Pontifícia Universidade Católica
do Rio de Janeiro

Fernando Gomes Mattar

Essays on female labor supply

Tese de Doutorado

Thesis presented to the Programa de Pós–graduação em Economia of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Economia.

Advisor: Prof. Gabriel Lopes de Ulyssea

Rio de Janeiro
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Abstract

Gomes Mattar, Fernando; Lopes de Ulyssea, Gabriel (Advisor). **Essays on female labor supply**. Rio de Janeiro, 2018. 113p. Tese de doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This thesis is comprised of three chapters. In the first two I analyze how fertility decisions and the presence of young children in the household affect the labor supply of women on the intensive margin and their labor market allocation across different types of work such as formal, informal and self-employment. To do so, I use Brazilian data, motivated by the relative rigidity of the labor legislation in Brazil, the high prevalence of informal work and the scarcity of part-time jobs in the formal sector. In the first chapter I specify a reduced-form, dynamic discrete choice model of sequential labor supply decisions in two margins, labor force participation and work in the formal sector. I use longitudinal household survey data representative of the Brazilian population for the years 2012-2017 to estimate the model. The results show that, among low-education women, observed fertility is endogenous with respect to the decision to work in the formal sector, and that the presence of children in the household has a significant negative effect on the probability of having a formal job conditional on labor force participation. No such evidence is found for college-educated women. In light of these results, in the following chapter I develop an estimable structural model of life-cycle labor supply and fertility decisions and estimate it using data for the years 2002-2015. I then perform counterfactuals on the estimated model in order to isolate the effect of fertility on female labor market informality and to show the partial equilibrium effects of increasing the availability of part-time work in the formal sector and of increasing the duration of maternity leave. In the third chapter, I present a new approach to handle the "initial conditions problem" in dynamic binary choice models with individual unobserved heterogeneity. I assess the performance of my method relative to existing approaches using a set of simulation experiments and show that it displays relatively better precision and only slightly worse accuracy.

Keywords

Female labor supply; Fertility; Informality; Life-cycle dynamics;

Resumo

Gomes Mattar, Fernando; Lopes de Ulyssea, Gabriel. **Ensaios sobre oferta de trabalho feminina**. Rio de Janeiro, 2018. 113p. Tese de Doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Esta tese é composta de três capítulos. Nos dois primeiros, eu estudo como decisões de fertilidade e a presença de filhos no domicílio afetam a oferta de trabalho das mulheres na margem intensiva e sua alocação entre diferentes tipos de trabalho: formal, informal e conta-própria. Tal análise é feita utilizando dados do mercado de trabalho brasileiro, devido à relativa rigidez da legislação trabalhista no Brasil, a alta prevalência de trabalho informal e a escassez de empregos com jornada em meio-período no setor formal. No primeiro capítulo, eu especifico um modelo dinâmico de escolha discreta, em forma reduzida, para a decisão sequencial de oferta de mão-de-obra e trabalho no setor formal. O modelo é estimado utilizando dados longitudinais da Pesquisa Nacional por Amostra de Domicílios Contínua de 2012 a 2017. Os resultados mostram que, entre as mulheres com educação secundária ou inferior, as variáveis observadas de fertilidade são endógenas com respeito à decisão de trabalhar no setor formal e a presença de filhos no domicílio tem um impacto negativo sobre a probabilidade de ter um emprego formal condicional à participação no mercado de trabalho. Nenhum efeito desta natureza é encontrado para as mulheres com ensino superior completo. Em face destes resultados, no capítulo seguinte eu desenvolvo um modelo estrutural estimável de decisões de oferta de trabalho e fertilidade no ciclo de vida. Este modelo é estimado com dados da Pesquisa Nacional por Amostra de Domicílios e da Pesquisa Mensal do Emprego de 2002 a 2015. São realizados exercícios contrafactuais utilizando o modelo estimado de modo a isolar o efeito da fertilidade sobre a informalidade entre as mulheres e para analisar os efeitos de equilíbrio parcial de se aumentar a disponibilidade de trabalho meio-período no setor formal e de se aumentar a duração de licença maternidade. No terceiro capítulo, eu proponho uma nova abordagem ao “problema de condições iniciais” em modelos dinâmicos de escolha binária com heterogeneidade individual não-observada. Eu avalio o desempenho deste método utilizando dados artificiais e mostro que seu desempenho é satisfatório relativamente a outras abordagens existentes.

Palavras-chave

Oferta de trabalho feminina; Fecundidade; Informalidade; Mercado de trabalho; Ciclo de vida;

Table of contents

List of figures

List of tables

1
The effect of fertility on labor market informality among skilled and unskilled female workers

1.1
Introduction

In this chapter I analyze how the the presence of children in the household affects labor market informality rates of women. There is a large literature that studies the extent to which fertility and child care responsibilities influences female labor supply. While the majority of previous work has focused on the participation and annual hours margins of labor supply of married women, some papers have also considered how women choose between jobs or occupations associated with different levels of schedule flexibility and shorter or longer regular hours.

The main contribution of this chapter is in showing that not only do women self-select into self-employment and part-time work in response to childcare responsibilities like a number of previous papers have already shown (see Related literature below), but also that in some labor markets this self-selection may involve leaving the formal sector altogether and trading-off not only wages, but also job benefits and social security coverage for the greater labor supply flexibility. At the same time, this study also helps to understand whether the informal sector in developing countries with rigid labor markets may work as a buffer for working mothers to avoid costly gaps in employment following the birth of a child.

In a seminal work on labor market informality and the segmentation hypothesis using Brazilian data, Maloney (2004) shows that a large share of men and women work in the informal sector voluntarily. However, while men almost never cite competing household chores as a reason for this choice, 31% of self-employed and 13% of informally employed women do. In addition, household surveys show that, in Brazil, full-time jobs constitute an overwhelming proportion of total jobs in the formal sector. Also, both informal jobs and self-employment are significantly prevalent, responding in 2015 for about 20 and 23%, respectively, of total private-sector employment among urban workers aged 18 to 65. Within these employment categories, part-time work (between 15 and 34 weekly hours) is almost six times as prevalent than within the formal sector. Moreover, while college-educated female workers are relatively more likely to be employed in non-domestic jobs in the formal sector than college-educated men, the opposite is true among low education individuals, which make up over 80% of the Brazilian labor force.

Like many other developing countries, Brazil regulates the labor market extensively and, during the period covered by the analysis in this chapter in particular[1], there were aspects of the labor legislation that prevented some of the labor costs and mandatory employment benefits from being scaled down proportionally in jobs with shorter working hours. For instance, the duration of paid vacation to which workers are entitled to is only reduced for employees that work under 25 hours a week.[2] Moreover, hourly wages of workers hired by a firm under a part-time regime must be proportional to the respective wages of full-time employees in the same occupation. However, if there are expected fixed costs related to, for example, hiring, training, firing, and other specific employment benefits[3], then the marginal costs of hiring a part-time worker would be higher (see Reis and Costa, 2016). Moreover, legislation prohibits the demand of overtime hours from part-time workers which reduce the flexibility of work from the perspective of the employer. Plausibly, these are some of the reasons for the overwhelming concentration of full-time jobs in the formal sector. In a similar context, Del Boca (2002) finds that institutional rigidities in the Italian labor market also reduce the availability of part-time work opportunities, which in turn lowers the overall female labor force participation, especially among mothers. Del Boca and Sauer (2009) and Michaud and Tatsiramos (2011) also find that female labor force participation depends on institutional factors that affect the overall flexibility of the labor market and the supply of child-care services.

In this setting, if there is a relative preference for part-time and more flexible work among mothers, coupled with limited demand for part-time work

[1]In July 2017, Brazil enacted a significant reform of labor legislation which sought to flexibilize the demand of labor. An analysis of the impact of this reform on the relationship between fertility and female labor supply is beyong the scope of this chapter and will be left for future work, since the period covered here is mostly prior to the implementation of this reform.

[2]Any formal job with more than 25 weekly hours is not considered part-time and a worker in such a job is then entitled to a full month of paid vacation a year. For part-time jobs, the duration of paid vacation depends on the number of weekly hours worked, with 18 days for 23-25 hours, 16 days for 21-22, 14 days for 16-20 hours, 12 days for 11-15 hours, 10 days for 6-10 hours and 8 days for up to 5 weekly hours.

[3]For example, legislation requires an employer either to arrange transportation or to provide transport vouchers to its employees and cover the costs that exceed 6% of their base salary. Other examples are benefits not provided for in the legislation but agreed in collective bargaining for some occupations, such as the requirement to provide life insurance policies to employees.

in the formal sector, these women may disproportionately self-select to the informal sector. This mechanism is of interest because formal and informal jobs in Brazil differ in many aspects. First, average wages are significantly higher in the formal sector. Second, informal workers in general do not have access to the social security system, which offers valuable benefits such as unemployment insurance, paid maternity leave and paid sick leave. Even though informal workers may voluntarily make social security contributions, this is relatively uncommon, particularly among low-education workers. Formal work also carries additional benefits such as minimum wages, paid vacation and a "13th salary". On the other hand, informality offers the opportunity to at least partly avoid income taxes, which may be desirable for workers on the upper part of the earnings distribution. Third, it's possible that informal jobs, like self-employment, may offer on average greater flexibility than formal jobs not only in terms of work hours but also schedule and location, which may be desirable for conciliation of labor supply with child care. Fourth, informal workers are relatively more likely to transition into unemployment or non-participation.

The differences presented above suggest that the higher the willingness to pay for flexibility and/or shorter hours among women, the more consequential the trade-off between formal and informal work would be in terms of earnings, human capital formation, employment benefits, social security coverage and labor force attachment.

The empirical analysis of this question poses a number of difficulties. First, given the nature of the decision process, it is important to model decisions to participate and to work in the formal sector jointly, because of the known effect of fertility on the extensive margin of labor supply. Failure to address the endogeneity of the participation decision may induce selection bias in the estimated effects of fertility on the decision to work in the formal sector. Second, it is also possible that underlying preferences with respect to different types of labor market activities are different between women with higher or lower desired and realized fertility. Third, frictions in the labor market and non-time separable individual preferences may give rise to persistence in labor market status which can affect estimates of how the effect of fertility depends on children's age. For instance, in the presence of high state dependence, a short-lived effect can be propagated forward in time, in which case an empirical model that ignores this persistence would spuriously suggest that the effect is longer-lived.

For these reasons, in order to investigate the proposed relationship it would be ideal to have longitudinal data on labor supply and fertility decisions over many periods and covering a long time span, and preferably for a country with a substantial informal sector and rigid labor legislation that limits the flexibility of labor supply in the formal sector. Unfortunately, longer panels with labor supply information are generally based on administrative data and, therefore, do not track unregistered workers. Another possible strategy would be to explore some source of exogenous variation in fertility, such as in Angrist and Evans (1998), although this only provides estimates of effects for specific child parities and it is reasonable to assume that the effect of moving from zero to one child is much more consequential than from moving from two to three children. With this in mind, I choose to follow a parametric approach inspired by the work of Hyslop (1999) on female labor force participation and use longitudinal data from the "Pesquisa Nacional por Amostra de Domicílios Contínua" (PNAD-C). The PNAD-C is a quarterly household survey representative of the Brazilian population, which allows me to incorporate the above permanent individual unobserved heterogeneity and first-order state-dependence into the analysis. To the best of my knowledge, this is the only paper to use a dynamic framework to look at the relationship between women's fertility and labor market informality in developing countries, where the informal sector is relatively much larger.

Methodologically, this paper is closely related to works by Hyslop (1999), Del Boca and Sauer (2009), Michaud and Tatsiramos (2011), Prowse (2012) and Semykina (2018), that have specified and estimated dynamic discrete choice models of female labor supply in reduced form, incorporating state dependence and serial correlation in the unobservables.

My results indicate a systematic relationship between the presence of children in the household and lower probability of holding a non-domestic job in the formal sector among women with high school education or less. Unsurprisingly, the estimated effect is stronger when the sample is restricted to married women. For college educated women, however, no effect is found. In light of Ribar (1992), that shows that market and non-market child-care goods and services are substitutes and have, respectively, positive and negative income elasticity, these results suggest that college-educated women in Brazil may have lower willingness to pay for flexibility due to greater access to child-care goods and services.

The estimated effect among low-education women is sizable, being of similar magnitude to the effect of children on participation, which is known to be economically relevant. Moreover, counterfactual exercises on the estimated model suggest that absent the direct effect of children and the correlation between fertility and unobserved preferences, the gap between men and women without college education in terms of probability of having a non-domestic formal job (conditional on participation) would shrink by between 40 and 60%. This provides suggestive evidence that children may be a relevant constraint for low-education women when searching for work. Moreover, this effect is dynamically reinforced due to state-dependence in labor market status. Hence, policies that increase access to child-care for women of lower socioeconomic status or that, alternatively, increase labor supply flexibility in jobs in the formal sector, may have spillover effects of reducing informality rates and the burden of fertility on labor market outcomes.

The chapter is organized as follows: section 2 presents the data, sample selection criteria and descriptive statistics of the sample; section 3 presents the empirical strategy and discusses the econometric issues that must be addressed given the nature of the data; section 4 presents and discusses the empirical results; section 5 provides an assessment of bias in the results due to possible misspecification of the empirical model and section 6 concludes.

1.2 Related literature

First of all, this chapter is related to the literature that explains the causes of labor market informality in developing countries from the perspective of labor supply. Maloney (2004) and Perry et al. (2007) provide an extensive analysis of this issue in the context of Latin America and find that a relevant fraction of workers voluntarily choose to participate in the informal sector of the labor market, trading-off lower wages for job amenities. For women in particular, conciliation of work with competing household chores and child-care responsibilities is cited by a significant number of self-employed or informal female workers as a reason for not working in formal salaried jobs, which points that one of the amenities valued by women is the relatively higher flexibility of labor supply of informal work arrangements.

In addition, this chapter relates to an extensive literature that studies the extent to which fertility and child care responsibilities influences female labor supply. For instance, a number of studies have looked at the relationship between motherhood and self-employment. Macpherson (1988), Connelly (1992), Edwards and Field-Hendrey (2002), Wellington (2006), Lim (2015a), Lim (2015b) and Semykina (2018) all find that women with children are relatively more likely to work in self-employment. Edwards and Field-Hendrey (2002) also report that women with children are relatively more likely to work from home. Boden (1999) shows that self-employed mothers are more likely than men to cite flexibility of schedule and family related reasons as motivation for becoming self-employed. Hundley (2000) and Gurley-Calvez et al. (2009) find that self-employed women are more involved with housework and spend more time in child-care related activities than women employed in regular jobs. Lombard (2001) and Gimenez-Nadal et al. (2012) find evidence that self-employment provide greater flexibility in the allocation of hours of work than wage-and-salary jobs. Lastly, Lim (2015b) finds evidence that self-employment is used by some mothers as a buffer to avoid nonemployment following the birth of a child.

A related strand of this literature has looked at the relationship between fertility and part-time work, with similar qualitative results. Francesconi (2002) finds that married women with young children are more relatively more likely to work part-time, although in his study the lifetime utility gains from substituting nonemployment with part-time work around and following child-birth are insubstantial. Paull (2008) finds that women are relatively more likely to transition to part-time work following the birth of a child and also that mothers are more likely to reenter the labor market in part-time jobs following nonemployment spells. Similarly, Prowse (2012) finds that shocks that increase part-time work subsequently lead to lower levels of nonemployment among mothers, compared with shocks that increase full-time work. Flabbi and Moro (2012), using an equilibrium search and bargaining framework, find evidence that women value part-time work relatively more than men. Lastly, Bick (2016) finds that subsidizing child-care has a small effect on female labor force participation, but a large effect in increasing the share of women working full-time.

Other works have examined how heterogeneity in terms of labor supply flexibility between occupations relates to the allocation and outcomes of women in the labor market. Goldin (2014) finds strong evidence that some occupations display large compensating wage differentials for long hours and inflexible schedules and, consequently, also display the largest gender wage gaps. Cortés and Pan (2016), find that the occupations with the highest rates of overtime work by men are also the ones with the lowest shares of married women with young children. Adda et al. (2017) show that women with children self-select into more child-friendly occupations and even that occupational choice very early in a woman's career is related to unobserved preferences toward future fertility.

Some related papers have also looked more directly to the demand for specific aspects of work flexibility. Edwards (2014) provides evidence that women value schedule flexibility, and more so if they have children and the younger these children are. Wiswall and Zafar (2017) find that skilled women have a higher willingness to pay than men for shorter hours and part-time work availability within a same job. Mas and Pallais (2017) find that not all, but a substantial fraction of women have high willingness to pay for various aspects of work flexibility. According to Mas and Pallais, the aspects of work flexibility for which women display higher willingness to pay are the ability to avoid irregular and unpredictable working hours and the ability to work from home.

Lastly, this study is also related to works such as Altonji and Paxson (1988), Altonji and Paxson (1992) and Blundell et al. (2008) that have shown that working hours are relatively constrained within jobs and that adjustments in the intensive margin of labor supply usually occur through job changes. Euwals (2001), in particular, find that women who want to adjust their working hours are more likely to succeed in doing so if they change jobs than otherwise. Felfe (2012) shows that women tend to adjust working hours down following the birth of a child, but to a larger extent if they change employers, and that these job changes also tend to involve adjustments in terms of level of stress and schedule flexibility. These amenities are generally traded-off for lower wages.

1.3 Data

I use data from the "Pesquisa Nacional por Amostra de Domicílios Contínua" (henceforth PNAD-C) collected by the Brazilian Institute of Geography and Statistics (IBGE). The PNAD-C is a household survey conducted on a quarterly frequency since the first quarter of 2012. Households are interviewed for five consecutive quarters following a rotating pattern.[4] The survey is representative at the national level. I use all PNAD-C waves up to the last quarter of 2017.

The PNAD-C does not readily provide information on marriage status or the number of children of women. In order to construct these variables, I use the household identifier and the reported status of each person within a household. I keep only women who report their status within their family being "head of household", "spouse or partner of the head of household", "child of both the head of household and spouse" or "child of only the head of household", because it is not possible to link parents and children for the remaining classifications. Observations are linked across waves using the household identifier and the date of birth. Any inconsistency in the date of birth across interviews results in

[4]In wave t a fifth of households which entered the sample in wave $t-5$ is replaced by a new set that will, in turn, remain in the sample until wave $t+4$

that observation being dropped. This also means that I only keep observations with non-missing date of birth information.

I restrict the sample to women living in urban areas and between the ages of 25 and 45, in order to focus on the main childbearing years and the ages during which women display higher labor force participation, and also to avoid the main ages of college attendance. I abstract from possible endogenous attrition and keep only observations with all five interviews[5]. Furthermore, I exclude full-time students by dropping all women who in any period report both attending an educational institution and not participating in the labor market. I drop observations whose maximum educational level was zero years or who report being illiterate in any period, since only about 30% of these women participate in the labor market and among these half are either domestic workers or work for no compensation. I also drop those who in any period report working for no compensation. Finally, I drop same-sex couples and married/cohabiting women whose age difference with their partner is greater than 20 or whose partner's age is below 18.

After these adjustments, the final sample consists of 119,500 women, and 597,500 person-quarter observations. I also focus on a sub-sample of women who are: 1) continuously married or living with the same partner for all five interviews; 2) listed either as "head of household" or "spouse or partner of the head of household" and 3) whose partner is a labor force participant in all periods. I conduct the analysis for this sub-sample because many previous studies of the relationship between fertility and labor supply in the literature focus only on married women. This sub-sample consists of 70,005 women, and 350,025 person-quarter observations. Lastly, in order to focus on labor supply decisions conditional on participation, I further restrict these samples to women who are labor force participants in all five interviews. With this sample restriction I attempt to avoid the issue of endogenous labor force participation by focusing on women who have higher labor force attachment. This restriction results in a sub-sample of 71,252 women, and 356,260 person-quarter observations, when applied to the full sample and 38,819 women, and 194,095 person-quarter observations when applied to the sample of continuously married/cohabiting women.

Table 1.1 presents some descriptive statistics on the full sample and the sub-samples considered. Monetary values are expressed in August 2017 Brazilian reais (R$), computed using the state-quarter specific deflators provided by the IBGE. Column 1-4 correspond, respectively, to the full-sample, the sub-

[5]Unfortunately, a comparison of observable characteristics between women with or without attrition in the data is still pending.

sample of women that participated in the labor market in all five periods, the sub-sample of women continuously married or living with a partner, and the one that combines these two restrictions. Some patterns deserve attention: imposing continuous labor force participation (columns 2 and 4) skews the sample toward higher proportion of white women, higher education, lower fertility and higher non-labor income. Moreover, the proportion of women married or living with a partner drops from 73.2% in the full sample to 67% among women that participated in all periods. Lastly, the samples of continuously married women (columns 3 and 4) display higher fertility and higher non-labor income.

In table 1.2, I present estimates of the age at first birth of women in the sample, by educational level. To compute these estimates, I use only information on women up to the age of 31, for which the probability of a child having exited the household is lower.[6] The average age at first birth is 20.7 years for women with at most secondary education and 23.8 years for women with college education. The sub-samples of women continuously married or living with a partner have slightly higher average age at first birth for both educational levels.

Tables 1.3 and 1.4 present statistics on the employment composition of the sample, by educational level. A job is classified as formal if the worker reports having a signed labor contract ("carteira assinada").[7] Domestic workers, whether formal or informal, employers and military/tenured public employees are categorized separately. College educated women display much higher labor force participation, with only 3.7% not participating in any of the periods observed, and 82.4% participating in all periods, compared with the respective figures of 17.5% and 51.7% for women with high school education or less. As expected, formal jobs, either in the private sector or in the form of tenured employment in the public sector, are much more prevalent among high-education women, and unemployment rates are lower. Lastly, almost 12% of women with high school education or less report being domestic workers.

Table 1.5 presents the distribution of usual weekly hours worked by type

[6]In the PNAD-C, I can only attribute to a woman the number and ages of her children that are currently living in her household. In this way, if a woman had a child at age 13, for example, at age 31 this child would be 18 years old and may have moved out of the household, in which case I could mistakenly calculate the age at the first birth of this woman based on the age of the second oldest child.

[7]Having a signed labor contract means that her employer is required to comply with regulations concerning, for example, the maximum length of a work day and the rate of over-time compensation, among many others, has to collect social security contributions on behalf of the worker and deduct income taxes directly into the paycheck. Also such a worker has access to all employment benefits provided for by the current labor legislation, such as annual paid vacation, unemployment insurance, severance compensation, among others. In general terms, a job with a signed labor contract is a registered, regulated job, as opposed to an unregistered, informal work arrangement.

of labor market activity. To compute these figures, I consider only workers who report usually working between 10 and 60 hours a week. Formal employment is by far the type of work with least variation in weekly hours. It is the only category with less than 10% of women working part-time. It is notable that the share of part-time workers in formal jobs is even lower than among employers. Even though this does not constitute clear evidence, it is strongly suggestive that there is lack of demand for part-time work in the formal sector. As argued in the previous section, this is likely due to institutional rigidities. On the other hand, over 46% self-employed women report working less than 35 hours a week. Moreover, Panels B and C show that the distribution of hours in all types of activities is skewed toward part-time work among women with children under 11.

Finally, table 1.6 presents some figures discussed in the previous section regarding the differences between formal and informal jobs. In this table, I include tenured public employees in the "Formal" category, and self-employed, employers and domestic workers in the "Informal" category. Montly income is lower among informal workers, and these are more likely to transition into unemployment and non-participation over time. The last panel also shows that only 34% of informal workers with high school education or less make social security contributions.

1.4 Empirical specification

1.4.1 Model I

Let the indicator variable $p_{it} = 1$ if individual i participates in the labor market in interview t and $p_{it} = 0$ otherwise. Given that the data has a rotating panel structure, note that $t = \{0, 1, 2, 3, 4\}$ here indicates the interview number rather than actual date. Similarly, let $f_{it} = 1$ if this individual reports being in a non-domestic formal job and $f_{it} = 0$ if she reports being in any of the other employment categories (including unemployment). Consider then the following reduced-form specification for the sequential decision rule over labor supply, which is largely inspired by the empirical specification used in Hyslop (1999):

$$p^*_{it} = X'_{it}\beta + \gamma_1 p_{i(t-1)} + \gamma_2 f_{i(t-1)} + u^p_{it} \tag{1-1}$$
$$f^*_{it} = X'_{it}\lambda + \mu_1 p_{i(t-1)} + \mu_2 f_{i(t-1)} + u^f_{it} \tag{1-2}$$
$$p_{it} = \begin{cases} 1 \text{ iff } p^*_{it} > 0 \\ 0 \text{ otherwise} \end{cases}$$
$$f_{it} = \begin{cases} 1 \text{ iff } f^*_{it} > 0 \text{ and } p_{it} = 1 \\ 0 \text{ otherwise} \end{cases}$$

where X_{it} is a vector of personal and household characteristics that may affect participation and work decisions, such that equation (1-1) may then be viewed as a first order approximation to the difference between indirect utility conditional on participation and indirect utility conditional on non-participation. Equation (1-2) may be interpreted analogously with the indirect utilities being conditional on participation and being or not in a formal job. The unobserved terms u^p_{it} and u^f_{it} are assumed to have the following structure:

$$\begin{aligned} u^p_{it} &= \alpha_i + \varepsilon^p_{it} \\ u^f_{it} &= \eta_i + \varepsilon^f_{it} \end{aligned} \tag{1-3}$$

in which $(\varepsilon^p_{it}, \varepsilon^f_{it})$ are transitory error components, assumed to be independent of X_{it}, α_i, η_i and each other, serially uncorrelated and distributed $N(0,1)$. In this way, the model above allows for inter-temporal linkages in the decision process both through state dependence (which may arise due to frictions in the labor market and/or preferences that are not time-separable), captured by the lags $p_{i(t-1)}$ and $f_{i(t-1)}$, and through the random effects vector (α_i, η_i), which captures unobserved time-invariant heterogeneity in preferences and/or productivity. Furthermore, I adopt a correlated random effects (CRE) specification, following Chamberlain (1984), in which the individual effects (α_i, η_i) in (1-3) are allowed to be correlated with the fertility covariates in X_{it}:[8]

$$\begin{aligned} \alpha_i &= \delta_1(\overline{\#\text{Ch0–2}})_i + \delta_2(\overline{\#\text{Ch3–5}})_i + \delta_3(\overline{\#\text{Ch6–11}})_i + \delta_4(\overline{\#\text{Ch12–17}})_i + \widetilde{\alpha}_i \\ \eta_i &= \zeta_1(\overline{\#\text{Ch0–2}})_i + \zeta_2(\overline{\#\text{Ch3–5}})_i + \zeta_3(\overline{\#\text{Ch6–11}})_i + \zeta_4(\overline{\#\text{Ch12–17}})_i + \widetilde{\eta}_i \end{aligned} \tag{1-4}$$

[8]In the empirical application, the only other non-deterministic time-varying explanatory variables are the income of the husband and his unemployment status. I explicitly assume that unobserved heterogeneity is independent of these variables. I acknowledge that this hypothesis should be tested, although I have not done so yet.

where the $\overline{(\#\mathrm{Ch})}_i$ variables above are the averages over t of the number of children in that respective age range, and $(\widetilde{\alpha}_i, \widetilde{\eta}_i)$ are independent of X_{it} and joint normally distributed with covariance matrix Σ. A simple test for the endogeneity of observerd fertility with respect to unobserved preferences follows by testing the joint hypotheses that the correlated random effects parameters in (1-4) are zero.

The structure assumed above for the joint distribution of the random effects implies the following serial and cross correlation pattern for the errors of the model, conditional on the fertility variables in X_{it}:

$$\begin{aligned}
\mathrm{corr}(u^p_{it}, u^p_{is}) &= \frac{\sigma^2_{11}}{1+\sigma^2_{11}} \\
\mathrm{corr}(u^f_{it}, u^f_{is}) &= \frac{\sigma^2_{22}}{1+\sigma^2_{22}} \\
\mathrm{corr}(u^p_{it}, u^f_{is}) &= \frac{\sigma_{12}\sigma_{11}\sigma_{22}}{\sqrt{(1+\sigma^2_{11})(1+\sigma^2_{22})}}
\end{aligned}$$

where σ_{ij} are the respective entries of the covariance matrix Σ. Note that, under this assumption, $\mathrm{corr}(u^p_{it}, u^p_{is})$ and $\mathrm{corr}(u^f_{it}, u^f_{is})$ are also the fraction of the total variance of the unobservable components in (1) and (2) due to the time-invariant random effects.

Given the survey nature of the data and that observations are linked across waves only on the basis of date of birth, it is likely that there is some degree of classification error in the observed sequences of labor supply decisions. Keane and Sauer (2009), while revisiting work by Hyslop (1999), demonstrate the importance of addressing the issue of classification error in a dynamic model of female labor supply such as the one used here. They show that incorporating this feature is broadly justified by the improvement of the model fit and it affects not only point estimates of the coefficients of interest, but also overturns the conclusion in Hyslop (1999) of exogeneity of observed fertility with respect to labor force participation decisions of married women.

In light of these results, I adopt the following specification for the measurement error process. Note that there are 3 possible decisions in each period: non-participation, informal work and formal work. Given that observations span 5 periods, there are a total of 3^5 possible observed decision sequences. I assume then that the observed sequence equals the true sequence with probability $1-\xi$, and with probability $\xi/(3^5-1)$ any of the other sequences is observed instead. Hence, the structure of the error process depends, rather parsimoniously, on a single parameter ξ, which is estimated. Note, however, that this assumption imposes that all sequences other than the true one are equally likely to be observed. While acknowledging this limitation, this modelling choice is motivated by the great tractability that it lends to the estimation method. Because of the presence of lagged dependent variables, a richer structure for the measurement error process would require the likelihood implied by the model to be simulated, which would greatly increase the computational burden of estimation. Lastly, Hausman, Abrevaya and Scott-Morton (1998) show that in a classification error framework such as this, identification of the error rate ξ requires only that the probability of the observed sequence be increasing in the probability of the true sequence, i.e. $\xi < (3^5 - 1)/3^5$.

Under the assumptions made, it is clear that p_{i0} and f_{i0} in the first period of observed data for each person are not independent of the individual effects (α_i, η_i) and cannot be taken as exogenous. This is a common issue in dynamic nonlinear models where the stochastic process is not observed from the start or cannot be assumed to be in a steady-state, and is generally termed the initial conditions problem. Heckman (1981) proposes a solution to this problem which involves treating the initial condition as random and specifying a reduced form approximation to its conditional distribution in terms of the initial period covariates, in addition to letting the error in this approximation be freely correlated with the errors in the subsequent periods. I follow this approach, which completes the specification of the empirical model.

The likelihood contribution by individual i in this model is:

$$L_i = \frac{\xi}{3^5 - 1} + \left(1 - \frac{3^5 \xi}{3^5 - 1}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_{t=0}^{4} \Phi(m_{it})[\Phi(q_{it})^{p_{it}}] d\Phi(\widetilde{\eta}) d\Phi(\widetilde{\alpha}) \tag{1-5}$$

$$\begin{aligned}
m_{i0} &= (2p_{i0} - 1)[X'_{i0}\beta_0 + \theta_p(Z'_i\delta + \widetilde{\alpha}_i) + \theta_f(Z'_i\zeta + \widetilde{\eta}_i)] \\
q_{i0} &= (2f_{i0} - 1)[X'_{i0}\lambda_0 + \phi_p(Z'_i\delta + \widetilde{\alpha}_i) + \phi_f(Z'_i\zeta + \widetilde{\eta}_i)] \\
m_{it} &= (2p_{it} - 1)(X'_{it}\beta + \gamma_1 p_{i(t-1)} + \gamma_2 f_{i(t-1)} + Z'_i\delta + \widetilde{\alpha}_i) \\
q_{it} &= (2f_{it} - 1)(X'_{it}\lambda + \mu_1 p_{i(t-1)} + \mu_2 f_{i(t-1)} + Z'_i\zeta + \widetilde{\eta}_i)
\end{aligned}$$

where Z_i is the vector of correlated random effects in (1-4), $\Phi(\cdot)$ is the standard normal CDF and $(\theta_p, \theta_f, \phi_p, \phi_f)$ flexibilize the correlation between the errors of the approximation to the conditional distribution of the initial conditions and the subsequent periods.

1.4.2
Model II

This specification follows directly from the one presented above, but is intended to be estimated using just the data on women who continuously participate in the labor market over the observed period. This specification is adopted in an attempt to avoid the issue of endogenous labor force participation by focusing on women who have higher labor force attachment, and also to check the robustness of the results obtained under the previous specification. Accordingly, let $f_{it} = 1$ if woman i reports being in a non-domestic formal job in interview t and $f_{it} = 0$ otherwise. The decision rule is then specified as:

$$f_{it}^* = X_{it}'\lambda + \mu f_{i(t-1)} + u_{it} \tag{1-6}$$

$$f_{it} = \begin{cases} 1 \text{ iff } f_{it}^* > 0 \\ 0 \text{ otherwise} \end{cases}$$

$$u_{it} = \eta_i + \varepsilon_{it} \tag{1-7}$$

where ε_{it} is a serially uncorrelated shock, independent of η_i and X_{it}, distributed $N(0,1)$. Considering the framework in Model I, it is clear that if the number of young children in the household correlates negatively with the probability of labor force participation and if (α_i, η_i) are positively (negatively) correlated, then imposing continuous participation will generate a selected sample in which women with young children have, on average, higher (lower) values of the individual effect η_i than in the unrestricted sample. Therefore, this sample restriction mechanically induces correlation between the number of children in each age category and η_i. Like in the previous model, I address this issue by adopting the same correlated random effects approach as in (1-4) but note, however, that correlation between the individual effect and fertility variables in this specification may be due to sample selection rather than endogeneity of fertility with respect to labor supply.

With regard to classification error, in this specification there are 2^5 possible observed sequences. Like above, I assume that the observed sequence equals the true sequence with probability $1-\xi$, and with probability $\xi/(2^5-1)$ any of the other possible sequences is observed instead.

Lastly, the issue of endogeneity of initial conditions is also present here, so I adopt the same approach as in the previous speficication. In this way, the contribution to the likelihood by individual i in this model is:

$$L_i = \frac{\xi}{2^5-1} + \left(1 - \frac{2^5\xi}{2^5-1}\right) \int_{-\infty}^{\infty} \prod_{t=0}^{4} \Phi(q_{it}) d\Phi(\tilde{\eta}) \tag{1-8}$$

$$q_{i0} = (2f_{i0} - 1)[X_{i0}'\lambda_0 + \phi(Z_i'\zeta + \tilde{\eta}_i)]$$

$$q_{it} = (2f_{it} - 1)(X_{it}'\lambda + \mu f_{i(t-1)} + Z_i'\zeta + \tilde{\eta}_i)$$

where Z_i is the vector of correlated random effects as in (1-4), $\Phi(\cdot)$ is the standard normal CDF and ϕ flexibilize the correlation between the error of the approximation to the conditional distribution of the initial conditions and the subsequent periods.

1.4.3 Estimation

I conduct estimation of models I and II separately for women with college education and those with at most secondary education, motivated by the differences in the employment composition between these groups, as seen in tables 1.3 and 1.4, and because informal work, especially self-employment, may be of a different nature for highly educated individuals. Moreover, I estimate both specifications on the full sample which includes single women and on the sub-sample of women who 1) report being continuously married to or cohabiting with the same partner; 2) are listed either as "head of household" or "spouse or partner of the head of household" and 3) whose partner is a labor force participant in all periods (henceforth, this sub-sample will be referred to as the "continuously married" sample).

I choose to classify as "formal" only non-domestic workers who report having a signed labor contract ("carteira assinada"). This means that, in the low education sample, domestic workers with a signed labor contract (about 25% of all domestic workers) are classified as informal. In the main empirical exercises, I drop observations that report being tenured public employees in any of the observed periods, despite the fact that these are registered workers. Tenured public employees are workers who are recruited to the public sector through an objective selection process which is commonly highly competitive and requires extensive preparation. These jobs are regulated by special labor legislation and, in general, provide greater job security, better job benefits and higher salaries, especially at the federal and state levels. Due to the significant barriers to entry imposed by the selection process, these jobs may not be an effective work option for many people. In the appendix, I present the results of the empirical exercises with tenured public employees included in the "formal"category.

In all specifications, the fertility profile of a woman is captured by four variables containing the number of children aged 0–2, 3–5, 6–11 and 12–17 years. I include as explanatory variables in X_{it} a quadratic in age, race[9], state fixed-effects[10] and a yearly linear trend. Estimation of the models on the

[9] I use two dummies: the first indicating that the individual reports being white in all five interviews, and the second indicating that the individual does not report being white in any of the interviews and reports being black at least once.

[10] In order to reduce the number of parameters to be estimated, I group some neighboring states that have relatively lower population and, hence, fewer observations in the data.

low education sample includes an indicator for having completed high school. Whenever using the full sample, I include dummies for marriage/living with partner and for being the spouse of or the head of household itself. I control for the level of and shocks to non-labor income using usual monthly earnings in all jobs of the husband/partner[11] and an indicator for whether he is unemployed. Lastly, when using the "continuously married" samples, I further control for permanent non-labor income by including an indicator for the partner being college educated.

Under the structure assumed in (1-3) and (1-7) for the unobservables, models I and II can be estimated by maximum likelihood using Gauss-Hermite quadrature to approximate the integrals in (1-5) and (1-8) to any degree of desired accuracy, as proposed by Butler and Moffitt (1982). For model I, I use a spectral decomposition of the covariance matrix of the random effects and 8-point quadrature in each dimension. For model II, I use 16-point quadrature. [12] To maximize the likelihood function, I employ the BHHH algorithm, by Berndt et al. (1974).

To alleviate concerns about the algorithm converging to a local maximum I repeat the estimation using different combinations of starting values for the parameters. To estimate model I for each of the sub-samples, I use as starting values for the parameters of the initial conditions and of the subsequent periods the coefficients estimated in a pooled static probit of participation on the same set of explanatory variables in X_{it}, and a pooled static probit of formal work conditional on participation, and set the random-effect and state-dependence coefficients to zero. Next, I use the resulting estimates to repeat estimation for four different sets of starting values based on them: add to or subtract from each of the coefficients 60% of their absolute values; multiply all the coefficients by 1.6 or 0.4. The same maximum was attained in all cases. The same procedure is employed for model II, but the first set of starting values is obtained by a pooled static probit of formal work estimated on the sub-sample of women that participate in the labor market in all five periods.

[11]Specifically, I control for $\text{Log}(1+Y)$, where Y is expressed in R$1000 of August 2017.

[12]Estimates are insensitive to increasing the number of nodes of the quadrature to 10 in each dimension for model I and 20 for model II when performing estimation on the low-education/continuously-married/no public workers sub-sample. In fact, the number of nodes used is actually greater than necessary, since estimates are also insensitive to reducing the number of nodes to 6 and 12 respectively. Nevertheless, there was no need to further simplify the estimation strategy because estimation times were still reasonable using 8 and 16 nodes.

1.5 Results

In this section I present the results of the estimation of the models specified in the previous section.

I begin by showing the results obtained by estimating models that ignore dynamics in labor supply of any kind and also classification error. Clearly, absent these features the log-likelihood becomes additively separable and the models are simple probits on the pooled sample and, as such, are subject to selection bias and gross misspecification of the errors. Tables 1.7 and 1.8 present these results for low- and high-education women, respectively. Columns (1) and (3) display results for the full and continuously married sub-samples, respectively and in columns (2) and (4) the samples are restricted to women who participate in all five periods.

A strong negative effect of young children (ages 0–2) on labor force participation is estimated for both low- and high-education women. On the other hand, the respective effect on formal work probability is estimated to be weaker than for children aged between 3 and 11. In columns (2) and (4), the coefficient on young children is precisely estimated to be zero. Moreover, results do not indicate that children have a relatively stronger impact on the labor supply of married women.

Although not reported here, for all samples considered, I also omit each of the relevant features separately, in order to test the null hypotheses of no state dependence, no unobserved time-invariant heterogeneity and no classification error. Each null is strongly rejected under a likelihood ratio test[13].

Moving on, table 1.9 present estimates from the dynamic models for women with at most secondary education. Note that including dynamics and classification error greatly improves the likelihood of the empirical model. Column (1) presents the coefficients estimated from model I on the full sample. As expected, the presence of young children in the household has a negative effect on labor force participation, but the magnitude of the coefficients is much smaller compared to the static models. The coefficient for children of ages 0-2 remains large, but for children older than 6 the effect becomes insignificant. Being married or living with a partner also has a strong negative effect on labor force participation, which suggests that married women tend to specialize in home production. There is also evidence of an “added worker effect”, that is, a negative shock to non-labor income, captured by the coefficient of the dummy for husband/partner unemployment, increases the probability of participation.

[13]When testing the null of no state dependence, I let the coefficients of the initial conditions be equal to the subsequent periods.

The coefficients on first-order state dependence confirm that participation is persistent and that formal workers are relatively less likely to exit to non-participation. Time-invariant heterogeneity is found to account for two thirds of the variance of the unobservables. Note that, under the assumption of serially uncorrelated transitory errors, this is also the overall degree of serial correlation in the unobservables.

With respect to the probability of being employed in a formal job conditional on participation, the presence of children in the household also displays a negative effect. The coefficients on are notably different from the ones in the static models. For children aged 0-2, the effect is smaller than in the participation equation, and of equal magnitude for children 3-5. However, the effect of children on formal work probability persists up to the 6-11 category, and is insignificant only after age 12. The coefficients are all close to -0.1 and do not appear to depend much on children's age up until age 11. An effect of this size translates into a reduction of up to four percentage points per child in the probability of working in the formal sector. Considering that about 40% of low-education women participating in the labor force are in the formal category as defined here, a simple back of the envelope calculation suggests a relative effect of up to -10%. In the formal equation, the coefficients of state dependence indicate that formal work is highly persistent, and also that a woman is more likely to enter the formal sector if she already participated in the labor market in the previous period than otherwise. This implies then that informal workers are relatively more likely to exit to non-participation and, once there, they are relatively less likely to reenter the labor force in the formal sector. Time-invariant heterogeneity is estimated to account for three quarters of the total error variance.

The correlation coefficient between the random-effects is estimated to be positive but small, at 0.09. The estimate of the classification error rate indicates that about 2% of observed sequences are erroneously reported at least at some point. The last important result to highlight is the rejection of the null hypothesis of no correlation between observed fertility variables and the individual effects. In other words, observed fertility is found to be endogenous not only with respect to the participation decision, but also with respect to the decision to work in the formal sector.

Column (2) in Table 1.9 presents the results from estimation of model II, which considers only women that participated in the labor market in all five observed periods and takes participation as exogenous. Estimates corroborate the finding of a negative effect of fertility on the conditional probability of formal work but, unlike the results from model I, this effect appears to be slightly stronger for younger children, even though differences are not statistically significant. The magnitude of the coefficients, between -0.01 and -0.14, imply an absolute reduction of up to 4–5.7 percentage points in that probability and relative reduction of up to 9–12%. The estimates of the degree of state dependence, the relative importance of unobserved heterogeneity and the classification error rate are all of very similar magnitude to the ones in column (1). Exogeneity of fertility variables is also rejected, but this may be to due sample selection as discussed in the previous section.

Columns (3) and (4) display results from models I and II, respectively, estimated on the ”continuously married” sample. The results are qualitatively very similar, but the point estimates of the fertility variables are larger in general, which suggests, unsurprisingly, that children play a relatively more important role in labor supply decisions of married women.

Table 1.10 presents the results for college-educated women. Fertility is found to have a stronger negative effect on labor force participation than compared to low-education women, and which does not die out as children age. However, despite the negative point estimates, the effects of children on the conditional probability of having a formal job are all statistically equal to zero, which is in sharp contrast to the results of the static models. Compared to low-education women, the individual random-effects are found to account for a larger share of the total variance of the errors related to the decision to work in a formal job, at 80%. Another difference is the higher correlation between both dimensions of unobserved heterogeneity, at 0.29[14]. The error rate is estimated at 1.6%. The null hypothesis of exogeneity of fertility with respect to unobserved preferences is rejected for both equations. However, in the case of formal work, this result is not robust to the inclusion of public employees as formal workers, as can be seen in table A.2 in the appendix. Hence, there is no conclusive evidence that college-educated women with higher observed fertility have different latent preferences toward work in the formal sector.

Overall, the results suggest that the presence of children in the household may not be a relevant constraint for the decision regarding formal/informal employment among college-educated women. Two possible reasons for this finding are: formal jobs for skilled workers may offer greater flexibility on average compared to jobs for unskilled workers; college-educated women may have greater access and more resources to spend on goods and services that reduce the need to seek more flexible work arrangements. The results in Ribar (1992) support the second of these hypothesis. He finds evidence that

[14] I do not know the underlying reason for this difference and it probably deserves further investigation

market and non-market child-care are substitutes and that market child care is a normal good while non-market is an inferior good. Nonetheless, further investigation of these possible mechanisms is certainly an interesting avenue for future research.

Next, I conduct a series of counterfactual exercises using the estimated models for low-education women in order to quantify how their labor market allocation would respond if the conditional probability of working in a formal job was independent of fertility. To do so, I make 500 draws of the vectors of the transitory errors and of the random-effects in accordance with their estimated correlation. Using the observed explanatory variables, I then simulate 500 sequences of participation and formal work choices and compute their relative frequency at period 5. Using the same set of draws, I repeat the simulations first setting to zero the CRE coefficients of the formal equation, next the coefficients of the direct effect of fertility, and finally both at the same time, and compute the participation and formal work relative frequencies at period 5. Table 1.11 present the results of these counterfactuals. Each column corresponds to the respective counterpart estimated in table 1.10.

In column (1), the observed participation and formal work frequencies are 66.6 and 39.2%, and their simulated counterparts are 66.3 and 38.2%, respectively. When I turn off the CRE coefficients, the frequency of formal work increases to 41.3%, and when I turn off the direct effect of fertility, the frequency rises to 39.8%. Finally, when both channels are turned off, the share rises to 43.1%. Note that participation rates also increase due to the mutual state dependence effects, but only to a very small degree. To put these results into perspective, I also compute the male frequency of formal work, subject to same sample restrictions. The male rate is 50.3%, which implies that absent the effects of fertility on formal work probability, the gap between men and women in this respect would shrink by almost 45%. Column (2) display the counterfactuals based on the sample that participates in all periods. The observed and simulated frequencies are 46.3 and 45.8%, respectively. With no endogeneity, the share increases to 47.8% and with no direct effect of children, to 47.6%. The combined counterfactual gives 49.6%. The respective male frequency is 52.4%, so that for this sub-sample the male-female gap would shrink by just over 60%. Columns (3) and (4) display the respective counterfactuals for continuously-married women and are qualitatively similar, although I do not make comparisons with men.

Finally, note that given the short span of the data for each women, it is possible that unobserved heterogeneity is capturing not latent preferences, but rather heterogeneity in human capital accumulation across the life cycle.

This is especially relevant because, even though the specification allows for correlated random effects, for the older women in our sample it is likely that some have children that have already exited the household. As children have a strong effect on labor force participation, it would be desirable to control for differences in human capital due to previous spells of non-participation and informal employment. A possible strategy would be to narrow the age-interval of the sample to a lower one, such as 24–34 years of age, in order to reduce the number of women with children older than 18. While this would make the results less general, it would allow the inclusion of controls such as a quadratic in the number of years since the birth of the first child, to help account for such individual differences in the history of human capital accumulation. This is an interesting exercise but will be left for future work.

1.6 Assessment of bias

The structure assumed in (1-3) and (1-7) is possibly misspecified because it imposes the serial and cross correlations in the joint distribution of transitory components of the errors to be zero. Relaxing these assumptions would introduce in the likelihood function implied by the model high-dimensional integrals for which numerical approximation is not a viable option. Thus, it would require resorting to simulation of the likelihood function, which produces consistent and asymptotically normal estimates only if the number of simulations increases at least as fast as $\sqrt{N}$ (Train, 2003). Unfortunately, for the sample sizes considered, this would greatly increase the computational burden of estimation.

Moreover, Keane (1992) demonstrates the lack of practical identification of the full covariance matrix in the multinomial probit model in the absence of exclusion restrictions, even though the model is formally identified. This result suggests that a fully flexible correlation structure of the unobservable components, in particular the contemporaneous correlation of the transitory errors in model I, would likely not be identifiable due to the lack of valid exclusion restrictions within a given decision period.

In this way, in order to provide some insight on the level of bias in the estimates of the effect of children on the probability of working in a formal job due to misspecification of the joint distribution of the errors, I conduct a series of simulations in which I estimate models I and II on artificial data generated using a richer covariance structure for the unobservables. The data generating process used in these simulations is described below:

$$p^*_{it} = \quad 1.0 - 0.3x_{it} + 0.2p_{i(t-1)} + 0.2f_{i(t-1)} + \alpha_i + \varepsilon^p_{it} \tag{1-9}$$

$$f^*_{it} = -0.5 - 0.2x_{it} - 0.2p_{i(t-1)} + 0.4f_{i(t-1)} + \eta_i + \varepsilon^f_{it} \tag{1-10}$$

$$p_{it} = \begin{cases} 1 \text{ iff } p^*_{it} > 0 \\ 0 \text{ otherwise} \end{cases}$$

$$f_{it} = \begin{cases} 1 \text{ iff } f^*_{it} > 0 \text{ and } p_{it} = 1 \\ 0 \text{ otherwise} \end{cases}$$

where x_{it} is an exogenous regressor and the errors follow AR(1) processes: $\varepsilon^j_{it} = \rho_j \varepsilon^j_{it} + \nu^j_{it}$, $j = (p, f)$. The innovations (ν^p_{it}, ν^f_{it}) are serially uncorrelated, independent from x_{it}, α_i and η_i and joint normally distributed with variance $(1 - \rho^2_j)$ and correlation coefficient ψ. The time-invariant random effects (α_i, η_i) are also joint normally distributed with correlation coefficient equal to 0.5 and variances equal to 2, so that the random-effects account for two thirds of the total variance of the unobservables.

The start of the process is at $t = -24$ with the lagged dependent variables and errors set to zero, but data is only "observed" from $t = 1$ up to $t = 5$, so that the problem of initial conditions is present. The exogenous regressor is generated by a Nerlove process of the form: $x_{it} = 0.125 + 0.02t + 0.9x_{i(t-1)} + U[-0.25, 0.25]$ with $x_{i-25} \sim U[-1, 2]$ so that it is subject to a high degree of persistence and also contains a deterministic time-trend. Finally, the random effects are correlated with x_{it} such that $E[\alpha_i] = -0.5\overline{x}_i$ and $E[\eta_i] = -0.25\overline{x}_i$, where $\overline{x}_i$ is the individual average of x_{it} from $t = -24$ through $t = 5$[15].

First, I generate 10000 individual sequences of the exogenous regressor. Keeping the values for x_{it} fixed, I build 100 datasets for each of the following eighteen combinations of parameters $\rho_j = \{0.9, 0.6, -0.25\}$ and $\psi = \{-0.4, 0.4\}$. These values capture moderate, high or negative serial correlation in the transitory shocks and positive or negative contemporaneous correlation of the innovations. I then estimate models I and II[16] on each dataset using only data from $t = 1$ up to $t = 5$ and compute the relative bias in the estimate of the coefficient of x_{it} in equation (1-10).

Table 1.12 presents the results of these simulations. First, when there is no misspecification, estimates of the coefficient of the exogenous regressor in equation (1-10) from model I are unbiased and from model II are only about 5% greater than the true value. Also, if dynamics are ignored, estimates are

[15] $\overline{x}_i$ is demeaned by the grand mean, so that the random effects have mean zero in the population.

[16] Model II is estimated only on observations with $p_{it} = 1$ in all five "observed" periods.

clearly biased to a significant degree. When the joint distribution of the errors is misspecified, however, estimates using either model are moderately biased, in general. Mean relative biases were bounded between 55 and -41% for model I, and between 39 and -21% for model II. There is no apparent pattern that point to a specific correlation structure inducing more or less bias.

Nevertheless, the lower panel of Table 1.12 shows that in the overwhelming majority of replications, estimates are less than 100% greater than the true value. This suggests that the true value of the coefficients of children on formal work probability would likely fall within the 95% confidence interval even if their respective point estimates were to be overestimated.

Although not reported here, it should be clear that ignoring serial correlation in the transitory components of the errors when it is in fact present greatly biases the estimates of the state-dependence coefficients, as these act as fitting parameters in the misspecified model. For this reason, the degree of true state dependence in labor supply is possibly smaller than the results presented in the previous section suggest if the errors have positive serial correlation.

This exercise, albeit limited in scope, support the findings in the previous section that, for women with at most secondary education, fertility has a negative impact on the probability of having a formal job conditional on labor force participation.

1.7 Conclusion

In this chapter, I analyze how the allocation of women in the formal or informal sectors of the Brazilian labor market depends on the number of children present in the household. For this purpose, I use a reduced form specification of the sequential decision over labor force participation and choice between formal or informal work. The empirical framework allows for intertemporal linkages in the decision process both through state-dependence and time-invariant individual heterogeneity.

The results indicate that fertility has a negative effect on the probability of having a non-domestic formal job conditional on labor force participation among women with at most secondary education. The magnitude of the estimated coefficients suggest that one additional child in the household has an absolute effect of up to minus four percentage points in this conditional probability. Moreover, I find that, for these low-education women, fertility is not exogenous with respect to the decision over working in the formal or informal sector. A series of counterfactual exercises on the estimated models indicate that the gap relative to men would shrink by over 40% if this dimension of labor supply was independent of fertility. Considering that formal jobs are mostly full-time, these findings are in line with the hypothesis that child-care responsibilities increase the relative preference for part-time and more flexible work arrangements which, in this setting, may drive self-selection of women with children into informality. With the framework used here, however, it is not possible to rule out the alternative explanation that these effects are due to employer discrimination against women with children or that endogeneity of observed fertility is due to heterogeneity in the history of human capital accumulation. With respect to college-educated women, I find that fertility has no effect on the decision to work in the formal sector, a result that is in line with Ribar (1992) and suggests that college-educated women in Brazil may have lower willingness to pay for flexibility due to greater access to child-care goods and services.

Last, I conduct a series of simple simulation exercises designed to assess the severity of bias induced by misspecification of the covariance structure of the errors of the model. These simulations indicate that, for the types of misspecification tested, the estimated coefficients are, in general, moderately biased, but not so much as to put their true values outside the 95% confidence interval. If there is no misspecification, the simulation exercise suggests that estimates are accurate.

In future work I will revisit these exercises, incorporating more recent data from the PNAD-C and narrowing the age-interval of the samples in order to test whether my results are robust to controlling for number of years since the first birth.

Table 1.1: Sample characteristics

	Full sample (1)	Always part. (2)	Married (3)	Married Always part. (4)
Age	34.9	35.0	35.0	35.2
	(0.02)	(0.02)	(0.02)	(0.03)
Race (%)				
Black – all periods	3.1	3.3	2.8	2.8
	(0.05)	(0.06)	(0.06)	(0.08)
Black – at least once	11.3	11.0	10.1	9.6
	(0.09)	(0.11)	(0.11)	(0.15)
White/Asian – all periods	33.1	37.6	35.2	40.6
	(0.14)	(0.20)	(0.17)	(0.25)
Education (%)				
Less than high school	28.7	20.3	28.6	19.7
	(0.13)	(0.15)	(0.18)	(0.21)
High school	45.5	44.0	46.7	44.6
	(0.14)	(0.20)	(0.20)	(0.25)
College	25.8	35.6	24.7	35.7
	(0.12)	(0.17)	(0.16)	(0.24)
Married/cohabiting (%)	73.2	67.0	100	100
	(0.13)	(0.19)	–	–
Number of children				
aged 0-2 years	0.16	0.12	0.20	0.16
	(0.001)	(0.001)	(0.002)	(0.002)
aged 3-5 years	0.19	0.16	0.22	0.20
	(0.001)	(0.001)	(0.002)	(0.002)
aged 6-17 years	0.88	0.78	0.95	0.86
	(0.003)	(0.003)	(0.004)	(0.004)
Partner's monthly	2678	2958	2802	3088
earnings (R$)	(12)	(16)	(15)	(18)
Sample size	119,500	71,252	70,005	38,819

Notes: All moments are computed based on the 1st interview. Bootstrap standard errors in parenthesis based on 500 draws.

Table 1.2: Average age at first birth

	Full sample	Always part.	Married	Married Always part.
	(1)	(2)	(3)	(4)
Education group				
High school or less	20.7	20.7	20.9	21.0
	(0.03)	(0.03)	(0.03)	(0.05)
College	23.8	23.7	24.3	24.2
	(0.07)	(0.08)	(0.08)	(0.10)

Notes: All moments are computed based on the 1st interview. Bootstrap standard errors in parenthesis based on 500 draws.

Table 1.3: Employment composition - High school or less

	Full sample	Always part.	Married	Married Always part.
	(1)	(2)	(3)	(4)
Type of work (%)				
Formal	25.4	43.2	23.8	43.5
	(0.15)	(0.23)	(0.18)	(0.32)
Informal	6.6	9.0	5.8	8.4
	(0.08)	(0.14)	(0.10)	(0.17)
Self-employment	12.8	16.8	13.1	18.2
	(0.12)	(0.17)	(0.14)	(0.23)
Domestic worker	11.7	16.4	10.0	15.0
	(0.11)	(0.17)	(0.13)	(0.23)
Employer	1.5	2.6	1.8	3.4
	(0.04)	(0.08)	(0.06)	(0.11)
Military/Public	4.1	7.4	4.0	7.9
	(0.06)	(0.12)	(0.09)	(0.17)
Unemployment	6.4	4.6	5.5	3.6
	(0.08)	(0.10)	(0.10)	(0.12)
Non-participation	31.6	0	35.9	0
	(0.16)	–	(0.21)	–
Total	100	100	100	100
Participate all periods (%)	51.7	100	47.4	100
	(0.17)	–	(0.23)	–
Never participates (%)	17.5	0	21.0	0
	(0.14)	–	(0.18)	–
Sample size	88,658	45,852	52,707	24,960

Notes: All moments are computed based on the 1st interview. Bootstrap standard errors in parenthesis based on 500 draws.

Table 1.4: Employment composition - College educated

	Full sample	Always part.	Married	Married Always part.
	(1)	(2)	(3)	(4)
Type of work (%)				
Formal	34.1	38.9	30.9	36.0
	(0.26)	(0.29)	(0.35)	(0.41)
Informal	10.2	10.7	9.3	9.9
	(0.17)	(0.19)	(0.23)	(0.26)
Self-employment	8.9	9.0	8.7	8.8
	(0.16)	(0.18)	(0.21)	(0.23)
Domestic worker	0.5	0.5	0.4	0.4
	(0.04)	(0.05)	(0.05)	(0.05)
Employer	4.1	4.7	5.3	6.2
	(0.11)	(0.13)	(0.17)	(0.21)
Military/Public	28.4	33.3	30.6	36.7
	(0.26)	(0.30)	(0.34)	(0.42)
Unemployment	4.5	2.9	3.5	2.0
	(0.12)	(0.11)	(0.14)	(0.12)
Non-participation	9.3	–	11.4	–
	(0.17)	–	(0.24)	–
Total	100	100	100	100
Participate all periods (%)	82.4	100	80.1	100
	(0.22)	–	(0.31)	–
Never participates (%)	3.7	0	5.3	0
	(0.10)	–	(0.16)	–
Sample size	30,842	25,400	17,298	13,859

Notes: All moments are computed based on the 1st interview. Bootstrap standard errors in parenthesis based on 500 draws.

Table 1.5: Distribution of weekly hours by type of work

	Formal	Informal	Self-emp.	Domestic	Employer	Public
Usual weekly hours worked			(A) Full Sample			
10-34	9.1	37.3	46.2	44.0	13.1	30.7
	(0.16)	(0.53)	(0.46)	(0.51)	(0.66)	(0.41)
35-48	84.9	55.7	41.7	50.7	63.1	64.1
	(0.20)	(0.53)	(0.43)	(0.51)	(1.00)	(0.47)
49-60	6.0	7.0	12.0	5.3	23.8	5.3
	(0.13)	(0.26)	(0.28)	(0.28)	(0.85)	(0.20)
			(B) Women without children under 11			
10-34	8.2	36.3	41.6	39.6	9.6	29.6
	(0.22)	(0.73)	(0.64)	(0.73)	(0.90)	(0.59)
35-48	85.6	56.3	45.5	53.9	64.9	64.6
	(0.27)	(0.74)	(0.67)	(0.73)	(1.52)	(0.65)
49-60	6.2	7.4	12.9	6.5	25.5	5.9
	(0.17)	(0.38)	(0.46)	(0.37)	(1.35)	(0.32)
			(C) Women with children under 11			
10-34	10.0	38.3	49.9	48.2	15.7	31.7
	(0.23)	(0.71)	(0.62)	(0.71)	(1.00)	(0.63)
35-48	84.2	55.1	38.7	47.6	61.8	63.6
	(0.28)	(0.73)	(0.56)	(0.71)	(1.26)	(0.62)
49-60	5.7	6.6	11.4	4.2	22.6	4.7
	(0.17)	(0.37)	(0.36)	(0.30)	(1.11)	(0.28)

Notes: All moments are computed based on the 1st interview. Bootstrap standard errors in parenthesis based on 500 draws.

Table 1.6: Formal and informal work statistics

	Formal	Informal
Average monthly income by education (R$)		
High school or less	1285	1001
College	3034	2704
Transition rates into non-participation (%)		
Quarterly	1.5	9.2
Yearly	5.3	13.2
Transition rates into unemployment (%)		
Quarterly	2.9	3.4
Yearly	3.2	4.3
Share of workers that contribute to the social security system (%)		
High school or less	100	34.2
College	100	66.3

Notes: Formal category include tenured public employees. Informal category includes domestic workers, self-employed and employers. Average monthly income is expressed in R$ of August 2017, and exclude people working less than 10 or more than 60 weekly hours, and outliers (incomes under R$100 and over R$30000)

Table 1.7: Static models – High school or less

				Continuously married		
	Full sample		Always part.	Full sample		Always part.
	(1)		(2)	(3)		(4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.529***	-0.068***	0.002	-0.572***	-0.058***	0.017
	(0.006)	(0.008)	(0.010)	(0.007)	(0.010)	(0.012)
# Children 3 – 5	-0.260***	-0.152***	-0.101***	-0.290***	-0.153***	-0.102***
	(0.005)	(0.007)	(0.008)	(0.006)	(0.008)	(0.010)
# Children 6 – 11	-0.125***	-0.141***	-0.128***	-0.141***	-0.144***	-0.130***
	(0.003)	(0.004)	(0.005)	(0.004)	(0.005)	(0.006)
# Children 12 – 17	-0.002	-0.061***	-0.059***	-0.021***	-0.077***	-0.074***
	(0.003)	(0.004)	(0.005)	(0.004)	(0.005)	(0.006)
Married / cohab.	-0.505***	-0.068***	0.025*			
	(0.008)	(0.009)	(0.010)			
Partner college educated				-0.069***	-0.039**	-0.026
				(0.011)	(0.013)	(0.015)
Log partner's earnings	-0.086***	0.038***	-0.001	-0.147***	0.062***	0.007
	(0.005)	(0.007)	(0.008)	(0.007)	(0.010)	(0.011)
Partner unemployed	0.178***	-0.035*	-0.031	0.120***	-0.011	-0.038
	(0.013)	(0.016)	(0.019)	(0.017)	(0.021)	(0.024)
Average Log-likelihood	-5.016		-3.210	-5.074		-3.247
Sample size	83,481		41,132	49,690		22,240

Notes: All specifications include as controls: a quadratic in age, race, completed secondary education, state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses. (*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)

Table 1.8: Static models – College educated

	Full sample (1)		Always part. (2)	Continuously married: Full sample (3)		Continuously married: Always part. (4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.518***	-0.033*	-0.010	-0.522***	-0.011	0.010
	(0.013)	(0.013)	(0.015)	(0.015)	(0.015)	(0.016)
# Children 3 – 5	-0.288***	-0.137***	-0.125***	-0.304***	-0.134***	-0.115***
	(0.013)	(0.013)	(0.014)	(0.015)	(0.015)	(0.016)
# Children 6 – 11	-0.153***	-0.097***	-0.090***	-0.181***	-0.122***	-0.117***
	(0.010)	(0.009)	(0.010)	(0.012)	(0.011)	(0.012)
# Children 12 – 17	-0.093***	-0.109***	-0.100***	-0.110***	-0.110***	-0.088***
	(0.011)	(0.010)	(0.011)	(0.013)	(0.013)	(0.014)
Married / cohab.	-0.184***	0.178***	0.228***			
	(0.022)	(0.018)	(0.019)			
Partner college educated				0.007	-0.010	-0.020
				(0.015)	(0.014)	(0.015)
Log partner's earnings	-0.172***	-0.138***	-0.150***	-0.191***	-0.158***	-0.169***
	(0.009)	(0.009)	(0.010)	(0.012)	(0.011)	(0.012)
Partner unemployed	0.198***	-0.136***	-0.133***	0.141*	-0.215***	-0.231***
	(0.051)	(0.037)	(0.041)	(0.062)	(0.047)	(0.051)
Average Log-likelihood	-4.733		-3.305	-4.955		-3.319
Sample size	19,524		14,727	10,591		7,550

Notes: All specifications include as controls: a quadratic in age, race, state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses. (*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)

Table 1.9: Estimation results – High school or less

				Continuously married		
	Model I		Model II	Model I		Model II
	(1)		(2)	(3)		(4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.225***	-0.117**	-0.143**	-0.259***	-0.200***	-0.246***
	(0.025)	(0.041)	(0.051)	(0.031)	(0.053)	(0.069)
# Children 3 – 5	-0.096***	-0.107**	-0.126**	-0.126***	-0.163***	-0.223***
	(0.024)	(0.037)	(0.046)	(0.031)	(0.051)	(0.066)
# Children 6 – 11	-0.033	-0.096***	-0.097**	-0.020	-0.126**	-0.168**
	(0.020)	(0.030)	(0.036)	(0.026)	(0.042)	(0.054)
# Children 12 – 17	0.018	-0.035	-0.032	0.014	-0.071*	-0.070
	(0.017)	(0.024)	(0.029)	(0.022)	(0.034)	(0.043)
Married / cohab.	-0.788***	-0.168***	-0.038			
	(0.021)	(0.029)	(0.035)			
Partner college educated				0.499***	-0.157**	-0.146*
				(0.020)	(0.051)	(0.062)
Log partner's earnings	-0.079***	0.074***	0.049	-0.180***	0.135***	0.078*
	(0.013)	(0.021)	(0.025)	(0.019)	(0.029)	(0.036)
Partner unemployed	0.301***	-0.092*	-0.134**	0.187***	-0.022	-0.131
	(0.024)	(0.039)	(0.049)	(0.033)	(0.053)	(0.067)
Lagged participation (p_{t-1})	0.683***	0.445***		0.710***	0.416***	
	(0.013)	(0.030)		(0.017)	(0.039)	
Lagged formal job (f_{t-1})	0.774***	1.507***	1.653***	0.783***	1.574***	1.764***
	(0.023)	(0.021)	(0.035)	(0.028)	(0.029)	(0.050)
RE Variance	2.025***	3.069***	3.164***	2.146***	3.231***	3.452***
	(0.042)	(0.121)	(0.148)	(0.057)	(0.170)	(0.232)
RE Correlation	0.088***			0.087***		
	(0.014)			(0.017)		
Measurement error rate	0.018***		0.025***	0.018***		0.030***
	(0.001)		(0.002)	(0.001)		(0.003)
Average Log-likelihood	-3.052		-1.578	-3.041		-1.537
Wald statistics						
H_0: δ CRE = 0	507.93 (0.000)			370.92 (0.000)		
H_0: ζ CRE = 0	40.43 (0.000)		26.22 (0.000)	19.28 (0.001)		17.46 (0.002)
H_0: All CRE = 0	560.05 (0.000)			394.96 (0.000)		
Sample size	83,481		41,132	49,690		22,240

Notes: All specifications include as controls: a quadratic in age, race, completed secondary education, state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses except for p-values for Wald statistics. Tenured public employees excluded. (*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)

Table 1.10: Estimation results – College educated

	Model I (1)		Model II (2)	Continuously married Model I (3)		Continuously married Model II (4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.308***	-0.047	-0.090	-0.349***	0.035	-0.005
	(0.063)	(0.071)	(0.079)	(0.073)	(0.081)	(0.092)
# Children 3 – 5	-0.235***	-0.076	-0.044	-0.250**	-0.026	0.012
	(0.072)	(0.076)	(0.084)	(0.085)	(0.091)	(0.102)
# Children 6 – 11	-0.269***	-0.111	-0.085	-0.289***	-0.002	0.008
	(0.072)	(0.070)	(0.078)	(0.089)	(0.092)	(0.103)
# Children 12 – 17	-0.231***	-0.040	-0.004	-0.240**	0.062	0.109
	(0.068)	(0.066)	(0.073)	(0.083)	(0.086)	(0.100)
Married / cohab.	-0.464***	0.150*	0.258***			
	(0.061)	(0.065)	(0.071)			
Partner college educated				-0.030	-0.139*	-0.201***
				(0.051)	(0.061)	(0.063)
Log partner's earnings	-0.117***	-0.170***	-0.169***	-0.158***	-0.181***	-0.173***
	(0.024)	(0.029)	(0.031)	(0.033)	(0.038)	(0.040)
Partner unemployed	0.547***	-0.196*	-0.188	0.405**	-0.266*	-0.296*
	(0.107)	(0.090)	(0.099)	(0.141)	(0.115)	(0.131)
Lagged participation (p_{t-1})	0.787***	0.499***		0.868***	0.605***	
	(0.037)	(0.077)		(0.049)	(0.107)	
Lagged formal job (f_{t-1})	0.507***	1.089***	1.273***	0.591***	1.154***	1.292***
	(0.054)	(0.031)	(0.060)	(0.072)	(0.046)	(0.084)
RE Variance	1.906***	4.287***	3.694***	2.238***	4.701***	4.396***
	(0.095)	(0.215)	(0.223)	(0.150)	(0.361)	(0.376)
RE Correlation	0.292***			0.242***		
	(0.031)			(0.041)		
Measurement error rate	0.016***		0.022***	0.017***		0.020**
	(0.002)		(0.006)	(0.003)		(0.007)
Average Log-likelihood	-2.809		-1.767	-2.817		-1.685
Wald statistics						
H_0: δ CRE = 0	73.33 (0.000)			49.88 (0.000)		
H_0: ζ CRE = 0	18.60 (0.001)		15.09 (0.005)	22.13 (0.000)		12.99 (0.011)
H_0: All CRE = 0	92.53 (0.000)			72.14 (0.000)		
Sample size	19,524		14,727	10,591		7,550

Notes: All specifications include as controls: a quadratic in age, race, state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses except for p-values for Wald statistics. Tenured public employees excluded.
(*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)

Table 1.11: Counterfactual exercises

	Full sample (1)		Always part. (2)	Continuously married Full sample (3)		Continuously married Always part. (4)
	Participation	Formal	Formal	Participation	Formal	Formal
Observed	66.6 (0.2)	39.2 (0.2)	46.3 (0.2)	62.1 (0.2)	39.3 (0.3)	46.9 (0.3)
Simulated	66.3	38.2	45.8	61.7	38.3	46.0
No CRE	66.6	41.3	47.8	62.0	41.0	47.2
No direct effect	66.5	39.8	47.6	62.0	41.1	49.5
Neither	66.8	43.1	49.6	62.2	44.0	50.6
Men		50.3 (0.2)	52.4 (0.2)			

Notes: Simulations based on 500 randomly drawn sequences of the unobservables. Frequencies computed at the last observed period. Bootstrap standard errors in parentheses for observed frequencies.

Table 1.12: Simulation results

	$(\varrho_p, \varrho_f, \psi)$	Mean relative bias	
		Model I	Model II
Baseline	(0.0 , 0.0 , 0.0)	-0.021 (0.019)	0.053 (0.025)
Static		-0.376 (0.012)	-0.445 (0.015)
1:	(0.9 , 0.9 , 0.4)	0.037 (0.031)	0.143 (0.030)
2:	(0.9 , 0.9 , -0.4)	0.112 (0.032)	-0.004 (0.027)
3:	(0.9 , 0.6 , 0.4)	-0.405 (0.026)	-0.210 (0.028)
4:	(0.9 , 0.6 , -0.4)	-0.189 (0.020)	-0.159 (0.019)
5:	(0.9 , -0.25 , 0.4)	0.094 (0.019)	0.243 (0.020)
6:	(0.9 , -0.25 , -0.4)	0.036 (0.024)	0.078 (0.024)
7:	(0.6 , 0.9 , 0.4)	0.241 (0.037)	0.090 (0.033)
8:	(0.6 , 0.9 , -0.4)	0.512 (0.038)	0.247 (0.036)
9:	(0.6 , 0.6 , 0.4)	-0.245 (0.023)	0.004 (0.023)
10:	(0.6 , 0.6 , -0.4)	-0.019 (0.022)	-0.115 (0.023)
11:	(0.6 , -0.25 , 0.4)	0.107 (0.025)	0.270 (0.026)
12:	(0.6 , -0.25 , -0.4)	0.195 (0.020)	0.156 (0.023)
13:	(-0.25 , 0.9 , 0.4)	0.290 (0.043)	0.025 (0.039)
14:	(-0.25 , 0.9 , -0.4)	0.547 (0.034)	0.394 (0.035)
15:	(-0.25 , 0.6 , 0.4)	-0.256 (0.022)	-0.163 (0.025)
16:	(-0.25 , 0.6 , -0.4)	0.114 (0.022)	0.116 (0.028)
17:	(-0.25 , -0.25 , 0.4)	-0.046 (0.019)	0.040 (0.024)
18:	(-0.25 , -0.25 , -0.4)	0.287 (0.021)	0.335 (0.026)
	Aggregate:		
	Mean	0.078	0.082
	95th percentile	0.723	0.626
	99th percentile	1.106	0.903

Notes: True value of the coefficient = -0.2. ϱ_p and ϱ_f are the degree of serial correlation in the transitory components of the errors and ψ is the correlation coefficient of the innovations. The number of observations is 10000. Statistics computed based on 100 replications for each combination of parameters. Jackknife standard errors in parentheses. Aggregate statistics do not include baseline simulations.

2
Labor market rigidity, female life-cycle labor supply and fertility

2.1
Introduction

In almost all developing countries, informal employment[1] is the norm rather than the exception, according to the 2018 report "Women and men in the informal economy: a statistical picture", by the International Labour Organization. In this report, tackling informality throughout the world is presented as an urgent need, due to the fact that the informal sector is characterized by higher rates of poverty, lower productivity, lower incomes, and an overall socioeconomic vulnerability of its workers. Moreover, in over half of the countries analyzed, women are over-represented in the urban/non-agricultural informal sector and, in contrast to men, women in the informal sector "are more often found in more vulnerable situations, for instance as domestic workers, home based-workers or contributing family workers". Neri (2002), in an extensive analysis of the informal sector in Brazil, also highlights the strong correlation between labor market informality and the poor socioeconomic background and outcomes of its workers.

In the previous chapter, I have shown that among women of low educational achievement in Brazil, informality is partly linked to the presence of children in the household. Household survey data on the allocation of women between part-time and full-time work in the formal and informal sectors suggest that this may be due to inflexibility of labor supply and insufficient demand for part-time work in the former. However, the reduced-form methodology adopted there does not allow for a more thorough analysis of the consequences of this relationship between fertility and labor market informality.

In this chapter, then, I develop a structural life-cycle model that endogenizes fertility, labor force participation and full/part-time work decisions in three distinct sectors of the labor market: formal, informal and self-employment in a forward-looking framework. The structural approach allows me to analyze

[1]Informality is defined in general terms as employment relationships that are unregistered with labor regulation authorities and, as such, do not comply with labor legislation and in which there is no collection of social security contributions.

the impact that self-selection into informality due to the presence of children in the household and the scarce demand for part-time work in the formal sector has on human capital accumulation and lifetime labor income. In the model, the presence of a child in the household carries utility costs that depend on how young the child is and which can vary depending on the nature and intensity of work of the mother. This is meant to capture differences in the average level of labor supply flexibility of formal/informal jobs in a broad sense. Moreover, labor income in each sector is modelled as a function of labor market experience, which evolves depending on the intensity of labor supply and the duration of career interruptions, so that children affect present and future income. Furthermore, I assume that entry into formal part-time work is restricted due to insufficient demand by the firm side of the labor market, motivated by the low prevalence of this category observed in the data.

Considering the result in the previous chapter that children do not affect the rates of informality among women with college education in Brazil, the model is estimated using household survey data on Brazilian women with at most completed secondary education. The estimated model is able to match well the main features of the data. The results show that the presence of young children in the household reduce the utility associated with working in the formal sector, relative to the utility derived from working in the informal sector or being self-employed. Furthermore, having young children also affects relative preferences between full and part-time work. This effect exacerbates the self-selection of mothers of young children into informality, given that there is little demand for part-time work in the formal sector.

I then use the estimated model to perform four counterfactual exercises that show the extent to which stimulating flexibility in the formal sector can be a policy tool to improve the labor market outcomes of women.

The first two scenarios illustrate how maternity leave is actually not very effective in preventing women from dropping out of the labor market following childbirth but, on the other hand, is very effective in preventing working mothers from dropping out of the formal sector, particularly those working full-time jobs. Nevertheless, the overall value of maternity leave in terms of lifetime labor income and human capital accumulation is found to be small.

Next, I simulate a version of the model in which fertility is completely shut off. This scenario illustrates the extent to which fertility influences the allocation of women in the labor market across the life-cycle. The results show that children impose a heavy burden in terms of lifetime labor income, 3.5% and 9.2% for high school graduates and dropouts, respectively, which is partly due to the fact that mothers switch to the lower paying informal sector to be able to work part-time. Besides earning lower incomes on average in the informal sector, these women also experience a higher number of career interruptions, which subsequently hurt future earnings potential. This exercise also shows that women compensate later in life for missed human capital accumulation in the period of active fertility. Before age 45, child-care responsibilities increase non-participation, unemployment, informal and self-employment and part-time work. Full-time work in the formal sector, on the other hand, would be up to 2 percentage points higher in the mid-20s for women with high school education and up to 5 percentage points higher for women without.

Lastly, I perform a counterfactual designed to approximate the effects of implementing a policy that increases demand for part-time work in the formal sector. The aggregate impact of such a policy on lifetime labor income is found to be positive, but small at the level of increased demand for this type of work that is assumed. I find that if the probability of receiving a formal part-time job offer was twice its estimated value, lifetime labor income would increase by just under 1%. Moreover, the rates of formal work are increased throughout the life-cycle, but particularly so for mothers of young children. This result illustrates how the implementation of measures that facilitate demand of part-time work in the formal sector can be a way to improve the labor market outcomes of women and to reduce the burden that children impose on female labor supply.

The remainder of the chapter is structured as follows. Section 2 presents the specification of the empirical model. Section 3 describes the estimation methodology and discusses identification of the parameters of the empirical model under the structure assumed. Section 4 describes the data used to estimate the model. In Section 5, I discuss the fit of the model to the data and the parameter estimates obtained. Section 6 describes the results of the counterfactual exercises and section 7 concludes.

2.2
Related literature

This paper contributes to a literature that studies the role that labor supply flexibility has on preventing and/or shortening fertility-related career interruptions and reducing the motherhood penalty on labor market outcomes of women. For instance, papers such as Goldin and Katz (2008, 2011) and Herr and Wolfram (2012) use data on high-skilled women in the U.S. to show that women in occupations with greater flexibility are less likely to interrupt their careers and have briefer interruptions if they do so. Goldin (2014) shows that, in the U.S., occupations with greater degree of labor supply flexibility are also the ones with the smallest gender wage gaps. From a different perspective, the early seminal work by Polachek (1981) argues that intermittent labor force participation behavior by women is an important factor in explaining differences in occupational distribution by gender. Adda et al. (2017) estimate a life-cycle model of fertility and occupational choice using German administrative data and show that occupational switching after having children contribute to the overall career costs of children. Moreover, they show that women self-select early on to careers with greater labor supply flexibility based on predicted/desired future fertility and underlying preferences for having children.

My main contribution is in showing that, in a developing country where jobs in the formal sector provide, on average, less flexibility, demand for this amenity following the birth of a child increases labor market informality among women. On one hand, informal employment works as a buffer for some women to avoid dropping out of the labor market but, on the other, informality is also characterized by lower incomes and higher transition rates into unemployment and non-participation. My results also suggest that, in developing countries where a large fraction of the labor market can already be said to be relatively flexible in terms of labor supply, stimulating flexibility in the formal sector might not have strong effects on overall labor force participation, but might be very effective in improving informality rates among women.

This work is also related to a number of papers that model fertility and female labor supply jointly in a life-cycle framework, such as Hotz and Miller (1988), Francesconi (2002), Sheran (2007) , Keane and Wolpin (2010), Edwards (2014), Lim (2015) and Adda et al. (2017). My paper is the first to adopt a similar methodology as the ones above to study the relationship between fertility and labor market informality. In this way, this work is also related to papers on informality such as by García (2015), Ulyssea (2010), Meghir et al. (2015) and Narita (2017). García (2015) estimates a life-cycle model of educational decisions and choice of working in the formal or informal sector for men, using Chilean data. He shows that non-wage characteristics of formal jobs are, on average, valued more by workers compared to non-wage characteristics of informal jobs, and this difference is an important factor in explaining the allocation of workers between formal and informal sectors. My work shows that, for women, preferences for these non-wage characteristics depend strongly on the presence of young children. The work by Meghir et al. (2015) estimates a equilibrium search model of firm and worker informality using Brazilian data and shows that increasing enforcement would have small effects on employment and reduce worker informality, increasing welfare. In contrast, Ulyssea (2010) calibrates a model of the same nature to match a set of descriptive statistics of the Brazilian labor market and arrives at the opposite conclusion that increasing enforcement against informal firms would increase unemployment and decrease welfare. My results suggest that even if such an increase in enforcement would be welfare-improving, it would likely be detrimental to labor force attachment of mothers of young children.

Lastly, this chapter contributes to a literature that analyzes the impact of maternity leave policies on female labor market outcomes. More specifically to the case of Brazil, this paper is related to the works by Carvalho, Firpo and Gonzaga (2006) and Machado and Pinho Neto (2016) which find that maternity leave policy is not effective in increasing participation in the long-term, a result that is in line with the diminute effects on the life-cycle profile of female labor force participation found in counterfactual simulations that vary the length of ML using the model estimated here.

2.3 The empirical model

In this section, I describe the dynamic discrete choice model of labor supply and fertility decisions.

2.3.1 Setup

The agent is a woman and the decision period begins at age 18. Time is discrete and each period lasts one year. I model decisions up to age 59[2], at which point the decision is static. I adopt a partial equilibrium framework, in which the demand side of the labor market is assumed to be exogenous and agents behave independently.

In each period, a woman decides whether or not to participate in the labor market and, in case she does, whether to work full-time or part-time in one of three work options: formal, informal or self-employment, or to remain unemployed. Moreover, at each age up to 44, she also decides whether to have a child, in which case the child is born in the following period. From age 45 on, fertility is shut off and decisions are made only with respect to labor supply.

[2]This is motivated by the fact that, in Brazil, for most of the sample period, a female worker that has contributed to the social security system for a minimum of 15 years is eligible for retirement at age 60, whereas below this age the worker is only eligible if she has a minimum of 30 years of contributions.

Preferences are defined over consumption (c), number of children ever had (n), age of the youngest child (age^K), marriage/cohabitation status (m), current and past labor market choice (j) and conception choice (b). Let flow utility be denoted by u. At any period, a woman maximizes the expected present value of remaining lifetime utility:

$$U_\tau = E\left[\sum_{t=\tau}^{T} \delta^{t-\tau} u(c_t, n_t, age_t^K, m_t, j_t, j_{t-1}, b_t, \Upsilon_t, \varepsilon_t) \,\middle|\, \Omega_t \right] \tag{2-1}$$

with respect to the labor market choice $j = \{0, 1, 2, ..., 7\}$ and also the conception choice $b = \{0, 1\}$ if she is under 45. Labor market choices are indexed as: (0) out of labor force, (1) unemployment, (2) formal full-time, (3) formal part-time, (4) informal full-time, (5) informal part-time, (6) self-employment full-time and (7) self-employment part-time. Ω_t is the vector of observed state variables, ε_t is the vector of shocks to labor and non-labor income and Υ_t is a vector of choice-specific preference shocks. Finally, δ is the discount factor. The expectation is taken with respect to the distribution of the Υ and ε shocks and the stochastic evolution of the state space Ω.

The observed state space vector $\Omega_t = [x_t, n_t, age_t^K, m_t, j_{t-1}]$ contains labor market experience x, in addition to fertility, marriage and past labor supply. Number of children and age of the youngest child are discrete states that evolve according to the conception choice, while labor market experience evolves according to labor supply decisions. Marital/cohabiting status is assumed to be exogenous and to evolve stochastically depending solely on a woman's age. I allow for five discrete states: single or married with a husband with/without college education who is employed/unemployed.

2.3.2 Earnings and consumption

I assume that there is no savings or borrowing.[3] Moreover, without data on consumption or expenditures, it is not possible to separately identify the monetary costs of child-rearing from their direct utility effects. In this way, I do not explicitly model expenditures and thus consumption differs from aggregate household income only due to a consumption equivalence scale, as shown below:

$$c_t = \rho_t(y_t + y_t^H + UI_t) \tag{2-2}$$

where y_t is labor income of the woman, y^H is the husband's income, UI is unemployment insurance and ρ is the consumption equivalence scale which

[3]This is a standard assumption in life-cycle labor supply models that are estimated without consumption/savings data. Considering that the model will be estimated only using data for women without college education, this assumption is less problematic.

depends on whether the woman is married or not. Labor income is determined by labor market experience x, and type of employment.

$$\log(y_t) = \beta_0^k + \beta_1^k \log(1 + x_t) - \beta_2^k p_t + \sigma^k \varepsilon_t^k \tag{2-3}$$

where $k = \{f, i, s\}$ indexes formal, informal and self-employment, respectively, $p = \{0, 1\}$ is an indicator for part-time work and ε^k is an income shock. This functional specification is employed, as opposed to a standard Mincer-form, to ensure monotonicity and concavity of earnings with respect to labor market experience and also to reduce the number of parameters to need to be estimated. y^H is zero if the woman is single or if the husband is unemployed. Otherwise, husband's earnings are determined as a function of the wife's age[4] and his education level:

$$\log(y_t^H) = \nu_0 + \nu_1 \log(1 + t) + \nu_2 hc_t + \sigma^H \varepsilon_t^H \tag{2-4}$$

where hc is an indicator for husband with college degree.

2.3.3 Within-period timing of decisions

The timing of the decision process is as follows. In the beginning of the period the woman observes whether she enters or exits a relationship and whether her partner is unemployed. In addition, she also observes two preference shocks, one specific to the "out of labor force" state and another common to all labor force participation states. She then proceeds to decide whether to participate in the labor market based on the expected value of the remaining preference and income shocks. Next, if she decides to stay out of the labor force, she observes the conception and non-conception preference shocks and decides whether to have a child be born in the beginning of the following period. In case she participates, I assume that formal part-time work is only available with certainty in case the woman was already working in such a job in the previous period. Otherwise, this category is only available with a constant and exogenous probability π. This parameter can be interpreted as a probability of receiving an offer for a formal part-time job and it is arguably the parameter that would be directly affected by policies that flexibilize labor demand in the formal sector.[5] She then observes the two conception shocks

[4] Modelling the husband's earnings process as exogenous and as a function of wife's characteristics is a common approach in the literature.

[5] This modelling choice is reminiscent of "consideration sets" models of consumer behavior, in which the available options for a consumer are not deterministic, such as in Eliaz and Spiegler (2011). However, in this literature this feature is included in order to relax the underlying hypothesis of perfect consumer rationality and complete information. In my case, the motivation is to capture limited demand for formal part-time work on behalf of firms.

and the remaining six or seven labor supply preference shocks, each specific to one of the available labor supply choices and proceeds to make a decision on both dimensions. The limited availability of formal part-time work is the only frictional aspect of the labor market that is modelled. In this way, preference shocks are also intended to capture shocks such as involuntary layoffs. After both decisions have been made, she observes shocks to her income and to her husband's earnings in case he is employed.[6] After the realization of decisions and outcomes, the remaining state variables are updated and the woman enters the subsequent decision period.

2.3.4 State space dynamics

The probabilities of entering and exiting a marital/cohabiting relationship are given, respectively, by:

$$\Pr\left(m_t \neq 0 \mid m_{t-1} = 0\right) = \left[1 + \exp\left(\zeta_0 + \zeta_1 t + \zeta_2 t^2\right)\right]^{-1} \tag{2-5}$$

$$\Pr\left(m_t = 0 \mid m_{t-1} \neq 0\right) = \xi \tag{2-6}$$

with $m_t = 0$ representing singlehood. When a woman enters a relationship, her partner's education is determined according to an exogenous constant probability ζ_3. Moreover, every period her husband may be hit with an unemployment shock, also according to an exogenous probability μ_h or μ_l for husbands with or without college education, respectively. There are no direct transitions between partner's educational level, so this feature remains fixed for the duration of a relationship.

The evolution of the number of children and age of the youngest child is straightforward:

$$n_{t+1} = n_t + b_t \tag{2-7}$$

$$age^K_{t+1} = (age^K_t + 1)(1 - b_t) \tag{2-8}$$

An important piece of the model is the evolution of labor market experience. Given the lack of data on full work histories for each woman, I impose a rather stylized structure for this process. I assume that experience is not specific to each type of employment and that one year of full-time work increases experience by one unit, while part-time work accumulates at exactly half the

[6]There is a reasonable alternative within-period timing structure in which the conception preference shocks are realized simultaneously with the participation preference shocks. In this case, the subsequent decision between different types of employment would be made with the conception decision already fixed. In the future, I will test whether this modelling decision is relevant for the fit of the model and the associated counterfactuals.

rate of full-time work. Moreover, I assume that experience depreciates if a woman does not work for two consecutive periods. The rate of depreciation is assumed to be the same as the rate of accumulation of part-time work. In this way, short spells (1 period) out of work preserve experience, but after two consecutive years out of work, each additional year erases the equivalent of half a year of full-time work experience. This structure is convenient because it preserves the number of reachable state space points while also introducing a penalty in terms of future earnings potential for long spells out of work.

$$x_{t+1} = \begin{cases} x_t + 1 & \text{if working full-time} \\ x_t + 0.5 & \text{if working part-time} \\ x_t - 0.5 & \text{if not working for two consecutive periods} \\ x_t & \text{otherwise} \end{cases} \tag{2-9}$$

2.3.5 Flow utilities

The per-period flow of utility is specified below. With some abuse of notation, let m_t be an indicator variable for being married/cohabiting. The utility of staying out of the labor force is:

$$\begin{aligned} u_t^0 = {} & c_t + \alpha_1 n_t + \alpha_2 n_t^2 + \alpha_3 c_t n_t + \gamma_0 I_{[j_{t-1}=0]} + \gamma_8 I_{[j_{t-1}=1]} + \\ & (\eta_1 + \eta_2 m_t + \upsilon_{1t}^b) b_t + \upsilon_{0t}^b (1 - b_t) + \phi_{0t} \end{aligned} \tag{2-10}$$

where γ are first order state dependence parameters included to allow the model to match the observed transition rates between labor market states. The η parameters are the intercepts that control the probability of conceiving a child in a given period, depending on marriage status, so that married women can have a higher probability of having a child, as observed in the data. υ^b are the conception preference shocks and ϕ_0 is the preference shock for non-participation. Similarly, the flow utility of being unemployed is:

$$\begin{aligned} u_t^1 = {} & \psi_0 + \psi_1 m_t + c_t + \alpha_1 n_t + \alpha_2 n_t^2 + \alpha_3 c_t n_t + \gamma_1 I_{[j_{t-1}=1]} + \\ & (\eta_1 + \eta_2 m_t + \upsilon_{1t}^b) b_t + \upsilon_{0t}^b (1 - b_t) + \phi_{1t} + \upsilon_{1t} \end{aligned} \tag{2-11}$$

where the ψ parameters are the intercepts that control the probability of labor force participation, also dependent on marriage status. υ_1 is the preference shock for staying unemployed in the labor supply decision conditional on participation.

For the remaining employment options, utility also depends on the age of the youngest child. Moreover, let f_t, i_t, s_t be indicator variables for formal, informal and self-employment.

$$\begin{aligned} u_t^j = {} & \psi_0 + \psi_1 m_t + \psi_j + (1+\rho_k)c_t + \alpha_1 n_t + \alpha_2 n_t^2 + \alpha_3 c_t n_t + \\ & (\theta_0 c_t + \theta_1 m_t + \theta_j) g(age_t^K) I_{[n_t>0]} + \\ & \gamma_j I_{[j_{t-1}=j]} + \gamma_9 I_{[j_{t-1}>1]} + \gamma_{10} f_t f_{t-1} + \gamma_{11} i_t i_{t-1} + \gamma_{12} s_t s_{t-1} + \\ & (\eta_1 + \eta_2 m_t + \upsilon_{1t}^b) b_t + \upsilon_{0t}^b (1-b_t) + \phi_{1t} + \upsilon_{jt} \end{aligned} \tag{2-12}$$

The ψ_j parameters control the probability of choosing each of the employment options. In order to allow for non-labor income and marriage status to influence the decision between formal, informal and self-employment, I let the marginal utility of consumption to depend on the type of work via the ρ_k parameters. Finally, I allow age of the youngest child to affect utility non-linearly through the function $g(\cdot)$. I choose the following functional form for g:

$$g(age_t^K) = \frac{12 - age_t^K}{12 + \kappa \, age_t^K} \tag{2-13}$$

which is motivated by the finding in the previous chapter that the presence of children older than 11 in the household does not appear to influence labor supply. This functional form gives $g(0) = 1$ and $g(12) = 0$, with κ controlling how fast the effect declines with age of the youngest child. If $\kappa = 0$ the effect decays linearly and if $\kappa > 0$ the effect is convex. By bounding the effect between zero and one, the θ_j parameters control the relative importance of the effect of young children on each employment category.

2.3.6 Distributional assumptions

I assume that the income shocks ε are distributed i.i.d. standard normal and the preference shocks are distributed i.i.d. extreme value type I. Note that the model above relaxes independence of irrelevant alternatives by allowing the utility of participation/non-participation and conception/non-conception decisions to be correlated. Also note that the scale of utility is fixed by the scale of consumption, which is expressed in monetary units. More specifically, I express consumption in R$12000 units and, thus, all parameters are expressed in this same scale. In this way, the variances of the preference shocks υ^b, υ, ϕ also need to be estimated. The assumption that income shocks are only realized after decisions have been made and that preference shocks are distributed extreme value type I means that the probabilities of choosing each option can be calculated analytically, which greatly simplifies the solution of the model.

2.3.7 Permanent unobserved heterogeneity

Lastly, I allow for permanent unobserved heterogeneity in preferences in the form of three discrete mass points, as in Heckman and Singer (1984). One of the types is allowed to have lower preference for labor force participation, with a different ψ_0 parameter. The second type is allowed to have lower marginal utility from children with a different α_1 parameter. The inclusion of these unobserved types is important to match the observed frequencies of women that never work and that never have children, by age.

This concludes the empirical specification of the model, which has 70 parameters that need to be estimated.

2.4 Estimation

In this section I describe the estimation procedure for the empirical model specified above. In order to achieve a better overall fit to the data and to reduce the dimensionality of the estimation problem, I estimate the model separately for women with completed high school education and women with less than high school education. I conduct estimation in two steps.

2.4.1 Parameters estimated outside the model

In the first step, I calibrate/estimate the parts of the model that are assumed to be exogenous, namely, the marriage and divorce probabilities and the husband's earnings process, in order to reduce the computational burden of estimation. The latter is estimated using OLS regressions where the dependent variable is the logarithm of the deflated sum of gross earnings from all of the husband's jobs on the logarithm of the wife's age (to ensure monotonicity), and a dummy for the husband having a college degree. Moreover, I fix the probabilities of husband's unemployment to the respective observed frequencies of being either unemployed or out of the labor force among husbands with or without college education.

The parameters of the marriage and divorce probabilities are calibrated through simulation in order to match some features of the data. To identify the probability of entering marriage/cohabitation as a function of age and the separation probability I compute the frequencies of marriage/cohabitation by age and educational level using data from the PNAD. Furthermore, from 2011 on, the PNAD provides information on whether a woman that is not married/cohabiting has ever lived with a partner or not. This information is crucial because using just the cross-sectional distribution of marriage/cohabitation to identify parameters of the transition process is not enough, as there would be a high and a low turnover process that would likely be equivalent. For this reason, I also include as targets the frequencies of single women that have ever cohabited or not with a partner, by age and educational level, which is enough to separate the desired low turnover process.[7]

In figure 2.1 I present the fit between the calibrated process and the empirical life-cycle profile of marriage/cohabitation. The overall fit of the profile is satisfactory, although the rate of cohabitation is underestimated for women without high school education at early ages. To further assess the validity of the calibration, I compare some statistics implied by this approximation with information on marriage and divorce obtained from the "Statistics of the Civil Registry" report by the IBGE. Note that this report concerns only legal marriages and divorces irrespective of cohabitation, while the calibrated process concerns any form of cohabitation spousal relationship. The calibrated process generates average ages at the start of the first relationship of 27 and 24 for women with and without high school education, respectively, and 25.5 overall. The 2002 and 2015 civil registry reports show that the average ages at first marriage in Brazil those years was 26.7 and 27, respectively. Moreover, the average duration of a marriage/cohabitation relationship implied by the model is around 13.4 years for both educational levels, while the 2011 and 2015 civil registry reports both present an average duration of legal marriages of 15 years. Considering this, the model appears to provide a reasonable approximation to the dynamics of marriage, divorce and cohabitation.

With respect to unemployment insurance, in Brazil this benefit is paid for a limited number of months and its value is determined as a non-linear function of the income of the worker prior to being laid-off, subject to a floor and a ceiling. To approximate the UI function, I compute the deflated averages of the benchmark levels of income specified by the labor legislation from 2002 to 2015 that are used to calculate the value of the monthly installments. With

[7]Marriage/cohabitation status is defined based on three variables in the data. The first is a person's status within their family, that can be one of the following: "Head of the family", "Spouse of the head of the family", "Son or daughter", "Other relative", "Partner of the son or daughter", "Pensionist" and "Domestic worker". The second variable is one that classifies the type of a family, which can be: "Couple without children", "Couple with children", "Single mother without children", "Single mother with children" and "Other type of family". The third variable is the one referred above that directly answers the question of whether a person is living with a partner, but that is only present in the data from 2011 on. I choose to classify a woman as being in a married/cohabiting relationship if her status within her family is "Head of the family" or "Spouse of the head of the family" and her family type is "Couple" with or without children. This classification agrees with the direct answer to the cohabitation variable for 98.8% of individuals. For the 1.2% of observations that this procedure disagrees, I correct the classification based on the cohabitation variable.

these, I set the floor and ceiling and interpolate in-between using a quadratic. Over the period from 2002 to 2015, the Brazilian social security system would pay 4 or 5 monthly installments of UI to unemployed workers that had worked at least 12 or 24 months in a formal job, respectively, within the previous 36 months, provided that these workers had been fired without cause. As a simplification, in the model I assume that UI is always paid in 4.5 installments following a transition out of a formal job, regardless of how many periods the individual had been employed in the formal sector. Moreover, because the idiosyncratic shocks to income are assumed to be realized only after decisions are made and are not part of the state space, I also assume that UI is computed based on expected rather than realized income.

I choose not to model maternity leave as a distinct state, because mandatory ML in Brazil lasts for four months and the time period of the model is a year. Given that the age of the youngest child enters the flow utility function non-linearly as shown in the previous section, I instead model ML by linearly approximating the integral of equation (2-13) over $age_{it}^{K} \in [0, \frac{1}{3}]$ as a function of the curvature parameter κ and correct the utility effect of a child age zero on women working in the formal sector appropriately. However, I do not model any sort of job protection for women when they choose to conceive or in the period in which the child is born.

For the household consumption equivalence scale, I adopt the inverse square root scale and set the marginal utility of aggregate household income to $\frac{1}{\sqrt{2}}$ for married/cohabiting women. Finally, I assume a discount factor parameter of 0.9.[8]

2.4.2 Model solution and the MSM estimator

In the second step, conditional on the exogenous features described above, I estimate the remaining parameters of the model using the method of simulated moments (MSM), as proposed by McFadden (1989). In a finite horizon, dynamic discrete choice framework such as mine, the model is solved using backward recursion from the final decision period back to the first. Solving the model amounts to computing the expected value of the maximum over choice-specific value functions at each point of the state space. Here, the assumption that preference shocks are independent and identically distributed extreme value type I is very convenient, because the integral over their support can be evaluated analytically using the familiar log-sum formula. For

[8]While the literature usually adopts a yearly discount factor closer to 0.95, this value is probably large for Brazil, particularly for agents of low socio-economic status. Using the average of real interest rates from 2002 and 2015 would imply a discount factor ≈ 0.94.

a thorough discussion on the advantages and disadvantages of this empirical assumption, see Train (2003). Considering that I model decisions beginning at age 18, I assume that every woman enters the model unmarried, out of the labor force and with zero labor market experience. The only observed heterogeneity in the initial conditions are the number of children and age of the youngest child, which I set to match the empirical fertility profile of 18 year old women with and without completed high school education in 1992[9]. Finally, I also assume that fertility prior to 18 years of age is exclusively unplanned and exogenous and, in this way, the distribution of unobserved types is independent of the initial fertility profile. Note that this assumption is not as strong as it appears because unobserved heterogeneity in fertility is introduced only in the marginal utility of children, and not in the intercept or variance of the preference shocks, which capture, among other things, heterogeneity in contraceptive preferences and behavior.

I solve the model for a limited number of reachable state space points. I keep track of labor market experience up to 25 years of full-time work. Hence, it is implicitly assumed that labor market experience in excess of 25 years is not valued by employers. Given the functional form of the income process, the value function is close to linear for higher levels of experience. In this way, above 10 years of experience, I solve the model exactly at every other point and interpolate linearly in-between. I keep track of number of children from 0 up to 4 and age of the youngest child from 0 to 12. Women with 4 children can still choose to have a child, in which case the age of the youngest child is set to zero in the following period but the number of children is not incremented.

Having calculated the expected value of the maximum over choice-specific value function at every point of the state space for a given parameter vector, I simulate a number of complete life-cycle histories of endogenous decisions, states and outcomes. This gives me a simulated panel with which I compute a vector of moments that can be directly compared to analogous empirical moments. Estimation then amounts to searching for the parameter vector that minimizes a distance function between the simulated and empirical moments. Let the parameter vector that fully describes the model be θ. The MSM estimator is defined as:

$$\hat{\theta} = \text{argmin}_{\theta}[\mathbf{m}(\theta) - \hat{\mathbf{m}}]^T W^{-1}[\mathbf{m}(\theta) - \hat{\mathbf{m}}] \tag{2-14}$$

where $\mathbf{m}(\theta)$ is the vector of simulated moments, $\hat{\mathbf{m}}$ is the vector of estimated

[9]I use data from the PNAD 1992 because of possible cohort effects in the fertility behavior of women and because I use the 1973 cohort as a reference cohort in the computation of all the moments in the data. Note that there isn't a 1991 edition of the PNAD because it was a Census year.

empirical moments and W is a positive semi-definite weighting matrix. I set W as a diagonal matrix of the reciprocals of the variances of each sample moment. This means that more precisely estimated moments receive greater weight in the objective function. I discuss moment selection and identification of the parameters of the model below.

2.4.3 Moments and identification

In order to implement the MSM estimator and identify the parameters of the model, a sufficiently rich set of statistics that are sensitive to each parameter needs to be selected. Even though it is not possible to give a rigorous proof of identification for the parameters of the model, I provide an heuristic discussion of which types of moments identify each group of parameters.

Average of log-income in formal, informal and self-employment by age and by part-/full-time work identify the parameters of the income process. Together with average log-income, average income in levels identify the variances of the income shocks because of the assumption that income is distributed log-normally.

The share of childless women by age identifies the type proportion of women with low marginal utility for children. Average age at last birth and the share of women with a child aged zero conditional on marriage status, age and number of children older than zero identify the η parameters, the variance of the conception shock and the concavity of the utility function with respect to the number of children. Furthermore, average number of children by age also helps to discipline the fertility parameters so that the model produces a reasonable life-cycle fertility profile.

The shares of women in each of the labor supply categories by age, marriage status, number of children and age of youngest child identify the ψ, ρ_k, θ, κ, and α_3 parameters. The share of women that reports never having worked identifies the type proportion of women with low preference for labor force participation.

The 1-year transition matrix between labor market states identifies the γ first order state dependence parameters. I also include average duration of non-employment spells by age in order to help identify the persistence in non-participation and unemployment and the variance of labor supply preference shocks. Identification of the probability π of receiving a formal part-time job offer is more intricate. Essentially, what identifies this probability are the differential transition rates between full and part-time work within the formal sector. In the data, there is substantial transition out of formal part-time work and into formal full-time work, but very little in the opposite direction. Although this identification is clearly conditional on the assumption of a common first order state dependence parameter for work in the formal sector (γ_{10}), I see no reason to assume that persistence in formal work would be asymmetric in the way required to reproduce the observed transition rates, especially when no such pattern is seen within the other types of employment.

Table 2.1 lists and describes the moments chosen to identify the parameters of the model. In total, 277 moments for each educational level are used to identify the 58 parameters that are not fixed prior to estimation.

2.5 Data

I use two data sources in order to compute the moments that are used to estimate the model. The first is the "Pesquisa Nacional por Amostra de Domicílios" (National Household Sampling Survey - PNAD) which is a cross-sectional household survey collected by the Brazilian Institute of Geography and Statistics (IBGE) between the years of 2002 and 2015 that is representative of the Brazilian population. The PNAD data is important because it contains information on labor supply of informal workers and also information on women's fertility such as the number of children ever had and the age of the youngest child. The second data source is the "Pesquisa Mensal do Emprego" (Monthly Employment Survey - PME) which is a rotating panel survey of households in six of the main metropolitan regions in Brazil[10] that follows a 4-8-4 pattern[11] and contains complete information only on labor supply of individuals, but not fertility. The PME survey is important because it provides the 1-year transition rates between labor market states that identify the state dependence parameters of the model. I also use PME data between the years 2002 and 2015. Although these household surveys were also collected before 2002, this interval was chosen because the most recent PNAD wave with detailed fertility information is 2015, while the PME survey had its methodology extensively reformulated after 2002.

While it is certainly not ideal to estimate a life-cycle model with only repeated cross-sectional and short panel data covering only a 14-year interval, there is no long longitudinal data with information on informal labor supply and also on women's fertility. In the absence of better suited alternatives, I estimate the model using the PNAD data as a pseudo-panel and supplement it with transition rates extracted from the PME. This strategy suffers from

[10]São Paulo, Rio de Janeiro, Belo Horizonte, Salvador, Recife and Porto Alegre

[11]A household is surveyed for 4 consecutive months, leaves the sample for 8 months and returns for another 4 consecutive months.

the problem of disentangling the pure life-cycle trends that are important to the model from the secular and cohort trends that ideally should be controlled for. For this reason, instead of trying to match moments covering the entire life-cycle, which would involve comparing women born at very different points in time, as a compromise I restrict my data to women born between 1967 and 1978 and match moments only for the age interval of 26 to 45, which is the most critical period of interaction between child-rearing and female labor supply.

Furthermore, to make the two data sources compatible, I only keep observations in the PNAD from the same six metropolitan regions that are covered by the PME survey. In the final sample I also drop women that: report being illiterate or having zero years of completed education; are attending an educational institution or are retired; are working for no compensation; report monthly salaries above 30,000 or below 50 Brazilian reais of 2015; report working less than 8 or more than 72 hours a week in their main job. After applying these restrictions, the PNAD sample consists of 80,958 women with at most completed secondary education, born between 1967 and 1978, aged 26 to 45 and observed between 2002 and 2015.

The PNAD survey has the last week of September of each year as the reference date. Considering this, the PME sample is constructed based on households interviewed between the second week of August and second week of November of a given year. I link individuals across PME waves using the household identifier, female gender and exact date of birth. After linking individuals, I search for pairs of interviews in years t and $t+1$ that are separated by an interval as close to 12 months as possible. If the only pairs of interviews in t and $t+1$ are separated by intervals shorter than 11 months (48 weeks) or longer than 13 months (56 weeks), that observation is dropped from the sample. If, for a given individual, two pairs of interviews are separated by the same interval, I keep the pair that has the second interview closest in date to the last week of September of that year. Given that each woman is followed for two years in the PME sample, I keep women born between 67 and 79 and aged between 25 and 45. Furthermore, I apply the same restrictions described in the previous paragraph to the PME sample. The final PME sample consists of 40,654 women observed for two consecutive years between 2002 and 2015 (a total of 81,308 observations).

Before computing the moments, I reweigh the data so that every age-education cell receives the same weight. This is done because the number of women in the data varies with age, while my sample simulated in the model is a balanced panel. The moments are calculated by a simple linear regression of the dependent variable on a set of dummies for the relevant conditioning cells, and controlling for state, race and year fixed effects, and a linear cohort trend centered on the 1973 cohort. Given that most moments are conditional on age, in order to avoid issues of multicolinearity I constrain year fixed effects to have mean zero and be orthogonal to a linear time trend, following the methodology proposed by Deaton (1997).

Finally, I deflate incomes using the INPC price index, which is better suited for low-income families than the more conventional IPCA index. Moreover, I incorporate many job benefits of formal workers directly into their reported monthly incomes: I deduct income taxes, add the FGTS benefit,[12] and add the monthly equivalent of the 13th salary and 1/3 vacation bonus to which formal workers are entitled to. However, social security and pension contributions are not deducted and thus are assumed to be equivalent to current consumption.

Unlike in the previous chapter, I consider as "formal" all private sector jobs that have a signed labor contract ("carteira de trabalho assinada"), including domestic jobs because I explicitly model employment benefits in the formal sector. Moreover, I also classify tenured public employees as "formal" because the data is not longitudinal and the observed frequency of public employment varies with age, so dropping these workers might affect the distribution of unobserved characteristics by age. In the self-employment category, I include workers that report being employers. Finally, part-time work is defined as habitually working less than 35 hours a week in the main job.

2.6 Results

In the following subsections I discuss the results of the estimation of the empirical model. First, I discuss the fit of the model to the data. Second, I present and dicuss the parameters estimates.

2.6.1 Model fit

The estimated model is able to fit well the most important features of the data, such as the overall allocation and transition rates of women between labor market states and the life-cycle profile of fertility. The overall quality of the fit is similar for both educational levels. Statistically, however, the model

[12]This benefit is a contribution that the employer makes at 8% of gross income that can be withdrawn as severance pay in the event of the worker being fired without cause.

is clearly rejected by the data because most target moments are estimated with high precision. In individual Wald tests, equality between target and simulated moments cannot be rejected at the 95% significance level for 49% of the moments. 58% of simulated moments are within a 10% deviation from their respective targets, 78% are within a 25% deviation and 88% are within a 50% deviation. Most simulated moments that fall outside this range are very low transition rates or probabilities that are very sensitive to small absolute deviations. For example, the moment with the worst fit under this criteria is the transition rate from part-time self-employment into a part-time formal job for women without high school education, where the empirical target is 0.2% and the simulated moment is 1.3%. The moment with the largest statistic for the test of equality between the empirical target and the simulated counterpart is the share of non-participation among women aged 42-45 with completed high school and with zero children, where the target is 14% and the model generates a share of 6.5%. This demonstrates that the model does not produce any gross discrepancies when compared to the data.

In figures 2.2 and 2.3 I present the simulated life-cycle profile of fertility, labor force participation and employment allocation and their respective data counterparts. One problematic result is that the model misses the increasing trend of non-participation with age. Although this may be due to problems with the data and the impossibility of perfectly disentangling age, period and cohort effects, most likely this occurs because the specification of per-period utility is linear in consumption and independent of age, so that the increase in earnings potential due to experience accumulation decreases the relative value of non-participation over the life-cycle. As a consequence of this poor fit of the participation rates, the model generates a smaller mass of women that never work than the data suggests, although it is possible that this information is reported with error in the data. In the data, 8.5% and 13% of high school graduates and dropouts aged 42-45, respectively, report having never worked. The simulated counterparts are 2.6% and 6.5%. On the other hand, the model captures well other important features such as the increasing rate of self-employment with age and even the high rate of unemployment for young workers which is not a targeted moment in the estimation.

Figures 2.4 and 2.5 show how the model also captures the relationship between labor supply and fertility well, which is the most important feature that the model intends to reproduce. Finally, figure 2.6 shows the fit of the evolution of labor income by age and the overall distribution of income. The model over-predicts average incomes in the late 20s and under-predicts in the early 40s. Nevertheless, the overall distribution of log-income for women aged

26-45 displays a very good fit.

Tables 2.2 and 2.3 present the fit of the transition matrix between labor market states and show that the simulated transition matrix approximates well its main patterns. The main discrepancies are that the model generates too much transition from informal and self-employment into formal full-time work, and too few transitions from informal and self-employment to non-participation.

Although there is certainly room for refinement of the model and improvement of the fit to the data, I believe the model reasonably approximates the overall dynamics of life-cycle labor supply and fertility and is able to provide meaningful counterfactual exercises.

2.6.2 Parameter estimates

Tables 2.4 and 2.5 present the parameter estimates of the model. The first interesting result is that the curvature parameter κ of the function $g(\cdot)$ which governs how fast the utility cost of children on working women declines with age of the youngest child is estimated at around 4.1. This implies that a child aged 1 or 2, carries about two thirds or half, respectively, of the cost that a child age 0 does. The cost of young children on working women is reduced if the work is part-time. Moreover, the utility cost of children is higher if the job is formal, which suggests that this work category is relatively worse in terms of labor supply flexibility. As expected, the type of work in which the utility costs of children are smaller is part-time self-employment.

Among women without high school education, incomes in self-employment and formal work are more dependent on labor market experience than informal jobs. The part-time penalty is estimated at -15%, -37% and -60.5% in formal, informal and self-employment, respectively. Among high school graduates, incomes in self-employment are the more highly correlated with labor market experience. The part-time penalty is -5%, -37% and -67% in formal, informal and self-employment, respectively.

The distribution of unobserved types is found to be correlated with educational level. About 18% of women with completed high school are estimated to have lower preference for labor force participation, and about 22% of those without. The respective figures for the "low marginal utility of children" type are 29% and 26%. Interestingly, in the simulated model, women of low preference for work are found to have slightly greater overall fertility and also to work more part-time compared to the baseline type, despite having preferences that only differ in one parameter. Moreover, all of the women that end up never working in any year are of the low preference for work type.

The probability of receiving a part-time job offer in the formal sector is estimated to be very low, around 13-15%. Another very intuitive pattern in the estimates is that the intercepts of the labor supply decision and first-order state dependence parameters indicate imply that it is easiest to enter the labor market from unemployment or non-participation in the informal sector. Self-employment display the lowest intercepts and highest level of state-dependence, which might indicate that there are substantial entry costs in becoming self-employed.

Finally, the standard deviation of the participation utility shock is estimated at just under 1.5 for both educational levels. Given that the scale of the utility is set at 12000 Brazilian reais, this means that the shocks have standard deviations of just under 18000. This value is substantial, indicating that unobserved factors are an important source of variation in labor force participation decisions. These values correspond to about 19% and 32% of total discounted lifetime income for women with or without high school education, respectively. The standard deviation for the labor supply shock conditional on participation is smaller, at about 0.96. In general, the parameter estimates are very consistent between high school graduates and dropouts, with the exception of the standard deviation of the conception shock. For the first group, it is estimated to be 1.13 and for the second, 0.66. When taken as a ratio to discounted lifetime labor income, however, the values are very similar, at 14.3% and 14.5%. These estimates suggest that unobserved factors play a more important role in relative terms for the labor supply decisions of women with lower education, which is intuitive. However, unobserved factors are responsible for a similar degree of variation in fertility decisions of women of both educational levels.

2.7 Counterfactual exercises

In this section I present the results of four counterfactual exercises: (1) lowering maternity leave from 4 to 0 months; (2) increasing maternity leave from 4 to 6 months; (3) turning off fertility; (4) doubling the probability of receiving a part-time job offer for a formal job. Exercises (1) and (2) have the goal of assessing on what dimensions of female labor supply maternity leave is an effective policy. Exercise (3) seeks to determine the role that fertility behavior and child-care responsibilities has on female labor market informality across the life-cycle. Finally, the goal of exercise (4) is to illustrate the potential effects of implementing labor market policies that increase the demand for part-time work in the formal sector. However, an analysis of the specific policy instruments that could be used to achieve such an effect will be left for future work.

The counterfactual scenarios are evaluated in terms of the present value of lifetime labor income discounted by the rate of 0.9 assumed in the estimation of the model, accumulation of labor market experience at ages 45 and 59, and the labor market allocation by age, number of children and age of the youngest child.

2.7.1 The effect of maternity leave

In the first set of counterfactual exercises I analyze the effect of maternity leave on female labor supply. First, I simulate the model under the assumption that there is no maternity leave as I model it. In this scenario the average of the present value of lifetime earnings is 0.93% smaller than in the baseline for women with completed high school education and 0.86% smaller for women without. This effect is partly due to slightly lower labor market experience accumulation before age 45, which is the period of active fertility behavior. Average experience at age 45 is 0.95% lower for both educational levels (this is about 2 months of equivalent full-time work), but shows no difference from the baseline at age 59. The other part of the effect on labor income is due to a higher rate of informality before age 45 as seen in the graphs in figures 2.7 and 2.8.

Perhaps surprisingly, maternity leave is found to have negligible effects on average labor force participation across the life-cycle. There are two main reasons for this: first, the fact that labor force participation is mostly explained in the model by permanent unobserved heterogeneity and not by the presence of children. Moreover, women with lower preference for labor supply also have higher overall fertility; second: at only 4 months and applying to just the formal sector of the labor market, maternity leave is not economically very relevant. The bulk of its effect is in lowering unemployment, which is a state conditional on participation. Without maternity leave, the deviation from the baseline in terms of unemployment peaks in the mid-20s, which is the period that concentrates the highest number of births. Unemployment around these ages is 0.8 percentage points higher for women with high school education and 0.6 p.p. higher for women without.

As expected, without maternity leave the share of women working in the formal sector is lower. The deviation from the baseline also peaks in the mid-20s, when formality is around 2 p.p. lower for both educational levels. No difference is found on the distribution of part-time work across the life-cycle.

Despite these relatively small life-cycle effects, maternity leave is indeed effective in influencing labor supply as a function of fertility. In the following comparisons, I focus solely on women under 45. In figures 2.9 and 2.10 I show the effect of maternity leave on labor market allocation as a function of the number of children ever had. Maternity leave is found to matter more for the allocation of women with high school education, despite having lower overall fertility. Formality is around 2 p.p. lower for mothers in this education group and 1 p.p. lower for mothers with lower education.

Figures 2.11 and 2.12 present the effects conditional on the age of the youngest child. Despite the negligible impact of ML on average rates of participation across the life-cycle, it does indeed increase LFP following the birth of a child, at 1.8 p.p. and 1.3 p.p. for women with or without HS education, respectively. Most importantly, rates of formality are 10 p.p. lower under the no ML scenario immediately following the birth of a child. Another interesting finding is that the share of part-time work among formal workers increases, which suggests that the women that would drop out of the formal sector following a birth are mostly full-time workers.

Next, I perform a similar exercise in which I increase the time of ML from 4 to 6 months. Similarly, this generates a slight increase in the present value of lifetime labor income of 0.6%. Average experience at age 45 is increased by the same amount. Higher maternity leave increases the rate of formal work by just over 1 p.p. during the 20s as shown in figures 2.13 and 2.14. Moreover, labor force participation and unemployment are reduced by around 1 p.p. and 1.5 p.p., respectively, for both educational levels following the birth of a child. Formality is increased by 5 and 6 p.p. for women with and without high school education, respectively, immediately after the birth of a child. This positive effect on formality persists until the child is aged 4, when the rate is still around 1 p.p. higher than in the baseline, as can be seen in figures 2.17 and 2.18.

These exercises suggest that maternity leave has a small influence on aggregate rates of labor force participation, unemployment, lifetime income and overall experience accumulation. In fact, the dimension in which this policy is most effective is in preventing women from dropping out of the formal sector following the birth of a child. In this way, maternity leave works by substantially reducing the impact that inflexibility of labor supply in the formal sector has on women with young children.

2.7.2 Labor supply without fertility

In the following counterfactual exercise, I quantify the impact that fertility has on life-cycle labor supply, the allocation of women in the labor market and its associated outcomes. To do so, I simulate the model assuming zero fertility in the initial conditions and that the utility of making a conception choice is negative infinity. In this way, fertility is completely eliminated and this is perfectly anticipated in the computation of the value function. Under this scenario, for women with high school education the present value of lifetime income is 3.5% higher, labor market experience is 3.5% higher at age 45 but only 0.3% higher at age 59. For women without high school education, lifetime income is 9.3% higher, experience is 7.2% higher at age 45 and 0.8% higher at age 59. These results show that women in the model compensate the missed human capital accumulation during the period of active fertility by working more at later ages after the critical period of child-rearing responsibilities. Note, also, that this exercise does not take into account child-care costs, only the impact on gross income.

Next, figures 2.19 and 2.20 show how the allocation of women in the labor market deviates under the no fertility scenario from the baseline. All labor market states display a similar and expected pattern, in which the more flexible states such as non-participation, self-employment and part-time work in general are less prevalent before age 45. This pattern reverses after age 45, which corroborates the result above that women seek to compensate for missed opportunities in human capital accumulation after completing fertility. This explains why full-time work in the formal sector is substantially lower between ages 45-55 under the no fertility scenario: when there is relatively more room for accumulating/recovering human capital and increasing future income, this state is relatively more attractive because it is the one with highest temporal persistence. Conversely, in the mid-20s, the rates of formal work would be up to 2 p.p. higher among high school graduates and up to 5 p.p. higher among high-school dropouts. The magnitude of these differences are consistent with the results in chapter 1. Lastly, note also the large difference in the rate of part-time work among self-employed women under the no fertility scenario, which demonstrates the extent of self-selection of mothers to labor market activities with greater flexibility.

In short, this exercise demonstrates how children are a relevant constraint in the labor supply decisions of women not only in terms of participation and full/part-time work. In fact, in a setting in which the formal sector of the labor market is relatively less flexible, child-care responsibilities and motives of family-work conciliation also push mothers into informality, adding to the negative effect of children on lifetime labor income and job stability.

2.7.3 Increasing demand for formal part-time work

In this last counterfactual, I simulate the model with a probability of receiving a formal part-time job offer of twice its estimated value: from 13 and 15% for women with and without HS education, respectively, to 26 and 30%. This exercise intends to approximate the partial equilibrium effects of a policy directed toward increasing the demand for part-time work in the formal sector. Precisely because of the partial equilibrium nature of the model, I refrain from assuming a larger increase in this probability, as a more effective policy in this dimension would arguably also impact other parameters of the model.

Present value of lifetime income would increase very slightly under this scenario, by 0.9% and 0.7% for high school graduates and dropouts, respectively. Accumulated experience at age 45, on the other hand, would decrease by about 0.6% and remain unchanged at age 59. The income gains would come solely from increased rate of formal work, which is better remunerated.

In figures 2.21 and 2.22, I present the effects of increasing demand for formal part-time work on the labor market allocation of women across the life-cycle. The magnitudes of the effects are similar to the higher maternity leave scenario. Unemployment decreases by between 0.5 and 1 p.p. in the early 20s and the rate of formal work increases by around 2 p.p. throughout the life-cycle for women of both educational levels. As expected, the main difference is the increased share of part-time work in the formal sector, between 5 and 6 p.p. higher than in the baseline. No effect is found on the distribution of part-time work in informal or self-employment. This is due to the fact that the preference shocks for part-time work in different sectors are assumed to be uncorrelated, a hypothesis which I intend to relax in the future. Indeed, it is reasonable to expect that higher availability of part-time work in the formal sector would substitute part-time work in the informal sector to a larger extent than full-time. Moreover, if formal and informal part-time work are in fact close substitutes, no decrease in experience such as the one found here should be expected and the overall effect on income would likely be larger.

Figures 2.23 through 2.26 present the effects of such a policy on the relationship between labor supply and fertility. Consistent with the previous exercise, the women that would most be affected are the ones with the highest number of children and that have young children. The rates of formal work immediately following the birth of a child would be 4 and 3 p.p. higher for high school graduates and dropouts, respectively. In contrast with the 6-month maternity leave scenario, the rates of formal work remain 2 p.p. higher than in the baseline even as the child ages.

The main takeaway of this exercise is to show that there is a substantial excess supply of workers willing to switch to part-time work in the formal sector if given the opportunity, especially mothers of young children. In this way, policies directed toward increasing flexibility and demand for part-time work in the formal sector should be effective in increasing employment rates, job stability and reducing labor market informality among women, particularly following the birth of a child.

2.8 Conclusion

In this chapter I specified a life-cycle model of fertility and female labor supply and estimated it with data on Brazilian women with at most completed secondary education. The model endogenizes both the labor force participation margin and the decision between working full or part-time in the formal or informal sectors or being self-employed.

The model is able to fit the data satisfactorily. The results show that full-time work in the formal sector is the worst category in terms of the utility costs arising from the presence of young children in the household. I interpret this finding as evidence that the average degree of labor supply flexibility in jobs in the formal sector is the lowest compared the informal jobs and self-employment. Moreover, the demand for part-time work in the formal sector is estimated to be highly limited. The estimated probability for receiving an offer for a part-time job in the formal sector in a given year is only about 15%. This lack of part-time work opportunities in the formal sector contributes to the relationship between motherhood and informality, because part-time work is also a highly valued job amenity by mothers of young children.

I perform four counterfactual exercises on the estimated model. In the first two, I show that maternity leave has a small impact on discounted lifetime labor income and overall labor experience accumulation. ML is found to not very effective in affecting labor force participation following childbirth, but very effective in preventing the share of women working full-time jobs in the formal sector from dropping sharply among mothers of young children. In the second exercise, I quantify the role that children play on the allocation of women in the labor market across the life-cycle. Realized fertility is responsible for a decrease of 3.5% in discounted lifetime labor income and labor market experience at age 45 for women with completed high school education, and for a decrease of 9.3% in lifetime income and 7.2% in experience at 45 for women without high school education. However, experience at age 59 is not very affected by realized fertility, which shows that women compensate for missed human capital accumulation early in life due to child-care responsibilities by working more later on as their children age.

In the last counterfactual, I increase the probability of receiving an offer for a part-time job in the formal sector to twice its estimated value. This exercise is intended to approximate the partial equilibrium effects of a policy directed toward increasing the demand for part-time work in the formal sector. The aggregate impact of such a policy on lifetime labor income is found to be positive, but small. Discounted lifetime labor income would increase by just under 1%, and experience at age 45 would, in fact, decrease slightly. This is due to the fact that I do not model correlation between preference shocks related to the part-time work options in the different sectors of the labor market, a hypothesis that I intend to relax in the future. Aggregate labor force participation rates are also largely unaffected throughout the life-cycle and only slightly better following childbirth. Despite these small effects, the rates of formal work are increased throughout the life-cycle, and particularly so when there are young children in the household.

I interpret these results as evidence that, in developing countries where a large fraction of the labor market can already be said to be relatively flexible in terms of labor supply, stimulating flexibility in the formal sector might not have strong effects on overall labor force participation. However, such policies might be very effective in improving informality rates among women, which is associated with better labor market outcomes.

Finally, it is important to note that this conclusion is largely dependent on the fact that, in the model, unobserved heterogeneity with respect to labor supply is modelled only in preferences for labor force participation and not in preferences for work in formal, informal or self-employment. However, the nature of the data available for estimating this model does not offer sources for credible identification of permanent unobserved heterogeneity of this kind, unless the first-order state dependence structure is abandoned. This is certainly an issue that deserves further consideration, even though I would argue that first-order state dependence is a more natural way of generating the observed 1-period transition rates than resorting to permanent unobserved heterogeneity for this purpose.

Figure 2.1: Marriage process calibration fit

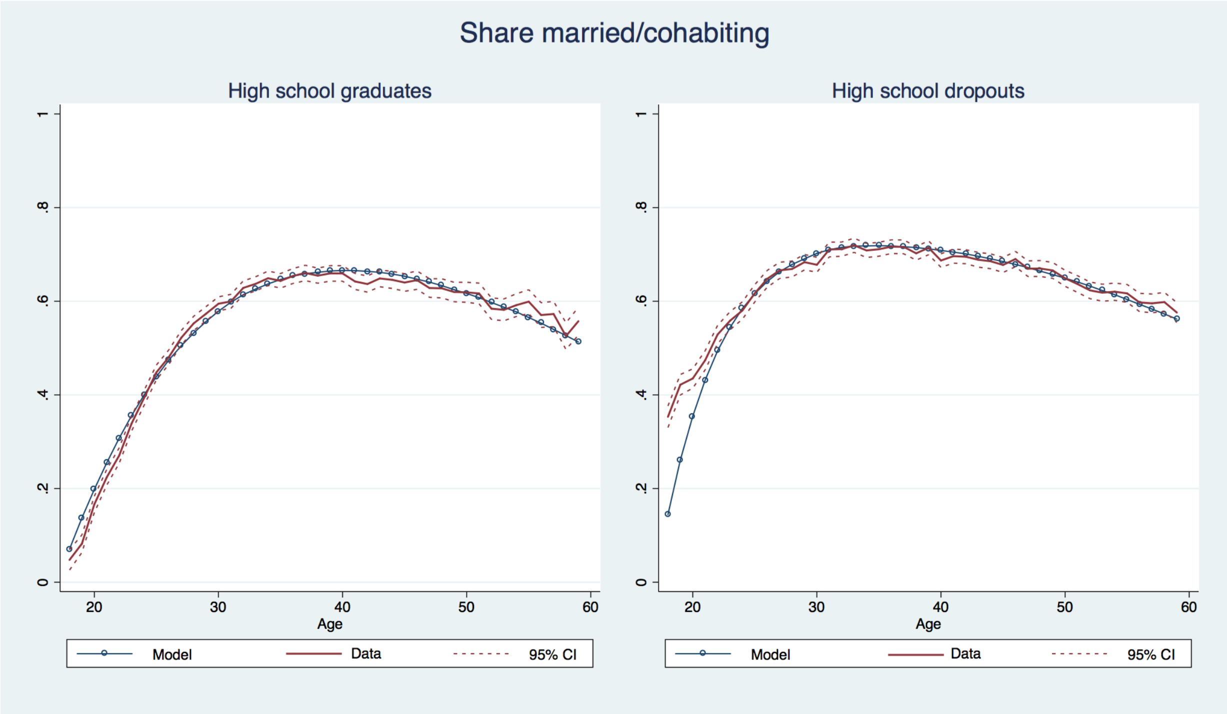


Figure 2.2: Model fit: Labor market allocation by age

Figure 2.3: Model fit: Fertility by age

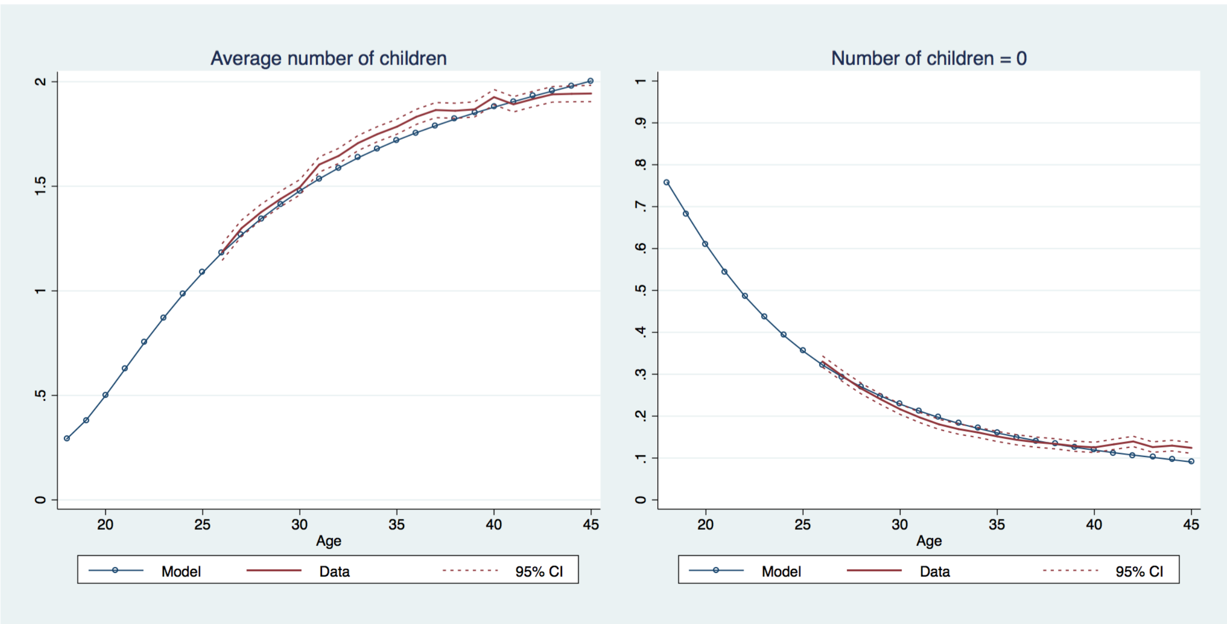


Figure 2.4: Model fit: Labor market allocation by number of children

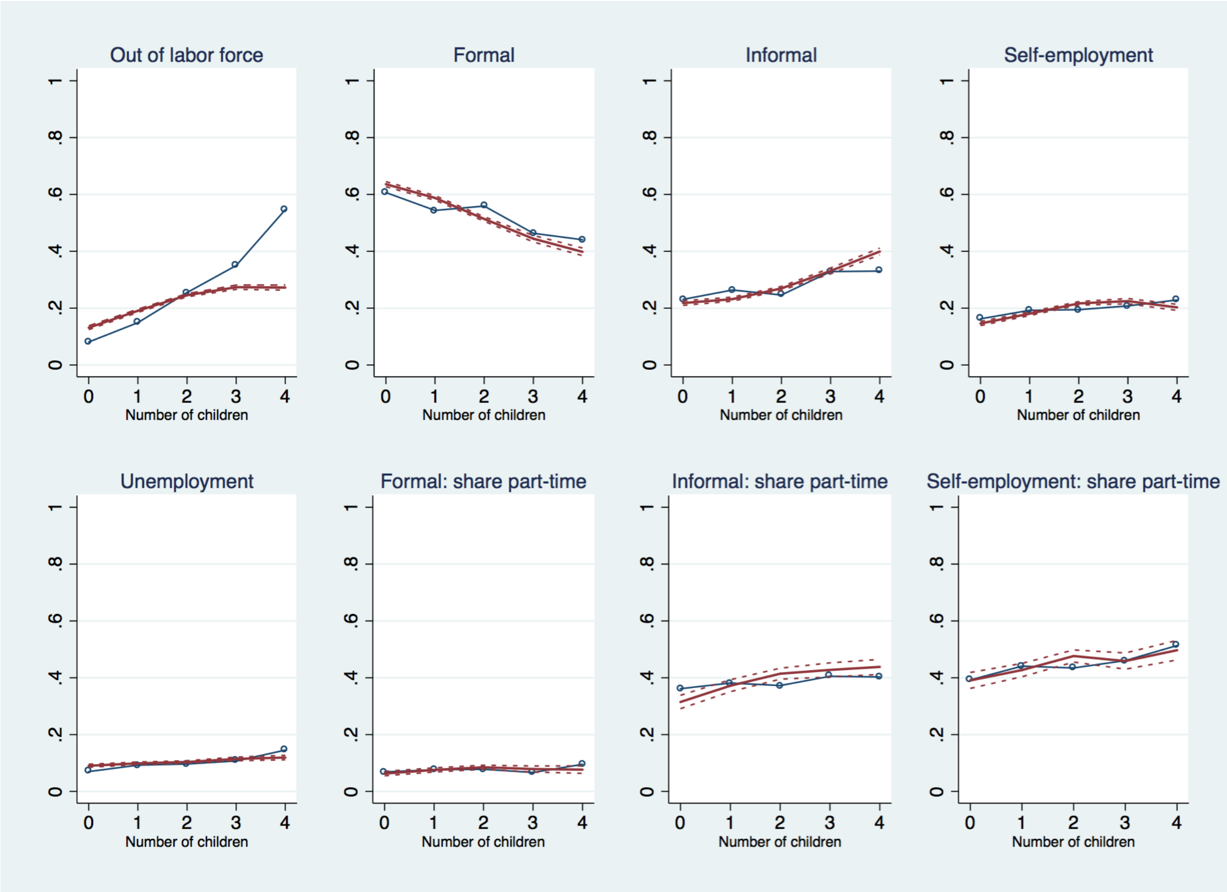


Table 2.1: Listing and description of moments

Description	Conditioning	Count
Fertility:		
Avg. no. of children	Age	5
Share with zero children	Age	5
Share with child aged zero	Age, Single	5
Share with child aged zero	Age, Married, No. of children older than 0	15
Avg. age at most recent birth	At age 45	1
Labor market allocation:		
Share out of labor force	Age	5
	Single / Unemployed husband	2
	Employed husb. with HS/College educ.	2
	Age of youngest child	5
	No. of children*	4
Share in: unemployment / formal / informal / self-employment**	Age	20
	Single / Married	8
	Age of youngest child	20
	No. of children*	16
Share part-time among: formal / informal / self-employed	Age	15
	Single / Married	6
	Age of youngest child	15
	No. of children*	12
Labor market dynamics:		
Transition rates from j_{t-1} to j_t	–	64
Share that never worked	Age	5
Avg. no. of years since last had a job	Age	5
Income:		
Avg. Log income	Age, Formal/Informal/Self-empl.	15
	Full/Part-time, Formal/Informal/Self-empl.	6
Avg. income in level	Age, Formal/Informal/Self-empl.	15
	Full/Part-time, Formal/Informal/Self-empl.	6
Total:		277

Notes: Age groups are: [26-29], [30-33], [34-37], [38-41], [42-45]
Age of youngest child groups are: 0, [1-2], [3-5], [6-11], 12+
Number of children groups are: 0, 1, 2, 3+

* These moments are computed only for women in the [42-45] age interval and whose youngest child over 11, in order to try to capture the residual correlation between labor market decisions and completed fertility after the critical period of child-care passes.

** Unemployment rate is computed conditional on labor force participation and shares in formal/informal/self-empl. are computed conditional on employment.

Figure 2.5: Model fit: Labor market allocation by age of youngest child

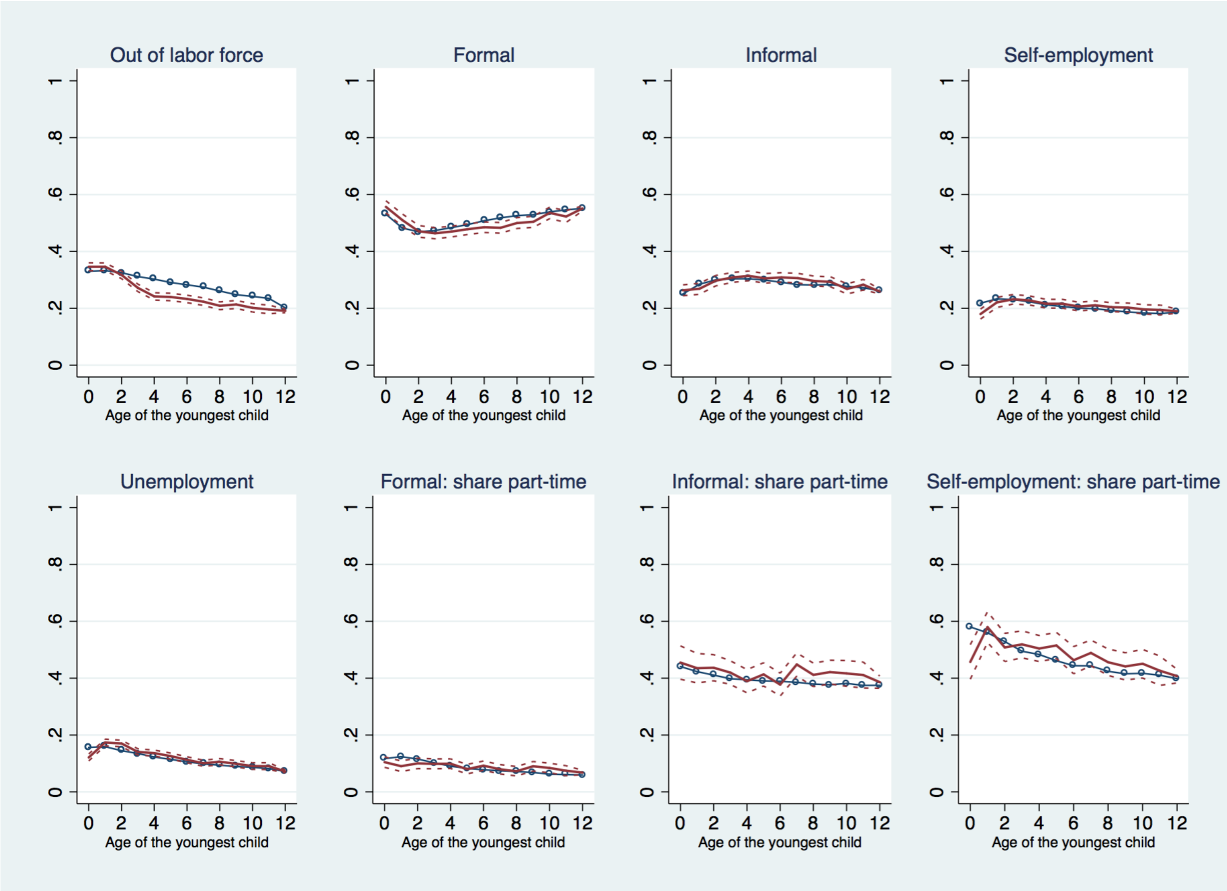


Figure 2.6: Model fit: Income

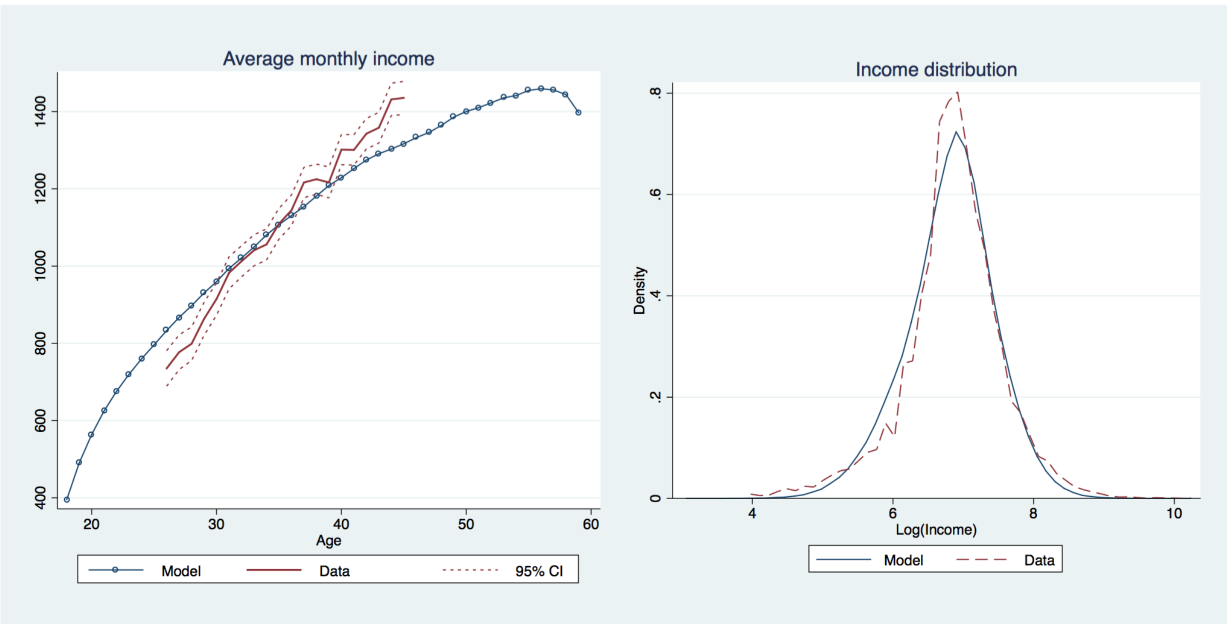


Table 2.2: Model fit: Transition rates - High school graduates

	Labor market status in period *t*							
	OLF	Unemp.	Formal FT	Formal PT	Informal FT	Informal PT	Self FT	Self PT
Status in *t-1*								
OLF	**66.7**	**8.2**	**10.3**	**1.1**	**5.7**	**3.0**	**2.4**	**2.7**
	69.1	8.4	8.4	0.8	3.8	2.6	3.2	3.8
	(0.53)	(0.33)	(0.47)	(0.18)	(0.36)	(0.27)	(0.31)	(0.27)
Unemp.	**35.3**	**31.1**	**14.6**	**1.8**	**6.9**	**3.5**	**3.3**	**3.4**
	31.1	23.9	23.4	2.3	7.6	3.8	3.3	4.5
	(0.97)	(0.60)	(0.85)	(0.33)	(0.65)	(0.49)	(0.56)	(0.49)
Formal FT	**3.2**	**3.2**	**77.0**	**3.0**	**5.2**	**2.6**	**3.1**	**2.8**
	7.2	4.4	79.5	2.6	3.4	1.0	1.0	0.9
	(0.49)	(0.30)	(0.43)	(0.17)	(0.33)	(0.24)	(0.28)	(0.25)
Formal PT	**4.4**	**4.2**	**33.6**	**40.5**	**6.5**	**3.4**	**3.7**	**3.7**
	5.1	4.8	37.7	43.0	3.2	4.0	0.6	1.7
	(1.60)	(0.99)	(1.40)	(0.55)	(1.07)	(0.81)	(0.93)	(0.81)
Informal FT	**8.4**	**7.1**	**25.0**	**2.8**	**36.2**	**9.7**	**5.6**	**5.2**
	11.4	5.8	28.2	1.4	35.0	6.1	8.9	3.2
	(1.08)	(0.67)	(0.94)	(0.37)	(0.72)	(0.54)	(0.63)	(0.55)
Informal PT	**8.3**	**7.0**	**23.9**	**2.7**	**19.3**	**28.3**	**5.4**	**5.1**
	17.2	6.6	16.9	6.2	13.4	29.8	4.4	5.5
	(1.59)	(0.98)	(1.39)	(0.54)	(1.06)	(0.80)	(0.92)	(0.81)
Self FT	**4.0**	**4.0**	**15.0**	**1.9**	**6.2**	**3.2**	**56.6**	**9.3**
	12.9	2.8	6.9	0.3	6.2	1.8	57.0	12.0
	(1.01)	(0.63)	(0.88)	(0.35)	(0.68)	(0.51)	(0.59)	(0.51)
Self PT	**6.8**	**5.7**	**20.2**	**2.5**	**8.9**	**4.4**	**13.3**	**38.3**
	20.7	5.6	5.7	0.8	4.5	4.1	20.5	38.1
	(1.34)	(0.83)	(1.17)	(0.46)	(0.90)	(0.68)	(0.78)	(0.68)

Notes: Blue/Bold – Model; Maroon – Data; Standard errors in parentheses.

Table 2.3: Model fit: Transition rates - High school dropouts

	Labor market status in period *t*							
	OLF	Unemp.	Formal FT	Formal PT	Informal FT	Informal PT	Self FT	Self PT
Status in *t-1*								
OLF	**71.3**	**6.8**	**7.1**	**0.6**	**6.1**	**3.8**	**1.9**	**2.5**
	74.4	5.7	4.8	0.3	4.7	3.8	2.7	3.6
	(0.45)	(0.28)	(0.40)	(0.16)	(0.30)	(0.23)	(0.26)	(0.23)
Unemp.	**38.9**	**29.4**	**11.0**	**0.9**	**8.5**	**5.2**	**2.8**	**3.3**
	37.3	23.4	12.0	0.8	12.3	6.6	3.7	3.9
	(1.07)	(0.66)	(0.93)	(0.37)	(0.71)	(0.54)	(0.62)	(0.54)
Formal FT	**5.5**	**4.2**	**70.3**	**1.9**	**7.5**	**4.6**	**2.9**	**3.2**
	8.9	4.6	74.5	1.8	6.3	2.0	0.9	0.9
	(0.60)	(0.37)	(0.53)	(0.21)	(0.40)	(0.30)	(0.35)	(0.31)
Formal PT	**8.3**	**5.6**	**31.5**	**29.6**	**10.4**	**6.3**	**3.9**	**4.4**
	10.9	5.5	36.5	29.1	6.3	7.8	0.5	3.5
	(2.47)	(1.53)	(2.16)	(0.85)	(1.65)	(1.24)	(1.43)	(1.26)
Informal FT	**9.3**	**6.7**	**15.7**	**1.4**	**44.5**	**13.9**	**4.1**	**4.5**
	15.6	5.2	17.6	0.8	43.0	10.0	5.4	2.5
	(0.92)	(0.57)	(0.80)	(0.31)	(0.61)	(0.46)	(0.53)	(0.47)
Informal PT	**8.4**	**6.0**	**14.0**	**1.3**	**20.4**	**42.8**	**3.4**	**3.8**
	19.9	5.6	8.6	1.7	19.4	38.2	3.0	3.5
	(1.19)	(0.73)	(1.04)	(0.41)	(0.79)	(0.60)	(0.69)	(0.60)
Self FT	**6.0**	**4.1**	**10.9**	**1.0**	**7.7**	**4.7**	**56.5**	**9.2**
	16.2	3.4	3.1	0.3	8.4	3.2	54.1	11.4
	(1.07)	(0.66)	(0.93)	(0.37)	(0.71)	(0.54)	(0.62)	(0.54)
Self PT	**9.0**	**6.2**	**14.2**	**1.3**	**10.5**	**6.3**	**10.8**	**41.6**
	27.6	3.7	2.8	0.2	5.0	6.1	16.2	38.4
	(1.33)	(0.82)	(1.16)	(0.45)	(0.89)	(0.67)	(0.77)	(0.67)

Notes: Blue/Bold – Model; Maroon – Data; Standard errors in parentheses.

Table 2.4: Parameter estimates - High school graduates

Paramater	Estimate	Parameter	Estimate	Parameter	Estimate
Utility:		θ_5	-1.2965	**State dependence:**	
ρ_f	0.1884	θ_6	-1.7689	γ_0	0.7037
ρ_i	0.0582	θ_7	-0.8106	γ_1	2.4231
ρ_s	0.3279			γ_2	1.5227
ψ_0	1.3332	**Unobserved types:**		γ_3	0.3209
ψ_1	-2.0396	% Type 2	0.1765	γ_4	0.9022
ψ_2	-0.9870	% Type 3	0.2910	γ_5	1.5663
ψ_3	-0.7708	ψ_0 (Type 2)	-2.9024	γ_6	2.4984
ψ_4	-0.3757	α_1 (Type 3)	0.3226	γ_7	1.5526
ψ_5	-1.0896			γ_8	2.3100
ψ_6	-3.0174	**Income:**		γ_9	0.2541
ψ_7	-1.8785	$\beta^f{}_0$	6.1801	γ_{10}	1.4500
α_1	1.3012	$\beta^f{}_1$	0.3856	γ_{11}	0.8743
α_2	-0.3654	$\beta^f{}_2$	-0.0497	γ_{12}	1.4935
α_3	0.0323	$\beta^i{}_0$	5.8772		
η_1	-10.0591	$\beta^i{}_1$	0.3640	**Formal part-time prob.**	
η_2	8.9954	$\beta^i{}_2$	-0.3723	π	0.1508
κ	4.1303	$\beta^s{}_0$	5.9910		
θ_0	0.1958	$\beta^s{}_1$	0.4570	**Preference shocks:**	
θ_1	0.1029	$\beta^s{}_2$	-0.6697	Std. dev. ϕ	1.4900
θ_2	-1.9039	σ_f	0.4739	Std. dev. υ^b	1.1271
θ_3	-0.8532	σ_i	0.5520	Std. dev. υ	0.9689
θ_4	-1.6182	σ_s	0.6653		

Notes: The calculation of standard errors is in progress

Table 2.5: Parameter estimates - High school dropouts

Paramater	Estimate	Parameter	Estimate	Parameter	Estimate
Utility:		θ_5	-1.1655	**State dependence:**	
ρ_f	0.2478	θ_6	-1.5768	γ_0	0.8446
ρ_i	0.0645	θ_7	-1.0057	γ_1	2.0984
ρ_s	0.4119			γ_2	1.5747
ψ_0	1.1790	**Unobserved types:**		γ_3	0.3481
ψ_1	-2.2908	% Type 2	0.2218	γ_4	0.9385
ψ_2	-0.9756	% Type 3	0.2632	γ_5	1.7229
ψ_3	-0.8743	ψ_0 (Type 2)	-3.0578	γ_6	2.6725
ψ_4	-0.2558	α_1 (Type 3)	0.4612	γ_7	1.6355
ψ_5	-0.8567			γ_8	1.9949
ψ_6	-2.9086	**Income:**		γ_9	0.2718
ψ_7	-1.8078	$\beta^f{}_0$	5.9402	γ_{10}	1.2992
α_1	1.7817	$\beta^f{}_1$	0.3859	γ_{11}	0.9576
α_2	-0.3180	$\beta^f{}_2$	-0.1484	γ_{12}	1.5385
α_3	0.0374	$\beta^i{}_0$	5.7626		
η_1	-7.4678	$\beta^i{}_1$	0.3250	**Formal part-time prob.**	
η_2	6.1231	$\beta^i{}_2$	-0.3673	π	0.1324
κ	4.0936	$\beta^s{}_0$	5.7992		
θ_0	0.3582	$\beta^s{}_1$	0.3892	**Preference shocks:**	
θ_1	0.1188	$\beta^s{}_2$	-0.6048	Std. dev. ϕ	1.4494
θ_2	-2.0407	σ_f	0.2519	Std. dev. υ^b	0.6609
θ_3	-1.2064	σ_i	0.4867	Std. dev. υ	0.9595
θ_4	-1.5903	σ_s	0.5794		

Notes: The calculation of standard errors is in progress

Figure 2.7: Counterfactual exercise 1

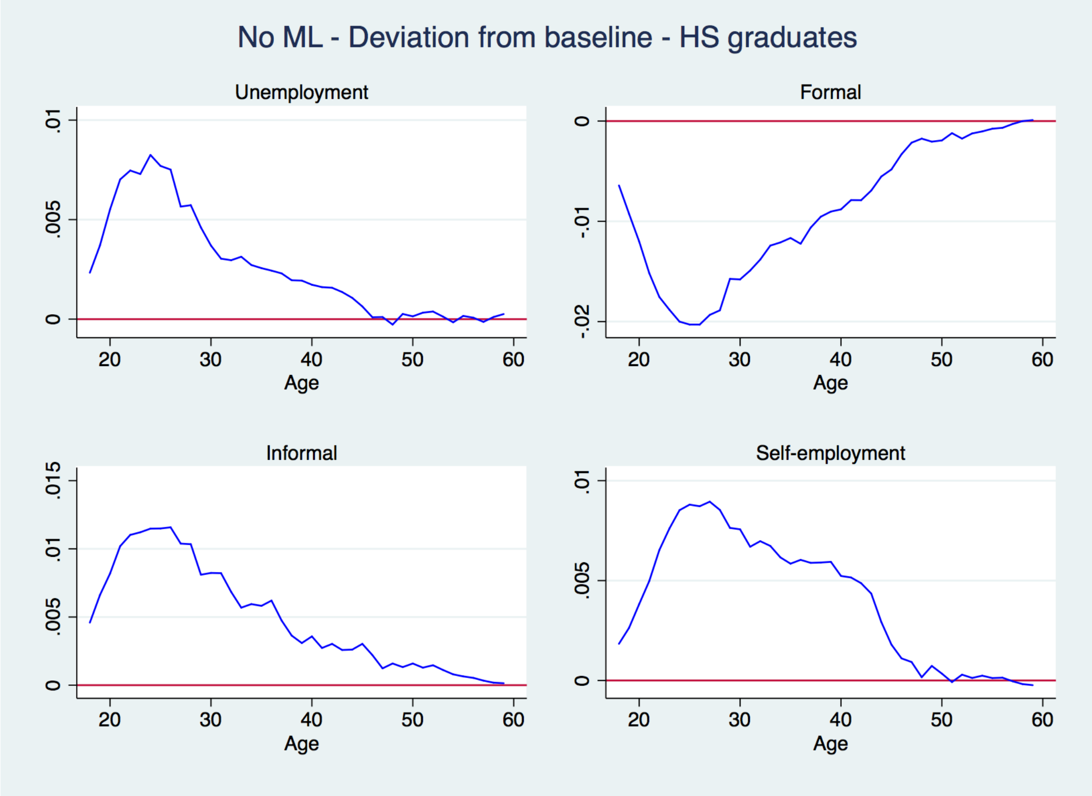


Figure 2.8: Counterfactual exercise 1

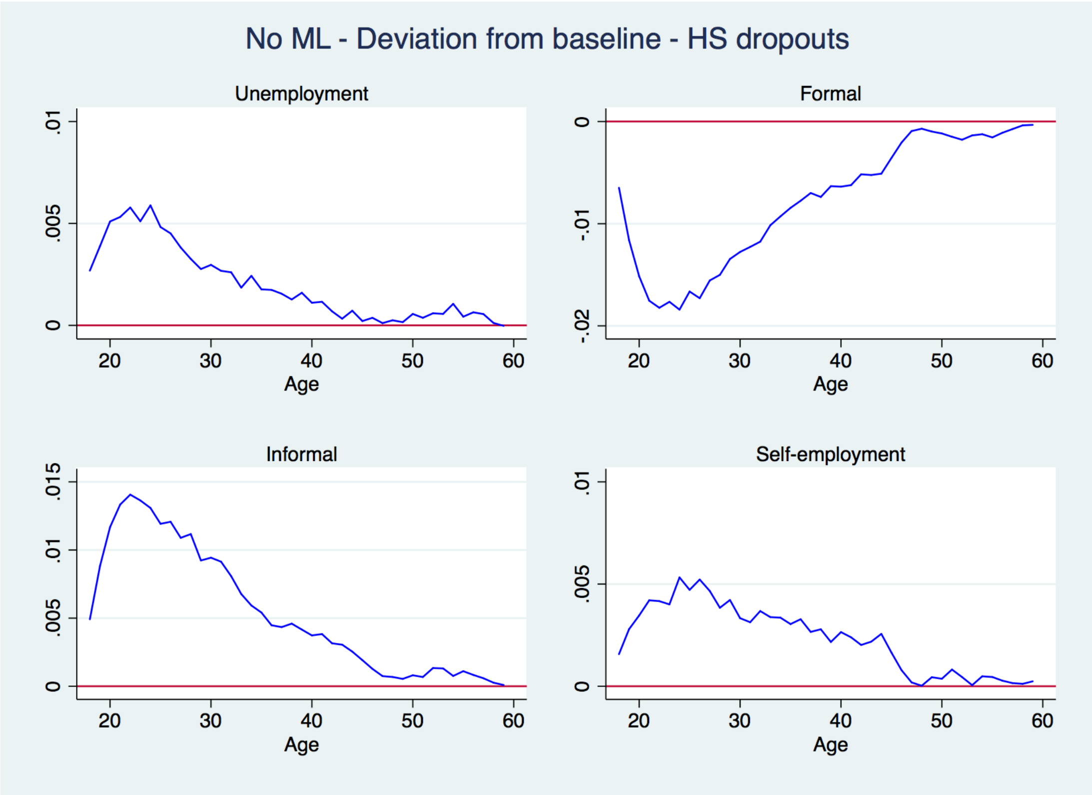


Figure 2.9: Counterfactual exercise 1

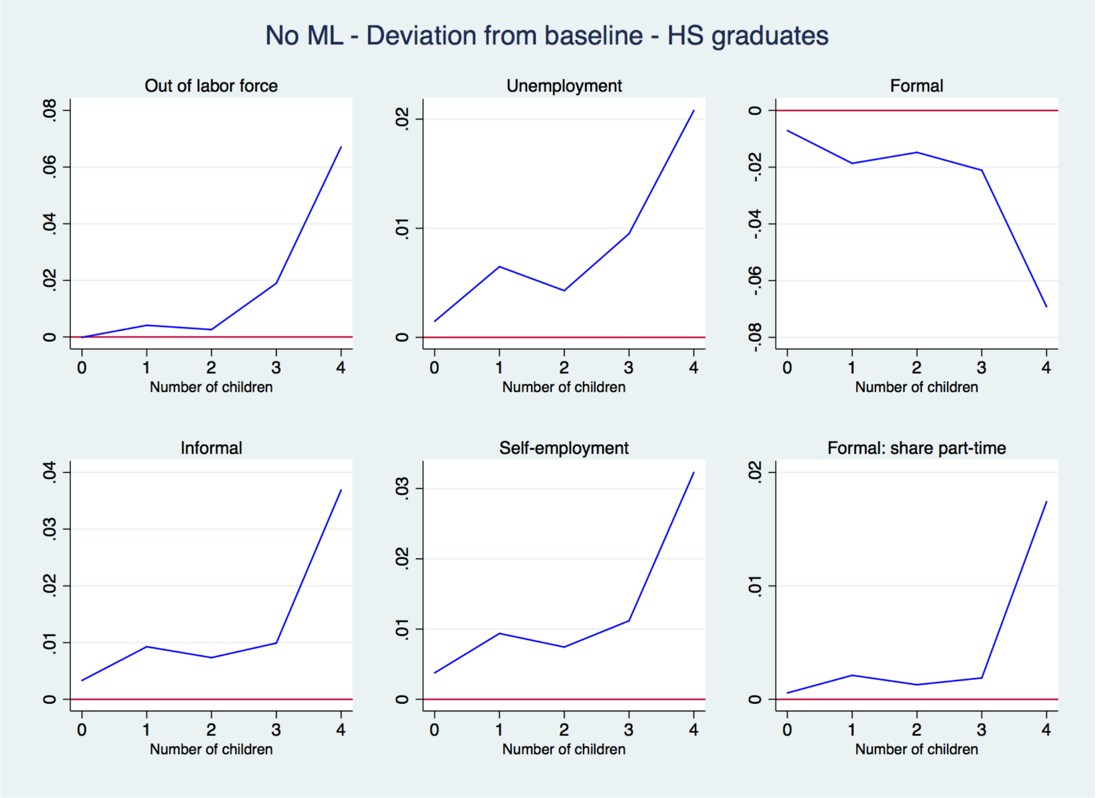


Figure 2.10: Counterfactual exercise 1

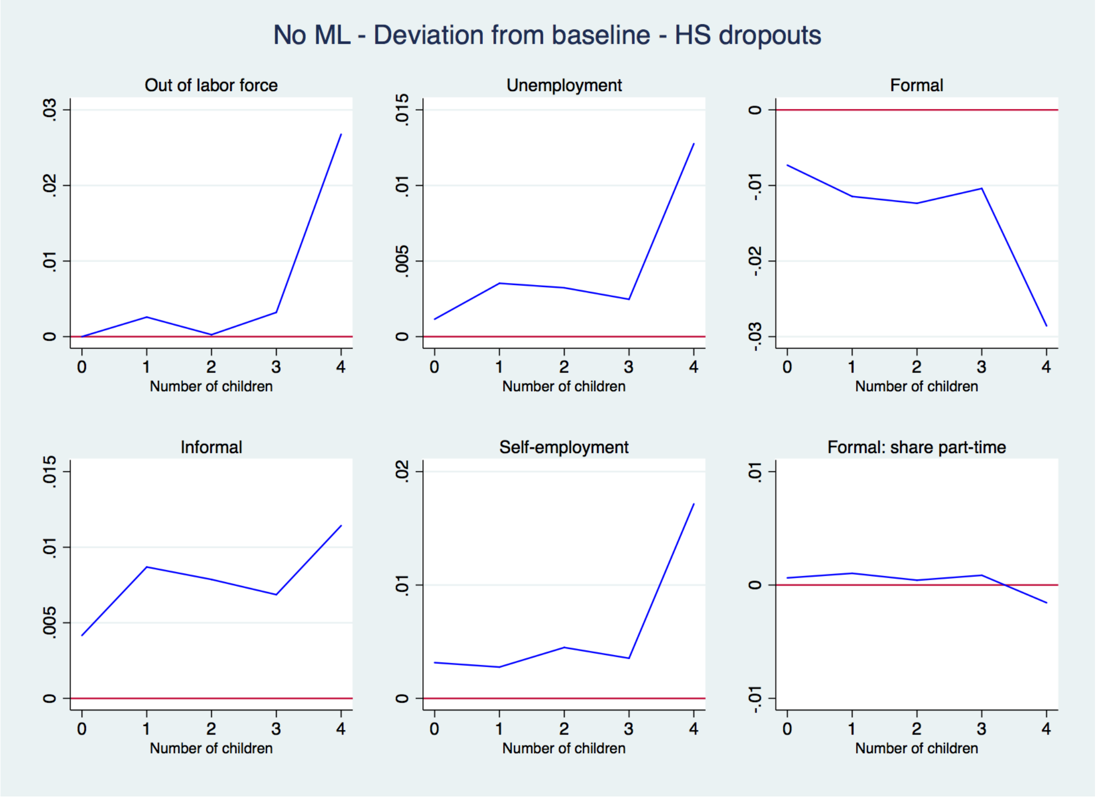


Figure 2.11: Counterfactual exercise 1

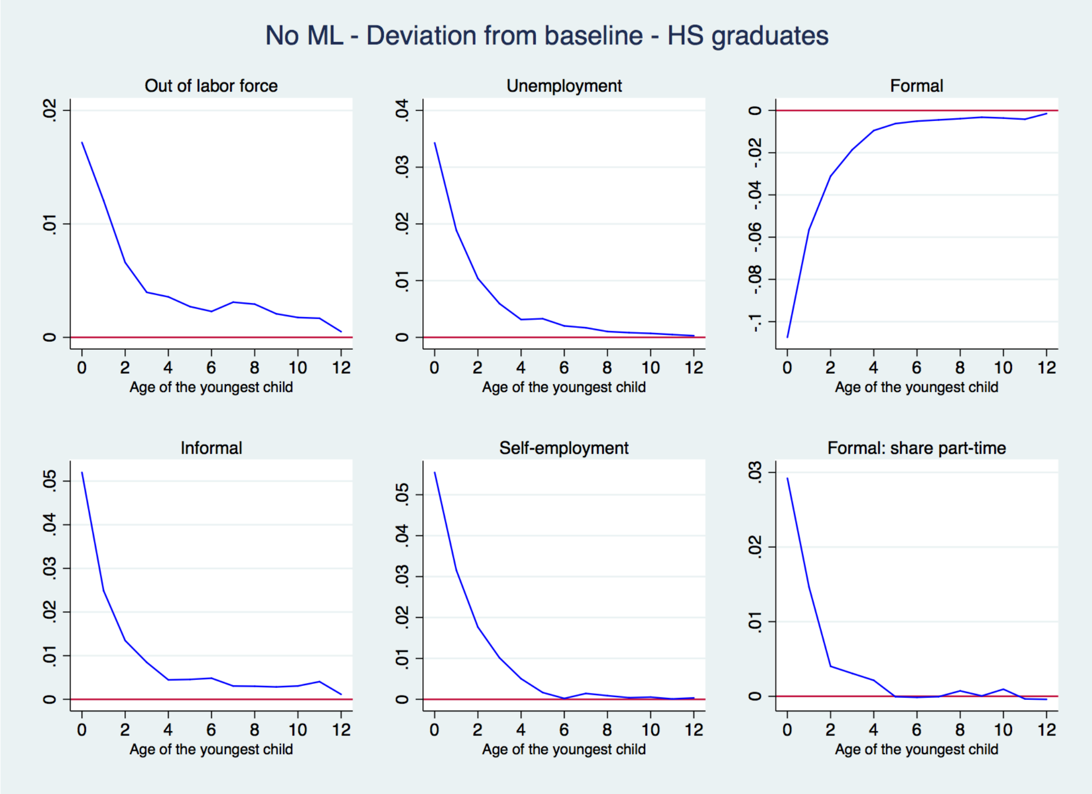


Figure 2.12: Counterfactual exercise 1

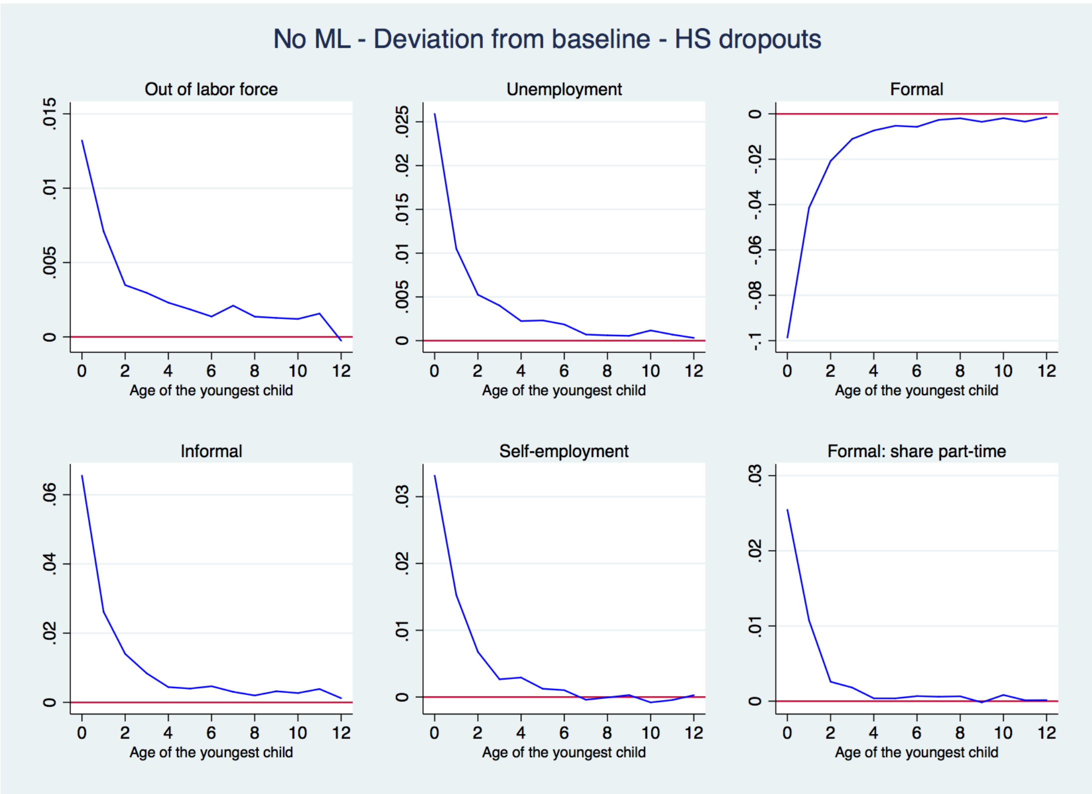


Figure 2.13: Counterfactual exercise 2

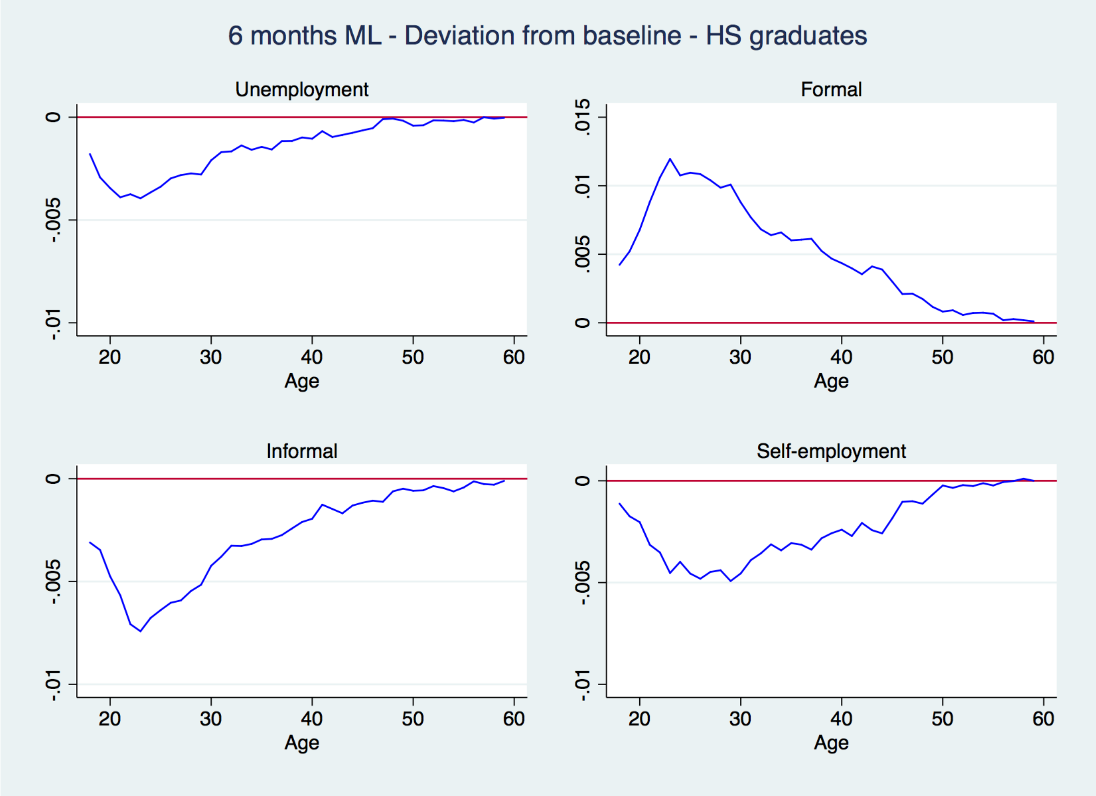


Figure 2.14: Counterfactual exercise 2

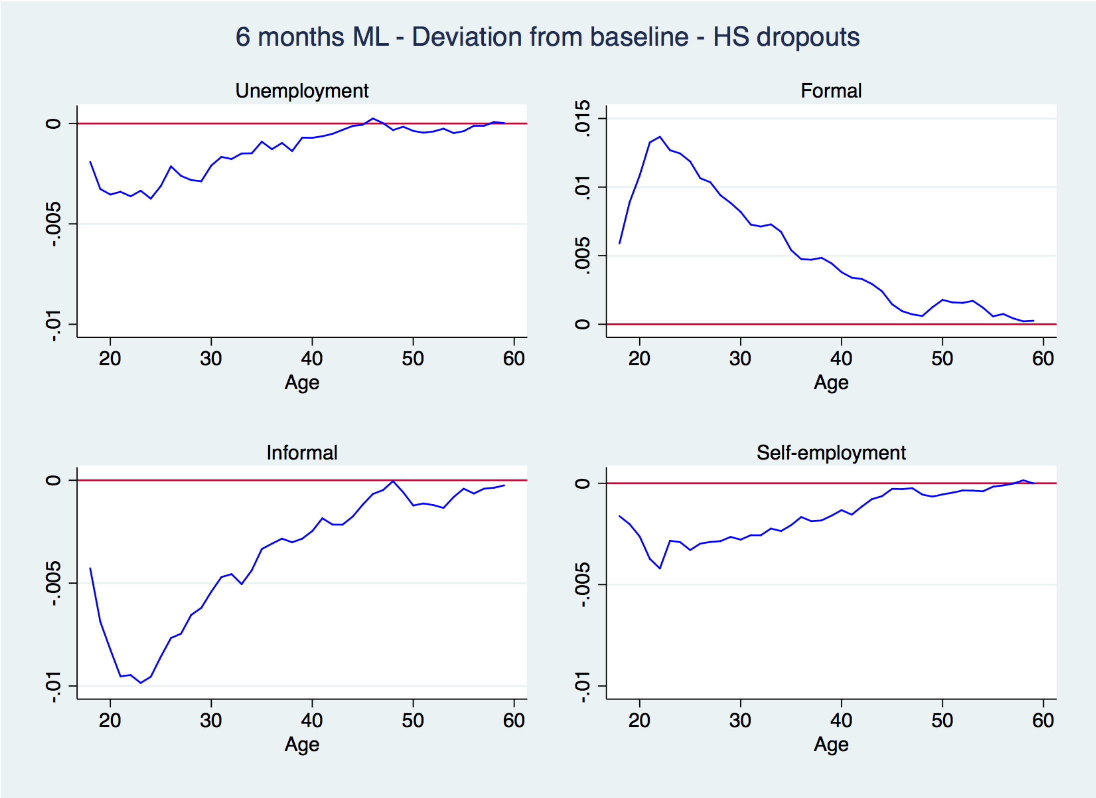


Figure 2.15: Counterfactual exercise 2

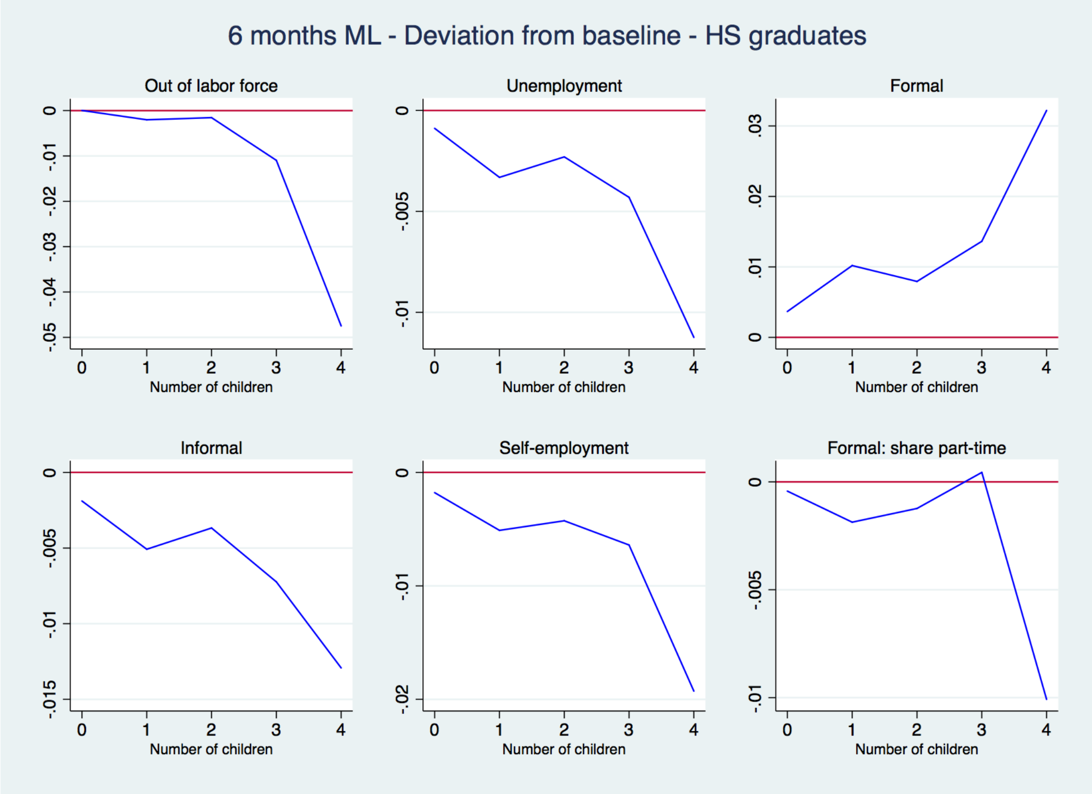


Figure 2.16: Counterfactual exercise 2

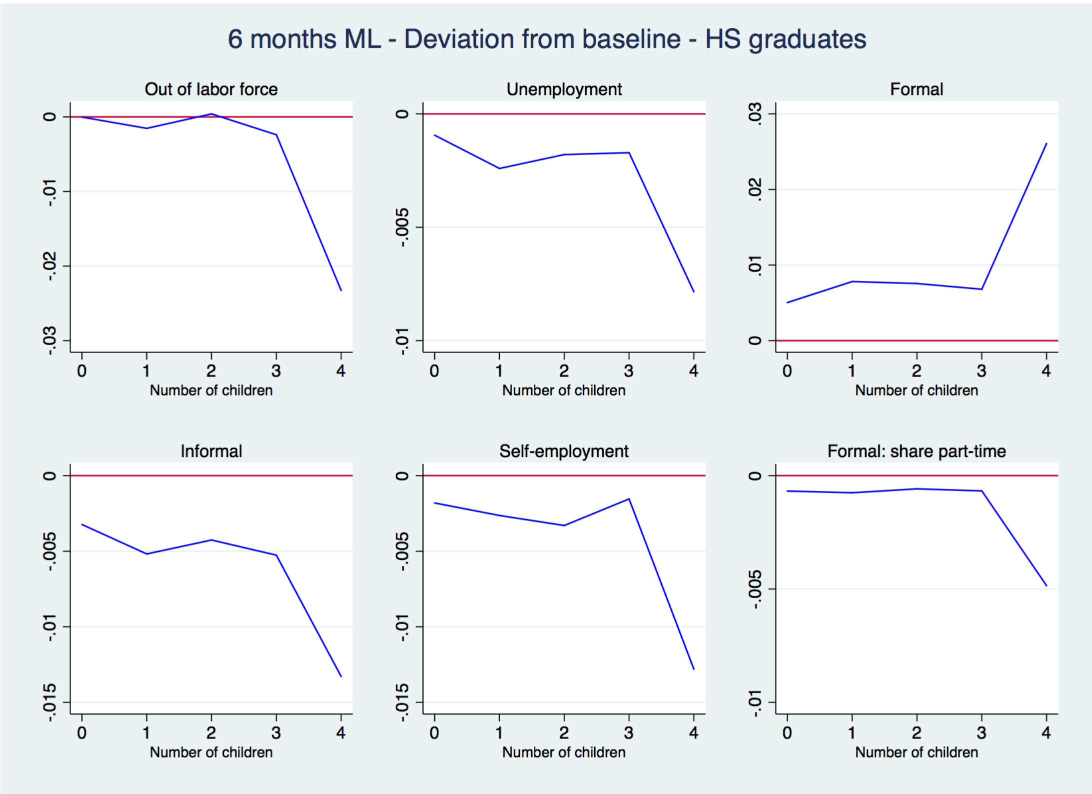


Figure 2.17: Counterfactual exercise 2

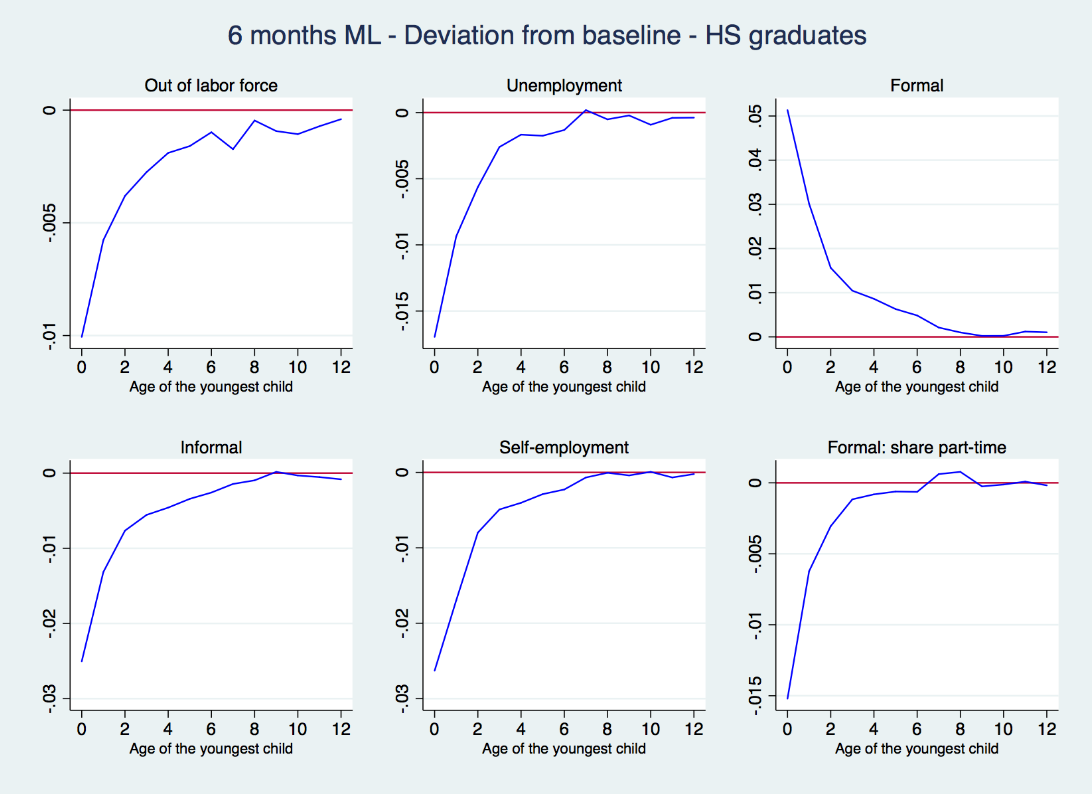


Figure 2.18: Counterfactual exercise 2

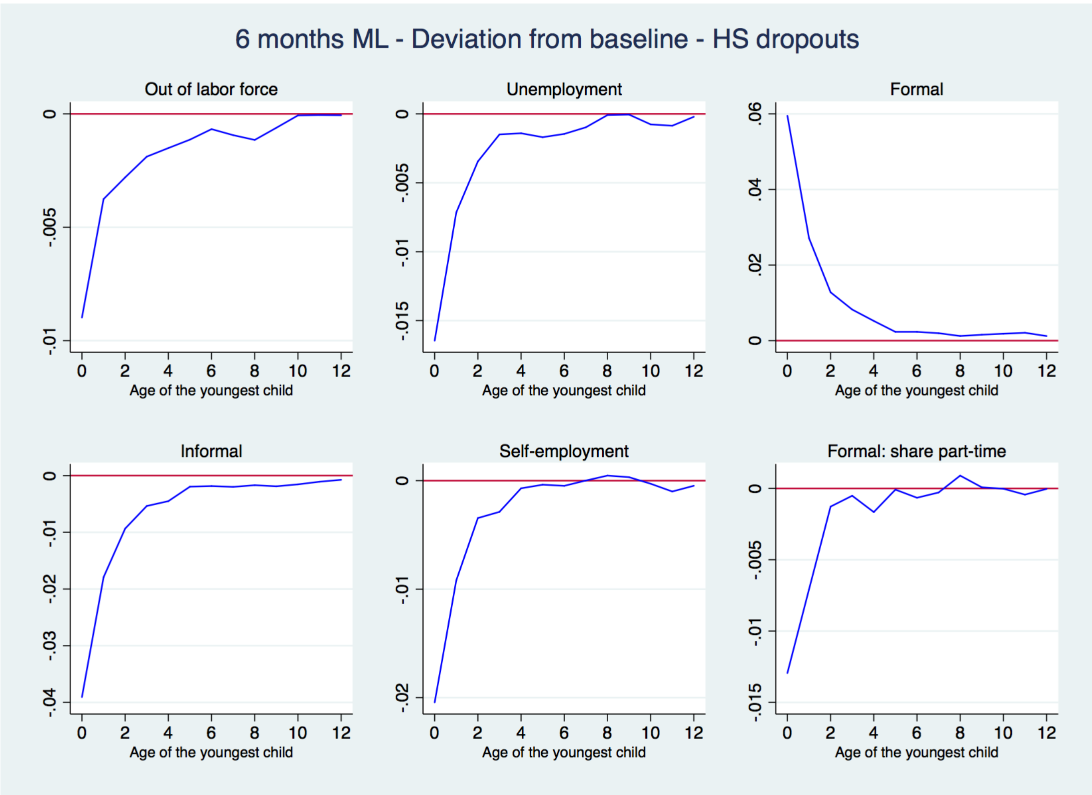


Figure 2.19: Counterfactual exercise 3

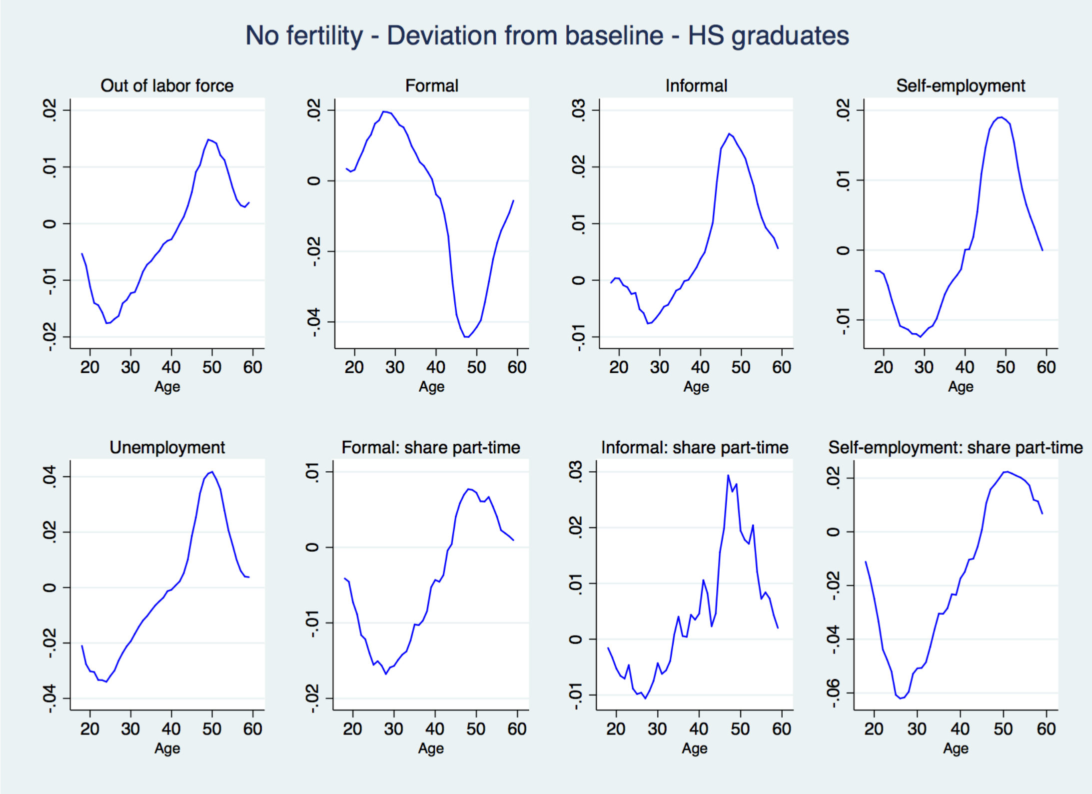


Figure 2.20: Counterfactual exercise 3

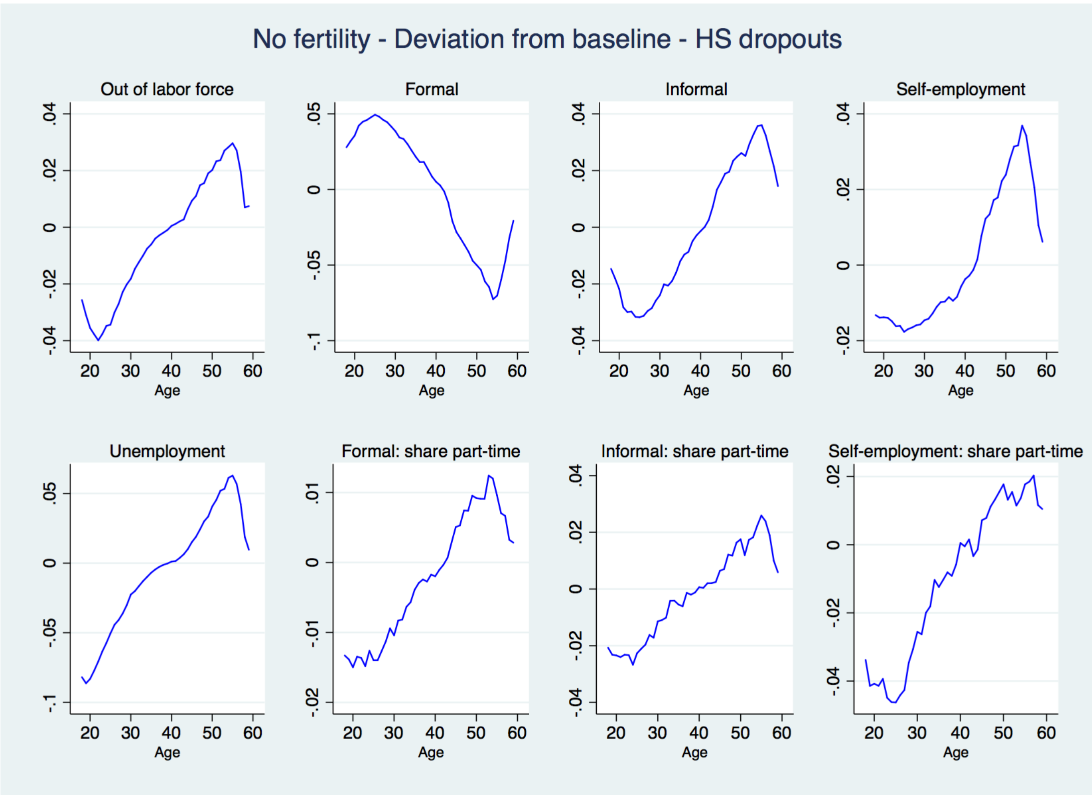


Figure 2.21: Counterfactual exercise 4

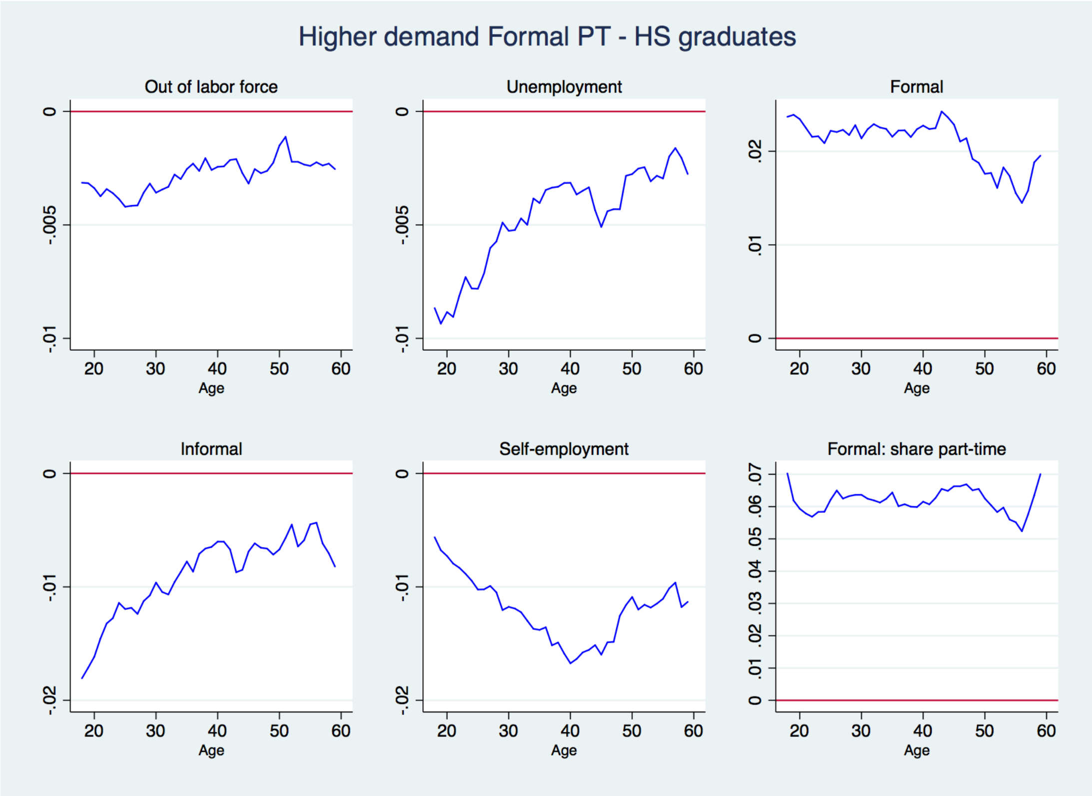


Figure 2.22: Counterfactual exercise 4

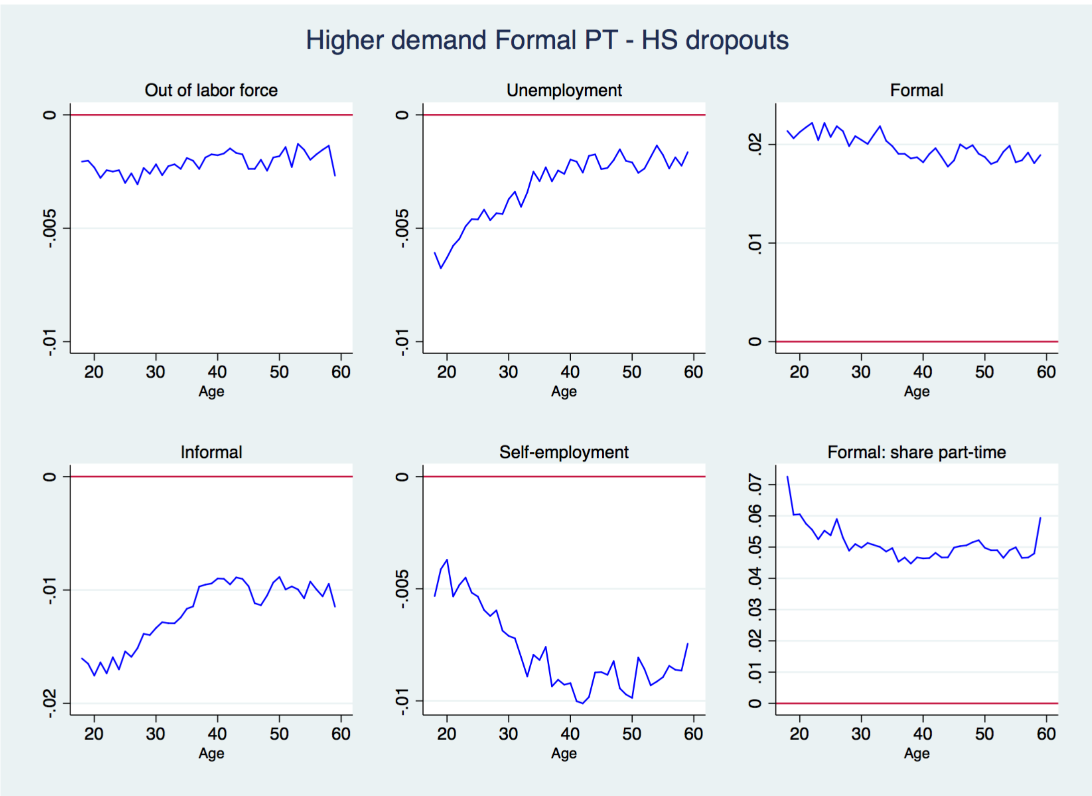


Figure 2.23: Counterfactual exercise 4

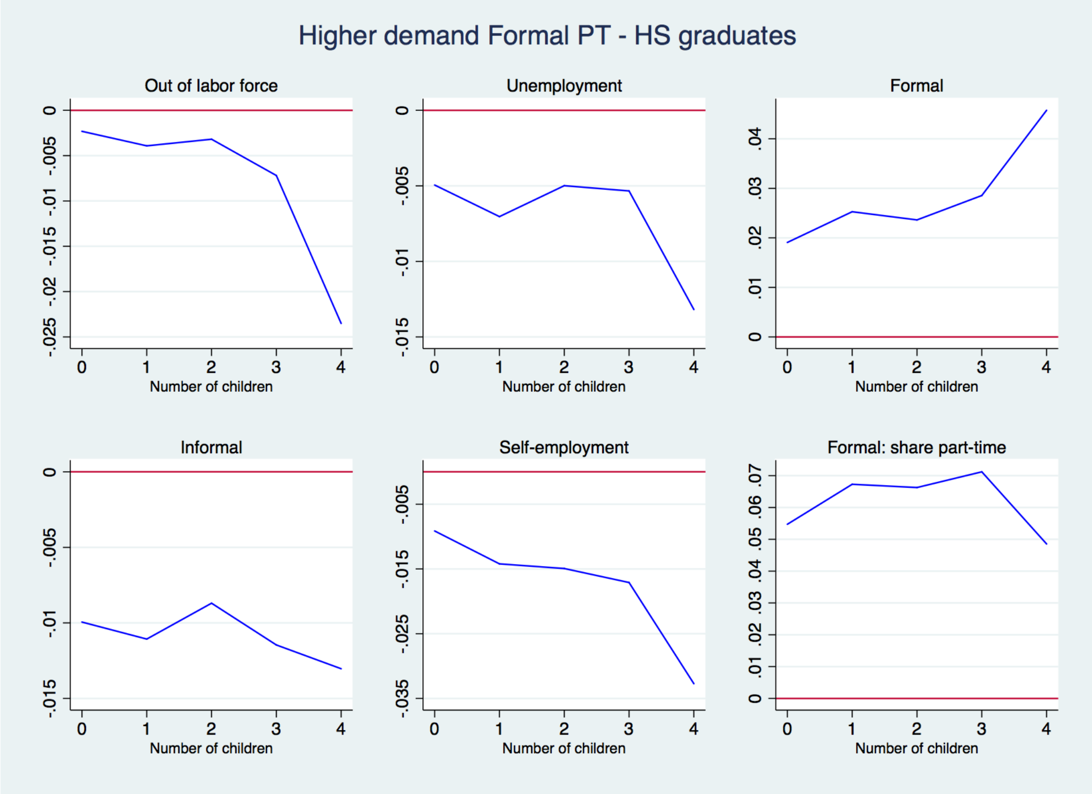


Figure 2.24: Counterfactual exercise 4

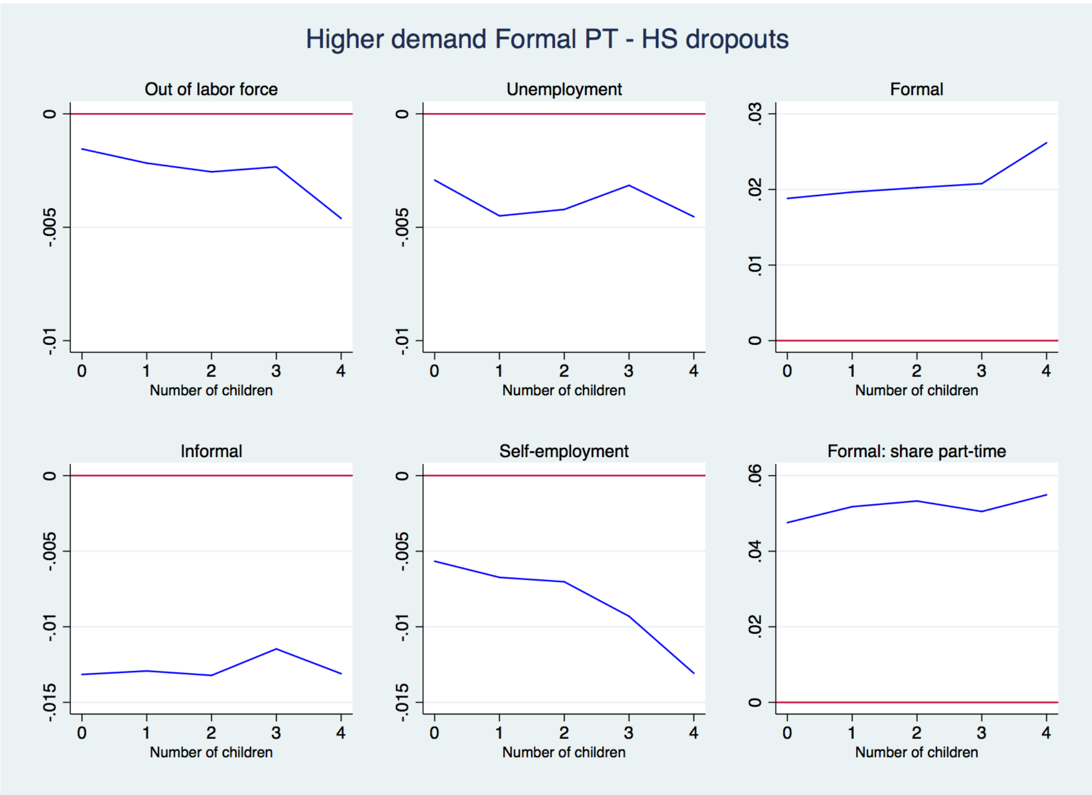


Figure 2.25: Counterfactual exercise 4

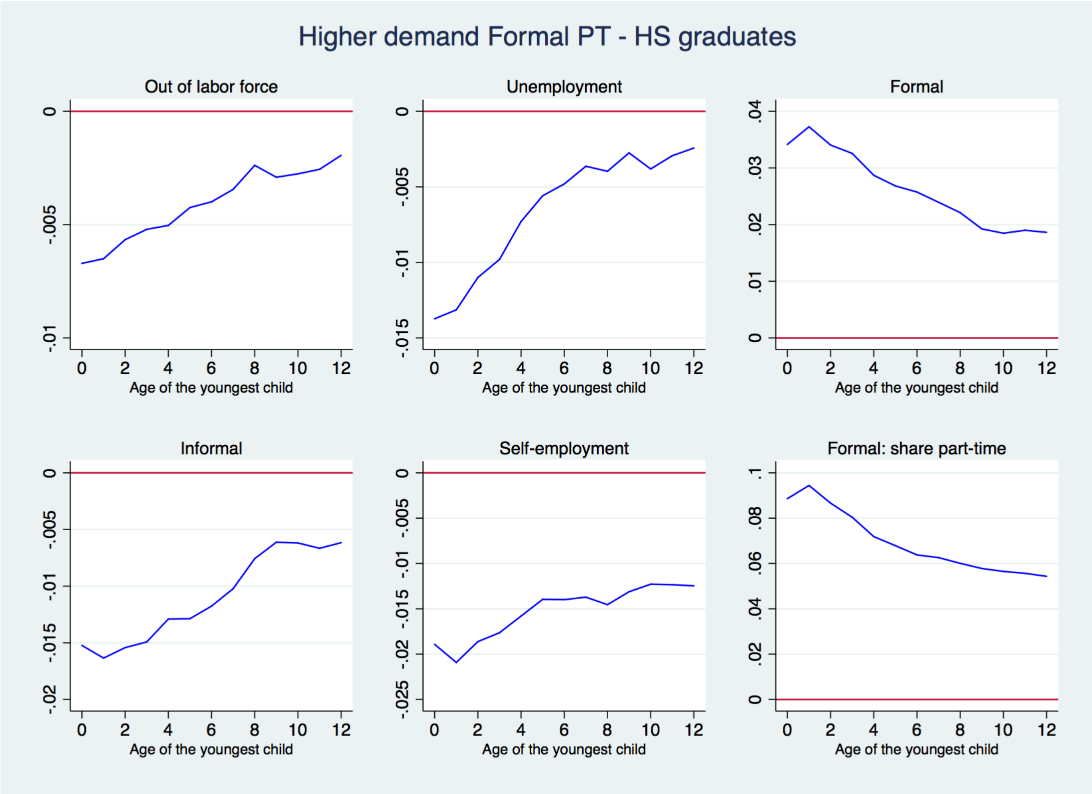
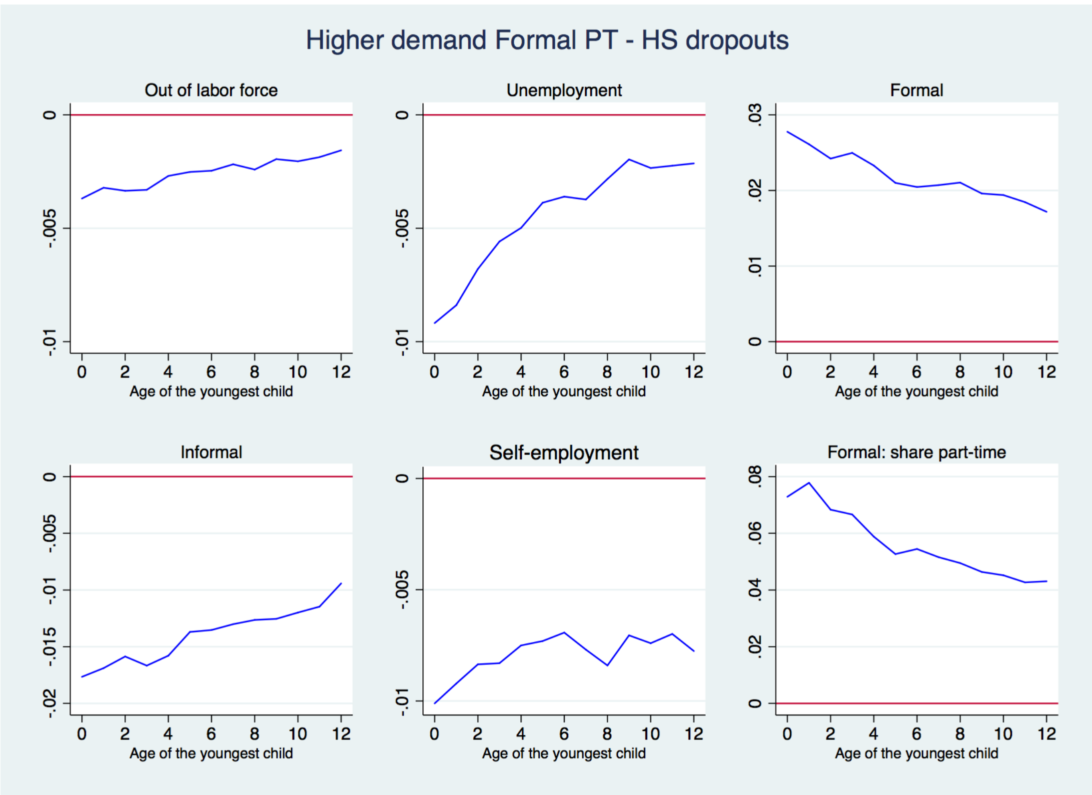


Figure 2.26: Counterfactual exercise 4



3
An alternative approach to the initial conditions problem in dynamic, binary response panel data models with unobserved heterogeneity

3.1
Introduction

In this chapter I propose a new estimation strategy for parametric dynamic panel data models where the dependent variable is discrete. Estimation of such models is complicated by the well-recognized problem of initial conditions, an issue that typically arises when the available panel data does not cover the entire history of the stochastic process. In such dynamic panel data models, the inclusion of time-constant unobserved heterogeneity in the error term implies that the lag of the dependent variable in the first period when choices are modelled is an endogenous explanatory variable. Hence, the presence of lags of the dependent variable preclude the standard procedure of marginalizing the likelihood function with respect to the parametric distribution assumed for the random-effect. At the same time, naive assumptions such as taking the first lag as exogenous, or even modelling the first period choice while assuming the unobserved lag to be zero can deliver seriously biased estimates for the coefficients of exogenous covariates and state dependence.

The first solution to the problem of initial conditions was proposed by Heckman (1981). His suggestion was to treat the dependent variable in the first observed period as random and to specify a reduced-form approximation to its distribution conditional on the permanent unobserved individual heterogeneity term. Later, Wooldridge (2005) proposed a simple alternative solution to the same problem that proved to be very attractive in that it could be implemented in a very simple way with standard statistical software. His suggestion was to approximate the density of the random-effect conditional on the dependent variable in the first period and the exogenous covariates in all periods. One slight shortcoming of the Wooldridge method is that the coefficients of time-constant explanatory variables become unidentifiable if they are also assumed to be included in the approximation of the conditional density of the random-effect.

The method that I propose is closely related to the Heckman approach in that it consists of approximating the conditional distribution of the first observation of the dependent variable. However, instead of specifying a separate reduced-form model for this purpose, my approach builds on the fact that the only missing information that prevents modelling the choice in the initial period within the same structure specified for the subsequent periods is the unobserved lag of the dependent variable. Hence, what I propose is to integrate over the known discrete distribution of this unobserved lag, i.e., to marginalize the likelihood function with respect to it. In other words, this strategy consists of introducing another margin of unobserved heterogeneity with a discrete support, in the spirit of Heckman and Singer (1984). It is clear that the distributions of the unobserved lag and the random-effect will be correlated and thus the crucial step in the implementation of this idea is the choice of how to model this correlation. In the more detailed description below, I show how the specification used for the empirical model itself can be exploited in a natural way to model this correlation, with only minor adaptation due to the issue of missing data on the time-varying exogenous covariates.

Although the idea behind this strategy should be applicable to other classes of dynamic discrete choice models, in this chapter I focus on the simple case of a random-effects probit model with first order state dependence. Some examples of prominent applications of this simple model are Hyslop (1999) and Chay and Hyslop (2000). The analysis of a broader class of discrete choice models will be left for future work.

After presenting the new approach, I conduct a series of Monte-Carlo experiments to assess its relative performance with respect to the standard approaches by Heckman (1981) and Wooldridge (2005). The performance of the three methods is compared in terms of mean relative bias and root mean squared error of the estimates of the coefficients of state dependence and of a single exogenous time-varying covariate.

The results show that the method proposed here fares well in terms of root mean squared error in comparison to the usual approaches by Heckman and Wooldridge, especially when the length of the panel is short. However, my approach appears to deliver slightly biased estimates. Hence, there is a bias/variance trade-off between the three strategies and no approach appears to clearly dominate the others in both accuracy and precision of the estimates. Further investigation of the underlying econometric theory is warranted to determine the asymptotic behavior of the estimates obtained under this new method and is the immediate next step in this project.

Intuitively, the lower variance under my method is likely related to the fact that the empirical model has fewer parameters than Heckman's and Wooldridge's methods. In fact, the number of parameters can be an issue in practical applications when the available sample is small. In typical applications of Heckman's method, as many additional paramaters need to be estimated as the number of exogenous covariates plus two. In typical applications of Wooldridge's method, as many additional parameters need to be estimated as twice the number of time-varying exogenous covariates plus one. In my approach, only one additional parameter is estimated. In a last simulation exercise, I assess the relative performance of my method under a more complex benchmark model, with three exogenous covariates, two being time-varying and one constant. The results show that performance of my method is very satisfactory.

The chapter is organized as follows. In section 2, I first describe the initial conditions problem in the context of a simple dynamic random-effects probit model, and then present the usual approaches to dealing with the issue and my alternative method. In section 3, I describe the Monte-Carlo exercise used to assess the relative performance of my proposed method and the standard approaches and discuss the results. Section 4 concludes.

3.2 Related literature

This work contributes to a literature that studies estimation methods for dynamic, non-linear panel data models with permanent individual unobserved heterogeneity that is modelled parametrically. Naturally, the papers that are closest in spirit to this one are the aforementioned seminal works by Heckman (1981) and Wooldridge (2005).

This paper also contributes to the literature that has examined the relative performance of these methods in numerical experiments and using real data and that has sought to provide guidance on their practical implementation. Using real longitudinal data on welfare participation and labor force participation, Chay and Hyslop (2000) show that assumptions that the initial conditions are exogenous or that the stochastic process is in equilibrium are both rejected due to the relative poor fit. Moreover, estimates of their coefficients of interest under these assumptions are in drastic disagreement with the ones obtained Heckman's method to handle the initial conditions. Arulampalam and Steward (2009) describe a shortcut implementation of Heckman's method that can be done using the GLLAMM package for Stata. Moreover, they also conduct a series of simulation exercises to assess the relative performance of the Heckman's and Wooldridge's methods, and an additional two-step approach by Orme (2001) that is closely related to Heckman's method but that achieves a great reduction in computation time.[1] Their results suggest that all three methods perform equally well in general. Akay (2012) conducts a series of numerical experiments that suggest that Wooldridge's method performs well in small samples when the number of observed periods is relatively long ("longer than 5-8 periods") but that in shorter panels Heckman's approach should be preferred. Finally, Rabe-Heskett and Skrondal (2013) show that performance with the Wooldridge method depends on how the relationship between the individual effect and the exogenous covariates is modelled. In Akay's paper, for example, the individual effect is modelled in such a way that the coefficients of the exogenous covariate in all periods are constrainted to be equal. Rabe-Heskett and Skrondal show that relaxing this assumption and letting the exogenous covariate in the first period have a differential effect substantially improves the performance of Wooldridge's method.

My contribution to this literature is in proposing a new estimation method that appears to be more robust than the standard approaches based on my Monte-Carlo experiments, while apparently being biased only to a very small degree.

3.3 The dynamic RE Probit model and the initial conditions problem

Let y denote a binary response variable that is observed for individual units indexed by $i = 1, ..., N$ over multiple periods indexed by $t = 1, ..., T$. Consider the following model for the joint distribution of $(y_{i1}, ..., y_{iT})$ conditional on a set of exogenous explanatory variables denoted by x_{it}:

$$\begin{aligned} y_{it}^* &= \gamma y_{it-1} + x_{it}'\beta + \varepsilon_{it} \\ y_{it} &= \mathbf{1}[y_{it}^* > 0] \\ \varepsilon_{it} &= \alpha_i + u_{it} \end{aligned}$$

where ε_{it} is an error term that is composed of a constant individual effect α_i and a time-varying idiosyncratic shock u_{it}. α_i and u_{it} are assumed to be independent of each other and of x_{it}. Moreover, u_{it} is assumed to be serially uncorrelated and i.i.d. distributed according to a standard normal distribution $N(0, 1)$. α_i is also assumed to be normally distributed, with distribution $N(0, \sigma^2)$. This is the random-effects probit model with first order state-dependence.

[1]While acknowledging Orme's contribution, his method has not been as widely used, and computation time is not a serious issue for the simple numerical experiments conducted in my paper, which is why I chose to not include it in these experiments.

If the first period of observed data does not coincide with the start of the stochastic process, then it is clear that y_{i1} will be correlated with the unobserved individual effect α_i. This correlation, precisely, is what is termed the initial conditions problem.

If y_{i1} and α_i were independent, then the model above could be estimated by maximizing the likelihood function below:

$$L = \prod_{i=1}^{N} \int_0^{\infty} \prod_{t=2}^{T} \left\{ \mathbf{\Phi}[(2y_{it} - 1)(\gamma y_{it-1} + x_{it}'\beta + \sigma\alpha)] \right\} d\mathbf{\Phi}(\alpha)$$

where $\mathbf{\Phi}(\cdot)$ is the standard normal CDF.

Heckman's method consists of specifying an auxiliary, reduced-form model for the conditional distribution of y_{i1} with respect to the individual effect:

$$y_{i1} = \mathbf{1}[z_{i1}'\lambda + \theta\alpha_i + u_{i1} > 0]$$

where z_{i1}' is a set of observed explanatory variables, θ is a parameter that flexibilizes the correlation between the unobserved terms in the first and subsequent periods and, finally, u_{i1} is a standard idiosyncratic shock that is assumed to be distributed $N(0,1)$. Under this approach, the individual likelihood function of the model becomes:

$$L_i = \int_0^{\infty} \mathbf{\Phi}[(2y_{i1} - 1)(z_{i1}'\lambda + \theta\alpha)] \prod_{t=2}^{T} \left\{ \mathbf{\Phi}[(2y_{it} - 1)(\gamma y_{it-1} + x_{it}'\beta + \sigma\alpha)] \right\} d\mathbf{\Phi}(\alpha)$$

Wooldridge's approach consists, instead, of specifying an approximation for the conditional density of the unobserved individual effect in terms of the dependent variable in the first period and the exogenous covariates in all periods. Following the suggestion in Rabe-Heskett and Skrondal (2013), this approximation is specified as follows:

$$\alpha_i = \delta_0 + x_{i1}'\delta_1 + \bar{x}_i'\delta_2 + \mu y_{i1} + \eta_i$$

where $\bar{x}_i'$ are the time averages of the exogenous covariates and η_i is a new individual effect that is orthogonal to y_{i1} and distributed normal $N(0,\omega)$. Substituting this expression for α_i back in the model and, recognizing that the intercept δ_0 is not separately identifiable, the individual likelihood fucntion under this approach becomes:

$$L_i = \int_0^{\infty} \prod_{t=2}^{T} \left\{ \mathbf{\Phi}[(2y_{it} - 1)(\gamma y_{it-1} + x_{it}'\beta + x_{i1}'\delta_1 + \bar{x}_i'\delta_2 + \mu y_{i1} + \omega\eta)] \right\} d\mathbf{\Phi}(\eta)$$

My suggestion is closely related to Heckman's approach in that it involves modelling the conditional distribution of dependent variable in the initial period. Note that if one has complete data on the response variable and the exogenous covariates covering periods 1 through T, then the only missing information that prevents one from modelling the conditional distribution of y_{i1} using the empirical model exactly as it is specified is that the lag of the response variable y_{i0} is unobserved. Considering this, and given that the support of the distribution of y_{i0} is known to be $\{0,1\}$, what I propose is to assume that y_{i0} is another dimension of unobserved heterogeneity and, hence, the likelihood function can be marginalized with respect to its distribution.

Let $p = \Pr[y_{i0} = 1]$. If we assumed that y_{i0} and α_i were independently distributed, then the individual contribution to the likelihood function could be written as:

$$L_i = \int_0^\infty \Big\{ p\mathbf{\Phi}[(2y_{i1}-1)(\gamma + x'_{i1}\beta + \sigma\alpha)] + \\ (1-p)\mathbf{\Phi}[(2y_{i1}-1)(x'_{i1}\beta + \sigma\alpha)] \Big\} \prod_{t=2}^{T} \Big\{ \mathbf{\Phi}[(2y_{it}-1)(\gamma y_{it-1} + x'_{it}\beta + \sigma\alpha)] \Big\} d\mathbf{\Phi}(\alpha)$$

Obviously, however, y_{i0} and α_i are not independent, in the same way that correlation between y_{i1} and α_i gave rise to the initial conditions problem in the first place. The crucial step in the specification of this method, then, is in how to model the correlation between y_{i0} and α_i. In the numerical exercises presented below, I chose to model p as:

$$p_i = \mathbf{\Phi}(\kappa + \alpha_i)$$

where κ is an auxiliary parameter. Intuitively, it would be ideal to also include a coefficient for the individual effect but, in some simulation tests, the likelihood function was almost flat with respect to this coefficient. I acknowledge that this aspect certainly deserves further and, especially, more rigorous investigation. Nevertheless, the numerical results under this approach as it is are satisfactory enough that they deserve to be considered.

3.4 Monte-Carlo exercise

In this section I describe the simulation exercises designed to assess the performance of the method proposed here. The benchmark data generating process is the same as the one previously used in the related papers:

$$y_{it}^* = \beta_0 + \beta_1 x_{it} + \gamma y_{it-1} + \alpha_i + u_{it}$$
$$y_{it} = \mathbf{1}[y_{it}^* > 0]$$

where u_{it} is generated as i.i.d. $N(0,1)$ and α_i is generated as $N(0,\sigma^2)$. The stochastic process begins at $t = -25$ but data is only "observed" in periods $t = 1, ..., T$. The exogenous covariate is generated as a Nerlove process as follows:

$$x_{it} = 0.1t + 0.5x_{it-1} + U[-0.5, 0.5]$$

with $x_{i,-25} \sim U[-3, 2]$. The individual sequences of the exogenous covariate are held fixed through replications. $y_{i,-25}^*$ is generated as $N(0,1)$. In the baseline experiments, the parameter vector used is $(\beta_0, \beta_1, \gamma, \sigma) = (1.0, -1.0, 0.5, 1.0)$. Experiments are conducted for all combinations of $T = \{3, 4, 5, 7\}$ and $N = \{200, 500, 1000\}$ and are based on 400 replications of this data generating process.

Next, I conduct another set of experiments with $N = 500$ and $T = \{3, 4, 5\}$. In the first set, I change the σ parameter to 2. In the second, I change the state-dependence parameter to $\gamma = 0.75$. In the third, I change the coefficient on the exogenous covariate to $\beta_1 = 0.2$. Finally, in the last set, I use a different process for the exogenous covariate, with higher serial correlation:

$$x_{it} = 0.125 + 0.02t + 0.9x_{it-1} + U[-0.25, 0.25]$$

with $x_{i,-25} \sim U[-1, 2]$.

In a final experiment, I increase the complexity of the model by including one additional time-varying exogenous covariate and one time-constant dummy covariate. The data generating process for this experiment is:

$$y_{it}^* = \beta_0 + \beta_1 x_{it} + \beta_2 z_{it} + \beta_3 d_i + \gamma y_{it-1} + \alpha_i + u_{it}$$
$$y_{it} = \mathbf{1}[y_{it}^* > 0]$$

where d_i is a constant dummy covariate equal to one for 25% of observations. The time varying covariates are generated both as Nerlove processes like the ones above:

$$x_{it} = 0.1t + 0.5x_{it-1} + U[-0.5, 0.5]$$
$$z_{it} = 0.125 + 0.02t + 0.9z_{it-1} + U[-0.25, 0.25]$$

with $x_{i,-25} \sim U[-3, 2]$ and $z_{i,-25} \sim U[-1, 2]$. The number of observations in this experiment is set at 500 and the length of the panel is set at 5. The parameter vector for this experiment is set as $(\beta_0, \beta_1, \beta_2, \beta_3, \gamma, \sigma) = (1.0, -1.0, 0.5, -0.5, 0.5, 1.5)$.

In table 3.1 I present the results of the Monte-Carlo experiments of the benchmark data generating process for different combinations of number of periods and sample sizes. Average relative biases are only larger than 10% when the length of the panel is 3 and sample size is 200 for all three methods. Even though my approach appears to have larger average relative biases overall, when the length of the panel is at least 4 and the sample size at least 500, their magnitude is at most 3%. The clear advantage of my approach is that estimates display the lowest root mean squared error in all experiments, with the difference being particularly noticeable when the panel is very short, at only 3 periods. Note also that in the experiment with the largest sample, with 1000 observations covering 7 periods, average relative biases under my method were below 1% and very close to the results obtained under Heckman's or Wooldridge's method.

In table 3.2, I alter the data generating process as explained above. First, I increase the variance of the random-effect. This strongly affects average relative biases of the state-dependence coefficient under Heckman's method and mine, and it increases RMSE of the estimates relative to the respective lower RE variance experiments for all methods. While Wooldridge's method display lower relative biases, RMSE of its estimates are much larger when the panel is short. Increasing the degree of state-dependence has a much lower impact on the accuracy and precision of the estimates, but RMSE are in fact somewhat larger when compared to the respective experiments in table 3.1. Next, I reduce the magnitude of the coefficient of the exogenous covariate to -0.2. Average relative biases for this coefficient increase noticeably under all three methods, but RMSE is not affected. Lastly, I increase the degree of serial correlation of the exogenous covariate as described above. Average relative biases in all three methods remain unaffected, but RMSE errors become larger. Note that in table 3.2, again, my method displays the lowest RMSE for both coefficients in all cases.

Finally, in table 3.3 I conduct a single experiment with a more complex model, with three exogenous covariates, two time-varying and one constant dummy. I compare relative biases and RMSE of the coefficients on the two time-varying covariates and state-dependence. Once again, average relative biases under my method were larger for the coefficients of both exogenous covariates, but RMSE were the lowest for all three coefficients.

3.5 Conclusion

In this chapter I presented an alternative solution to the problem of initial conditions in dynamic binary response panel data models with unobserved individual heterogeneity. Estimates obtained in the simulation exercises under the method proposed here display lower root mean squared error when compared to the standard approaches by Heckman (1981) and Wooldridge (2005), especially when the length of the panel is short. On the other hand, estimates appear to be biased, although only to a small degree. Even though this work is still very preliminary, the numerical exercises presented here suggest that the idea behind this method certainly deserves further consideration and development. Moreover, in future work I intend to extend this approach to a other classes of non-linear models where the dependent variable is discrete.

Table 3.1: Monte-Carlo experiments

		β_1		γ	
Experiment	Method	Relative bias (%)	RMSE	Relative bias (%)	RMSE
1) T = 3, N = 200	Heckman	12.06	0.450	-24.54	0.546
	Wooldridge	11.02	0.605	-14.90	0.434
	Mine	8.28	0.306	-10.77	0.332
2) T = 4, N = 200	Heckman	2.14	0.291	3.78	0.288
	Wooldridge	2.49	0.331	3.09	0.297
	Mine	2.99	0.223	4.27	0.241
3) T = 5, N = 200	Heckman	-0.17	0.201	3.23	0.192
	Wooldridge	-0.75	0.217	1.98	0.192
	Mine	2.06	0.192	1.46	0.182
4) T = 7, N = 200	Heckman	1.07	0.128	-0.42	0.140
	Wooldridge	0.93	0.132	-1.24	0.142
	Mine	2.06	0.118	-1.22	0.135
5) T = 3, N = 500	Heckman	3.16	0.236	-3.86	0.261
	Wooldridge	2.04	0.307	-1.95	0.280
	Mine	5.97	0.197	-5.80	0.230
6) T = 4, N = 500	Heckman	-0.13	0.155	0.27	0.179
	Wooldridge	-0.75	0.178	-1.03	0.182
	Mine	1.74	0.136	-2.19	0.157
7) T = 5, N = 500	Heckman	1.11	0.118	1.92	0.123
	Wooldridge	0.51	0.135	1.14	0.124
	Mine	3.01	0.108	-0.17	0.117
8) T = 7, N = 500	Heckman	-0.26	0.079	-0.69	0.085
	Wooldridge	-0.44	0.082	-1.10	0.085
	Mine	0.51	0.076	-1.46	0.083
9) T = 3, N = 1000	Heckman	0.13	0.176	-0.71	0.181
	Wooldridge	-0.09	0.231	1.96	0.195
	Mine	2.63	0.126	-3.24	0.161
10) T = 4, N = 1000	Heckman	-0.17	0.113	2.92	0.116
	Wooldridge	-0.87	0.135	1.49	0.117
	Mine	1.92	0.100	-0.12	0.108
11) T = 5, N = 1000	Heckman	0.13	0.083	0.80	0.093
	Wooldridge	-0.31	0.092	-0.05	0.094
	Mine	1.12	0.077	-0.38	0.089
12) T = 7, N = 1000	Heckman	-0.11	0.055	0.81	0.061
	Wooldridge	-0.34	0.057	0.22	0.061
	Mine	0.68	0.049	-0.22	0.059

Notes: All experiments based on 400 Monte-Carlo replications of the DGP. True values of β_1 and γ are -1.0 and 0.5, respectively.

Table 3.2: Monte-Carlo experiments

Experiment	Method	β_1 Relative bias (%)	β_1 RMSE	γ Relative bias (%)	γ RMSE
13) $\sigma = 2$, T = 3	Heckman	0.53	0.284	20.30	0.329
	Wooldridge	3.91	0.385	-3.92	0.432
	Mine	4.07	0.229	12.43	0.268
14) $\sigma = 2$, T = 4	Heckman	-4.79	0.206	23.40	0.208
	Wooldridge	-0.16	0.226	3.78	0.212
	Mine	-0.57	0.174	18.06	0.180
15) $\sigma = 2$, T = 5	Heckman	-3.12	0.153	17.52	0.166
	Wooldridge	0.86	0.165	-2.03	0.155
	Mine	-0.78	0.136	14.24	0.150
16) $\gamma = 0.75$, T = 3	Heckman	2.64	0.265	1.20	0.331
	Wooldridge	0.62	0.340	0.85	0.365
	Mine	6.07	0.222	-1.91	0.281
17) $\gamma = 0.75$, T = 4	Heckman	1.86	0.184	-1.46	0.200
	Wooldridge	1.79	0.196	-2.07	0.207
	Mine	3.37	0.161	-2.59	0.179
18) $\gamma = 0.75$, T = 5	Heckman	0.51	0.135	0.28	0.151
	Wooldridge	0.22	0.139	-0.15	0.150
	Mine	2.34	0.121	-1.50	0.144
19) $\beta = -0.2$, T = 3	Heckman	7.86	0.206	-3.21	0.350
	Wooldridge	9.67	0.310	-2.49	0.350
	Mine	10.05	0.168	-0.42	0.285
20) $\beta = -0.2$, T = 4	Heckman	4.76	0.147	-3.50	0.193
	Wooldridge	6.18	0.181	-3.58	0.195
	Mine	7.87	0.131	-5.00	0.169
21) $\beta = -0.2$, T = 5	Heckman	1.83	0.117	0.43	0.145
	Wooldridge	2.08	0.125	0.15	0.146
	Mine	6.13	0.110	-1.17	0.139
22) Alternative x_{it} , T = 3	Heckman	1.01	0.307	-0.55	0.330
	Wooldridge	1.28	0.536	-2.13	0.336
	Mine	5.44	0.230	-5.07	0.262
23) Alternative x_{it} , T = 4	Heckman	-0.23	0.189	1.42	0.195
	Wooldridge	0.86	0.273	0.26	0.200
	Mine	3.40	0.174	-2.91	0.174
24) Alternative x_{it} , T = 5	Heckman	0.44	0.154	0.46	0.136
	Wooldridge	-1.31	0.201	-0.65	0.138
	Mine	3.37	0.147	-3.03	0.131

Notes: All experiments based on 400 Monte-Carlo replications of the DGP. N = 500.

Table 3.3: Monte-Carlo experiments - DGP with 3 exogenous covariates

		β_1		γ		β_2	
Experiment	Method	Relative bias (%)	RMSE	Relative bias (%)	RMSE	Relative bias (%)	RMSE
25) 3 Exog. Covariates	Heckman	0.11	0.100	4.06	0.095	1.63	0.129
N = 500, T = 5	Wooldridge	-0.00	0.121	-0.28	0.101	0.69	0.177
	Mine	1.99	0.095	0.67	0.090	3.98	0.126

Notes: All experiments based on 400 Monte-Carlo replications of the DGP. True values of β_1, β_2 and γ are -1.0, 0.5 and 0.5, respectively.

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A
Appendix to Chapter 1

In this appendix I present the results of the estimation of models I and II without dropping tenured public employees form the sample and classifying these as "formal" workers. These results can be found in tables A.1 and A.2. All the coefficients associated with the fertility variables are of similar magnitude and display the same patterns.

Table A.1: Estimation results – High school or less – Tenured public employment classified as formal

				Continuously married		
	Model I		Model II	Model I		Model II
	(1)		(2)	(3)		(4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.215***	-0.136***	-0.172***	-0.248***	-0.201***	-0.239***
	(0.024)	(0.039)	(0.048)	(0.031)	(0.050)	(0.063)
# Children 3 – 5	-0.089***	-0.110**	-0.132**	-0.119***	-0.135**	-0.170**
	(0.024)	(0.036)	(0.044)	(0.030)	(0.048)	(0.060)
# Children 6 – 11	-0.032	-0.096***	-0.096**	-0.019	-0.101*	-0.119*
	(0.020)	(0.028)	(0.034)	(0.025)	(0.040)	(0.050)
# Children 12 – 17	0.016	-0.039	-0.038	0.013	-0.061	-0.053
	(0.016)	(0.023)	(0.027)	(0.022)	(0.032)	(0.039)
Married / cohab.	-0.763***	-0.115***	0.048			
	(0.020)	(0.028)	(0.033)			
Partner college educated				-0.122***	-0.064	-0.089
				(0.033)	(0.049)	(0.060)
Log partner's earnings	-0.079***	0.062**	0.026	-0.182***	0.113***	0.046
	(0.013)	(0.020)	(0.024)	(0.018)	(0.028)	(0.034)
Partner unemployed	0.295***	-0.112**	-0.151***	0.179***	-0.045	-0.148*
	(0.024)	(0.037)	(0.045)	(0.032)	(0.050)	(0.062)
Lagged participation (p_{t-1})	0.701***	0.513***		0.727***	0.486***	
	(0.013)	(0.030)		(0.017)	(0.038)	
Lagged formal job (f_{t-1})	0.810***	1.371***	1.500***	0.829***	1.425***	1.577***
	(0.024)	(0.019)	(0.032)	(0.029)	(0.026)	(0.045)
RE Variance	1.981***	3.502***	3.481***	2.094***	3.620***	3.729***
	(0.040)	(0.118)	(0.134)	(0.055)	(0.164)	(0.200)
RE Correlation	0.130***			0.120***		
	(0.015)			(0.017)		
Measurement error rate	0.018***		0.025***	0.018***		0.028***
	(0.001)		(0.002)	(0.001)		(0.003)
Average Log-likelihood	-3.060		-1.639	-3.055		-1.603
Wald statistics						
H_0: δ CRE = 0	496.19 (0.000)			358.99 (0.000)		
H_0: ζ CRE = 0	35.47 (0.000)		28.20 (0.000)	19.51 (0.001)		31.38 (0.000)
H_0: All CRE = 0	541.90 (0.000)			383.44 (0.000)		
Sample size	88,658		45,852	52,707		24,960

Notes: All specifications include as controls: a quadratic in age, race, completed secondary education state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses except for p -values for Wald statistics.
(*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)

Table A.2: Estimation results – College educated – Tenured public employment classified as formal

				Continuously married		
	Model I		Model II	Model I		Model II
	(1)		(2)	(3)		(4)
Dependent variable	p_t	f_t	f_t	p_t	f_t	f_t
# Children 0 – 2	-0.299***	-0.008	-0.029	-0.304***	0.041	-0.005
	(0.055)	(0.054)	(0.058)	(0.063)	(0.063)	(0.068)
# Children 3 – 5	-0.182**	-0.080	-0.053	-0.158*	-0.055	-0.038
	(0.063)	(0.059)	(0.062)	(0.073)	(0.071)	(0.076)
# Children 6 – 11	-0.224***	-0.081	-0.050	-0.216**	-0.056	-0.053
	(0.062)	(0.054)	(0.057)	(0.076)	(0.069)	(0.073)
# Children 12 – 17	-0.223***	-0.048	-0.029	-0.275***	-0.035	-0.036
	(0.058)	(0.048)	(0.049)	(0.072)	(0.064)	(0.068)
Married / cohab.	-0.409***	0.202***	0.275***			
	(0.053)	(0.049)	(0.048)			
Partner college educated				-0.065	-0.188***	-0.047
				(0.039)	(0.046)	(0.044)
Log partner's earnings	-0.109***	-0.159***	-0.136***	-0.129***	-0.138***	-0.145***
	(0.020)	(0.022)	(0.022)	(0.027)	(0.028)	(0.030)
Partner unemployed	0.503***	-0.176**	-0.187**	0.342**	-0.204*	-0.262**
	(0.095)	(0.067)	(0.071)	(0.121)	(0.091)	(0.098)
Lagged participation (p_{t-1})	0.877***	0.386***		0.967***	0.499***	
	(0.035)	(0.064)		(0.046)	(0.083)	
Lagged formal job (f_{t-1})	0.420***	0.761***	0.812***	0.449***	0.753***	0.845***
	(0.054)	(0.023)	(0.036)	(0.064)	(0.032)	(0.045)
RE Variance	1.688***	4.714***	4.008***	1.863***	5.370***	4.558***
	(0.077)	(0.161)	(0.152)	(0.104)	(0.232)	(0.238)
RE Correlation	0.436***			0.408***		
	(0.033)			(0.037)		
Measurement error rate	0.012***		0.004	0.011***		0.012
	(0.002)		(0.006)	(0.002)		(0.006)
Average Log-likelihood	-2.579		-1.714	-2.559		-1.636
Wald statistics						
H_0: δ CRE = 0	66.42 (0.000)			52.94 (0.000)		
H_0: ζ CRE = 0	2.91 (0.574)		3.59 (0.465)	5.47 (0.242)		2.45 (0.654)
H_0: All CRE = 0	68.94 (0.000)			58.15 (0.000)		
Sample size	30,842		25,400	17,298		13,859

Notes: All specifications include as controls: a quadratic in age, race, state fixed-effects and a linear yearly trend. Columns (1) and (2) also include an indicator for not being the spouse of or the head of household itself. Standard errors in parentheses except for p-values for Wald statistics. (*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$)