Pontifícia Universidade Católica
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Stability and Perturbativity constraints on Higgs Portal Models

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Física of PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Ciências – Física.

Advisor: Prof. Gero Arthur Hubertus Thilo Freiherr Von Gersdorff

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Abstract

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The Standard Model is one of the most successful theories in particle physics. With the discovery of the Higgs boson, a new pathway has been opened to investigate possible new physics interacting through the Higgs portal, including scenarios motivated by dark matter and baryogenesis. Supposing there is a neutral scalar state in the Standard Model coupled to it only through the Higgs portal, we investigate the potential stability and the Landau poles of the extended Standard Model potential. We focus on the regime in which the scalars are primarily generated via an off-shell Higgs. We predict the available parameter space to probe the theory for different mass values.

Keywords

Standard Model; Physics Beyond the Standard Model; ; Potential Stability; Effective Potential; Renormalization Group Equations; Dark Matter.

Resumo

Marinho Pereira de Melo, Marcus Vinícius; Freiherr Von Gersdorff, Gero Arthur Hubertus Thilo. **Vínculos de Estabilidade e Perturbatividade em Modelos de Portal de Higgs**. Rio de Janeiro, 2018. 81p. Dissertação de Mestrado – Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro.

O Modelo Padrão é uma das teorias mais bem sucedidas da física de partículas. Com a descoberta do bóson de Higgs, além de ter sido uma demonstração robusta do poder preditivo do Modelo Padrão, foi aberto um novo caminho para a investigação de nova física interagindo por meio do portal de Higgs, incluindo cenários motivados por matéria escura e bariogênese. Investigamos a estabilidade do potencial e os pólos de Landau do Modelo Padrão sob efeito da interação entre o bóson de Higgs e uma partícula escalar. Focamos no regime onde os escalares são gerados primariamente via um off-shell Higgs. Prevemos o espaço de parâmetros disponível para acessar a teoria em diferentes valores de massa do campo escalar.

Palavras-chave

Física Além do Modelo Padrão; Estabilidade do Potencial; Potencial Efetivo; Equações do Grupo de Renormalização; Matéria Escura.

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The game of science is, in principle, without end. He who decides one day that scientific statements do not call for any further test, and that they can be regarded as finally verified, retires from the game.

Karl Popper, *The Logic of Scientific Discovery.*

1
Introduction

There are two widely accepted scientific theories which, if unified, could offer a viable theoretical description of our universe. In chronological order, the first theory is General Relativity, broadly known even among the general public. It describes the gravitational field as a geometric property of space and time, or space-time, since they are no longer independent physical quantities. The second theory is the Standard Model. It is the theory which best explains the physics of elementary particles and its fundamental interactions.

The Standard Model in particle physics has been found to be in remarkable agreement with experiments. For instance, in the fermionic sector we have the discovery of the top and charm and in the bosonic sector the discovery of the W and Z bosons, the direct measurement of CP violation in K and B systems. In addition to that, there is also the discovery of one of the most awaited particle predicted by the Standard Model, the Higgs boson (1)(2)(3).

However, despite the staggering demonstration of predictive power shown by the Standard Model, we must also take into account its problems. For instance, if we assume that the Standard Model laws are true up to very high energies, the precise value of the Higgs mass is rather intriguing, because it implies that for energies in the neighbourhood of 10^{10}GeV the Higgs self-coupling could possibly become negative, allowing the Standard Model potential to have arbitrarily large negative energy values, that is, the electroweak vacuum would no longer be absolutely stable (4)(5)(6)(7), but metastable with a lifetime longer than the age of the Universe for decay either via zero temperature quantum fluctuations or thermal fluctuations. For a more recent discussion on this topic, reference (8) has recently updated the famous stability/metastability phase diagram of the Standard Model. Besides that, there are several other observed phenomena which the theory cannot explain

(i) Neutrino masses and the mixture of flavour states. In the Standard Model the global baryon symmetry $U(1)_B$ as well as each of the lepton numbers are anomalous, but the linear combination $B - L$ is anomaly free. If we assume the existence of only left-handed neutrinos, in principle we could write down a Majorana mass term. However, such term violates the $B-L$

symmetry by two units and since this symmetry is non-anomalous, by construction, it prevents neutrino masses not only at tree level, but to all orders in perturbation theory and also forbids neutrino masses even at the nonperturbative level. Therefore, the renormalizable Standard Model gives the exact prediction $m_\nu^i = 0$, and, as a consequence, neutrinos would not oscillate.

(ii) There is evidence that the mass density of the Universe is made up of some nonbaryonic form of dark matter (9) (preferably cold dark matter (10)), which is neutral, colourless and massive.

(iii) The slight excess of matter over antimatter. The parameters of the Standard Model do not allow the correct prediction of the abundance of matter over antimatter, but, in principle, it has all the necessary ingredient (11) just not in the right quantity.

And, of course, there are more fundamental features of nature that help us build up a case suggesting the incompleteness of the Standard Model

(i) The impossibility of treating quantum gravity at energy scales greater than the Planck scale. What are the gravitational quantum states and how are they coupled to the Standard Model at the Planck scale?

(ii) The hierarchical value observed among the cosmological constant and the other scales. How come it is so small, given that it receives contributions from QCD and the electroweak sector?

(iii) The hierarchy problem involving the fine-tuning in the Higgs mass, whose pole mass m_p is approximately 125GeV. If we assume that the Standard Model is valid up to the Planck scale $\Lambda \approx 10^{19}$GeV, then $m_p^2/\Lambda^2 \approx 10^{-34}$.

(iv) In the current Universe, the density of matter, radiation and energy are approximately equal.

(v) Although fermion masses are free parameters in the Standard Model, there is a hierarchy among the masses. Is there any underlying guiding principle explaining this fact?

(vi) The charge quantization. The very first Grand Unified Theory, investigated in (12), based on the $SU(5)$ can in principle explain this, but its predictions does not agree with the experimental data. For instance, this GUT predicts $\sin^2\theta_W = 0.21$ at the $W-$boson mass, whereas the experimentally observed value is $\sin^2\theta_W = 0.23161 \pm 0.00018$. Nonetheless, we could still use it as a framework to investigate other Grand Unified Theories.

(vii) The existence of exactly three generations of fermions. The measured value of the Z boson width agrees very well the existence of three generations of fermions.

(viii) The $3 + 1$ structure of space-time.

Despite all those evidences, performing changes in the Standard Model is no simple task. It has a delicate structure based on fundamental symmetry principles, some of which are shared even at the quantum level. Moreover, the new theory must also be in agreement with the results already explained by the Standard Model. Among all the possibilities, the simplest modification allowed by the renormalizable Standard Model is an additional neutral scalar state. In particular, the discovery of the Higgs boson could possibly provide a window into the new physics associated with such extension. This has to do with the rigidity of the spontaneous symmetry breaking pattern limiting the allowed interactions of the Higgs boson. So, it makes the Higgs particle one of the most sensitive states in the Standard Model in such a manner that if any new physics manifested itself in this sector, the deviations from the Standard Model would be patently clear. Having said that, we shall try to explore this window into the New Physics by evaluating its consequences on the potential stability and Landau Poles, in a general case and for some particular applications.

This dissertation is organized as follows. We begin in Chapter 2 by reviewing the building principles of the Standard Model, the guidelines to construct a Lagrangian based upon phenomenological grounds and in the Higgs sector we are going to describe the spontaneous symmetry breaking mechanism. In Chapter 3 we review the relevant theoretical tools to our investigation, the background field method and the concept of renormalization group equations. In Chapter 4, we use those techniques to calculate the Standard Model potential including one-loop effects and, then, employ it in the derivation of the $\beta-$functions for the Standard Model. In Chapter 5 we introduce the New Physics and investigate the stability of the modified Standard Model potential. Next we employ the mathematical machinery developed in the earlier chapters to evaluate the modifications caused by the scalar extension in the effective potential and the β-functions. In Chapter 6 we present the available parameter space for the theory. We conclude in Chapter 7 and discuss some results in the literature.

2
The Standard Model

Before talking about any physics beyond the Standard Model, we need to first define what we mean by Standard Model. Please note that a more profound and detailed exposition can be found in the references (13)(14). The fundamental elements that the theory is built on are essentially two. First we have the observed matter — three generations of leptons and quarks —, the Higgs doublet, and gauge fields. Second, all renormalizable (marginal or relevant) interactions allowed by the field content and gauge symmetries; this has to do with Gell-Mann's Totalitarian Principle, which states that "Everything not forbidden is compulsory". The seminal work of references (15)(16) corroborates the efficiency of employing this line of reasoning. If we are to propose any sort of New Physics, then what we are bound to do is to introduce anything beyond this recipe, that is, new fields or irrelevant operators.

Moreover, the Standard Model of particle physics is the most precise theory ever built to describe the properties and interactions of elementary particles. In the latter, the theory provides a solid basis to study three of the four fundamental interactions: electromagnetism, weak interaction and strong interaction. In other words, the Standard Model is made up of fundamental particles which are either the building blocks of matter, called fermions, or the mediator of interactions, called bosons.

As far as we know, the former set of particles is composed of twelve fermions; six quarks, three neutral fermions and three charged fermions, while the latter is known to have four types of bosons. Among the boson type, we have eight different gluons, each corresponding to a generator of the $SU(3)_C$ gauge group, which are the mediators of the strong force. There are three weak bosons associated with the $SU(2)_L$ gauge group and one gauge boson corresponding to the $U(1)_Y$ gauge group, the hypercharge quantum number. The last boson is the Higgs boson. It plays a special role which is going to be discussed soon. The fermions come in three generations of leptons and quarks and within each generation they are divided into left-handed $SU(2)$ doublets and right-handed singlets. After this brief introduction, below we shall introduce the mathematical structure of the Standard Model.

In mathematical terms, the Standard Model is a quantum field theory with a local gauge symmetry group

$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

which is spontaneously broken into

$$G_{\text{SM}} \to SU(3)_C \times U(1)_{\text{EM}}.$$

Furthermore, there are with matter fields, comprising three fermion generations or families, (observables suggest that three is the magic number), each consisting of five representations of the local gauge group:

$$Q_{L_i}(3,2)_{1/6}, \quad u_{R_i}(3,1)_{2/3}, \quad d_{R_i}(3,1)_{-1/3}, \quad \ell_{L_i}(1,2)_{-1/2}, \quad e_{R_i}(1,1)_{-1}. \tag{2-1}$$

This notation is a short form of indicating which representation of the gauge group, G_{SM}, is being carried by a given particle. For instance, consider an arbitrary state $X(i,j)_k$, the index i is associated with the $SU(3)_C$, the index j denotes the $SU(2)_L$ representation and k defines the hypercharge, $U(1)_Y$. The chiral nature of the Standard Model, i.e. parity violation, manifests itself through the different representations of the fermions, left-handed fermions transform as doublets under $SU(2)$, while right-handed fermions transform as singlets. The left-handed quarks grouped into $SU(2)$ doublets are given by

$$Q^w_{L_i} = \begin{pmatrix} u^w_L \\ d^w_L \end{pmatrix}_i , \tag{2-2}$$

where i labels the family, w is a superscript assigned to remind us that these are weak eigenstates, that is, they change into each other through absorption or emission of a gauge boson. The right-handed quarks are singlets; $u^w_{R_i}$ and $d^w_{R_i}$.

The left-handed leptons are grouped into $SU(2)$ doublets

$$\ell^w_{L_i} = \begin{pmatrix} \nu^w_{e_L} \\ e^w_L \end{pmatrix}_i . \tag{2-3}$$

The right-handed lepton and right-handed neutrinos, $e^w_{R_i}$ and $\nu^w_{R_i}$, respectively, are an $SU(2)$ singlet. So far, the index w has been used just to illustrate the nature of these doublets, but hereafter to avoid visual cluttering, we shall drop this notation. In order to complete our set of necessary particles, the last one we must add is a complex scalar H carrying the $(1,2)_{1/2}$ representation of

$G_{\rm SM}$. It can be checked that the aforementioned representation is the smallest one satisfying the required spontaneous symmetry breaking pattern and also gauge invariance. This complex $SU(2)$ doublet of scalar fields

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{2-4}$$

is known as the Higgs boson.
The hypercharge, Y, assignments are chosen so that the charge operator Q is given by $Q = T^3 + Y$, where T^3 is the third component of the weak isospin. The hypercharge of each one of the particles above are fully described in equation (2-1). Next we shall focus on each specific sector of the Standard Model.

2.1 The Standard Model Lagrangian

In order to appreciate the Standard Model in its full-fledged form, we shall first note that the most general renormalizable Lagrangian with a scalar and fermions may be decomposed into four pieces

$$\mathcal{L}_{\rm SM} = \mathcal{L}_K + \mathcal{L}_\Psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_H. \tag{2-5}$$

The term $\mathcal{L}_K$ describes the propagation through space-time of the matter content in vacuum, as well as gauge interactions. The second term has the fermion masses. The third term generates the Yukawa interactions. The last piece of our Lagrangian is reserved to the Higgs doublet. When writing down any of these terms, we should always bear in mind that the Lagrangian must be locally invariant under the group of internal symmetries.

2.1.1 Kinetic Sector

The gauge boson degrees of freedom needed by the local symmetry are

$$G^\mu_a(8,1)_0, \quad W^\mu_a(1,3)_0, \quad B^\mu(1,1)_0, \tag{2-6}$$

the index a has to do with the non-Abelian structure of the Standard Model. The number of gauge bosons (vector fields) is equal to the number of generators of the corresponding gauge group. Next, we employ these gauge bosons to write down their corresponding field strength

$$
\begin{aligned}
G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f_{abc} G_b^\mu G_c^\nu \\
W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g \epsilon_{abc} W_b^\mu W_c^\nu \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu
\end{aligned}
$$

where f_{abc} and ϵ_{abc} are the structure constants of $SU(3)_C$ and $SU(2)_L$, respectively. Next we define the covariant derivative as

$$D_\mu = \partial_\mu - ig' Y B_\mu - ig W_\mu^a T^a - i g_s G_\mu^b L^b. \tag{2-7}$$

Here the symbol Y denotes the $U(1)_Y$ hypercharges. The T^a stands for the generators of the $SU(2)_L$, the set of Pauli matrices $\sigma^a/2$, for doublets, in which $a = (1, 2, 3)$. The element, L^b, represents the generators of the $SU(3)_C$, the set of Gell-Mann matrices $\lambda^b/2$, for triplets, where b runs from one to eight. It is important to bear in mind that the form of the covariant derivative is associated with the field it is being applied to. Explicitly, the covariant derivative acting on the various fermions and scalars fields are given by

$$
\begin{aligned}
D_\mu Q_{L_i} &= \left(\partial_\mu - ig' B_\mu Y_Q - ig W_\mu^a T^a - i g_s G_\mu^a L^a\right) Q_{L_i}, \\
D_\mu \ell_{L_i} &= \left(\partial_\mu - ig' B_\mu Y_\ell - ig W_\mu^a T^a\right) \ell_{L_i}, \\
D_\mu H &= \left(\partial_\mu - ig' B_\mu Y_H - ig W_\mu^a T^a\right) H, \\
D_\mu u_{R_i} &= \left(\partial_\mu - ig' B_\mu Y_u - i g_s G_\mu^a L^a\right) u_{R_i}, \\
D_\mu d_{R_i} &= \left(\partial_\mu - ig' B_\mu Y_d - i g_s G_\mu^a L^a\right) d_{R_i}, \\
D_\mu e_{R_i} &= \left(\partial_\mu - ig' B_\mu Y_e\right) e_{R_i}.
\end{aligned}
$$

Now, we write down the specific form of the Lagrangian involving the gauge-invariant kinetic operators

$$
\begin{aligned}
\mathcal{L}[X_{\mu\nu}^a, \Psi_j, H] = &- \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
&+ i \bar{Q}_{L_j} \not{D} Q_{L_j} + i \bar{\ell}_{L_j} \not{D} \ell_{L_j} + i \bar{u}_{R_j} \not{D} u_{R_j} + i \bar{d}_{R_j} \not{D} d_{R_j} \\
&+ i \bar{e}_{R_j} \not{D} e_{R_j} + (D_\mu H)^\dagger (D^\mu H).
\end{aligned}
$$

Those left- and right-handed fermions are in different representations, so we are not allowed to write down a mass term such as $\bar{f}_L f_R + h.c.$, Dirac mass term, because the fermions are assigned to chiral representations of the gauge symmetry. Moreover, the gauge symmetry also forbids the presence of mass terms for the gauge bosons. Again, it spoils the gauge invariance. Nevertheless, if we were to simply add a mass term for the gauge bosons, the Lagrangian can still be made gauge invariant by introducing a Stückelberg field. Although this method can mimic the Higgs mechanism in the $U(1)_Y$ hypercharge sector of the Standard Model, for a non-Abelian gauge theory the Lagrangian ceases to be renormalizable. Therefore, to solve the problem of having massive gauge bosons and fermions masses, we shall employ the spontaneous symmetry breaking. In the next section we will discuss how this can be done.

2.1.2 Spontaneous Symmetry Breaking

The concept of spontaneous symmetry breaking is one of the most important aspects of a quantum field theory. While it might have seemed impossible to build up a theory of massive gauge bosons without explicit breaking the gauge symmetry, by implementing the spontaneous symmetry breaking idea we can circumvent such problem. By definition, when a system undergoes a process of spontaneous symmetry breaking, it means that the Lagrangian is invariant under a given symmetry, but the ground state of the theory is not. The difference between this and an explicit symmetry breaking is that in the latter there was never a symmetry to begin with in the system.

Additionally, the consequence of having a spontaneous symmetry breaking in the theory depends on the nature of the symmetry. For instance, consider a continuous global symmetry such as $\phi(x) \to e^{i\alpha}\phi(x)$ for any real constant α, an automatic implication of breaking this symmetry is the appearance of massless particles and the manifestation of long-range correlations. On the other hand, the resulting physics from breaking a gauged symmetry such as $\phi(x) \to e^{i\alpha(x)}\phi(x)$ with an associated massless gauge field $A_\mu(x)$ involves a more elaborated scheme, in which when the theory is in the broken phase, the gauge boson will acquire mass and there will be no massless particles.

To illustrate this concept, let us work out a simple case. Consider the following Lagrangian of a self-interacting complex scalar field at tree-level

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial_\mu\phi)^\dagger(\partial^\mu\phi) - \frac{1}{2}m^2\phi^\dagger\phi - \frac{1}{4}\lambda(\phi^\dagger\phi)^2, \tag{2-8}$$

where we have used a normalization such that when we write down the ϕ field as a linear combination of two real scalar, $\phi_1 + i\phi_2$, it will be canonically normalized. This Lagrangian clearly is endowed with a $U(1)$ global symmetry, that is,

$$\mathcal{L}[\phi(x)e^{i\theta}] = \mathcal{L}[\phi]. \tag{2-9}$$

The ground state of the theory is situated at the critical points of the potential, which also results in the lowest of all possible values for the potential. So, to determine the former, we must find the solutions $\bar{\phi}_j$ such that the first-order derivatives vanish

$$\frac{\partial V(\phi)}{\partial \phi_j} = [m^2 + \lambda(\bar{\phi}_i \bar{\phi}_i)]\bar{\phi}_j = 0. \tag{2-10}$$

In the equation above, (2-10), assuming $\lambda > 0$, two cases are to be considered. First, when the mass parameter is positive, there is a single ground state and the vacuum state still enjoys the same symmetry present in the Lagrangian. However, in the second case, when $m^2 < 0$, the zero field configuration is no longer a viable solution and we say that the field has acquired a nonzero vacuum expectation value. Some caution must be taken here, the first impression we have is that this is no longer a physical particle, but a tachyon. Indeed, the spacelike momentum allows to communicate faster than the speed of light, but this problem has to do with the fact that the theory is spontaneously broken, as soon as we expand the potential around one of its minima, the physics will be well defined again. In particular, the ground state of this potential is degenerate

$$\bar{\phi}_1^2 + \bar{\phi}_2^2 = -\frac{m^2}{\lambda}. \tag{2-11}$$

This solution is more transparent if we parametrize it in terms of an angle θ and a radius f, i.e. $\bar{\phi}_j = f\cos(\theta - (j-1)\pi/2)$ [1]. We immediately find the solution

$$f = \sqrt{-\frac{m^2}{\lambda}}, \tag{2-12}$$

which denotes the radius of the 2-dimensional sphere of minima.

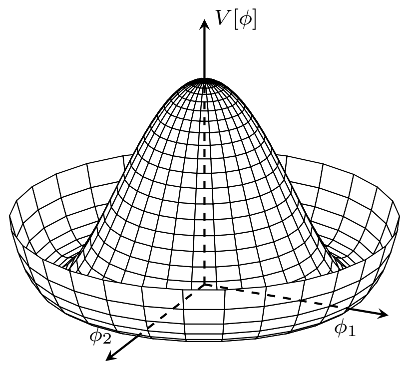

Figure 2.1: Mexican hat potential. The potential V depends on the complex field ϕ and the $U(1)$ symmetry allows us to perform rotations around the vertical axis. There are two frequencies of oscillation; the radial (longitudinal) mode is massive and the angular (transversal) is massless.

[1] $\cos(\theta - \pi/2) = \sin\theta$

The mass matrix in a tree-level approximation evaluated at the critical point $\bar{\phi}$ is

$$\frac{\partial^2 V}{\partial \phi_j \phi_k} = -2m^2 \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}, \tag{2-13}$$

with eigenvalues 0 and $-2m^2$. As we expected, the spontaneous symmetry breaking generated one massless particle, known as Goldstone boson.

Consider the quadratic expansion of the potential

$$V(\phi + \bar{\phi}) - V(\bar{\phi}) = \frac{1}{2!}\sum_{ij} \phi_i \frac{\partial^2 V(\bar{\phi})}{\partial \phi^2}\phi_j \tag{2-14}$$

and perform a rotation $\phi \to R\phi$, where R is a symmetric matrix that diagonalizes the mass matrix

$$V(\phi + \bar{\phi}) - V(\bar{\phi}) = \frac{1}{2}\sum \phi_i m_i^2 \phi_i. \tag{2-15}$$

In analogy with the harmonic oscillator, this equation shows that there will be oscillations in a flat region associated with massless particles and also oscillations in regions where there is a nonzero curvature, these are the regions where massive particles are.

Finally, we shall verify that indeed, when the mass parameter is negative, the potential defined at one of its minima will display no violation of special relativity. In determining this and also the new interactions, we perturb the potential around the stable vacuum

$$\phi = f + \sigma + i\chi, \quad f^2 = -\frac{m^2}{\lambda}, \tag{2-16}$$

where σ and χ are real scalar fields with a zero vacuum expectation value. Inserting this expression into the Lagrangian yields

$$\begin{aligned} \mathcal{L} =& (\partial_\mu \sigma)^2 + (\partial_\mu \chi)^2 - \frac{1}{2}(2\lambda f^2)\sigma^2 - \lambda f \sigma^3 - \lambda f \sigma \chi^2 \\ & - \frac{1}{4}\lambda\sigma^4 - \frac{1}{4}\lambda\chi^4 - \frac{1}{2}\lambda\sigma^2\chi^2. \end{aligned} \tag{2-17}$$

After performing the shift, we no longer have a mass term corresponding to the existence of tachyons plaguing the Lagrangian. Moreover, when the terms are read off from the Lagrangian, we confirm that one field is massive, σ, having a mass proportional to the vacuum expectation value, and the other one is massless, χ. The general case of spontaneous symmetry breaking of a continuous symmetry states that if a group G with N generators is spontaneously broken down to a group H with M generators, then there will be $N - M$ Goldstone bosons.

Next we consider a Lagrangian invariant under a local symmetry and investigate the consequences of a having a spontaneous symmetry breaking when such symmetry is broken. In here the theorem presented above is not completely valid. Moreover, we shall see that the problem of massless gauge bosons is fixed by the Goldstone bosons. To study what is the new pattern of spontaneous symmetry breaking, we must elevate the derivative to a covariant derivative and include, of course, the gauge boson corresponding to the gauge group. Let us perform this in a $U(1)$ gauge theory with a self-interacting scalar field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D^\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{2}m^2\phi^\dagger\phi - \frac{1}{4}\lambda(\phi^\dagger\phi)^2, \tag{2-18}$$

where $D_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative. We repeat here the same idea employed above. If $m^2 < 0$, then the scalar field has a nonvanishing vacuum expectation value $f^2 = -m^2/\lambda$. In here we again parametrize ϕ in terms of new fields σ and χ, but we do not use a simple shift $\phi \to f + \sigma + i\chi$. Instead, we find more useful to exploit an exponential parametrization for the Goldstone bosons (16), that is,

$$\phi = e^{i\chi}(f + \sigma). \tag{2-19}$$

This parametrization is much more convenient, because it facilitates displaying the particle spectrum. For instance, we can rotate away the exponential and obtain

$$\phi = f + \sigma. \tag{2-20}$$

Inserting this into the Lagrangian, the terms involving only the gauge field are

$$\mathcal{L} \sim -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(e^2f^2)A_\mu^2. \tag{2-21}$$

The breaking of the gauge symmetry has led to a nonzero mass for the gauge boson, $m_A^2 = e^2f^2$, and the Goldstone boson has vanished from the Lagrangian. Typically the number of degrees of freedom in a system is conserved. In the initial Lagrangian (2-18), the gauge boson A_μ has two physical degrees of freedom and the complex field also has two degrees of freedom. After writing the Lagrangian in the broken basis, we finished with a massive gauge boson, which now has an additional longitudinal degree of freedom and a real scalar field σ, which has one degree of freedom. In the literature it is usually said that the Goldstone boson has been eaten by the gauge boson, so the later could acquire a nonzero mass. A more thoroughly study of this phenomenon has been presented in the references (17)(18)(19)(20)(21).

To conclude this discussion, we also point out that the physical predictions must not depend on the specific chosen parametrization. Moreover, had we chosen a different parametrization, for instance a shift around the degenerate vacuum, both Lagrangian would be connected by a gauge transformation. This suggests that before performing any calculations we should first specify the gauge-fixing terms, in order to remove these arbitrariness from the theory. Discussing this is beyond the scope of this work, so we indicate the relevant literature involving this topic (4)(22).

2.1.3 Higgs Sector

The most general gauge-invariant renormalizable potential is

$$\mathcal{L}_H = -m^2(H^\dagger H) - \lambda(H^\dagger H)^2 = -V_H, \tag{2-22}$$

where $\lambda > 0$. Writing $H = \frac{1}{\sqrt{2}}(H_1 + iH_2, H_3 + iH_4)^t$, we have

$$V_H = \frac{1}{2}m^2 \sum_i H_i H_i + \frac{1}{4}\lambda \sum_j H_j H_j \sum_k H_k H_k. \tag{2-23}$$

If $m^2 > 0$ the minimum is at $\bar{H} = 0$. If $m^2 < 0$ the potential has two degenerate minima $\bar{H}^2 = -m^2/\lambda = v^2$ and the $SU(2) \times U(1)$ symmetry is broken down to a $U(1)$ subgroup. Because the Higgs potential depends only on $H^\dagger H$, the orientation of the minimum will not matter, as we have seen when we studied spontaneous symmetry breaking. We follow the conventional approach and fix the minimum to lie along the direction of the real component of H, that is

$$\bar{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2-24}$$

As a consequence of this spontaneous symmetry breaking, we have that there exists only one linear combination of generators such that the vacuum state is annihilated, which corresponds to the unbroken subgroup $Q = T_3 + Y$.

2.1.4 Yukawa Sector

In quantum electrodynamics, left- and right-handed fermions are connected by a Dirac mass term. However, such a term would explicitly break the Standard Model $SU(2)$ symmetry. The way out of this conundrum is to employ the Higgs doublet to write down the fermion masses, but we must bear in mind that, in this manner, fermion masses will only show up after the electroweak symmetry breaking. In particular, we write down the term

$$\mathcal{L}_{\text{Yuk}} \supset -y\bar{\ell}_L H e_R + \text{h.c.}. \tag{2-25}$$

Note that, after the symmetry breaking, such term will generate a mass term $-m_e(\bar{e}_L e_R + \text{h.c.})$, where $m_e \sim y_e v$. So, we see that in this framework, the charged leptons and down-type quarks will get masses and no additional symmetry breaking of $SU(2)$ is required.

To work out the form of the terms of the remaining masses, we need to bear in mind that all those terms must be neutral under the hypercharge. Thus, we exploit an exclusive property of the $SU(2)$ group, which is the possibility of writing down a complex conjugate field such that it transforms as a doublet. This new object is

$$\widetilde{H} = i\sigma_2 H^*. \tag{2-26}$$

Now we can write terms that give masses to neutrinos and the up-type quarks, including all three generations, indexed by i and j, we then have

$$\mathcal{L}_{\text{Yuk}} = -Y^d_{ij}\bar{Q}_{L_i} H d_{R_j} - Y^u_{ij}\bar{Q}_{L_j}\widetilde{H} u_{R_j} - Y^e_{ij}\bar{\ell}_{L_i} H e_{R_j} - Y^\nu_{ij}\bar{\ell}_{L_i}\widetilde{H}\nu_{R_j} + \text{h.c.}. \tag{2-27}$$

Although we have included right-handed neutrinos above, it is worth remarking that they have not been observed and, as we discussed in the previous chapter, neutrino masses are a phenomenon beyond the Standard Model. We shall focus first on the quark masses and then on the lepton and neutrino masses.

The Yukawa matrices Y^u and Y^d are general 3×3 matrices with complex entries, not necessarily hermitian. So, it has a lot more information than we are able to measure. Had we had no gauge interactions, these matrices could just be diagonalized and the only physical parameters would be the masses.

After the symmetry breaking, in matrix form, the quark mass terms become

$$\mathcal{L} = -\frac{v}{\sqrt{2}}(\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R) + \text{h.c.}. \tag{2-28}$$

To diagonalize these matrices, we perform bi-unitary rotations, generically written as

$$Y_d = U_d M_d K_d^\dagger, \quad Y_u = U_u M_u K_u^\dagger, \tag{2-29}$$

where U and K are unitary matrices. Hence, in this basis, the Yukawa terms are

$$\mathcal{L} = -\frac{v}{\sqrt{2}}(\bar{d}_L U_d M_d K_d^\dagger d_R + \bar{u}_L U_u M_u K_u^\dagger u_R) + \text{h.c.}. \tag{2-30}$$

Now we may perform a change of basis for the right-handed quarks $d_R \to K_d d_R$ and $u_R \to K_u u_R$ and the left-handed quarks $d_L \to U_d d_L$ and $u_L \to U_u u_L$. As a consequence of this change of basis, we have removed the U and K and all the Yukawa terms are now diagonal. What we just did is known as going to the mass basis. In the mass basis, the Lagrangian reads

$$\mathcal{L} = -m_d^j \bar{d}_L^j d_R^j - m_u^j \bar{u}_L^j u_R^j + \text{h.c.}, \tag{2-31}$$

where the masses are

$$m_d^j = \frac{v}{\sqrt{2}}(y_d, y_s, y_b), \quad m_u^j = \frac{v}{\sqrt{2}}(y_u, y_c, y_t). \tag{2-32}$$

We have already many evidences confirming the existence of massive left-handed neutrinos, but if we additionally assume that right-handed neutrinos also exist in nature, then the most general renormalizable mass term in the lepton sector we can write down is

$$\mathcal{L} = -Y_{ij}^e \bar{\ell}_{L_i} H e_{R_j} - Y_{ij}^\nu \bar{\ell}_{L_i} \widetilde{H} \nu_{R_j} - iM_{ij}(\nu_{R_i})^c \nu_{R_j} + \text{h.c.}. \tag{2-33}$$

These three terms require some explanation.

After electroweak symmetry breaking, the first term generates the conventional charged lepton masses. To diagonalize the Yukawa matrix, the course of action here is the very same one we have applied in the quark sector, performing the bi-unitary rotation we end up with the expression

$$\mathcal{L} \sim -m_e^j \bar{e}_{L_j} e_{R_j} + \text{h.c.}, \quad m_e = \frac{v}{\sqrt{2}}(y_e, y_\mu, y_\tau). \tag{2-34}$$

In the second term, we note that the left-handed fermion field ℓ_L and the Higgs antidoublet carry the same weak and hypercharge quantum numbers. Therefore, the right-handed neutrino is not allowed to carry any weak or hypercharge quantum numbers, besides zero. For this reason, this neutrino is usually referred as a sterile neutrino.

The last term explores the possibility of neutrinos being Majorana type particles. The notation $\nu_{R_i}^c = \nu_{R_i}^t \sigma_2$ denotes the charge conjugate Weyl spinor of the right-handed neutrino. When both right and left-handed massive neutrinos are present, it is conceivable that they receive a mass contribution from a Dirac mass term and a Majorana mass term. This leads to a nondiagonal 2×2 mass matrix, which when diagonalized has two nonzero eigenvalues. Assuming that there is a mass scale $M = m_R$, where m_R is the mass parameter associated with the right-handed neutrino, which is much larger than the Dirac and the left-handed neutrino mass parameter. Then, the mass eigenstates will have two very distinct values. One very light, inversely proportional to the scale M, and the other very heavy, directly proportional to the scale M. Clearly there is much more of this model than we can present in here, thus for a more profound discussion of neutrino masses we refer to the literature (13)(14)(23)(24).

2.2 Spectrum

We now study the particle spectrum. Based on the previous discussions, the renormalizable part of Standard Model Lagrangian is given by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger (D^\mu H)\\ &+ i\bar{Q}_{L_j}\not{D}Q_{L_j} + i\bar{\ell}_{L_j}\not{D}\ell_{L_j} + i\bar{u}_{R_j}\not{D}u_{R_j} + i\bar{d}_{R_j}\not{D}d_{R_j} + i\bar{e}_{R_j}\not{D}e_{R_j}\\ &- Y^d_{ij}\bar{Q}_{L_i}Hd_{R_j} - Y^u_{ij}\bar{Q}_{L_j}\widetilde{H}u_{R_j} - Y^e_{ij}\bar{\ell}_{L_i}He_{R_j} + \text{h.c.}\\ &- m^2(H^\dagger H) - \lambda(H^\dagger H)^2.\end{aligned} \tag{2-35}$$

As before, we shift the field to the vacuum state and use an exponential parametrization for the Goldstone bosons,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix} \exp\left(\frac{2i\pi^a X^a}{v}\right), \tag{2-36}$$

where X^a are the broken generators of $SU(2)\times U(1)$; T_1, T_2 and $T_3 - Y$. Since we are primarily interested in evaluating the particle spectrum, it is more convenient to use the unitary gauge. After rotating away the Goldstone bosons, we have

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}. \tag{2-37}$$

The Higgs mass can be obtained by plugging (2-37) into the Higgs potential, the result is

$$m_h^2 = 2\lambda v^2, \tag{2-38}$$

whose experimental value is (25)

$$m_h = 125.09 \pm 0.24\text{GeV}. \tag{2-39}$$

Going to the gauge boson sector, we can find the mass terms for them by plugging the vacuum expectation value into $D_\mu H$,

$$\mathcal{L}_{M_V} = \frac{g^2v^2}{8}\left[(W^1_\mu + iW^2_\mu)(W^\mu_1 - iW^\mu_2) + \left(\frac{g'}{g}B_\mu - W^3_\mu\right)^2\right]. \tag{2-40}$$

These are the mass terms associated with the three massive gauge bosons coming from the spontaneous symmetry breaking. We originally had four unbroken generators, but after the Higgs field developed a nontrivial vacuum strcuture, we ended up having three out of the four generators spontaneously broken, so three of the four gauge bosons will acquire a mass, and one will remain massless.

To diagonalize the masses in the gauge boson sector, we rotate the fields B_μ and W^3_μ by an angle θ_W,

$$\begin{pmatrix} B \\ W^3 \end{pmatrix}_\mu = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_w \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}_\mu , \tag{2-41}$$

with

$$\tan\theta_w = \frac{g'}{g}. \tag{2-42}$$

With these definitions plugged into equation (2-40), we have

$$\mathcal{L}_{\mathcal{M}_\mathcal{V}} = \frac{1}{2}\left(\frac{g^2v^2}{4}\right) W^+_\mu W^\mu_- + \frac{1}{2}\left(\frac{(g'^2+g^2)v^2}{4}\right) Z_\mu Z^\mu, \tag{2-43}$$

where we have used $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$ and as we anticipated the photon is massless, $m_A^2 = 0$. The experimental values of the gauge boson masses are given by (25)

$$m_W = 80.385 \pm 0.015\text{GeV}, \quad m_Z = 91.1876 \pm 0.0021\text{GeV} \tag{2-44}$$

and we can use that

$$\frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \tag{2-45}$$

to determine $\sin^2\theta_W$:

$$\sin^2\theta_W = 0.2229 \pm 0.0004. \tag{2-46}$$

The fermion masses were already obtained when we discussed the Yukawa sector. Their experimental values are (25)

$$\begin{aligned}
&m_e = 0.510998946(3)\text{MeV}, \quad m_\mu = 105.6583745(24)\text{MeV}, \quad m_\tau = 1776.86(12)\text{MeV} \\
&m_u = 2.2^{+0.6}_{-0.4}\text{MeV}, \quad m_c = 1.27 \pm 0.03\text{GeV}, \quad m_t = 173.2 \pm 0.09\text{GeV} \\
&m_d = 4.7^{+0.5}_{-0.4}\text{MeV}, \quad m_s = 96^{+8}_{-4}\text{MeV}, \quad m_b = 4.18^{+0.04}_{-0.03}\text{GeV}.
\end{aligned}$$

The up, down and strange-quark masses are given at the scale $\mu = 2$GeV. The charm and bottom-quark masses are the running masses in the modified minimal subtraction scheme for $m_c(\mu = m_c)$ and $m_b(\mu = m_b)$, respectively, and the top-quark mass is derived from direct measurement.

We summarise the results derived in this section in the table 2.1 with the mass eigenstates of the Standard Model with their masses in units of v. In the absence of spontaneous symmetry breaking, the gauge bosons and fermions are protected from acquiring masses by the gauge invariance and chiral symmetry, respectively. The Higgs mass being proportional to v is just a manifestation of the fact that the Standard Model has a single dimensionful parameter.

Particle	Spin	Color	Q	Mass [v]
$W^\pm$	1	1	±1	$g/2$
Z	1	1	0	$\sqrt{g^2+g'^2}/2$
A	1	1	0	0
G	1	8	0	0
h	0	1	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	1	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	1	0	0
u,c,t	1/2	3	2/3	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	3	-1/3	$y_{d,s,b}/\sqrt{2}$

Table 2.1: Particle spectrum. The building blocks of the Standard Model along with their quantum numbers and masses. Only the $SU(3)\times U(1)$ symmetry survives the spontaneous symmetry breaking

2.3 Interactions

We finally conclude our discussion about the general structure of the Standard Model in this session, in which we shall the discuss the interactions of the fermions and the scalar mass eigenstates.

The interactions of the Standard Model mediated by the gauge bosons $W^\pm$, Z and A can be described in terms of currents. The left-handed $SU(2)$ currents are

$$j_a^\mu = \sum_\psi \bar{\psi}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\frac{\sigma_a}{2}\psi, \tag{2-47}$$

where $a = 1,2,3$ runs over the generators and the ψ are either the lepton doublets

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}, \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}, \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix} \tag{2-48}$$

or the quark doublets

$$\begin{pmatrix}u\\ d'\end{pmatrix}, \quad \begin{pmatrix}c\\ s'\end{pmatrix}, \quad \begin{pmatrix}t\\ b'\end{pmatrix} \tag{2-49}$$

with the flavour eigenstates q' being related to the mass eigenstates q via the unitary Cabibbo-Kobayashi-Maskawa matrix

$$q'_i = V_{ij}q_j. \tag{2-50}$$

This relation comes from the rotations we have performed to write down the Yukawa sector in the mass basis. In the kinetic sector, the terms involving the B_μ and W_μ^a gauge bosons are

$$\mathcal{L} \sim \left(\bar{u}_L \bar{d}_L\right)_i \left[i\partial\!\!\!/ + \begin{pmatrix} \frac{g'}{6}B\!\!\!/ + \frac{g}{2}W\!\!\!/^3 & \frac{1}{\sqrt{2}}W\!\!\!/^+ \\ \frac{1}{\sqrt{2}}W\!\!\!/^- & \frac{g'}{6}B\!\!\!/ - \frac{g}{2}W\!\!\!/^3 \end{pmatrix}^{\mu} \right] \begin{pmatrix} u_L \\ d_L \end{pmatrix}. \quad (2\text{-}51)$$

Clearly the only elements sensitive to the flavour rotations are the charged gauge boson couplings,

$$\bar{u}W\!\!\!/^+ d_L \to \bar{u}_L U_u^\dagger W\!\!\!/^+ U_d d_L, \quad \bar{d}W\!\!\!/^- u_L \to \bar{d}_L U_d^\dagger W\!\!\!/^- U_u u_L, \quad (2\text{-}52)$$

where we conventionally define $V = U_u^\dagger U_d$. There is no unique representation of this matrix V. We usually order the up quarks and the down quarks by their masses, that is, $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The complex unitary matrix is then given by

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Such matrix has nine real degrees of freedom. If V were a real matrix, then it would be an $O(3)$ matrix, with three rotation angles. Thus, we have three rotation angles and six phases in V. Considering that the fermion masses have a $U(1)^6$ global symmetry

$$u_{L_j} \to \mathrm{e}^{i\beta_j} u_{L_j}, \quad u_{R_j} \to \mathrm{e}^{i\beta_j} u_{R_j}, \quad d_{L_j} \to \mathrm{e}^{i\alpha_j} d_{L_j}, \quad d_{R_j} \to \mathrm{e}^{i\alpha_j} d_{R_j},$$

we can eliminate five phases. In here j denotes the generation and there is no summation over this index. The most general CKM matrix can be written as

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, these angles θ_{ij} correspond to rotations in the ij-flavour planes and δ is the phase responsible for all CP-violating phenomena in flavour-changing processes in the Standard Model. The numerical values for the angles and phase are $\theta_{23} = 2.36° \pm 0.08°$, $\theta_{13} = 0.20° \pm 0.02°$, $\theta_{12} = 13.02° \pm 0.04°$ and $\delta = 69° \pm 5°$ (13).
The interaction Lagrangian is then

$$\mathcal{L}_{\text{int}} = \frac{e}{\sin\theta_W}(W_+^\nu J_\nu^- + W_-^\nu J_\nu^+) + \frac{e}{\sin\theta_W} Z_\nu J_Z^\nu + eA_\nu J_{\text{EM}}^\nu, \quad (2\text{-}53)$$

where

$$J_Z^\nu = \frac{1}{\cos\theta_W}\left(J_3^\nu - \sin^2\theta_W J_{\mathrm{EM}}^\nu\right) = \frac{1}{\cos\theta_W}\sum_i \left[\bar{\psi}_i\gamma^\nu T^3\psi_i - \sin^2\theta_W Q_i\psi_i\gamma^\nu\psi_i\right]. \tag{2-54}$$

The charged currents have been defined as

$$J_\pm^\mu = \frac{j_1^\mu \mp i j_2^\mu}{\sqrt{2}}. \tag{2-55}$$

The electromagnetic current couples to quarks and leptons, all charged fermions will interact with the photon

$$J_{\mathrm{EM}}^\mu = \sum_i \left(\frac{2}{3}\bar{u}_i\gamma^\mu u_i - \frac{1}{3}\bar{d}_i\gamma^\mu d_i - \bar{e}_i\gamma^\mu e_i\right). \tag{2-56}$$

The interactions involving the Higgs boson are

$$\begin{aligned}\mathcal{L}_h = &-\frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4 + m_W^2 W_\nu^- W_+^\nu\left(\frac{2h}{v}+\frac{h^2}{v^2}\right) + \frac{1}{2}m_Z^2 Z_\nu Z^\nu\left(\frac{2h}{v}+\frac{h^2}{v^2}\right)\\ &-\frac{h}{v}\left(\sum_i m_i\bar{\psi}_L\psi_R + \text{h.c.}\right),\end{aligned}$$

where the summation over i comprises all the charged fermions.
Concluding, all colored fermions interact with the gluon

$$\mathcal{L}_g = -\frac{1}{2}g_s\bar{q}\lambda_a \not{G} q. \tag{2-57}$$

Within the Standard Model, we have then five types of interaction, which we summarise in table 2.2

Interaction	Fermions	Force Carrier	Coupling
Strong	u, d	g	g_s
Electromagnetic	u, d, ℓ	A	eQ
CC weak	$\bar{u}d, \bar{l}\nu$	$W^\pm$	gV, g
NC weak	all	Z	$e(T_3 - s_W^2 Q)/(s_W c_W)$
Yukawa	u, d, ℓ	h	y_q

Table 2.2: The fundamental interactions within the Standard Model.

3
Theoretical Tools

3.1
The Effective Potential

So far, we have introduced the foundational structure of the Standard Model all at tree-level and this should be enough in case we wanted to qualitatively understand the simplest possible features the model can exhibit. However, such approximation does not allow us to make accurate and precise predictions, because all the considered physics only accounts for process occurring at the classical level, when in fact there might be effects at the quantum level providing contributions that are large enough to heavily affect the theoretical predictions. To answer the question whether quantum corrections may invalidate the drawn conclusions from the tree-level, be it stability conditions associated with the coupling parameters at different energy scales or extrema points in the potential, one should consider the effective potential (13)(26)(27). An additional feature of the effective potential is that it allows us to derive the β-functions of a theory without having to explicitly evaluate Feynman diagrams. The physical meaning of this quantity should be understood as the potential energy of the involved fields taking into account all the quantum corrections, which is nothing but the sum over all one-particle irreducible connected vacuum-vacuum amplitudes in the presence of an external current.

A very useful object in this approach is the generating functional

$$\begin{aligned}\mathcal{Z}[J] &= \langle \text{VAC}, \text{out} | \text{VAC}, \text{in} \rangle \\ &= \int \left[\prod_{s,y} d\phi^s(y) \right] \exp\left(i\mathcal{I}[\phi] + i \int d^4x \phi^r(x) J_r(x) \right).\end{aligned} \tag{3-1}$$

This is a quantum field theory with an action $\mathcal{I}[\phi]$ in which it was added an external source term $\phi^r J_r$, where ϕ^r can be a composite object or a fermionic field and J_r is a classical current. Now, assume that this generating functional represents the sum of all possible diagrams

$$\mathcal{Z}[J] = \sum_{n_i} D_{n_i}, \tag{3-2}$$

where n_i denotes the number of connected components. Moreover, the diagrams D_{n_i} are produced by sewing together other connected diagrams

$$D_{n_i} = \prod_i \frac{1}{n_i!}(iC_i)^{n_i}, \tag{3-3}$$

the $n_i!$ comes from the fact that a diagram consisting of n_i connected components have $n_i!$ possible permutations of vertices that simply permute all the vertices in one connected component with the vertices in another, it is merely a symmetry factor to avoid overcounting the diagrams. Putting together equations (3-2) and (3-3), we have

$$\mathcal{Z}[J] = \prod_i \sum_{n_i} \frac{1}{n_i!}(iC_i)^{n_i}. \tag{3-4}$$

Hence, the sum of all graphs is

$$\mathcal{Z}[J] = \exp\left(iW[J]\right), \tag{3-5}$$

where $W[J]$ is the sum of all connected vacuum-vacuum amplitudes, where we have excluded diagrams that differ only by a permutation of vertices.

Next, exploiting the equation (3-5), we write down the functional

$$W[J] = -i\log\left\{\int\left[\prod_{s,y} d\phi^s(y)\right]\exp\left(i\mathcal{I}[\phi] + i\int d^4x\phi^r(x)J_r(x)\right)\right\}. \tag{3-6}$$

The effective action can then be defined by the Legendre transform of $W[J]$

$$\Gamma[\phi] = W[J_\phi] - \int d^4x\phi^r(x)J_{\phi r}(x), \tag{3-7}$$

where $J_{\phi r}(x)$ is an implicit functional of ϕ_r defined as the solution to

$$\frac{\delta W[J]}{\delta J_s(y)} = \phi^s_J(y). \tag{3-8}$$

There is also a conjugated relation associated with W, it comes from varying the effective action with respect to ϕ

$$\frac{\delta\Gamma[\phi]}{\delta\phi^s(y)} = -J_{\phi s}(y). \tag{3-9}$$

This equation is equivalent to the familiar classical field equations, which just required that the actual action $\mathcal{I}$ be stationary. So the equation (3-9) is simply the equations of motion for the external field in the presence of a source, taking quantum effects into account. The generating functional $\mathcal{Z}$ is a very useful quantity in quantum field theory, because it allows us to generate Green's function via

$$\langle J|\phi^r(x_1)\cdots\phi^r(x_n)|J\rangle = (-i\hbar)^n \frac{1}{\mathcal{Z}[J]}\frac{\partial^n \mathcal{Z}[J]}{\partial J_r(x_1)\cdots\partial J_r(x_n)}. \tag{3-10}$$

We commonly set the source to zero after taking the derivatives, this turns the above equation into vacuum matrix elements. In comparison with that, if we let $J \neq 0$, then the generated Green's functions for the fields are under the effect of a background given by a classical current J. We have shown above that the functional W encodes all the information associated with connected diagrams. We now verify how this particular feature manifests itself in the calculation of matrix elements. If we can take derivatives of the source terms and generate Green's functions, lets suppose that the same idea is true for W, that is,

$$\frac{\partial^n W}{\partial J_1\cdots\partial J_n} \sim \langle J|\phi(x_1)\cdots\phi(x_n)|J\rangle_{\text{connected}}. \tag{3-11}$$

If we single out the $n-$th derivative, use the equation (3-6) and evaluate the $n-$th and $(n-1)-$th derivatives, we get

$$(-i\hbar)^{-2}\left(\frac{\partial^{n-2}}{\partial J_1\cdots\partial J_{n-2}}\right)(\langle J|\phi_{n-1}\phi_n|J\rangle - \langle J|\phi_1|J\rangle\langle J|\phi_2|J\rangle). \tag{3-12}$$

Thus, by taking the derivatives, we obtained two correlation functions. The first two-point correlation function encodes all the information about the interacting theory, it has contributions from all possible diagrams, connected or not. Whereas, the second term, contains all the possible disconnected diagrams. The relation presented in equation (3-12) between the former and the latter functions validates the fact, at least to the order we checked, that W indeed generates only connected diagrams. We can convince ourselves that this claim is true by noting that when taking derivatives of W with respect to J the same structure are going to manifest itself at every order, the full Green's function minus disconnected parts. The full form of equation (3-11) reads

$$(-i\hbar)^n\frac{\partial^n W[J]}{\partial J_1\cdots\partial J_n} = -i\hbar\,\langle J|\phi(x_1)\cdots\phi(x_n)|J\rangle_{\text{connected}}. \tag{3-13}$$

Even though we have learned what the functional W represents and the underlying physics behind it, it is not immediately obvious that its Legendre transform is going to reproduce the complete set of one-particle irreducible diagrams. The general idea is that what we did for the action $\mathcal{I}$ is also going to hold for the effective action Γ, except that now, such an effective action, when used at the tree-level is going to have only one particle irreducible diagrams. In mathematical terms, we have

$$W[J] = \lim_{\hbar\to 0}(-i\hbar)\int\left[\prod_{s,y} d\phi^s(y)\right]\exp\left[(i/\hbar)\left(\Gamma[\phi] + i\int d^4x\phi^r(x)J_r(x)\right)\right]. \tag{3-14}$$

Because of the highly oscillatory behaviour of the integral in the limit $\hbar\to 0$, it isolates the tree-level contributions, as a consequence the path integral is going to be dominated by field configurations that extremize the action. This leads the functional integration to

$$W[J] = \Gamma[\phi_J] + \int d^4x J\phi_J, \tag{3-15}$$

which is the inverse Legendre transformation of the equation (3-7). What also is not obvious is that the effective action $i\Gamma[\phi_0]$ can be calculated for some fixed $\phi_0^r(x)$ as the sum of one-particle irreducible graphs for the vacuum-vacuum amplitude, evaluated with a shifted action

$$\exp(i\Gamma[\phi_0]) = \int_{1PI} \left[\prod_{s,y} d\phi^s(y)\right] \exp(i\mathcal{I}[\phi + \phi_0]). \tag{3-16}$$

The required calculations to prove the validity of equations (3-14) and (3-16) are explained in much more details in the references (26)(27).

3.1.1 Calculation of the Effective Potential

Consider a general renormalizable theory of a single real scalar ϕ, with an action

$$\mathcal{I}[\phi] = -\int d^4x \left[\frac{1}{2}\phi(\Box + m^2)\phi + \frac{1}{4!}g\phi^4\right] \tag{3-17}$$

and suppose we want to calculate $\Gamma[\phi_0]$, where ϕ_0 is an homogeneous field configuration. Using the formalism described in the previous section, we perform a shift $\phi \to \phi + \phi_0$ in the action

$$\begin{aligned}\mathcal{I}[\phi + \phi_0] = &- \mathcal{V}_4 \left(\frac{1}{2}m^2\phi_0^2 + \frac{1}{4!}g\phi_0^4\right) - \frac{1}{2}\int d^4x\phi \left(\Box + \frac{1}{2}m^2 + \frac{1}{2}g\phi_0^2\right)\phi \\ &- \frac{g}{4!}\int d^4x(4\phi^3\phi_0 + \phi^4),\end{aligned}$$

where $\mathcal{V}_4$ is the four-dimensional hypervolume and we have dropped interaction terms proportional to ϕ (they do not affect the one-particle irreducible diagrams). Then,

$$\exp(i\Gamma[\phi_0]) = \int_{\text{restr.}} \mathcal{D}\phi \exp(i\Gamma^{(0)}[\phi_0] + i\Gamma^{(1)}[\phi_0] + i\Gamma^{(2)}[\phi_0]), \tag{3-18}$$

where the superscript in the effective action means that only i-loop diagrams are contributing to the final result. The symbol $\mathcal{D}\phi$ is the short notation of the product integral employed above. The subscript 'restr.' indicates that only a restricted set of field configurations are to be integrated over, the ones contributing to one-particle irreducible diagrams. Otherwise, we could just shift the field $\phi \to \phi - \phi_0$ yielding an effective action independent of the background field.

Employing that the background field is space-time-independent, fixing $m^2_{\text{eff}}(\phi_0) = m^2 + g\phi_0^2/2$ as the field-dependent effective mass and focusing on the tree-level and one-loop contributions, we have

$$\begin{aligned} \exp(-i\mathcal{V}_4 V_{\text{eff}}(\phi_0)) =& \exp(-i\mathcal{V}_4 V_0(\phi_0)) \\ & \times \int_{\text{restr.}} \mathcal{D}\phi \exp\left[-\frac{i}{2}\int d^4x \phi \left(\Box + m^2_{\text{eff}}(\phi_0)\right)\phi\right], \end{aligned} \tag{3-19}$$

where V_0 is the zero-loop potential

$$V_0(\phi_0) = \frac{1}{2}m^2\phi_0^2 + \frac{1}{4!}g\phi_0^4.$$

To include the one-loop contributions, we must evaluate the functional integral

$$\exp(i\Gamma^{(1)}[\phi_0]) = \int_{\text{restr.}} \mathcal{D}\phi \exp\left[-\frac{i}{2}\int d^4x \phi \left(\Box + m^2_{\text{eff}}(\phi_0)\right)\phi\right].$$

The result is

$$i\Gamma^{(1)}[\phi_0] = \log\det\left(\frac{iK}{\pi}\right)^{-1/2} = -\frac{1}{2}\text{tr}\log\left(\frac{iK}{\pi}\right), \tag{3-20}$$

where the matrix K is given by

$$K_{x,y} = \left(\frac{\partial^2}{\partial x^\lambda \partial y_\lambda} + m^2_{\text{eff}}(\phi_0)\right)\delta^4(x-y). \tag{3-21}$$

Next, to calculate the trace in equation (3-21), we diagonalize the matrix K by passing to momentum space

$$K_{p,q} = (-p^2 + m^2_{\text{eff}})\delta^4(p-q).$$

The logarithm of a diagonal matrix is just the diagonal matrix with logarithms in its main diagonal entries. This allows us to write equation (3-20) in the form

$$i\Gamma^{(1)}[\phi_0] = -\frac{\mathcal{V}_4}{2}\int \widetilde{d^4p}\log\left(-p^2 + m^2_{\text{eff}}(\phi_0)\right), \quad \widetilde{d^4p} = d^4p/(2\pi)^4.$$

The problem now is that this integral is hideously ultraviolet divergent. We can either insert a high energy cut-off or we can take the derivative with respect to m^2_{eff} three times and integrate thrice after solving the integral. We here choose to impose a hard cut-off $p_E < \Lambda$, where p_E is the Euclidean momentum. After we Wick rotate it and integrate over the angles of the four-dimensional hypersphere, then the integral reads

$$\int \widetilde{d^4p} \log\left(1 - \frac{m_{\text{eff}}^2(\phi_0)}{p^2}\right) = \frac{2\pi^2}{(2\pi)^4} i \int_0^\Lambda dp_E p_E^3 \log\left(1 + \frac{m_{\text{eff}}^2(\phi_0)}{p_E^2}\right)$$
$$= i\frac{m_{\text{eff}}^4(\phi_0)}{32\pi^2} \log\left(\frac{m_{\text{eff}}^2(\phi_0)}{\Lambda^2}\right) + \cdots, \quad m_{\text{eff}}(\phi_0) \ll \Lambda,$$

where we have dropped constant terms. Since the physics content resides in the logarithm, the ellipsis has been used to replace the various divergent terms which can be removed through renormalization. At this point, inserting the derived results in equation (3-19), the effective potential is then

$$V_{\text{eff}}(\phi_0) = V_0(\phi_0) + \frac{m_{\text{eff}}^4(\phi_0)}{64\pi^2} \log\left(\frac{m_{\text{eff}}^2(\phi_0)}{\Lambda^2}\right). \tag{3-22}$$

Had we considered a field of a different nature other than a scalar, this final expression would be mildly changed. More generally, a useful formula for general contributions is

$$V_{\text{eff}}(\phi) = V_0(\phi) + \sum_{i=1}^{N} (-1)^{2s_i} \frac{n_{i,d}}{64\pi^2} m_{\text{i,eff}}^4 \log\left(\frac{m_{\text{i,eff}}^2(\phi_0)}{\Lambda^2}\right). \tag{3-23}$$

The additional parameters comes from a generalization of the Gaussian integral. The term $(-1)^{2s_i}$ involves the particle's spin, in particular, for fermions the one-loop contributions are negative and for bosons is positive. The second parameter $n_{i,d}$ represents the degrees of freedom, a neutral scalar is 1, a complex scalar has two degrees of freedom, so $n_d = 2$, a Dirac fermion of spin 1/2 particle has two spin states and also antiparticle states, thus $n_d = 4$ and so on.

One important application of this technique is to the effective potential of the Standard Model. The Higgs field couples to the gauge bosons $W_{\pm}^{\mu}$ and Z^{μ}, to the left-handed fermions in the Yukawa sector and to itself. Among the fermions, we are going to consider only the top quark, because it has the largest coupling parameter, being close to one. In this approximation, it can be shown that the effective potential is

$$\begin{aligned} V_{\text{eff}} &= V_0 + V_{\text{H}} + V_{\text{GB}} + V_{\text{F}} \\ &= \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \frac{(m^2+3\lambda h^2)^2}{64\pi^2}\log\left(\frac{m^2+3\lambda h^2}{\Lambda^2}\right) + \frac{3(m^2+\lambda h^2)^2}{64\pi^2}\log\left(\frac{m^2+\lambda h^2}{\Lambda^2}\right) \\ &+ \frac{6g^4h^4}{1024\pi^2}\log\left(\frac{g^2h^2}{4\Lambda^2}\right) + \frac{3(g^2+g'^2)^2h^4}{1024\pi^2}\log\left(\frac{(g^2+g'^2)h^2}{4\Lambda^2}\right) - \frac{3y_t^4h^4}{64\pi^2}\log\left(\frac{y_t^2h^2}{2\Lambda^2}\right), \end{aligned} \tag{3-24}$$

in agreement with the literature (4).

3.2 Renormalized Perturbation Theory

In the previous section we introduced the concept of effective potential and derived a general formula for an homogeneous background field configuration with which we obtained the Standard Model effective potential. When investigating the effective potential, we observed the existence of a energy scale dependence of the parameters at one-loop level and in this section we shall investigate the dependence of the couplings on this scale parameter. We shall not delve into the details of renormalization theory (4)(29)(30)(31), but instead we present the essential steps to exploit such technique in the present context. The key idea here is that, given an action $\mathcal{I}[\phi]$, made up of a (infinite) bare field ϕ_b and a set of bare parameters $\{m_b, g_{1,b}, \ldots, g_{n,b}\}$. The condition of renormalizability of a given theory requires that for every infinite coupling there should be a free parameter to absorb it. For instance, in this case, additionally to the field and coupling parameters, we would need an extra of $n + 2$ free parameters, one for the bare field, one for the bare mass and n for the coupling parameters.

Consider the action $\mathcal{I}$ for a scalar field theory

$$\mathcal{I}[\phi] = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi_b)^2 - \frac{1}{2} m_b^2 \phi_b^2 - \frac{1}{4!} g_b \phi_b^4 \right). \tag{3-25}$$

The idea is that we want the renormalization parameters as a function of the renormalized quantities; mass and coupling parameters. The bare field is conventionally rescaled by the wavefunction renormalization parameter Z_{phi}

$$\phi_b \to Z_\phi \phi_r \tag{3-26}$$

$$\mathcal{I}[\phi] = \int d^4x \left(\frac{1}{2} Z_\phi (\partial_\mu \phi_r)^2 - \frac{1}{2} Z_\phi m_b^2 \phi_r^2 - \frac{1}{4!} Z_\phi^2 g \phi_r^4 \right). \tag{3-27}$$

Then, we define the renormalized quantities as

$$m_b^2 Z_\phi = m_r^2 Z_m, \quad g_b Z_\phi^2 = g_r Z_g. \tag{3-28}$$

Plugging the above defintions into the bare action, we find as a result the renormalized action:

$$\begin{aligned} \mathcal{I}_\epsilon[\phi] = \int d^4x - \frac{1}{2}\phi_r(\Box + m_r^2)\phi_r - \frac{1}{4!} g_r \phi_r^4 \\ - \frac{1}{2}\phi_r[(Z_\phi - 1)\Box + \delta m_r^2]\phi_r - \frac{1}{4!}(Z_g - 1)\phi_r^4, \end{aligned} \tag{3-29}$$

where

$$
\begin{aligned}
Z_\phi &= 1 + a_1(\epsilon)g_r + a_2(\epsilon)g_r^2 + \cdots \\
Z_g &= 1 + b_1(\epsilon)g_r + b_2(\epsilon)g_r^2 + \cdots \\
\delta m^2 &= m^2(c_1(\epsilon)g_r + c_2(\epsilon)g_r^2 + \cdots),
\end{aligned}
\tag{3-30}
$$

where ϵ is an arbitrarily small free parameter, defined via the relation $d = 4-\epsilon$. The renormalization parameters allowed us to separate the action into two parts (3-29), one has a finite value and the other absorbs the infinities. Next consider the generating functional of proper functions $\Gamma[\phi]$, expanded in powers of g_r

$$\Gamma[\phi] = \Gamma_0[\phi] + g_r\Gamma_1[\phi] + \cdots . \tag{3-31}$$

At tree-level, no counterterm is required, because the action is bounded

$$\lim_{\epsilon\to 0} \mathcal{I}_\epsilon[\phi] < \infty. \tag{3-32}$$

At one-loop order, the contributions from the tree-level action come as a consequence of the Gaussian functional integration

$$\Gamma_1[\phi] \to \frac{1}{2}\mathrm{tr}\log\left[1 + (\Box + m_r^2)^{-1}\frac{g_r\phi_r^2}{2}\right]. \tag{3-33}$$

We expand it in powers of ϕ_r and, at the one-loop level, find two divergent contributions. The first contribution, (a), has has two external legs and the second contribution, (b) has four external legs. In terms of integral, we can represent those contributions as, respectively,

$$(a) = \frac{g_r\mu^{(4-d)/2}}{4}\int \widetilde{d^dk}\frac{1}{p^2 - m_r^2 + i\epsilon} = i\frac{m_r^2 g_r}{32\pi^2\epsilon} + \mathcal{O}(\epsilon^0) \tag{3-34}$$

and

$$(b) = \frac{g_r^2\mu^{4-d}}{16}\int \widetilde{d^dk}\frac{1}{(p^2 - m_r^2 + i\epsilon)((p-q)^2 - m_r^2 + i\epsilon)} = i\frac{g_r^2}{128\pi^2\epsilon} + \mathcal{O}(\epsilon^0), \tag{3-35}$$

where $\widetilde{d^dk} = d^dk/(2\pi)^d$. To regulate the ultraviolet divergence we employed as regularization scheme the dimensional regularization. In this scheme, we evaluate the integral in d dimensions and then analytically continue it to $4-\epsilon$ dimensions.

Hence, the divergent part of the generating functional of proper vertices is

$$\Gamma_1^{\mathrm{div}}[\phi_r] = \frac{1}{32\pi^2\epsilon}\int d^4x\left(m_r^2 g_r\phi_r^2 + \frac{1}{4}g_r^2\phi_r^4\right). \tag{3-36}$$

It is important observing in here that, although we do not have regulator-dependent divergent constants, in particular power law divergences, such feature is only a consequence of the chosen regularization scheme. By any means it implies that we have no fine-tuning in the theory.

In the absence of counterterms such contributions would spoil the validity of the quantum field theory. Thus, in order to make the theory valid, we must next introduce the counterterms to absorb those contributions and tame the infinities. The counterterm Lagrangian at one-loop order

$$\frac{1}{2}(Z_\phi - 1)(\partial_\mu \phi_r)^2 - \frac{1}{2}\delta m_r^2 \phi_r^2 - \frac{1}{4!}(Z_g - 1)\phi_r^4 = \frac{1}{2}\delta\phi(\partial_\mu \phi_r)^2 - \frac{1}{2}\delta m_r^2 \phi_r^2 - \frac{1}{4!}\delta g \phi_r^4. \tag{3-37}$$

At leading order, the counterterms contributes additively to $\Gamma[\phi_r]$. It is possible then to associatively eliminate the divergences and renormalize the ϕ^4 theory. Focusing only on the condition of finiteness of correlation functions, we find that the counterterms are given by

$$\begin{aligned} \delta m^2 &= \frac{m_r^2}{32\pi^2 \epsilon} g_r, \\ \delta g &= \frac{1}{128\pi^2 \epsilon} g_r^2. \end{aligned} \tag{3-38}$$

Now notice that the renormalized values are functions of μ, but the bare values have no μ dependence, then we must have

$$\begin{aligned} \mu \frac{dg_b}{d\mu} &= \mu^{\epsilon/2} Z_g g_r \left(\frac{\epsilon}{2} + \frac{1}{Z_g}\mu\frac{dZ_g}{d\mu} + \frac{1}{g_r}\mu\frac{dg_r}{d\mu} \right) = 0, \\ \mu \frac{dm_b^2}{d\mu} &= Z_m m_r^2 \left(\frac{1}{Z_m}\mu\frac{dZ_m}{d\mu} + \frac{1}{m_r^2}\mu\frac{dm_r^2}{d\mu} \right) = 0. \end{aligned} \tag{3-39}$$

For the quartic coupling parameter, g_r, at leading order in g_r the renormalization field parameter does not depend on the one-loop contributions, hence we have

$$\mu \frac{dg_r}{d\mu} = -\frac{\epsilon}{2} g_r. \tag{3-40}$$

At next order, the one-loop becomes relevant and so we must include the counterterm in our calculation

$$16\pi^2 \mu \frac{dg_r}{d\mu} = -8\pi^2 \epsilon g_r + 3g_r^3 = \beta_g. \tag{3-41}$$

In deriving this result equation (3-40) has been used and we introduced the conventional notation for this quantity which is formally called $\beta-$function. A $\beta-$function is a quantity in which the energy scale dependence of the coupling parameter is implicitly encoded, implying that it only depends on its coupling parameter.

As an independent check, we next exploit the results derived when we investigated the effective potential. For a homogeneous background field configuration, to determine the effective potential we concluded that it was only need to specify the nature of the involved field, spin and degrees of freedom, and the effective mass of the field theory. In a ϕ^4 theory, the effective mass is trivially given by

$$m_{\text{eff}}^2 = m_r^2 + \frac{1}{2} g_r \phi_r^2. \tag{3-42}$$

The terms associated with the four vertex in the effective potential are

$$V_{\text{eff}}(\phi_r) \sim \frac{1}{4!}\delta g \phi_r^4 + \frac{g_r^2 \phi_r^4}{256\pi^2} \log\left(\frac{g_r \phi_r^2}{2\Lambda^2}\right). \tag{3-43}$$

One efficient way of determining what are the counterterms is to insert a term μ^2/μ^2, where μ is a energy variable, in the logarithm and separate it into two parts

$$V_{\text{eff}}(\phi_r) \sim \frac{1}{4!}\delta g \phi_r^4 + \frac{g_r^2 \phi_r^4}{256\pi^2} \log\left(\frac{\mu^2}{\Lambda^2}\right) + \frac{g_r^2 \phi_r^4}{256\pi^2} \log\left(\frac{g_r \phi_r^2}{2\mu^2}\right). \tag{3-44}$$

The counterterm is then given by

$$\delta g = -\frac{3g_r^2}{32\pi^2} \log\left(\frac{\mu^2}{\Lambda^2}\right). \tag{3-45}$$

Now the scheme to derive the differential equation associated with the constant quantity g_b does not involve explicitly the energy variable, that is,

$$\mu\frac{dg_b}{d\mu} = Z_g g_r \left(\frac{1}{Z_g}\mu\frac{dZ_g}{d\mu} + \frac{1}{g_r}\mu\frac{dg_r}{d\mu}\right) = 0 \tag{3-46}$$

and the differential equations is then

$$16\pi^2 \mu\frac{dg_r}{d\mu} = 3g_r^3 = 16\pi^2 \beta_g. \tag{3-47}$$

In the limit $\epsilon \to 0^+$ the equation (3-41) is in complete agreement with the result above. In the next chapter we shall put to good use the tools developed in here to investigate the vacuum stability of the Standard Model and the validity of the one-loop approximation and the relevance of other quarks in the running coupling.

4
Paralipomena on the Standard Model

In order to investigate the vacuum instability of the Standard Model, we need to understand the dependence of the Higgs self-interaction coupling, λ, on an arbitrary energy scale, μ. We know from the previous sections that such information can be precisely found in the $\beta-$functions. In the Standard Model case, the required $\beta-$functions form a set of differential equations, called renormalization group equations, which we can numerically solve and use to study the behaviour of the gauge couplings, the top quark coupling and the Higgs self-interaction coupling. Later we are going to verify our assumption that the top quark contribution is dominant over the other quarks.

Recall that the general effective potential has the form

$$V_{\text{eff}}(h) = V_0(h) + V_{1-\text{loop}}(h). \tag{4-1}$$

For the Standard Model, the term V_0 is the tree-level potential and the one-loop contributions come from the gauge bosons ($W^{\pm}$ and Z), the top quark and the scalar h. We here profit from the derived equation (3-24), because to find the renormalization group equations, we can use the scale arbitrariness of the effective potential; the coupling parameters and fields can be used to absorb the effects of changing the scale. If the effective potential cannot be affected by a change in the energy scale, then its total derivative must vanish, that is,

$$\mu\frac{dV}{d\mu} = \frac{\partial V_0}{\partial \lambda}\beta_\lambda + \frac{\partial V_0}{\partial m^2}\beta_{m^2} + \gamma_h h\frac{\partial V_0}{\partial h} + \mu\frac{\partial V_{1-\text{loop}}}{\partial \mu} = 0, \tag{4-2}$$

where γ_h is the anomalous dimension determining how the field normalization varies as the renormalization scale changes. Matching the powers of h^2 and h^4, we have

$$\beta_\lambda + 4\gamma_h\lambda = \frac{1}{8\pi^2}\left[12\lambda^2 + \frac{3g^4}{8} + \frac{3(g^2+g'^2)^2}{16} - 3y_t^4\right] \tag{4-3}$$

and

$$\beta_{m^2} + 2\gamma_h m^2 = \frac{3}{4\pi^2}m^2\lambda. \tag{4-4}$$

The anomalous dimension can be derived from the two-point correlation function $\langle H(x)H^\dagger(y)\rangle$ and is given by (4)

$$\gamma_h = \frac{1}{64\pi^2}(9g^2 + 3g'^2 - 12y_t^2). \tag{4-5}$$

Then, equations (4-5) and (4-3) yield the renormalization function

$$16\pi^2\beta_\lambda = 24\lambda^2 + \frac{3g^4}{4} + \frac{3(g^2+g'^2)^2}{8} - 6y_t^4 + \lambda(-9g^2 - 3g'^2 + 12y_t^2). \tag{4-6}$$

We next address the gauge coupling parameters. In place of evaluating the running coupling associated with each gauge group, we find it easier to consider a more general framework, the $SU(N \geq 1)$ gauge group, and then specialize for the Standard Model gauge groups. In a one-loop approximation, the gauge couplings have $\beta-$functions given by (32)

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2}. \tag{4-7}$$

This form is valid for $\beta-$functions of Abelian and non-Abelian Yang-Mills theories. For an arbitrary compact semisimple Lie group, the coefficients β_0 can be expressed in terms of characteristic parameters of the involved Yang-Mills symmetry group, that is,

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f, \tag{4-8}$$

where we have included the matter fields contribution, C_A is the quadratic Casimir in the adjoint representation, that is, $f^{acd}f^{bcd} = C_A\delta^{ab}$ and T_F is the Dynkin index of the fundamental representation of the symmetry group, satisfying $\mathrm{tr}(T^aT^b) = T_F\delta^{ab} = \delta^{ab}/2$ and n_f is the number of flavours.

For a theory to be asymptotically free the $\beta-$function the dominant term must be negative in the large μ limit. The equation (4-8) yields then a bound for the number of flavours

$$n_f < \frac{11}{2}N, \tag{4-9}$$

which for $n_f = 6$ is certainly satisfied for any $N \geq 2$. In particular, this constraint implies that the running couplings associated with the $U(1)$ gauge group will always diverge at some point. In the Standard Model this is not a problem, because in the region where such couplings diverge, the Standard Model is not valid.

Now, including a scalar field, these equations are slightly modified. Particularly, in the Standard Model case with n_H Higgs doublet in the electroweak sector, the coefficients become (14)

$$\beta_0^{U(1)} = -\frac{2}{3}n_f - \frac{1}{10}n_H,$$
$$\beta_0^{SU(2)} = -\frac{2}{3}\left[(11 - n_f) - \frac{1}{4}n_H\right], \tag{4-10}$$
$$\beta_0^{SU(3)} = 11 - \frac{2}{3}n_f.$$

If we consider the Standard Model as an effective field theory, n_f should be understood as the number of quarks existing below the threshold energy.

Thus, using the coefficients (4-10) in the Yang-Mills $\beta-$function (4-7), we get

$$\beta_{g'} = \frac{41}{96\pi^2}g'^3, \quad \beta_g = -\frac{19}{96\pi^2}g^3, \quad \beta_{g_s} = -\frac{7}{16\pi^2}g_s, \tag{4-11}$$

where in the hypercharge gauge coupling it has been used the Grand Unified Theory normalization, $g' \to \sqrt{5/3}g'$.
All that remains now is to write the top quark Yukawa coupling (33)

$$16\pi^2\beta_{y_t} = \left(\frac{9}{2}y_t^2 - \frac{17}{12}g'^2 - \frac{9}{4}g^2 - 8g_s^2\right)y_t. \tag{4-12}$$

We now have all the Standard Model $\beta-$functions. Unfortunately, these one-loop $\beta-$functions are not robust enough to make predictions on the running coupling, because at two-loop level there are substantial corrections to the running. We shall check this by numerically solving the renormalization group equations. Furthermore, we also shall check the validity of our assumption regards the irrelevance of the other quarks in comparison with the top quark.

To solve this system of differential equations, we first renormalize all quantities at the mass of the top quark $m_t = 173.21$ (7) We then numerically

$\mu = m_t$	λ	y_t	g	g'
Tree-level	0.12917	0.99561	0.65294	0.34972
One-loop	0.12774	0.95113	0.64754	0.35940
Two-loop	0.12604	0.94018	0.64779	0.35830

Table 4.1: The Standard Model parameters computed up to two-loop level in the modified minimal subtraction scheme at the top quark mass.

solve the system of differential equations using these renormalized quantities as initial values.

Including the next heaviest quark to the renormalization group equations, the bottom quark, which can be found in the reference (7). We make a plot with the numerical solution and compare with the equations with only the top quark included. The result is shown in figure 4. The plot confirms that indeed not including the other quarks can provide a good estimation of the running coupling.

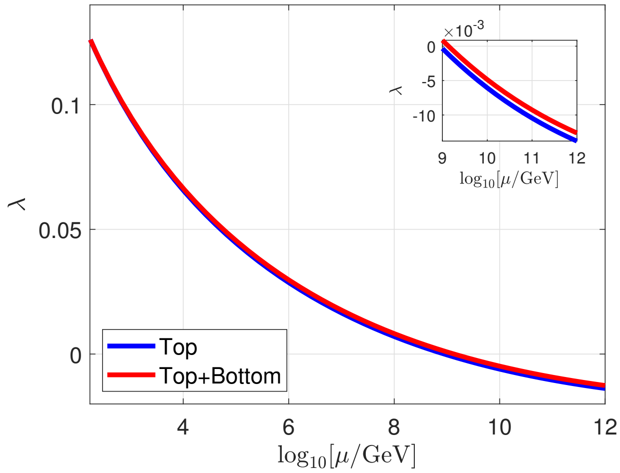

Figure 4.1: The renormalization group evolution of λ in the presence of only the top quark and including also the bottom quark. Inset shows how small is the difference between the two cases.

As we stated, the one-loop approximation does not suffice to provide a precise picture of the running coupling. Using the $\beta-$functions calculated in the reference (7), we display the plot of the running coupling λ in figure 4 for a one, two and three-loop approximation.

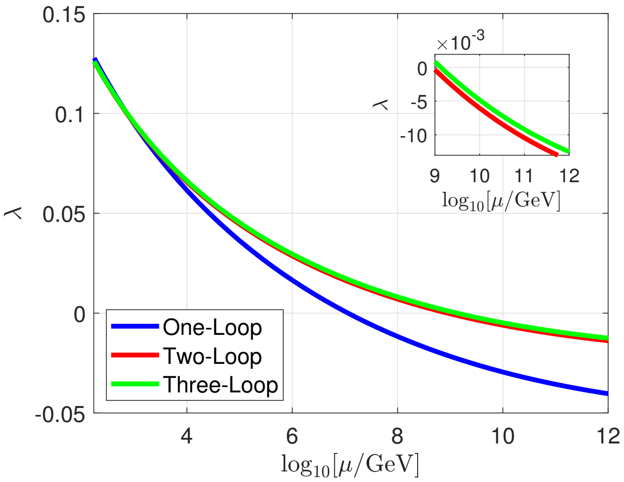

Figure 4.2: The qualitative behaviour of λ is the same, but quantitatively the two-loop approximation provides a substantial contribution to the running coupling.

Based on the numerical solution and the plot, in a any reliable calculation involving the running couplings of the Standard Model, we should include at least two-loop contributions. For instance, in a one-loop approximation, the onset of instability manifests itself around an energy scale of order $\mathcal{O}(10^7)$, whereas in a two-loop calculation this occurs around $\mu = 10^9$GeV. Also, such deviation could significantly impact the predicted values of a given observable. The main reason for this difference comes from the contributions of the top Yukawa coupling of order $\mathcal{O}(y_t^6)$ and from the three gauge coupling parameters, the latter have many positive contributions with coefficients greater than order $\mathcal{O}(1)$.

5
New Physics through the Higgs Portal

Recall that the Higgs potential is

$$V(H) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2, \tag{5-1}$$

writing the Higgs doublet as $H = \frac{1}{\sqrt{2}}\begin{pmatrix} H_1 + iH_2 \\ H_3 + iH_4 \end{pmatrix}$, the prefactor is there to keep the kinetic term canonical, then the potential reads

$$V(H_i) = \frac{1}{2}m^2(H_iH_i) + \frac{1}{4}\lambda(H_jH_j)^2. \tag{5-2}$$

That is, the potential is actually invariant under a larger $SO(4)$ symmetry, under which the quadruplet (H_1, H_2, H_3, H_4) transform in the fundamental representation. Having said that, the Higgs can be thought of as carrying two $SU(2)$ symmetries, the usual $SU(2)_L$ and the other is conventionally called $SU(2)_R$. Of course the Higgs field only enjoys this full symmetry in its own sector and before the spontaneous symmetry breaking takes place. Such process breaks down the $SU(2)_L \times SU(2)_R$ group to one of its subgroups, the $SU(2)_V$. Using this representation, we now consider the existence of an extended Higgs potential. Thanks to the low dimensionality of the Higgs operator, $H^\dagger H$, the Higgs sector can easily accommodate a new marginal or relevant operator of the form $\mathcal{O}(H^\dagger H)$, where $\mathcal{O}$ is a gauge-invariant operator with dimension lower than or equal to two. For instance, in our conservative approach, the additional operator is $\mathcal{O} = S^2$, where S is a massive scalar field which is neutral under the Standard Model gauge group. This Higgs Portal presents itself as an entirely new pathway to access the physics Beyond the Standard Model. Such portals are important to be studied not only from pragmatic grounds, being the simplest extension allowed by the symmetries, but also from theoretical grounds (34). A classical example is to consider the massive particle S as a cold dark matter candidate, where the interaction strength is tuned by a coupling parameter ρ. In this case, the scalar must be a stable particle, this condition is satisfied if the Lagrangian is invariant under the transformation $S \to -S$. As a consequence of this $\mathbb{Z}_2$ symmetry, the singlet S is not allowed to acquire a vacuum expectation value. Such scenario is particularly interesting, because if we assume that all the relic density of cold dark matter is composed of scalars S, then for a given M_S there is an coupling parameter ρ fixed by the physical mass (35)(36)(37)(38)(39)(40).

Another relevant application of such portal is in the study of baryogenesis. Although the Standard Model contains the necessary ingredients to realize a first-order phase transition (11); CP-violation, baryon number violation and interactions out of thermal equilibrium, the parameters are such that the electroweak phase transition is too weak to generate the required departure from equilibrium. At earlier times, the Universe was hot enough to allow for nonperturbative saddle-point solutions of the field equations associated with the Standard Model gauge group to form. These solutions are called sphalerons. Such solutions allow transitions to topologically inequivalent $SU(2)$ vacua, with distinct baryon number. This mechanism is expected to keep the matter-antimatter quantity in balance, as any deviations will be washed out by the sphalerons. However, as the Universe expands, the temperature cools down and as soon as temperature approaches its critical value, the bubbles of electroweak broken phase starts to form (41); at this point broken and unbroken phase are nearly degenerate. Within the bubbles, provided the CP violation is strong enough, the net baryon number will no longer be washed out by sphalerons; since the electroweak symmetry is broken, the sphaleron transition rates are strongly suppressed (42)(43)(44). In the typical approach, the validity of this scenario demands that $v_c/T_c \geqslant 1$, where v_c is the vacuum expectation value of the Higgs field evaluated in the broken phase at the critical temperature. Such condition is not satisfied by the parameters of the minimal Standard Model, its electroweak phase transition is not strong enough. A possible way to circumvent this phase transition weakness is to include Beyond Standard Model states (45)(46) or corrections to the Standard Model (47), note that such corrections rise naturally if, in the former case, the singlet scalar is heavy enough to be integrated out.

In addition to that, since we live in an era rich with data to challenge our models, we can also keep an eye on other promising alternatives. For instance, it is also possible to have a vector Higgs portal. Let V be a gauge boson of mass M_V. The most general renormalizable Lagrangian for such portal we can write down is then

$$\mathcal{L}_V = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}M_V^2 V_\mu V^\mu - \frac{1}{4}\lambda_V (V_\mu V^\mu)^2 - g_V v V_\mu V^\mu h - \frac{1}{2}g_V V_\mu V^\mu h^2,$$
$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu.$$

This is a renormalizable model, but with a broken gauge invariance. On the other hand, we can also consider fermionic Higgs portal, which is gauge invariant, but nonrenormalizable. Both portals have been recently reviewed in reference (40).

Besides these examples, there has been many studies in the direction of collider physics, in which the main focus is to determine the parameter space where the deviations from current measurements of the parameters of the Higgs sector are expected to be. This is a rather thrilling possibility, because although New Physics may deform the Standard Model and manifests itself through the Higgs sector, any new possibility must also satisfy the Electroweak Precision Measurements. In this line of research the far more challeging scenario to be probed is the one in which the scalar does not acquire a nonzero vacuum expectation value and it is heavier than the Higgs boson (48). In case the scalar has a nontrivial vacuum field configuration, the possibilities of experimental verification are considerably increased. For instance, in the mass eigenbasis, the conventional Higgs is a linear combination of two fields H_1 and H_2, so all the observables of the Higgs physics will have a manifestation of the new physics through the mixing angle of the Higgs and the scalar field (49)(50)(51)(52)(53)(54)(55).

5.1 Potential Stability

The extended potential in the tree approximation is

$$V(\chi) = \frac{1}{2}\sum_{ij}\chi_i\mathcal{M}^2_{ij}\chi_j + \frac{1}{4}\sum_{kl}\chi_k^2\mathcal{C}_{kl}\chi_l^2, \tag{5-3}$$

where we have introduced the column vector of fields χ, a matrix $\mathcal{M}^2$ in which its entries are the mass parameters and a matrix $\mathcal{C}$ with the couplings associated with quartic interactions,

$$\chi = \begin{pmatrix} S \\ H \end{pmatrix}, \quad \mathcal{M}^2 = \begin{pmatrix} M^2 & 0 \\ 0 & m^2 \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} \kappa/6 & \rho/2 \\ \rho/2 & \lambda \end{pmatrix}. \tag{5-4}$$

We find that to study the vacuum stability, the unitary gauge provides a neat framework and so we shall employ it in order to investigate the vacua of the extended potential. Next, we assume that the potential has, at least, continuous second-order partial derivatives at a stationary point $\bar{\chi}$ and we expand it around this critical point

$$V(\chi) = V(\bar{\chi}) + \sum_i \frac{\partial V(\bar{\chi})}{\partial \chi_i}(\chi_i - \bar{\chi}_i) + \frac{1}{2}\sum_{jk}(\chi_j - \bar{\chi}_j)\frac{\partial^2 V}{\partial\chi_j\partial\chi_k}(\chi_k - \bar{\chi}_k), \tag{5-5}$$

performing a shift $\chi \to \chi + \bar{\chi}$, the expansion reads

$$V(\chi+\bar{\chi}) - V(\bar{\chi}) = \sum_i \frac{\partial V(\bar{\chi})}{\partial \chi_i}\chi_i + \frac{1}{2}\sum_{jk} \chi_j \frac{\partial^2 V}{\partial \chi_j \partial \chi_k}\chi_k. \tag{5-6}$$

If a differentiable scalar potential has a stationary point at $\bar{\chi}$, the nature of the stationary point is determined by the algebraic sign of the difference $V(\chi+\bar{\chi}) - V(\bar{\chi})$ for χ near $\bar{\chi}$. At a stationary point, $\text{grad}V(\bar{\chi}) = 0$, and the Taylor expansion becomes

$$V(\chi+\bar{\chi}) - V(\bar{\chi}) = \frac{1}{2}\sum_{jk} \chi_j \frac{\partial^2 V}{\partial \chi_j \partial \chi_k}\chi_k. \tag{5-7}$$

Then, the needed information about the algebraic sign is to be found in the quadratic form of the expansion. The coefficients of the quadratic form are the second-order partial derivatives evaluated at $\bar{\chi}$. The $n \times n$ matrix of second-order derivatives is called the Hessian matrix and is denoted by $\Upsilon(\chi)$.

The nature of the stationary points is determined by the eigenvalues of the Hessian matrix. Since the quadratic terms are dominant over higher-order contributions, it seems reasonable to expect that the algebraic sign of equation (5-7) is defined by the quadratic form. Moreover, nature of these points is given by the eigenvalues of the Hessian matrix. There are four possible cases

- If all the eigenvalues of $\Upsilon(\bar{\chi})$ are positive, V has a relative minimum at $\bar{\chi}$
- If all the eigenvalues of $\Upsilon(\bar{\chi})$ are negative, V has a relative maximum at $\bar{\chi}$
- If $\Upsilon(\bar{\chi})$ has both positive and negative eigenvalues, V has a saddle point at $\bar{\chi}$
- If some eigenvalue is zero, then no conclusion can be drawn from the eigenvalues of the Hessian matrix and higher order derivatives are required to treat such examples

Next we go a step further and exploit the fact that we have a 2×2 Hessian matrix, because in such cases the nature of the stationary point can be determined by the algebraic sign of the second derivative $\partial^2 V/\partial S^2$ and the determinant of the Hessian matrix.
For instance, let $\tilde{\chi}$ be a stationary point of a scalar potential $V(\chi)$ with continuous second-order partial derivatives and fix

$$A = \frac{\partial^2 V}{\partial S^2}, \quad B = \frac{\partial^2 V}{\partial S \partial H}, \quad C = \frac{\partial^2 V}{\partial H^2}. \tag{5-8}$$

Then, the determinant of the Hessian matrix is

$$\det \Upsilon(\tilde{\chi}) = AC - B^2 = \Delta. \tag{5-9}$$

This matrix has two eigenvalues λ_1 and λ_2 and they related by the equations

$$\lambda_1 + \lambda_2 = A + C, \quad \lambda_1 \lambda_2 = \Delta. \tag{5-10}$$

Therefore, there are two possibilities

- if $\Delta < 0$ the eigenvalues have opposite signs, so V has a saddle point at $\tilde{\chi}$.
- if $\Delta > 0$ the eigenvalues have equal signs, so V has a relative minimum at $\tilde{\chi}$ when $A > 0$ and a relative maximum at $\tilde{\chi}$ when $A < 0$.

This test provides no information for $\Delta = 0$, but, as far as we are concerned, such case is not relevant to our study. Now we apply this mathematical machinery to investigate the vacuum stability of the extended potential.

First we determine the extrema of the potential (5-3), that is, we find its stationary points. To do so, it was discussed above that in this situation, we want to find a point $\bar{\chi}^2$ such that all first order partial derivatives vanish

$$\frac{\partial V(\bar{\chi})}{\partial \chi_k} = \bar{\chi}_k \left(\mathcal{M}_k^2 + \sum_j \mathcal{C}_{kj} \bar{\chi}_j^2 \right) = 0. \tag{5-11}$$

Unfortunately, we must also determine the nature of these stationary points. In particular, we want no vacuum in the theory with a runaway direction. The cornerstone *sine qua non* to avoid such problem is assuming that the Higgs and the scalar self-couplings never turn negative for any energy scale

$$\lambda(\Lambda) > 0, \quad \kappa(\Lambda) > 0. \tag{5-12}$$

Under the assumptions (5-12), when the field configurations of h and S are large enough such that the quartic terms outrun the quadratic terms, the potential becomes

$$\begin{aligned} 4V(h,S) &\approx \begin{pmatrix} S^2 & h^2 \end{pmatrix} \begin{pmatrix} \kappa/6 & \rho/2 \\ \rho/2 & \lambda \end{pmatrix} \begin{pmatrix} S^2 \\ h^2 \end{pmatrix} \\ &= \left(\sqrt{\frac{\kappa}{6}} S^2 - \sqrt{\lambda} h^2 \right)^2 + \left(\rho + \sqrt{\frac{2\kappa\lambda}{3}} \right) h^2 S^2. \end{aligned} \tag{5-13}$$

Since the first term can never be negative, we shall only focus on the second term. There are two cases we must investigate in here. When ρ is positive, there is no immediate danger to the theory. However, as the energy scale grows, the running coupling λ flows to negative values around $\mu = 10^9$TeV, allowing for a runaway direction. Therefore, we conclude that in order to maintain the theory safe from directions in which the potential can assume arbitrarily large negative values, we must impose the following set of conditions

$$\lambda(\Lambda) > 0, \quad \kappa(\Lambda) > 0. \tag{5-14}$$

In the second case, when ρ is negative. This coupling may outrun the product $\kappa\lambda$ allowing for runaway directions. Then, we must impose that the coefficient of the h^2S^2 remains positive for all energy scales

$$\rho + \sqrt{\frac{2\kappa\lambda}{3}} > 0. \tag{5-15}$$

If we fix $\rho = -\rho' > 0$, then, after a few algebraic operations, the condition above assumes a more familiar form

$$\det \mathcal{C}(\Lambda) > 0 \tag{5-16}$$

and the set of conditions required to avoid runaway directions is

$$\kappa(\Lambda)\lambda(\Lambda) - \frac{3}{2}\rho^2(\Lambda) > 0, \quad \kappa(\Lambda) > 0. \tag{5-17}$$

In one-loop approximations, we note that the running coupling κ is always positive for any energy scale. Concluding that this statement is true it is not obvious, but later it will become clear when we present the $\beta_\kappa-$function. So, if $\det \mathcal{C}$ is positive, then necessarily the running coupling λ is also positive. This implies that when investigating the behaviour of the determinant is sufficient to determine under which values of the coupling parameters the theory is stable. Exploiting these assumptions, we find the extrema points.

Now we must bear in mind that for a positive ρ, if the the mass parameter $\mathcal{M}_k^2$ associated with the particle χ_k is positive, then the extremum is trivial, the minimum of $V(\chi)$ is at the point $\chi = 0$. On the other hand, for a negative ρ a more involved analysis is required and we shall deal with it below, because it is now possible to find nontrivial real solutions for the extrema points. For $\mathcal{M}_k^2$ negative, regardless of the sign ρ might have, the extremum is not going to be trivial at the point $\bar{\chi}_k \neq 0$, that is,

$$\mathcal{M}_k^2 + \sum_j \mathcal{C}_{kj}\bar{\chi}_j^2 = 0. \tag{5-18}$$

This same equation also holds for the case where the mass parameters are positive, but ρ is negative. The case in which no field acquires a vacuum expectation value there is nothing to be evaluated. In the electroweak vacuum, we have

$$\bar{S} = 0, \quad \bar{H}^2 = -\frac{m^2}{\lambda}. \tag{5-19}$$

Again, the mass parameter must be negative $m^2 < 0$ and without any loss of generality we have chosen the vacuum expectation value along the real component of the Higgs doublet. If only the scalar S has a nontrivial vacuum structure, then

$$\bar{S}^2 = -\frac{6M^2}{\kappa}, \quad \bar{H} = 0, \tag{5-20}$$

where $M^2 < 0$. Finally, in the most general case both fields have a nonzero vacuum expectation value

$$\begin{pmatrix} \bar{S}^2 \\ \bar{H}^2 \end{pmatrix} = \frac{1}{\det \mathcal{C}} \begin{pmatrix} -\lambda & \rho/2 \\ \rho/2 & -\kappa/6 \end{pmatrix} \begin{pmatrix} M^2 \\ m^2 \end{pmatrix}. \tag{5-21}$$

Thus, we have the solutions listed in table 5.1, where the subscripts denote which vacuum we are addressing in our discussions.

Vacuum Expectation Value	$\bar{S}^2 = 0$	$\bar{S}^2 \neq 0$
$\bar{H}^2 = 0$	$(0,0)_1$	$(-6M^2/\kappa, 0)_3$
$\bar{H}^2 \neq 0$	$(0, -m^2/\lambda)_2$	$(\det \mathcal{C})^{-1}(-\lambda\widetilde{M}^2, -\kappa\widetilde{m}^2/6)_4$

Table 5.1: The field configurations which satisfy the condition grad$V = 0$.

Now that we know stationary points of the potential, we must classify these points. In order to understand the complete vacuum structure of the theory, we find it helpful to use the Cartesian plane $M^2 \times m^2 = \{(M^2, m^2) \in \mathbb{R}^2 : M^2 \in \mathbb{R}, m^2 \in \mathbb{R}\}$ and to define two new quantities

$$\widetilde{m}^2 = m^2 - 3\frac{\rho M^2}{\kappa}, \tag{5-22}$$

$$\widetilde{M}^2 = M^2 - \frac{1}{2}\frac{\rho m^2}{\lambda}, \tag{5-23}$$

with which we cut the mass plane into eight regions, each one of them corresponding to a different field configurations of the vacuum expectation value. However, we will see below that not all of them are interesting or valid (for instance, since all the coupling parameters are real, we must drop solutions in which $\bar{\chi_j}^2 < 0$).

The mass matrix in the tree approximation is then

$$\frac{\partial^2 V(\bar{\chi})}{\partial\chi_l \partial\chi_k} = \delta_{kl}\left(\mathcal{M}_k^2 + \sum_j \mathcal{C}_{kj}\bar{\chi}_j^2\right) + 2\bar{\chi}_k \mathcal{C}_{kl}\bar{\chi}_l, \tag{5-24}$$

which is equivalent to the Hessian matrix Υ. The first vacuum is trivial, there is no symmetry breaking

$$\Upsilon(0,0) = \text{diag}(M^2, m^2). \tag{5-25}$$

In the second and third vacuum, $m^2 < 0$ and $M^2 < 0$, respectively. So, at least one of the eigenvalues of the Hessian will be positive,

$$\Upsilon\left(\bar{S}^2 = 0, \bar{H}^2 = -\frac{m^2}{\lambda}\right) = \text{diag}\left(M^2 - \frac{1}{2}\frac{\rho m^2}{\lambda}, -2m^2\right) = \text{diag}(\widetilde{M}^2, -2m^2), \tag{5-26}$$

$$\Upsilon\left(\bar{S}^2 = -6\frac{M^2}{\kappa}, \bar{H}^2 = 0\right) = \text{diag}\left(-2M^2, m^2 - 3\frac{\rho M^2}{\kappa}\right) = \text{diag}(-2M^2, \widetilde{m}^2). \tag{5-27}$$

In the fourth vacuum both fields are going to acquire a nonzero vacuum expectation value, and now we have a nontrivial Hessian matrix

$$\Upsilon(\bar{S}^2, \bar{H}^2) = \frac{1}{3}\begin{pmatrix} \kappa\bar{S}^2 & 3\rho\bar{S}\bar{H} \\ 3\rho\bar{S}\bar{H} & 6\lambda\bar{H}^2 \end{pmatrix} \tag{5-28}$$

with eigenvalues

$$M^2_{\pm} = -\frac{\lambda\kappa}{6\det\mathcal{C}}\left[\widetilde{M}^2 + \widetilde{m}^2 \pm \sqrt{(\widetilde{M}^2 - \widetilde{m}^2)^2 + 6\frac{\rho^2\widetilde{M}^2\widetilde{m}^2}{\lambda\kappa}}\right]. \tag{5-29}$$

However, studying the sign of those eigenvalues may prove to be a laborious task and since further investigations are needed to determine which kind of extremum the fields are situated, we can profit from the discussion above about 2×2 Hessians with nondiagonal terms. As previously demonstrated, to determine the nature of a stationary point, we can evaluate the sign of the determinant

$$\det[\Upsilon(\bar{S}^2, \bar{H}^2)] = \frac{2}{3}\frac{\lambda\kappa}{\det\mathcal{C}}\widetilde{M}^2\widetilde{m}^2 \tag{5-30}$$

and the term $\kappa\bar{S}^2$ which at the critical point reads

$$\kappa\bar{S}^2/6 = -\frac{\lambda\kappa}{\det\mathcal{C}}\widetilde{M}^2. \tag{5-31}$$

So, in case the other parameters are positive, namely κ, λ and $\det\mathcal{C}$, this amounts to say that the nature of the stationary points will be fixed by the sign of $\widetilde{M}^2$ and $\widetilde{m}^2$. Next, we present an analysis of the nature of the potential evaluated in its extrema points in the $M^2 \times m^2$ plane. At all times we will assume that when ρ is positive we have $\lambda > 0$, $\kappa > 0$ and when ρ is negative $\det\mathcal{C} > 0$, $\kappa > 0$. Otherwise the potential would not be bounded from below, the condition imposed on the determinant is particularly important for the case in which coupling parameter ρ is negative, when ρ is positive the potential do not develop a runaway when $\det C < 0$.

Region/Field Configuration	$(0,0)_1$	$(0,\bar{H}^2)_2$	$(\bar{S}^2,0)_3$	$(\bar{S}^2,\bar{H}^2)_4$
I	Minimum	0	0	0
II	Saddle	Minimum	0	0
III	Maximum	Minimum	Saddle	0
IV	Maximum	Saddle	Saddle	Minimum
V	Maximum	Saddle	Minimum	0
VI	Saddle	0	Minimum	0

Table 5.2: The nature of the stationary points for when the coupling ρ and the determinant are both positive quantities. Although the lines given by $\widetilde{m}^2 = 0$ and $\widetilde{M}^2 = 0$ cut the mass plane into eight regions. In the quadrant $M^2 > 0, m^2 > 0$, the fourth vacuum does not have real vacuum expectation values. The entries with zeros indicate that there is no real solution.

In the first situation, we study the vacuum stability assuming that the Higgs-Scalar interaction parameter ρ is positive. One immediate conclusion we can draw from the diagram 5.1 and the table 5.2 is that the local minima are not competing with each other, only one of them can correspond to a local minima at a given region. If quantum corrections do not provide large logarithmic contributions, this results should also hold at one-loop level.

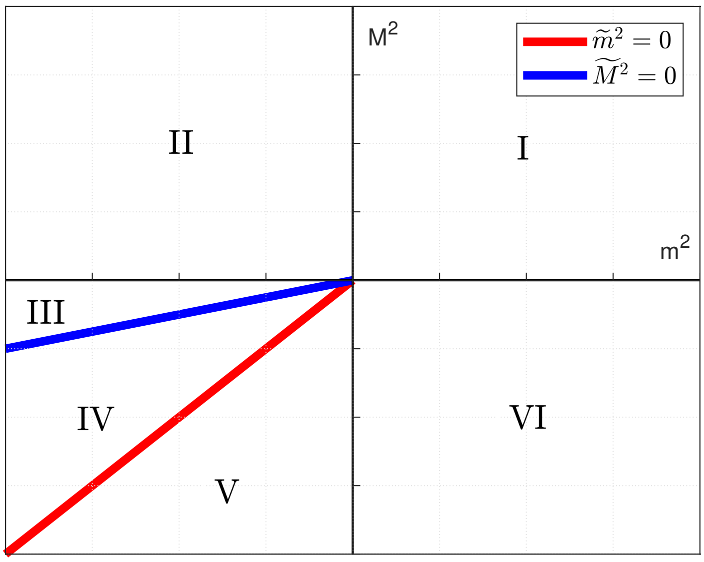

Figure 5.1: Phase diagram for when the coupling ρ and the determinant are both positive. The red and blue lines are given by (5-22) and (5-23), respectively.

When ρ is positive the determinant can assume negative values. In table 5.3 we show the results of our investigation under the assumption that $\det \mathcal{C} < 0$.

Region/Field Configuration	$(0,0)_1$	$(0,\bar{H}^2)_2$	$(\bar{S}^2,0)_3$	$(\bar{S}^2,\bar{H}^2)_4$
I	Minimum	0	0	0
II	Saddle	Minimum	0	0
III	Maximum	Minimum	Saddle	0
IV	Maximum	Minimum	Minimum	Saddle
V	Maximum	Saddle	Minimum	0
VI	Saddle	0	Minimum	0

Table 5.3: The nature of the stationary points for when ρ is positive and the determinant is negative. When $\det \mathcal{C} < 0$ the lines coming from $\tilde{M}^2 = 0$ and $\tilde{m}^2 = 0$ are inverted.

What is remarkable in this result is that when the determinant is allowed to be negative, the electroweak vacuum competes with the singlet vacuum. In addition, the case when both fields, H and S, acquire a nonzero vacuum expectation value is no longer a minimum.

The second situation we also address is the one in which ρ is negative. The analysis is almost similar. Putting to good use the diagram 5.2 and the table 5.4, we find that the minima of each vacua are no competing with each other. However, in this case, the region associated with the fourth field configuration extends to a larger parameter region, it has three possible minima.

Region/Field Configuration	$(0,0)_1$	$(0,\bar{H}^2)_2$	$(\bar{S}^2,0)_3$	$(\bar{S}^2,\bar{H}^2)_4$
I	Minimum	0	0	0
II	Saddle	Minimum	0	0
III	Saddle	Saddle	0	Minimum
IV	Maximum	Saddle	Saddle	Minimum
V	Saddle	0	Saddle	Minimum
VI	Saddle	0	Minimum	0

Table 5.4: When the interaction parameter ρ is negative, the electroweak vacuum no longer has a competition with the third field configuration.

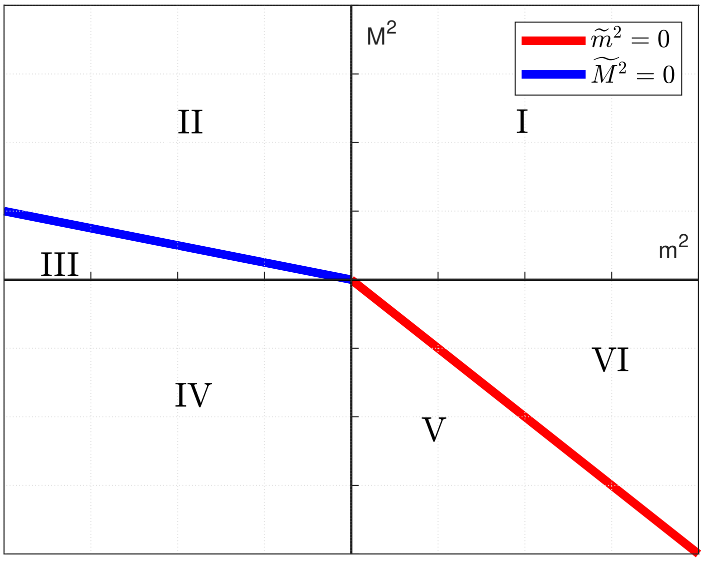

Figure 5.2: Phase diagram for the theory when $\rho < 0$.

5.2 Effective Potential

It has already been discussed the strategy we must exploit in order to calculate the effective potential. In here we shall put to good use these previous discussions. We first introduce the relevant part of the nonminimal Standard Model action, that is, the one involving the Higgs sector carrying along the new physics

$$\begin{aligned}\mathcal{I} =& \frac{1}{2}\sum_i(\partial_\mu H_i)^2 + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m^2\sum_i H_iH_i - \frac{1}{2}M^2S^2 \\ &- \frac{1}{4}\lambda\sum_i H_iH_i\sum_j H_jH_j - \frac{1}{4!}\kappa S^4 - \frac{1}{4}\rho\sum_i H_iH_iS^2.\end{aligned} \tag{5-32}$$

Hitherto we shall omit the summation signs. Next, we calculate the mass matrix in a tree-level approximation which is made up of the second-order derivatives of the action (5-32). The diagonal terms are

$$\frac{\delta^2\mathcal{I}}{\delta S(x)\delta S(x')} = -\left(\Box + M^2 + \frac{1}{2}\rho H_iH_i + \frac{1}{2}\lambda_S S^2\right)\delta(x-x') \tag{5-33}$$

$$\frac{\delta^2\mathcal{I}}{\delta H_j(x)\delta H_k(x')} = -\left\{\delta_j^k[\Box + m_h^2 + \frac{1}{2}\rho S^2 + \lambda(H_iH_i)] + 2\lambda H_jH_k\right\}\delta(x-x') \tag{5-34}$$

and the off-diagonal terms are

$$\frac{\delta^2 \mathcal{I}}{\delta S(x)\delta H_j(x')} = -\rho H_j S \delta(x-x') \tag{5-35}$$
$$\frac{\delta^2 \mathcal{I}}{\delta H_k(x)\delta S(x')} = -\rho H_k S \delta(x-x'). \tag{5-36}$$

Note that the functional derivatives are to be taken at the background field. In particular, for the Higgs field, without any loss of generality, we can always use the $SO(4)$ symmetry to set the classical solutions as $\bar{H}_3 = v$ and $H_1 = H_2 = H_4 = 0$ to all orders. By doing so, we do not even need to turn on a background for any Higgs field, besides H_3. In what follows, we shall assume that there exists a turned on background field only for $H_3 = h$. Hence, the diagonal terms of the mass matrix are

$$(\mathcal{M}^2_{\text{eff}})_{00} = M^2 + \frac{1}{2}\kappa S^2 + \frac{1}{2}\rho h^2, \quad (\mathcal{M}^2_{\text{eff}})_{33} = m^2 + 3\lambda h^2 + \frac{1}{2}\rho S^2$$
$$(\mathcal{M}^2_{\text{eff}})_{11} = m^2 + \lambda h^2 + \frac{1}{2}\rho S^2 = (\mathcal{M}^2_{\text{eff}})_{22} = (\mathcal{M}^2_{\text{eff}})_{44}$$

and the off-diagonal entries are

$$(\mathcal{M}^2_{\text{eff}})_{03} = (\mathcal{M}^2_{\text{eff}})_{30} = \rho h S.$$

Now that we have evaluated the second-order functional derivatives and have shown explicitly the mass matrix at tree-level, we focus on the action. Recalling that first-order terms cannot contribute to the one-particle irreducible diagrams, our action is reduced to a tree-level part and a one-loop contribution

$$\mathcal{I}[\chi + \bar{\chi}] = \mathcal{I}[\bar{\chi}] + \frac{1}{2}\sum_{ij}\int d^4x \chi_i(x)[\delta_{ij}\Box + (\mathcal{M}^2_{\text{eff}})_{ij}]\chi_j(x). \tag{5-37}$$

Before performing the Gaussian integration, we must bear in mind that if the operator $\delta_{ij}\Box + (\mathcal{M}^2_{\text{eff}})_{ij}$ is not positive definite, then the functional integration diverges. In any case, this is not a hindrance in our calculations, because a judicious rotation of the integration contour can resolve such problem. After performing the functional integration, we obtain

$$V_{\text{eff}}[h,S] = V[h,S] + \frac{1}{64\pi^2}\text{tr}\left(\widetilde{\mathcal{M}}^4_{\text{eff}} \log\left(\frac{\widetilde{\mathcal{M}}^2_{\text{eff}}}{\mu^2}\right)\right), \tag{5-38}$$

where $\widetilde{\mathcal{M}}_{\text{eff}}$ is the diagonalized effective mass matrix.

5.3 BSM β-functions

Now that we have obtained the extended effective potential, the next step is developing a way to understand the evolution of the new parameters, κ and ρ, over some finite range of energies. As discussed in previous chapters, this can be done by exploiting the concept of renormalization group equations. In here, focusing only on the affected degrees of freedom of the Standard Model, we shall derive $\beta-$functions from the effective potential in presence of the new physics. Additionally, we provide the guidelines on how to obtain the same results using counterterms.

First, let us find out what are the counterterms. Starting out from the extended potential

$$V(H,S) = \frac{1}{2}m^2 H^\dagger H + \frac{1}{2}M^2 S^2 + \lambda(H^\dagger H)^2 + \frac{1}{4!}\kappa S^4 + \frac{1}{2}\rho(H^\dagger H)S^2. \quad (5\text{-}39)$$

The renormalized fields are conventionally related to the bare fields by their corresponding field strength renormalization parameters Z_H and Z_S, the Higgs and the scalar, respectively. We apply to the potential the following set of simple modifications, we rescale the field intensity, send the mass parameters and the coupling parameters to their bare value

$$\begin{aligned} &H \to Z_H^{1/2} H, \quad m \to m_B, \quad \lambda \to \lambda_B \\ &S \to Z_S^{1/2} S, \quad M \to M_B, \quad \kappa \to \kappa_B, \quad \rho \to \rho_B. \end{aligned} \quad (5\text{-}40)$$

Hence, after the transformations, the extended potential becomes

$$\begin{aligned} V(H,S) =& \frac{1}{2}m_B^2 Z_H H^\dagger H + \frac{1}{2}M_B^2 Z_S S^2 + \frac{1}{2}\rho_B Z_H Z_S (H^\dagger H)S^2 \\ &+ \lambda_B Z_H^2 (H^\dagger H)^2 + \frac{1}{4!}\kappa_B Z_S^2 S^4. \end{aligned} \quad (5\text{-}41)$$

Now that we have assembled together mass and coupling parameters with the field strength renormalization, we define how the infinite bare parameters are related to the X_R quantities, renormalized couplings, by a parameter renormalization Z_X:

$$\begin{aligned} &Z_H m_B^2 = m^2 + \delta m^2, \quad Z_S M_B^2 = M^2 + \delta M^2 \\ &Z_H^2 \lambda_B = \lambda + \delta\lambda, \quad Z_S^2 \kappa_B = \kappa + \delta\kappa, \quad Z_H Z_S \rho_B = \rho + \delta\rho. \end{aligned} \quad (5\text{-}42)$$

Next we want to expand these coupling around some tree-level value, which can be taken to be any available convenient choice. The expansions are conventionally written as

$$Z_X = 1 + \delta X. \quad (5\text{-}43)$$

When a theory is independent of some quantity, it amounts to say that we can setup a homogeneous differential equation based on that quantity. For instance, say X is an observable that do not depend on a quantity Y, then

$$\frac{dX}{dY} = 0. \tag{5-44}$$

In particular, for our case, there is in fact a system of homogeneous differential equations based on the μ-independence of the bare values, that is,

$$\begin{aligned} \mu\frac{d\rho_B}{d\mu} &= (\rho+\delta\rho)\mu\frac{d}{d\mu}\left(\frac{1}{Z_H Z_S}\right) + \frac{1}{Z_H Z_S}\mu\frac{d(\rho+\delta\rho)}{d\mu} = 0, \\ \mu\frac{d\kappa_B}{d\mu} &= (\kappa+\delta\kappa)\mu\frac{d}{d\mu}\left(\frac{1}{Z_S^2}\right) + \frac{1}{Z_S^2}\mu\frac{d(\kappa+\delta\kappa)}{d\mu} = 0. \end{aligned} \tag{5-45}$$

For the sake of brevity, we decided to show only relevant and beyond standard model coupling parameters. Hence, in a first-order approximation the renormalization group equations are

$$\begin{aligned} &\gamma_{m^2} = \mu\frac{\partial\delta H}{\partial\mu} - \frac{\mu}{m^2}\frac{\partial\delta m^2}{\partial\mu}, \quad \gamma_{M^2} = \mu\frac{\partial\delta S}{\partial\mu} - \frac{\mu}{M^2}\frac{\partial\delta M^2}{\partial\mu} \\ &\beta_\lambda = 2\lambda\mu\frac{\partial\delta H}{\partial\mu} - \mu\frac{\partial\delta\lambda}{\partial\mu}, \quad \beta_\kappa = 2\kappa\mu\frac{\partial\delta S}{\partial\mu} - \mu\frac{\partial\delta\kappa}{\partial\mu} \\ &\beta_\rho = \rho\left(\mu\frac{\partial\delta H}{\partial\mu} + \mu\frac{\partial\delta S}{\partial\mu}\right) - \mu\frac{\partial\delta\rho}{\partial\mu}. \end{aligned}$$

In the derivation of the above equations, we combined the convention defined in equation (5-43) with the consequence of variable-independence introduced in equation (5-45). Since we will not employ Feynman diagrams to calculate the $\beta-$functions, we shall only point out the necessary calculations. In order to diagrammatically obtain the $\beta-$functions for the nonminimal Standard Model, we must evaluate the four-vertex diagrams associated with the scalar S and the Higgs-Scalar interaction. Furthermore, the latter also provides an additional contribution to β_λ.

Fortunately, we can avoid these calculations, because we have already set the stage to go through all the calculations when we evaluated the effective potential in the previous chapter. There is only one caveat in this scheme. When we calculated the effective potential, we restricted ourselves to an homogeneous field configurations, so we have no means to compute the field strength renormalization within this framework. For the Higgs field, we can employ perturbation theory and evaluate the relevant diagrams associated with the propagator renormalization, the Landau gauge is the most convenient gauge to perform the calculations. The scalar field has no complications, because the involved loops in its renormalization do not depend on any external momentum.

First, we observe that the evaluated effective potential (5-38) is a combination of the tree-level interaction (classical Lagrangian) with the addition of one-loop effects (quantum corrections), that is,

$$V_{\text{eff}} = V_0 + V_1, \tag{5-46}$$

where V_0 is the tree-level potential and V_1 are the one-loop quantum corrections. As we have discussed, the effective potential cannot be affected by a change in the energy parameter μ. Recalling the chain rule and taking the total derivative of V_{eff}, we obtain

$$-\mu\frac{\partial V_1}{\partial\mu} = \sum_{ij}\frac{\partial V_0}{\partial C_i}\beta_{C_i} + \sum_i \mathcal{M}_i^2\frac{\partial V_0}{\partial \mathcal{M}_i^2}\gamma_{\mathcal{M}_i^2} + \sum_i \chi_i\frac{\partial V_0}{\partial\chi_i}\gamma_{\chi_i}, \tag{5-47}$$

where $C_j = (\lambda, \kappa, \rho)_j$, $\mathcal{M}^2$ is the tree-level mass matrix and $\chi_i = (h, S)_i$. Inserting the effective potential into the equation above and considering the aforementioned definitions, the equation (5-47) becomes

$$\begin{aligned}\frac{1}{32\pi^2}\text{tr}(\mathcal{M}^4_{\text{eff}}) = &\frac{1}{2}(m^2\gamma_{m^2} + 2m^2\gamma_H)h^2 + \frac{1}{2}(M^2\gamma_{M^2} + 2M^2\gamma_S)S^2 \\ &+ \frac{1}{4}(\beta_\lambda + 4\lambda\gamma_H)h^4 + \frac{1}{4!}(\beta_\kappa + 4\kappa\gamma_S)S^4 + \frac{1}{4}[\beta_\rho + 2\rho(\gamma_H+\gamma_S)]h^2S^2.\end{aligned}$$

Matching the h^2, S^2, h^4, S^4 and h^2S^2 coefficients yield

$$16\pi^2 m^2\gamma_{m^2} = 12\lambda m^2 + M^2\rho - 32\pi^2 m^2\gamma_H, \tag{5-48}$$
$$16\pi^2 M^2\gamma_{M^2} = M^2\kappa + 4m^2\rho - 32\pi^2 M^2\gamma_S, \tag{5-49}$$
$$16\pi^2\beta_\lambda = \frac{1}{2}\rho^2 + 16\pi^2\beta_\lambda^{\text{SM}}, \tag{5-50}$$
$$16\pi^2\beta_\kappa = 3\kappa^2 + 12\rho^2 - 64\pi^2\kappa\gamma_S, \tag{5-51}$$
$$16\pi^2\beta_\rho = \rho(12\lambda + 4\rho + \kappa) - 32\pi^2\rho(\gamma_H + \gamma_S). \tag{5-52}$$

where it has been used that

$$\gamma_H = \frac{1}{2}\mu\frac{d\log Z_H}{d\mu}, \quad \gamma_S = \frac{1}{2}\mu\frac{d\log Z_S}{d\mu}, \tag{5-53}$$

where the $\gamma-$functions are called an anomalous dimension. This terminology has to do with the fact that in a quantum theory, we typically have deviations from the classical scaling behaviour. The interactions involving the propagator of the scalar do not generate contributions which are dependent on the external momenta at one-loop order, thus there is no divergences to be renormalized, which means that $\gamma_S = 0$. Whereas, the anomalous dimension of the Higgs field is

$$\gamma_H = \frac{1}{16\pi^2}\left[\frac{3}{4}(3g^2 + g'^2) - 3y_t^2\right]. \qquad (5\text{-}54)$$

In this nonminimal Standard Model, besides ρ and κ, we must also determine the $\beta-$functions for the gauge coupling parameters g', g, g_s, the Higgs quartic coupling λ and from the fermion sector we only include the running coupling from the heaviest quark, y_t. At the one-loop level, the gauge couplings and the Yukawa coupling y_t run unaffected by the new degrees of freedom, but, as we have seen, the Higgs self-interaction receives an additional contribution from the extended sector and the β-function for λ becomes

$$16\pi^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + (12y_t^2 - 3g'^2 - 9g^2)\lambda + \frac{3}{8}(g'^2 + g^2)^2 + \frac{3}{4}g^4 + \frac{1}{2}\rho^2. \quad (5\text{-}55)$$

Hence, we now have all the $\beta-$functions and the anomalous dimensions in the one-loop approximation for the nonminimal Standard Model. In the gauge sector, we have

$$16\pi^2\mu\frac{dg_1}{d\mu} = \frac{41}{6}g'^3, \quad 16\pi^2\mu\frac{dg_2}{d\mu} = -\frac{19}{6}g^3, \quad 16\pi^2\mu\frac{dg_3}{d\mu} = -7g_s^3.$$

The $\beta-$function for the top quark is unaffected

$$16\pi^2\mu\frac{dy_t}{d\mu} = \left(\frac{9}{2}y_t^2 - \frac{17}{12}g'^2 - \frac{9}{4}g^2 - 8g_s^2\right)y_t.$$

And finally, the $\beta-$functions for the Higgs quartic coupling, mildly modified by the New Physics, and new coupling parameters

$$\begin{aligned}
16\pi^2\mu\frac{d\lambda}{d\mu} &= 24\lambda^2 - 6y_t^4 + (12y_t^2 - 3g'^2 - 9g^2)\lambda + \frac{3}{8}(g'^2 + g^2)^2 + \frac{3}{4}g^4 + \frac{1}{2}\rho^2\theta(\mu - M),\\
16\pi^2\mu\frac{d\rho}{d\mu} &= \left(12\lambda + 4\rho + \kappa + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2\right)\rho\theta(\mu - M),\\
16\pi^2\mu\frac{d\kappa}{d\mu} &= \left(3\kappa^2 + 12\rho^2\right)\theta(\mu - M).
\end{aligned}$$

In order to hold on the effects of the new degrees of freedom on the Standard Model until we reach the energy scale in which $\mu = M$, we have inserted a Heaviside function θ along with the new physics parameters.

6
Results

Now that we have introduced all the necessary mathematical machinery, we investigate the parameter space of the theory. We shall focus on the case where only the Higgs field has a nontrivial vacuum structure, the $\mathbb{Z}_2$ symmetry $S \to -S$ is preserved. One immediate consequence of this assumption is that the scalar field does not have a decay mode (possibly stable at cosmological time scale), which makes it a viable candidate for cold dark matter. Having said that, we expand the extended potential around the electroweak vacuum. In the unitary gauge, we have

$$V[h,S] = \frac{1}{2}m_h^2 h^2 + \frac{1}{2}M_S^2 S^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4!}\kappa S^4 + \frac{1}{4}\rho h^2 S^2 + \lambda v h^3 + \frac{1}{2}v\rho h S^2, \quad (6\text{-}1)$$

where the physical masses are

$$m_h^2 = 2\lambda v^2, \quad M_S^2 = M^2 + \frac{1}{2}\rho v^2. \quad (6\text{-}2)$$

Note that we have decided to use the unitary gauge, because it is the most convenient gauge to investigate the particle spectrum.

In order to have a stable potential, we have already discussed that, when ρ is positive, the Higgs and the scalar self-couplings must be positive for all energy scales

$$\lambda(\Lambda) > 0, \quad \kappa(\Lambda) > 0. \quad (6\text{-}3)$$

When ρ is negative, we must impose that the determinant associated with the quartic couplings, equation (6-4), is never negative, otherwise the potential will not be bounded below, allowing for runaway directions

$$\det \mathcal{C}(\Lambda) = \kappa(\Lambda)\lambda(\Lambda) - \frac{3}{2}\rho^2(\Lambda) \geq 0, \quad (6\text{-}4)$$

where for the sake of both simplicity and physical relevance, we have dropped a scaling constant; the roots of the determinant are not affected by a global scaling factor. In our stability analysis we do not watch out for κ, because this running coupling is always positive for any given energy scale. Also, as long as the determinant is positive, the coupling parameter λ is consequently positive. These two remarks suggest that if the determinant is positive, then we already have a stable theory.

To determine the parameter space, we define an energy scale Λ as the scale at which the determinant vanishes for a given set of initial values $(\kappa_0 > 0, \rho_0 < 0)$, then we find the critical ρ associated with this set. When the coupling parameter ρ is positive, we repeat the former approach, but now we are interested in the scale at which $\lambda(\Lambda) = 0$. In addition, when $\rho < 0$, this running coupling grows much slower than in the former case, because now the ρ^2 in the $\beta-$functions is competing against the top Yukawa, for instance. Interestingly enough, this coupling parameter delays the onset of the instability of the electroweak vacuum, $\lambda(\Lambda) < 0$.

Suppose that ρ is negative and consider the simplest possible case, the one in which the determinant is evaluated at the fixed energy scale $\Lambda = M$, where M is the mass parameter. We show below in figure (6.1) the plot for this case

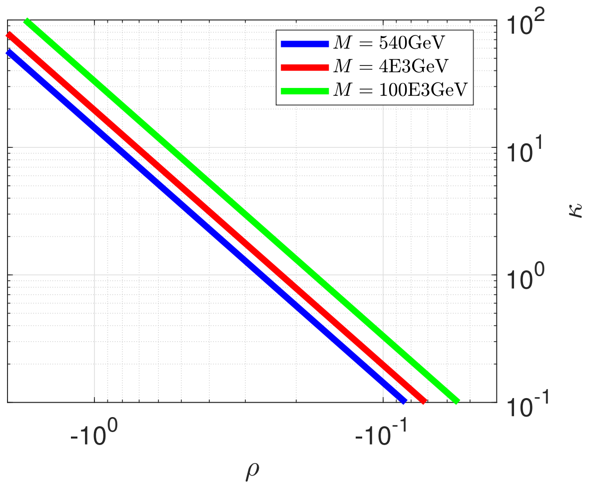

Figure 6.1: The minimum necessary value of κ to have a stable electroweak vacuum. The instability curves in this case are simply given by $\kappa(\Lambda) = \frac{3\rho^2(\Lambda)}{2\lambda(\Lambda)}$

Recalling that a positive determinant is required in order to have an electroweak stable vacuum, we immediately observe that for $\mathcal{O}(1)$ values of the coupling parameter ρ, the stability condition forces the values of the scalar self-interaction coupling parameter κ to be dangerously large and this has to do with the inevitable smallness of the initial value of λ.

Next, we must exclude the region where the running coupling of the scalar self-interaction κ reaches a Landau pole. In general, it is required that we find the scale for which the coupling parameter diverges, but, given the complexity in the numerical integrations of the system of differential equations, that would be rather inconvenient. Instead, we use as rough estimation for the Landau poles the region where one-loop perturbation theory breaks down.

In calculations of Feynman diagrams, each loop contributes with a geometrical factor N_d, which is made up of the area of a $d-1$-dimensional hypersphere divided by a factor $(2\pi)^d$ (coming from the volume element $\widetilde{d^dk}$):

$$N_d = \frac{2}{(4\pi)^{d/2}\Gamma(d/2)}. \tag{6-5}$$

Hence, the contributions to β_κ coming from two-loop diagrams can be can written as a quadratic factor N_d and a constant factor α_2:

$$\beta_\kappa \sim N_d \alpha_1 \kappa^2 \left(1 + \frac{N_d \alpha_2}{\alpha_1}\kappa,\right) \tag{6-6}$$

where we have included only the relevant coupling. The equation we have used to determine the region where perturbation theory breaks down is[1]

$$\kappa(\Lambda) - 8\pi^2 = 0. \tag{6-7}$$

Again, we want to fix an initial value κ^0 and solve the renormalization group equations for the other values of ρ^0, until we find an initial value of the scalar-Higgs interaction such that the equation (6-7) is satisfied.

We note that because of the large mass values, $M_S \gg v$, and for the sake of simplicity, we have used the mass parameter M^2 in place of the physical mass parameter $M_S^2 = M^2 + \rho v^2/2$. The largest deviation from the true value happens when the scalar mass is 1TeV, $M_S = 1\text{TeV}$, and the coupling parameter ρ is around 1, $\rho \approx 1$. The estimated deviation is

$$1 - \frac{\rho v^2}{2M^2} = 0.97, \tag{6-8}$$

which would mildly change the parameter space. As a result of the described procedure, we obtain the parameter space of the theory for various masses and instability scales.

We observe that in the plots within the figures 6.5, 6.6, 6.7 and 6.8, negative ρ, the parameter space is strongly dependent on the scale Λ. As we increase Λ, the coupling parameter λ decreases and, even though, the coupling parameter κ is always increasing, we have that the determinant starts to be dominated by the Higgs-scalar interaction coupling parameter ρ, which outruns the product $\kappa(\Lambda)\lambda(\Lambda)$. As a consequence of the interplay among these three

[1]It can be checked that α_2/α_1 is approximately 0.45. Such value would provide only a mild change in the parameter space.

running couplings, the parameter space shrinks as we increase the energy scale Λ.

When ρ is positive, the parameter space is constrained by the stability condition $\lambda(\Lambda) > 0$, which is only violated when we explore energy scale around 10^6TeV, the typical instability scale present in the Standard Model. In this case, the parameter is displayed in figure 6.4. What is remarkable, in this situation, is that because the $\beta_\lambda-$function receives a ρ^2 contribution from the new state S, even if we do not consider the possibility of having a scalar equipped with a self-interaction term, it is possible to stabilize the electroweak vacuum with a small valued running coupling ρ. It shown in figure 6.2 the running coupling λ as a function of the energy scale μ with a fixed $\kappa = 0$. We vary the initial value of the running coupling ρ, starting from $\rho_0 = 0$ (Standard Model).

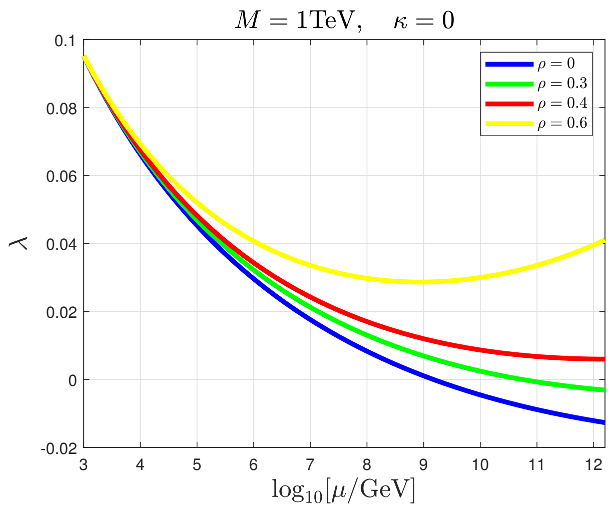

Figure 6.2: When $\rho = 0$, we have the Standard Model restored. At a given energy scale, as we increase ρ, the running coupling λ increases until we reach a critical ρ with which λ can be made always positive.

In this case, the instability region occupies only a small fraction of the $\kappa\rho-$plane, see figure 6.4. As the energy scale grows, the parameter space will enjoy a continuous decreasing of its size up to some scale Λ_*, after that λ starts to increase and the instability region will shrink, until it completely vanishes when λ becomes positive again.

In the figures 6.5, 6.6, 6.7, 6.8 and 6.4, we investigate the parameter space combining the information extracted from the Landau Poles and the Instability Curves. The yellow and green shaded areas correspond to the parameter space, bounded in the upper limit by the Landau Pole and in the lower limit by the Instability Curve.

For $\rho < 0$, it is shown in figure 6.3 various contours for different values of Λ. For each fixed instability scale Λ, we find at small couplings, solutions where the determinant turns negative. However, there is a region in which the determinant never turns negative for any scale, and hence in this region the theory is absolutely stable for any couplings, the dashed line denotes the boundary of absolute stability.

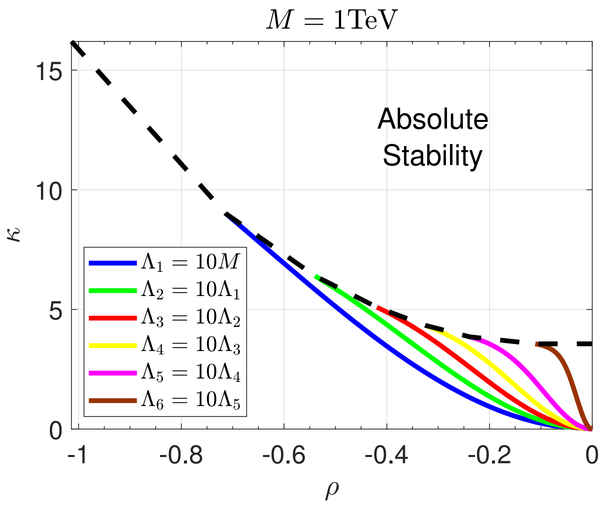

Figure 6.3: The region above the instability curves represents the parameter space for which the theory is not plagued by runaway directions in the potential. Above the dashed line the determinant never turns negative.

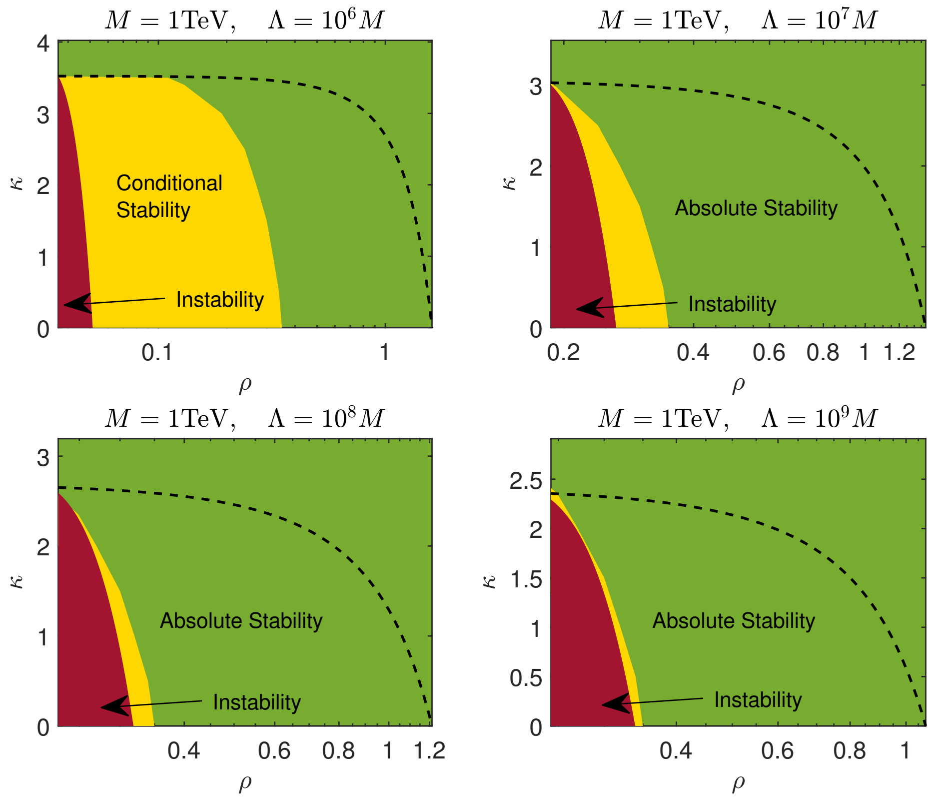

Figure 6.4: Constrained parameter space when ρ is positive. The yellow shaded region indicates that the theory only stable for $\mu \leq \Lambda$. The red shaded region denotes the region where the runnning coupling λ is negative. The theory is absolutely stable in the green shaded region. The dashed line is the boundary of Landau poles region. To avoid visual cluttering, the label *Conditional Stability* has been omited in some plots.

The gradual shrinking of the parameter, as we increase the energy scale, indicates that such theory does not provide any sort of stabilization mechanism for the electroweak vacuum. Recalling that $\det \mathcal{C} = \kappa\lambda - \frac{3}{2}\rho^2$, we observe that the determinant is going to be inevitably negative at large energies, typically around 10^6TeV, because in the neighbourhood of this energy scale the Higgs self-coupling becomes negative. For $\rho > 0$, energy scales below the typical instability scale of the Standard Model are uninteresting, because, as far as we know, they are only constrained by the Landau Pole of the theory. In the plots shown within the figure 6.4, we focus on the energy scales where the instability starts to manifest itself in the potential. From the plots within the figure 6.4 and the considerations associated with a positive ρ, it follows that even a small contribution from the Higgs-scalar coupling parameter may be enough to provide stability for the electroweak vacuum.

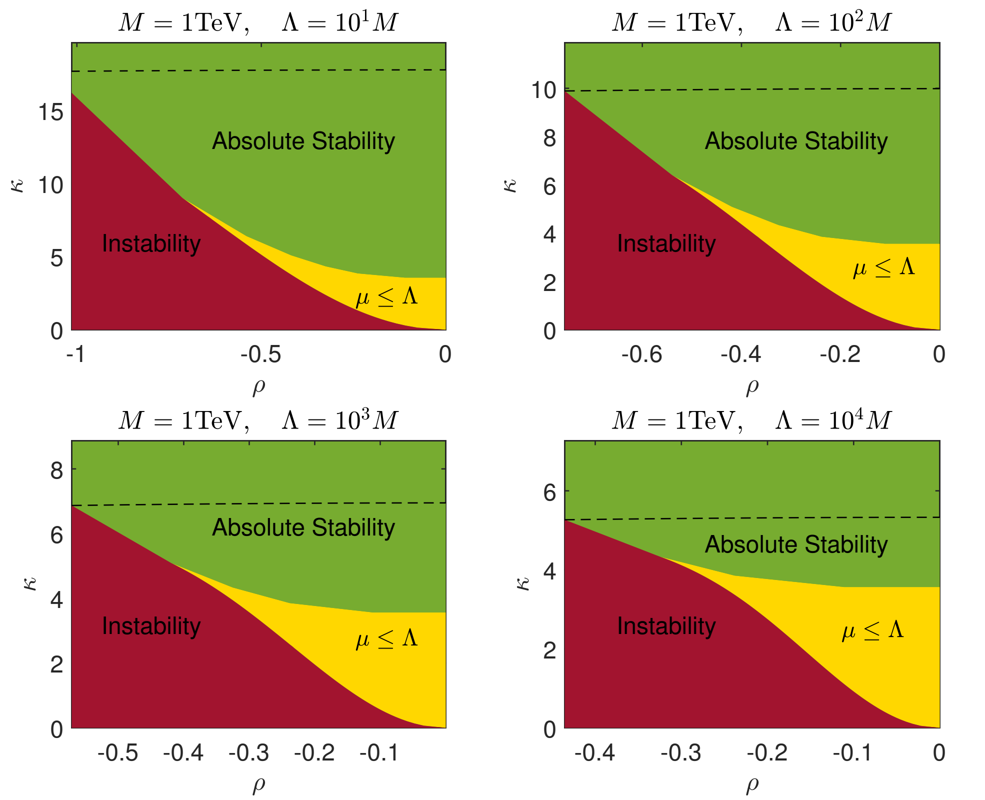

Figure 6.5: Limits on κ as a function of ρ, for different values of Λ and fixed mass parameter M. The red shaded region indicates when $\det \mathcal{C}(\Lambda) < 0$. The yellow shaded region indicates conditional stability, the potential is stable only when $\mu \leq \Lambda$. The potential is absolutely stable in the green shaded region. The dashed line denotes the boundary of Landau poles region.

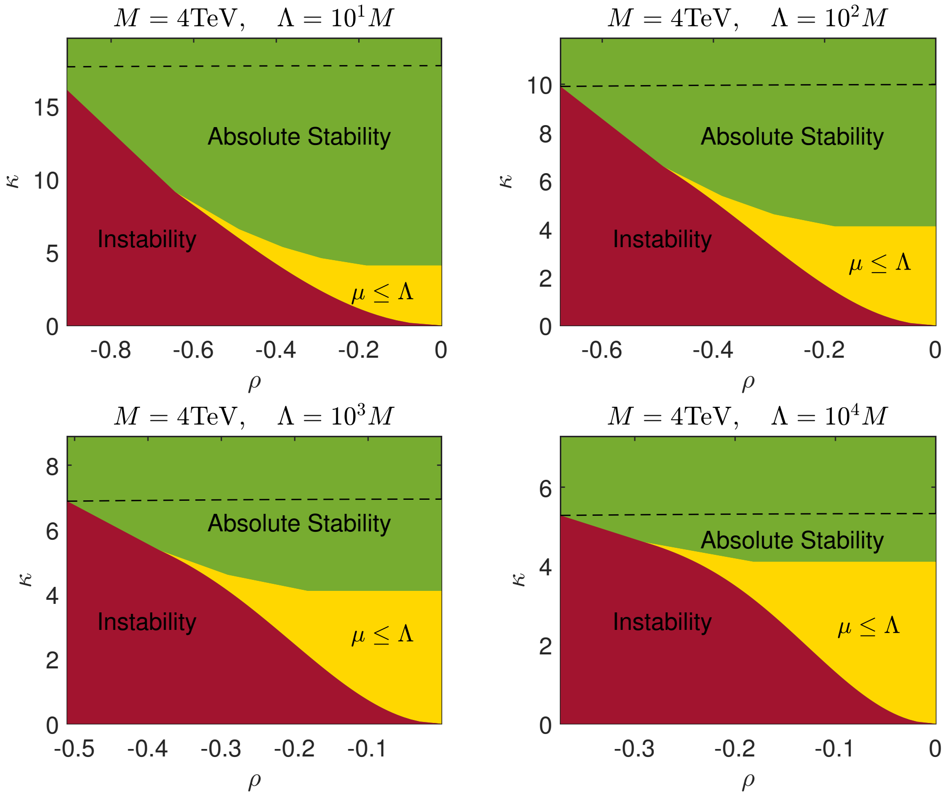

Figure 6.6: See figure 6.5 for explanations.

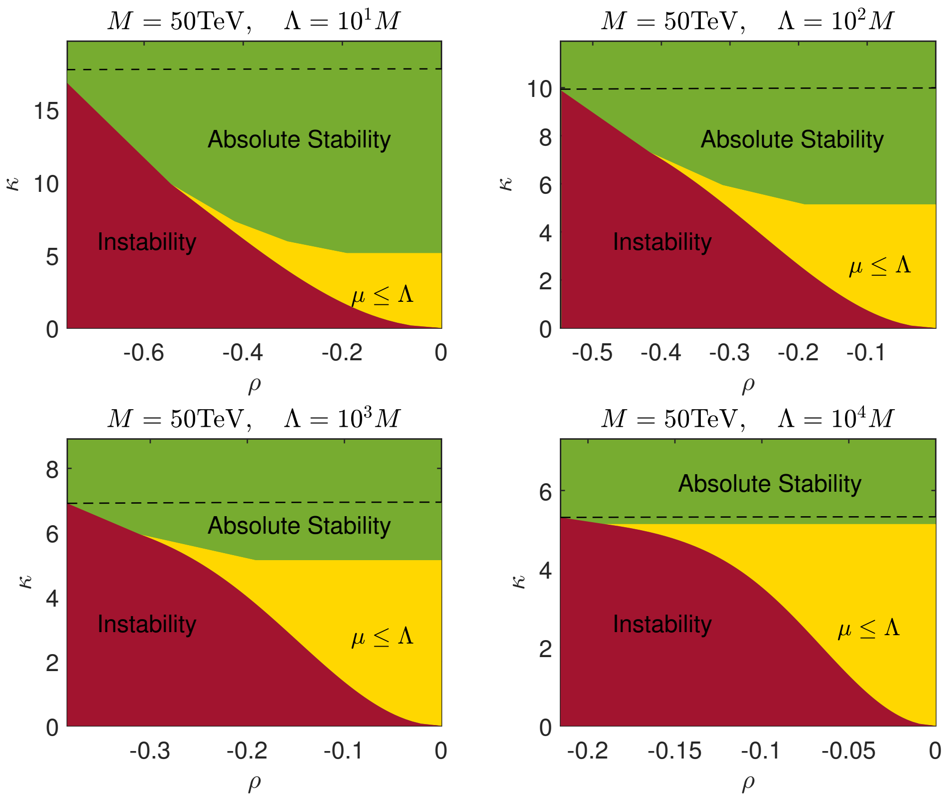

Figure 6.7: See figure 6.5 for explanations.

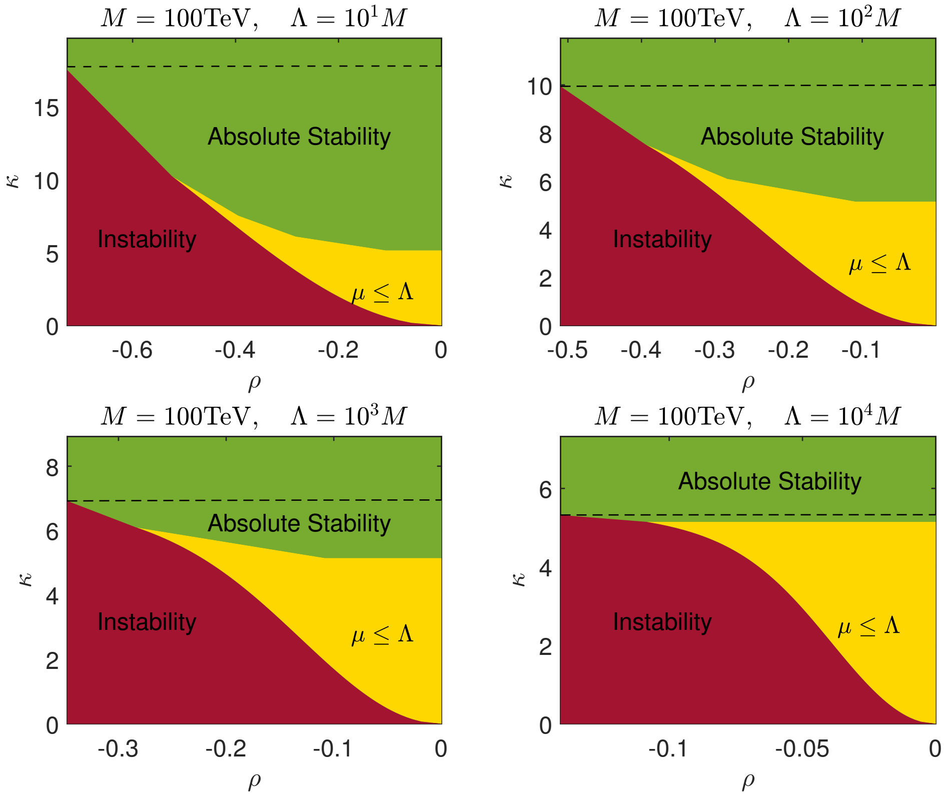

Figure 6.8: See figure 6.5 for explanations.

6.1 Applications

In reference (39), it was used the Planck result $\Omega_{\text{cdm}}h^2 = 0.1198 \pm 0.003$ to derive relic abundance of S with the assumption that all the cold dark matter is composed of this scalar field. As demonstrated in (35), this approach leads to a precise relation between the interaction coupling parameter ρ and the physical mass parameter of the scalar field. Building on that, we employ the results from reference (39) to fix an initial condition for the Higgs-scalar coupling parameter which is the maximum acceptable value of ρ_0 for which all the cold dark matter is made up of only the real scalar and we constrain the allowed parameter space. In addition, since the coupling parameter ρ only contributes to the differential equation solved in references (35)(39) through the scalar annihilation cross section, we assume the possibility of having an either positive or negative ρ. In here, the previous scheme has some minor modification. Now we shall fix the coupling parameter ρ and the mass parameter and we find the parameter space associated with the instability scale Λ and the self-interaction coupling parameter κ.

For $\rho > 0$, the required criteria for stability are $\lambda > 0$ and $\kappa > 0$ at all energy scales.

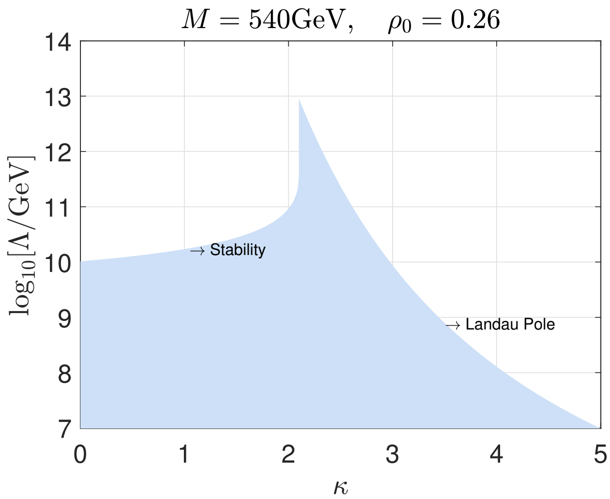

Figure 6.9: The relic density of dark matter is composed exclusively by the real scalar, $\Omega_S = \Omega_{\text{cdm}}$. The positive coupling parameter raises the instability scale of the Standard Model, allowing for a larger parameter space.

In this case, λ only reaches the instability region around 10^{10}TeV. The steepness around $\kappa = 2$ has to do with the fact that β_λ depends quadratically on ρ^2, and the interplay between β_ρ and β_κ helps building up the running coupling λ, consequently delaying the onset of the instability and for values of κ greater than 2, the parameter space becomes bounded essentially only by the Landau Poles. For $\rho < 0$, the required criteria for stability are $\det \mathcal{C}(\Lambda) > 0$ and $\kappa(\Lambda) > 0$ for all $\Lambda > 0$.

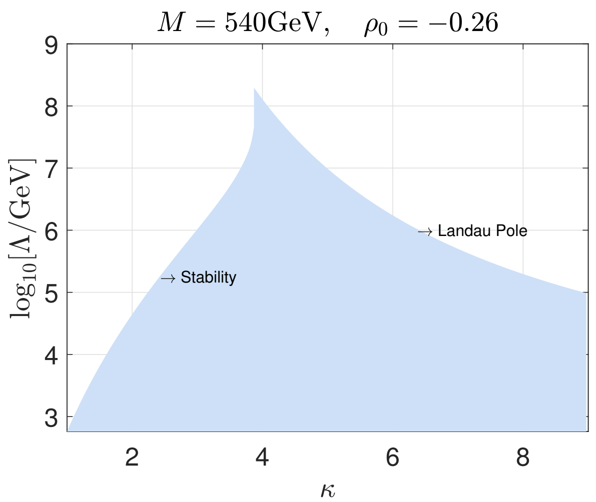

Figure 6.10: See figure 6.9 for explanations. When ρ is negative, the instability kicks in earlier than in the former case, reducing the parameter space.

In this case, the parameter space enjoys a larger region of the Cartesian plane. A negative ρ has a slower running than a positive ρ, such feature of the former running coupling can be seen through the inclination of the Landau Pole curve. In comparison with the former case, we also point out that the instability in the potential manifests itself much earlier than before. The determinant is typically dominated by the running coupling ρ.

Recently there has been some development in the direction of larger masses (40), $M_S > 1$TeV. For these larger masses, the associated values of ρ are typically greater than one. When we have a positive ρ, such values lead to an absolutely positive λ, the parameter space is constrained only by the Landau Poles. On the other hand, when ρ is negative, the determinant is always negative, except for large values of κ; if $M_S = 4$TeV, then the estimated minimum value for κ is around 40.

In reference (56), it was considered the limit $M_S \gg v$, at one-loop the scalar S was integrated out and the resulting Lagrangian was matched to the full theory. Within the framework of this effective field theory, the fractional change in the Zh associated production cross section relative to the Standard Model was calculated

$$\frac{\sigma_{Zh} - \sigma_{Zh}^{\mathrm{SM}}}{\sigma_{Zh}^{\mathrm{SM}}} = \frac{\rho^2 v^2}{16\pi^2 m_h^2}\left[1 + \frac{1}{4\sqrt{\tau_S^2 - \tau_S}}\log\left(\frac{1 - 2\tau_S - 2\sqrt{\tau_S^2 - \tau_S}}{1 - 2\tau_S + 2\sqrt{\tau_S^2 - \tau_S}}\right)\right] = \delta\sigma_{Zh},$$

where

$$\tau_S = \frac{m_h^2}{4M_S^2}.$$

Employing the same data used to make figure 6.5, but before making the plot we transform the coupling parameter $\rho \to \sqrt{\rho^2}$, we obtain figure 6.11. The values exhibited in the plot are way beyond the current reach of the measurements of the Higgs physics. For larger masses the numbers get even smaller.

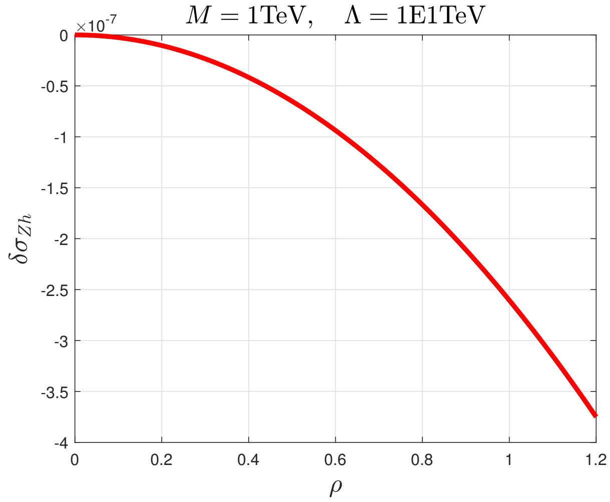

Figure 6.11: Indirect limits. The deviations from the Standard Model of this cross section tend to get even smaller for larger values of M.

7
Conclusion

The discovery of the Higgs boson allows us to explore a whole new possibility of New Physics through the Higgs Portal. In this dissertation, we investigated the parameter space when the singlet S is heavier than the Higgs boson and when the $\mathbb{Z}_2$ symmetry is not broken, the latter assumption being particularly relevant for cold dark matter . The general conclusion we can draw from this investigation is that in such models, in regions where the potential instability is present, the constrained parameter space only allows for feeble interactions between the Higgs and the scalar S. In addition, when ρ is positive, the new state present in this nonminimal Standard Model helps increasing the running coupling λ, making the electroweak vacuum absolutely stable even for small values of ρ, $0 \leq \rho \leq 1$. The case in which ρ is negative constrains the parameter space all the way up to the typical instability scale of the Standard Model. The disadvantage of this case is reflected in the impossibility of having a positive determinant, when λ becomes negative, typically around $\Lambda \approx 10^6$TeV. In addition, usually the running coupling ρ becomes the dominant term in the determinant, making it not possible to have a stable theory with a strong interaction between the Higgs and the scalar S. The scalar S is still a viable cold dark component in both cases, but in the latter case it will not help elevating the current status of electroweak vacuum from metastable to stable.

In particular, employing results from (40); the derived relic density for large masses, $M_S > 1$TeV, associated with current experimental constraints, we see that the proposed model of a real scalar is not a viable candidate for dark matter in the large mass limit, when ρ is negative. Such large masses associated with a Higgs-scalar interaction coupling parameter of approximately $\rho \geqslant -1$ has important consequences for the theory. The required condition, $\det \mathcal{C}(\Lambda) > 0$, to have vacuum stability is only achieved for a very large value of $\kappa_0 \geqslant 70$, implying a very low cutoff due to the proximity of the Landau Pole for κ. When ρ is positive and larger than one, it considerably helps increasing the values of the running coupling λ. In this case, the advantage is two-fold, we have a candidate for cold dark matter with a precise relation between the running coupling ρ and the physical mass and we also have the possibility of a stable electroweak vacuum associated with this result.

Although the scalar can only interact through the Higgs portal, when the physical mass parameter M_S is greater than v, the scalar field S can be integrated out, modifying the Zh coupling through a dimension six operator. The evaluated corrections and discussions are demonstrated in the reference (48). Using the data from our stability investigation, we concluded from figure 6.11 that the typical corrections for the Standard Model observable σ_{Zh}, the Zh production cross section, coming from the Higgs Portal are of order $\mathcal{O}(10^{-8})$. This result is not very promising, because even for the best of all possible scenarios we have studied, the corrections are way beyond the current precision measurements. What we can learn from this study is that this scenario is very challenging to be verified with collider physics. The interactions through the Higgs portals are feeble, leading to vanishingly small corrections to important Standard Model observables.

The scenario when both fields acquire a nonzero vacuum expectation value, which we have omitted here, may provide a better possibility of a direct or indirect measurements of the Higgs portal observables in terms of collider physics. It also seems to be possible to render the electroweak vacuum absolutely stable, thanks to tree-level threshold corrections for the Higgs quartic coupling (49). However, when the $\mathbb{Z}_2$ symmetry is broken, the Lagrangian is allowed to have terms such as h^2S, as a consequence, the scalar S is no longer stable. This invalidates the possibility of it being a component of cold dark matter.

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