasible solution, if it remains in this phase until the beginning of *IB* coincides with the beginning of planning horizon, then the last iteration of the heuristic would be equivalent to solving the original problem (*IB* is equal to planning horizon).

The *relax-and-fix* heuristic and the modifications implemented we described above were based on the index *t* (time) just for illustration purposes. One may use the same rationale but using another index to choose the integer variables that will compose the *Integer Block*, *Fixed Block* and *Continuous Block*, such as ports *i* or vessels *v*. We have implemented and tested the heuristic using the index time *t* and using index ports *i*. Next, we present some preliminary tests to compare different ways of selecting variables to fix and relax during the *relax-and-fix* heuristic.

6.2.1. Different ways of selecting variables in the *relax-and-fix* heuristic

In order to check whichever index is more efficient when selecting variables to fix and relax during the heuristic approach, we decided to perform some comparison tests. For these tests, we used an instance based on the Coari-Manaus sub-system, when the crude oil is produced in Coari terminal and Manaus is the consumer port. There are 5 vessels available to transport the crude oil during a planning horizon of 20 days. We consider uncertainty in the time spent at both ports and create 25 different levels of robustness. It means that for each level of robustness, we allow a deviation in time spent at ports happens more often in both ports (details about robust optimization approach are in Chapter 6). However, by this point, we are not interested in assess the uncertainty aspect of the model. We used the levels of robustness just to differentiate the instances among them and increase the number of tests.

In these tests, we compare the *relax-and-fix* heuristic using indexes time *t* and ports *i* for selecting variables with the robust optimization FCNF formulation. We execute 25 instances, one for each level of robustness. We describe in Figure 9 the objective function value obtained using the robust optimization FCNF formulation (blue line), and the deviation of the objective function using the *relax-and-fix* heuristic with index *t* (red bar) and index *i* (blue bar) for selecting variables to fix and relax during the heuristic. When we use the index *i* to select variables, the objective function values obtained by the *relax-and-fix* heuristic are much lower than when using the index *t*.

On average, the use of index *i* in the *relax-and fix* heuristic provide solutions 3.5% higher than the objective function of the RO-FCNF, while the average deviation of the heuristic using index *t* is 14.1%. The maximum deviation from RO-FCNF objective function obtained by the heuristic using index *i* is 11.2% while when we use index *t*, the maximum deviation is 46.8%.

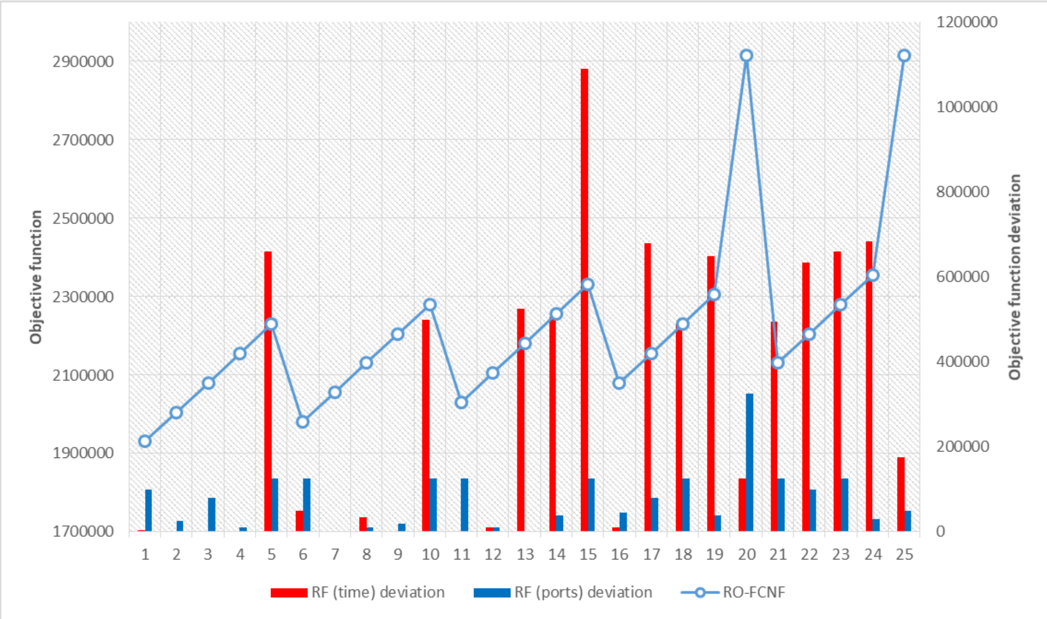

Figure 9: Deviation in the objective function from *relax-and-fix* heuristic to RO-FCNF formulation, using different indexes to select variables.

Regarding computational time, Figure 10 presents computational time of RO-FCNF (grey bar) formulation against, *relax-and-fix* using index *i* (orange bar) and using index *t* (blue bar). The reduction in computational time from RO-FCNF formulation to the *relax-and-fix* heuristic using index *i* is of 96% and using index *t* is of 98%.

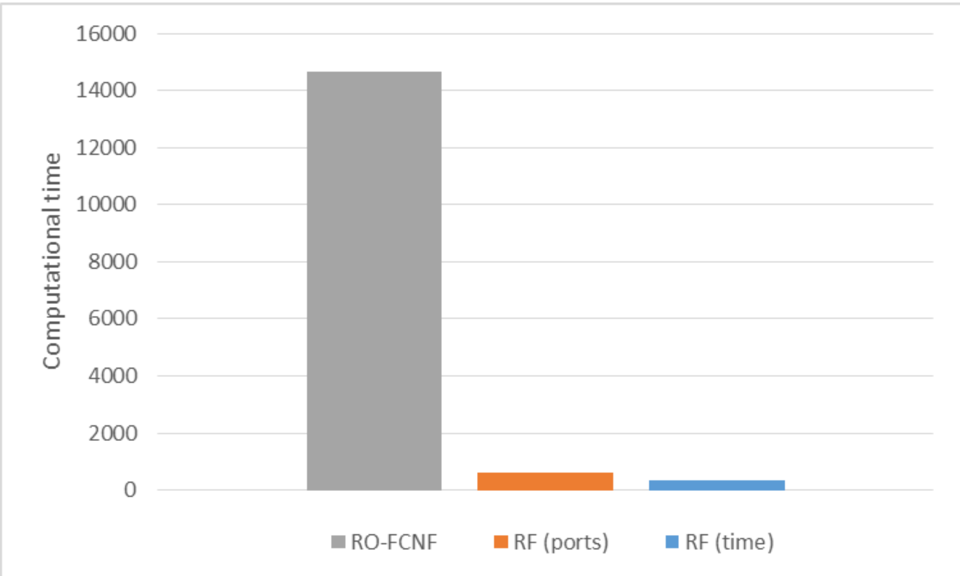

Figure 10: Computational time of RO-FCNF and relax-and-fix heuristic using

different indexes to select variables.

Considering the large difference in the objective function value between both ways of selecting variables in the relax-and-fix heuristics and the considerable reduction in computational time obtained by both of them, we decided to use the index i to select variables in the relax-and-fix heuristic.

The *relax-and-fix* heuristic was adopted to find feasible solutions faster than using only the MIP solver. However, according to Uggen et al. (2013), the solution found by the *relax-and-fix* heuristic is often worse than the one obtained using an MIP solver. Once the *relax-and-fix* heuristic finds a feasible solution, we adopt a *fix-and-optimize* heuristic to improve the quality of the solution obtained.

6.3. Fix-and-optimize heuristic

The *fix-and-optimize* heuristic was proposed independently by Gintner et al. (2005) and Pochet and Wolsey (2006). In the latter, the method was called *exchange*, designed to improve the *relax-and-fix* heuristic. However, the name *fix-and-optimize* used by the former was adopted in the literature. After these first studies, Helber and Sahling (2008), Seeanner et al. (2013), Dorneles et al. (2014) and Toledo et al. (2015) adopted fix-an-optimize for solving multi-level lot-sizing and scheduling problems. We have not found any register in literature of the use *fix-and-optimize* heuristic in MIR problems. The basic idea of *fix-and-optimize* heuristic is very similar to *relax-and-fix*, where several MIP problems need to be solved iteratively. At each iteration, some integer variables are fixed to an initial value and the problem is solved using an MIP solver This process is repeated aiming to improve the initial solution until a stop condition is reached.

Following Pochet and Wolsey (2006), we first use the *relax-and-fix* heuristic to build an initial solution. Then, we further improve it by applying the *fix-and-optimize* heuristic. We define two blocks: a *Fixed Block* and a *Free Block*. The *Fixed Block* is a subset of integer variables, where they are set to the values of the initial solution. All other variables compose the complementary subset *Free Block*. In the *Free Block*, variables are only constrained by their original domains (integer or continuous). Then, we solve this MIP sub-problem and compare the value of its objective function with the value of the best objective function obtained up to that moment (in the first iteration, it is the objective function obtained by the *relax-and-*

fix heuristic). If a better solution is obtained, we set this solution as the new best solution. If it is worse, then we discard this solution. This process is then repeated, changing the variables of the *Fixed Block* at each iteration and always comparing the value of the objective function with the best solution obtained before. As the number of different *Fixed Blocks* may be enormous due to the combinatorial characteristic of the variables, we define a stop criterion that halts execution if, after *n* successive iterations, the best solution is not improved.

Algorithm 4 gives a pseudocode of the *fix-and-optimize* heuristic described. Here, *P* represents the total problem defined over $t := \{0, . . . , Horizon\}$ time periods with integer variables *x* and continuous variables *y*. *FB* is the *Fixed Block*, *IS* is the initial solution provided by the *relax-and-fix* heuristic and *BS* registers the best solution obtained during the heuristic. The auxiliary parameter *rep_it* counts the number of iterations without any improvement in the objective function and *n* is the maximum number of consecutive iterations allowed without improvement in the objective function.

Algorithm 4: Fix-and-optimize heuristic (P(x(t), y(t)),FB, IS, BS, n, rep_it)

```
Define FB;
Define n;
IS:= RF Solution;
BS:= Obj. Func. of Initial Solution;
FixIntegerVariables (x(t| t ϵ FB);
rep_it:=1;
while rep_it <= n do
  Solve P(x(t), y(t));
  If Obj. func. of P < BS then
    BS:= Obj. Func. of P;
    rep_it:=1
  else
    rep_it:=it+1;
  endif;
  Redefine Fixed Block;
endwhile;
GetLastSolution BS
```

Algorithm 4: Pseudo code for fix-and-optimize heuristic.

Just as in the *relax-and-fix* heuristic, one may use different forms to define which variables will compose the *Fixed Block*. We implemented the heuristic defining the

Fixed Block selecting integer variables according indexes time *t* and vessels *v*. Next, we compare both ways of selecting variables to be fixed during *fix-and-optimize* heuristic.

6.3.1. Different ways of selecting variables in the *fix-and-optimize* heuristic

In this phase of the test, we compare the use of index *t* and *v* for selecting the variables that will be fixed during the *fix-and-optimize* heuristic. We set the same initial solution for both alternatives and execute the heuristic for the same 25 instances of the tests of Section 6.2.1. Figure 11 describes the initial solution for all instances (grey line) and the objective function obtained by the *fix-and-optimize* heuristic using two different ways of selecting variables: using index *t* (red line) and index *v* (blue line). Note that the *fix-and-optimize* heuristic improved the objective function of almost every instance. However, the improvement obtained when the *fix-and-optimize* used index *v* is clearly greater than the improvement provided by the heuristic when using index *t*. On average, the use of index *v* provided a reduction of 11.0% of the objective function and the use of index *t* 4.6%. The maximum improvement in one instance was 31.9% and 21.9% for index *v* and index *t*, respectively.

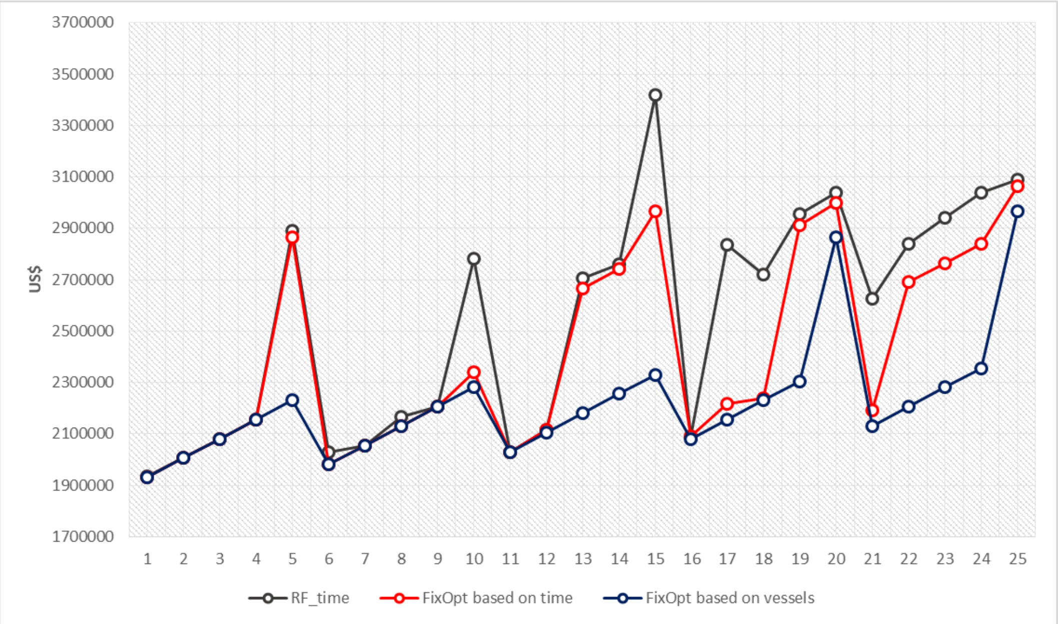

Figure 11: Comparison of different ways of selecting variables in the *fix-and-optimize* heuristic.

Concluding these preliminary tests, we decided to use the index *v* in the *fix-and-*

optimize heuristic. At each iteration, we choose a group of *m* vessels and all the integer variables related to those vessels were set to the *Fixed Block*. At next iteration, we change the vessels of such group and solve the problem again until the stop condition interrupts this routine.

In order to check the performance of the heuristic presented here, we test them using the same set of instances of Chapter 4. In next section, we present the computational results obtained.

6.4. Computational experiments

In this section, we repeat some tests of Chapter 4 but using the approach based on the heuristics *relax-and-fix* and *fix-and-optimize*, which we refer to as the "heuristic approach" hereinafter. We implemented this approach using AIMMS 3.13 and solved the problems using the GUROBI 5.5. The computational experiments were performed on an Intel Core i7 CPU with 8.0 GB RAM.

Prior to presenting the results, we highlight that some of the heuristic parameters (the size of the *Fixed Block*, the *pace* and *return* parameters and the number of iterations in the *fix-and-optimize* heuristic) can be adjusted according to the instance being solved. Depending on the adjustment of these parameters, the results or computational time of the approach may vary. In these tests, we select integer variables to compose the *Fixed*, *Integer* and *Continuous Block* using the index ports *i* (in the *relax-and-fix* phase). The size of *Integer Block* varied according to the number of ports of each instance, the *pace* and r*eturn* parameters are set as 2 and 1 port, respectively. For each iteration of the *relax-and-fix* phase, we established a time limit of 1000 seconds or a relative optimality gap of 20%, whichever occurs first. The *relax-and-fix* procedure finishes when a feasible solution is obtained. Note that the main purpose of the *relax-and-fix* heuristic is to find a feasible solution, not necessarily reaching optimality. In the *fix-and-optimize* heuristic, we fixed the values of the integer variables according to the index *v* (of vessels). At each iteration, the size of *Fixed Block* is defined according to the number of available vessels at each instance. The *fix-and-optimize* heuristic finishes after 10 iterations without any improvement in the objective function. We established a time limit of 200 seconds for each iteration of the *fix-and-optimize* procedure.

To check the performance of the heuristic approach, we first compare its results to the results obtained using the MIP solver. As we implemented the heuristic approach over the FCNF + K + M + W formulation, then we compare its results with the results obtained by the same formulation in Chapter 4. In Table 7, the results displayed in the columns *FCNF + K + M + W Objective function*, *FCNF + K + M + W lower bound*, *Optimality gap* and *Computational time* refer to results obtained for this formulation during the tests conducted in Chapter 4. In the last four columns of Table 7, we display: the results obtained using the heuristic approach, the optimality gap (calculated over the FCNF + K + M + W lower bound), the relative distance to the best solution obtained in Chapter 4 and the computational time.

Instances	FCNF + K + M + W	FCNF + K + M + W Lower Bound	Optimality gap	Computational time	Heuristic Approach solution	Optimality gap*	Δ% Distance to best solution	Computational time
03fpso2term3ves15days	726,045	726,045	0.0%	0	726,045	0.0%	0.000%	2
03fpso3term3ves15days	913,260	913,260	0.0%	0	913,260	0.0%	0.000%	3
04fpso2term4ves15days	1,535,460	1,535,460	0.0%	1	1,535,460	0.0%	0.000%	9
04fpso3term4ves15days	1,030,730	1,030,730	0.0%	0	1,030,730	0.0%	0.000%	7
05fpso2term5ves15days	2,233,560	2,056,130	7.9%	3600	2,233,560	7.9%	0.000%	1,556
06fpso3term6ves15days	2,309,635	2,109,235	8.7%	3600	2,309,635	8.7%	0.000%	764
07fpso3term6ves15days	2,569,010	2,448,525	4.7%	3600	2,650,110	7.6%	3.060%	2,451
08fpso4term6ves15days	2,915,410	2,544,910	12.7%	3600	2,943,280	13.5%	0.947%	3,511
09fpso5term6ves15days	3,893,375	3,893,375	0.0%	2875	3,893,375	0.0%	0.000%	3,324
10fpso5term7ves15days	3,226,085	2,769,895	14.1%	3600	3,225,995	14.1%	-0.003%	5,862
03fpso2term3ves30days	3,397,100	3,397,100	0.0%	1525	3,397,235	0.0%	0.004%	279
03fpso3term3ves30days	3,832,185	3,259,290	14.9%	3600	3,857,420	15.5%	0.654%	1,070
04fpso2term4ves30days	5,502,695	4,930,830	10.4%	3600	5,502,985	10.4%	0.005%	2,613
04fpso3term4ves30days	4,908,275	4,205,935	14.3%	3600	5,211,085	19.3%	5.811%	6,515
05fpso2term5ves30days	NA	5,931,880	NA	7200	6,903,710	14.1%	NA	6,020
06fpso3term6ves30days	NA	6,990,465	NA	7200	NA	NA	NA	-

* calculated over the FCNF + K + M + W ower bound

Table 7: Results from relax-and-fix and fix-and-optimize approach compared to FCNF+K+M+W results.

Notice that the heuristic approach reached optimality in the 8 of the 16 instances (the same instances for which optimality was proved in Chapter 4). It is important to highlight that optimality was not proved during the execution of the heuristic approach, but comparing to the best lower bounds obtained in Chapter 4. It is not possible to calculate optimality gap during execution of heuristic approach as we are always solving sub-problems of the original problem. Therefore, we cannot guarantee that the lower bound obtained in each sub-problem is a valid bound for the original problem. In 10 of the 16 instances, the heuristic approach reached or reduced the best solution found by FCNF + K + M + W formulation. In one instance, the heuristic approach obtained a feasible solution that any of the exact formulations used in Chapter 4 could find. In other 6 instances where the heuristic approach could not reach the best solution obtained in Chapter 4, the average increase on optimality gap was 1.74%. For the last instance, neither the heuristic approach nor the exact methods could find a feasible solution. Table 8 summarizes the tests presented in Table 7. Notice that the main advantage of the heuristic approach is associated with computational time, as it reduces total computational time in 24% when compared to the time spent during FCNF + K + M + W tests.

	Heuristic Approach
Reduce obj. func. Value	2
Reach obj. func. Value	8
Increase obj. func. value	6
Average relative increase in Obj. Func*	1.74%
Total reduction in computational time	24%

*Considering only the 6 instances where objective function increases

Table 8: Heuristic approach tests summary.

After comparing the heuristic approach performance with the results of FCNF + K + M + W, we decided to check how each phase of the approach (*relax-and-fix* and *fix-and-optimize* heuristics) contribute to the results obtained. Table 9 presents the results and computational time obtained in each phase and compare how much each phase contribute for the final result. Nearly 40% of total computational time was spent during the *relax-and-fix* phase. However, despite having spent more time, the *fix-and-optimize* phase improved the result obtained by the *relax-and-fix* phase in almost every instance (except for the first, where the *relax-and-fix* phase had already reached optimality). On average, the *fix-and-optimize* phase improved the solution of the first phase in 7.8% (with a maximum improvement of 24.1%). The last column of Table 9 presents the contribution of *fix-and-optimize* heuristic in total computational time of the heuristic approach. As mentioned before, this phase of the heuristic approach usually consumes more computational time, on average 54% of total computational time of the heuristic approach.

Instances	Relax-and-fix solution	Computational time (relax-and-fix)	Fix-and-optimize solution	Computational time (fix-and-optimize)	Δ % solution value	Δ% computational time
03fpso2term3ves15days	726,045	2	726,045	-	0.0%	0.0%
03fpso3term3ves15days	925,280	2	913,260	1	-1.3%	33.3%
04fpso2term4ves15days	1,592,120	4	1,535,460	5	-3.6%	55.6%
04fpso3term4ves15days	1,122,135	5	1,030,730	2	-8.1%	28.6%
05fpso2term5ves15days	2,343,125	137	2,233,560	1,419	-4.7%	91.2%
06fpso3term6ves15days	2,339,495	178	2,309,635	586	-1.3%	76.7%
07fpso3term6ves15days	2,848,385	744	2,650,110	1,707	-7.0%	69.6%
08fpso4term6ves15days	3,214,130	1,528	2,943,280	1,983	-8.4%	56.5%
09fpso5term6ves15days	4,557,080	1,570	3,893,375	1,754	-14.6%	52.8%
10fpso5term7ves15days	4,252,750	1,854	3,225,995	4,008	-24.1%	68.4%
03fpso2term3ves30days	3,930,110	175	3,397,235	104	-13.6%	37.3%
03fpso3term3ves30days	4,113,575	409	3,857,420	661	-6.2%	61.8%
04fpso2term4ves30days	5,773,435	817	5,502,985	1,796	-4.7%	68.7%
04fpso3term4ves30days	5,663,165	4,513	5,211,085	2,002	-8.0%	30.7%
05fpso2term5ves30days	7,833,050	1,524	6,903,710	4,496	-11.9%	74.7%
06fpso3term6ves30days	NA		NA		NA	NA

Table 9: Results and computational time obtained in the heuristic approach.

We conclude that the heuristic approach has demonstrated to be an efficient method for solving MIR problems. Although it cannot prove optimality, whenever we compare its results with the solutions obtained using an exact method (FCNF + K + M + W), we found positive evidences concerning the quality of these solutions. The heuristic approach reached the FCNF + K + M + W objective function value in 8 instances, reduces its value twice and it performed slightly worse in other 6 instances (1.74% worse on average). Moreover, the total computational time spent during the tests was 24% smaller, which consists of a considerable decrease in computation time.

We also note that the two phases of the heuristic approach were important to reach the solutions obtained. While the *relax-and-fix* phase constructs initial solutions, the *fix-and-optimize* phase improves them. Therefore, we believe that this approach is a good alternative for solving MIR problems, especially for larger instances, where conventional formulations could not return feasible solutions.

We have discussed the best deterministic approach to deal with MIR problems and an alternative heuristic approach. However, as we presented in Section 2.2, the uncertainty aspect is vital for MIR real-life problems. In next Chapter, we present a brief review of literature regarding MIR problems considering uncertainty parameters, robust optimization and a robust approach for MIR problems.

7. A robust approach for maritime inventory routing optimization

There are few studies in literature that features uncertainty in MIR problems. Most of them deal with uncertainty in production/consumption rates or in travel time through adoption of inventory safety levels in the inventory planning (Halvorsen-Weare et al., 2013; Agra et al., 2105; Assis and Campnogara, 2016). If a ship arrives late at a loading port, production in that port may stop due to shortage in storage capacity. To mitigate the possibility of such situations, Christiansen and Nygreen (2005), Christiansen et al. (2007) and Papageourgious et al. (2014) proposed to set an upper safety stock level that is below the storage capacity, and a lower safety stock level that is above a specified lower storage capacity. Any diversion of the inventory from this range of safety stock limits was penalized.

In order to address the uncertainty aspect in sailing time and production level, Halvorsen-Weare et al. (2013) proposed a solution method adopting several robust strategies and a simulation model to evaluated solutions. They claimed that robust strategies added value to the solutions with lower expected costs. Agra et al. (2105) presented a computational study based on real-life instances where a stochastic MIR problem with uncertain sailing times and unpredictable waiting times at ports. A two-stage stochastic programming model with recourse was presented where the first-stage decisions consisted of routing, loading and discharging decisions and the second-stage decisions consisted of scheduling and inventory decisions. Assis and Campnogara (2016) presented a MILP model for planning the trips of dynamic positioned tankers with variable travel time in a problem of crude oil transportation from offshore production sites to onshore terminals. To address large-sized instances, they proposed a combination of a MILP formulation and heuristics such as relax-and-fix and rolling-horizon.

As mentioned before, in the MIR problem we considered in this study, the time spent at ports represents about 75% of the fleet available time, while travel time is about 25%. Additionally, travel distances are short (taking 1 to 3 days on average) and variations in vessel's speed have little impact in travel time. Therefore, we decided to consider uncertainty in the time spent at ports. Hence, the problem at hand is to minimize transportation costs of carrying crude oil from offshore production sites to onshore terminals and keep inventory levels within operational bounds at ports, without knowing exactly how long each vessel will stay at each port due to delays in the total time spent at ports.

In next Section, we present a brief overview about optimization under uncertainty.

7.1. Modeling uncertainty – a brief review

Sahinidis (2004) claims that a key difficulty in optimization under uncertainty is dealing with a large variety of uncertain possibilities that frequently leads to prohibitively large-scale optimization models. Decision-making under uncertainty is often further complicated by the presence of integer decision variables that model logical and other discrete decisions in multi-period settings. Approaches to optimization under uncertainty have followed a variety of modeling philosophies, including minimization of expected values, deviations from goals, maximum costs, and optimization of soft constraints. According to Sahinidis (2004), the main approaches reported in literature are stochastic programming (recourse models, robust stochastic programming, and probabilistic models), robust optimization, fuzzy programming (flexible and probabilistic programming) and stochastic dynamic programming.

According to Sahinidis (2004), the objective of stochastic programming is to minimize the first-stage variable costs and the expected value of the random second-stage costs. Gorissen et al. (2015) claims that two important aspects have to be noted prior to the adoption of stochastic programming approach are that it assumes that the true probabilistic distribution of uncertain data is known or is possible to be estimated, and its reformulation usually increases the complexity of the problem.

Another approach to deal with uncertainty is the probabilistic programming (or chance constraints). According to Sahinidis (2004), the philosophy of this approach is that infeasibilities are allowed up to a certain extent. In this case, the focus is on the reliability of the system and it is expressed as a minimum requirement on the probability of satisfying constraints. However, Prékopa (1971) has shown that unless the cases where the right-hand uncertain vector has a log-concave multivariate probability density function, in general, the probability constraints leads to a non-convex feasible set. Pangoncelli et al. (2009) claims that even for simple functions (e.g, linear problems), chance constrained problems may be extremely difficult to solve numerically and their feasible sets can be non-convex. Techniques to convert the non-convex feasible set into a convex-set approximation usually require the generation of scenarios, which leads to increase computational complexity.

On the other hand, Gorissen et al. (2015) claims that robust optimization is popular because it does not rely on previous knowledge of probability distribution and because of its computational tractability for many classes of problems. A drawback of the robust optimization approach is that it usually leads to very conservative solutions. Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004) proposed different methodologies to deal with the level of conservativeness of the solution.

Although we have access to a historical data of time spent at ports, we decided to adopt a robust optimization approach for two main reasons. First, MIR problems are very complex, especially for real-life instances as we have seen in Section 3.1 and in the results of computational experiments in Section 5.1 and 5.2, and computational tractability is an advantage of robust optimization. The second reason is that stochastic optimization provides an optimal solution considering the expected value for a set of possible scenarios and their probability of realization. In MIR problems, this might lead to infeasible plans for the real problem, thus compromising the usefulness of the model. As feasibility is crucial for MIR problems and computational performance is also an important issue, we decided to adopt a robust optimization technique to address uncertainty in MIR problem. In next section, we present a brief review on robust optimization.

7.2. Robust optimization literature review

According to Gorissen et al. (2015), robust optimization (RO) is a relatively young and active research field, and has mainly developed in the last 15 years. Its concepts and techniques are very useful in practice, since they are tailored to information at hand and typically lead to tractable formulations. However, few works have been published for real-life applications and there are still much more potential than what has been exploited hitherto.

According to Gabrel et al. (2014), uncertainty impacts the solution feasibility in many problems. In these cases, robust optimization seeks to obtain a solution that will be feasible for a given realization taken by the unknown coefficients. However, complete protection from adverse realizations often comes at the expense of a severe deterioration in the objective function. To make the robust methodology appealing to business practitioners, RO thus focuses on obtaining a solution that will be feasible for any realization taken by the unknown coefficients within a smaller "realistic" set, called the uncertainty set, which is centered on the nominal values of the uncertain parameters. The specific choice of the set plays an important role in ensuring computational tractability of the robust problem and limiting deterioration of the objective function.

A pioneer in RO, Soyster (1973) proposes a linear optimization model to construct a solution that is feasible for all data that belong in a convex set. The resulting model produces solutions that are too conservative in the sense that it gives up too much of optimality for the nominal problem in order to ensure robustness. To address the issue of over conservatism, Ben-Tal and Nemirovski (2000) proposed a less conservative model by considering uncertain linear problems with ellipsoidal uncertainties, which involve solving the *robust counterparts* of the nominal problem in the form of conic quadratic problems. However, a practical drawback of such an approach is that it leads to nonlinear, although convex, models, which are more demanding computationally than the earlier linear models by Soyster (1973). Bertsimas and Sim (2004) propose an approach for robust linear optimization that retains the advantages of the linear framework of Soyster (1973) and offers full control on the degree of conservatism for every constraint. Additionally, unlike Ben-Tal and Nemirovski (2000), the robust counterparts proposed by Bertsimas and Sim (2004) are linear optimization problems, and thus their approach readily generalizes to discrete optimization problems.

Bertsimas and Sim (2004) approach protects the model against violation of constraint i deterministically, when only a prespecified number Γ_i of the coefficients changes; in other words, it guarantees that the solution is feasible if up to Γ_i uncertain coefficients change. The parameter Γ is introduced in order to adjust the solution robustness against the decision-maker conservatism. It is also known as the budget of uncertainty, which reflects the decision-maker's attitude towards uncertainty. As this budget increases, the model is more protected against deviations in the uncertain parameter.

According to Gabrel et al. (2014), in recent years a major area of research has been developed in robust optimization. In order to deal with the information revealed over time directly into the model, many theoretical works are incorporating two-stage decision problems in robust optimization. In multiple-stage optimization, an important assumption of RO paradigm, i.e., the decisions are “here and now”, can be relaxed and “wait and see” decisions are incorporated into the modeling. Some decision variables can therefore be adjusted at a later moment in time according to a decision rule, which is a function of the uncertain data. According to Gorissen et al. (2015), adjustable robust optimization (ARO), as multi-stage robust optimization is known, is less conservative than the classic RO approach, since it yields more flexible decisions that can be adjusted according to the realized portion of data a given stage. Both Gabrel et al. (2014) and Gorissen et al. (2015) presented more details about ARO.

7.3. Fleet fixed costs and robust optimization

Prior to present the robust approach for MIR problem, it is important to elucidate some important changes in the MIR problem we adopted from now on in this study.

Usually, MIR problems are planning problems of operational or tactical levels and they consider a given and fixed fleet of ships (which is also the case we presented here in Chapters 2 and 3 - real problems and deterministic formulations, respectively). As the fleet is given and fixed over the planning horizon then fixed costs of the fleet are commonly disregarded in the objective function. For example, if one has a fleet of 5 ships, there are no differences between using 1, 3 or all 5 ships in the planning horizon, in terms of fixed costs, because hiring costs will be paid regardless for the entire planning horizon.

7.3.1. Fixed cost reformulation

In robust optimization MIR problem (RO-MIR), the aim continues to be minimizing transportation costs, while finding a feasible solution that maintains inventory levels between operational bounds. However, in RO-MIR, when delays in time spent at ports happen, the proposed ship scheduling consumes more days of the fleet vessels. Thus, one way of protecting the solution against delays in time spent at ports is "consuming" more vessels, that is, using vessels for a longer period for the same voyage. Thus, we considered important not to ignore fixed costs (hiring costs) in the RO-MIR problem. In this way, we include in the problem the fixed cost of the fleet in order to measure the impact of uncertainty in transportation costs.

It might seen contradictory to consider the hire cost if the entire fleet was already hired as a time charter, i.e. for a long time horizon. However, PETROBRAS MIR problem is enormous in terms of number FPSOs, vessels and onshore terminals. Therefore, prior to scheduling the fleet, a planning team divide the vessels in groups and assign each group of vessels to a geographic region, where those vessels will transport crude oil from the FPSOs to onshore terminals. These geographic regions are named subsystem of the entire MIR problem and are presented in Figure 12.

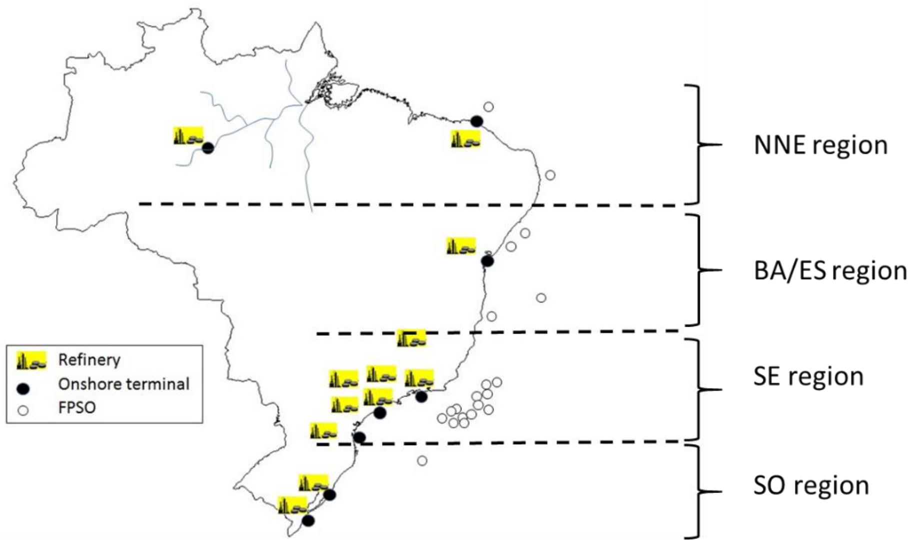

Figure 12: The fleet is divided in regions and assigned to each subsystems of PETROBRAS MIR problem.

The scheduling team have flexibility to change vessels from one subsystem to another during the day-by-day activity, if necessary. However, it is important for the planning team to make sure that the number of vessels assigned to each subsystem is enough to offload the entire crude oil production, but not too large such that vessels become idle during the planning horizon. In other words, the planning team is responsible for fleet sizing decisions as well as for maintain service level (i.e., keep inventory levels at FPSOs and onshore terminals within operational range).

Therefore, indicating the number of vessels for a specific subsystem during a specific planning horizon does not mean hiring or delivering a new vessel, but reallocating a vessel from one subsystem to another. In practice, the main decision supported by this proposed robust methodology for MIR problems is sizing the fleet for each geographic subsystem for the next short-term planning horizon. While scheduling decisions indicated by the robust methodology will be important to ascertain feasibility of a given plan, the most important decision concerns the number of vessels dedicated to each subsystem for the next short-term planning horizon. Even though we have a historical data of the time spent at each port, we cannot know in advance when the deviation at time spent at port will occur. In other words, despite our ability to estimate how long vessels will stay at each port during the entire planning horizon (according to historical database), we cannot know exactly in which visit the time spent by a vessel at a port will deviate from its nominal value. Such information is vital for scheduling decisions.

Next, we present some additional elements that will be used to model fixed costs.

Parameters

$TIME$ The last time period of the planning horizon T,

$HIRE_v$ The hire fee paid in a daily basis to vessels owner,

Variables

nh_v^{NT} Binary variable that indicates whether a vessel stayed idle during the entire planning horizon or not.

Objective function

$$min \sum_{v\in V} \sum_{i\in N\cup\{O_v\}} \sum_{j\in N\cup\{DE_v\}} \sum_{t\in T} C_{ijv}^{T} x_{ijvt} + \sum_{v\in V} \sum_{i\in N} \sum_{t\in T} C_{iv}^{P} x_{ijvt} + \sum_{v\in V} \sum_{i\in N} \sum_{t\in T} C_{v}^{W} w_{ivt} + \sum_{v\in V} (1 - nh_v^{NT}) HIRE_v TIME \quad (69)$$

Constraints

$$nh_v^{NT} = \sum_{i=O_v} \sum_{j=DE_v} \sum_{t\in T} x_{ijvt} \qquad \forall v \in V \quad (70)$$

In (69), we add to the original objective function the fixed cost for every vessel that is utilized during the planning horizon. Constraints (70) ensure that if a ship goes directly from the artificial origin node to the artificial destination node, then it is not utilized for that planning horizon.

Once we stablish the particularity of RO-MIR problems, we may present the robust optimization approach for dealing with RO-MIR problems.

7.4. Methodology

The proposed robust model is based on Bertsimas and Sim (2004) robust approach. As mentioned before, Bertsimas and Sim (2004) approach provides the possibility of controlling the level of robustness and a linear robust counterpart, which allows it to be generalized to discrete optimization problems.

To quantify explicitly the relationship between the level of robustness of the solution and the probability of constraint violation, we associate Bertsimas and Sim (2004) approach with a Monte Carlo simulation process. Simulating the uncertain parameter time spent by vessels at port, one is able to estimate the probability of infeasibility for the solution of each level of robustness provided by the robust optimization model.

Our robust optimization approach has three phases. First, we apply Bertsimas and Sim's (2004) approach to the FCNF combined with valid inequalities (K+M+W) in order to obtain the robust optimization fixed charge network flow (RO-FCNF). In the second phase, we execute RO-FCNF model for each level of conservatism, varying the budget of uncertainty Γ (or level of robustness). For each level of Γ, we will obtain a robust solution that consists of a solution that will remain feasible even if the realization of the uncertain parameter occurs in its worst case in every visit of vessel v to port i (the maximum number of visits defined by Γ). In the third phase, we execute a Monte Carlo simulation process to estimate the probability of infeasibility for each level of Γ. We execute the deterministic FCNF model plus valid inequalities (K+M+W) 100 times, simulating the realization of the uncertain parameters according to a probabilistic distribution of historical data base; and fixing the number of vessel required and vessel availability (protection against infeasibility) provided by the RO-FCNF, for each level of Γ. Figure 13 illustrates the methodology we adopted.

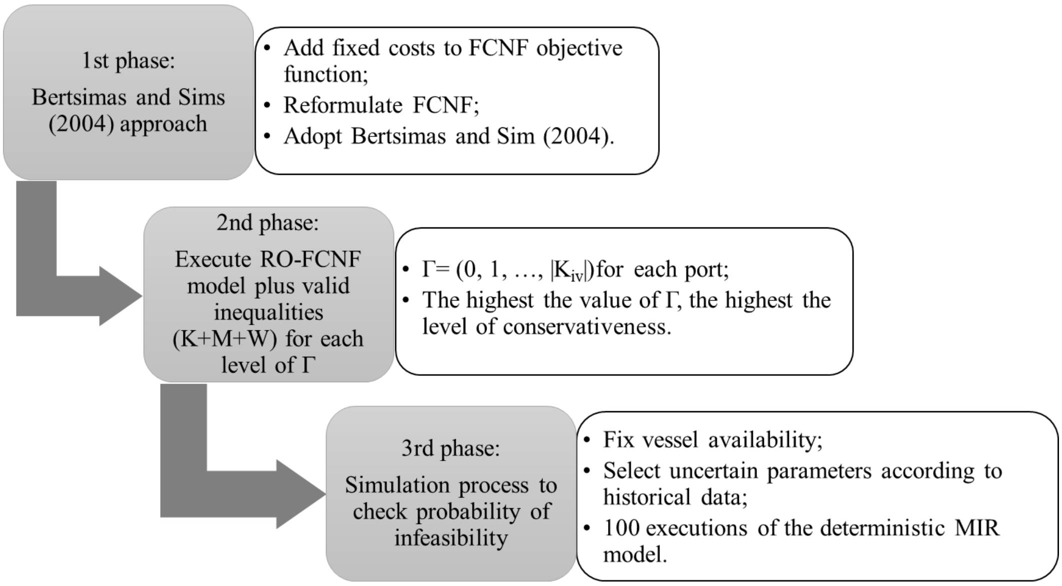

Figure 13: Methodology flux of robust optimization approach.

In the next three sub-sections, we describe the mathematical reformulation we apply to the FCNF model, the Bertsimas and Sim (2004) approach to obtain the RO-FCNF model and the simulation phase to identify the trade-off between infeasibility level and the impact on the objective function.

7.4.1. Mathematical reformulation

The proposed robust optimization model is an extension of the FCNF deterministic model presented in Section 4.3 defined by (24) – (26), (30) – (31), (35) – (38) and (42) – (54) and adjusted by (69)-(70) in section 7.3.1. However, before applying Bertsimas and Sim (2004) robust approach, we have to perform adjustments in the FCNF formulation. Next, we present some additional sets, parameters and variables included in this version.

Additional sets:

K_{iv} Set of coefficients OH_{iv} that are subject to parameter uncertainty.

Additional parameters:

OH_{iv} Nominal time spent by vessel v at port i,

OHD_{iv} Maximum deviation observed for the time spent by vessel *v* at port i,

$\widetilde{OH}_{iv}$ Random variable representing the time spent by vessel *v* at port i,

Γ_{iv} Represents the maximum number of coefficients parameters of constraints (i,*v*) that can deviate from their nominal value. It adjusts the robustness of the decision. The parameter $\Gamma_{iv} \in [0, |K_{iv}|]$. Hence, the parameter Γ_{iv} protects against deviations in up to $\lfloor \Gamma_{iv} \rfloor$ of these coefficients. In other words, we stipulate that nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely affect the solution.

Additional auxiliary variable:

k_{ijvt} Auxiliary variable representing the scaled deviation of time spent at port $k_{ijvt} = \left(\widetilde{OH}_{iv} - OH_{iv}\right)/OHD_{iv}$,

In order to model the time spent at ports properly, we add a new set of constraints.

$$\sum_{t \in T} o_{ivt} \geq \sum_{t \in T, j \in N} OH_{iv} x_{ijvt}, \quad \forall i \in N, v \in V, \qquad (71)$$

Constraints (71) determine that the number of time periods a vessel *v* spends at port *i* must be equal to or greater than the product of a nominal time spent at port $OH_{i,v}$ and the number of visits of vessel *v* at port *i*. Note that constraints (71) control the total time spent at ports by vessel *v* during the entire planning horizon. For modeling purposes, it is sufficient to represent how many time periods each vessel will require to perform all assigned activities during the planning horizon. This information is then used to support the fleet sizing decision under uncertainty. The scheduling decision will be treated properly in the 3th phase of the methodology, in which we simulate realizations of parameter $OH_{i,v}$ to assess the performance of the solution obtained from the robust optimization model.

7.4.2. Bertsimas and Sim robust approach applied to FCNF formulation

To consider uncertainty in parameter OH_{iv}, we reformulate constraints (71) in a stochastic version (71s), where OH_{iv} may deviate up to its maximum deviation OHD_{iv} according to its scalar deviation k_{ijvt}:

$$\sum_{t\in T} o_{ivt} \geq \sum_{t\in T, j\in N} OH_{i,v}x_{ijvt} + \sum_{t\in T, j\in N} OHD_{iv}x_{ijvt}k_{ijvt}\ , \forall i \in N, v \in V \quad (71s)$$

In order to build the robust counterpart of the model, it is necessary to reformulate constraints (71s). First, we adopt the robust paradigm, which considers that uncertainty will behave as worse as possible. In other words, (71s) indicates that the solution would be feasible for every realization of the time spent at port.

$$\Phi(x,\Gamma) = Max_k\{\sum_{t\in T, j\in N} OHD_{iv}\, x_{ijvt}k_{ijvt} \,|\, \sum_{t\in T, j\in N} k_{ijvt} \leq \Gamma_{iv}, k_{ijvt} \geq 0\} \quad (72)$$

Note that (72) protects against the uncertainty for every visit of a vessel *v* at a port *i* and it is known as protection function. We disregard $k_{ijvt} \in [-1{,}0]$, because we intend to protect only against the worst cases. Moreover, the parameter Γ_{iv}, introduced to adjust the model robustness against the conservatism of the solution, can be understood as the maximum number of the uncertain parameters that can deviate from their nominal values. Γ_{iv} may take values in the interval [0,|*K*|], where | *K*| represents the maximum number of visits a port *i* may receive during the entire planning horizon.

The robust counterpart of (71) is

$$\sum_{t\in T} o_{ivt} \geq \sum_{t\in T, j\in N} OH_{i,v}x_{ijvt} + \Phi(x,\Gamma), \forall i \in N, v \in V \quad (73)$$

Applying the robust optimization technique developed by Bertsimas and Sim (2004), we formulate an auxiliary problem (74-76). Its objective is to maximize the sum of all deviations over the set of all admissible realizations of the uncertain parameters.

$$Max_k \sum_{t\in T, j\in N} OHD_{iv}\, x^*_{ijvt}k_{ijvt} \quad (74)$$

Subject to:

$$\sum_{j,t} k_{ijvt} \leq \Gamma_{iv}, \forall i, v \quad (75)$$

$$0 \leq k_{ijvt} \leq 1, \forall i, j, v, t \quad (76)$$

If $\Gamma_{iv} = 0$, the k_{ijvt} for all *i,j,v,t*, are forced to be 0, so that random variable $\widetilde{OH}_{iv}$ are equal to their mean value $OH_{i,v}$ and there is no protection against uncertainty. On the other hand, when $\Gamma_{iv} = |K|$, the k_{ijvt} for all *i,j,v,t*, are forced to be 1 (in this particular problem) and constraints (71s) is completely protected against uncertainty, which yields a very conservative solution. For values between 0 and $|K|$, the decision-maker can make a trade-off between the protection level and the degree of conservatism of the solution.

Following the same rationale of Bertsimas and Sim (2004), the dual of model (74)-(76) is stated as follows:

$$Min_{\pi,\rho}\pi_{iv}\Gamma_{iv} + \sum_{t\in T, j\in N}\rho_{ijvt} \quad (77)$$

Subject to:

$$\pi_{iv} + \rho_{ijvt} \geq OHD_{iv}x_{ijvt}, \quad (78)$$

$$\rho_{ijvt} \geq 0, \forall i, j, v, t \quad (79)$$

$$\pi_{iv} \geq 0 \quad (80)$$

This dual problem has two dual variables (π_{iv}, ρ_{ijvt}) that are associated to constraints (75) and (76), respectively. By strong duality, as model (74)-(76) is feasible and bounded for all $\Gamma_{iv} \in [0,|K_{iv}|]$, then the dual problem (77)-(80) is also feasible and their objective function values coincide.

Substituting (77)-(80) in constraints (73), the following robust linear set of constraints are obtained.

$$\sum_{t\in T, j\in N} OH_{iv}x_{ijvt} + \pi_{iv}\Gamma_{iv} + \sum_{t\in T, j\in N}\rho_{ijvt} - \sum_{t\in T} o_{ivt} \leq 0, \forall i \in N, v \in V, \quad (81)$$

$$\pi_{iv} + \rho_{ijvt} \geq OHD_{iv}x_{ijvt}, \quad (82)$$

$$\rho_{ijvt} \geq 0, \forall i, j, v, t \quad (83)$$

$$\pi_{iv} \geq 0 \quad (84)$$

Adding (81)-(84) to the original FCNF formulation, we have now the RO-FCNF formulation. This model minimizes transportation costs and ensure that up to Γ coefficients deviate their value from the mean time spent at port within the permitted interval ($[OH_{iv} - OHD_{iv}, OH_{iv} + OHD_{iv}]$), then the solution of the robust optimization model will remain feasible. In other words, the solution of this model is a robust solution.

We present the complete RO-FCNF formulation in the Appendix 1.

7.4.3. Probability of infeasibility

The third phase of the framework consists of assessing the robust solutions in terms of their probability of infeasibility for each level of robustness. In other words, we evaluate the chances of a robust solution being infeasible (when inventory level violates a maximum or minimum level) when one implements the protection provided by such level of robustness (protection means the number of vessels and vessel availability provided in each level of robustness). We simulate the realization of the uncertain parameter OH_{iv} 100 times according to its historical data and count how many times the problem becomes infeasible due to higher time spent at port and a lack of protection. As $\Gamma = 0$ leads to a deterministic solution and $\Gamma = K$ leads to a very conservative solution, we expect that the probability of infeasibility would be considerable for lower levels of robustness and that it would diminish as Γ increases.

For each level of Γ, we obtain a distinct ship routing and scheduling for the fleet. However, ship-scheduling decisions are very dynamic, as they are influenced by day-by-day uncertain events and adjusted with short notice as these uncertain events unfold. Therefore, we decided to discard the robust ship scheduling decisions (i.e., the exact time period and port each vessel has visited) and only consider as fixed the vessel availability obtained for each level of Γ. We name such vessel availability the protection given by the RO-FCNF for each level of robustness against infeasibility.

The vessel availability is given by the time between the time period in which a ship leaves the artificial origin node O_v and the time period the same ship enters the artificial destination node DE_v. For example, suppose that a vessel leaves its artificial origin node O_v at time period 5 to start its first voyage in the planning horizon and, at the end of the planning horizon, arrives at its artificial destination node DE_v at time period 15. In our simulation, it means that its availability window is from time period 5 to time period 15. We use constraints (85) and (86) to set this availability period, making sure that this vessel will not starts a voyage before time period 5 and it will have to finish its last voyage of the planning horizon until time period 15.

Next, we present additional constraints in the deterministic FCNF model used to set vessel availability for each level of Γ.

Sets

UNB(v) Subset of T that comprehend the time periods in the beginning of the planning horizon when each vessel is unavailable

UNE(v) Subset of T that comprehends the time periods at the end of planning horizon when each vessel is unavailable.

Constraints

$$\sum_{t \in UNB_v} x_{ijvt} = 0 \qquad \forall i, j \in N \text{ and } \forall v \in V \tag{85}$$

$$\sum_{t \in UNE_v} x_{ijvt} = 0 \qquad \forall i, j \in N \text{ and } \forall v \in V \tag{86}$$

Constraints (85) and (86) defines the period of vessel unavailability in the beginning and at the end of planning horizon, respectively. In other words, it fixes the availability period for each vessel of the fleet. It is important to highlight that constraints (85)-(86) are only used in the simulation phase of the robust optimization approach, because the unavailability periods (in the beginning and at the end of planning horizon) are only known after the model is solved for each level of Γ.

The probabilistic distribution of the uncertain parameter OH_{iv} used in the 3rd phase of the methodology is defined according to historical data obtained from PETROBRAS. Despite the parameter being indexed by the each port *i* and each vessel *v*, we consider that the time spent at port OH_{iv} depends only on the port *i*. There is no statistical dependence between the ship and the port

The aim of this phase is to support the decision by trading of level of robustness (or risk) and efficiency of the solution. One should seek for that solution that provides a good trade-off between probability of infeasibility and the impact on the objective function in terms of costs. Ideally, one should seek for solutions that provide high chances of feasibility and low impacts in the objective function (low increase in transportation costs.

8. Experimental results of robust approach

In this Chapter, we present some tests using the robust optimization approach described in the preview Chapters. The aim of the tests is to verify how the model behaves against uncertain deviations of the time spent at ports. The main hard constraints of the model are related to maintaining inventory levels between predefined upper and lower bounds. Delays in (un)loading operations due to deviations in the time spent at ports may lead inventory level to the minimum limit, at onshore terminals, or to the maximum limit, at production sites. The violation of maximum and minimum inventory limits turns the solution infeasible. Thus, the decision maker must define a robustness level, where the ship scheduling is robust enough to avoid the infeasibility. At the same time, it cannot be too conservative such that it unnecessarily increases the objective function value.

In Section 8.1, we present the instance used to assess the robust optimization approach and the probability distribution of the time spent at ports according historical data. In Section 8.2, we discuss the results of the numerical experiments performed, aiming at identifying which actions the model suggests in order to maintain feasibility of the problem while the level of robustness (Γ) increases.

8.1. Instance and historical probability distribution

We prepared an instance to represent the geographic south region of Brazil. In this subsystem there are 2 onshore terminals (Tedut and Tefran - that supplies one refinery each) and are supplied of crude oil from 4 different FPSOs, located in the southeast coast of Brazil. We allocate a potential fleet of 5 vessels of different sizes to supply these refineries during a planning horizon of 15 days. The model suggests how many vessels are required to supply such subsystem in each level of robustness. The uncertain parameter (time spent by vessels at each port) varies according to many reasons described in Section 2.2. We analyzed a two-year historical dataset to estimate the mean time spent at ports (OH_i) and the maximum deviation allowed in this mean time (OHD_i). As operations in FPSOs are more scattered than in onshore terminals, we analyzed the time spent at FPSOs as if they were only one and the time spent at each onshore terminal individually.

For Tefran onshore terminal, we analyzed 240 operations; the minimum data value was 0.5 days, the maximum data value was 14.7 days; sample mean was 1.8 days, the medium value was 1.0 day and the standard deviation was 2.1 days. Figure 14 presents the histogram for the data. We used the medium value to estimate the nominal time spent at Tefran onshore terminal (OH_{Tefran}) and defined the maximum allowed deviation (OHD_{Tefran}) as 3 days. It means that our robust model protects against infeasibility for up to 93% of the cases, where the time spent at Tefran onshore terminal reaches up to 4 days ($OH_{Tefran} + OHD_{Tefran}$).

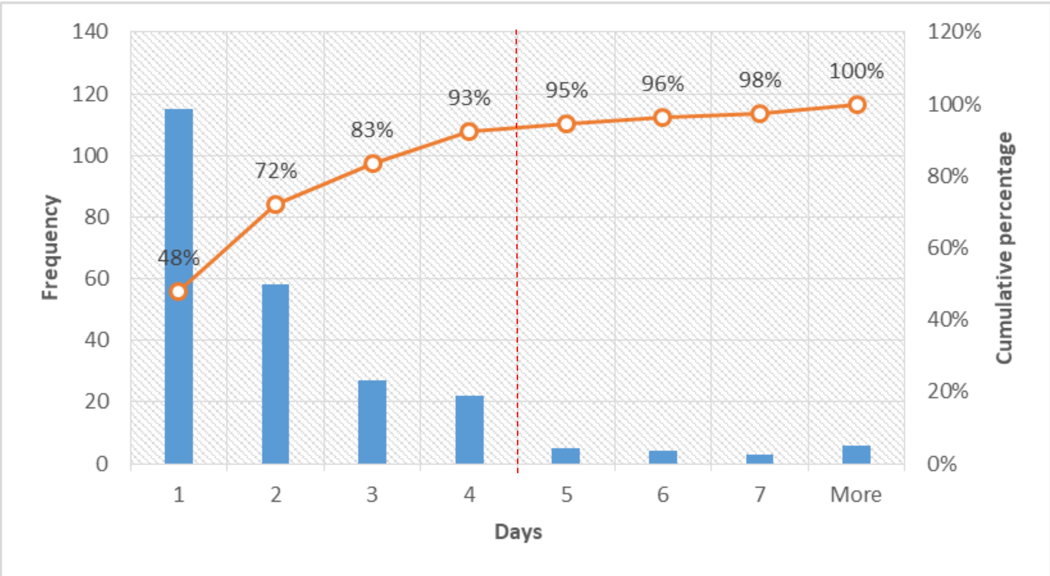

Figure 14: Two-year base Tefran onshore terminal port time histogram

For Tedut onshore terminal, we analyzed 187 operations; the minimum data value was 0.7 days, the maximum data value was 11.7 days; sample mean was 2.6 days, the medium value was 1.9 days and the standard deviation was 2.0 days. Figure 15 presents the histogram of the sample. We used the medium value to estimate the nominal time spent at Tedut onshore terminal (OH_{Tedut}) and defined the maximum allowed deviation (OHD_{Tedut}) as 4 days. It means that our robust model protects against infeasibility for up to 94% of cases where the time spent at Tedut onshore terminal reaches up to 6 days ($OH_{Tedut} + OHD_{Tedut}$).

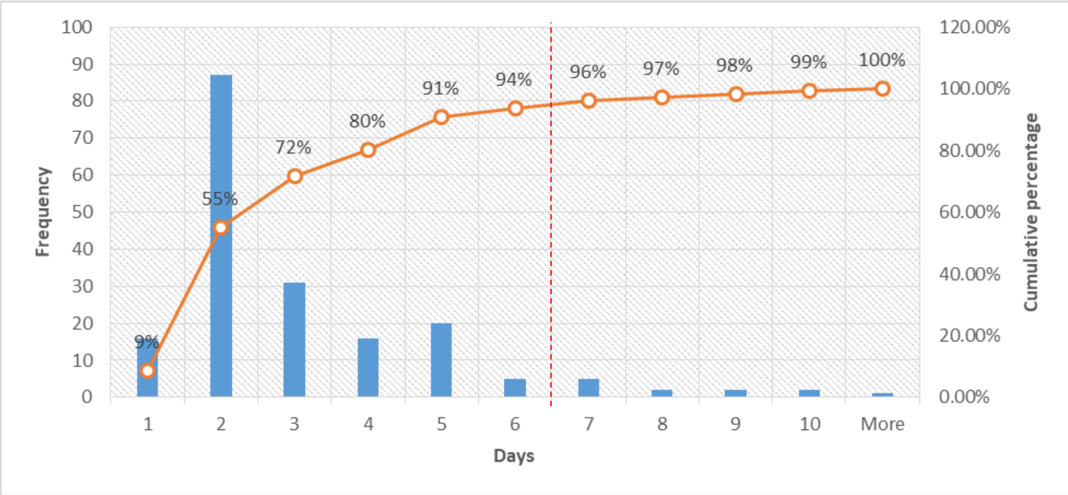

Figure 15: Two year base Tedut onshore terminal port time histogram.

For the FPSO, we analyzed 373 operations; the minimum data value was 0.5 days, the maximum data value was 7.5 days; sample mean was 1.1 days, the medium value was 0.9 day and the standard deviation was 0.9 days. Figure 16 presents the histogram of the sample. Note that operations at FPSOs suffers less variation in the total time spent by the vessels at port. We estimate the parameter $OH_i = 1,$ where $i \epsilon NP$ (NP is the set of production ports or FPSOs). As deviation in time spent at FPSOs is small (78% of operations last up to 1 day), we decided to consider deviation at FPSOs equal to zero ($OHD_{FPSOs} = 0$), which is the same of disregard uncertainty in time spent by vessels at FPSOs.

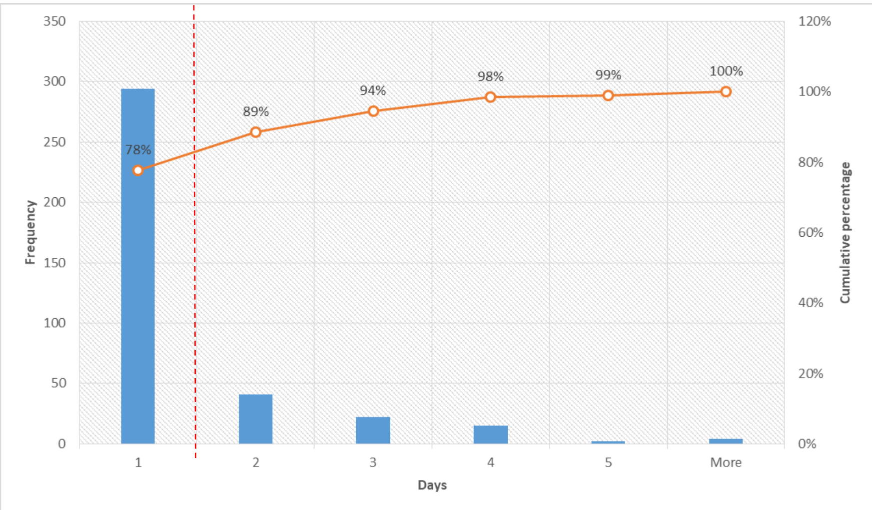

Figure 16: Two year base FPSOs port time- histogram.

Once we know the historical probabilistic behavior of the time spent at ports, we proceed to the robust optimization results in next section.

8.2. Computational results

In the robust optimization approach, the decision maker needs to determine the level of conservativeness he/she wants to admit. In these experiments, we demonstrate different operation and costs aspects of the robust solution for the worst case of realizations in each level of robustness (Γ). However, as the uncertainty occurs differently for each port, there are one Γ for each onshore terminal (Tefran and Tedut). The value of Γ_i may be interpreted as the maximum number of visits at a port i in which the time spent by the vessel may deviate from its mean value. For example, $\Gamma_{Tefran} = 0$ indicates that there is no deviation from the mean value of the port time in any operation at Tefran; and $\Gamma_{Tedut} = 3$ indicates that the time spent by a vessel at port will deviate from its nominal value to maximum deviation in up to 3 visits at Tedut port. According to consumption rates and initial inventory level at each onshore terminal, and loading capacity of the ships, we estimate the maximum number of visits at each port (the maximum Γ_i). The maximum Γ_i must be enough to supply the onshore terminals continuously and without letting inventory levels violate minimum bounds. In the instance we have considered, the estimative for the maximum value is $\Gamma_{Tefran} = 3$ and $\Gamma_{Tedut} = 4$. As uncertainty in the time spent at each port follows their own historical probabilistic distribution, we have to combine both Γ (for Tefran and Tedut) to define each level of robustness. Then, we have 20 levels of robustness. Next, we present results for each level of robustness.

We implemented the model using AIMMS 3.13 and solved them using the GUROBI 5.5. We used an Intel Core i7 CPU with 8.0 GB RAM. We implemented and tested the robust methodology using the FCNF+K+M+W formulation and using heuristic approach (also based on FCNF+K+M+W formulation). Subsection 8.2.1 compares the use of conventional MIP solver with the heuristic approach for solving the proposed robust model. Once identifying the best method for running the robust methodology (conventional MIP solving or heuristic approach), Subsection 8.2.2 analyzes some operational and costs behavior of the model for each level of robustness and the trade-off between robustness and probability of infeasibility.

8.2.1. Robust methodology using conventional MILP solving and heuristic approach

First, we executed the RO-FCNF model considering (K+M+W) valid inequalities using conventional MIP solving. We limited computational time in 3,600 seconds for each level of robustness. In Figure 17, we can assess the evolution of objective function value, lower bound value and optimality gap as we variate the level of Γ for (Tedut, Tefran). The bars represent the optimality gap for each level of robustness. The continuous line represents the best objective function for the worst case of realizations of time spent at port for each level Γ. The dashed line means the linear relaxed bound for each level of Γ. The chart is divided in 4 blocks, each block represents a level of Γ_{Tefran} (first block $\Gamma_{Tefran} = 0$, second block $\Gamma_{Tefran} = 1$, third block $\Gamma_{Tefran} = 2$ and fourth block $\Gamma_{Tefran} = 3$). For each level of Γ_{Tefran}, we variate level of Γ_{Tedut} from 0 to 4. We use this block scheme in most charts of this section to represent the variation in the level of robustness.

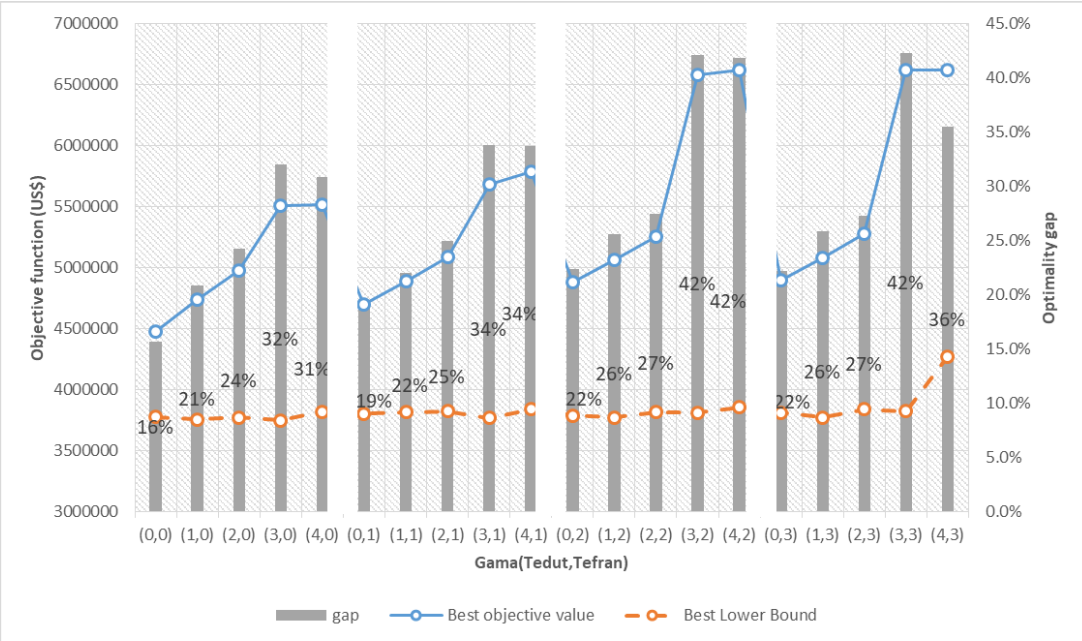

Figure 17: RO-FCNF using conventional MILP solving

Note that we didn’t reach optimality in any level of robustness. Despite that, for each block the objective function value has a tendency of growth due to the increase in the robustness level. Such behavior indicates that the more robust is the solution (ship scheduling), the more expensive is the transportation cost (objective function value).

Although the behavior of the objective function was expected, the lower bound provided by the linear relaxation of the problem are very low when compared to the objective function value, providing high optimality gaps. Such a situation is very adverse for robust optimization modeling, because one cannot ensure that the increase in the objective function value indeed due to the increase in the level of robustness or if there is a better solution “hidden” in the optimality gap.

This situation encourage us to solve the RO-FCNF using the heuristic approach presented in Chapter 5. For this test, the auxiliary parameters of the *relax-and-fix* heuristic were defined as follows: the size of *Integer Block* was 2 ports at the beginning of the heuristic, the *pace* and r*eturn* parameters are set to 1 each. For each iteration of the *relax-and-fix* phase, we established a time limit of 1000 seconds or a relative optimality gap of 15%, whichever occurs first. The use of heuristic approach within the robust methodology aims to find lower objective function values and/or reduce total computational time expended during the second phase of the methodology. Figure 18 presents the evolution of objective function using conventional MIP solving (continuous blue line), the evolution of objective function using the first phase of the heuristic approach (green dotted line) and the bars represent the relative distance between then.

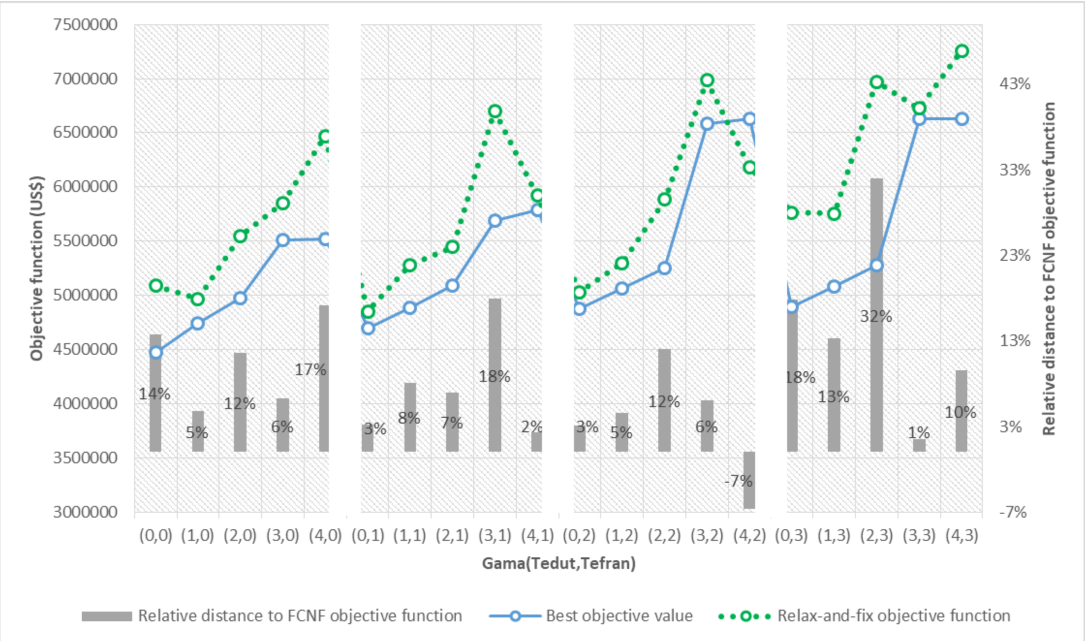

Figure 18: Objective function evolution for each level of robustness using *relax-and-fix* heuristic.

In general, the use of *relax-and-fix* heuristic (first phase of the heuristic approach) did not find better solutions than those obtained using the conventional MIP for almost every level of robustness (except by the level of robustness $\Gamma_{Tedut,Tefran} = (4,2)$). On average, the objective function value using the relax-and-fix heuristic was 9% worse than when the problem was solved using conventional MIP. However, the *relax-and-fix* heuristic prove to be an efficient alternative for finding initial solutions. In the levels of robustness that it does not improved objective function value, computational time was considerably reduced. The reduction in computational time, presented in Figure 19, was around 77%.

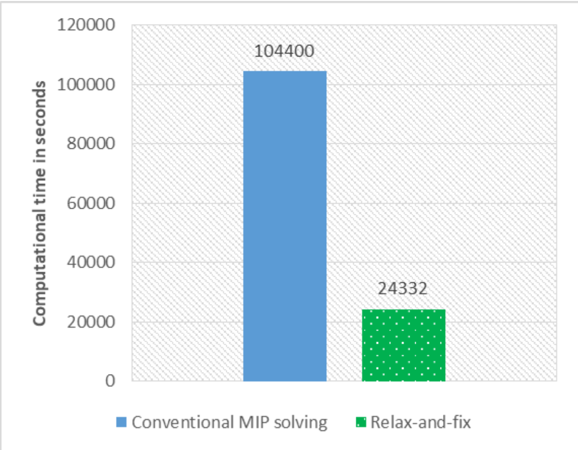

Figure 19: Computational time using conventional MIP solving and using *relax-and-fix* heuristic.

Next, we improved the initial solution provided by the *relax-and-fix* phase using the *fix-and-optimize* heuristic. At this phase, we fixed the values of the integer variables according to the index *v* (of vessels). At each iteration, the *Fixed Block* composed by 2 different vessels. The *fix-and-optimize* heuristic finishes after 10 iterations without any improvement in the objective function. We established a time limit of 250 seconds for each iteration of the *fix-and-optimize* procedure. Figure 20 presents the evolution of objective function using conventional MIP solving (continuous blue line), the evolution of objective function improved by the second phase of the heuristic - fix-and-optimize heuristic -. (orange dotted line) and the bars represent the relative distance between then.

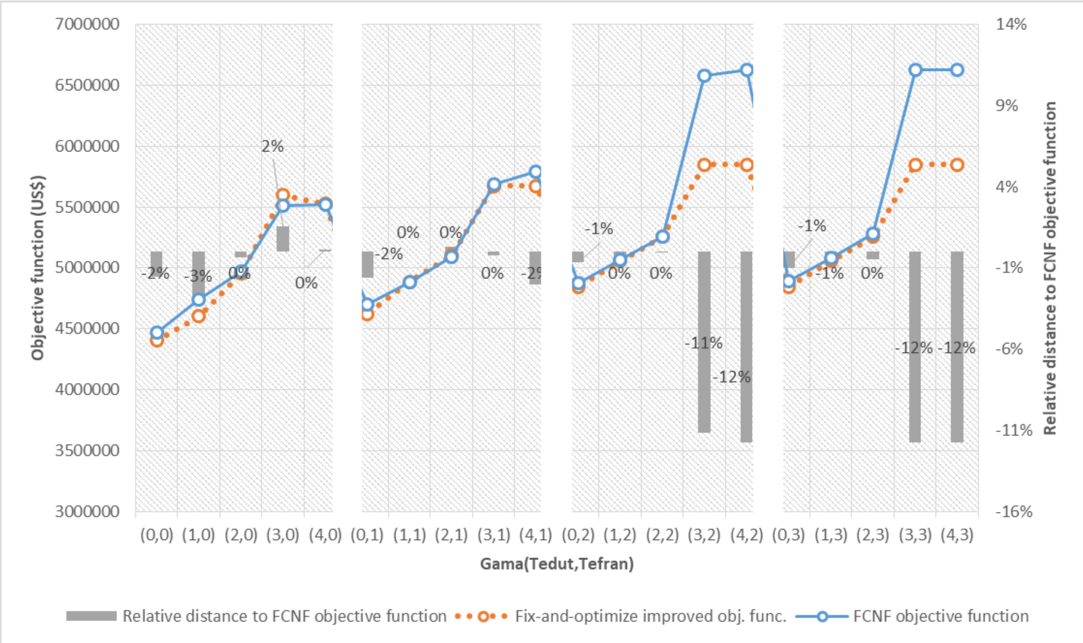

Figure 20: *Fix-and-optimize* improved objective function evolution for each level of robustness.

The heuristic approach was able to improve the objective function in most levels of robustness. On average, the objective function was improved in 2.8% and only on 3 levels of robustness the objective function value obtained was worse than using the conventional MIP. In addition to improve objective function, the total computational time of the heuristic approach solution was also reduced when compared to the conventional MIP solving. Figure 21 presents the total computational time using conventional MIP solving, and the two phases of the heuristic approach. The total reduction in computational time was about 33%.

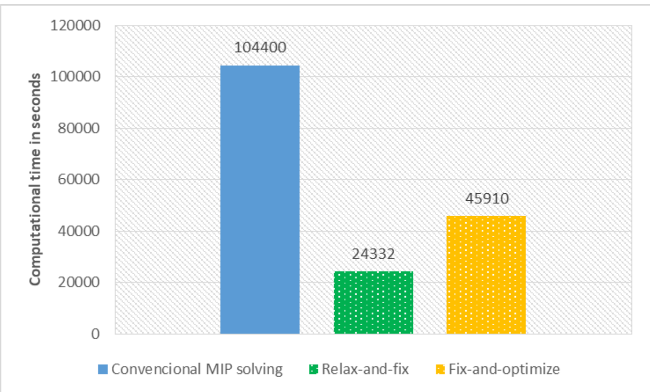

Figure 21: Computational time using conventional MIP solving, *relax-and-fix* and *fix-and-optimize* heuristics.

Even though the heuristic approach could not close the optimality gaps obtained using the conventional MIP solving (because we cannot consider the lower bounds obtained by the heuristic valid for the original problem), it provided better solutions for almost every level of robustness and reduced the total computational time. Thus, from now on we will consider the heuristic approach solution as a reference for all the analysis of the next section.

8.2.2. Operational and cost behavior in robust methodology

The objective function of RO-FCNF model is composed by four cost components: port fee, hiring cost, bunker cost and demurrage cost. We divide Figure 22 in four quadrants and each of them demonstrates how each cost component of the objective function behaves as we modify the level of robustness (a. port fee, b. hiring cost, c. bunker cost and d. demurrage cost).

In Figure 22.*a*, the port fee cost is represented by the blue area. We may note that it remains stable for every level of robustness, which means that the level of robustness has not much influence on this cost component. The number of visits on the onshore terminals do not suffer much oscillation. In Figure 22.*b*, note that hiring cost (represented by the dark blue area) is directly related to the number of vessel used. As the level of robustness increase, the use of vessels becomes more intensive (due to deviations in time spent at ports) and it is necessary more vessels to attend to the demand for transportation. However, after a certain level of robustness, increasing the number of vessel is no longer necessary. Note that in the last block ($\Gamma_{Tefran} = 3$), the evolution of the objective function value and number of vessels used are the same as in the third block ($\Gamma_{Tefran} = 2$). In Figure 22.*c*, we notice some variation in bunker cost (represented by the yellow area). Observe that the levels of robustness that presents the highest bunker costs are those where there were used only three vessels instead of four. It happens because using less vessels for the same number of voyages, they will need to perform more ballast sailing voyages (when the vessel sails empty after discharging at a consumption port to a production port, in order to start a new voyage). When a solution adopt more vessels for the same number of voyages, the number of ballast sailing trips is reduced and, consequently, bunker costs also decreases.

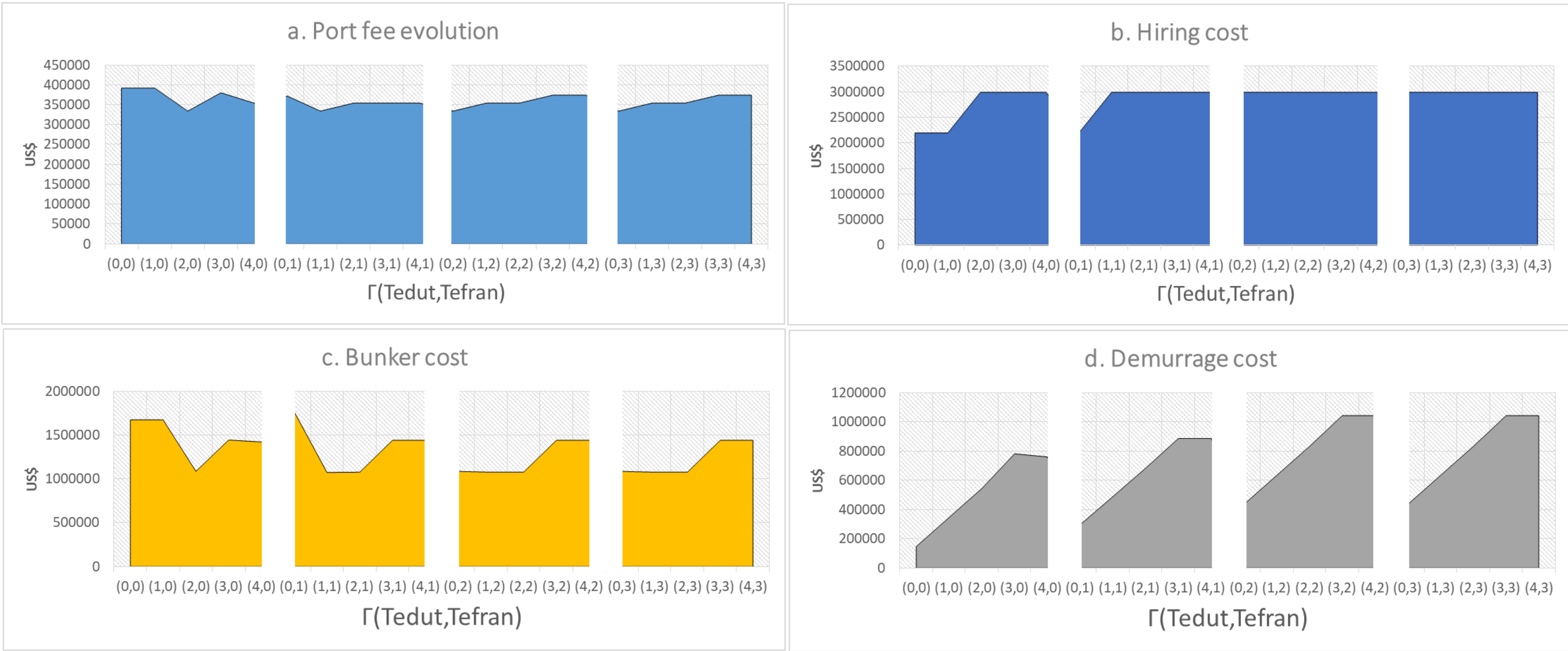

Figure 22: Objective function components and their behavior for each level of robustness.

The last cost component, represented in grey area of Figure 22.*d* is the demurrage rate. We may observe that the demurrage cost is directly related to the level of robustness. As the level of robustness increase, the model admits that a vessel will spend more time at ports, and it results in more demurrage expenses.

As the level of robustness represents a variation in the time spent at ports, Figure 23 illustrates such a parameter. Note that as the level of robustness increases, the time spent at onshore ports (measured in days) also increases. However, the number of visits at both ports suffers little variation. An exception for this behavior is between levels Γ(3,0) and Γ(4,0). As we have the problem of large optimality gaps, the solution obtained in the level of robustness Γ(3,0) certainly could be improved.

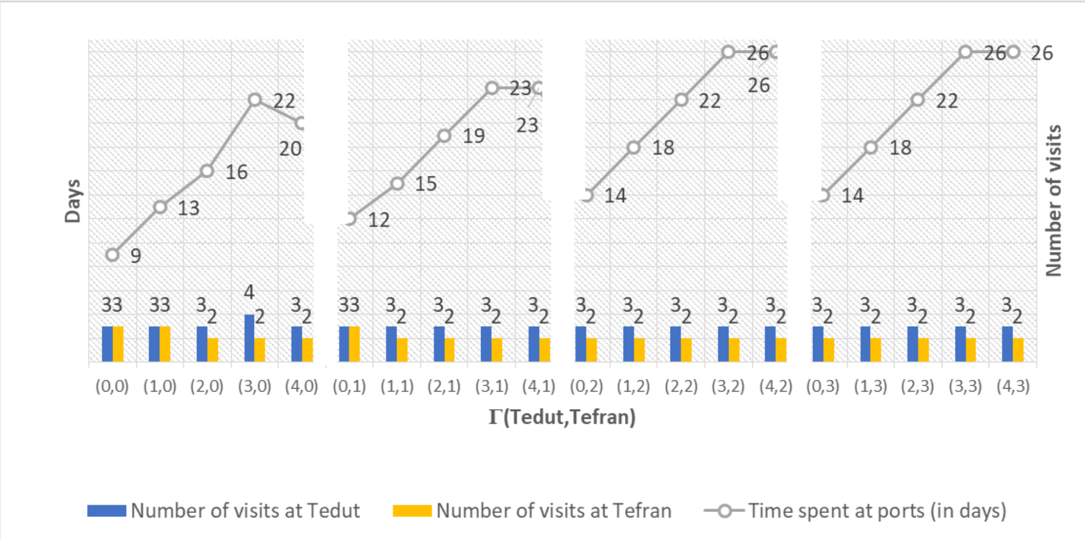

Figure 23: Total time spent at ports (in days) and number of visits at onshore terminals.

The time spent at ports is directly related to the demurrage costs and has a strong influence on the number of vessel needed. Regarding the demurrage cost, operation at ports is programmed to last no longer than one day. Thus, any extra day spent by the vessel at a port is penalized by the demurrage rate. Regarding the number of vessels, as they spent more time in ports (when level of robustness increase), more vessels are required to offloading the FPSOs, supply onshore terminals, and ensure that inventory levels at ports will not violate inventory operational limits (maximum and minimum).

The time spent at ports will also influence the vessel availability. It means that the total time vessel was used during planning horizon, independently if it was sailing, waiting, operating at a FPSO or an onshore terminal. We measure vessel availability from the moment the vessel leaves its artificial origin node until the moment when the vessel enters its destination node. In Figure 24, we present a relation between the number of vessels (line) and total vessel availability (bars), which means the sum of vessel availability of the fleet. Note that as level of robustness increases, the vessel availability also tends to increase. This is a way of protection against infeasibility, as the model requires more vessels for a more robust solution.

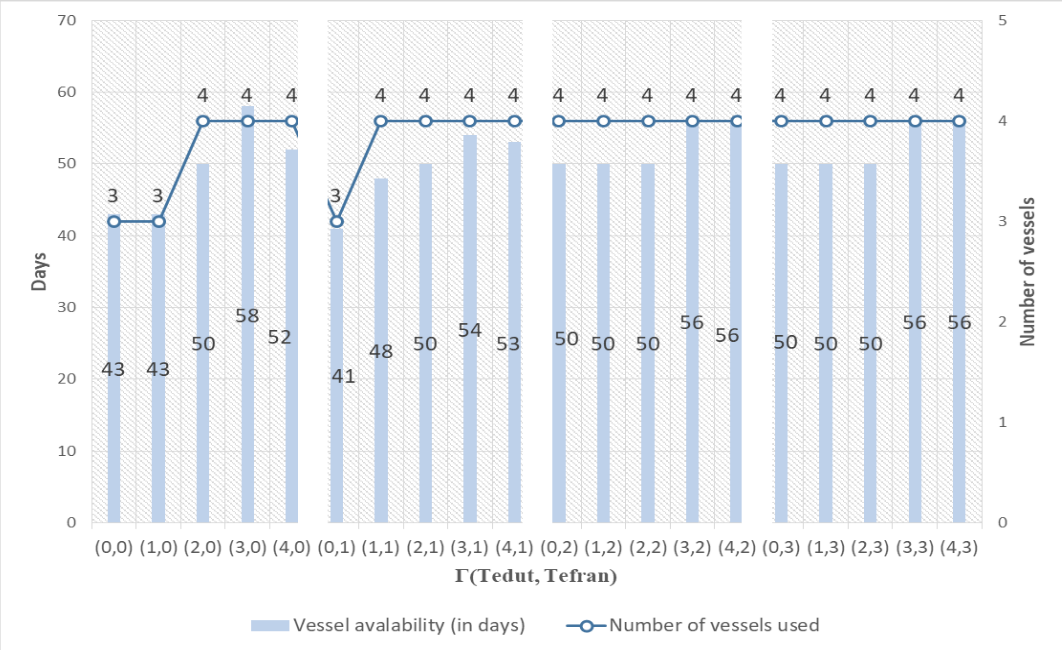

Figure 24: Number of vessels and total vessel availability.

Once it was observed how the model gets protection against infeasibility while the level of robustness (Γ) increases and what are the consequences of such protection on the objective function, we proceeded to verify the probability of infeasibility of each solution on each level robustness.

8.2.3. Probability of infeasibility in robust methodology

In order to verify the probability of infeasibility, we simulated 100 scenarios for the time spent at ports for each pair (port, vessel) according to the probability obtained by the historical data and presented in Section 8.1. For each scenario and each level of Γ, we execute the deterministic model FNCF+K+M+W fixing the fleet size and fleet availability defined for each level of Γ, as demonstrated in Section 7.3.3, constraints (82) – (83). The number of vessel required and the vessel availability represent the protection against infeasibility suggested by the RO-FCNF on each level of robustness. Figure 25 presents an example of protection suggested by the RO-FCNF in the level of robustness $\Gamma_{(Tedut,Tefran)} = (0{,}1)$. In this example, vessels MT Navion Stanvanger and MT Fortaleza Knutsen were not required during the planning horizon, while MT Elka Leblon and MT Rio Grande started their first voyage at time period 1 and finish their last voyage at 15. The vessel MT Angra dos Reis start at time period 5 and finish at 15. Summing how many days these three vessels were available, we have 41 days of vessel availability.

$\Gamma_{(Tedut,Tefran)} = (0{,}1)$

Number of vessels required	3	Planning horizon														
Time periods		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
MT Elka Leblon availability	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
MT Rio Grande availability	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
MT Angra dos Reis availability	11					1	1	1	1	1	1	1	1	1	1	1
MT Navion Stavanger availability	0															
MT Fortaleza Knutsen availability	0															
Total vessel availability	41	Time periods														

Figure 25: Vessel availability suggested by the RO-FCNF for the level of robustness $\Gamma_{(Tedut,Tefran)} = (0{,}1)$.

Figure 26 presents a comparisson between the worst-case objective function (green continuous line) for each level of Γ (provided by RO-FCNF model), the average objective function (black dotted line) obtained from the simulation of time spent at ports and the probability of infeasibility of each level of protection given by the RO-FCNF model (red dashed line). It is important to mention that during simulation phase, the execution of the deterministic model FNCF+K+M+W with fleet size and fleet availability fixed for each level of robustness provided optimal (or near optimal) results for every feasible solution obtained. We set a relative optimality gap of 3% as the stopping criterion for each execution.

Note that the more robust the solution, the higher is the chance of obtaining a feasible solution. We might also observe how conservative are the robust solutions proposed by the RO-FCNF model. When we observe the average objective function of simulated solutions, we note that they are almost ever lower than the objective function provided by the RO-FCNF. As the level of robustness increases, this difference also increases. As in the simulation phase we only fix the vessel availability and the number of vessels required, the only component of cost that is fixed is the hiring cost. Vessels will be re-scheduled according to the simulated parameter $OH_{i,v}$. Consequently, the cost components of objective function port fee, bunker cost and demurrage rates will be recalculated according to each solution.

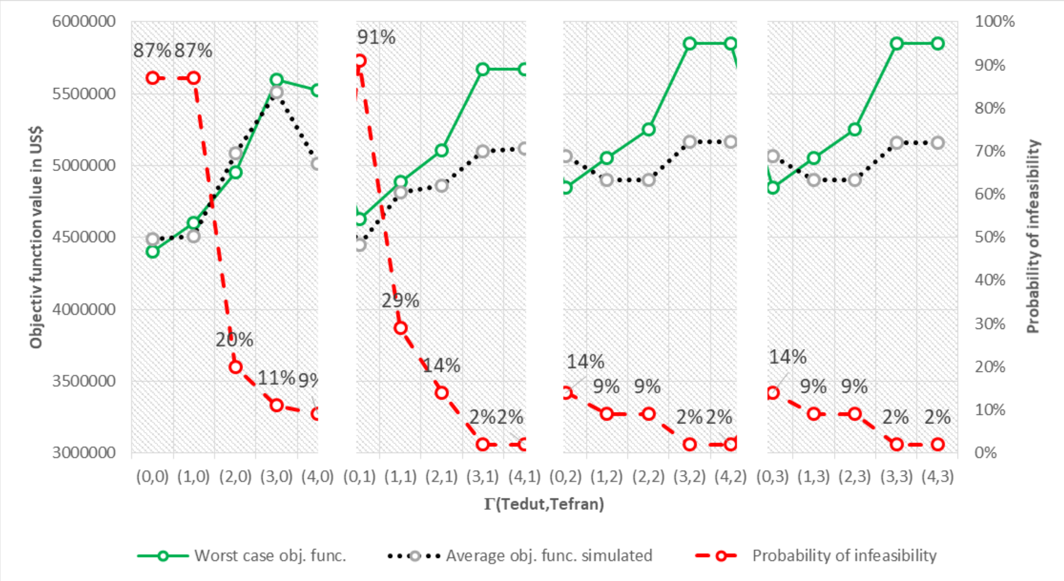

Figure 26: Evolution of objective function of worst case, average objective function simulated and probability of infeasibility.

Observing Figure 26, one may decide how much risk he/she is willing to accept. The only previous decision is how many vessels he/she will alocate to each subsystem and when each vessel must be available for that subsystem (vessel availability). The simulation phase represents the day-by-day scheduling decisions we present in Section 2.1.1. In practice, they are made on daily basis as uncertain parameters are revealed.

Table 10 presents for each level of robustness, the values of the worst case objective function (obtained in the second phase of the metodology by the RO-FCNF model), the average objective function simulated and probability of infeasibility (obtained during simulation phase). The last two columns present, respectively, the absolute and relative difference from the average objective function of each level of robustness to the risk neutral level (Γ=(0,0)), the one that has no protection against uncertainty.

Γ_{Tedut}	Γ_{Tefran}	Worst case objective function	Average objective function	Probability of infeasibility	Δ Average obj. func. (average obj. func. - average obj. func. of Γ = (0,0)	Δ% Average obj. func.
0		4,404,600	4,460,081	87%	-	-
1		4,603,000	4,508,385	87%	48,304	1.1%
2	0	4,954,780	5,089,379	20%	629,299	14.0%
3		5,596,530	5,510,351	11%	1,050,270	20.6%
4		5,527,290	5,013,689	9%	553,608	10.0%
0		4,625,200	4,445,756	91%	- 14,325	-0.3%
1		4,887,140	4,810,905	29%	350,824	7.9%
2	1	5,105,490	4,858,600	14%	398,520	8.3%
3		5,671,800	5,101,589	2%	641,509	13.2%
4		5,671,800	5,117,964	2%	657,883	12.9%
0		4,846,240	5,065,837	14%	605,757	11.8%
1		5,055,890	4,901,082	9%	441,001	8.7%
2	2	5,254,290	4,901,082	9%	441,001	9.0%
3		5,847,920	5,163,895	2%	703,814	14.4%
4		5,847,920	5,163,895	2%	703,814	13.6%
0		4,846,240	5,065,218	14%	605,137	11.7%
1		5,055,890	4,901,082	9%	441,001	8.7%
2	3	5,254,290	4,901,082	9%	441,001	9.0%
3		5,847,920	5,158,760	2%	698,679	14.3%
4		5,847,920	5,158,760	2%	698,679	13.5%

Table 10: Evolution of objective function and probability of infeasibility for each level of conservativeness.

Note that for the nominal problem, where Γ=(0,0) for (Tedut, Tefran), 87% of the scenarios simulated are infeasible, although the objective function reaches the lowest value, around US$ 4.4 million. It means that in 87% of the simulated scenarios, the fleet availability indicated in the solution of the nominal problem is not enough to carry the entire crude oil production from production sites to onshore terminals without letting inventory levels violate minimum and/or maximum limits. In other words, 87% of scenarios are infeasible due to inventory levels violation. When one increases the level of robustness, for example Γ=(3,1), the probability of infeasibility reduces to 2% and the increase in the average objecive function is around US$ 641 thousand, which corresponds to about 13% of increase in the value of the nominal problem objective function.

Regarding the trade-off decision between risk of infeasibility and transportation costs, Figure 27 plots an efficiency frontier for each level of robustness. The Y-axis represents the average objective function simulated, X-axis represents the probability of infeasibility and each dot represents a level of robustness. Analysing the efficiency frontier, one can easily identify the efficient solutions and the dominated solutions. The efficient solutions are those that for same probability of infeasibility has the lower transportation costs, and dominated solutions are the ones that presents higher transportation costs for the same probability of infeasibility. Another important advantage of the efficient frontier chart is that independently of how many ports the uncertainty is considered (what represents the number of Γ indexes), one can plot the results in a two axis chart. It lets the result analysis much easier for the decision maker than Figure 26, for example, where the chart must be split in blocks according to the number of Γ indexes.

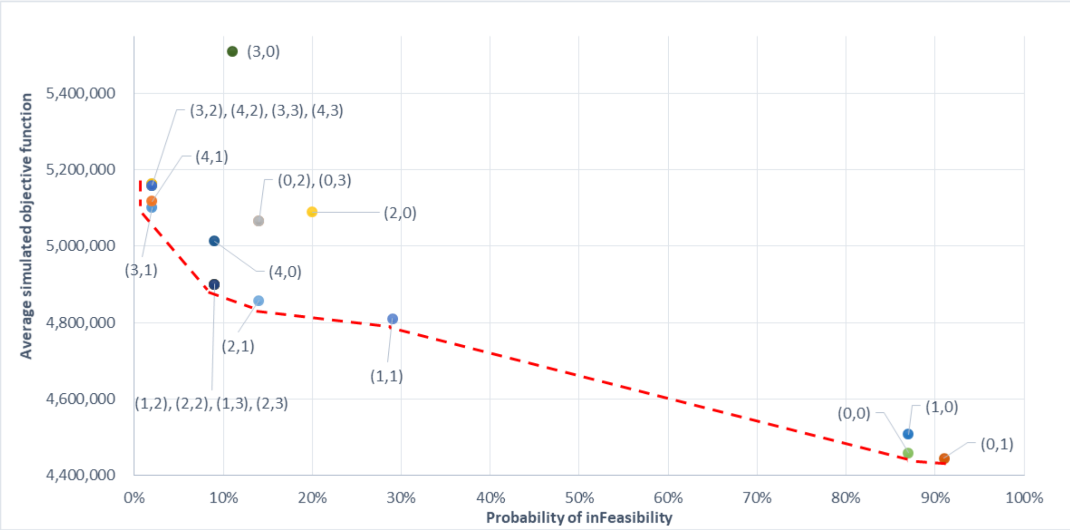

Figure 27: Efficiency frontier for levels of robustness.

Observing this chart, the decision maker may decide in advance which level of protection to give for a subsystem of the PETROBRAS MIR problem, considering two important aspects: probability of failing service (infeasibility) and increase in transportation costs (objective function). Such decision takes in consideration uncertainty in the time spent at ports. In this example, if one considers allowable about 10% of probability of infeasibility, than the levels of robustness (2,1), (1,2), (2,2), (1,3) and (2,3) are the ones that presents the lower transportation cost for such level of conservadorism. If one seeks for a more robust solution, it means, a solution with a lower probability of infeasibility, than the level of robustness (3,1) is definitly the best option. Such level is the one that associates the lower average transporation costs for the lower probability of infeasibility. In practice, such solution considers the use of 4 vessels, summing 54 time-periods of vessel availability. It also assumes that this level of protection is enough to accomplish 98% of the cases, considering the historical uncertainty of time spent at onshore terminals.

An important contribution of such robust metodology is suporting the decision maker in the ship scheduling activity. Giving in advance the risk of failing service level and how much the level of conservative admitted would impact transportation costs. Such trade-off analysis is fundamental for an efficient management of ship scheduling activity in a real-life MIR problem, such as the one faced by PETROBRAS.

9. Conclusion

We studied the special case of maritime transportation problem named maritime inventory routing and we described in detail a practical MIR problem faced by a Brazilian petroleum company. In order to find the most appropriate optimization model to support such a real problem decision-making, we research in the literature to identify a mathematical formulation that better models and optimize its ship-scheduling decisions.

As we found different formulations in literature, using different ways of using time aspect, we structured a framework that compares continuous time and discrete time and indicated which is the most appropriated for real-life MIR problems. As far as we known, this is the first time in literature that such a framework is presented in literature. Using a set of real-life instances (most of them harder than the ones found in literature), we concluded that continuous time, FCNF+K+M+W and FCNF+K+M+W+D formulations have the best computational performance. Additionally, we noted that continuous time formulation provides strong linear relaxation bounds. Therefore, for certain instances, the use of these bounds may reduce considerably the optimality gap obtained for FCNF+K+M+W and FCNF+K+M+W+D formulations. Our tests indicated a reduction of 65% for those instances that did not reach optimality.

However, even improving optimality gap, our tests indicated that conventional MILP formulations are not enough to prove optimality for some real-life instances due to their computational complexity. We proposed then a *relax-and-fix* and *fix-and-optimize* heuristic approach in order to improve solution and reduce computational time. Test indicated that the heuristic approach reached or improved the objective function value in most instances tested when compared to FCNF+K+M+W formulation and reduces the total computational time in 24%. The heuristic approach was also valuable to get better solutions and reduce computational time of the robust approach we propose. It is important to highlight that depending on the way of selecting variables to fix and relax during the heuristic execution, the solution could be different. Our tests indicated that the use of index ports *i* and vessels *v* provided better solutions than the use of index time *t*, utilized by Uggen (2013) and Rodrigues et al. (2016).

Nevertheless, identifying a deterministic formulation (or heuristic approach) for MIR problem is not enough to solve a practical MIR problem. Due to a large number of unforeseen occurrences that influence ship-scheduling decision, we proposed a robust optimization approach that considers uncertainty in time spent by vessels at ports and quantifies the probability of infeasibility for several solutions. Each solution optimized for a certain level of robustness. Our tests indicated that the use of such robust optimization approach to support ship-scheduling decisions contributes considerably to improve quality of the decision. It means that using the robust optimization approach, the decision maker can measure the risk of infeasibility and the impact in transportation costs for several solutions with different robust levels. Therefore, one may decide within a range of options, which solution represent the best trade-off between conservativeness and increasing in transportation costs. In other words, the proposed robust approach helps the decision maker to identify its willingness to take risks and measures the transportation costs of its decision. Such analysis represents one of the main contributions of this thesis to operational research and practice.

Summarizing, the main contributions of this thesis were: supports the decision maker to assign an efficient number of vessels for a certain MIR problem that avoids inefficiency caused by an oversized fleet; quantifies the risks of failing service level (infeasible solutions) and the impacts of level of conservativeness admitted in transportation costs; and provides an optimization decision support tool that helps to make ship-scheduling decisions, based on the risk of failing level of service.

Despite the results we obtained, there are still much space for development in the study of MIR problems, especially if considering uncertainty. As future work, we propose the application of valid inequalities in the continuous time formulation; and a better use of the linear lower bounds of the continuous time formulation. Regarding MIR problems under uncertainty, we recommend studying alternatives to reduce the optimality gaps obtained during the robust approach. Another opportunity of future research is the inclusion of new uncertain parameters in the model, such as production and consumption rates and vessel speed, for example.

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Appendix 1

The RO-FCNF formulation starts with the original FCNF formulation proposed by Agra et al. (2013). We add some valid inequalities also proposed by Agra et al. (2013) to strengthen formulation and improve computational performance. Finally, we reformulate the original FNCF in order to obtain the RO-FCNF formulation, considering uncertainty in the time spent by the vessels at ports.

Here we describe the mathematical formulation of the RO-FCNF model. In order to better reference with the text, we will use the same equation numbers of the text.

Sets:

N set of all ports indexed by i and j

T set of time periods indexed by t

V set of vessels indexed by v

NP set of loading ports indexed by i and j

ND set of discharge ports indexed by i and j

K_{iv} Is the set of coefficients OH_{iv} that are subject to parameter uncertainty.

Parameters:

C^T_{ijv} sailing cost of vessel v between ports i and j

C^W_v waiting cost of vessel v for each time period

C^P_{iv} port cost of port i for vessel v

o(v) position of vessel i in the beginning of planning horizon

d(v) artificial end node for each vessel

T_{ijv} sailing time of vessel v between ports i and j

B_{it} number of berths available in port i during time period

Q_v maximum amount of product to be (un)loaded at one time period of ship v

L^0_v inventory on board of vessel v in the beginning of the planning horizon

K_v vessel capacity

D_{it} demand rate in the discharge port i for each time period t

P_{it} production rate in the load port i for each time period t

SMX_{it} Upper bound of inventory level in port i for each time period t

SMN_{it} Lower bound of inventory level in port i for each time period t

S^0_i Inventory level at port i in the beginning of the planning horizon

$TIME$ The last time period of the planning horizon T,

$HIRE_v$ The hire fee paid in a daily basis to vessels owner,

OH_{iv} Mean time spent by vessel v at port i,

OHD_{iv} Maximum deviation observed for the time spent by vessel v at port i,

Γ_{iv} Represents the maximum number of coefficients parameters of constraints (i,v) that can deviate from their nominal value. It adjusts the robustness of the decision. The parameter $\Gamma_{iv} \in [0, |K_{iv}|]$. Hence, the parameter Γ_{iv} protects against deviations in up to $\lfloor \Gamma_{iv} \rfloor$ of these coefficients. In other words, we stipulate that nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely affect the solution.

Variables:

o_{ivt} where $o_{ivt} \in \{0,1\}$

equal to 1 if vessel v is operating (loading/unloading) in port i during time period t and 0 otherwise,

o^A_{ivt} where $o^A_{ivt} \in \{0,1\}$

indicates whether ship v starts to operate at port i in period t

o^{B}_{ivt} where $o^{B}_{ivt} \in \{0,1\}$

indicates whether ship v continue to operate at port i in period t

x_{ijvt} where $x_{ijvt} \in \{0,1\}$

equal to 1 if vessel v left port i to port j during time period t and 0 otherwise,

w_{ivt} where $w_{ivt} \in \{0,1\}$

equal to 1 if vessel v is waiting outside berth in port i during time period t and 0 otherwise,

f^{X}_{ijvt} that indicates the load on board ship v when traveling from port i to port j, leaving port i in period t,

f^{OA}_{ivt} that indicates the load on board ship v when starting to operate at port i in period t and has not operated in period t-1,

f^{OB}_{ivt} that indicates the load on board ship v before continuing to operate at port i in period t after having operated in time period t-1,

f^{W}_{ivt} that indicates the load on board ship v while waiting during time period t at port i.

q_{ivt} where $q_{ivt} \in R$

quantity (un)load from/to vessel v in port i during time period t,

s_{it} where $s_{it} \in R$

inventory level at port i during time period t.

nh_{v}^{NT} Binary variable that indicates whether a vessel stayed idle during the entire planning horizon or not.

Additional auxiliary variables

π_{iv} Robustness variable (dual problem),

ρ_{ijvt} Auxiliary robustness variable (dual problem).

Routing constraints

$$\sum_{j \in N \cup \{DE_v\}} \sum_{t \in T} x_{O_v jvt} = 1, \ \forall v \in V, \quad (25)$$

$$\sum_{i\in N\cup\{O_v\}}\sum_{t\in T} x_{iDE_v vt} = 1,\ \forall v \in V, \tag{26}$$

Constraints (25) and (26) guarantee that every ship leaves from its artificial origin port and finishes the voyage at its artificial destination port.

Sailing, waiting and operating time constraints

$$\sum_{j\in N\cup\{o(v)\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} = w_{ivt} + o^A_{ivt}, \forall v \in V,\ i \in N,\ t \in T, \tag{42}$$

$$o^A_{ivt-1} + o^B_{ivt-1} = o^B_{ivt} + \sum_{j\in N\cup\{d(v)\}} x_{ijvt}, \forall v \in V,\ i \in N,\ t \in T, \tag{43}$$

$$o^A_{ivt}, o^B_{ivt} \in \{0,1\},\ \forall v \in V,\ i \in N,\ t \in T, \tag{44}$$

$$o^A_{ivt} + o^B_{ivt} = o_{ivt},\ \forall v \in V,\ i \in N,\ t \in T, \tag{45}$$

$$\sum_{v\in V} o_{ivt} \le B_{it},\ \forall i \in N, t \in T, \tag{30}$$

Constraints (42) indicate the ship arrival at one port and (43) show when the ship sailing from one port. Constraints (44) indicates the variables o^A_{ivt} and o^B_{ivt} are binary. Constraints (45) provide the link between the old and the new operating variables. Constrains (30) give berth restrictions at each node and consequently waiting time.

Loading constraints

$$\sum_{j\in N\cup\{o(v)\}} f^X_{jiv,t-T_{jiv}} + f^W_{iv,t-1} = f^W_{ivt} + f^{OA}_{ivt}, \forall v \in V,\ i \in N,\ t \in T \tag{46}$$

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} + q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j\in N\cup\{d(v)\}} f^X_{ijvt},\quad \forall v \in V,\ i \in NP \cup \{o(v)\},\ t \in T, \tag{47}$$

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} - q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j\in N\cup\{d(v)\}} f^X_{ijvt},\quad \forall v \in V,\ i \in ND \cup \{o(v)\},\ t \in T, \tag{48}$$

$$f^X_{o(v)jvt} = L^0_v x_{o(v)jvt},\qquad \forall v \in V,\ j \in N \cup \{d(v)\},\ t \in T \tag{49}$$

The flow conservation constraints (46) – (49) ensure the load on board balance along every arc of the structure.

$$0 \le f^X_{ijvt} \le K_v x_{ijvt},\ \forall v \in V, i \in N \cup \{o(v)\}, j \in N \cup \{d(v)\},\ t \in T \tag{50}$$

$$0 \leq f_{ivt}^{OA} \leq K_v o_{ivt}^{A}, \quad \forall v \in V, i \in N, \ t \in T, \tag{51}$$

$$0 \leq f_{ivt}^{OB} \leq K_v o_{ivt}^{B}, \ \forall v \in V, i \in N, \ t \in T, \tag{52}$$

$$0 \leq q_{ivt} \leq Q_v o_{ivt}, \forall v \in V, i \in N, \ t \in T, \tag{53}$$

$$0 \leq f_{ivt}^{W} \leq K_v w_{ivt}, \forall v \in V, i \in N, \ t \in T. \tag{54}$$

The variables upper bounds and non-negativity constraints are expressed in (50) – (54). The quantity to be (un)loaded (q_{ivt}) must be less than the maximum amount of product to be (un)loaded of ship v (constraints 49).

Inventory control constraints

$$s_{i,t-1} + \sum_{v \in V} q_{ivt} = D_{it} + s_{it}, \ \forall i \in ND, t \in T, \tag{35}$$

$$s_{i,t-1} + P_{it} = \sum_{v \in V} q_{ivt} + s_{it}, \ \forall i \in NP, t \in T, \tag{36}$$

$$SMN_{it} \leq s_{it} \leq SMX_{it}, \ \forall i \in N, t \in T, \tag{37}$$

$$s_{i0} = S_i^0, \ \forall i \in N, \tag{38}$$

$$x_{ijvt} \in \{0,1\}, \forall v \in V, i \in N \cup \{o(v)\}, j \in N \cup \{d(v)\}, t \in T, \tag{39}$$

$$o_{ivt}, w_{ivt} \in \{0,1\}, \ \forall v \in V, i \in N, t \in T, \tag{40}$$

$$s_{it}, q_{ivt} \in \mathbb{R} \ \ \forall v \in V, i \in N, t \in T. \tag{41}$$

The inventory level at each port is controlled during every time unit of the planning horizon at load and discharge ports (constraints 35 and 36). Constraints (37) give the operational range within inventory levels must be and constraints (38) give initial inventory at each port. All binary variables are stated in constraints (39) and (40) and constraints (41) gives the continuous variables.

Identifying the utilized vessels

$$nh_v^{NT} = \sum_{i=O_v} \sum_{j=DE_v} \sum_{t \in T} x_{ijvt}, \quad \forall v \in V \tag{70}$$

Constraints (70) ensure that if a ship goes directly from the artificial origin node to the artificial destination node, then it is not utilized for that planning horizon.

Robust linear constraints

$$\sum_{t \in T, j \in N} OH_{iv} x_{ijvt} + \pi_{iv}\Gamma_{iv} + \sum_{t \in T, j \in N} \rho_{ijvt} - \sum_{t \in T} o_{ivt} \leq 0, \forall i \in N, v \in V, \tag{81}$$

$$\pi_{iv} + \rho_{ijvt} \geq OHD_{iv}x_{ijvt}, \quad (82)$$

$$\rho_{ijvt} \geq 0, \qquad \forall i, j, v, t \quad (83)$$

$$\pi_{iv} \geq 0, \qquad \forall i, v \quad (84)$$

Constraints (81) – (84) are the set robust linear constraints developed from Bertsimas and Sim (2004) approach.

Objective function:

$$min \sum_{v\in V}\sum_{i\in N\cup\{O_v\}}\sum_{j\in N\cup\{DE_v\}}\sum_{t\in T} C_{ijv}^{T}x_{ijvt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C_{iv}^{P}x_{ijvt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C_{v}^{W}w_{ivt} + \sum_{v\in V}(1 - nh_{v}^{NT})HIRE_{v}TIME \quad (69)$$

The minimization function (69) contains transportation costs, operation costs, waiting costs and hiring costs. The hiring costs was included in the original FCNF objective function in order to consider the cost of a new vessel in the fleet as the level of robustness increases.

Valid inequalities

To further strengthen the FCNF formulation, we have added some valid inequalities also proposed in Agra et al. (2013). The demonstration of how to get to these valid inequalities is explained in Agra et al. (2013).